Measurement of the $e^+e^- \to p\bar{p}$ cross section near the threshold with a high energy resolution

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Plan

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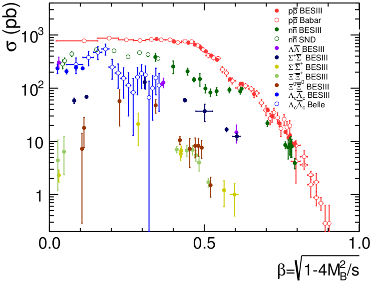
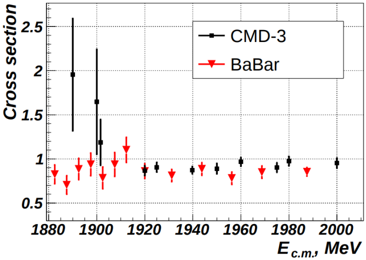
2. Peculiarity of the cross section measurement near the threshold
3. Two ways of detector modification
4. Experiment requirements
5. Fast simulation
6. Results
7. Conclusion

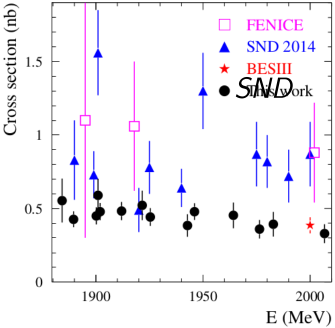
There are many processes ($e^+e^- \rightarrow p\bar{p}\gamma, p\bar{p}, n\bar{n}, \Lambda\bar{\Lambda}$ etc.) where strong enhancement near the threshold is observed.
This phenomenon attracts a lot of attention from theoretical and experimental point of view.



To date, there is no scientific consensus regarding the nature of the observed phenomenon.

Natl. Sci. Rev., **8**, 11, (2021)



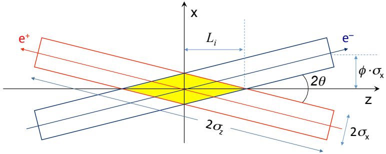


$e^+e^- \to p\bar{p}$
Phys. Let. B **759** p. 634 - 640 2016.
Phys. Rev. D. **87** p. 092005. 2013

$e^+e^- \to n\bar{n}$
Phys. Part. Nucl. **54** 4
624-629 (2023)

SND and CMD-3 at Novosibirsk VEPP-2000 collider.
We plan to continue this reasearch at VEPP-6!

Peculiarity of the cross section measurement near the threshold



Crab Waist scheme $\theta \sim 0.025$, $v_{c.m.s.} = \sin\theta c = 0.025c$

$$W = \sqrt{(P_0 + \delta P)^2} = W_0 + P_0^{\mu}\delta P_{\|\mu}/W_0 + (\delta P_{\perp}^2 + \delta P_{\|}^2)/2W_0$$

$$v^{\alpha}_{c.m.s.} = \frac{P_0^{\alpha} + \delta P_{\|}^{\alpha} + \delta P_{\perp}^{\alpha}}{\sqrt{(P_0 + \delta P_{\|} + \delta P_{\perp})^2}} = v^{\alpha}_{c.m.s.0} + \frac{\delta P_{\perp}^{\alpha}}{W_0}$$

Approximation: there is W invariant mass variation due to beam energy spread, but no velocity variation $\delta P_{\perp} = 0$, W^{rec} is function $v_{c.m.s.0}$.

Peculiarity of the cross section measurement near threshold

propagation of uncertainty

$$D(f(x)) = \begin{cases} \left[\frac{\partial f(x)}{\partial x}\right]^2\Big|_{x=\bar{x}} D(x), & if \frac{\partial f(x)}{\partial x}\Big|_{x=\bar{x}} \neq 0 \\ \left[\frac{\partial^2 f(x)}{\partial x^2}\right]^2\Big|_{x=\bar{x}} \frac{\overline{(x-\bar{x})^4}}{4}, & if \frac{\partial f(x)}{\partial x}\Big|_{x=\bar{x}} = 0 and \frac{\partial^2 f(x)}{\partial x^2}\Big|_{x=\bar{x}} \neq 0 \end{cases}$$

$$W = \frac{2m_p c^2}{\sqrt{1-\vec{v}^2/c^2}} \quad \frac{\partial W}{\partial \vec{v}} = \frac{2m_p \vec{v}}{(1-\vec{v}^2/c^2)^{3/2}} = \vec{0}\Big|_{W=2m_p}$$

At the threshold, resolution in invariant mass is substantially improved compared to the naive estimation $\frac{\sigma_W}{W}\Big|_{naive} \propto \frac{\sigma_E}{E_b}$, $\frac{\sigma_W}{W}\Big|_{real} \propto \frac{\sigma_E^2}{E_b^2}$!

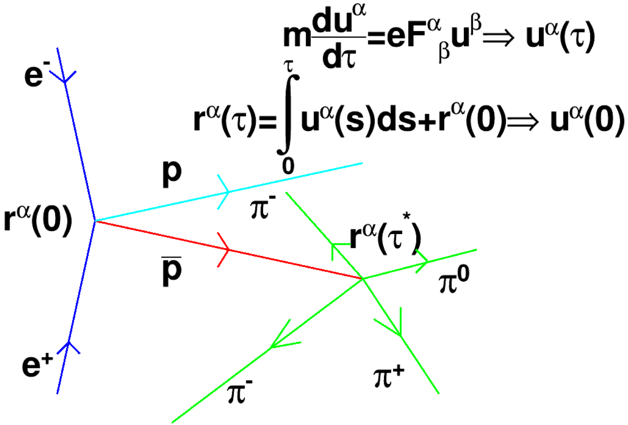
At the threshold non-relativictic approximation works well. This region is very interesting!

This effect for $e^-e^+ \to n\bar{n}$ was studied in Phys. Atom. Nucl. **86** 3 238-245 (2023)

Event reconstruction

1. At VEPP-2000 collider $p\bar{p}$ at the threshold have ~ 0 velocity. Due to this, the efficiency strongly depends on energy.
2. At a collider with Crab Waist scheme $p\bar{p}$ at the threshold have $\sim 0.03c$ velocity. $\bar{p}$ propagation inside the vacuum pipe up to annihilation point takes ~ 10 ns.
3. Stopping and annihilation in the material of vacuum pipe takes several ps. This is much shorter than $\bar{p}$ time of flight.
4. Reconstruction of the annihilation point coordinates and time (or four position) of the $\bar{p}$ allows to reconstruct event kinematics

Antiproton annihilation four-position determination through charged pions



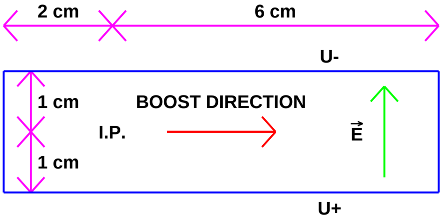
Two ways of detector modification

MED is magnetoelectric detector

scheme	Classical	MED
magnetic field	0.5 T	$0.05 \div 0.1$ T
electric field	-	+
beam pipe	$r \sim 3$ cm	very large
inner tracker	-	-

MED assumes full compensation of the electric field in c.m.s.

$$F^{\alpha}_{\beta} v^{\beta}_{c.m.s.0} = 0$$



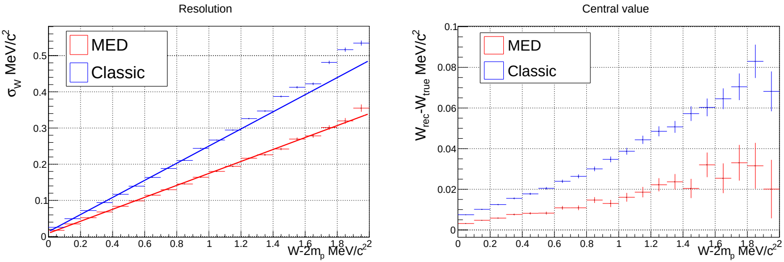
Experiment requirements

1. Luminosity measurement
2. I.P. position measurement
3. Beams energy and energy spread measurement with accuracy ~ 200 keV
4. Electromagnetic field configuration monitoring
5. Antiproton annihilation point and time reconstruction

1. Beam energy spread $\delta E_b/E_b = 6 \times 10^{-4}$, invariant mass variation $\sigma_W = 1.6$ MeV
2. Dispersion of the angular distribution in the beam $\sigma_{\theta_y} = 4.7 \times 10^{-4}$ $\sigma_{\theta_x} = 2.6 \times 10^{-4}$ radian
3. Vertical and horizontal beam sizes are 0.78 and 7.1 μm
4. Half of the beam intersection angle 0.025 radian
5. Beam energy E_b 938.563 ($p\bar{p}$ threshold), 939.163, 939.76, 940.363, 940.96 , 941.56 MeV
6. Magnetic field is 0.5 T Classical case, 0.05 T MED case.
7. Antiproton annihilation time resolution 100, 200 and 300 ps.

See talk by Anton Bogomyagkov 'Parameters optimization of e^+e^- crab waist colliders at low energy'.

Results invariant mass resolution



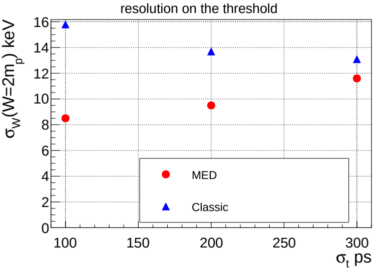
The resolution and central value weakly depend on

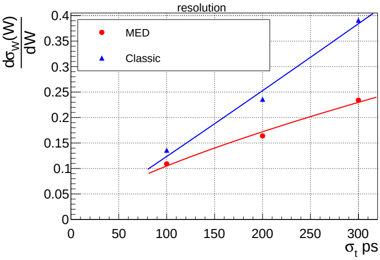
1. Beam energy
2. The accuracy of interaction point reconstruction (for realistic value of accuracy)
3. Value of magnetic field
4. The accuracy of energy measurement (for realistic value of accuracy)

The resolution as a function of W is linear.

Time resolution is 200 ps.

Results invariant mass resolution versus time resolution





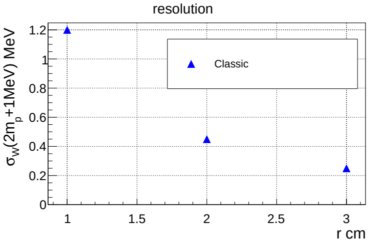
Time resolution of antiproton annihilation plays a key role for resolution of W. $c\sigma_t \sim 5 \div 10$ cm is much larger than resolution of coordinate annihilation.

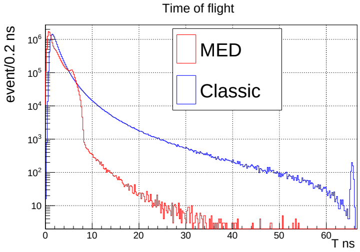
With time resolution degradation advantage of the MED scheme becomes more evident. $\frac{d\sigma_w(W)}{dW} = A\sigma_t^{\alpha}$

$\alpha = 1$ - Classical case, $\alpha = 0.7$ - MED.

$\frac{d\sigma_w(W)}{dW} \sim 0.1 \div 0.4$ dimentionless resolution parameter. Real chance to overcome beam energy spread!

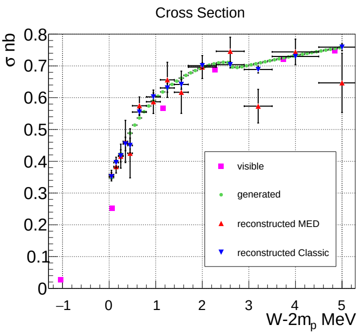
Results

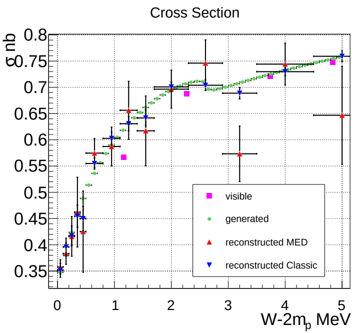




1. In classical size of the beam pipe strongly affects the resolution W.
2. For MED scheme, there is an advantage compared with the classical in annihilation time reconstruction. Bunch crossing (BC) period is too short 2.85 ns to associate event with paticular BC. It should be expanded in $2 \div 3$ times.
3. In MED case typical time is R/u=6cm/(0.75cm/ns)=8.0 ns. Luminosity lost by factor $2 \div 3$. In Classical case long tail in time distribution is determined by $\pi/\omega = \pi m_p/eB \sim$ 70 ns, 20 bunch crossing periods.

Results





Formfactors and cross section taken from Physical Review D. **106** 3 (2022).
Cross section reconstructed from fit of 2D distribution $(W_{rec};E_b)$.

There are six energy points. Data sample is equivalent to 2 months at $\frac{1}{3}$ design luminosity. Efficiency is $\frac{1}{6}$, CMD-3 based estimation.

Conclusion

1. The achievable resolution by invariant mass for the processes at the threshold is much less, the one estimated from the beam energy spread $\frac{\delta E_b}{E_b} \sim 10^{-3}$
2. We suggested two ways to study the process $e^+e^- \to p\bar{p}$ at the thresold
3. Using fast simulation, we estimated the $p\bar{p}$ invariant mass (W) resolution. The simulation confirms our first statement in this paticular case. The resolution scales as $\sigma_W = \frac{d\sigma_w(W)}{dW}(W - 2m_p)$, with $\frac{d\sigma_w(W)}{dW} = 0.1 \div 0.4$.
4. Most sensitive parameter for W resolution is $\bar{p}$ annihilation time resoltuion (or TOF resolution of $\pi^\pm$)
5. Reconstruction of the $e^+e^- \to p\bar{p}$ cross section demonstrates that the proposed procedure allows to overcome the resolution limitation from the beam energy spread

Peculiarity of the cross section measurement near the threshold

$$p_{e^+} = \begin{pmatrix} E_+ \\ 0 \\ E_+ \sin\theta \\ -E_+ \cos\theta \end{pmatrix} = \begin{pmatrix} E_0 e^{-\xi} \\ 0 \\ E_0 e^{-\xi} \sin\theta \\ -E_0 e^{-\xi} \cos\theta \end{pmatrix} p_{e^-} = \begin{pmatrix} E_- \\ 0 \\ E_- \sin\theta \\ E_- \cos\theta \end{pmatrix} = \begin{pmatrix} E_0 e^{\xi} \\ 0 \\ E_0 e^{\xi} \sin\theta \\ E_0 e^{\xi} \cos\theta \end{pmatrix}$$

$$E_0 = \sqrt{E_+ E_-} \;\; \xi = \log(E_-/E_+)/2 \;\; E_+ = E_0 e^{-\xi} \;\; E_- = E_0 e^{\xi}$$

$$W^2 = (p_{e^+} + p_{e^-})^2 = 4E_0^2 \cos^2\theta \;\; u^\alpha = \frac{p^\alpha_{e^+} + p^\alpha_{e^-}}{\sqrt{(p_{e^+} + p_{e^-})^2}}$$

$$u(\xi) = \cosh\xi \begin{pmatrix} \cos^{-1}\theta \\ 0 \\ \tan\theta \\ 0 \end{pmatrix} + \sinh\xi \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \cosh\xi \begin{pmatrix} \cosh\psi \\ 0 \\ \sinh\psi \\ 0 \end{pmatrix} + \sinh\xi \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$D(\xi) = \frac{\sigma_E^2}{2E^2}, \; \sigma_\xi \sim 10^{-3}, \delta\xi \sim 10^{-4}, \; \frac{\sigma_W}{W} = \frac{\sigma_E}{\sqrt{2}E_b}$$

$\sinh\psi = \tan\theta = 0.025 = v_{c.m.s.}/\sqrt{1 - v^2_{c.m.s.}}$, ξ is fixed, $E_0(W)$ is free.

$$p^\alpha_{e^+} + p^\alpha_{e^-} = W(\xi = \bar{\xi}) u^\alpha(\xi = \bar{\xi})$$

Equration of charged particle motion in static and uniform electro-magnetic field

$$m\frac{du^\mu}{d\tau} = zF^\mu_\nu u^\nu$$

$$F^\mu_\nu = \begin{pmatrix} 0 & E_x & E_y & E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix}$$

$$\lambda^4 - (\vec{E}^2 - \vec{B}^2)\lambda^2 - (\vec{E}\vec{B})^2 = 0$$

$$u^\alpha(\tau) = \Sigma_{i=0}^{i=3} c_i u_i^\alpha \exp \lambda_i \tau$$

$$r^\alpha(\tau) = r^\alpha(\tau = 0) + \Sigma_{i=0}^{i=3} c_i u_i^\alpha \begin{cases} [e^{\lambda_i \tau} - 1]/\lambda_i & if \ \lambda_i \neq 0 \\ \tau & if \ \lambda_i = 0 \end{cases}$$

If $\vec{E}\vec{B} = 0$, when two eigen values are equal to

Classical scheme magnetic field along z direction

$$u_0 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}\; u_+ = \begin{pmatrix}0\\ i/\sqrt{2}\\ 1/\sqrt{2}\\ 0\end{pmatrix}\; u_- = \begin{pmatrix}0\\ -i/\sqrt{2}\\ 1/\sqrt{2}\\ 0\end{pmatrix}\; u_z = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

$$\lambda_0 = 0, \lambda_+ = iz|B_z|, \lambda_- = -iz|B_z|, \lambda_z = 0, \omega = z|B_z|$$

$$u(\tau = 0) = \begin{pmatrix}\cosh\alpha\\ \sinh\alpha\sin\Theta\cos\phi\\ \sinh\alpha\sin\Theta\sin\phi\\ \sinh\alpha\cos\Theta\end{pmatrix}\; u(\tau) = \begin{pmatrix}\cosh\alpha\\ \sinh\alpha\sin\Theta\cos(\omega\tau+\phi)\\ \sinh\alpha\sin\Theta\sin(\omega\tau+\phi)\\ \sinh\alpha\cos\Theta\end{pmatrix}$$

$$r(\tau) = \begin{pmatrix}t\\x\\y\\z\end{pmatrix} = \begin{pmatrix}\cosh\alpha\tau\\ \sinh\alpha\sin\Theta[\sin(\omega\tau+\phi) - \sin(\phi)]/\omega\\ \sinh\alpha\sin\Theta[-\cos(\omega\tau+\phi) + \cos(\phi)]/\omega\\ \sinh\alpha\cos\Theta\tau\end{pmatrix}$$

Classical scheme magnetic field along z direction

$$\sinh\alpha\sin\Theta = \sinh\beta$$

$$t^2(\tau) - z^2(\tau) = \tau^2[\cosh^2\alpha - \sinh^2\alpha\cos^2\Theta] =$$
$$= \tau^2[\cosh^2\alpha - \sinh^2\alpha(1-\sin^2\Theta)] = \tau^2[1+\sinh^2\alpha\sin^2\Theta] =$$
$$= \tau^2[1+\sinh^2\beta] = \tau^2\cosh^2\beta$$

Now we can write an equation on β:

$$(x^2+y^2)\omega^2/4 = \sinh^2\alpha\sin^2\Theta\sin^2(\omega\tau/2) =$$
$$= \sinh^2\beta\sin^2(\omega\sqrt{t^2-z^2}/2/\cosh\beta)$$

The solution provides initial momentum of $\bar{p}$ and allows one to reconstruc the event $e^+e^- \to p\bar{p}$.

Scheme 'MED'

$$u_0 = \begin{pmatrix} \cosh\psi \\ 0 \\ \sinh\psi \\ 0 \end{pmatrix} u_+ = \begin{pmatrix} \sinh\psi/\sqrt{2} \\ i/\sqrt{2} \\ \cosh\psi/\sqrt{2} \\ 0 \end{pmatrix} u_- = \begin{pmatrix} \sinh\psi/\sqrt{2} \\ -i/\sqrt{2} \\ \cosh\psi/\sqrt{2} \\ 0 \end{pmatrix} u_z = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$u_0 = u(\xi = 0), \lambda_0 = 0, \lambda_+ = iz|\sqrt{B_z^2 - E_x^2}|, \lambda_- = -iz|\sqrt{B_z^2 - E_x^2}|,$$

$$\lambda_z = 0, \omega = z\sqrt{B_z^2 - E_x^2}$$

$$u_c = \begin{pmatrix} \sinh\psi \\ 0 \\ \cosh\psi \\ 0 \end{pmatrix} u_s = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

u_0, u_c, u_s, u_z ortogonal system.

$$u(\tau=0)=\cosh\beta u_0+\sinh\beta[\sin\Theta\{\sin\phi u_c+\cos\phi u_s\}+\cos\Theta u_z]=$$

$$=\begin{pmatrix}\cosh\psi\cosh\beta+\sinh\psi\sinh\beta\sin\Theta\sin\phi\\ \sinh\beta\sin\Theta\cos\phi\\ \cosh\psi\sinh\alpha\sin\Theta\sin\phi+\sinh\psi\cosh\beta\\ \sinh\beta\cos\Theta\end{pmatrix}$$

$$u(\tau)=\cosh\beta u_0+$$

$$+\sinh\beta[\sin\Theta\{\sin(\omega\tau+\phi)u_c+\cos(\omega\tau+\phi)u_s\}+\cos\Theta u_z]=$$

$$=\begin{pmatrix}\cosh\psi\cosh\beta+\sinh\psi\sinh\beta\sin\Theta\sin(\omega\tau+\phi)\\ \sinh\beta\sin\Theta\cos(\omega\tau+\phi)\\ \cosh\psi\sinh\beta\sin\Theta\sin(\omega\tau+\phi)+\sinh\psi\cosh\beta\\ \sinh\beta\cos\Theta\end{pmatrix}$$

Scheme 'MED'

$$r(\tau) = \begin{pmatrix} \cosh\psi\cosh\beta\tau + \sinh\psi\sinh\beta\sin\Theta\sin\phi[-\cos(\omega\tau+\phi)+\cos\phi]/\omega \\ \sinh\beta\sin\Theta[\sin(\omega\tau+\phi)-\sin\phi]/\omega \\ \cosh\psi\sinh\beta\sin\Theta[-\cos(\omega\tau+\phi)+\cos\phi]/\omega + \sinh\psi\cosh\beta\tau \\ \sinh\beta\cos\Theta\tau \end{pmatrix}$$

$$T(\tau) = r_\mu(\tau)u_0^\mu, X(\tau) = r_\mu(\tau)u_s^\mu, Y(\tau) = r_\mu(\tau)u_c^\mu, Z(\tau) = r_\mu(\tau)u_z^\mu$$

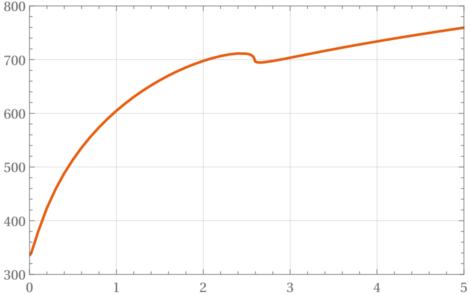
$$T(\tau) = \cosh\beta\tau, \; Y(\tau) = -\sinh\beta\sin\Theta[-\cos(\omega\tau+\phi)+\cos\phi]/\omega$$
$$X(\tau) = -\sinh\beta\sin\Theta[\sin(\omega\tau+\phi)-\sin\phi]/\omega Z(\tau) = -\sinh\beta\cos\Theta\tau$$

$$Z/T = -\tanh\beta\cos\Theta, \; t + \sinh\psi Y = \cosh\psi\cosh\beta\tau$$
$$X^2(\tau) + Y^2(\tau) = 4\sinh^2\beta\sin^2\Theta\sin^2[\omega\tau/2]/\omega^2 =$$
$$= 4\sinh^2\beta\sin^2\Theta\sin^2[\omega(t+\sinh\psi Y)/2/\cosh\beta/\cosh\psi]/\omega^2$$

Scheme 'MED'

$$X^2(\tau) + Y^2(\tau) =$$
$$= 4\sinh^2\beta \sin^2\Theta \sin^2[\omega(t + \sinh\psi Y)/2/\cosh\beta/\cosh\psi]/\omega^2$$

Equation has unknown parameter β. Approximation $\omega \to 0$ provides the solution:
$X^2(\tau) + Y^2(\tau) = \tanh^2\beta \sin^2\Theta(t + \sinh\psi Y)^2/\cosh^2\psi$.
$\tanh^2\beta = Z^2/T^2 + \cosh^2\psi(X^2 + Y^2)/(t + \sinh\psi Y)^2$
$W = 2m_p + m_p[\tanh^2\beta + O(\tanh^4\beta)]$.
$\vec{v}^2 = \vec{r}^2/t^2 = (x^2 + y^2 + z^2)/t^2$
The typical value of the magnetic field that can be compensated by electrical is $0.05 \div 0.1$ T. This is enoght to measure momentum of charged pions $\delta p/p \sim 3 \div 6\%$ for annihilation time reconstruction.



Fine structure of the cross sections of e^+e^- annihilation near the thresholds of $p\bar{p}$ and $n\bar{n}$ production. Nucl. Phys. A. **977** 60 - 68 (2018)
$N\bar{N}$ production in e^+e^- annihilation near the threshold revisited. Physical Review D. **106** 3 (2022).
The resolution of invariant mass at the threshold $n\bar{n}$ classical is 0.8 MeV/c^2, MED is 0.5 MeV/c^2.