Analogy between gravitational and gauge field Wilson line operators

玉苏普江·萨拉木

喀什大学

2025年7月27日

Outline

- Nonlocal Regularization of UV divergence
- The Relation between the nonlocality and Wilson line operators
- Nucleon -pion nonlocal PS interaction in the curved space-time
- Loop corrections to the EMT& gauge invariance
- Nucleon gravitational form factors in nonlocal chiral perturbation theory
- Conclusions

1. Nonlocal Regularization of UV divergence

- UV arises from the ill-defined nature of the product of two local field operators at the same space-time point.
- Nonlocal interactions ameliorate the UV aspects of QFT at short distances and small time scales.

E. Di Grezia,Mod.Phys. 6, 201–218 (2009)
L. Buoninfante, Nucl. Phys. B 944,114646 (2019)

- Nonlocal Regularization of UV Divergence

$$L^{local}{}_{int}[\phi(x),\phi^*(x)] \rightarrow L^{nonlocal}_{int}[\int d^4yF(x-y)\phi(y),\phi^*(x)]$$

↓

gauge invaricne is violated

- Gauge transformation of scalar field

$$\phi'(x) = U(x)\phi(x)$$

- Gauge transformation of bilocal operator

$$\phi^{\dagger}{}'(y)\phi'(x) \;\Rightarrow\; \phi^{\dagger}(y)U^{\dagger}(y)U(x)\phi(x)$$

- Introduction of the Wilson line

$$\phi^{\dagger}(y)W(y,x)\phi(x)$$

- Gauge transformation of the gauge field Wilson line operator

$$W'(y,x) = U(y)W(x,y)U^{\dagger}(x)$$

- With the help of Wilson line operator， nonlocal operator $\phi^{\dagger}(y)W(y,x)\phi(x)$ is gauge invariant.

- Parameterization of Gauge Field Wilson line operator： Solusion of Parallel Transport Equation

$$\frac{dW[y(t),x]}{dt} = -igA_{\mu}(t)W[y(t),x] \Longrightarrow W(y,x) = P\mathrm{Exp}[-ig\int_x^y dz^{\mu} A_{\mu}(z)]$$

2.The Relation between the Nonlocality and Wilson Loop Operators

- Nonlocal fields

$$\begin{cases} \Phi(x,a,A) = W(x,a,A)\phi(x+a) \\ \tilde{\Phi}(x,a,A) = e^{a.(\partial - igA)} e^{-a.\partial} \phi(x+a) \end{cases}$$

Nonlocal fields have the same gauge transformation behaviors

- Definition of the gauge Field Wilson operator

$$W(x,a,A) \equiv e^{a.(\partial - igA)} e^{-a.\partial} = \mathrm{Exp}[-ig\int_x^{x+a} dz^\mu A_\mu(z)]$$

- Gravitational Parallel Transport Equation

G. Modanese, Phys.Rev. D49 (1994)
H. W. Hamber   Phys.Rev.D76,2007

$$\frac{d}{dt}V^{\mu} + \Gamma^{\mu}{}_{\rho\sigma}\frac{dx^{\rho}}{dt}V^{\sigma} = 0$$

- Gravitational Wilson line operator

$$W^{a}_{\beta}(\mathrm{x}, y) = exp\left[\int_{x}^{y} dz^{\mu}\Gamma^{\alpha}_{\mu\beta}(z)\right]$$

Not consistent with double copy correspondence of gauge field and gravity

Alternatively, gravitational Wilson line operator is defined in terms of the path length of a massive particle in curved spacetime.

$$W(x_A, \mathrm{x}_B) = \exp\left(im\int_{\gamma}\sqrt{-g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}}\, d\tau\right) \approx \exp\left(i\frac{\kappa}{2}\int \mathrm{h}^{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} ds\right)$$

H. W. Hamber, Nucl.Phys. B435 (1995)
J. S. Dowker, Proc. Phys. Soc. 92 (1967)

$$
\begin{cases}
\Phi(x,a,A) = W(x,a,\mathrm{h})\phi(x+a) \\
\tilde{\Phi}(x,a,A) = e^{e_b^{\mu} a^b \partial_\mu} e^{-a.\partial}\phi(x+\mathrm{a}) \approx e^{a.\partial - \frac{\kappa}{2} h^{\mu\nu} a_\mu \partial_\nu} e^{-a.\partial}\phi(x+\mathrm{a})
\end{cases}
$$

Weak field approximation

$$
\begin{cases}
g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \mathrm{O}(\kappa^2), \\
\sqrt{-g} = 1 + \frac{1}{2}\kappa h + \mathrm{O}(\kappa^2), \\
e_a^{\mu} = \delta_a^{\mu} - \frac{\kappa}{2}\eta_{a\lambda} h^{\lambda\mu} + \mathrm{O}(\kappa^2)
\end{cases}
$$

$\eta^{\mu\nu}$ —— flat space-time backroud

$h^{\lambda\mu}$ ——curvaed space-time backroud

- Gravitational Wilson line operator

$$
W(x,a,h) = e^{a.\partial - \frac{\kappa}{2} h^{\mu\nu} a_\mu \partial_\nu} e^{-a.\partial} = \mathrm{Exp}[-\frac{\kappa}{4}\int_x^{x+a} h^{\mu\nu}(z) dz_{\{\mu}\partial_{\nu\}}]
$$

3.Nucleon -pion nonlocal PS interaction in the curved space-time

Nucleon -pion nonlocal PS interaction

$$L_{\text{int PS}}^{\text{local}} = \Big[\frac{i}{2}\bar{N}(x)(x)\gamma^{\mu}D_{\mu}N(x) - \frac{i}{2}D_{\mu}\bar{N}(x)(x)\gamma^{\mu}N(x) - m\bar{N}(x)N(x) + \delta m\bar{N}(x)N(x)\Big]$$

$$+\frac{1}{2}[\partial_{\mu}\pi(x).\partial^{\nu}\pi(x) - m_{\pi}^{2}\pi(x).\pi(x) + \delta m_{\pi}^{2}\pi(x).\pi(x)] - ig\bar{N}(x)\gamma^{5}\tau.\pi(x)N(x),$$

Nucleon -pion nonlocal PS interaction in the curved space-time

$$S_{\text{PS}}^{\text{nonlocal}} = \int d^{4}x\sqrt{-g(x)}\Big[\frac{i}{2}\bar{N}(x)e_{a}^{\mu}(x)\gamma^{a}\nabla_{\mu}N(x) - \frac{i}{2}\nabla_{\mu}\bar{N}(x)e_{a}^{\mu}(x)\gamma^{a}N(x)$$

$$-m\bar{N}(x)N(x) + \delta m\bar{N}(x)N(x)\Big] + \frac{1}{2}\int d^{4}x\sqrt{-g(x)}\,[\frac{g^{\mu\nu}(x)}{2}n\partial_{\{\mu}\pi(x).\partial_{\nu\}}\pi(x)$$

$$-m_{\pi}^{2}\pi(x).\pi(x) + \delta m_{\pi}^{2}\pi(x).\pi(x)] - ig\int d^{4}x\int d^{4}aF(a)\sqrt{-g(x)}\bar{N}(x)\gamma^{5}W(x,a,h)\tau.\pi(x+a)N(x),$$

Defination of energy–momentum tensor

$$T_{\mu\nu} = -\frac{2}{\kappa}\frac{\delta S}{\delta h^{\mu\nu}}$$

Energy-momentum tensor for Nucleon-pion PS interaction

$$T_{\mu\nu}(x) = \frac{i}{4}\left[\bar{N}(x)\gamma_{\{\mu}\partial_{\nu\}}N(x) - \partial_{\{\mu}\bar{N}(x)\gamma_{\nu\}}N(x)\right] - \eta_{\mu\nu}\left[\frac{i}{2}\bar{N}(x)\gamma^{\alpha}\partial_{\alpha}N(x)\right.$$

$$\left. - \frac{i}{2}\partial_{\alpha}\bar{N}(x)\gamma^{\alpha}N(x) - m\bar{N}(x)N(x) + \delta m\bar{N}x)N(x)\right] + \frac{1}{2}\partial_{\{\mu}\pi(x).\partial_{\nu\}}\pi(x)$$

$$- \frac{\eta_{\mu\nu}}{2}\left[\partial^{\alpha}\pi(x).\partial_{\alpha}\pi(x) - m_{\pi}^{2}\pi(x)g\pi(x) + \delta m_{\pi}^{2}\pi(x)g\pi(x)\right]$$

$$-ig\int d^{4}aF(a)n\left[-\eta_{\mu\nu}\,\bar{N}(x)\gamma_{5}\tau.\pi(x+a)N(x) + \frac{1}{2}\int_{0}^{1}dt\bar{N}(x)a_{\{\mu}\partial_{\nu\}}\gamma_{5}\tau.\pi(x+a-at)N(x)\right]$$

4.Loop corrections to the EMT& gauge invariance

Definition of Gravitational Form Factors of Nucleon

$$\langle p' | T_{\mu\nu} | p \rangle = \Gamma_{\mu\nu}(p,q) = \bar{u}(p',m)[A(t)\frac{\gamma_{\{\mu}P_{\nu\}}}{2} + iB(t)\frac{P_{\{\mu}\sigma_{\nu\}\alpha}q^{\alpha}}{4m} + D(t)\frac{q_{\mu}q_{\nu} - \eta_{\mu\nu}q^{2}}{4m} + m\bar{c}(t)\eta_{\mu\nu}]u(p,m)$$

Kinematic variables:

$$P = \frac{(p+p')}{2}, q = p' - p, t = q^{2} = -Q^{2}$$

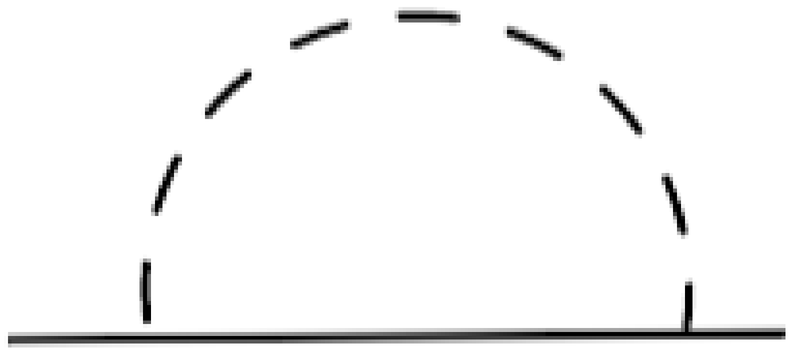
Gravitational Form Factos:

$$A(t), B(t), D(t), \bar{c}(t)$$

Gauge Invariance:

$$q_{\nu}\Gamma^{\mu\nu}(p,q) = 0 \quad \Longrightarrow \quad \sum_{i} \bar{c}_{i}(t) = 0$$

Pion one loop correction to the Nucleon self-energy



$$\Sigma(\not{p}, m) = -i\,3\,g^2 \int \frac{d^4k}{(2\pi)^4} \frac{\gamma_5(\not{p} - \not{k} + m)\gamma_5}{D_\pi(k) D_N(p-k)} \tilde{F}^2(k)$$

Local limit $\tilde{F}^2(k) = 1$

Nonlocal regulator

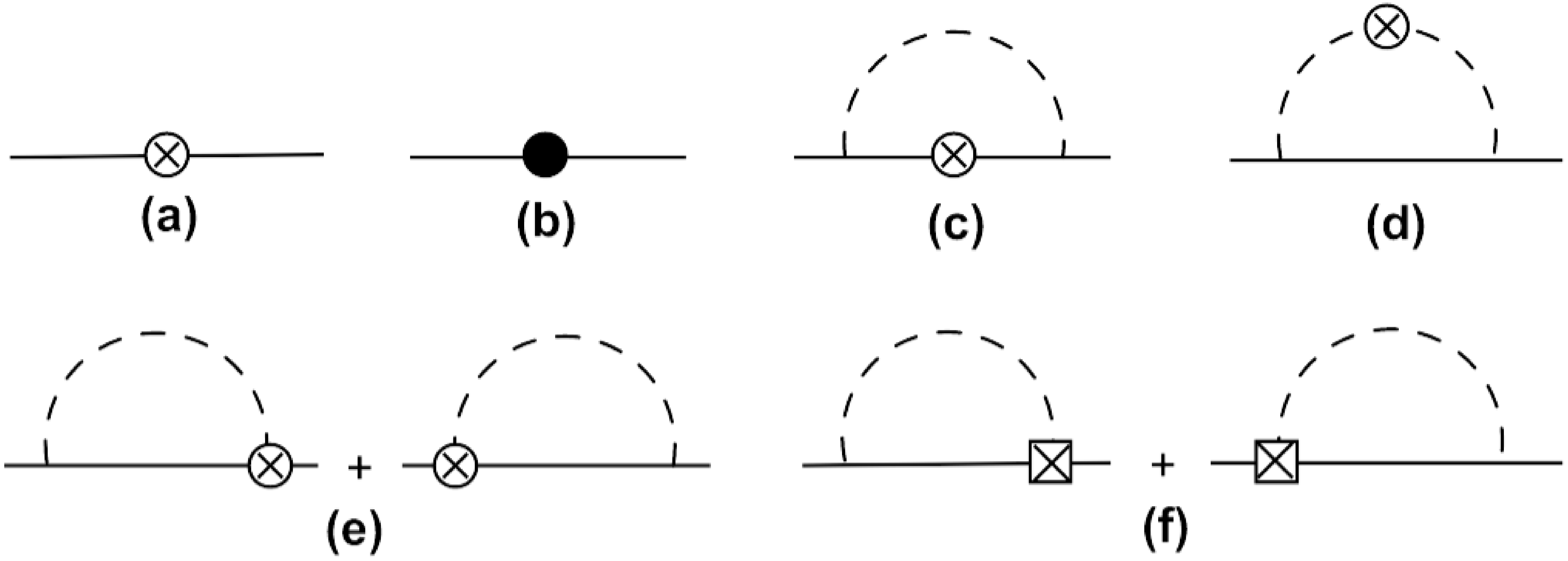
$$\tilde{F}(k) = \frac{(m_\pi^2 - \Lambda^2)^2}{(k^2 - \Lambda^2 + i\epsilon)^2} \quad \text{where} \quad \tilde{F}(k) = \int d^4a\, F(a) e^{-ik.a}$$

M.J. Musolf Z.Phys.C 61 (1994)
H. Forkel, Phys. Rev. C 50 (1994)

Mass counterterm and wavefunction renormalization constant

$$\delta m = \Sigma(\not{p} = m, m), \quad Z^{-1} = 1 - \left. \frac{d\Sigma(\not{p}, m)}{d\not{p}} \right|_{\not{p}=m}$$

Loop corrections to the EMT



solid lines ---- nucleon， dashed lines--- pions， ●-—- couterterm vertex

☒ ——guage link vertex， ⨁--matter-gravity coupling vertex

$$\Gamma^{\mu\nu}_{\mathrm{fig.2.a}} = \overline{u}(p')[\frac{P_{\{\mu}\gamma_{\nu\}}}{2} - \eta^{\mu\nu}(\not{P} - m)]u(p)$$

$$\Gamma^{\mu\nu}_{\mathrm{fig.2.b}} = -\eta^{\mu\nu}\delta m \overline{u}(p')u(p)$$

$$\Gamma^{\mu\nu}_{\mathrm{fig2.c}} = -3i\,g^2\overline{u}(p')\int\frac{d^4k}{(2\pi)^4}\gamma^5(\not{p}' - \not{k} - m)\left\{\frac{1}{2}(P-k)^{\{\mu}\gamma^{\nu\}} - \eta^{\mu\nu}(\not{P} - \not{k} - m)\right\}\frac{(\not{p} - \not{k} + m)\gamma^5}{D_N(p-k)D_N(p'-k)D_\pi(k)}\tilde{F}^2(k)u(p)$$

$$\Gamma^{\mu\nu}_{\mathrm{fig2.d}} = -3i\,g^2\,\overline{u}(p')\int\frac{d^4k}{(2\pi)^4}\left[k'^{\{\mu}k^{\nu\}} - \left(k.k' - m_\pi^2\right)\eta^{\mu\nu}\right]\frac{\gamma_5(\not{p} - \not{k} + m)\gamma_5}{D_\pi(k)D_\pi(k')D_N(p-k)}u(p)\tilde{F}(k)\tilde{F}(k').$$

$$\Gamma^{\mu\nu}_{\text{fig2.e}} = -3ig^2\overline{u}(p')\int\frac{d^4k}{(2\pi)^4}\left\{\frac{\eta^{\mu\nu}\gamma_5(\not p-\not k+m)\gamma_5}{D_\pi(k)D_N(p-k)}+\frac{\eta^{\mu\nu}\gamma_5(\not p'-\not k+m)\gamma_5}{D_\pi(k)D_N(p'-k)}\right\}\tilde{F}^2(k)u(p)$$

$$\Gamma^{\mu\nu}_{\text{fig2.f}} = 3ig^2\overline{u}(p')\int\frac{d^4k}{(2\pi)^4}\left\{\frac{\gamma_5(\not p-\not k+m)\gamma_5}{D_\pi(k)D_N(p-k)}\left[\frac{1}{2}k^{\{\mu}\frac{\partial}{\partial k^{\nu\}}}\int_0^1 dt\tilde{F}(k+qt)\right]\right.$$
$$\left.+\frac{\gamma_5(\not p'-\not k+m)\gamma_5}{D_\pi(k)D_N(p'-k)}\left[\frac{1}{2}k^{\{\mu}\frac{\partial}{\partial k^{\nu\}}}\int_0^1 dt\tilde{F}(k-qt)\right]\right\}\tilde{F}(k)u(p)$$

- Conservation of nonlocal EMT

$$q_\nu\Gamma^{\mu\nu}_{\text{fig.2.b}}+q_\nu\Gamma^{\mu\nu}_{\text{fig2.c}}+q_\nu\Gamma^{\mu\nu}_{\text{fig2.d}}+q_\nu\Gamma^{\mu\nu}_{\text{fig2.e}}+q_\nu\Gamma^{\mu\nu}_{\text{fig2.f}}=0$$

5.Nucleon gravitational form factors in nonlocal chiral perturbation theory

- Leading order order Nucleon-pion interactions

$$S^{(1),\mathrm{nl}}_{\pi N}=\int d^4x\int d^4l F(l)\sqrt{-g(x+l)}\Big[\frac{i}{2}\bar{B}(x)e^{\mu}_{a}(x+l)\gamma^{a}\nabla_{\mu}B(x)-\frac{i}{2}\nabla_{\mu}\bar{B}(x)e^{\mu}_{a}(x+l)\gamma^{a}B(x)-m\bar{B}(x)B(x)$$

$$+\delta_2\Big[\frac{i}{2}\bar{B}(x)e^{\mu}_{a}(x)\gamma^{a}\nabla_{\mu}B(x)-\frac{i}{2}\nabla_{\mu}\bar{B}(x)e^{\mu}_{a}(x)\gamma^{a}B(x)\Big]-\delta_m\bar{B}(x)B(x)\Big]-\frac{C^{(0)}_{B\phi}}{f_\phi}\int d^4x\int d^4a\int d^4l F(a)F(l)$$

$$\sqrt{-g(x+l)}\bar{p}(x)\gamma^{a}\gamma^{5}e^{\mu}_{a}(x+l)B(x)W^{\nu}_{\mu}(x,x+a)D_{\nu}\phi(x+a)+\mathrm{h.c.}-i\frac{C_{\phi\phi^\dagger}}{2f_\phi^2}\int d^4x\int d^4a\int d^4b\int d^4l F(a)F(b)F(l)$$

$$\sqrt{-g(x+l)}\bar{p}(x)\gamma^{a}e^{\mu}_{a}(x_l)p(x)\Big\{[W^{\nu}_{\mu}(x,x+a)D_{\nu}\phi(x+a)][W(x,x+b)\phi(x+b)]^\dagger-[W(x,x+b)\phi(x_b)][W^{\nu}_{\mu}(x,x_a)D_{\nu}\phi(x+a)]^\dagger\Big\}$$

Gravitainal Wilson line operator for vector field

$$W^{\nu}_{\mu}(x,y)\equiv\exp\Big\{\int_x^y\int d^4l F(l)\Big[-\frac{\kappa}{4}\delta^{\nu}_{\mu}h^{\alpha\beta}(z+l)dz_{\{\alpha}\partial_{\beta\}}-\Gamma^{\nu}_{\rho\mu}(z+l)dz^{\rho}\Big]\Big\}$$

■ Next-leading order order Nucleon-pion interactions

$$S_{\pi\mathrm{N}}^{(2),\mathrm{nl}} = 4c_1 m_\phi^2 \int d^4x \int d^4l F(l)\sqrt{-g(x+l)}\,\overline{p}(x)p(x) + c_1 m_\phi^2 \frac{C_{\phi\phi}^{(1)}}{f_\phi^2}\int d^4x\int d^4a\int d^4b\int d^4l F(a)F(b)F(l)\sqrt{-g(x+l)}\,\overline{p}(x)p(x)$$

$$[W(x,x+a)\phi(x+a)][W(x,x+b)\phi(x+b)]^\dagger + c_2 \frac{C_{\phi\phi}^{(2)}}{m^2 f_\phi^2}\int d^4x\int d^4a\int d^4b\int d^4l F(a)F(b)F(l)\sqrt{-g(x+l)}\,g^{\alpha\mu}(x+l)g^{\beta\nu}(x+l)$$

$$\left[\overline{p}(x)\nabla_\alpha\nabla_\beta p(x) + \nabla_\alpha\nabla_\beta\overline{p}(x)p(x)\right][W_{\{\mu}^{\rho}(x,x+a)D_\rho\phi(x+a)][W_{\nu\}}^{\lambda}(x,x+b)D_\lambda\phi(x+b)]^\dagger + c_3\frac{C_{\phi\phi}^{(3)}}{2f_\phi^2}\int d^4x\int d^4a\int d^4b\int d^4l F(a)F(b)$$

$$F(l)\sqrt{-g(x+l)}\,\overline{p}(x)g^{\mu\nu}(x+l)p(x)[W_{\{\mu}^{\alpha}(x,x+a)D_\alpha\phi(x+a)][W_{\nu\}}^{\beta}(x,x+b)D_\beta\phi(x+b)]^\dagger$$

■ Nucleon-gravity nonminmal coupling

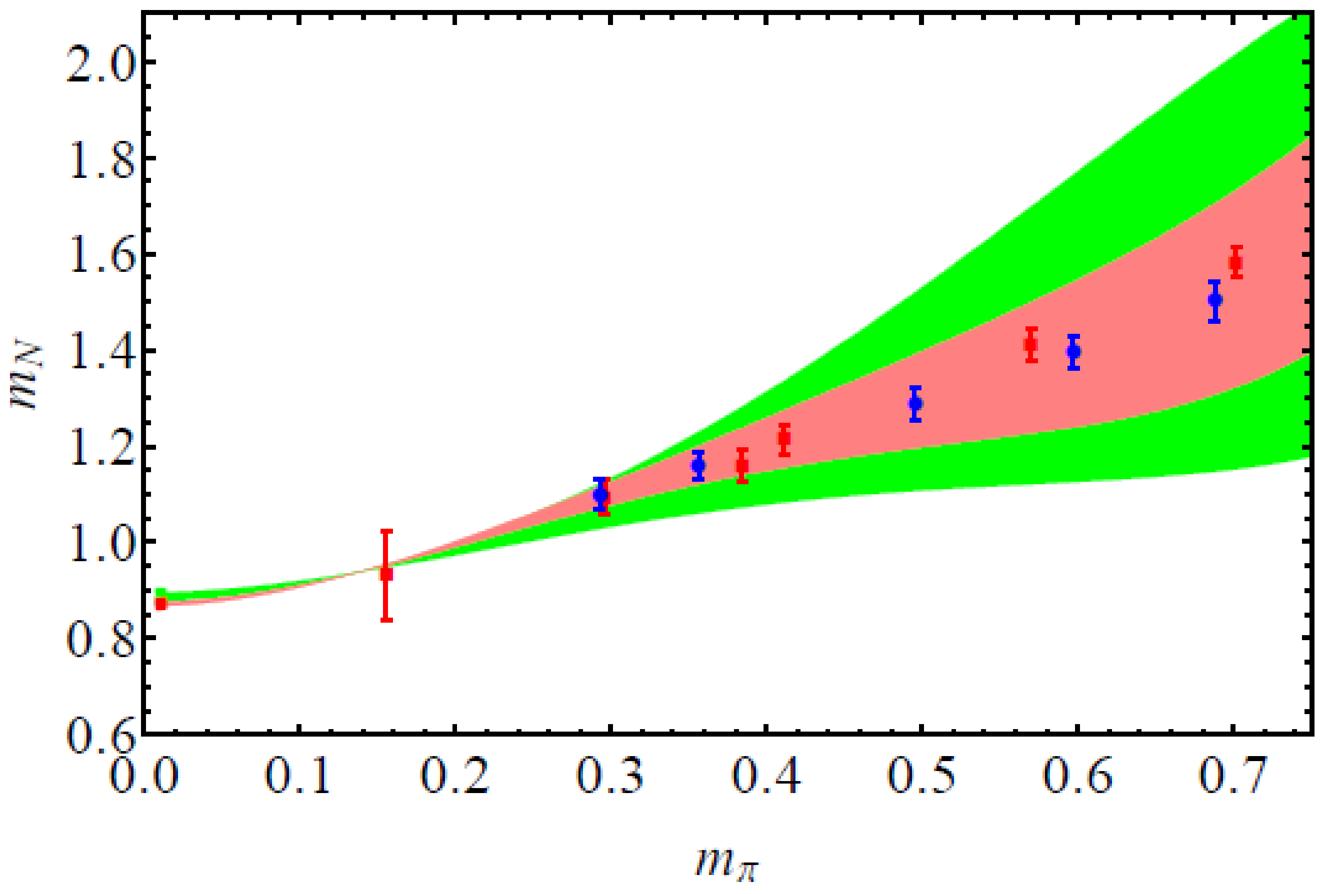
$$S_{\mathrm{non}}^{\mathrm{nl}} = \int d^4x\int d^4l F(l)\sqrt{-g(x+l)}\Big\{\frac{c_8}{8}R(x+l)\overline{B}(x)B(x) + i\frac{c_9}{m}R^{\mu\nu}(x+l)\Big[\overline{B}(x)e_\mu^a(x+l)\gamma_a\nabla_\nu B(x)$$

$$-\nabla_\nu\overline{B}(x)e_\mu^a(x+l)\gamma_a B(x)$$

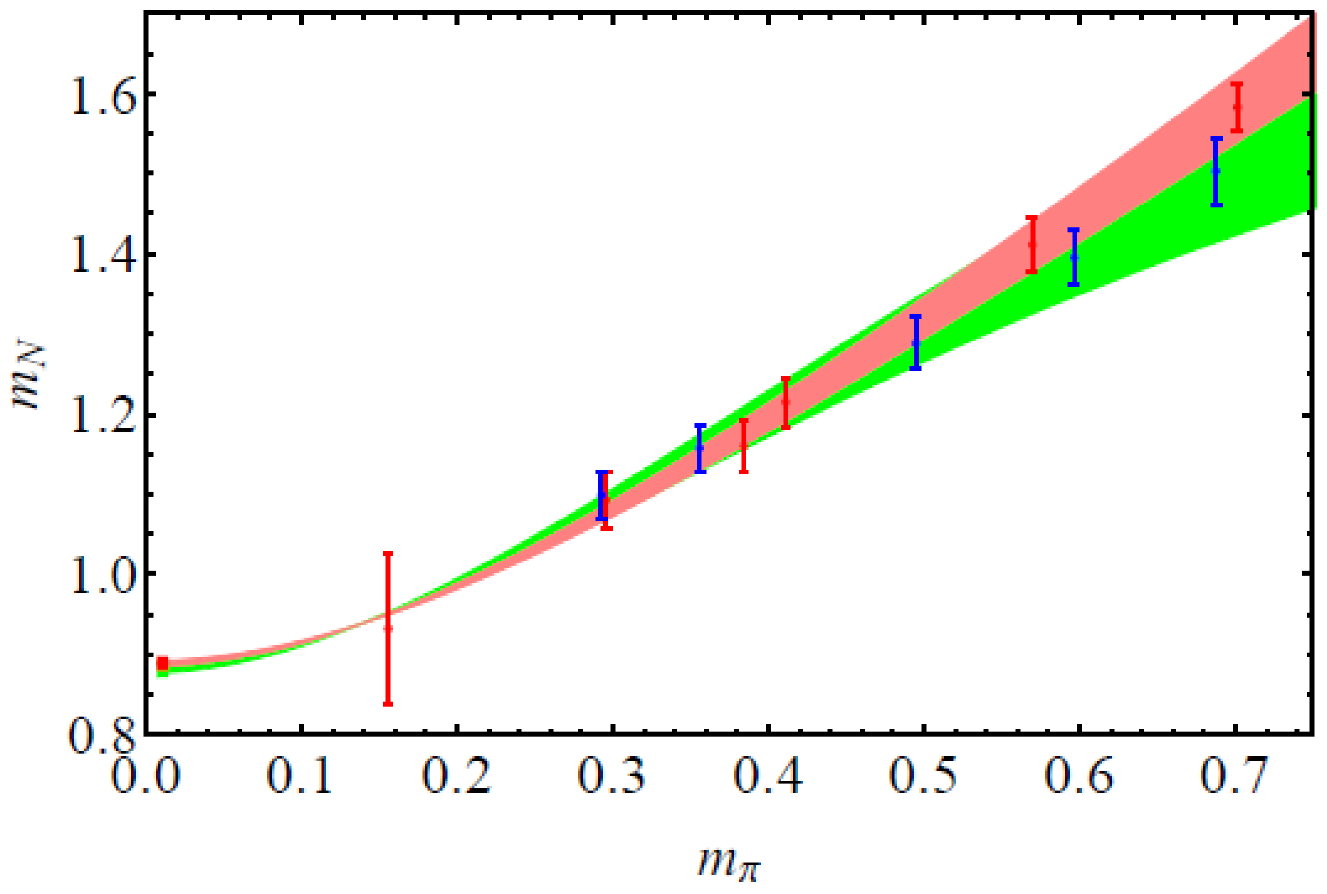
■ Pion mass depenency of Nucleon mass

$$m_N = m - 4m_\pi^2 c_1 + \Sigma(\not{p} = m, m) \quad \sigma_{\pi N} = m_\pi^2 \frac{\partial m_N}{\partial m_\pi^2}$$

Nex-leading order low-energy coupling constants (LECs) can be fitted to the Lattice data



Local Renormalization(EOMS method)



Nonlocal Renormalization

A. Walker-Loud. Phys. Rev. D 79 (2009)
S. Aoki Phys. Rev. D 79 (2009)

Determination of LECs with EOMS renormalization

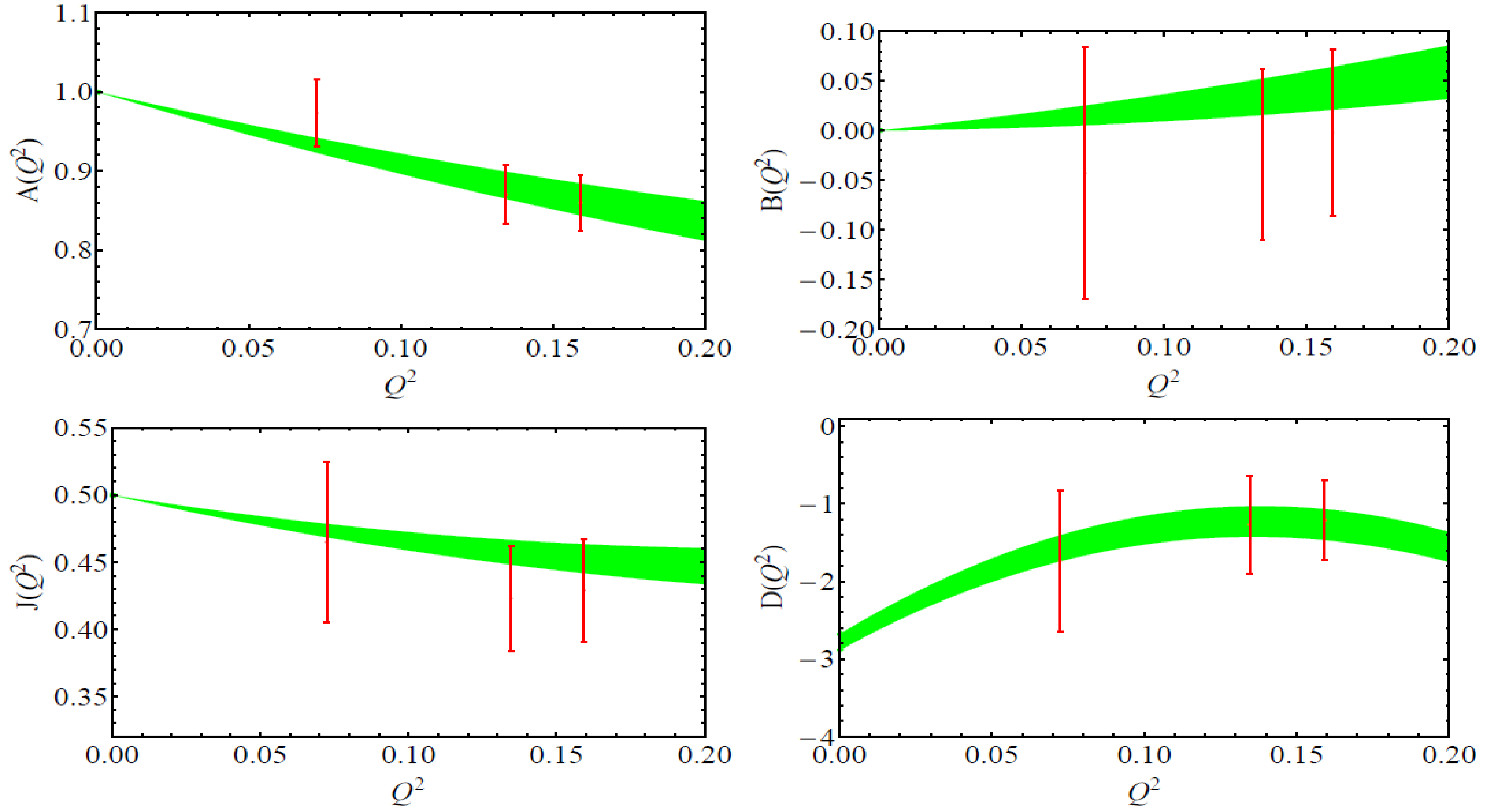
m(GeV)	c_1(GeV^{-1})	c_2(GeV^{-1})	c_3(GeV^{-1})	$\sigma_{\pi N}$(MeV)	χ^2/d.o.f
$m = 0.886 \pm 0.011$	$c_1 = -0.83 \pm 0.10$	$c_2 = 3.68 \pm 0.72$	$c_3 = -2.41 \pm 0.83$	$46.358 + 0.012$	0.18[64]
$m = 0.871 \pm 0.005$	$c_1 = -1.08 \pm 0.04$	$c_2 = 2.94 \pm 0.44$	$c_3 = -4.03 \pm 0.39$	$58.383 + 0.006$	0.037[65]

Determination of LECs in nonlocal framework

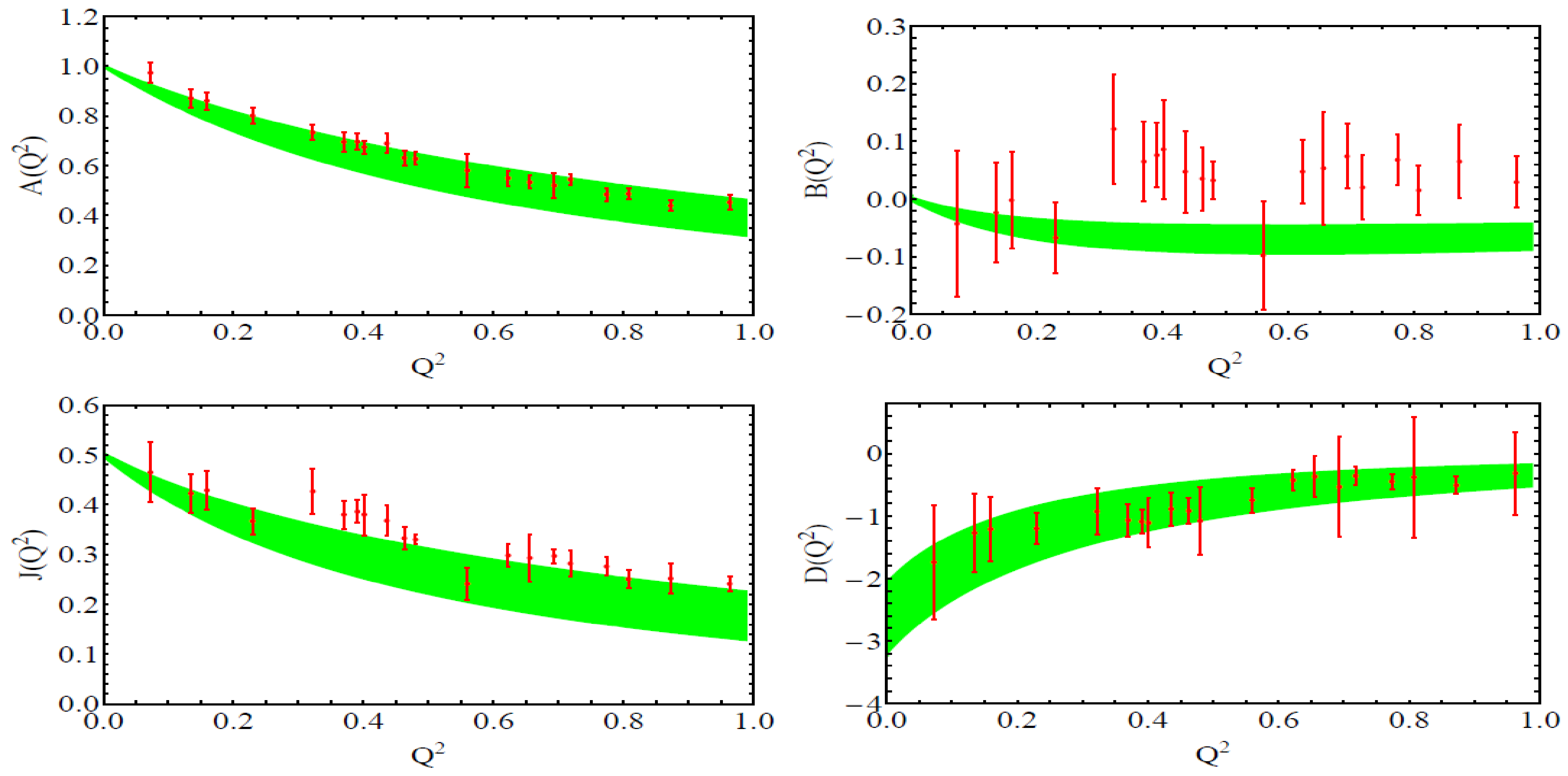
m(GeV)	c_1(GeV^{-1})	c_2(GeV^{-1})	c_3(GeV^{-1})	$\sigma_{\pi N}$(MeV)	χ^2/d.o.f
$m = 1.228 \pm 0.029$	$c_1 = -0.22 \pm 0.02$	$c_2 = -0.43 \pm 0.02$	$c_3 = -0.11 \pm 0.07$	45.223 ± 0.003	0.18[64]
$m = 1.311 \pm 0.049$	$c_1 = -0.12 \pm 0.04$	$c_2 = -0.88 \pm 0.03$	$c_3 = -0.22 \pm 0.13$	50.454 ± 0.003	0.37[65]

- In the nonlocal case, next-leading order LECs are very small compared to local one.

Q^2-dependency of local gravitational form factors $A(Q^2), B(Q^2), D(Q^2)$ and angular momentum $J(Q^2)$ compared with lattice data



Q^2-dependency of nonlocal gravitational form factors $A(Q^2), B(Q^2), D(Q^2)$ and angular momentum $J(Q^2)$ compared with lattice data



D. C. Hackett, Phys. Rev. Lett. 132 (2024)

- In the nonlocal case, Q^2 -dependency of gravitational form factors are in good agreement with lattice data over momentum transfer region $0 \le Q^2 \le 1\mathrm{GeV}^2$.

Conclusions

- The gravitational and gauge field Wilson line operators are derived from nonlocal field operators in an analogous way.
- With the help of gravitational Wilson line operator , the nonlocal EMT is still conserevd.
- In the nonlocal framework, LECs proved to be smaller than their local counterparts.
- Nonlocal renormalization the improves Q^2-dependency of gravitational form factors in the large momentum transfer region.

Thanks !