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FTCF2025, Huangshan, China

Opportunities for detecting the P-wave $\bar{D}D^*/D\bar{D}^*$ resonance in STCF

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Nov. 25th, 2025

Based on Phys.Rev.Lett. 133 (2024), 241903

Together with Zi-Yang Lin, Jun-Zhang Wang, Jian-Bo Cheng, Shi-Lin Zhu

Background

Experimental advances

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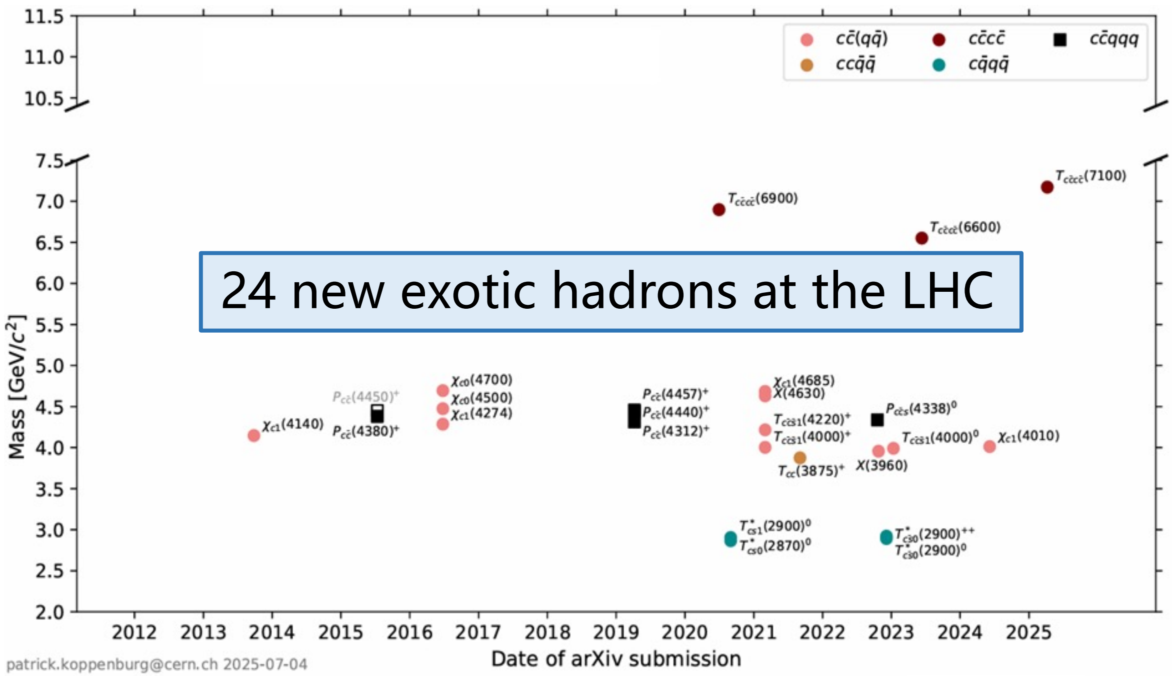

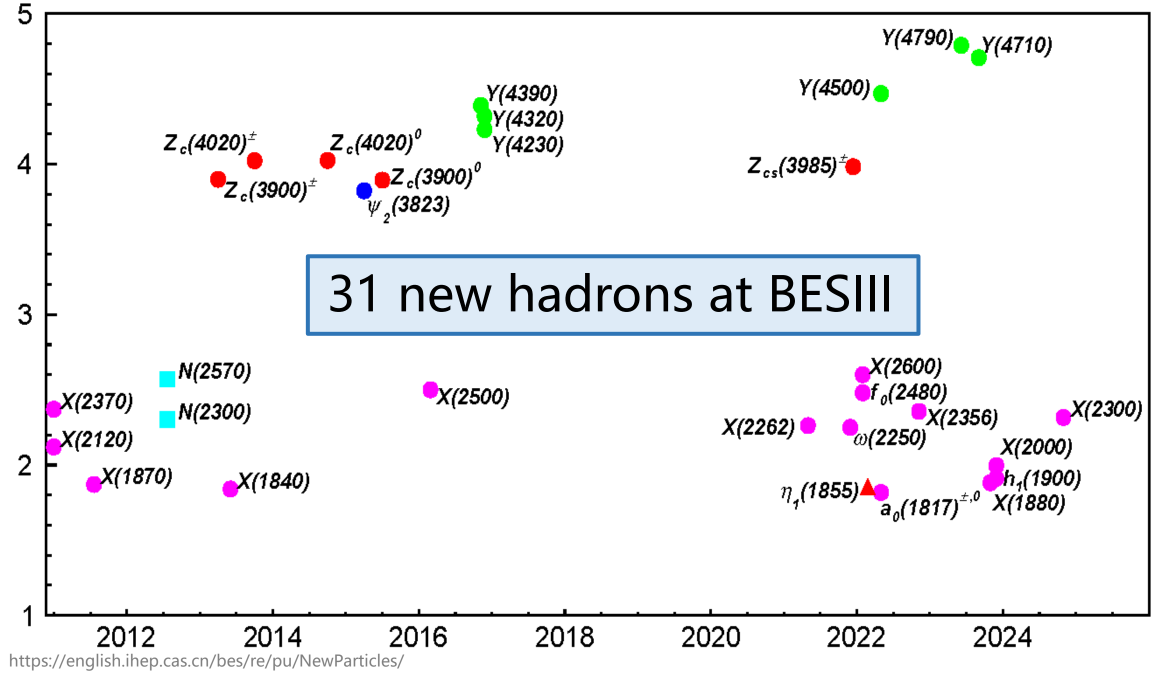

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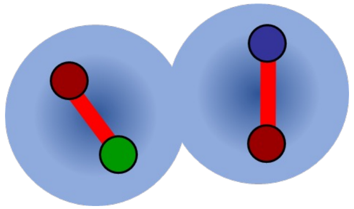
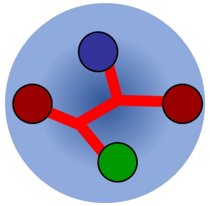
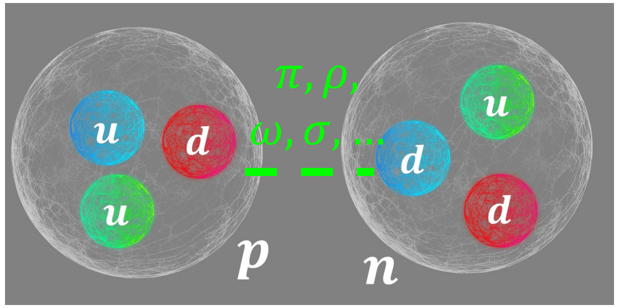

Molecular

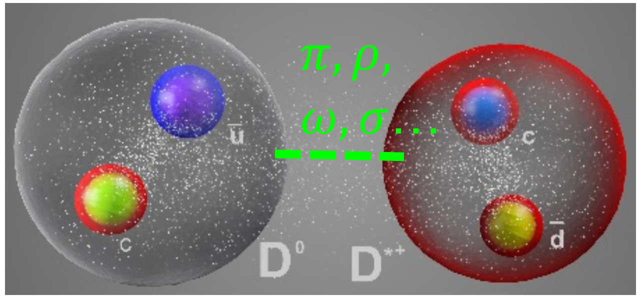
- **More and more states composed of at least 4 or 5 (anti)quarks**
- Multiquark states: Compact, or molecular or others?


Compact

Three super "stars"

- Hadonic molecule: deuteron as a typical example





- Great interest in DD^* and $\bar{D}D^*/D\bar{D}^*$ molecular states

	Quark contents	$I^G(J^{PC})$	Threshold	ΔM [MeV]	Γ [MeV]
$X(3872)$	$q\bar{q}c\bar{c}/c\bar{c}$	$0^+(1^{++})$	$D^0\bar{D}^{0*}$	$0.0068^{+0.1655}_{-0.17000}$	$0.380^{+0.412}_{-0.322}$
	The 1st charmonium-like state			BESIII:2023hml	
$Z_c(3900)$	$q\bar{q}c\bar{c}$	$1^+(1^{+-})$	$D\bar{D}^*$	11.3 ± 2.6	28.4 ± 2.6
	The 1st manifestly exotic charmonium-like state			PDG	
$T_{cc}(3875)$	$\bar{q}\bar{q}cc$	?	$D^{*+}D^0$	$-0.360^{+0.040}_{-0.040}$	$0.048^{+0.002}_{-0.014}$
	The 1st open double charm tetraquark state			LHCb:2021auc	

- S-wave molecular candidates

Experimental progress

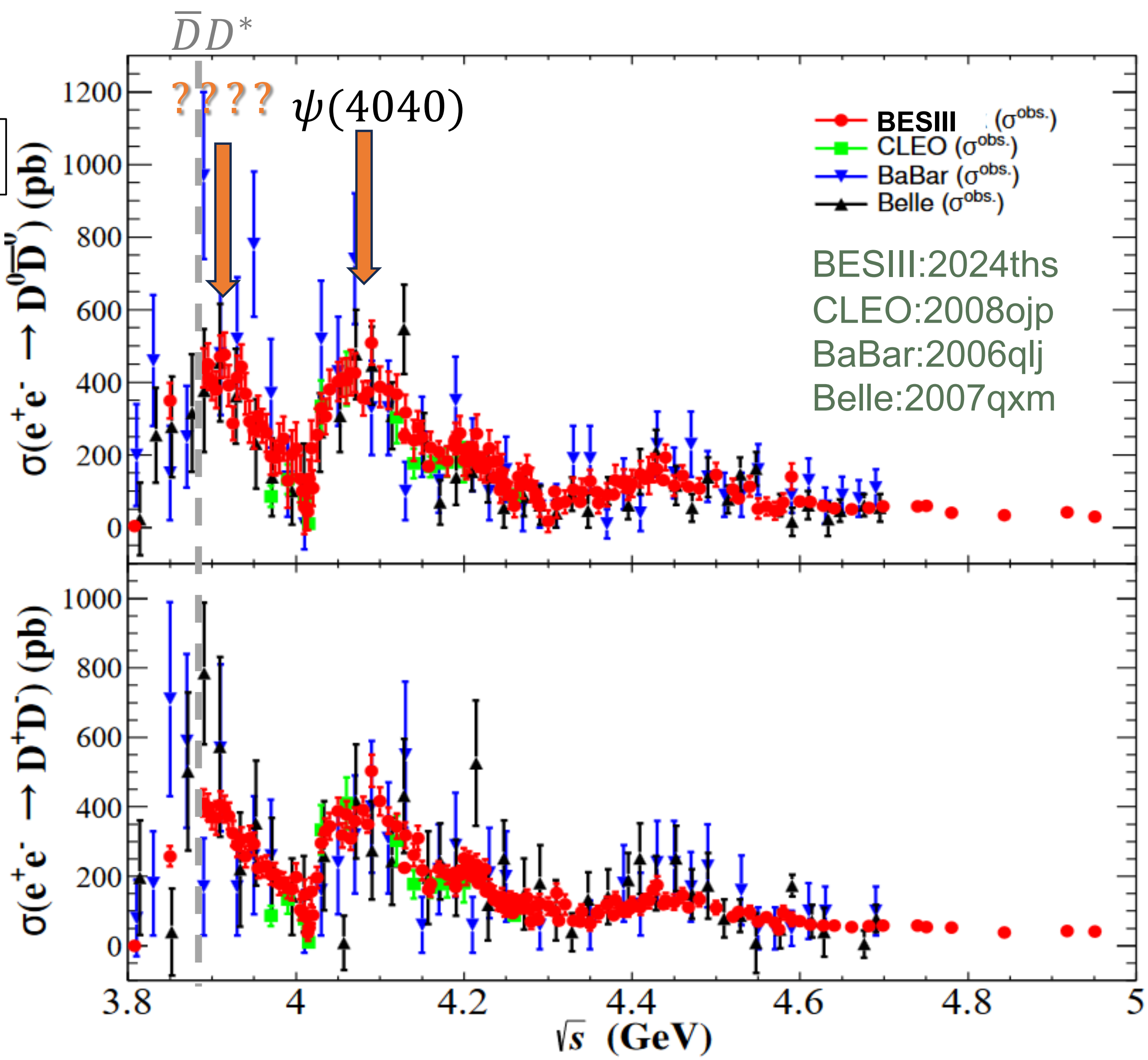
- An enhancement close to 3.9 GeV
  - Breit-Wigner fit: $M + \Gamma/2$ (MeV)

$$(3872.5 \pm 14.2 \pm 3.0) + (89.9 \pm 7.5 \pm 3.5)i$$

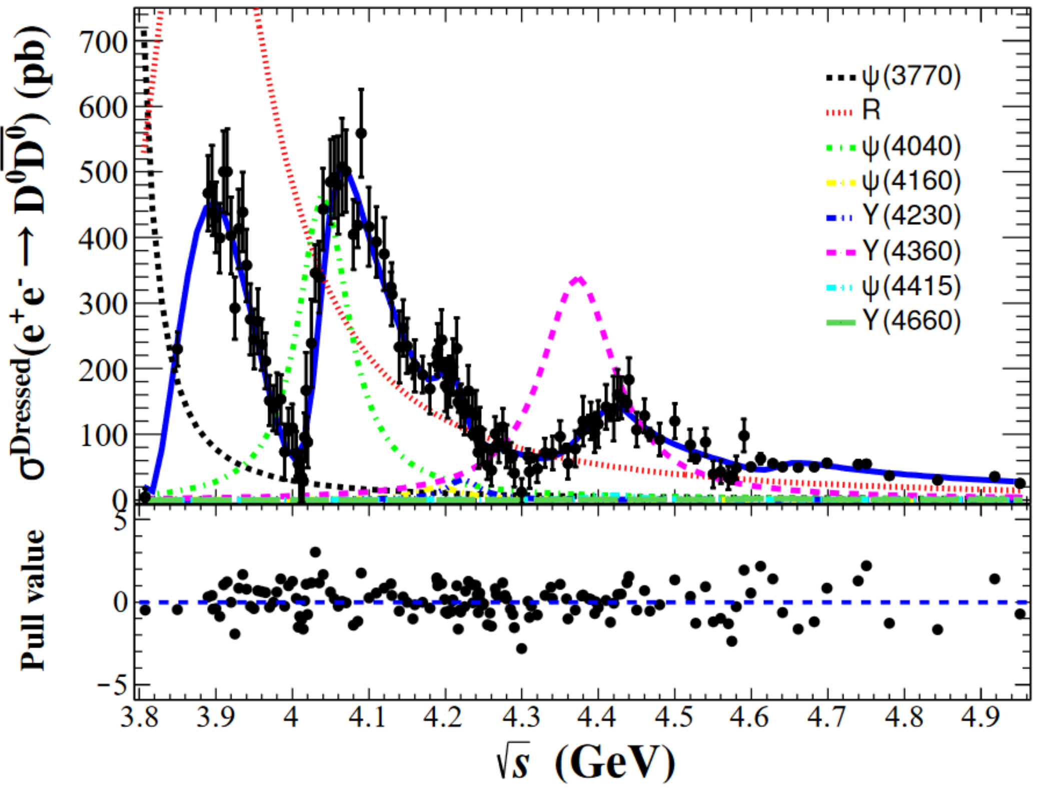
- New $D^*\bar{D}$ molecular states?
  - Close to $D^*\bar{D}$ threshold
  - Quantum number $J^{PC} = 1^{--}$, from virtual photon
  - P-wave $D^*\bar{D}$ state?

- Cornell model: enhancement at 3.9 GeV
  Eichten:1978tg, Eichten:1979ms
- Belle and BaBar: $G(3900)$
  - Fit with a Gaussian function
  - Not regarded as a resonance
  BaBar:2006qlj, Belle:2007qxm

- More precise data from BESIII
  BESIII:2024ths



Breit-Wigner fit from BESIII



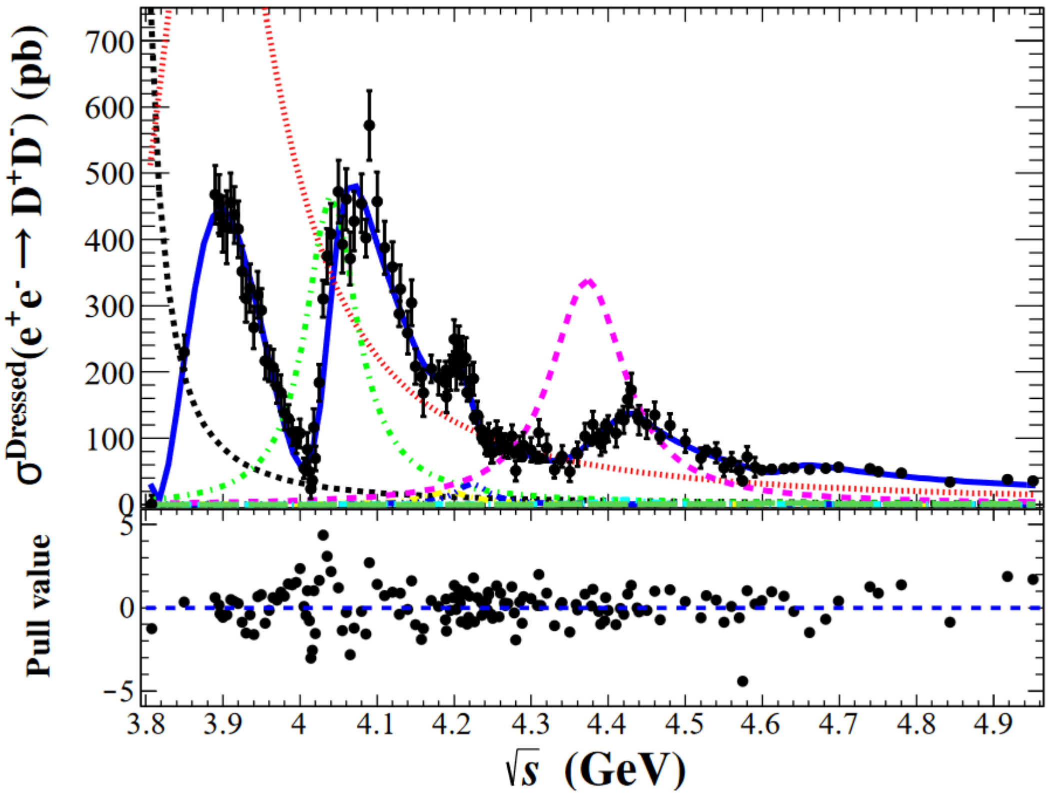


TABLE I. Fit results of the Born cross section, where the first uncertainties are the statistical and the second are systematic and S denotes the significance.

BESIII:2024ths

$e^+e^- \to D\bar{D}$								
Resonance	$\psi(3770)$	R	$\psi(4040)$	$\psi(4160)$	$Y(4230)$	$Y(4360)$	$\psi(4415)$	$Y(4660)$
Mass (MeV/c^2)	3773.7 (fixed)	3872.5±14.2±3.0	4039 (fixed)	4191 (fixed)	4222.5 (fixed)	4374 (fixed)	4421 (fixed)	4630 (fixed)
Width (MeV/c^2)	87.6 (fixed)	179.7±14.1±7.0	80 (fixed)	70 (fixed)	48 (fixed)	118 (fixed)	62 (fixed)	72 (fixed)
$\Gamma_{ee}\mathcal{B}$ (eV)	95-106	202-292	41-44	1-2	1-2	50-144	0-2	0-1
S(σ)	10	> 20	13	7	11	11	4	8
χ^2/d.o.f = 346/275				p-value = 0.002				

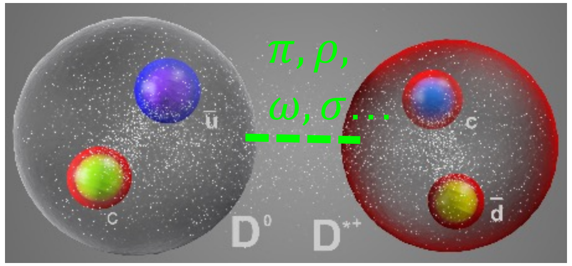
Contents

- **Textbook:** Why do P-wave systems favor resonance formation?

- **Model:** A unified framework explaining $X(3872)$, $Z_c(3900)$, $T_{cc}(3875)$ and $G(3900)$

- **Data:** Could the current data pin down the existence of $G(3900)$?

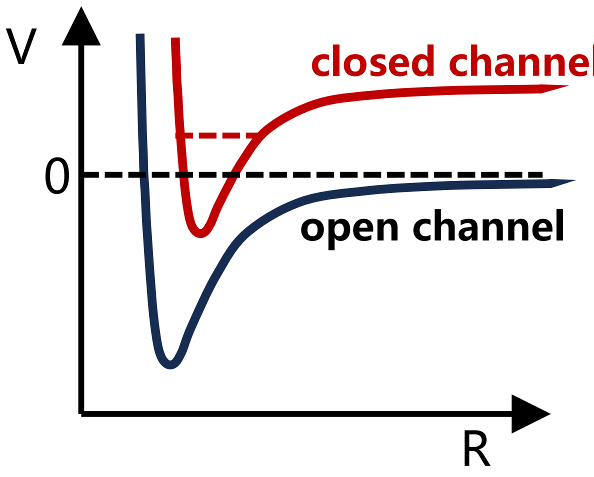
- **Predictions:** Opportunity in STCF



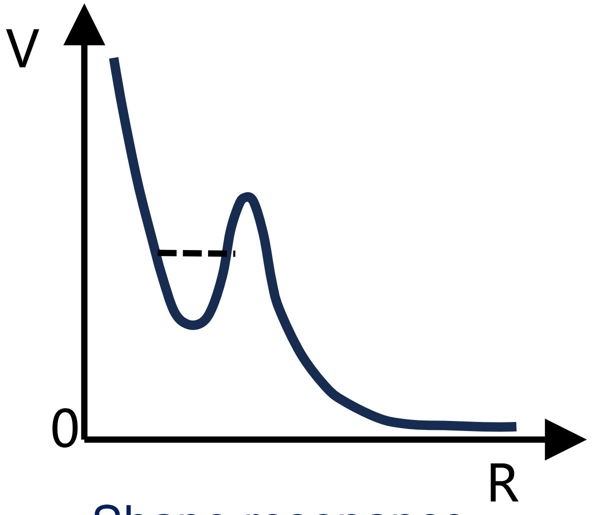
P-wave

P-wave resonance

Resonances



Feshbach resonance



Shape resonance

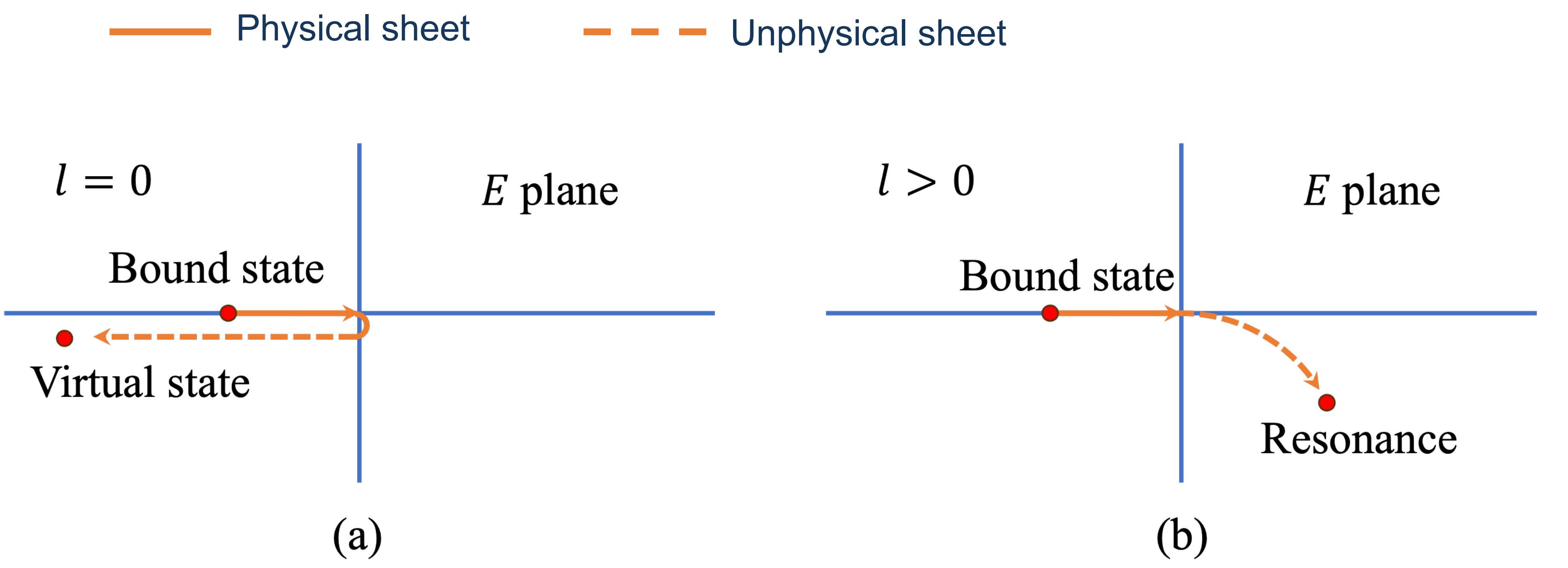
…

- Resonance: state with finite lifetime
- Feshbach resonance
  - ► E.g. P_c states could be bound states of $\Sigma_c\bar{D}^{(*)}$
  - ► Considering the $J/\psi p$ channel: resonance
- Shape resonance
  - ► Barrier
  - ► S-wave: $\Leftarrow$ potential with barrier, usually from nontrivial mechanism
  - ► Higher partial wave: centrifugal barrier
- Feshbach or shape? depending on schemes

From bound state to…

- For attractive potential giving bound state
  - ► $V \to \lambda V$, with $\lambda > 0$, decrease λ

Taylor, scattering theory textbook sec. 13-b P245

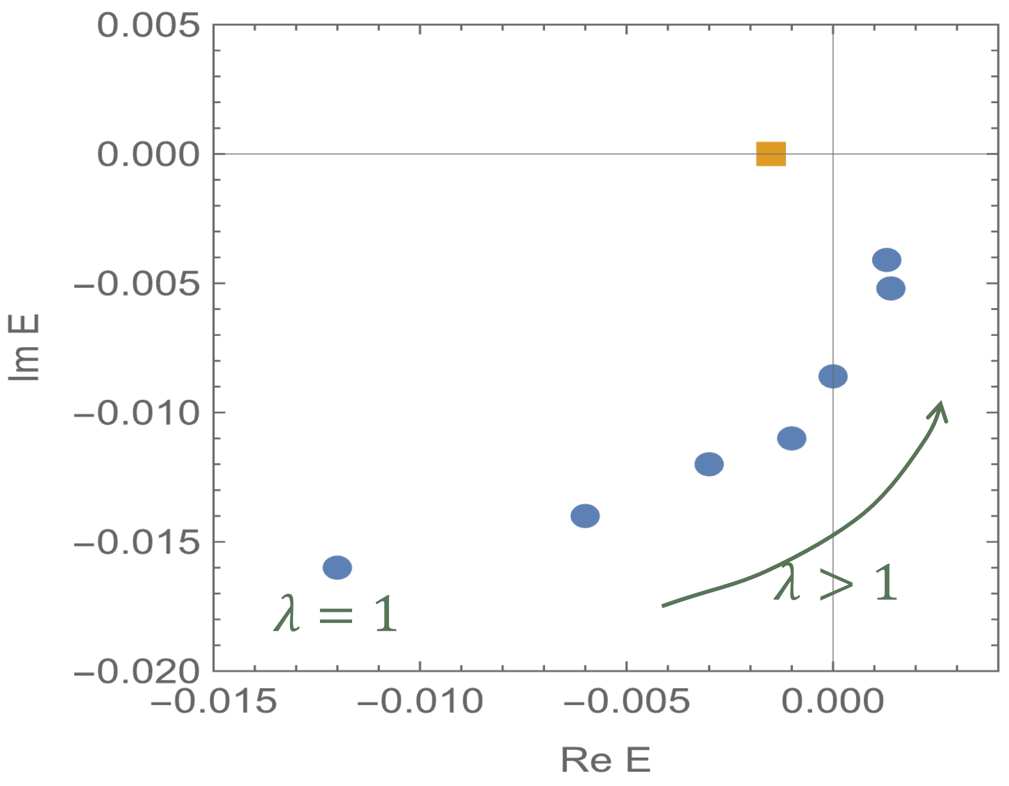


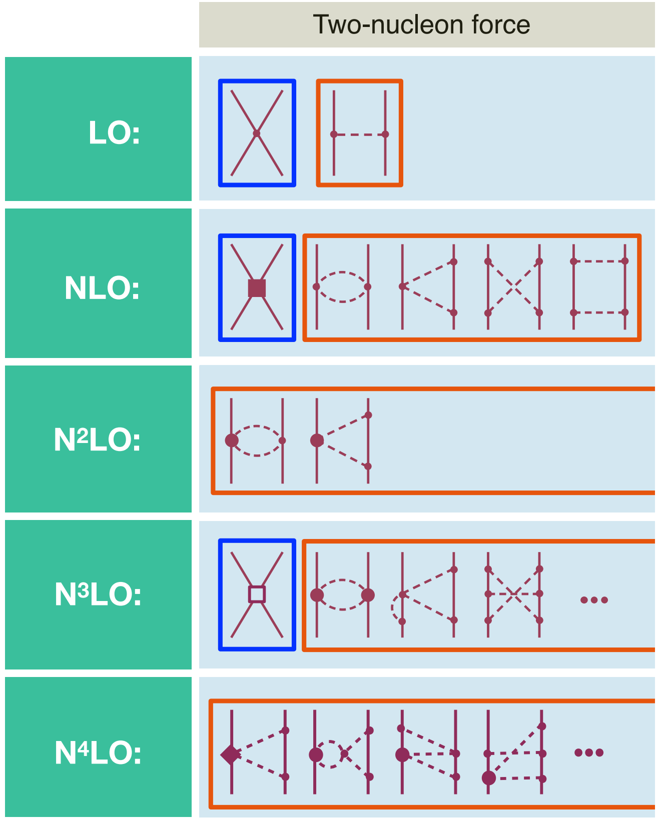
Potential barrier: centrifugal barrier for l>0

- Resonance: very common in $l > 0$ system
  - ► attractive systems that are not strong enough to form a bound state.

$^3P_0$ NN resonance pole

- $^3P_0$ NN resonance pole: $-0.012 - i0.016$
  - Using high precision chiral nuclear force
  - Seldom investigated
  - Hard to detect: no lower coupled-channel





P. Reinert, H. Krebs, and E. Epelbaum, Eur. Phys. J. A **54**, 86 (2018).

Dynamical model

Dynamical model

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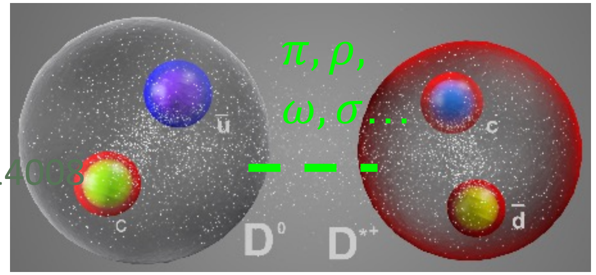
- Interactions: exchange: $\pi, \eta, \rho, \omega, \sigma$
  - ▶ Success in high precision nuclear force
  - ▶ S-wave and P-wave interactions are derived from the same Lagrangians
  - ▶ G-parity rules (particle-particle ⇔ particle-antiparticle)

Machleidt:2000ge



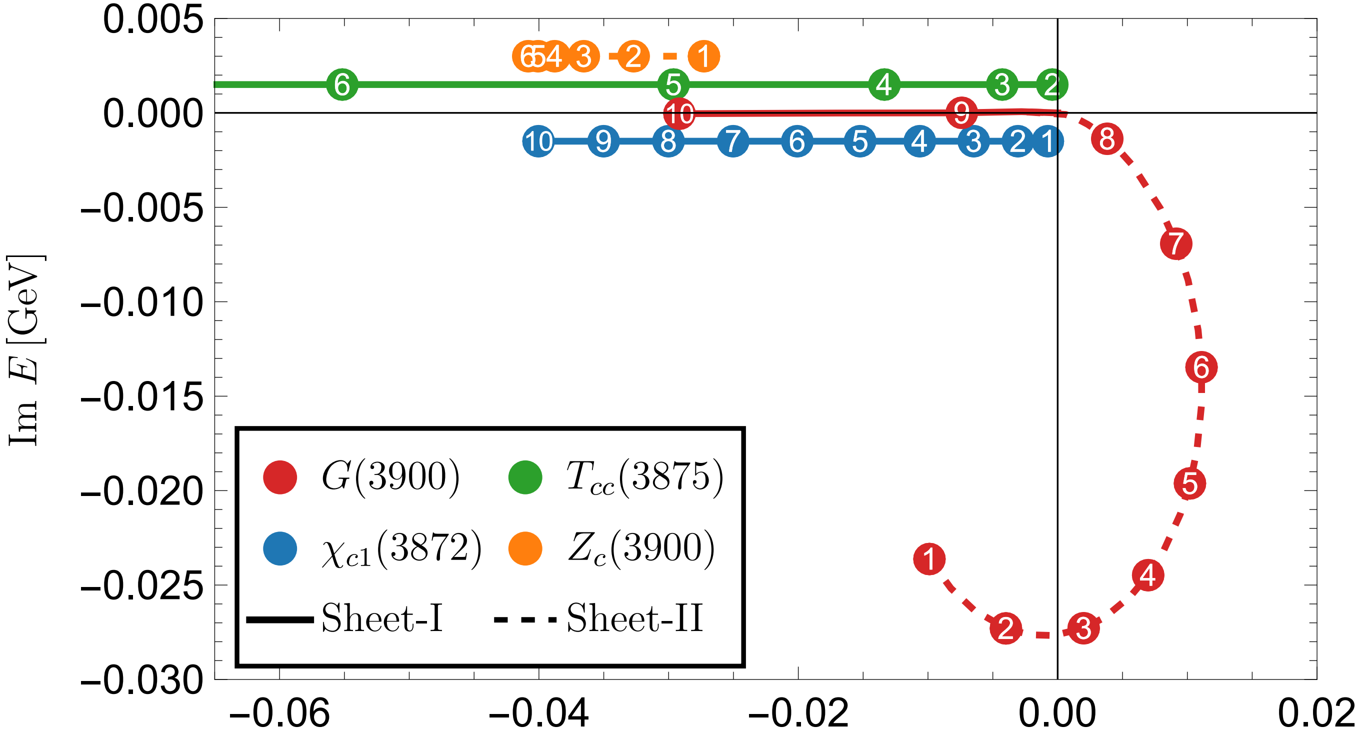
- Predicting the **molecular**-type T_{cc} N. Li, Z-F. Sun, X. Liu, S-L Zhu, *PRD*88(2013), 114008
  - ▶ More consistent with the exp. than compact scheme
  - ▶ 3 unknown coupling (2 for vector and 1 for scalar)

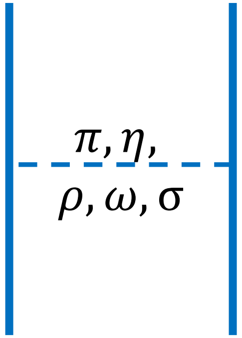
    vector meson dominance, Σ-model, light-cone sum rule, LQCD
  - ▶ OBE with Para. produce $X(3872)$ well $\bar{D}D^* \Rightarrow DD^*$
  - ▶ Results: a DD^* bound state with binding energy about 300 keV

- Regulator and cutoff

$$V(\boldsymbol{p}', \boldsymbol{p}) \rightarrow V(\boldsymbol{p}', \boldsymbol{p}) \frac{\Lambda^2}{\Lambda^2 + p^2} \frac{\Lambda^2}{\Lambda^2 + p'^2}, \text{ or } V(\boldsymbol{p}', \boldsymbol{p}) \rightarrow V(\boldsymbol{p}', \boldsymbol{p}) \left(\frac{\Lambda^2 - \mathrm{u}^2}{\Lambda^2 + q^2}\right)^2, \text{ or other options}$$

Scheme-I

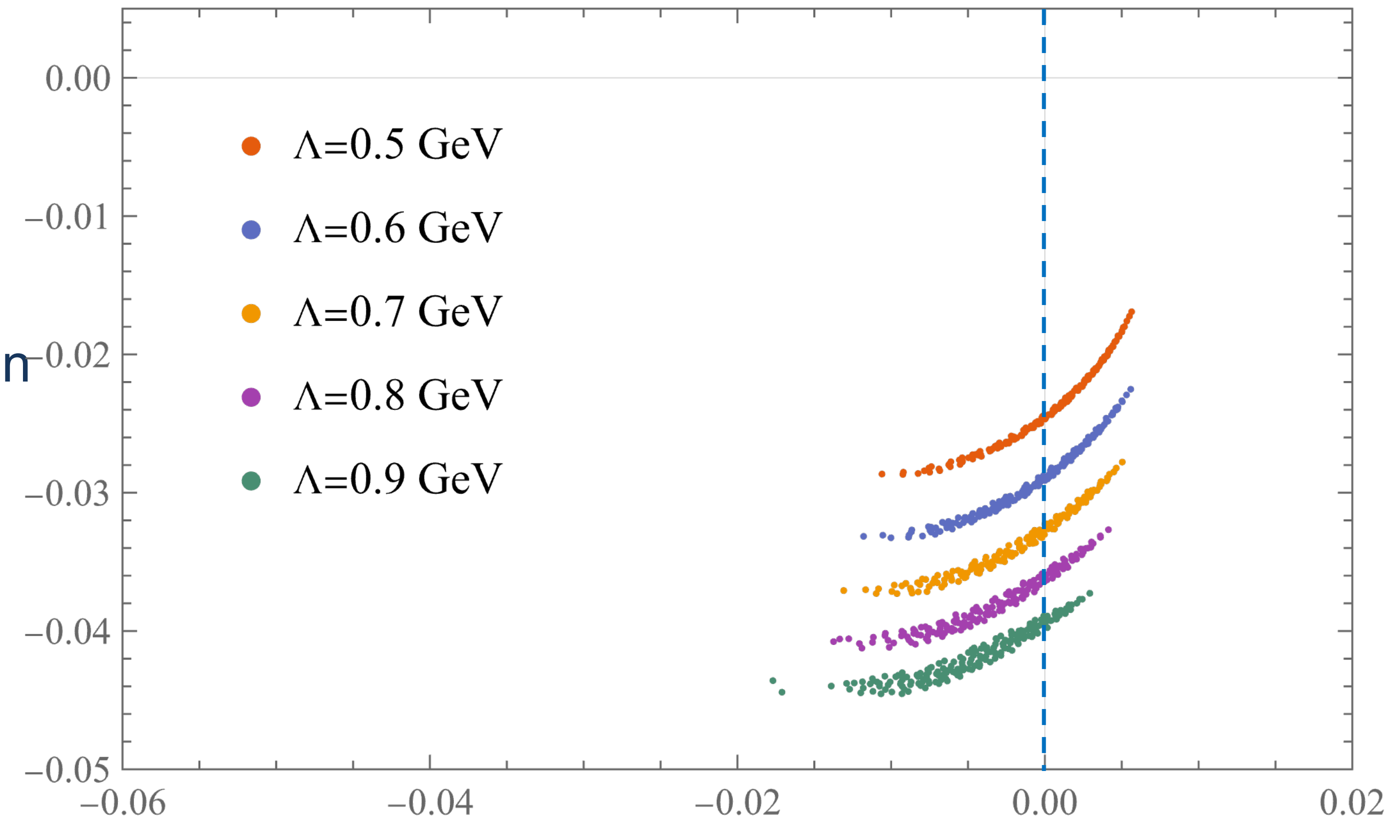




- Fix the coupling via pheno. models
- Vary Λ from 0.4 to 1.3 GeV (①−⑩)
- Constrained by $X(3872)$, $Z_c(3900)$, and $T_{cc}(3875)$
- $X(3872)$ and $T_{cc}(3875)$: bound states
- $Z_c(3900)$: virtual state
- $G(3900)$: ${}^3P_1$ $D^*\bar{D}/\bar{D}^*D$ resonance

Scheme-II

- Refit 3 unknown coupling parameters
  - ▶ Bound state $X(3872)$: $-4\sim 0$ MeV
  - ▶ Bound state $T_{cc}(3875)$: $-4\sim 0$ MeV
  - ▶ Virtual state $Z_c(3900)$: $-35\sim -15$ MeV
- For each cutoff, random pole positions in above ranges
- Numerically obtain three coupling constants
- Calculate the 1^{--}, ${}^3P_1$ $D^*\bar{D}/\bar{D}^*D$ poles



- Existence of $G(3900)$: robust
  - ▶ Different regulators
  - ▶ Vector-vector channel
  - ▶ 3-body effect

V_{OBE} constrained by $X(3872)$, $Z_c(3900)$, and $T_{cc}(3875)$ give rise to $G(3900)$

What can the data tell us?

K-matrix formalism fit

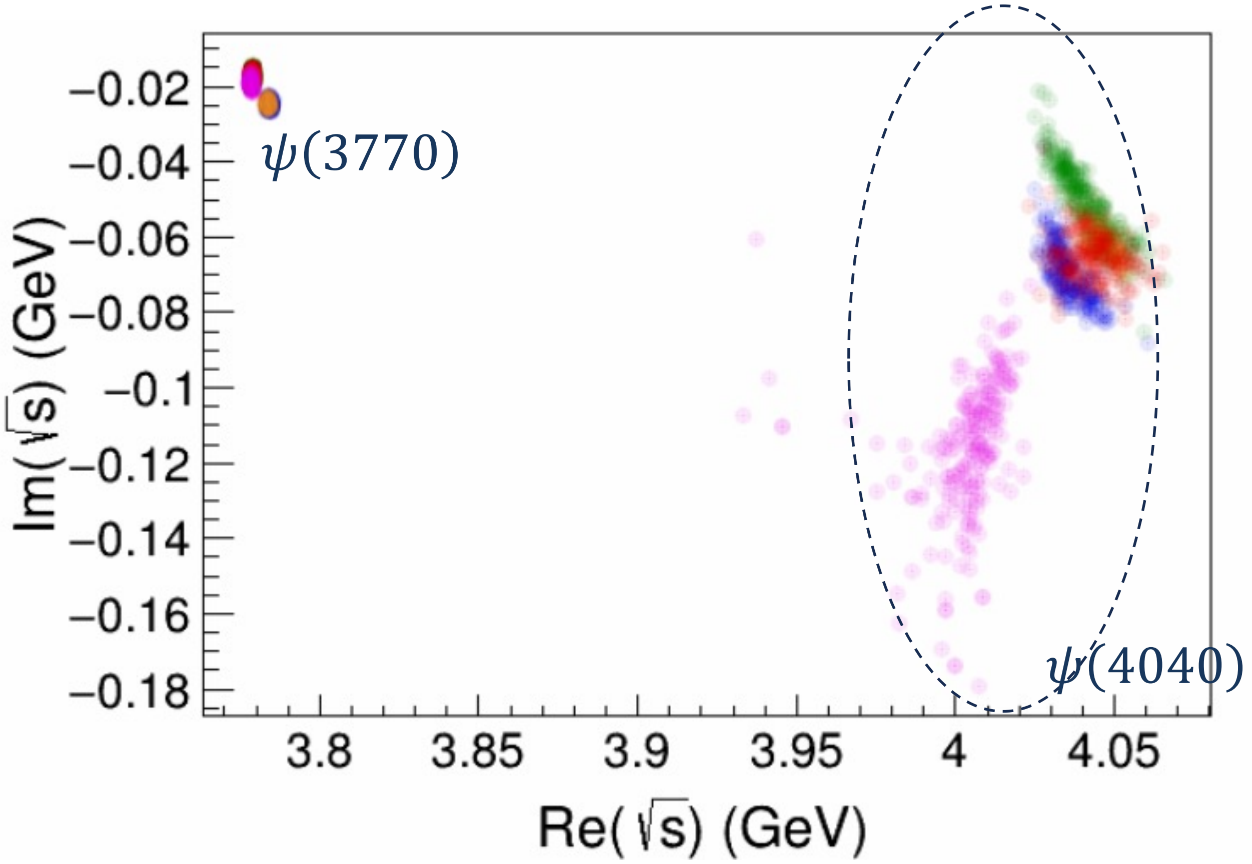
- Data: $e^+e^- \to D\bar{D}$, $\bar{D}D^*$, $D^*\bar{D}^*$ and hadrons
- K-matrix formalism

$$\mathcal{M}^{-1} = K^{-1} + C, \quad \text{Im } C(s) = -2\rho,$$

$$C(s) = C(s_0) - \frac{s - s_0}{\pi} \int_{s_0}^{\infty} ds' \frac{\text{Im}C(s)}{(s' - s)(s' - s_0)}$$

$$K = \frac{g^2}{m_R^2 - s} + f,$$

- 5 similar models (different colors in right fig)
- $G(3900)$ pole is not found



N. Hüsken, R. F. Lebed, R. E. Mitchell, E. S. Swanson, Y.-Q. Wang, and C.-Z. Yuan, Phys. Rev. D **109**, 114010 (2024).

Our improvments

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	Defects in [N. Hüsken et al]	Possible consequence	Our improvments
1	when $k^2 < 0$ In fit: Set $k = 0$ In searching pole: keep k	Not consistent; Θ function prevent analytial continuation	Always keep k
2	Using S-wave Chew-Mandelstam func. for P-wave system	Subtraction point dependence	Substraction-independence
3	No searching for pole below threshod	Might overlook some near threshold pole	Search pole below and above threshold
4	Gaussian regulator: $\exp[-z^2]$	Amplification for very negative k^2	Blatt-Weisskopf: $1/(1+z^2)$

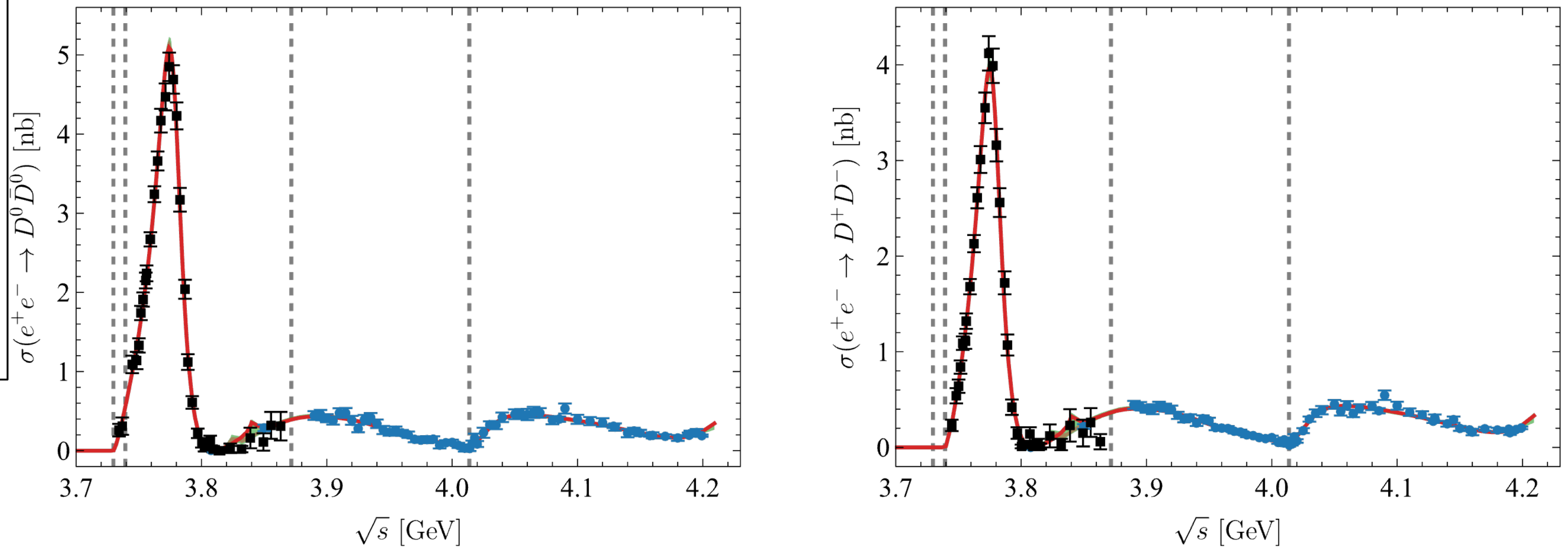
Closer to K-matrix formalisim in PDG

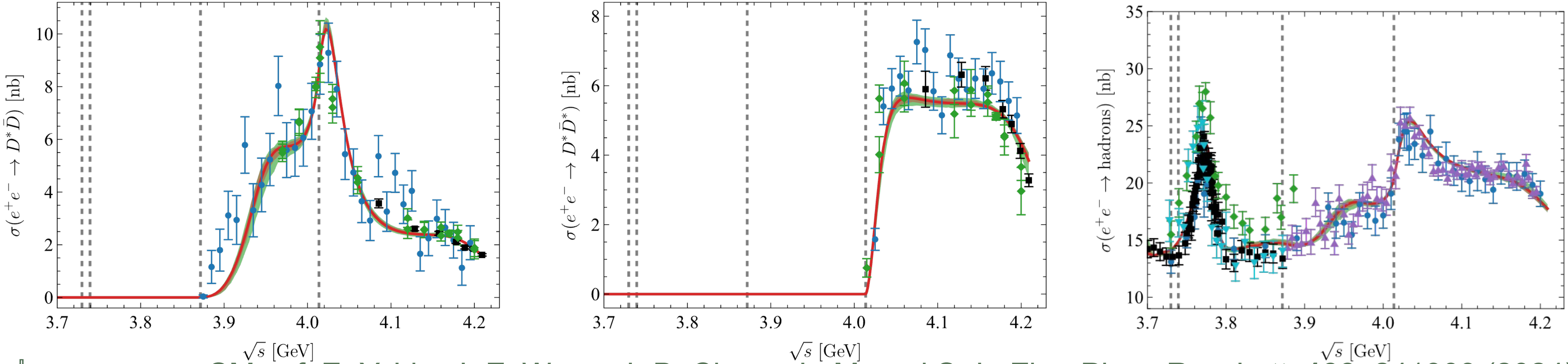
More details:

Supplemental Materials of Z.-Y. Lin, J.-Z. Wang, J.-B. Cheng, L. M, and S.-L. Zhu, Phys. Rev. Lett. **133**, 241903 (2024).

Our fit in the K-matrix formalism

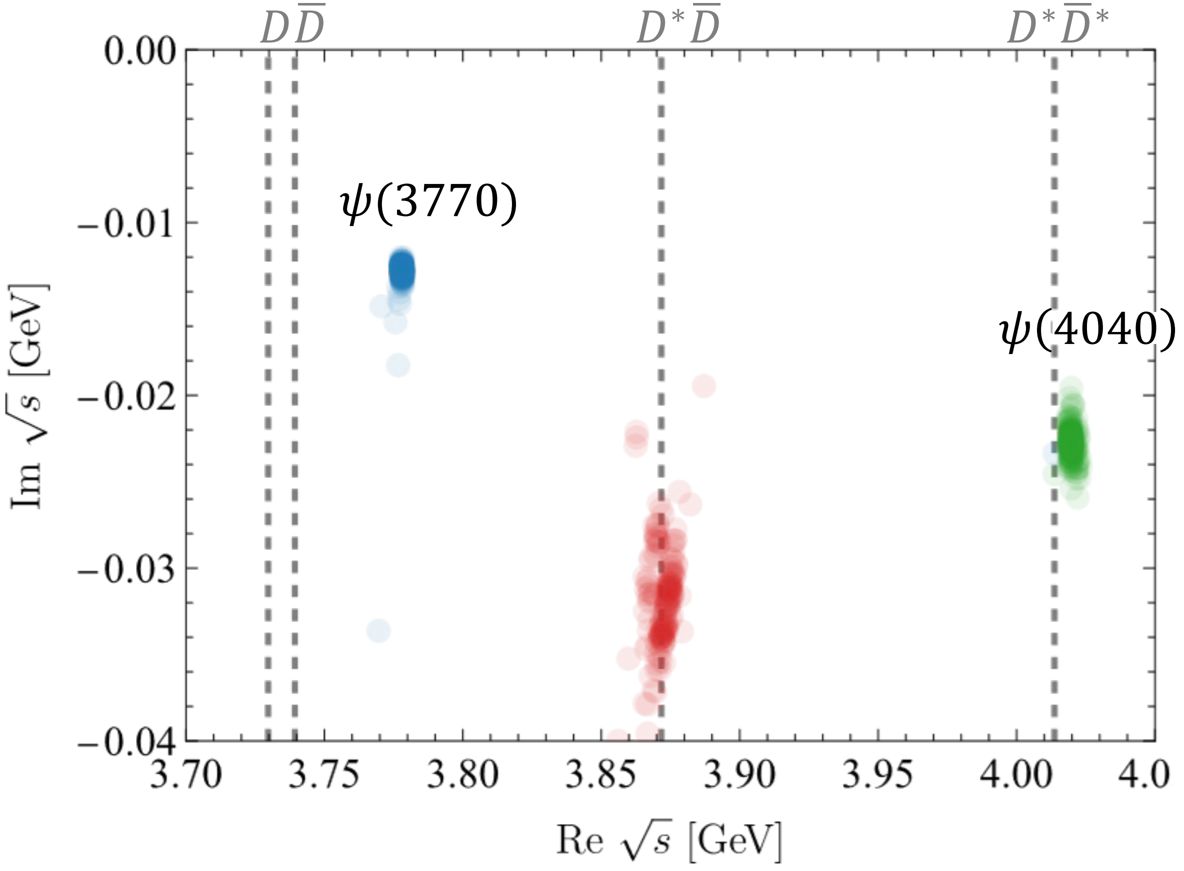
- Repair the three defects
- Fit the same data
- Our refit results
  - ▶ $\chi^2/\mathrm{dof} = 2.07$





SMs of Z.-Y. Lin, J.-Z. Wang, J.-B. Cheng, L. M, and S.-L. Zhu, Phys. Rev. Lett. **133**, 241903 (2024).

Our fit in the K-matrix formalism



Pole [MeV]

$$3869.2(67) - i29.0(52)$$

SMs of Z.-Y. Lin, J.-Z. Wang, J.-B. Cheng, L. M, and S.-L. Zhu, Phys. Rev. Lett. **133**, 241903 (2024).

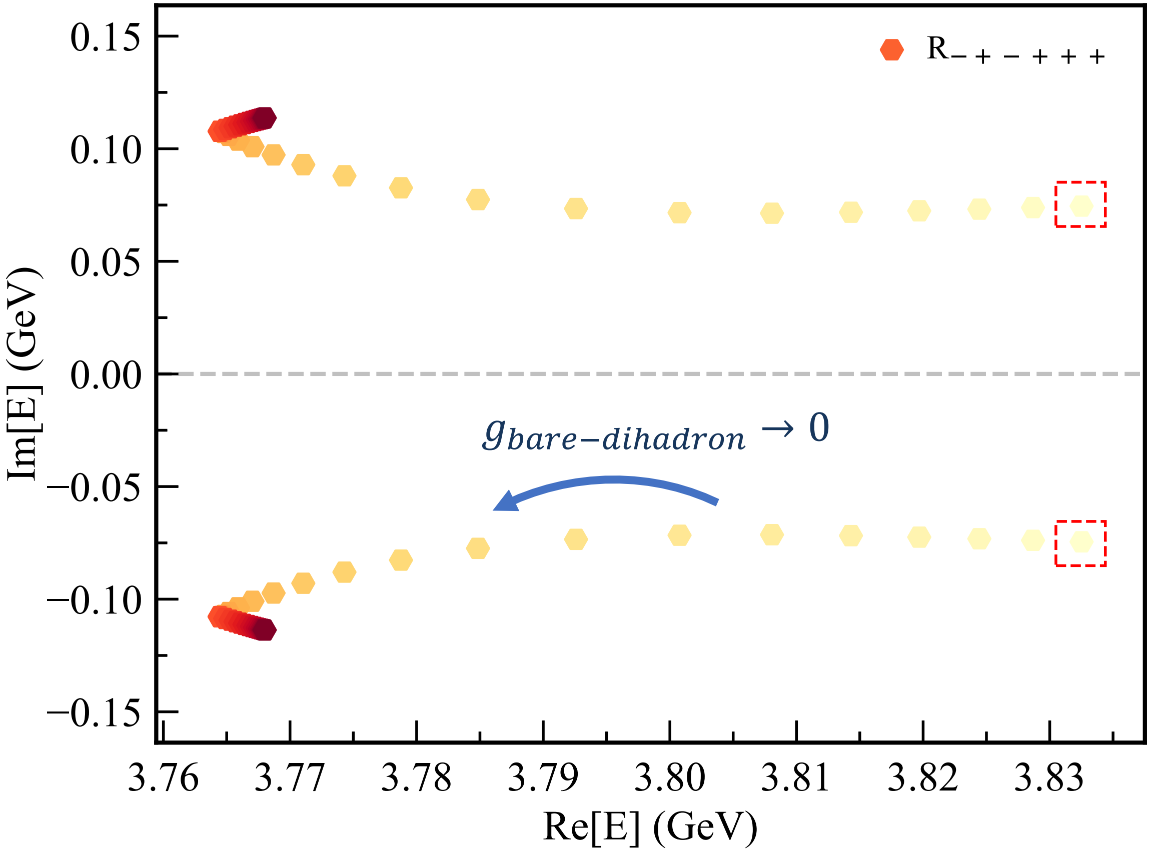
LSE formalism

- Lippmann-Schwinger equation formalism
- Heavy quark spin symmetry
- SU(3) flavor symmetry
- New data
  - ▶ $e^+e^- \to D\bar{D}, D\bar{D}^*, D^*\bar{D}^*, D_s\bar{D}_s, D_s\bar{D}_s^*, D_s^*\bar{D}_s{}^*$

Two solutions [MeV]

I $3832.6^{+0.9}_{-0.8} - 74.5^{+0.7}_{-2.2}\,i$

II $3883.9^{+0.4}_{-0.5} - 46.5^{+1.2}_{-1.2}\,i$



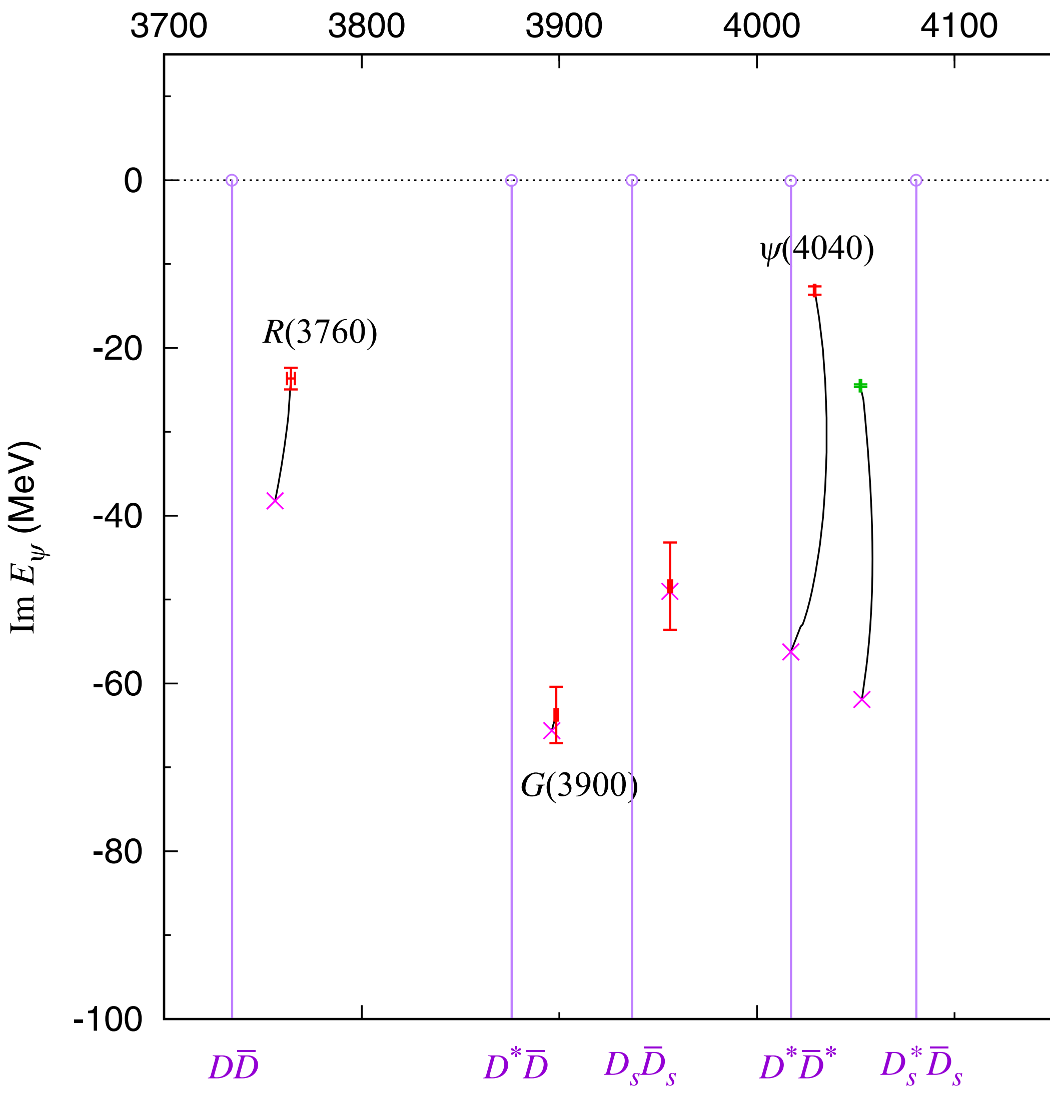
- Set $g_{bare-dihadron} = 0$
  - ▶ $G(3900)$: dynamically generated state

Q. Ye, Z. Zhang, M.-L. Du, U.-G. Meißner, P.-Y. Niu, and Q. Wang, The resonance parameters of the vector charmonium-like state G(3900), Phys. Rev. D **112**, 016015 (2025).

Global fit of $e^+e^- \to c\bar{c}$ processes

- $e^+e^- \to c\bar{c}$ processes in $\sqrt{s}$=3.75-4.7 GeV
- Coupled-channel global fit
- $G(3900)$: DD^* molecular state

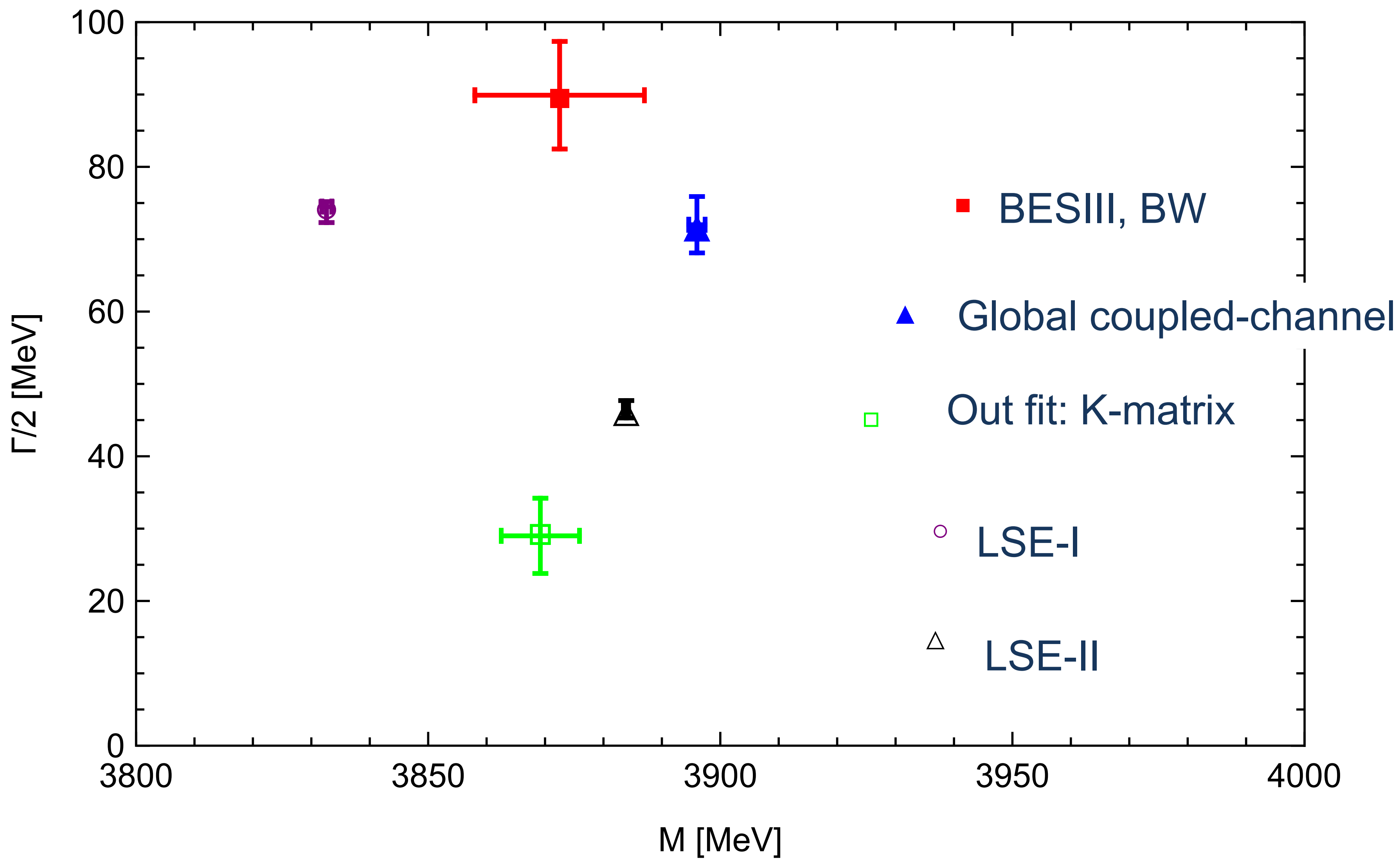
This work		PDG(ψ) [4], BESIII [16,38,88]		
M (MeV)	Γ (MeV)	M (MeV)	Γ (MeV)	
3764.2 ± 2.0	47.3 ± 2.6	3751.9 ± 3.8	32.8 ± 5.8	$^{r}D\bar{D}$
3780.2 ± 1.2	29.9 ± 2.3	3778.1 ± 0.7	27.5 ± 0.9	$\psi(3770)$
3898.4 ± 0.9	127.5 ± 6.7	3872.5 ± 14.2	179.7 ± 14.1	$^{r}D^*\bar{D}$
3956.1 ± 1.0	96.8 ± 10.4	$\cdots$	$\cdots$	$^{r}D_s\bar{D}_s$
4029.2 ± 0.4	26.3 ± 1.0	4039 ± 1	80 ± 10	$\psi(4040)$
4052.4 ± 0.4	49.0 ± 0.3	$\cdots$	$\cdots$	$^{v}D_s^*\bar{D}_s$
4192.2 ± 2.2	129.3 ± 4.2	4191 ± 5	70 ± 10	$\psi(4160)$
4216.2 ± 0.5	40.3 ± 1.0	$\cdots$	$\cdots$	$^{v}D_s^*\bar{D}_s^*$
4229.9 ± 0.9	46.4 ± 2.6	4222.5 ± 2.4	48 ± 8	$\psi(4230)$
4308.1 ± 2.2	138.2 ± 4.4	4298 ± 12	127 ± 17	$Y(4320)$
4346.2 ± 3.8	122.8 ± 6.7	4374 ± 7	118 ± 12	$\psi(4360)$



Switch off the bare-dihdaron coupling

S. X. Nakamura, X.-H. Li, H.-P. Peng, Z.-T. Sun, and X.-R. Zhou, Phys. Rev. D **112**, 054027 (2025).

Comparisions



We need:

- More Data
- Careful investigation of sys. uncertainties

Q. Ye, Z. Zhang, M.-L. Du, U.-G. Meißner, P.-Y. Niu, and Q. Wang,, Phys. Rev. D **112**, 016015 (2025).

Predictions

Predictions of meson-exchange model

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		$D\bar{D}^*, C=+$		$D\bar{D}^*, C=-$		DD^*	
		$I=0$	$I=1$	$I=0$	$I=1$	$I=0$	$I=1$
$\Lambda = 0.5$GeV	$1^+(^3S_1)$	-3.1^B, $\chi_{c1}(3872)$	-	-1.60^B ②	-35.6^V, $Z_c(3900)$	-0.41^B, $T_{cc}(3875)$	-
	$0^-(^3P_0)$	$-1.5-14.5i$ ③	-	-	-	$-9.6-9.7i$ ④	-
	$1^-(^3P_1)$	-	-	$-4.0-27.3i$, $G(3900)$ ①	-	$-31.7-70.6i$	-
	$2^-(^3P_2)$	$-42.6-39.4i$	-	$-21.3-50.7i$	-	$-37.8-40.9i$	-

B: bound state, V: virtual state

① Precision measurement of $e^+e^- \to \bar{D}D^*$ close to threshold

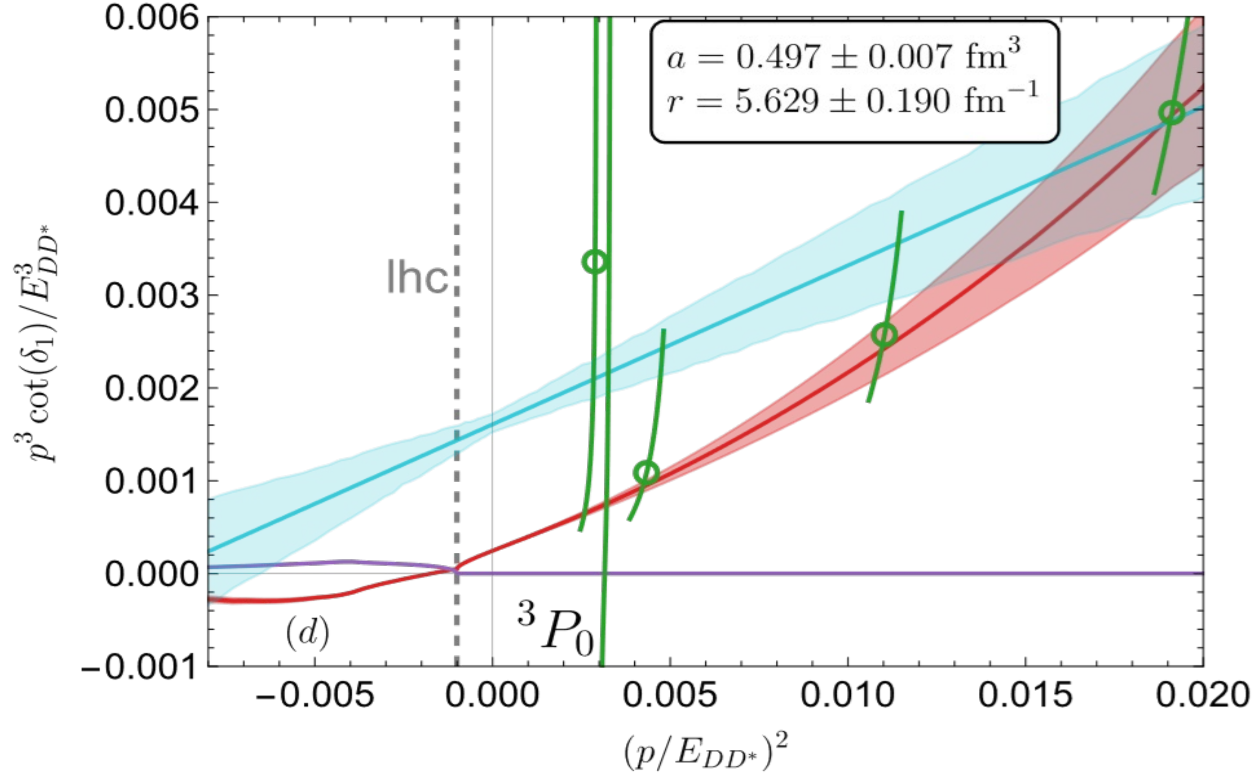
② Hidden charm final state: $\eta_c\omega$, $J/\psi\eta$, $J/\psi\pi\pi$

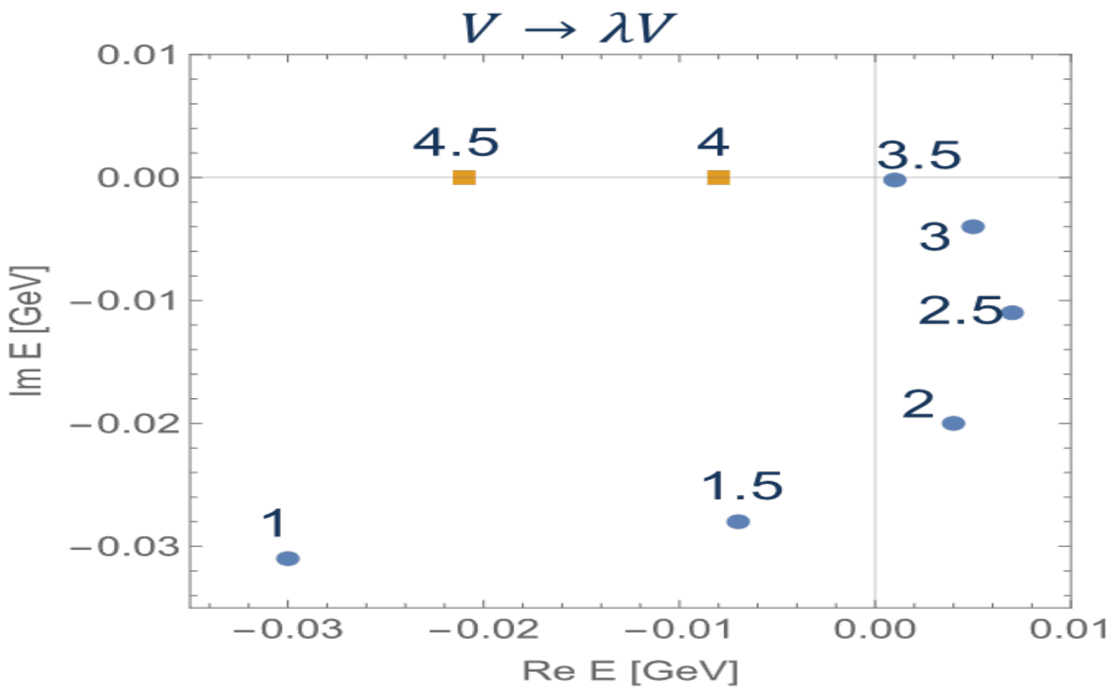
③ Hidden charm final state: $J/\psi\omega$, $\eta_c\pi\pi$, $\chi_{c1}\pi\pi$

④ Final state $DD\pi$

Hints from LQCD data

- Lattice QCD: ${}^3P_0\ DD^*$ resonance pole: $-0.030 - i0.031$





L. Meng, V. Baru, E. Epelbaum, A. A. Filin, and A. M. Gasparyan, PRD**109**, L071506 (2024).
M. Padmanath and S. Prelovsek, Phys. Rev. Lett. **129**, 032002 (2022).

- Hadron Spectrum Collaboration: attractive interaction in ${}^3P_0\ DD^*$ channel

T. Whyte, D. J. Wilson, and C. E. Thomas, Phys. Rev. D **111**, 034511 (2025).

Summary

- **Exp. :** An enhancement close to 3.9 GeV, $G(3900)$
- **Textbook:** P-wave system
  - ▶ Centrifugal barrier
  - ▶ Resonance: attractive systems that are not strong enough to form a bound state.
- **Model:** the meson-exchange model, $\bar{D}D^*/D\bar{D}^*$ $^3P_1$ resonance
  - ▶ Unified framework: $X(3872)$, $Z_c(3900)$, $T_{cc}(3875)$ and $G(3900)$
  - ▶ Same Lagrangians for S-wave and P-wave and for particle and antiparticle
- **Predictions:**
  - ▶ $[\bar{D}D^*]^{I=0}_{C=-1}$, $^3S_1$; $[\bar{D}D^*]^{I=0}_{C=+1}$, $^3P_0$; $[DD^*]^{I=0}$, $^3P_0$
  - ▶ Some lattice QCD hints
- **Data:**
  - ▶ 3 independent fittings favor the existence of $G(3900)$ pole
  - ▶ Inconsistent pole positions
- **STCF:** opportunities for detecting the $G(3900)$ and its partners

Thanks for your attentions!

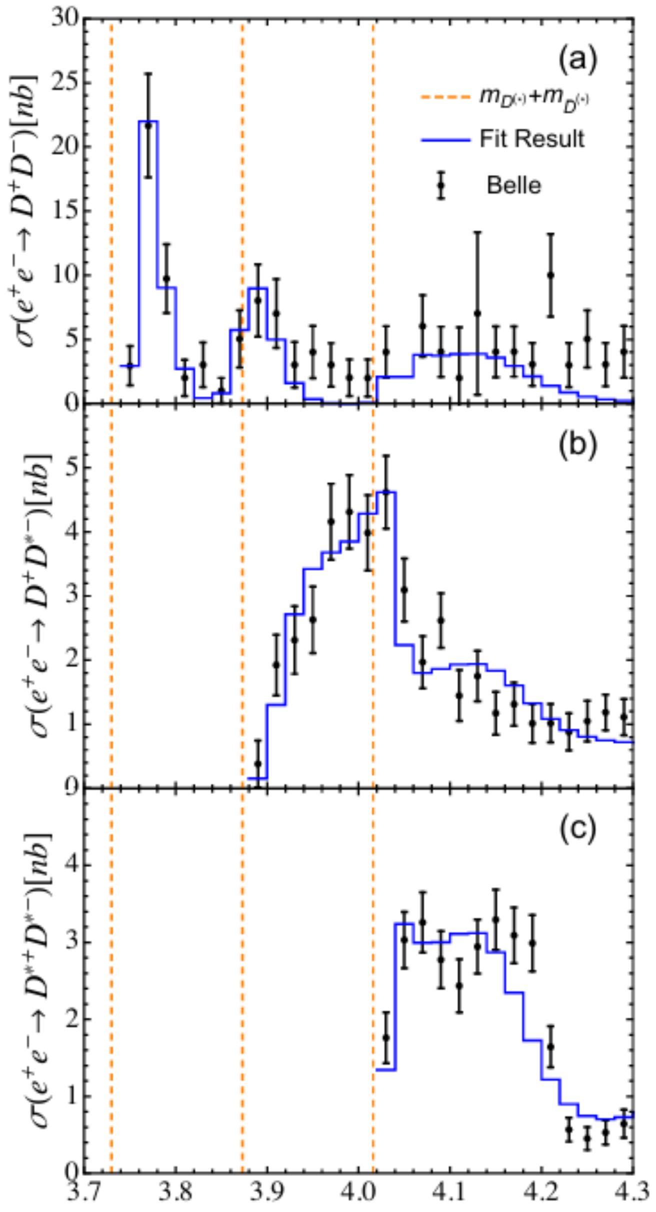
Backup

LSE formalism

- Lippmann-Schwinger equation formalism
- Coupled channel effect
- S-matrix unitarity and analyticity
- Belle data
- A resonance close to $\bar{D}D^*$ threshold

Sheet	Poles (GeV)	$\lvert g_{D\bar{D}}\rvert$	$\lvert g_{D\bar{D}^*}\rvert$	$\lvert g_{D^*\bar{D}^*_{s=0}}\rvert$	$\lvert g_{D^*\bar{D}^*_{s=2}}\rvert$
II	$3.764 \pm i0.006$	13.53	9.48	5.88	16.78
III	$3.879 \pm i0.035$	4.40	10.96	7.63	18.15
IV	$4.034 \pm i0.014$	2.90	2.23	12.52	12.85

$$
\begin{array}{llllll}
\mathrm{I} & \mathrm{Im}\, q_{D\bar{D}} > 0, & \mathrm{Im}\, q_{D\bar{D}^*} > 0, & \mathrm{Im}\, q_{D^*\bar{D}^*} > 0, & \text{for } E < 2m_D \,, \\
\mathrm{II} & \mathrm{Im}\, q_{D\bar{D}} < 0, & \mathrm{Im}\, q_{D\bar{D}^*} > 0, & \mathrm{Im}\, q_{D^*\bar{D}^*} > 0, & \text{for } 2m_D < E < m_D + m_{D^*} \,, \\
\mathrm{III} & \mathrm{Im}\, q_{D\bar{D}} < 0, & \mathrm{Im}\, q_{D\bar{D}^*} < 0, & \mathrm{Im}\, q_{D^*\bar{D}^*} > 0, & \text{for } m_D + m_{D^*} < E < 2m_{D^*} \,, \\
\mathrm{IV} & \mathrm{Im}\, q_{D\bar{D}} < 0, & \mathrm{Im}\, q_{D\bar{D}^*} < 0, & \mathrm{Im}\, q_{D^*\bar{D}^*} < 0, & \text{for } E > 2m_{D^*} \,.
\end{array}
$$



M.L. Du, U. G.Meissner and Q. Wang, Phys.Rev.D 94 (2016) 9, 096006

Recent progresses

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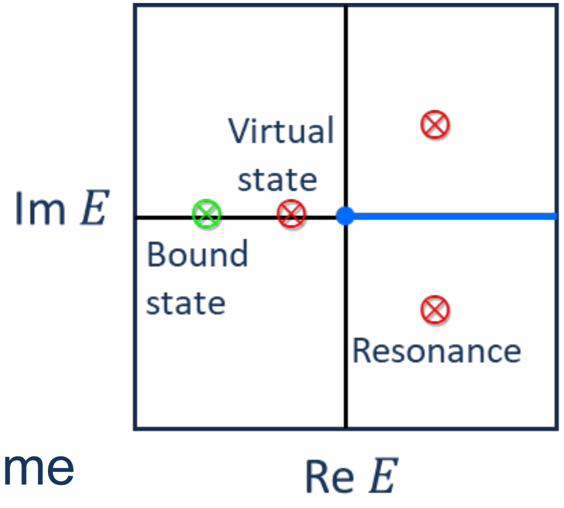
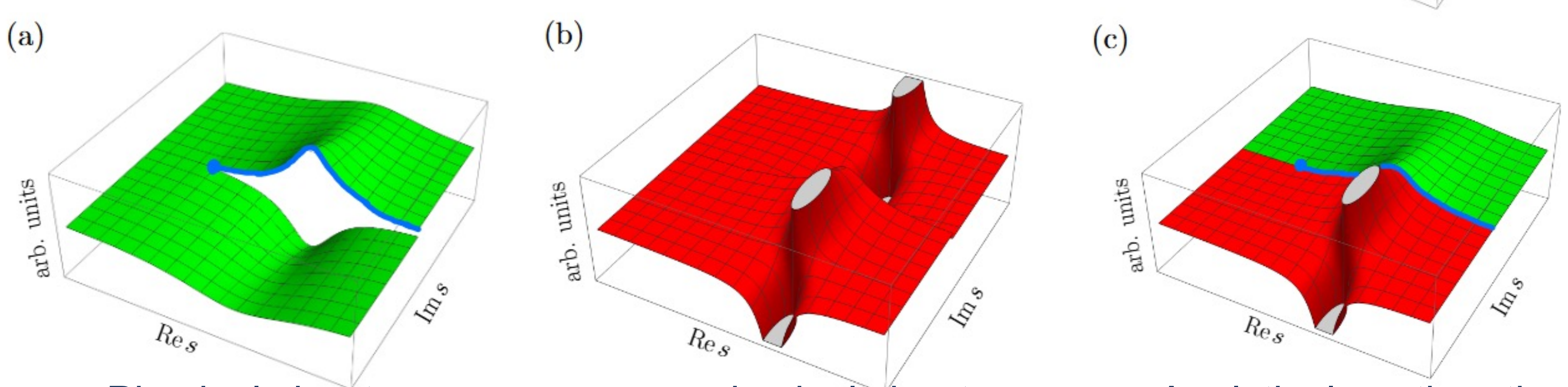
- P-wave double charm di-mesons
  - ► J.-L. Lu, M. Song, P. Wang, J.-Y. Guo, G. Li, and X. Luo,, Eur. Phys. J. C **85**, 920 (2025).
  - ► S.-D. Liu, Q. Wu, and G. Li, Phys. Rev. D **112**, 074002 (2025).
  - ► X.-X. Chen, Z.-M. Ding, and J. He, Phys. Rev. D **111**, 114008 (2025).
  - ► Z.-P. Wang, F.-L. Wang, G.-J. Wang, and X. Liu, Phys. Rev. D **110**, L051501 (2024).
- D-wave double charm di-mesons
  - ► K. Chen and J.-Z. Wang, arXiv:2508.11127 [hep-ph] (2025).
- *P*-wave double bottom
  - ► Z.-P. Wang, F.-L. Wang, G.-J. Wang, and X. Liu, arXiv:2505.03647 [hep-ph] (2025).
  - ► J.-Z. Wang, Z.-Y. Lin, J.-B. Cheng, LM, and S.-L. Zhu, arXiv:2505.02742 [hep-ph] (2025).
- Dibaryon
  - ► Y.-Y. Cui, X.-M. Tang, Q. Huang, and R. Chen, arXiv:2507.12958 [hep-ph] (2025).
- P-wave $\bar{D}K^*$
  - ► J.-Z. Wang, Z.-Y. Lin, B. Wang, L. Meng, and S.-L. Zhu, Phys. Rev. D **110**, 114003 (2024).
- …

Bound state, Virtual state and Resonance

- Single channel scattering as an example
- T-matrix: Unitary cut ⇒ multivalued function ⇒ Riemann sheets
- "States" ⇔ T-matrix poles
  - ▶ Bound state: real axis of physical sheet
  - ▶ Virtual state: real axis of unphysical sheet
  - ▶ Resonance: lower unphysical sheet
- Line shapes vary with processes, however, pole positions keeps the same
- Observables: bound state, $|T|^2$ with $E > 0$ in physical sheet
- Exact pole positions: general amplitudes satisfying physical constraints



Mizera:2023tfe; PDG



Physical sheet　　unphysical sheet　　Analytical continuation

Status of $Z_c(3900)$

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Refit Ex. data using amplitudes with exact unitarity.

solution I: resonance

Solution II: virtual state

M_{Z_c} (MeV)	$\Gamma_{Z_c}/2$ (MeV)	Ref.	Final state
3899 ± 6	23 ± 11	[1] (BESIII)	$J/\psi\,\pi$
3895 ± 8	32 ± 18	[2] (Belle)	$J/\psi\,\pi$
3886 ± 5	19 ± 5	[3] (CLEO-c)	$J/\psi\,\pi$
3884 ± 5	12 ± 6	[4] (BESIII)	$\bar{D}^*D$
3882 ± 3	13 ± 5	[5] (BESIII)	$\bar{D}^*D$
3894 ± 6 ± 1	30 ± 12 ± 6	Λ = 1.0 GeV	$J/\psi\,\pi, \bar{D}^*D$
3886 ± 4 ± 1	22 ± 6 ± 4	Λ = 0.5 GeV	$J/\psi\,\pi, \bar{D}^*D$
$3831 \pm 26^{+7}_{-28}$	virtual state	Λ = 1.0 GeV	$J/\psi\,\pi, \bar{D}^*D$
$3844 \pm 19^{+12}_{-21}$	virtual state	Λ = 0.5 GeV	$J/\psi\,\pi, \bar{D}^*D$

Below thresh. 30-40 MeV

M. Albaladejo, F. K. Guo, C. Hidalgo-Duque and J. Nieves, PLB755 (2016), 337-342

Global coupled-channel analysis of $e^+e^- \to c\bar{c}$

TABLE VI. $IJ^{PC} = 11^{+-}$ $D^*\bar{D} - D^*\bar{D}^* - J/\psi\pi - \psi'\pi - h_c\pi - \eta_c\rho$ coupled-channel scattering amplitude poles (unit:MeV). $Z_c(3900)$ and $Z_c(4020)$ are $D^*\bar{D}$ and $D^*\bar{D}^*$ virtual (resonance) poles in this work (PDG [4]).

$E_{Z_c}^{\text{This work}}$	$M_{Z_c}^{\text{PDG}}$	$\Gamma_{Z_c}^{\text{PDG}}$	
$(3837.7\pm7.4)+(19.4\pm1.6)i$	3887.1 ± 2.6	28.4 ± 2.6	$Z_c(3900)$
$(3989.9\pm5.6)+(26.1\pm4.3)i$	4024.1 ± 1.9	13 ± 5	$Z_c(4020)$

Virtual state: below thresh. 30-40 MeV

S. X. Nakamura, X. H.Li, H.P.Peng, Z.T.Sun and X.R.Zhou, PRD112 (2025), 054027

Three-coupled-channel analysis: $D\bar{D}^*$, $J/\psi\,\pi$, and $\rho\eta_c$

	Pole Position	Type	Scheme($\Lambda_{\pi J/\psi}$)
This work	3798.72 - 1.10i	Virtual	1(1.3GeV)
	3798.46 - 1.71i		1(1.5GeV)
	3798.12 - 2.26i		1(1.7GeV)
	3798.27 - 2.02i		2(1.5GeV)
	3797.80 - 2.64i		2(1.7GeV)

Virtual state Pole below threshod 80 MeV

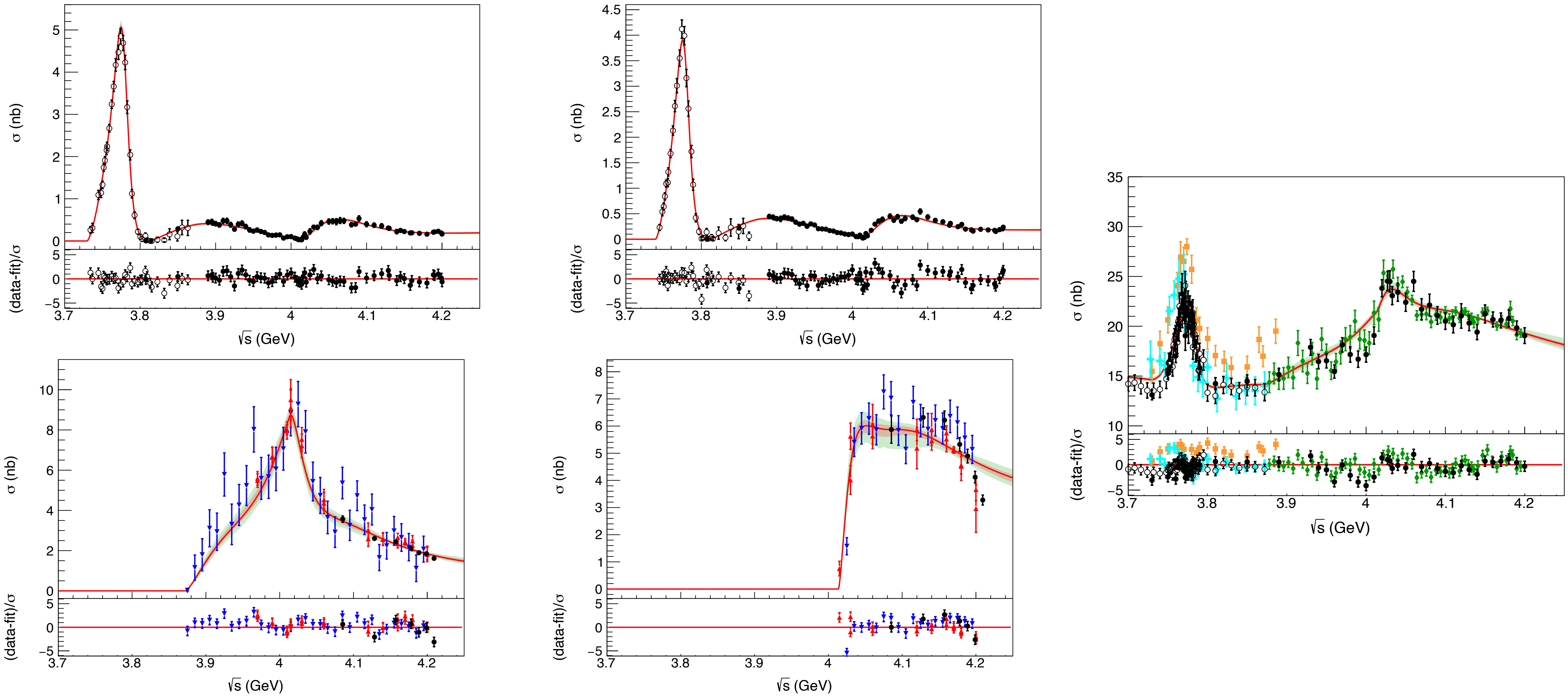
K.Yu, G.J.Wang, J.J.Wu and Z.Yang, PRD110 (2024), 114029

$\pi^+\pi^-$ and $J/\psi\pi^\pm$ mass spectra @ $e^+e^- \to J/\psi\pi^+\pi^-$
D^*D^{*-} mass spectrum @ $e^+e^- \to D^*D^{*-}\pi^+$

resonance:

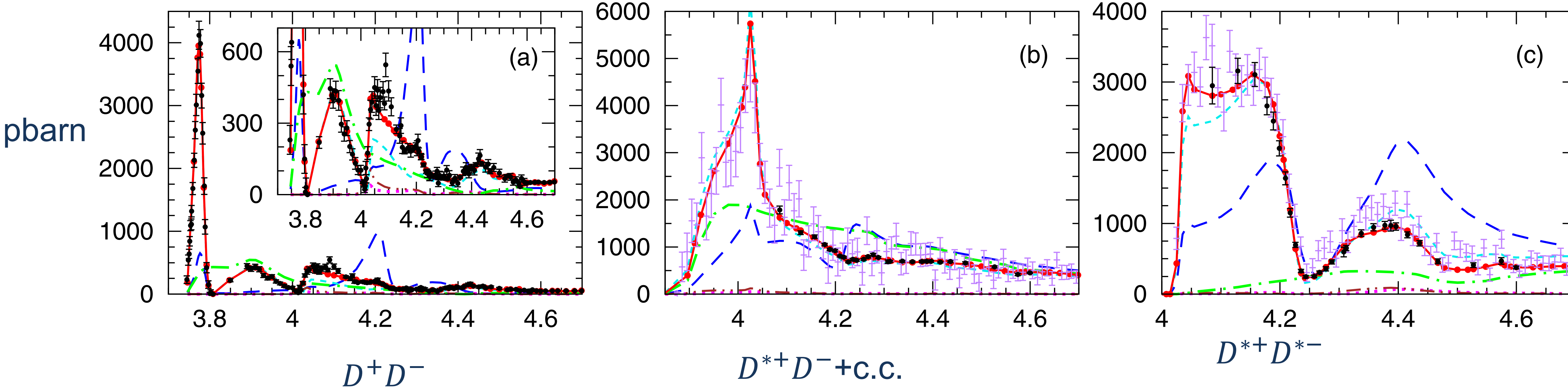
$$(3880.7 \pm 1.7 \pm 22.4) - i(17.9 \pm 0.7 \pm 7.7)\ \text{MeV}\,.$$

Y.H.Chen, M.L.Du and F.K.Guo, , *Sci.China Phys.Mech.Astron.* 67 (2024) 9, 291011



N. Hüsken, R. F. Lebed, R. E. Mitchell, E. S. Swanson, Y.-Q. Wang, and C.-Z. Yuan, Phys. Rev. D **109**, 114010 (2024).

Global fit



S. X. Nakamura, X.-H. Li, H.-P. Peng, Z.-T. Sun, and X.-R. Zhou, Phys. Rev. D **112**, 054027 (2025).

LSE fit

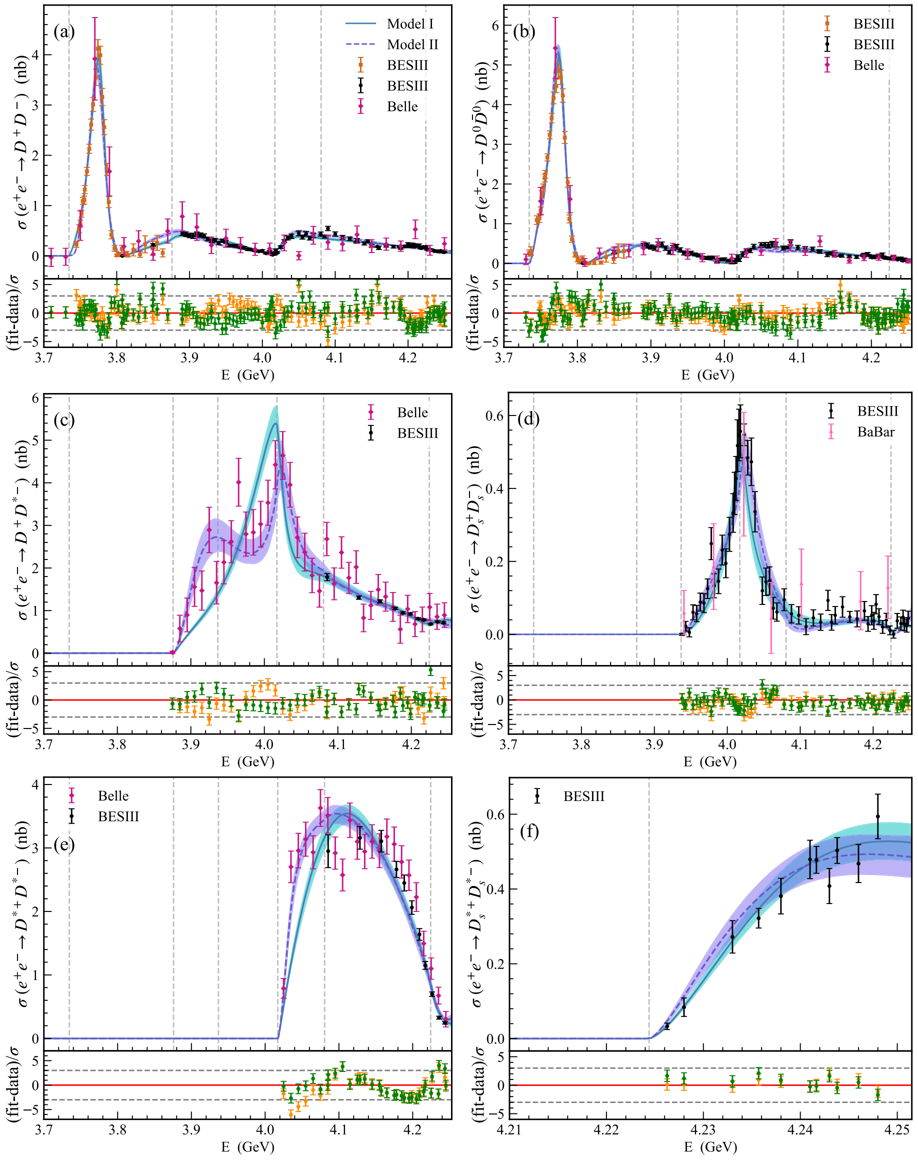


FIG. 2. The line shapes of model I (solid curve) and model II (dashed curve) in comparison with the experimental data. The $D\bar{D}$ data is from both BESIII [4,36] and Belle [1] collaborations. Panels (a)–(f) show the line shapes of the channels $e^+e^- \to D^+D^-$, $D^0\bar{D}^0$, D^+D^{*-}, $D_s^+D_s^-$, $D^{*+}D^{*-}$ and $D_s^{*+}D_s^{*-}$, respectively. The experimental data in the $D\bar{D}^*$ and $D^*\bar{D}^*$

Q. Ye, Z. Zhang, M.-L. Du, U.-G. Meißner, P.-Y. Niu, and Q. Wang, Phys. Rev. D **112**, 016015 (2025).

K-matrix formalism

- In principle, K-matrix formalism could meet the requirement of the analyticity and unitarity

$$\mathrm{Im}\mathcal{M}^{-1} = -2\rho, \qquad k = \frac{\sqrt{[s-(m_1-m_2)^2][s-(m_1+m_2)^2]}}{2\sqrt{s}}, \qquad \rho(s) = \frac{k}{8\pi\sqrt{s}}$$

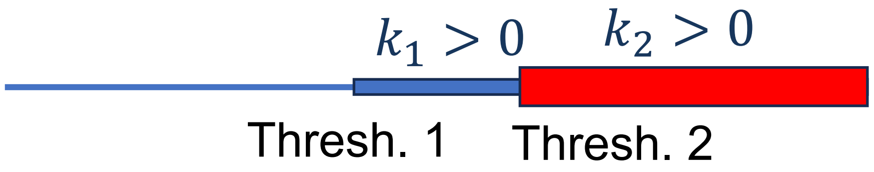
$$\mathcal{M}^{-1} = K^{-1} + C, \quad \mathrm{Im}\ C(s) = -2\rho,$$

$$K = \frac{g^2}{m_R^2 - s} + f, \qquad C(s) = C(s_0) - \frac{s-s_0}{\pi}\int_{s_0}^{\infty} ds' \frac{\mathrm{Im}C(s)}{(s'-s)(s'-s_0)}$$

- Three defects of K-matrix formalism in [arXiv: 2404.03896]

Different with that in PDG

  - ► Analyticity is not kept: $\mathcal{M}(s) = \mathcal{M}(k_1, k_2, \ldots)$, e.g. For $M_{th1}^2 < s < M_{th2}^2$, k_2 is set to zero
  - ► Subtraction dependence / regularization dependence for P-wave
  - ► Did not search the pole in the unphysical sheets below the thresholds



K-matrix formalism

- The unitarity: $\mathrm{Im}\mathcal{M}^{-1}=-2\rho$

$$k=\frac{\sqrt{[s-(m_1-m_2)^2][s-(m_1+m_2)^2]}}{2\sqrt{s}},\quad \rho(s)=\frac{k}{8\pi\sqrt{s}}$$

- K-matrix parameterization $\mathcal{M}^{-1}=K^{-1}+C,\quad \mathrm{Im}\,C(s)=-2\rho$
  - K is real, e.g. $K=\frac{g^2}{m_R^2-s}+f$
  - $C(s)$ determined by once-subtracted dispersion relation

$$C(s)=C(s_0)-\frac{s-s_0}{\pi}\int_{s_0}^{\infty}ds'\frac{\mathrm{Im}C(s)}{(s'-s)(s'-s_0)}$$

  - The subtraction-dependence is absorbed by the K
- P-wave and higher partial wave: threshold effect $\mathcal{M}\to p^{2l}$
  - PDG:

$$p^l\mathcal{M}_l^{-1}p^l=K^{-1}+C_l,\quad \mathrm{Im}C_l\to-2\rho p^{2l}$$

  - Swanson et al:

$$\mathcal{M}^{-1}=K^{-1}p^{-2l}+C_0(s)$$

- CM function VS one-loop diagram $C_0(s)\sim G(s)=\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2-m_1^2+i\epsilon}\frac{1}{(P-q)^2-m_2^2+i\epsilon},$
  - Subtraction ⇔ regularization
  - In BSE, absorbed by the Kernal or coupling constant

K-matrix formalism

- The unitarity: $\mathrm{Im}\mathcal{M}^{-1} = -2\rho$

$$k = \frac{\sqrt{[s-(m_1-m_2)^2][s-(m_1+m_2)^2]}}{2\sqrt{s}}, \quad \rho(s) = \frac{k}{8\pi\sqrt{s}}$$

- S-wave: K-matrix $\mathcal{M}^{-1} = K^{-1} + C, \quad \mathrm{Im}\, C(s) = -2\rho$
  - ► K is real, e.g. $K = \dfrac{g^2}{m_R^2 - s} + f$
  - ► $C(s)$ determined by once-subtracted dispersion relation

$$C(s) = C(s_0) - \frac{s-s_0}{\pi}\int_{s_0}^{\infty} ds' \frac{\mathrm{Im}C(s)}{(s'-s)(s'-s_0)}$$

  - ► The subtraction-dependence is absorbed by the K

    Set $g=0$, $C(s_0)$ can be absorbed by f

- P-wave and higher partial wave: threshold effect $\mathcal{M} \to p^{2l}$
  - ► PDG:

$$p^l \mathcal{M}_l^{-1} p^l = K^{-1} + C_l, \quad \mathrm{Im}C_l \to -2\rho p^{2l}$$

    for P-wave, the $C_l(s_0)$ can be absorbed by f

    Note: introducing the regulator, the once subtract. Is enough

  - ► Swanson et al: $\mathcal{M}^{-1} = K^{-1} p^{-2l} + C_0(s)$

    Set $g=0$, $C_0(s_0)$ cannot be absorbed by $f p^{-2l}$