# Generalized distribution amplitudes and gravitational form factors of hadrons

### Qin-Tao Song (Zhengzhou University)

### The 7th International Workshop on Future Tau-Charm Facilities, Huangshan, November 26, 2025

References: C. Lorce, B. Pire and Oin-Tao Song, PRD106 (2022), 094030.

B. Pire and Qin-Tao Song, PRD 107 (2023), 114014.

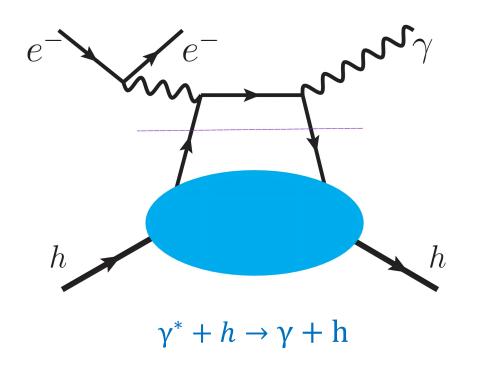
B. Pire and Qin-Tao Song, PRD 109 (2024), 074016.

Qin-Tao Song, O. V. Teryaev and S. Yoshida, PLB 868 (2025), 139797.

Jing Han, B. Pire and Qin-Tao Song, PRD 112 (2025), 014048. Jing Han, B. Pire and Qin-Tao Song, arXiv:2511.05970 [hep-ph]. > GPDs, GDAs and gravitational FFs

Outline: 
$$\triangleright$$
 Production of a scalar meson pair 
$$\begin{cases} \gamma^* \to M_1 M_2 \gamma \\ \gamma^* \gamma \to M_1 M_2 \end{cases}$$

### GPDs (Generalized parton distributions)



Deeply Virtual Compton Scattering (DVCS)

QCD collinear factorization

Hard part:  $\gamma^* q \rightarrow \gamma q$ 

Soft part: $qh \rightarrow qh$ , quark GPDs.

The GPDs are 3-D structure functions that offer opportunities to study a new aspect of nucleon structure: a nucleon tomography.

forward limit

GPDs ->

PDFs (Parton distribution functions )

#### GPDs and EMT

GPDs can help us to access the hadronic matrix elements of energy momentum tensor (EMT) indirectly.

- Proton spin puzzle
- ➤ EMT (gravitational) FFs of hadrons

Hadronic matrix elements of EMT:

$$\langle p', \vec{s}' | T_a^{\mu\nu} | p, \vec{s} \rangle$$

$$= \bar{u}(p', \vec{s}') \left[ A_a(t) \frac{P^{\mu}P^{\nu}}{M_N} \right]$$

$$+ D_a(t) \frac{\Delta^{\mu}\Delta^{\nu} - g^{\mu\nu}\Delta^2}{4M_N} + \bar{C}_a(t)M_Ng^{\mu\nu}$$

$$+ J_a(t) \frac{P^{\{\mu}i\sigma^{\nu\}\lambda}\Delta_{\lambda}}{M_N} - S_a(t) \frac{P^{[\mu}i\sigma^{\nu]\lambda}\Delta_{\lambda}}{M_N} \right] u(p, \vec{s}),$$

$$Uuark Augular Momentum (AM)$$

$$S_q(0): Quark Helicity contribution$$

$$J_q(0) - S_q(0)/2: Quark orbital AM$$

$$D_q(0): D-term (last global unknown charge)$$

$$X.D. Ji, PRL 78(1997), 610.$$

$$M. V. Polyakov and C. Weiss, PRD 60, 114017 (1999).$$

$$X. H. Cao, F. K. Guo, Q. Z. Li and D.L. Yao, Nature Commun. 16 (2025) no.1, 6979$$

Most hadrons are not stable, we can not use DVCS to study their GPDs. How to obtain EMT FFs for these unstable hadrons?

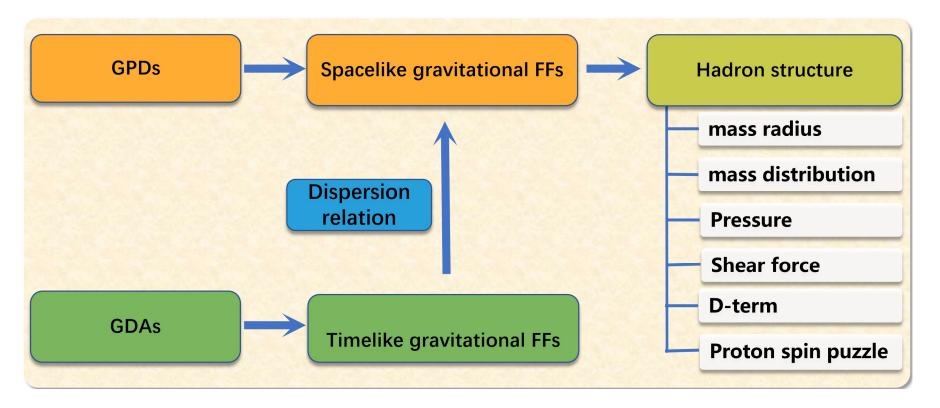
#### EMT FFs and unstable hadrons?

Generalized distribution amplitudes (GDAs) are the s-t crossed quantities of GPDs, the second moments of GDAs lead to timelike EMF FFs.

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL 81 (1998) 1782.

M. V. Polyakov, NPB 555 (1999) 231.

#### From GPDs and GDAs to hadron gravitational FFs:



M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) no.26, 1830025.

V. D. Burkert, L. Elouadrhiri, F. Girod, C. Lorce, P. Schweitzer and P. Shanahan, Rev. Mod. Phys. 95 (2023), 041002.

### GDAs are accessed in two-photon reactions

#### GDAs are also important inputs for decays of B mesons.

W. F. Wang, H. N. Li, W. Wang and C. D. Lu, PRD 91 (2015), 094024.

Y. Li, A. J. Ma, W. F. Wang and Z. J. Xiao, PRD 95 (2017), 056008.

S. Cheng, PRD 99(2019), 053005.

M. K. Jia, C. Q. Zhang, J. M. Li and Z. Rui, PRD 104 (2021), 073001.

J. W. Zhang, B. Y. Cui, X. G. Wu, H. B. Fu and Y. H. Chen, PRD 110(2024), 036015.

....

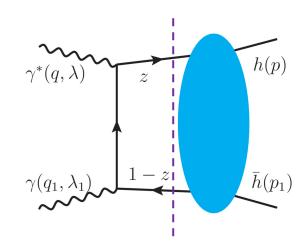
# GDAs are accessed in $\gamma^* \gamma \to hh$ , which can be measured in e<sup>+</sup>e<sup>-</sup> collisions (BESIII, Belle II, and STCF).

#### QCD collinear factorization

$$q^2 = -Q^2, \quad (q_1)^2 = 0,$$
  
$$Q^2 \gg s, \Lambda_{\text{OCD}}^2$$

Hard part:  $\gamma^* \gamma \rightarrow q \overline{q}$ 

Soft part: $q\bar{q} \rightarrow h\bar{h}$ , quark GDAs.



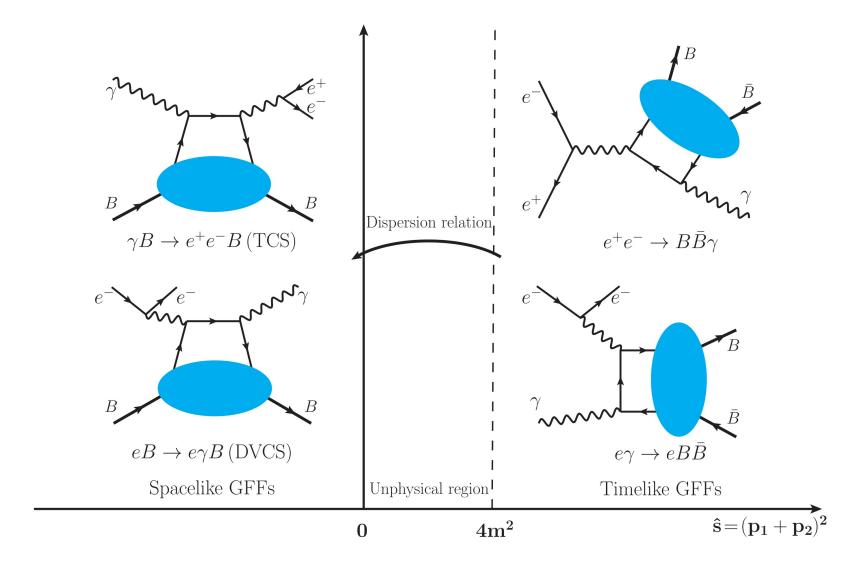
Quark GDA of a scalar hadron pair is defined as:

$$\Phi(z, \xi, s) = \int \frac{dx^{-}}{2\pi} e^{-izP^{+}x^{-}} \langle h(p)\bar{h}(p_{1}) | \bar{q}(x^{-})\gamma^{+}q(0) | 0 \rangle$$

M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.

C. Lorce, B. Pire and Qin-Tao Song, PRD 106 (2022), 094030.

### Hadron GDAs and GPDs are accessed in two-photon reactions



Spacelike and timelike EMT FFs are probed by different reactions, and the former ones can be obtained from the latter ones using dispersion relation.

Production of a scalar meson pair:  $\gamma^* \to M_1 M_2 \gamma$ and  $\gamma^* \gamma \to M_1 M_2$ 

# GDAs in $\gamma^* \gamma \rightarrow hh$

#### QCD collinear factorization

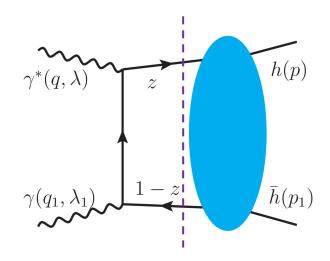
Hard part:  $\gamma^* \gamma \rightarrow q\bar{q}$ 

Soft part: $q\bar{q} \rightarrow h\bar{h}$ , quark GDAs.

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL 81 (1998) 1782.

M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.

M. V. Polyakov, NPB **555** (1999) 231.



Helicity amplitudes of a scalar meson pair:

$$A_{\lambda\lambda_1} = T_{\mu\nu} \epsilon^{\mu}(\lambda) \epsilon^{\nu}(\lambda_1)$$

$$q^2 = -Q^2, \quad (q_1)^2 = 0,$$

There are three independent helicity amplitudes:  $A_{++}$ ,  $A_{0+}$  and  $A_{+-}$ .

Leading twist amplitude:  $A_{++}$ 

Higher twist amplitudes:  $A_{0+}$  and  $A_{+-}$ 

Twist expansion:

Cross section=Leading-twist contribution + Higher-twist contribution

Suppressed by 1/Q or 1/Q<sup>2</sup>

At leading twist, only  $A_{++}$  appears in the cross section!

#### The extraction of pion GDAs and EMT FFs

If we neglect the higher-twist contribution, the cross section is given by  $A_{++}$ .

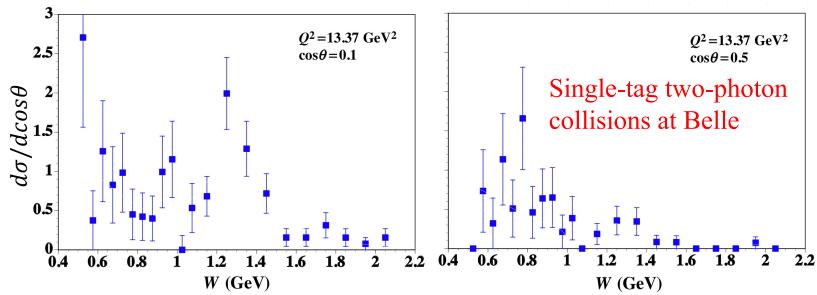
$$A_{++} = \sum_{q} \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z,\zeta,W^2) \longrightarrow \text{Pion GDA, leading twist.}$$

Similar to the meson distribution amplitudes (DAs), we have the simple asymptotic form of pion GDAs at  $Q^2 \rightarrow \infty$ :

$$\Phi(z, \cos\theta, s) = 18z(1-z)(2z-1)[\tilde{B}_{10}(s) + \tilde{B}_{12}(s) P_2(\cos\theta)]$$

The 3-D GDA is written as two 1-D functions, S-wave and D-wave.

In 2016, the cross sections of  $\gamma^* \gamma \to \pi \pi$  were released by Belle.



M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003.

### The extraction of pion GDAs and EMT FFs

We extracted the pion GDA and EMT FFs from Belle data using the asymptotic GDA.

From GDA to the gravitational FFs:

$$\int_{0}^{1} dz (2z - 1) \, \Phi_{q}^{\pi^{0}\pi^{0}}(z, \zeta, W^{2}) = \frac{2}{(P^{+})^{2}} \langle \pi^{0}(p_{1}) \, \pi^{0}(p_{2}) \, | \, T_{q}^{++}(0) \, | \, 0 \rangle.$$

$$\langle \pi(p_{2})\pi(p_{1}) \, | \, T_{q}^{\mu\nu} \, | \, 0 \rangle$$

$$= \frac{1}{2} \left[ \Theta_{1}(s) (sg^{\mu\nu} - P^{\mu}P^{\nu}) + \Theta_{2}(s) \Delta^{\mu}\Delta^{\nu} \right]$$

Two timelike EMT FFs for pions!

From the gravitational FFs to mass radius:

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 - 0.39 \text{ fm},$$

S. Kumano, Qin-Tao Song and O. Teryaev, PRD 97 (2018) 014020.

The mass radius is smaller than the pion charge radius:  $\sqrt{\langle r^2 \rangle} = 0.67 \text{fm}$ 

Mass radius:  $\sqrt{\langle r^2 \rangle} = 0.33$  fm by NJL model A. Freese and I. C. Cloet, PRC 100 (2019), 015201

### Kinematical higher-twist corrections

The errors are large for Belle measurements, where statistical errors are dominant. As precise measurements are expected at Belle II (or STCF), the leading-twist analysis is not enough for the meson GDAs.

Belle measurements:  $8 \text{ GeV}^2 < Q^2 < 24 \text{ GeV}^2$  $0.2 \text{ GeV}^2 < s < 4 \text{ GeV}^2$ 

Twist expansion:

Cross section=Leading-twist contribution + Higher-twist contribution

Suppressed by 1/Q (twist 3) or  $1/Q^2$  (twist 4), it could account for  $\sim 20\%$  in the cross section.

We include the kinematical higher-twist corrections to the GDA studies, where only the leading-twist GDAs are used.

V. M. Braun and A. N. Manashov, PRL 107(2011), 202001; JHEP 01 (2012), 085; PPNP 67 (2012), 162–167.

The kinematical corrections are included for recent DVCS measurements (GPDs).

F. Georges et al. [Jefferson Lab Hall A], PRL. 128 (2022), 252002.

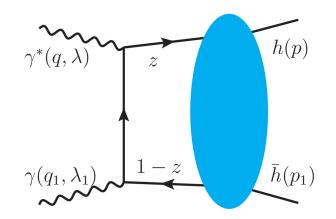
M. Defurne et al., Nature Communication 8(2017), 1408.

M. Defurne et al., Hall A collaboration, PRC92 (2015) no.5, 055202

# Kinematical higher-twist contributions in $\gamma^* \gamma \to M\overline{M}$

Helicity amplitudes of a scalar meson pair:

$$A_{\lambda\lambda_1}=T_{\mu\nu}\varepsilon^{\mu}(\lambda)\varepsilon^{\nu}(\lambda_1)$$



There are three independent helicity amplitudes:  $A_{++}$ ,  $A_{0+}$  and  $A_{+-}$ .

Leading twist amplitude: A<sub>++</sub>

Higher twist amplitudes:  $A_{0+}$  and  $A_{+-}$ 

We studied the kinematical higher-twist contributions for these helicity amplitudes in  $\gamma^* \gamma \to M \overline{M}$ .

### Helicity amplitudes (up to twist 4):

$$A^{(0)} = \chi \left\{ \left( 1 - \frac{s}{2Q^2} \right) \int_0^1 dz \, \frac{\Phi(z, \eta, s)}{1 - z} - \frac{s}{Q^2} \int_0^1 dz \, \frac{\Phi(z, \eta, s)}{z} \, \ln(1 - z) \right.$$

$$\left. - \left( \frac{2s}{Q^2} \eta + \frac{\Delta_T^2}{\beta_0^2 Q^2} \frac{\partial}{\partial \eta} \right) \frac{\partial}{\partial \eta} \int_0^1 dz \, \frac{\Phi(z, \eta, s)}{z} \left[ \frac{\ln(1 - z)}{2} + \text{Li}_2(1 - z) - \text{Li}_2(1) \right] \right\},$$

$$A^{(1)} = \frac{2\chi}{\beta_0 Q} \frac{\partial}{\partial \eta} \int_0^1 dz \, \Phi(z, \eta, s) \, \frac{\ln(1 - z)}{z},$$

$$A^{(2)} = -\frac{2\chi}{\beta_0^2 Q^2} \frac{\partial^2}{\partial \eta^2} \int_0^1 dz \, \Phi(z, \eta, s) \, \frac{2z - 1}{z} \, \ln(1 - z),$$

$$\eta = \cos \theta$$

C. Lorce, B. Pire and Qin-Tao Song, PRD 106 (2022), 094030

$$A_{++} = A^{(0)}$$
  
 $A_{0+} = -A^{(1)}\Delta \cdot \epsilon(-)$   $\longrightarrow \propto \Delta_T$   $\Delta$  is the relative momentum  $A_{-+} = -A^{(2)}[\Delta \cdot \epsilon(-)]^2$   $\longrightarrow \propto (\Delta_T)^2$  of final meson pair.

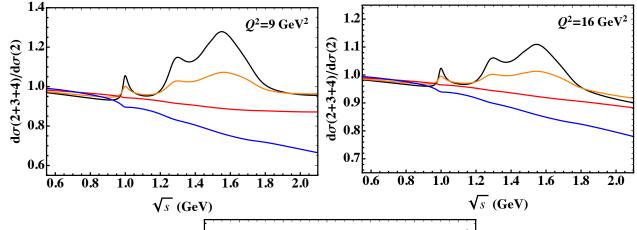
Asymptotic form of pion GDAs:

$$\Phi(z, \cos\theta, s) = 18z(1-z)(2z-1)[\tilde{B}_{10}(s) + \tilde{B}_{12}(s) P_2(\cos\theta)]$$

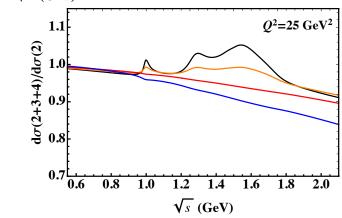
The nonvanishing helicity-flip amplitudes  $A_{0+}$  and  $A_{+-}$  indicate the existence of the D-wave GDAs.

### Ratios are estimated with the asymptotic $\pi \pi GDA$

Ratio=(twist-2+twist-3+twist-4 cross section)/ twist-2 cross section



Kinematics are chosen according to Belle (II) and STCF



Both types of  $\pi \pi$  GDAs indicate that the higher-twist kinematical contributions cannot be neglected if  $s > 1 \text{ GeV}^2$ 

Timelike EMT form factors

 $\Lambda \geq 3 \text{ GeV}^2$  is necessary for pion EMT form factor, PRD 97 (2018) 014020.

Spacelike form Dispersion factor t<0 relation:

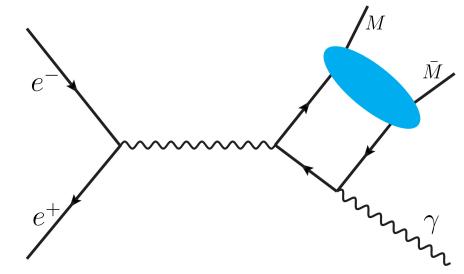
$$F(t) = \int_{4m^2}^{\Lambda} \frac{ds}{\pi} \frac{\text{Im}[F(s)]}{s - t - i\varepsilon}$$

Timelike form factor s>0

# Kinematical higher-twist contributions in $\gamma^* \to M\overline{M}\gamma$

Helicity amplitudes of a scalar meson pair:

$$A_{\lambda\lambda_1} = T_{\mu\nu}\epsilon^{\mu}(\lambda)\epsilon^{\nu}(\lambda_1)$$



There are three independent helicity amplitudes:  $A_{++}$ ,  $A_{0+}$  and  $A_{+-}$ .

Leading twist amplitude: A<sub>++</sub>

Higher twist amplitudes:  $A_{0+}$  and  $A_{+-}$ 

We also calculted the kinematical higher-twist contributions for the helicity amplitudes in  $\gamma^* \to M\overline{M}\gamma$ , the details can be found in our papers:

B. Pire and Q. T. Song, PRD 107 (2023), 114014

B. Pire and Qin-Tao Song, PRD 109 (2024), 074016

Production of a spin-1/2 baryon pair:  $\gamma^* \to B\bar{B}\gamma$  and  $\gamma^*\gamma \to B\bar{B}$ 

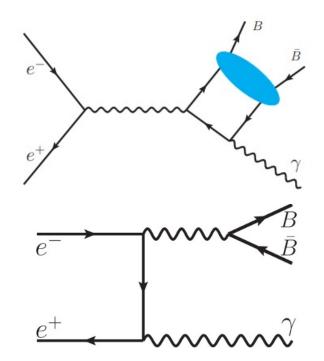
# Baryon-antibaryon GDAs in $e^+e^- \rightarrow B\bar{B}\gamma$ (BESIII and STCF)

There are two subprocesses in  $e^-e^+ \rightarrow B\bar{B}\gamma$ .

(1) QCD subprocess: $e^-e^+ \rightarrow \gamma^* \rightarrow B\bar{B}\gamma$ :

The  $B\bar{B}$  GDAs are involved, QCD collinear factorization.

(2) ISR subprocess:  $e^-e^+ \rightarrow \gamma^* \gamma \rightarrow B\bar{B}$ : The  $\gamma^* \rightarrow B\bar{B}$  vertex is described by the timelike EM FFs.



The timelike baryon EM FFs will be used to extract the baryon GDAs, and the FFs of the baryon octet family have been extensively studied at BESIII.

BESIII Collaboration, Nat. Phys. 17, 1200 (2021); PLB 817,136328 (2021); PRL130, 151905 (2023); PRL123, 122003 (2019); PRD 107,072005 (2023); PRL132, 081904 (2024); PRD 109,034029 (2024); PLB 820, 136557 (2021); PRD 103, 012005 (2021)......

See the talk of Prof. Alex Bobrov for proton EM FFs.

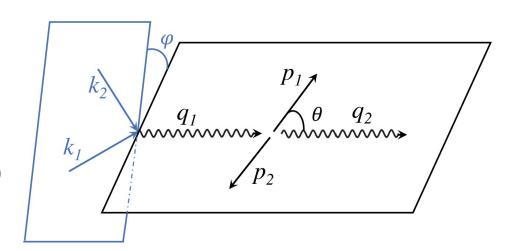
The process  $e^-e^+ \to B\bar{B}\gamma$  can be measured at BESIII, Belle II, and STCF. Actually, this process has been used for the recent measurements of baryon EM FFs.

$$e^-e^+ \rightarrow \Lambda \overline{\Lambda} \gamma$$
:

### Baryon-antibaryon GDAs

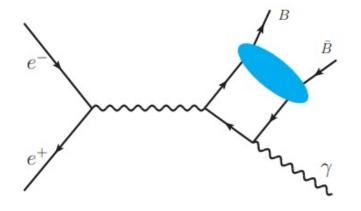
The center-of-mass frame of the baryon-antibaryon pair:

$$e^-(k_1)e^+(k_2) \to \gamma^*(q_1) \to B(p_1)\bar{B}(p_2)\gamma(q_2)$$



The subprocess  $e^-e^+ \rightarrow \gamma^* \rightarrow B\overline{B}\gamma$ , and the GDAs of a baryon-antibaryon pair:

$$\begin{split} P^{+} & \int dX^{-} e^{izP^{+} \cdot x^{-}} \langle \bar{B}(p_{2}) B(p_{1}) | \bar{q}(-x^{-}) \gamma^{+} \gamma_{5} q(0) | 0 \rangle \\ &= \Phi_{A}^{q}(z, \zeta_{0}, \hat{s}) \bar{u}(p_{1}) \gamma^{+} \gamma_{5} v(p_{2}) \\ &+ \Phi_{P}^{q}(z, \zeta_{0}, \hat{s}) \frac{P^{+}}{2m} \bar{u}(p_{1}) \gamma_{5} v(p_{2}), \\ P^{+} & \int dX^{-} e^{izP^{+} \cdot x^{-}} \langle \bar{B}(p_{2}) B(p_{1}) | \bar{q}(-x^{-}) \gamma^{+} q(0) | 0 \rangle \\ &= \Phi_{V}^{q}(z, \zeta_{0}, \hat{s}) \bar{u}(p_{1}) \gamma^{+} v(p_{2}) + \Phi_{S}^{q}(z, \zeta_{0}, \hat{s}) \frac{P^{+}}{2m} \bar{u}(p_{1}) v(p_{2}), \end{split}$$



#### Four baryon GDAs

M. Diehl, P. Kroll, and C. Vogt, EPJC26, 567 (2003)

#### **GDA** contribution

We define the timelike Compton FFs:

$$(\zeta_0)\mathcal{F}_i = \sum_q rac{e_q^2}{2} \int_0^1 dz rac{2z-1}{z(1-z)} \Phi_i^q\!(z,\zeta,\hat{s}) (i=V,S), \quad \mathcal{F}_{i'} = \sum_q rac{e_q^2}{2} \int_0^1 dz rac{1}{z(1-z)} \Phi_{i'}^q\!(z,\zeta,\hat{s}) (i'=A,P)$$

The hardon Tensor (leading twist) is given by

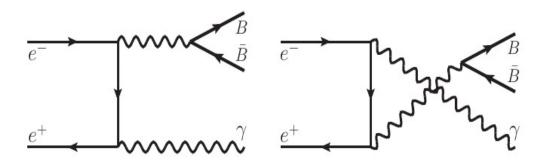
$$T_{\mu
u}=rac{-2}{\sqrt{2s}}igg\{g_T^{\mu
u}igg[\zeta_0\mathcal{F}_Var{u}(p_1)\gamma^+v(p_2)+\mathcal{F}_Srac{P^+}{2m}ar{u}(p_1)v(p_2)igg]-i\epsilon_T^{\mu
u}igg[\mathcal{F}_Aar{u}(p_1)\gamma^+\gamma^5v(p_2)+\mathcal{F}_Prac{P^+}{2m}ar{u}(p_1)\gamma^5v(p_2)igg]igg\}$$

Cross section:

$$egin{split} rac{d\sigma_{\mathrm{G}}}{d\hat{s}dud(\cos heta)darphi} &= rac{lpha_{\mathrm{em}}^3eta_0}{8\pi s^3}rac{1}{1+\epsilon}\Bigg[|\mathcal{F}_A|^2 - |\mathcal{F}_S|^2 + 2\mathrm{Re}(\mathcal{F}_A\mathcal{F}_P^*) + rac{\hat{s}\left(|\mathcal{F}_P|^2 + |\mathcal{F}_S|^2
ight)}{4m^2} \ &+ (eta_0)^2\cos^2 heta\Big[|\mathcal{F}_V|^2 + 2\mathrm{Re}(\mathcal{F}_S\mathcal{F}_V^*) - |\mathcal{F}_A|^2\Big] - (eta_0)^4\cos^4 heta|\mathcal{F}_V|^2\Bigg] \end{split}$$

#### ISR contribution

ISR process 
$$e^-e^+ \rightarrow \gamma^* \gamma \rightarrow B\bar{B} \gamma$$



Baryon EM FFs: 
$$\langle \bar{B}(p_2)B(p_1)|\, \bar{q}(0)\gamma^{\mu}q(0)\, |0
angle = F_V^q(\hat{s})\bar{u}(p_1)\gamma^{\mu}v(p_2) + F_S^q(\hat{s})rac{\Delta^{\mu}}{2m}\bar{u}(p_1)v(p_2),$$
  $G_E(\hat{s}) = F_V^q(\hat{s}) + (\tau - 1)F_S^q(\hat{s}), G_M(\hat{s}) = F_V^q(\hat{s})$ 

#### Cross section:

$$\begin{split} \frac{d\sigma_{\text{ISR}}}{d\hat{s}dud(\cos\theta)d\varphi} &= \frac{\alpha_{\text{em}}^{3}\beta_{0}^{3}}{4\pi s^{2}} \frac{1}{\epsilon\hat{s}} \left[ b_{0} + b_{1}\cos^{2}\theta + b_{2}\sin^{2}\theta + b_{3}\sin(2\theta)\cos\varphi + b_{4}\sin^{2}\theta\cos(2\varphi) \right] \\ b_{0} &= \left[ 1 - 2x(1-x)(1+\epsilon) \right] (2\lambda - 1)|G_{M}|^{2}, \\ b_{1} &= \left[ 1 - 2x(1-x)(1-\epsilon) \right] |G_{M}|^{2} + 4\epsilon x(x-1)(\lambda - 1) \left[ |G_{E}|^{2} - |G_{M}|^{2} \right], \\ b_{2} &= 2\epsilon x(x-1)|G_{M}|^{2} + \left[ 1 - 2x(1-x) \right] (\lambda - 1) \left[ |G_{E}|^{2} - |G_{M}|^{2} \right], \\ b_{3} &= \sqrt{\epsilon(1-\epsilon)} \sqrt{2x(x-1)} (2x-1) \text{sgn}(\rho) \left[ (\lambda - 1)|G_{E}|^{2} - \lambda |G_{M}|^{2} \right], \\ b_{4} &= 2\epsilon x(1-x) \left[ (\lambda - 1)|G_{E}|^{2} - \lambda |G_{M}|^{2} \right]. \end{split}$$

#### Interference term

The interference term of two subprocesses are also need to included.

$$\left|\mathcal{M}_{
m I}
ight|^2 = \mathcal{M}_{
m ISR}\mathcal{M}_{
m G}^* + \mathcal{M}_{
m ISR}^*\mathcal{M}_{
m G}$$

One can decompose the cross section according to its dependence on angles

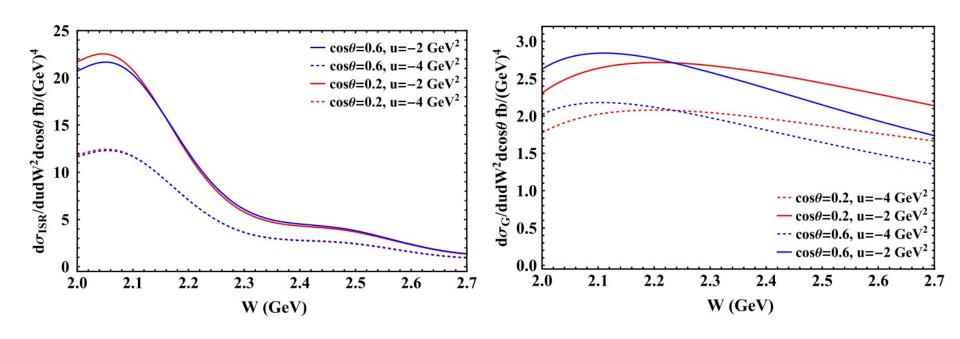
$$rac{d\sigma_{
m I}}{d\hat{s}dud(\cos heta)darphi} = rac{lpha_{
m em}^3eta_0}{8\pi s^2}rac{\sqrt{2}eta_0}{\sqrt{\hat{s}s\epsilon(1+\epsilon)}}ig[c_0\cos heta+c_1\cos^3 heta+c_2\sin heta\cosarphi+c_3\sin(2 heta)\cos heta\cosarphiig]$$

where the coefficients read

$$egin{aligned} c_0 = & 2 ext{sgn}(
ho) \sqrt{\epsilon(1-\epsilon)} \sqrt{2x(x-1)} [ ext{Re}(\mathcal{F}_V G_M^*) + ext{Re}(\mathcal{F}_S G_E^*)], \ c_1 = & 2(eta_0)^2 ext{sgn}(
ho) \sqrt{\epsilon(1-\epsilon)} \sqrt{2x(x-1)} [(\lambda-1) ext{Re}(\mathcal{F}_V G_E^*) - \lambda ext{Re}(\mathcal{F}_V G_M^*)], \ c_2 = & 2[1-(1-x)(1+\epsilon)] ext{Re}(\mathcal{F}_A G_M^*) + 2[1-(1-x)(1-\epsilon)] ext{Re}(\mathcal{F}_S G_E^*), \ c_3 = & (eta_0)^2 [1-(1-x)(1-\epsilon)] [(\lambda-1) ext{Re}(\mathcal{F}_V G_E^*) - \lambda ext{Re}(\mathcal{F}_V G_M^*)]. \end{aligned}$$

# Numerical estimate for $e^+e^- \rightarrow p\bar{p}\gamma$

We adopted the effective proton EM FF and GDA model for numerical estimate.

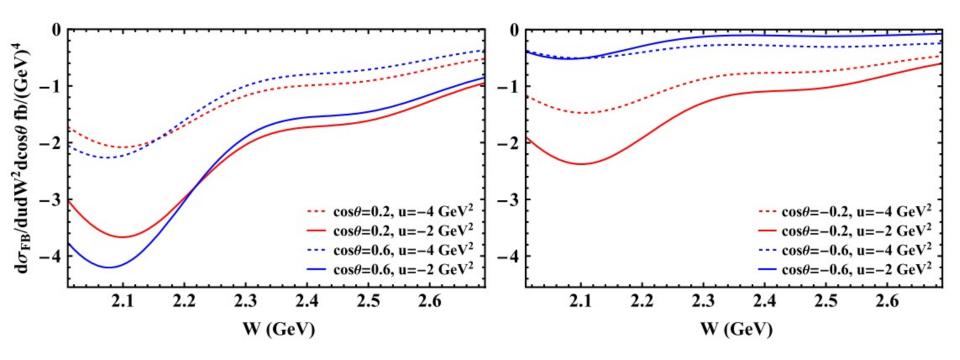


Estimate of ISR contribution

Estimate of  $e^+e^- \rightarrow \gamma^* \rightarrow B\overline{B}\gamma$  contribution

 $\sqrt{s}$  = 4 GeV is typical for BESIII and the proposed STCF.

### Numerical estimate



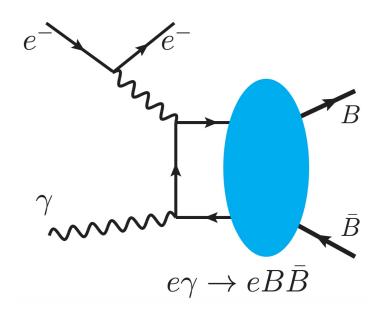
Estimate of the forward-backward contribution

The interference term is larger than pure QCD term, it will play an important role in the extraction of baryon GDAs.

# Baryon-antibaryon GDAs in $e\gamma \rightarrow eB\bar{B}$ (Belle II and STCF)

$$e(k_1)\gamma(q_2) \to e(k_2)B(p_1, S_1)\bar{B}(p_2, S_2)$$

The baryon-antibaryon GDAs:



The spin vectors  $S_1$  and  $S_2$  can be determined from subsequent decays of baryons such as  $\Lambda \to p\pi$ . The Spin physics has already been a hot topic at BESIII, with significant progress being made recently.

See the talk of Prof. Hai-Bo Li at SPIN2025 for Spin physics at BESIII!

By including baryon polarizations, we can access the imaginary parts of the products of two Compton FFs, thus, more information about GDAs is extracted. The details can be found in our paper:

### **Summary**

- > GDAs can be used to investigate the EMT FFs of unstable hadrons.
- We investigate the processes of  $e^-e^+ \to h\bar{h}\gamma$  and  $e\gamma \to eh\bar{h}$ , from which the hadron GDAs can be extracted.
- The measurements are possible at BESIII, Belle II, and STCF, the study of GDAs at BESIII and STCF can be a new research direction.