UNIVERSITÄT BONN

Precision study of η / η' decays (theory)

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FTCF2025, Huangshan

25/11/2025

η and η': symmetries & symmetry breaking

Example analyses: combining Standard Model and BSM physics

- $\eta \to \mu^+\mu^-$: *P* and *CP* violation — Sánchez-Puertas, JHEP **01** (2019) 031
- $\eta \to 3\pi$: quark masses & Dalitz plot asymmetries
  Akdag, Isken, BK, JHEP **02** (2022) 137
- $\eta \to \pi^0\ell^+\ell^-$: Standard Model & *C* violation
  Schäfer, Zanke, Korte, BK, Phys. Rev. D **108** (2023) 074025
  Akdag, BK, Wirzba, JHEP **03** (2024) 059
- $\eta' \to \pi^+\pi^-\gamma$: final-state universality & ρ–ω mixing
  Holz, Hanhart, Hoferichter, BK, Eur. Phys. J. C **82** (2022) 434
- $\eta^{(\prime)} \to \gamma\ell^+\ell^-$: transition form factor and *C*-odd asymmetries
  . . . + Herz BSc thesis 2023
- $\eta^{(\prime)} \to \pi^+\pi^-\ell^+\ell^-$: asymmetries
  Zillinger, BK, Sánchez-Puertas, JHEP **12** (2022) 001
  Herz BSc thesis 2023

η and η' properties

- quantum numbers $I^G\,J^{PC} = 0^+0^{-+}$

  $\longrightarrow$ *C*, *P* eigenstates, all additive quantum numbers are zero

  $\longrightarrow$ flavour-conserving lab for symmetry tests

- η: (largely) (pseudo-)Goldstone boson, $\Gamma_\eta = 1.31\,\text{keV}$

  $\longrightarrow$ all decay modes forbidden at leading order by symmetries

  (*C*, *P*, angular momentum, isospin/*G*-parity...)

- η': no Goldstone boson due to $U(1)_A$ anomaly, $\Gamma_{\eta'} = 196\,\text{keV}$

  $\longrightarrow$ still much narrower than e.g. ω, ϕ

- theoretical methods:
  - ▷ (large-N_c) chiral perturbation theory
  - ▷ dispersion theory (final-state interactions)
  - ▷ (sometimes) vector-meson dominance
- new experiments: JEF, REDTOP, super-η factory at HIAF

η (and η') physics

Channel	Expt. branching ratio	Discussion
$\eta \to 2\gamma$	39.41(20)%	chiral anomaly, η–η' mixing
$\eta \to 3\pi^0$	32.68(23)%	$m_u - m_d$
$\eta \to \pi^0\gamma\gamma$	$2.56(22) \times 10^{-4}$	χPT at $O(p^6)$, leptophobic B boson, light Higgs scalars
$\eta \to \pi^0\pi^0\gamma\gamma$	$< 1.2 \times 10^{-3}$	χPT, axion-like particles (ALPs)
$\eta \to 4\gamma$	$< 2.8 \times 10^{-4}$	$< 10^{-11}$ [54]
$\eta \to \pi^+\pi^-\pi^0$	22.92(28)%	$m_u - m_d$, C/CP violation, light Higgs scalars
$\eta \to \pi^+\pi^-\gamma$	4.22(8)%	chiral anomaly, theory input for singly-virtual TFF and $(g-2)_\mu$, P/CP violation
$\eta \to \pi^+\pi^-\gamma\gamma$	$< 2.1 \times 10^{-3}$	χPT, ALPs
$\eta \to e^+e^-\gamma$	$6.9(4) \times 10^{-3}$	theory input for $(g-2)_\mu$, dark photon, protophobic X boson
$\eta \to \mu^+\mu^-\gamma$	$3.1(4) \times 10^{-4}$	theory input for $(g-2)_\mu$, dark photon
$\eta \to e^+e^-$	$< 7 \times 10^{-7}$	theory input for $(g-2)_\mu$, BSM weak decays
$\eta \to \mu^+\mu^-$	$5.8(8) \times 10^{-6}$	theory input for $(g-2)_\mu$, BSM weak decays, P/CP violation
$\eta \to \pi^0\pi^0\ell^+\ell^-$		C/CP violation, ALPs
$\eta \to \pi^+\pi^-e^+e^-$	$2.68(11) \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \to \pi^+\pi^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for doubly-virtual TFF and $(g-2)_\mu$, P/CP violation, ALPs
$\eta \to e^+e^-e^+e^-$	$2.40(22) \times 10^{-5}$	theory input for $(g-2)_\mu$
$\eta \to e^+e^-\mu^+\mu^-$	$< 1.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \to \mu^+\mu^-\mu^+\mu^-$	$< 3.6 \times 10^{-4}$	theory input for $(g-2)_\mu$
$\eta \to \pi^+\pi^-\pi^0\gamma$	$< 5 \times 10^{-4}$	direct emission only
$\eta \to \pi^\pm e^\mp \nu_e$	$< 1.7 \times 10^{-4}$	second-class current
$\eta \to \pi^+\pi^-$	$< 4.4 \times 10^{-6}$ [55]	P/CP violation
$\eta \to 2\pi^0$	$< 3.5 \times 10^{-4}$	P/CP violation
$\eta \to 4\pi^0$	$< 6.9 \times 10^{-7}$	P/CP violation

- Standard Model tests
  - ▷ WZW/chiral anomaly
  - ▷ quark masses $m_u - m_d$
  - ▷ theory input for $(g-2)_\mu$
  - ▷ scalar resonance dynamics $f_0(500)$, $a_0(980)$
- BSM: symmetry tests
  - ▷ new sources of P/C and CP violation
- BSM: light mediators
  - ▷ dark photons, protophobic X-, leptophobic $U(1)_B$ bosons
  - ▷ light Higgs-like scalars
  - ▷ axion-like particles (ALPs)

Gan, BK, Passemar, Tulin 2020

Patterns of discrete symmetry breaking

- search for *CP* violation beyond the Standard Model (BSM)

Class	Violated	Conserved	Interaction
0		C, P, T, CP, CT, PT, CPT	strong, electromagnetic
I	C, P, CT, PT	T, CP, CPT	(weak, with no KM phase or flavor-mixing)
II	P, T, CP, CT	C, PT, CPT	
III	C, T, PT, CP	P, CT, CPT	
IV	C, P, T, CP, CT, PT	CPT	weak

- class II: *P*, *CP* violation
  - ▷ QCD θ-term; in general: electric dipole moments (EDMs)
  - ▷ $\eta^{(\prime)}$ decay examples: $\eta^{(\prime)} \to 2\pi$, $\eta^{(\prime)} \to \pi^+\pi^-\gamma^{(*)}$
    $\longrightarrow$ largely excluded indirectly via EDMs Gan et al. 2020
- class III: *C*, *CP* violation ["ToPe" = *T*-odd, *P*-even]
  - ▷ far less discussed & constrained
  - ▷ $\eta^{(\prime)}$ decay examples: $\eta^{(\prime)} \to 3\gamma$, $\eta^{(\prime)} \to \pi^0\gamma^*\ldots$

$\eta \to \mu^+\mu^-$: *P* and *CP* violation

- new class of *CP*-tests in Sánchez-Puertas 2019

$$\eta \to \mu^+\mu^- , \quad \eta \to \mu^+\mu^-\gamma , \quad \eta \to \mu^+\mu^- e^+e^-$$

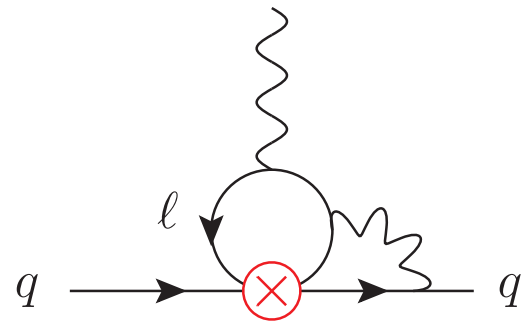
  CP-odd observables for the first two require muon polarisation

- *CP*-odd $\eta\gamma^*\gamma^*$ couplings ruled out from EDM constraints
- quark–lepton four-fermion operators (scalar–pseudoscalar):

$$\mathcal{L}_{\text{eff}} = \frac{1}{2v^2}\operatorname{Im} c^{2222}_{\ell edq}\Big[(\bar\mu\mu)\big(\bar s i\gamma^5 s\big) - \big(\bar\mu i\gamma^5\mu\big)(\bar s s)\Big] + [u\text{-, } d\text{-quarks}]$$

- EDMs only generated at two loops
  constraint for strange quarks weakest:

$$|\operatorname{Im} c^{2222}_{\ell edq}| < 0.04$$



- asymmetries in $\eta \to \mu^+\mu^-$ may be within reach at REDTOP
- testing CPV in $\eta \to \pi^0\mu^+\mu^-$ Escribano, Royo, Sánchez-Puertas 2022
  and $\eta \to \pi^+\pi^-\mu^+\mu^-$ Zillinger, BK, Sánchez-Puertas 2022

$\eta \to 3\pi$: quark masses

- $\eta \to 3\pi$ isospin violating; two sources in the Standard Model:

$$m_u \neq m_d \qquad\qquad e^2 \neq 0$$

- electromagnetic contribution small — Sutherland 1967

Baur, Kambor, Wyler 1996; Ditsche, BK, Meißner 2009

$$\eta \to \pi^+\pi^-\pi^0: \quad \mathcal{A}_c^{\mathsf{LO}}(s,t,u) = \frac{B(m_u - m_d)}{3\sqrt{3}F_\pi^2}\left\{1 + \frac{3(s-s_0)}{M_\eta^2 - M_\pi^2}\right\}$$

$$s = (p_{\pi^+} + p_{\pi^-})^2\ ,\ 3s_0 \doteq M_\eta^2 + 3M_\pi^2$$

- $\Delta I = 1$ relation between charged and neutral decay amplitudes:

$$\eta \to 3\pi^0: \quad \mathcal{A}_n(s,t,u) \ = \ \mathcal{A}_c(s,t,u) + \mathcal{A}_c(t,u,s) + \mathcal{A}_c(u,s,t)$$

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- relevance: (potentially) clean access to $m_u - m_d$

  but: large higher-order / final-state interactions

  $\longrightarrow$ require good theoretical Dalitz-plot description to extract normalisation

  Kampf et al. 2011, P. Guo et al. 2015, 2016, Colangelo et al. 2016...

$\eta \to 3\pi$: Dalitz plot asymmetries

- $\eta(I^G = 0^+) \to 3\pi(I^G = 1^-)$ breaks *G*-parity:
  - ▷ SM: *C* conserved, isospin broken
  - ▷ BSM: *C* broken, isospin either conserved or broken

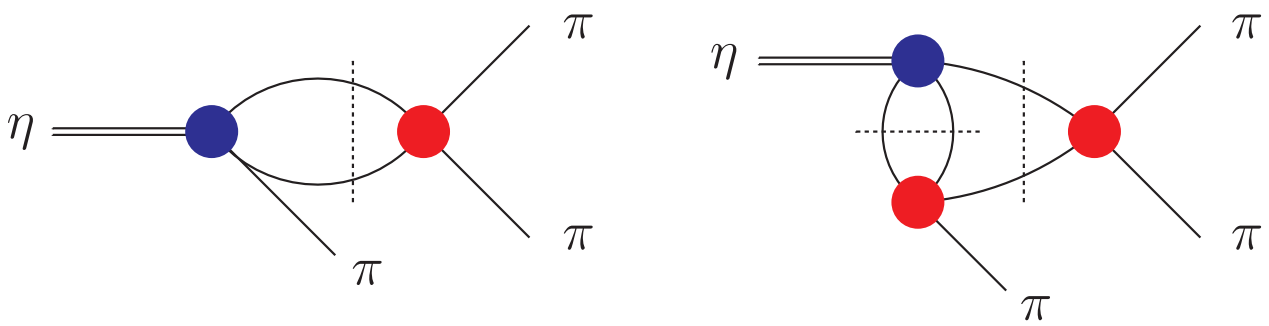
$$\mathcal{M}(s,t,u) = \mathcal{M}_1^{\not C}(s,t,u) + \mathcal{M}_0^{\not C}(s,t,u) + \mathcal{M}_2^{\not C}(s,t,u)$$

- interference: $\pi^+ \leftrightarrow \pi^-$ asymmetries linear in BSM couplings

  Gardner, Shi 2019

- follow SM strategy for hadronic amplitudes: Akdag, Isken, BK 2021

  analyse $\mathcal{M}_{0,2}^{\not C}(s,t,u)$ using dispersive Khuri–Treiman framework



$\eta \to 3\pi$: amplitude decomposition

- Bose symm.: even (odd) $\pi\pi$ isospin $\leftrightarrow$ even (odd) partial waves
- "reconstruction theorem": symmetrised partial-wave expansion

$$\mathcal{M}_1^C(s,t,u) = \mathcal{F}_0(s) + (s-u)\,\mathcal{F}_1(t) + (s-t)\,\mathcal{F}_1(u) + \mathcal{F}_2(t) + \mathcal{F}_2(u) - \frac{2}{3}\mathcal{F}_2(s)$$

$$\mathcal{M}_0^{\not C}(s,t,u) = (t-u)\,\mathcal{G}_1(s) + (u-s)\,\mathcal{G}_1(t) + (s-t)\,\mathcal{G}_1(u)$$

$$\mathcal{M}_2^{\not C}(s,t,u) = 2(u-t)\,\mathcal{H}_1(s) + (u-s)\,\mathcal{H}_1(t) + (s-t)\,\mathcal{H}_1(u) - \mathcal{H}_2(t) + \mathcal{H}_2(u)$$

$\longrightarrow$ rescattering for *S*- and *P*-waves Gardner, Shi 2019

cf. also Bernard et al. 2024 for $K \to 3\pi$

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- note: *C*-even/odd $\leftrightarrow$ even/odd under $t \leftrightarrow u$
- *T*-odd requires $\mathcal{M}_{0,2}^{\not C}$ relatively imaginary w.r.t. $\mathcal{M}_1^C$ at tree level

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- *T*-odd requires $\mathcal{M}_{0,2}^{\not C}$ relatively imaginary w.r.t. $\mathcal{M}_1^C$ at tree level
- Omnès solutions ($\mathcal{A}_I = \mathcal{F}_I, \mathcal{G}_I, \mathcal{H}_I$):

$$\mathcal{A}_I(s) = \Omega_I(s)\left(P_{n-1}(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{dx}{x^n}\,\frac{\sin\delta_I(x)\,\hat{\mathcal{A}}_I(x)}{|\Omega_I(x)|\,(x-s)}\right)$$

  - $P_{n-1}(s)$: subtraction polynomial, free parameters

$\eta \to 3\pi$: parameters, data

SM amplitude $\mathcal{M}_1^C$

- minimal subtraction scheme: 3 (real) constants
- "data" fit to
  - ▷ KLOE Dalitz plot $\eta \to \pi^+\pi^-\pi^0$ — KLOE 2016
  - ▷ A2 Dalitz plot $\eta \to 3\pi^0$ — A2 2018
  - ▷ chiral constraints [at $\mathcal{O}(p^4)$] — Colangelo et al. 2018

  $\longrightarrow$ $\chi^2/\text{dof} \approx 1.054$, works very well!

BSM amplitude $\mathcal{M}_1^C + \mathcal{M}_0^{\not C} + \mathcal{M}_2^{\not C}$

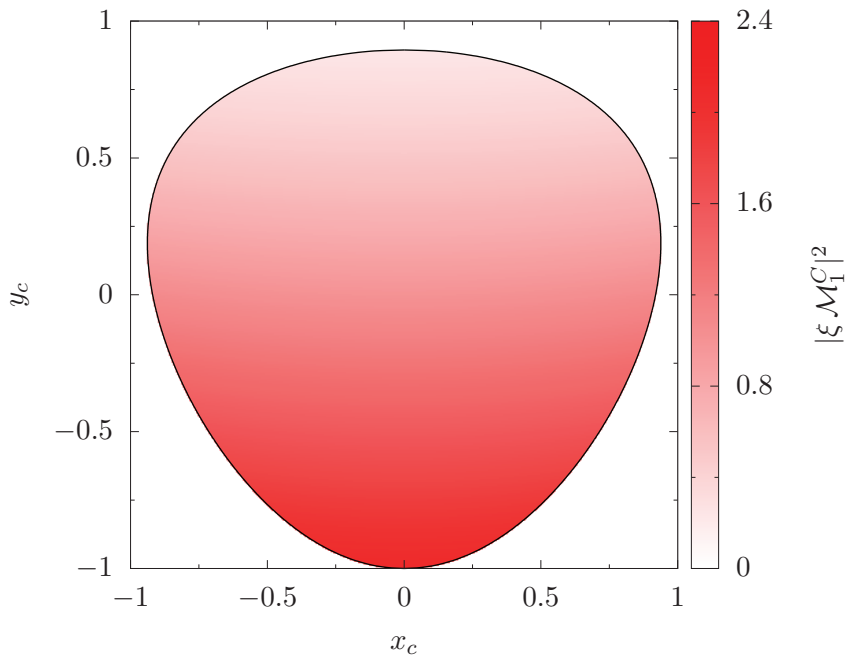
- by same assumptions: 1 imaginary subtraction each for $\mathcal{M}_{0,2}^{\not C}$ act as overall normalisation constants $\longrightarrow$ $\chi^2/\text{dof} \approx 1.048$
- all *C*-/*CP*-violating signals vanish within $(1-2)\sigma$

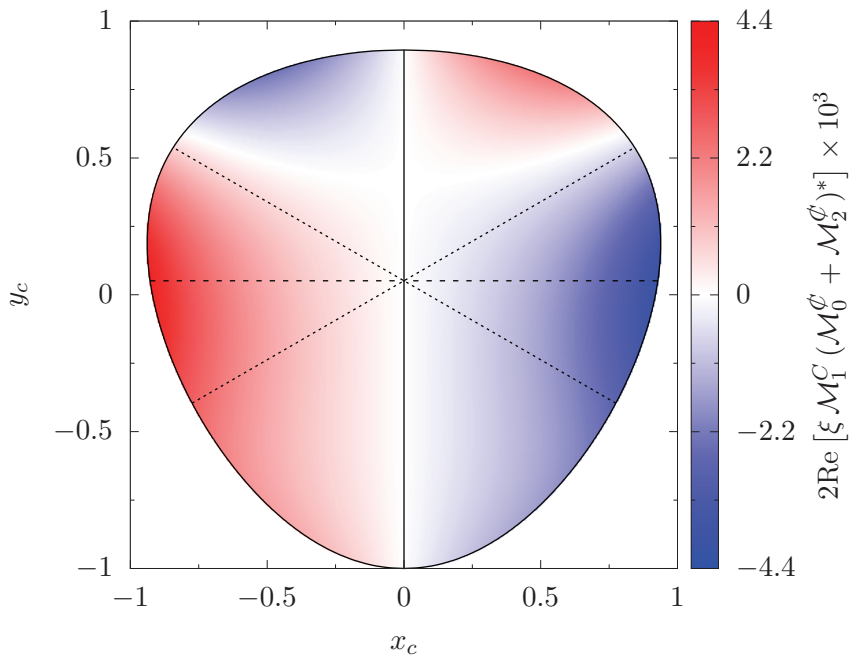
Akdag, Isken, BK 2021

$\eta \to \pi^+\pi^-\pi^0$: Dalitz plot asymmetries

- Dalitz plot decomposition (central fit result)

$$\left|\mathcal{M}_c\right|^2 \approx \left|\mathcal{M}_1^C\right|^2 + 2\mathrm{Re}\left[\mathcal{M}_1^C\,(\mathcal{M}_0^{\not C})^*\right] + 2\mathrm{Re}\left[\mathcal{M}_1^C\,(\mathcal{M}_2^{\not C})^*\right]$$





- asymmetries constrained to the permille level
- nonvanishing interference due to strong FSI phases!

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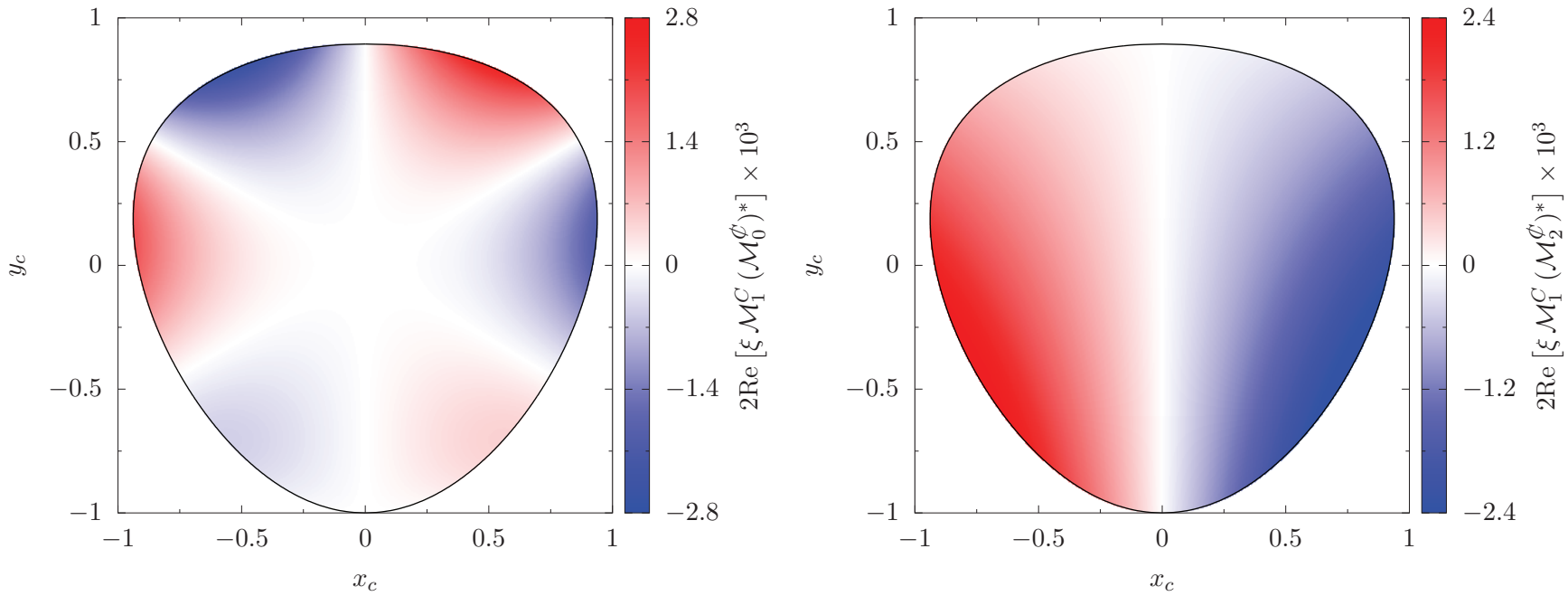


- asymmetries constrained to the permille level
- nonvanishing interference due to strong FSI phases!
- $\mathcal{M}_0^{\not C}$ and $\mathcal{M}_2^{\not C}$ lead to different interference patterns

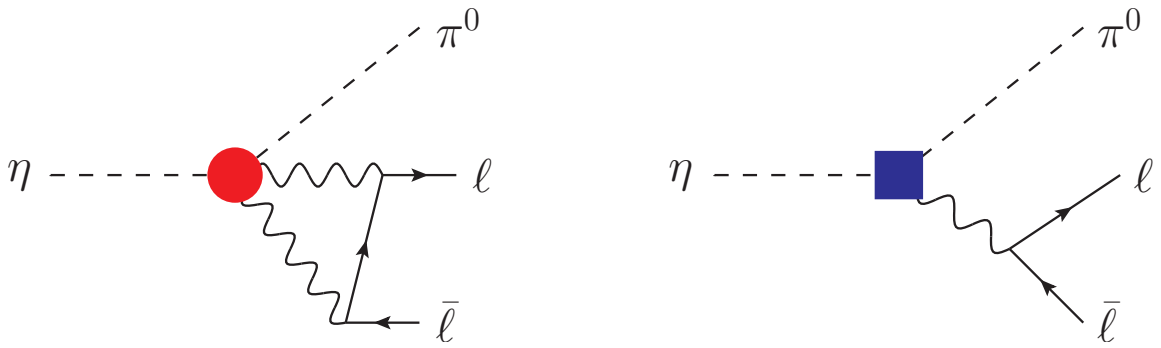
$\eta \to \pi^0 \ell^+ \ell^-$: Standard Model

- gauge invariance: $\eta(p) \to \pi^0(k)\gamma^*(q)$ vanishes for real photon:

$$\langle \pi^0(k)|j_\mu(0)|\eta(p)\rangle = e\big[q^2(p+k)_\mu - (M_\eta^2 - M_\pi^2)q_\mu\big]F_{\eta\pi^0}(q^2)$$

  $\longrightarrow$ can only measure dilepton decays

- *C*-even two-photon decay as Standard Model background:



e.g. $\mathcal{B}(\eta \to \pi^0 e^+ e^-) = 1.36(15) \times 10^{-9}$
$\mathcal{B}(\eta \to \pi^0 \mu^+ \mu^-) = 0.67(7) \times 10^{-9}$

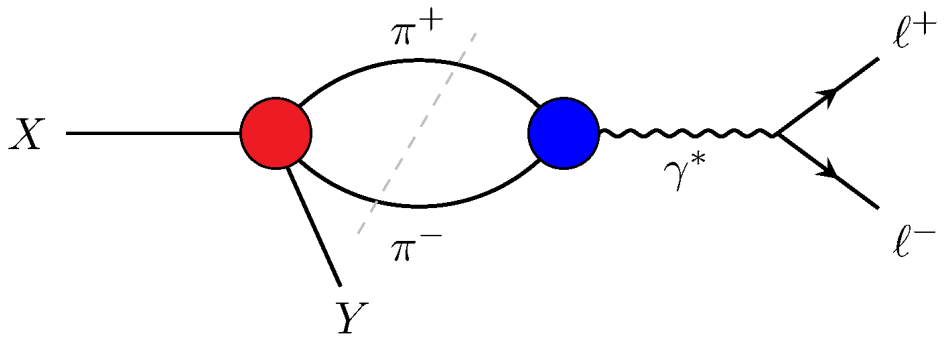
based on VMD model for $\eta \to \pi^0\gamma^*\gamma^*$ Schäfer, Zanke, Korte, BK 2023

$\longrightarrow$ 3 orders of magnitude below current experimental limits

$\eta \to \pi^0 \ell^+ \ell^-$: long-range *C* violation

- can correlate $\eta \to \pi^0\pi^+\pi^-$ *P*-wave and (isovector) $\eta \to \pi^0 \ell^+ \ell^-$:

Akdag, BK, Wirzba 2023



$$F_{XY}^{(1)}(s) = \frac{i}{48\pi^2} \int_{4M_\pi^2}^{\infty} dx\, \sigma_\pi^3(x) F_\pi^{V\,*}(x)\, \frac{f_{XY}(x)}{x-s}$$

pion form factor $F_\pi^V(s) = \Omega_1(s) = \exp\left\{ \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} dx \frac{\delta_1^1(x)}{x(x-s)} \right\}$

$\longrightarrow$ analogy to $\omega \to 3\pi \leftrightarrow \omega \to \pi^0\gamma^*$ Schneider, BK, Niecknig 2012

- add isoscalar form factor (ω-exchange) using isospin + vector-meson dominance

  $\longrightarrow$ hadronic long-range contribution to *C*-odd form factor

$\eta' \to \pi^+\pi^-\gamma$: final-state universality

- $\pi^+\pi^-$ in P-wave $\longrightarrow$ universal final-state interactions; ansatz:

$$\mathcal{F}_{\eta'\pi\pi\gamma}(s) = P(s) \times \Omega(s), \quad P(s) = \frac{A}{2}(1+\beta s+\gamma s^2), \quad s = M^2_{\pi\pi}$$

$$\Omega(s) = \exp\left\{ \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\delta_1^1(s')}{s'(s'-s)} \right\}$$

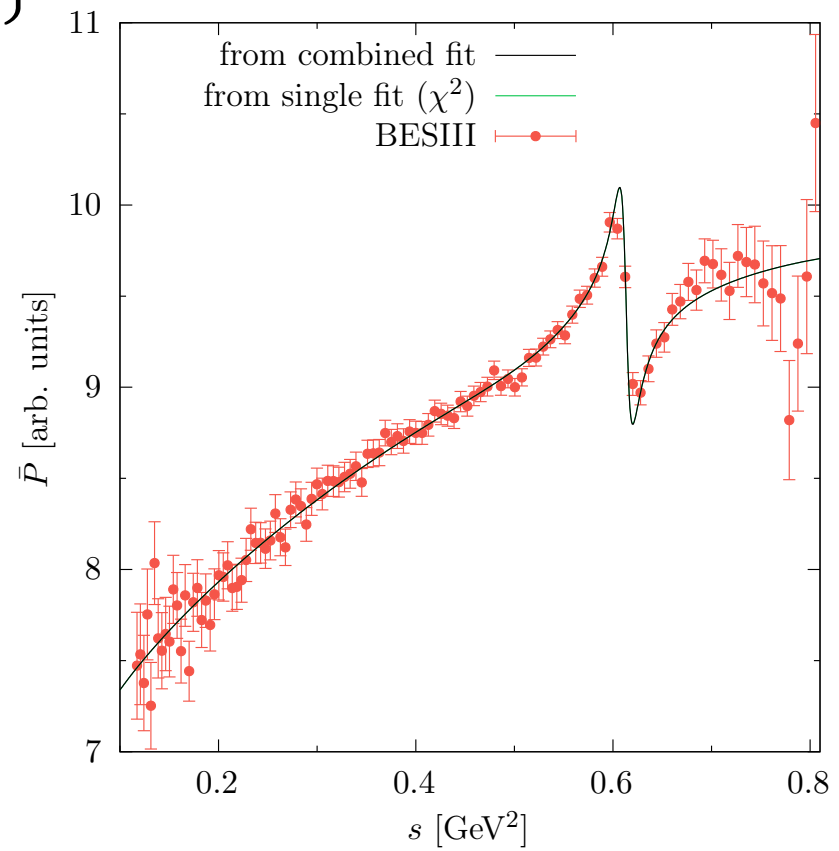
- divide data BESIII 2017
  by Omnès function $\Omega(s)$
  $\longrightarrow$ extract $P(s)$
- strong signal from ρ-ω mixing

  $$\epsilon_{\rho\omega} = 2.0(1) \times 10^{-3}$$

  perfect agreement w/ pion FF

  $$\epsilon_{\rho\omega} = 1.99(1) \times 10^{-3}$$

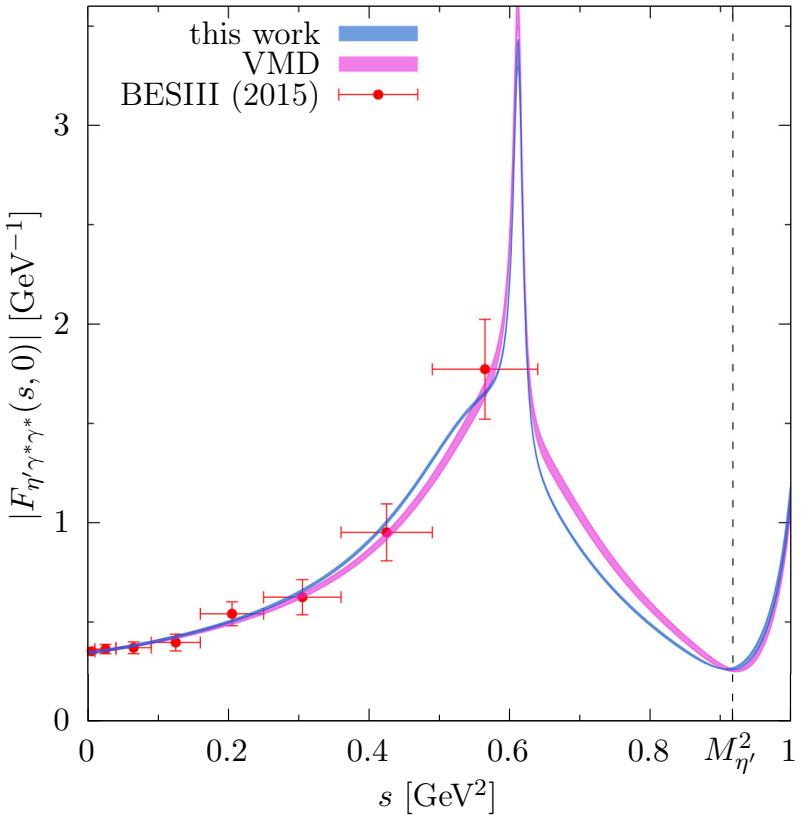
  Holz, Hanhart, Hoferichter, BK 2022

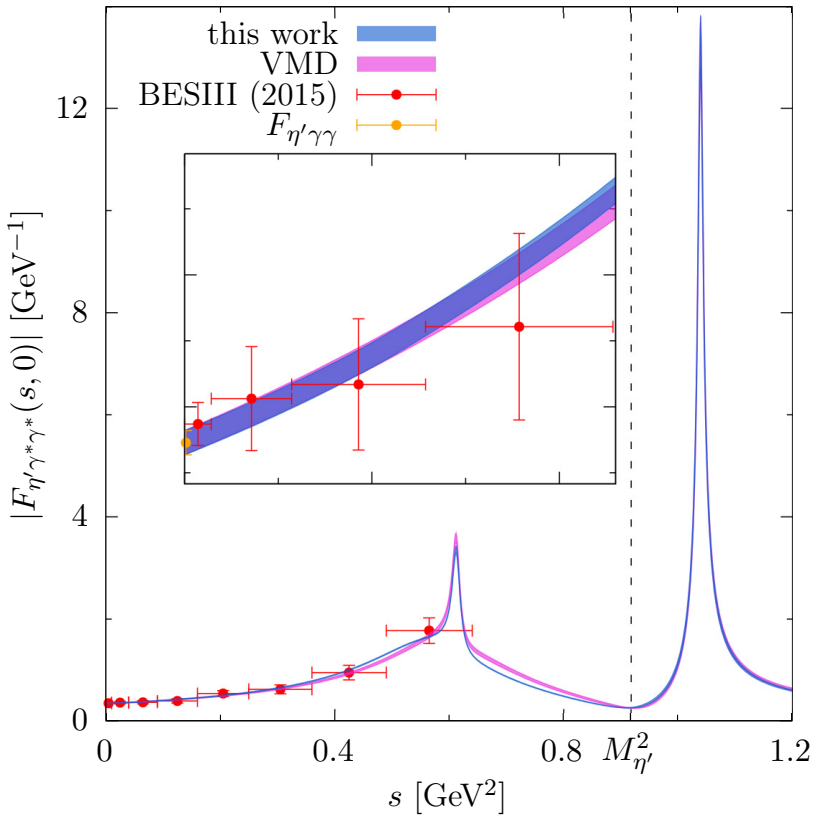


$\eta' \to \gamma\ell^+\ell^-$: transition form factor

$$F_{\eta'\gamma^*\gamma}(q^2,0) = F_{\eta'\gamma\gamma} + \frac{q^2}{12\pi^2}\int_{4M_\pi^2}^{\infty} dt\, \frac{q_\pi^3(t)[F_\pi^V(t)]^* F_{\eta'\pi\pi\gamma}(t)}{t^{3/2}(t-q^2)}$$

$$+ \Delta F^{I=0}_{\eta\gamma^*\gamma}(q^2,0)\ \text{[VMD]} \ + \ \text{[consistent isospin breaking]}$$





Holz, Hanhart, Hoferichter, BK 2022; data: BESIII 2015 (cf. update BESIII 2024)

$\eta^{(\prime)} \to \gamma\ell^+\ell^-$: *C*-odd Dalitz plot asymmetries

- Dalitz decays; SM: given by transition form factors $F_{\eta^{(\prime)}\gamma^*\gamma}(q^2)$
- *C*-odd effects can be induced by quark–lepton operators

$$\bar{\ell}\gamma^\mu\gamma_5\ell\,\bar{\psi}\overleftrightarrow{D}_\mu\gamma_5\psi \quad \text{etc.}$$

- Dalitz plot asymmetries from *C*-odd/*C*-even interference

$$\mathrm{Re}\left(\mathcal{M}_{\mathrm{BSM}}\mathcal{M}_{\mathrm{SM}}^\dagger\right) \propto \mathrm{Im}\, F_{\eta^{(\prime)}\gamma^*\gamma}(q^2) \qquad \text{Herz BSc 2023}$$

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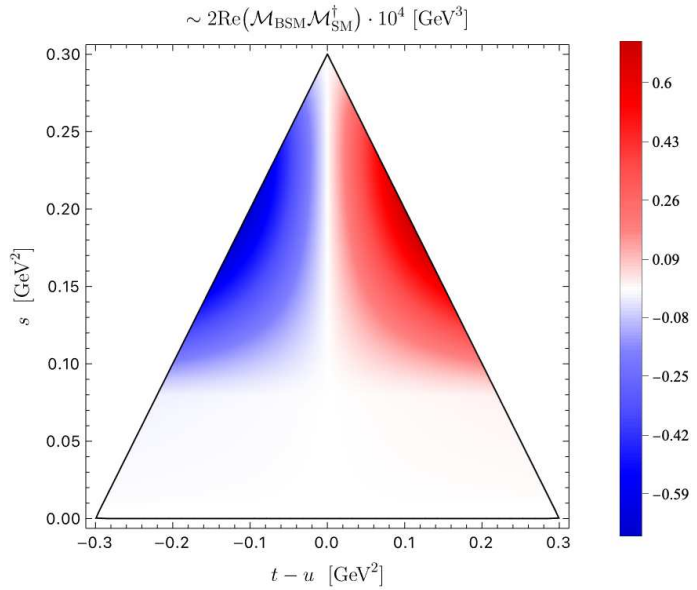
$\eta \to \gamma e^+ e^-$



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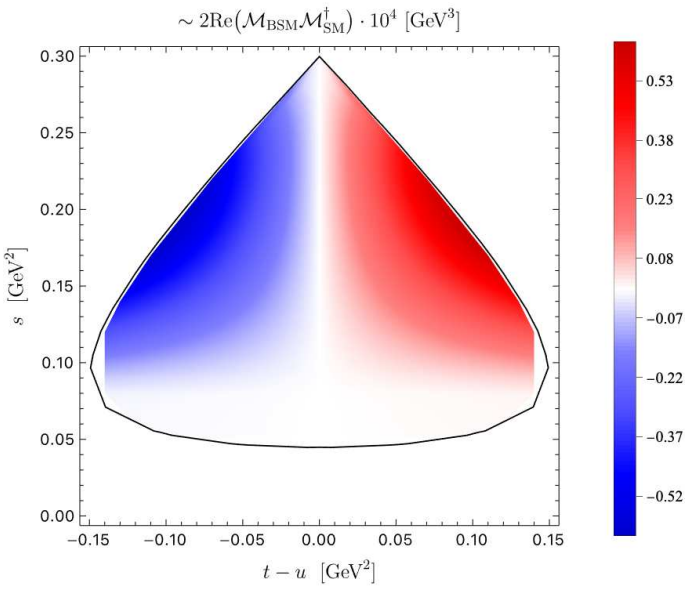

$\eta^{(\prime)} \to \gamma\ell^+\ell^-$: *C*-odd Dalitz plot asymmetries

- Dalitz decays; SM: given by transition form factors $F_{\eta^{(\prime)}\gamma^*\gamma}(q^2)$
- *C*-odd effects can be induced by quark–lepton operators

$$\bar{\ell}\gamma^\mu\gamma_5\ell\,\bar{\psi}\overleftrightarrow{D}_\mu\gamma_5\psi \quad \text{etc.}$$

- Dalitz plot asymmetries from *C*-odd/*C*-even interference

$$\mathrm{Re}\left(\mathcal{M}_{\mathrm{BSM}}\mathcal{M}_{\mathrm{SM}}^\dagger\right) \propto \mathrm{Im}\, F_{\eta^{(\prime)}\gamma^*\gamma}(q^2)$$ Herz BSc 2023

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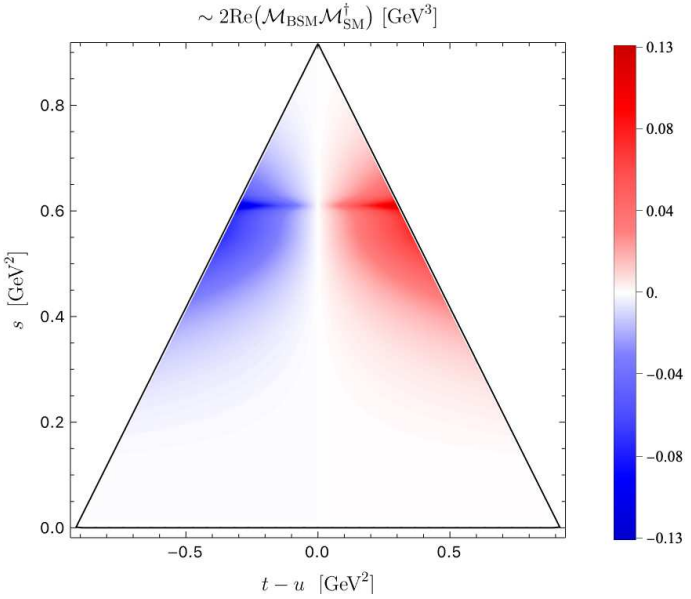

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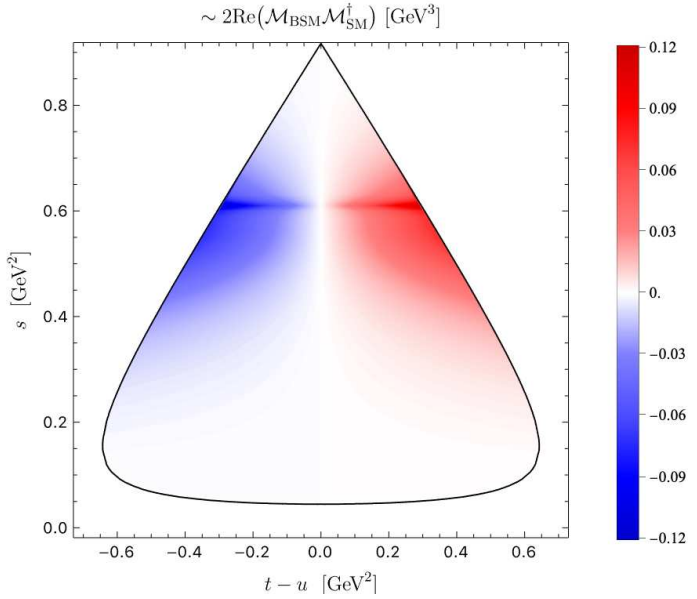

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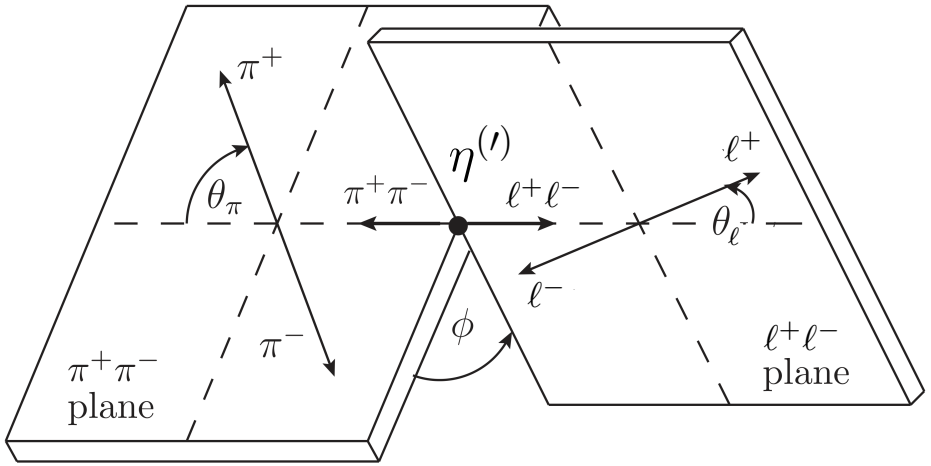
- consequences:
  - ▷ Asym$(\eta' \to \gamma\ell^+\ell^-) \gg$ Asym$(\eta \to \gamma\ell^+\ell^-)$
    $\longrightarrow \rho(770), \omega(782)$ resonances
  - ▷ Asym$(\eta^{(\prime)} \to \gamma\mu^+\mu^-) >$ Asym$(\eta^{(\prime)} \to \gamma e^+e^-)$
    $\longrightarrow$ dominance of photon pole
  - ▷ no such asymmetries generated for $\pi^0 \to \gamma\ell^+\ell^-$
    $\longrightarrow$ dilepton masses always below $\pi^+\pi^-$ cut

$\eta^{(\prime)} \to \pi^+\pi^-\ell^+\ell^-$: asymmetries



- SM amp. $\mathcal{F}(s, s_\ell) \propto f_1(s) \times \bar{F}(s_\ell)$ dispersively Holz et al. 2021, 2024
- *P*-odd, *C*-even operators induce plane–plane asymmetries

  $\propto \sin 2\phi$ [Gao 2002] or $\propto \sin\phi$ [Zillinger et al. 2022]
- *P*-even, *C*-odd operators can induce Herz BSc 2023
  - ▷ asymmetry in $\theta_\ell \to \theta_\ell + \pi$ $\propto \operatorname{Im} \bar{F}(s_\ell)$ ($\longrightarrow$ only η') *or*
  - ▷ asymmetry in $\theta_\pi \to \theta_\pi + \pi$ $\propto \operatorname{Im} f_1(s)$ ($\longrightarrow$ larger for η')

  (*C*-odd *P*-*D*-wave interference as in $\eta^{(\prime)} \to \pi^+\pi^-\gamma$)

$\longrightarrow$ exp. studies of more general asymmetries strongly advised!

cf. BESIII 2024

Summary: highlights in η and η' physics

Our (personal) recommended highlights selection:

Decay channel	Standard Model	Discrete symmetries	Light BSM particles
$\eta \to \pi^+\pi^-\pi^0$	light quark masses	C/CP violation	scalar bosons (also η')
$\eta^{(\prime)} \to \gamma\gamma$	η–η' mixing, precision partial widths		
$\eta^{(\prime)} \to \ell^+\ell^-\gamma$	$(g-2)_\mu$		Z' bosons, dark photon
$\eta \to \pi^0\gamma\gamma$	higher-order χPT, scalar dynamics		$U(1)_B$ boson, scalar bosons
$\eta^{(\prime)} \to \mu^+\mu^-$	$(g-2)_\mu$, precision tests	CP violation	
$\eta \to \pi^0\ell^+\ell^-$		C violation	scalar bosons
$\eta^{(\prime)} \to \pi^+\pi^-\ell^+\ell^-$	$(g-2)_\mu$		ALPs, dark photon
$\eta^{(\prime)} \to \pi^0\pi^0\ell^+\ell^-$		C violation	ALPs

$\longrightarrow$ decay channels that allow for synergies between

- Standard Model precision analyses
- discrete symmetry tests
- searches for light BSM particles

Gan, BK, Passemar, Tulin 2020

Spares

$\eta \to 3\pi$: effective BSM couplings

Akdag, Isken, BK 2021

- polynomial ambiguities $\longrightarrow$ subtractions no good observables
- define unambigous Taylor invariants & match to these:

$$\begin{aligned} \mathcal{M}_0^{\not C}(s,t,u) &= i\, g_0\, (s-t)(u-s)(t-u) + \mathcal{O}(p^8) \\ \mathcal{M}_2^{\not C}(s,t,u) &= i\, g_2\, (t-u) + \mathcal{O}(p^4) \end{aligned}$$

- fit corresponds to

$$g_0 = -2.8(4.5)\,\mathrm{GeV}^{-6}\,, \quad g_2 = -9.3(4.6)\times 10^{-3}\,\mathrm{GeV}^{-2}$$

$\longrightarrow$ sensitivity $|g_0/g_2| \sim 10^3\,\mathrm{GeV}^{-4} = \mathcal{O}(M_\pi^{-4})$

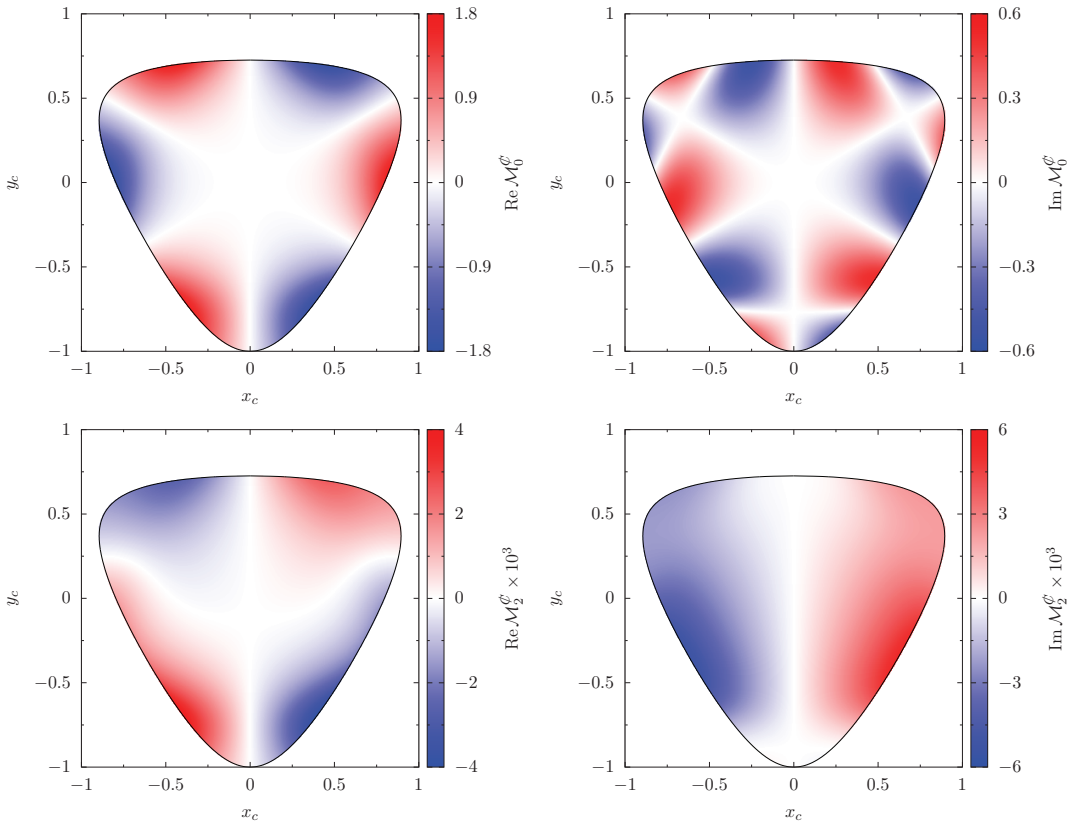
$\longrightarrow$ theoretical/chiral expectation: $|g_0/g_2| \sim \mathrm{GeV}^{-4}$

- small phase space ($M_\eta - 3M_\pi \sim M_\pi$) reduces sensitivity to $\mathcal{M}_0^{\not C}$

Generalisation to η' decays

$\eta' \to \pi^+\pi^-\pi^0$

- rather rare, $\mathcal{B} \sim 3.6 \times 10^{-3} \longrightarrow$ data not so precise BESIII 2016
- rescale $\eta \to \pi^+\pi^-\pi^0$ with same $g_{0,2} \longrightarrow$ more sensitive to g_0 by factor ~ 100



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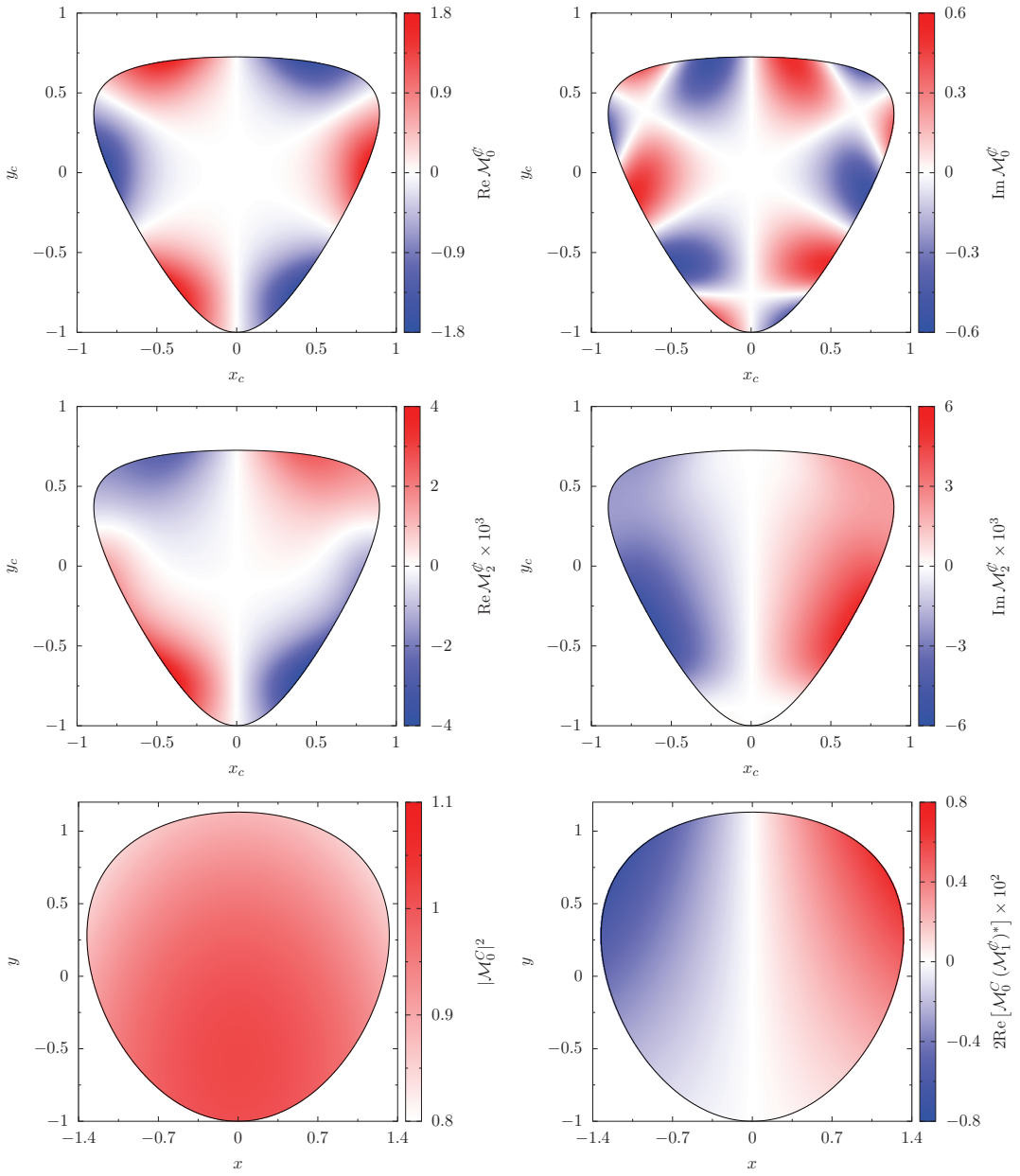

$\eta' \to \eta\pi^+\pi^-$

- SM conserves isospin
- C-odd op. $\Delta I = 1$ $\longrightarrow$ constrained at 10^{-2} BESIII 2017

Correlating $\eta \to \pi^0\pi^+\pi^-$ and $\eta \to \pi^0\ell^+\ell^-$

- isovector contribution scales with C-odd $\eta \to \pi^0\pi^+\pi^-$ couplings:

	isovector	isovector + isoscalar	experiment
$\mathcal{B}(\eta \to \pi^0 e^+ e^-)$	$< 20 \cdot 10^{-6}$	$< 29 \cdot 10^{-6}$	$< 7.5 \cdot 10^{-6}$
$\mathcal{B}(\eta \to \pi^0 \mu^+ \mu^-)$	$< 7.2 \cdot 10^{-6}$	$< 10 \cdot 10^{-6}$	$< 5.0 \cdot 10^{-6}$

- isovector and isoscalar contributions of comparable magnitude
- direct experimental limit stronger than indirect from $\eta \to \pi^0\pi^+\pi^-$
- refined regression to $\eta \to \pi^0\pi^+\pi^-$ with $\eta \to \pi^0\ell^+\ell^-$ as constraint:

$$|g_0| < 7.3\,\mathrm{GeV}^{-6} \quad \longrightarrow \quad |g_0| < 4.4\,\mathrm{GeV}^{-6}$$

- similar relations $\eta' \to \eta\pi^+\pi^- \leftrightarrow \eta' \to \eta\ell^+\ell^-$ worked out

Akdag, BK, Wirzba 2023

- beware: interference with short-range effects neglected
  (C-odd photon couplings, leptonic operators)