中国科学技术大学
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BESIII

Bose-Einstein Correlations study
A powerful probe to hadron source geometry

第十届 BESIII R 值与 QCD 强子结构
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Introduction

- Intensity interferometry (Hanbury-Brown/Twiss effect):
  - Origin in astrophysics, developed by Hanbury-Brown and Twiss in the 1950s, as means of determining the dimension of distant astronomical objects(galaxy or star). *Nature* 177, 27-29 (1956)
  - In subatomic physics, Bose-Einstein Correlations (BEC) first observed in pion emissions form proton-antiproton reactions by Goldhaber, Goldhaber, Lee and Pais. *Phys. Rev. Lett.* 3, 181-183 (1959) *Phys. Rev. Lett.* 120, 300-312 (1960)
  - After over a decade, G. Goldhaber and S. Goldhaber develop similar interferometry method to subatomic system. *Yad. Fiz.* 18, 656-666 (1973)
  - Since then, Significant theoretical development and widespread application in subatomic physics experiments, used to investigate the space-time evolution of elementary-particle and nuclear collisions. *Rept. Prog. Phys.* 66, 481-522 (2003)
  - In subatomic physics field, it is called BEC study or correlation femtoscopy (飞镜), as the scale of the hadron source is always determined in femtometer level.

Introduction

- Expression of boson interferometry:
  - Consider a scenario where particles are emitted from multiple discrete sources, each characterized by a probability amplitude $f_i(\vec{x}) = f_i \delta^3(\vec{x} - \vec{x}_i)$.
  - If $\psi(\vec{p}, \vec{x}_i)$ is the wave function of a particle emitted with momentum $\vec{p}$, on the plane wave assumption, $\psi(\vec{p}, \vec{x}_i) \sim e^{i(\vec{p}\vec{x}_i + \phi)}$.
  - Further assuming the incoherent emissions, the initial phase ϕ could set to 0, simplying the wave function $\psi(\vec{p}, \vec{x}_i) \sim e^{i\vec{p}\vec{x}_i}$.
  - Accordingly, the probability P of observing a particle with momentum $\vec{p}$ given by:

$$P = \sum_i |f_i \psi(\vec{p}, \vec{x}_i)|^2 .$$

  - Further more, if the source is assumed continuous in space, the probability expressed as an integral:

$$P = \int d^3\vec{x}\, |f(\vec{x})|^2 .$$

Introduction

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- Expression of boson interferometry:
  - Similarly, the joint probability of observing two particles with momenta $\vec{p}_1$ and $\vec{p}_2$ from $\vec{x}_1$ and $\vec{x}_2$ is given by:

$$P_{12} = \int d^3\vec{x}_1 d^3\vec{x}_2 \, |\psi_{12}|^2 \, |f(\vec{x}_1)|^2 \, |f(\vec{x}_2)|^2 \, ,$$

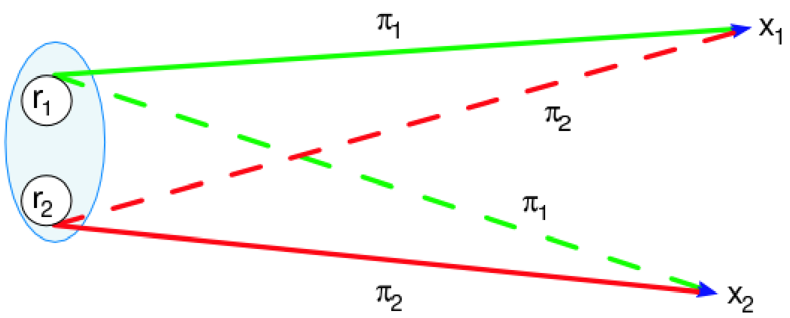
  - $\psi_{12} = \psi_{12}(\vec{p}_1, \vec{p}_2, \vec{x}_1, \vec{x}_2)$ is the two-particle wave function.
  - For identical bosons, the symmetrized ψ_{12} takes the form:

$$\psi_{12} = \frac{1}{\sqrt{2}} \left[e^{i(\vec{p}_1\vec{x}_1 + \vec{p}_2\vec{x}_2)} + e^{i(\vec{p}_1\vec{x}_2 + \vec{p}_2\vec{x}_1)} \right] .$$

  - Then, we can define the correlation function as:

$$R(\vec{p}_1, \vec{p}_2) = \frac{P_{12}}{P_1 P_2} = 1 + \frac{\int d^3\vec{x}_1 d^3\vec{x}_2 \cos\left[\Delta\vec{p}(\vec{x}_1 - \vec{x}_2)\right] |f(\vec{x}_1)|^2 \, |f(\vec{x}_2)|^2}{P_1 P_2},$$

With $\Delta\vec{p} = \vec{p}_1 - \vec{p}_2$.



Introduction

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- Expression of boson interferometry:
  - After integration (Fourier transformation), we obtain:

$$R(\Delta\vec{p}) = 1 + |F(\Delta\vec{p})|^2 .$$

  - Using the Lorentz-invariant parameter $Q^2 = -(q_1 - q_2)^2 = M^2 - 4m^2$, we obtain:

$$R(Q^2) = 1 + \left|F(Q^2)\right|^2 .$$

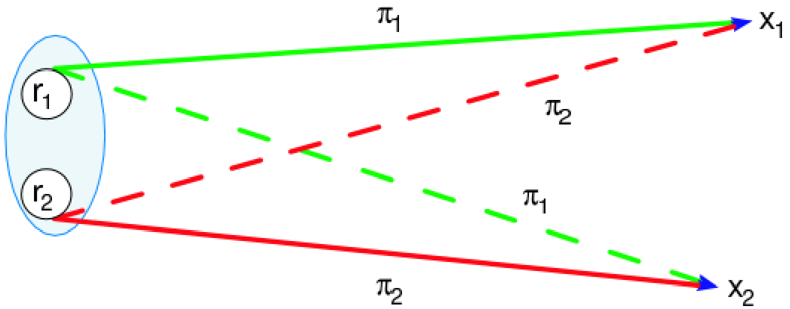
  - Assuming a spherically symmetric Gaussian distribution for the emitting source,

$$f(r) = f(0)e^{-\frac{r^2}{2r_0^2}} ,$$

  - The BEC function takes the form:

$$R(Q^2) = 1 + e^{-r^2Q^2} .$$

  - Above derivation under the ideal case of a fully incoherent source, due to the complexities of experiments, the expression of $\mathrm{R}(Q^2)$ more complex too.



Analysis method

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- Two particle correlation function:
  - Experimentally, BEC effect examined by measuring the two-particle correlation function:

$$C(Q^2) = \frac{\rho(Q^2)}{\rho_0(Q^2)},$$

  - $\rho(Q^2)$ is the distribution formed from the sample of all same-sign charged particle (SCP) pairs.
  - $\rho_0(Q^2)$ is the distribution of a reference sample, designed to exclude the BEC but include all other correlations.
  - Both ρ and ρ_0 are normalized to unity, i.e. they are probability density distributions.
  - To remove the bias due to the choice of reference sample, the $C^{data}(Q^2)$ corrected by dividing it by the corresponding distributions of MC $C^{MC}(Q^2)$, obtaining the double ratio:

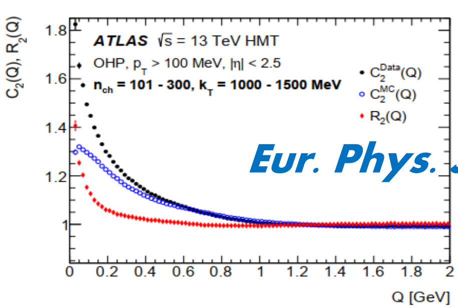
$$R(Q^2) = \frac{C^{\text{data}}(Q^2)}{C^{\text{MC}}(Q^2)} = \frac{\rho^{\text{data}}(Q^2)}{\rho_0^{\text{data}}(Q^2)} \Bigg/ \frac{\rho^{\text{MC}}(Q^2)}{\rho_0^{\text{MC}}(Q^2)},$$

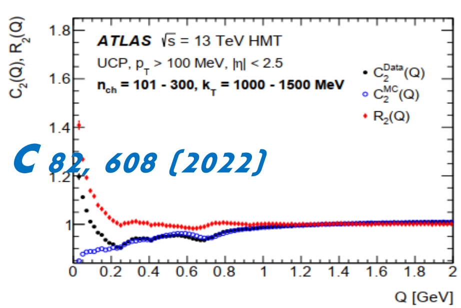
where the superscripts "MC" and "data" denote the distributions of MC and data.

Analysis method

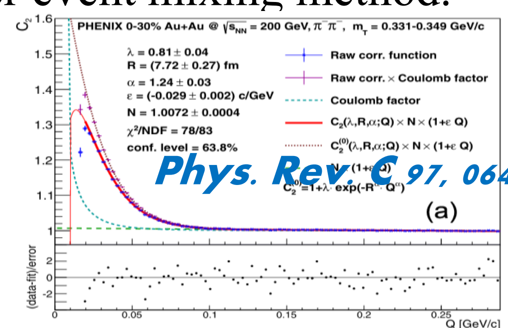
Reference sample:

- A well-chosen reference sample is crucial for an unbiased BEC signal.
- Ideally, the $\rho_0(Q^2)$ should include all correlations present in $\rho(Q^2)$ except those arising form BEC.
- To this end, several methods designed:
  - OCP sample: Formed with opposite-sign charged particles (OCP) from the same events.
  - OHP sample: Formed with SCP pairs from the same events, but inverts momentum of one particle.
  - ROTA sample: Formed with SCP pairs from the same events, but inverts transverse momentum of one particle.
  - OCPOHP sample: Formed with OCP pairs from the same events, and inverts momentum of one particle.
  - MIX sample: Form with the "event-mixing" method, particle pairs from different events.
- No one preferred in prior, usually chose according different situations of different experiment.
- LHC prefer OCP samples, nuclear collisions prefer event mixing method.





Eur. Phys. J. C 82, 608 (2022)



Phys. Rev. C 97, 064911 (2018)

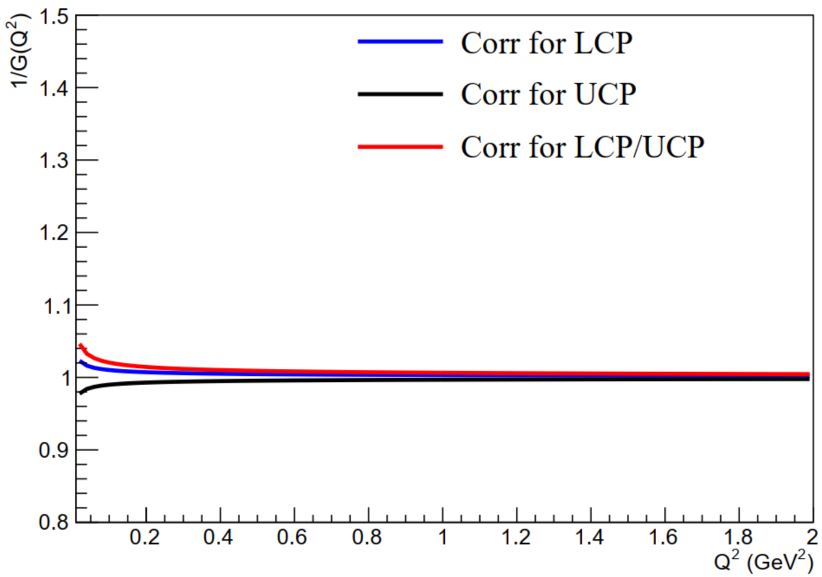
Analysis method

- Corrections due to the Coulomb interactions (Gamow factor):
  - Coulomb interactions between charged particles modify the relative momentum distribution, and thus distort the Q^2 distributions.
  - The effects, therefore, should be corrected, usually with the Gamow factors:

$$G(Q^2) = 2\pi\zeta/(\exp(2\pi\zeta) - 1),$$ ***Phys. Rev. C* 20, 2267 (1979)**

with $\zeta = \pm\alpha m_\pi/\sqrt{Q^2}$, positive sign for SCP and negative sign for OCP.

  - Only need apply to data, do not need to MC since no Coulomb interactions simulated in current MC samples.



Analysis method

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- Efficiency correction:
  - Specialized selection criteria used to obtain the $\pi^\pm\pi^\pm + X$ final states for BEC study.
  - Possible bias may induced by the selection criteria, and these effect could be correct using the efficiency curve:

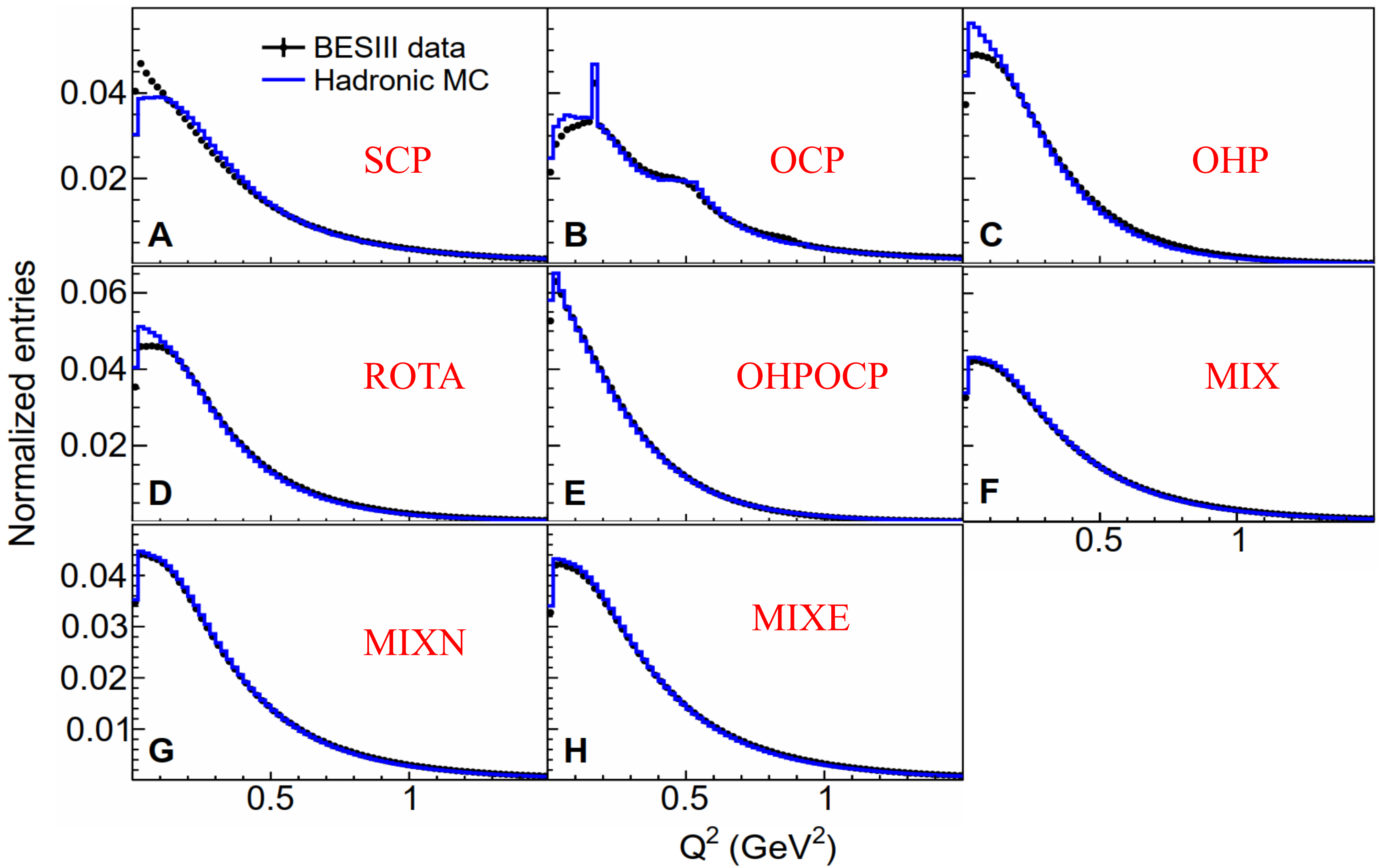
$$\frac{\rho_{\mathrm{Rec}}^{\mathrm{data}} / \rho_{0,\mathrm{Rec}}^{\mathrm{data}}}{\rho_{\mathrm{Rec}}^{\mathrm{MC}} / \rho_{0,\mathrm{Rec}}^{\mathrm{MC}}} \rightarrow \frac{(\rho_{\mathrm{Rec}}^{\mathrm{data}} / \varepsilon^{\mathrm{BEC\ MC}}) \Big/ (\rho_{0,\mathrm{Rec}}^{\mathrm{data}} / \varepsilon_0^{\mathrm{BEC\ MC}})}{(\rho_{\mathrm{Rec}}^{\mathrm{MC}} / \varepsilon^{\mathrm{MC}}) \Big/ (\rho_{0,\mathrm{Rec}}^{\mathrm{MC}} / \varepsilon_0^{\mathrm{MC}})} = \frac{\rho_{\mathrm{Rec}}^{\mathrm{data}} / \rho_{0,\mathrm{Rec}}^{\mathrm{data}}}{\rho_{\mathrm{Rec}}^{\mathrm{MC}} / \rho_{0,\mathrm{Rec}}^{\mathrm{MC}}} \Bigg/ \frac{\boldsymbol{\varepsilon}^{\mathbf{BEC\ MC}} / \boldsymbol{\varepsilon}_{\mathbf{0}}^{\mathbf{BEC\ MC}}}{\boldsymbol{\varepsilon}^{\mathbf{MC}} / \boldsymbol{\varepsilon}_{\mathbf{0}}^{\mathbf{MC}}},$$

$R(Q^2)$ Efficiency curve

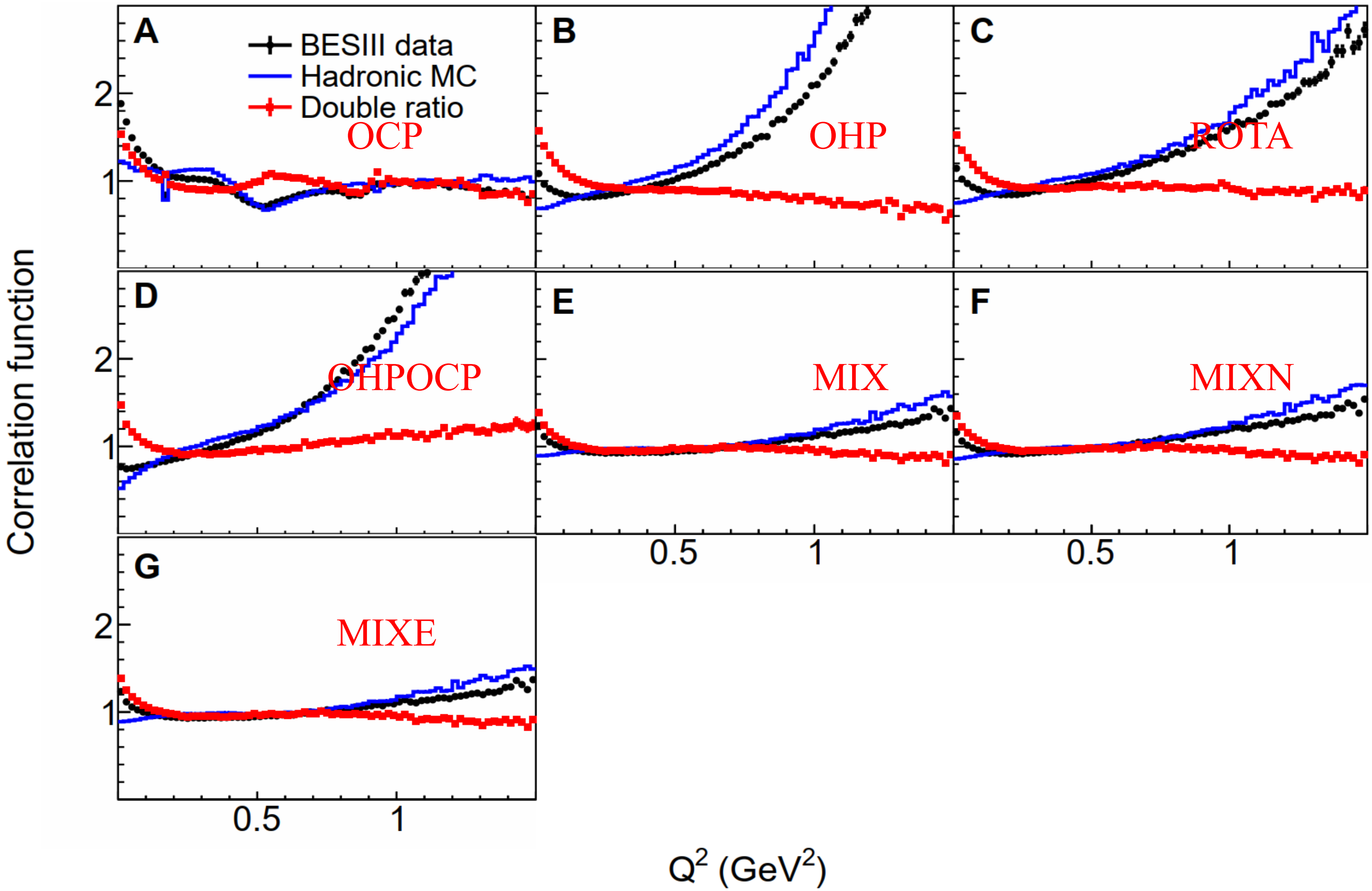
- Extraction of the 1-d BEC parameters, fitting the corrected $R(Q^2)$ with different source assumption:
  - Gaussian source: $R(\mathrm{Q}^2) = N(1 + \delta Q^2)(1 + \lambda e^{-r^2Q^2})$
  - Exponential source: $R(\mathrm{Q}^2) = N(1 + \delta Q^2)(1 + \lambda e^{-rQ})$
  - Lévy-type source: $R(\mathrm{Q}^2) = N(1 + \delta Q^2)(1 + \lambda e^{-(rQ)^\alpha})$

Here N is the normalized constant, λ is the incoherent parameter, r is the source size, and $(1 + \delta Q^2)$ accounts the residual nonBEC correlations not removed by reference sample and double ratio method.

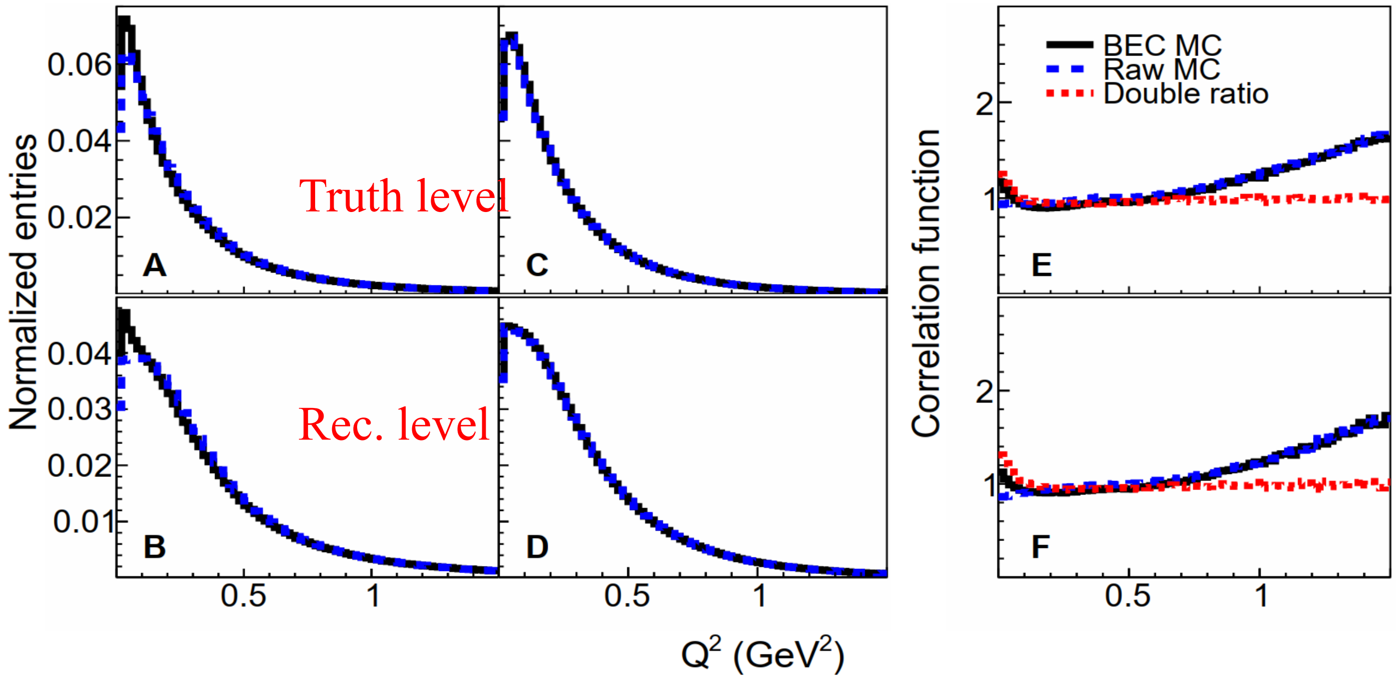
Q^2 distributions

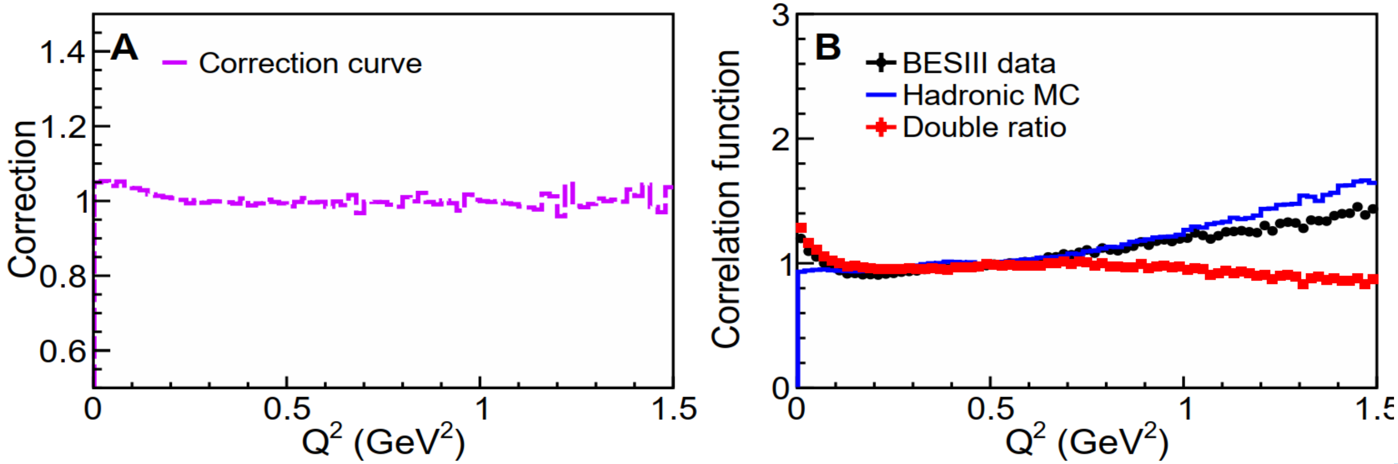


Correlation function

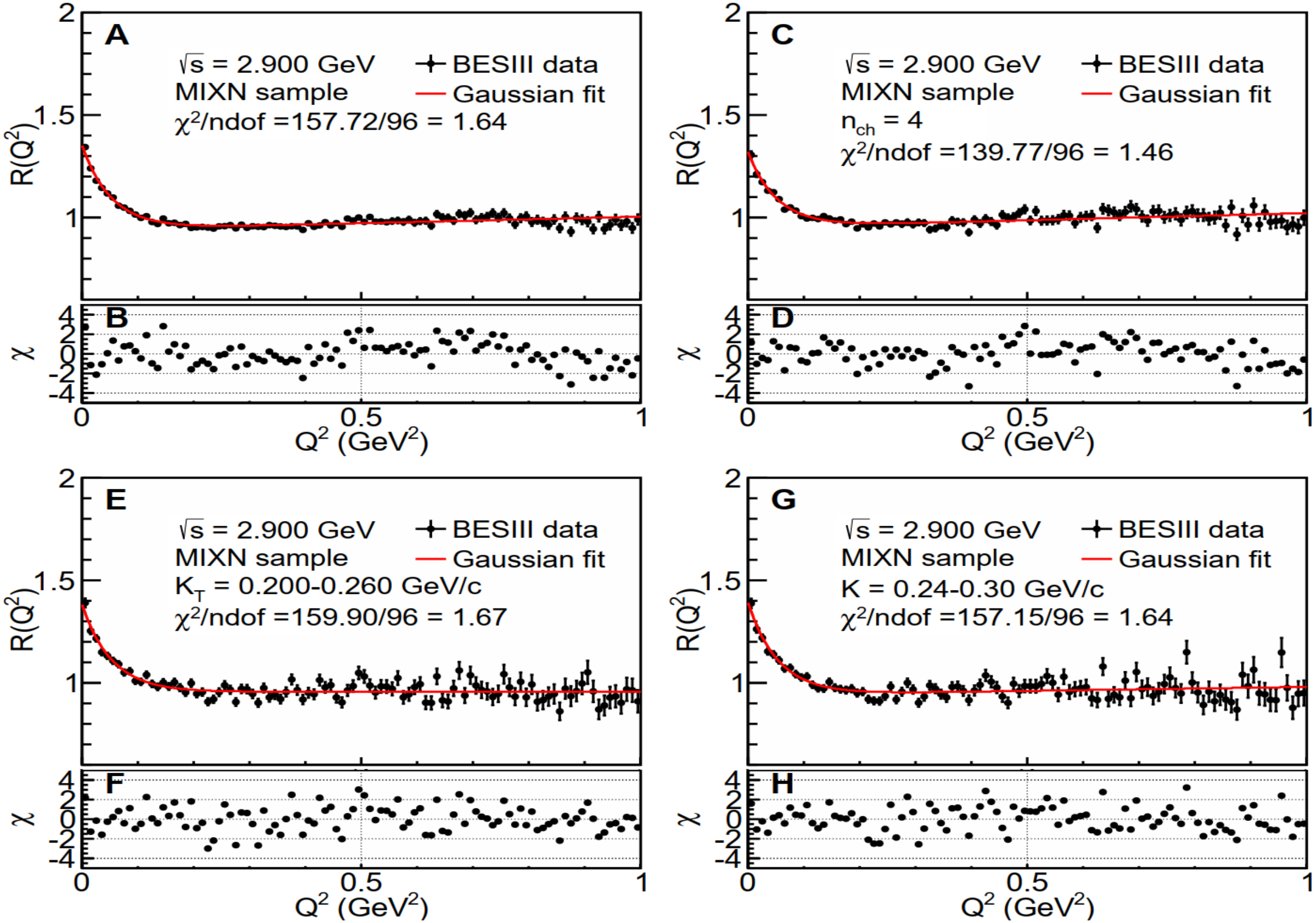


BEC MC distribution



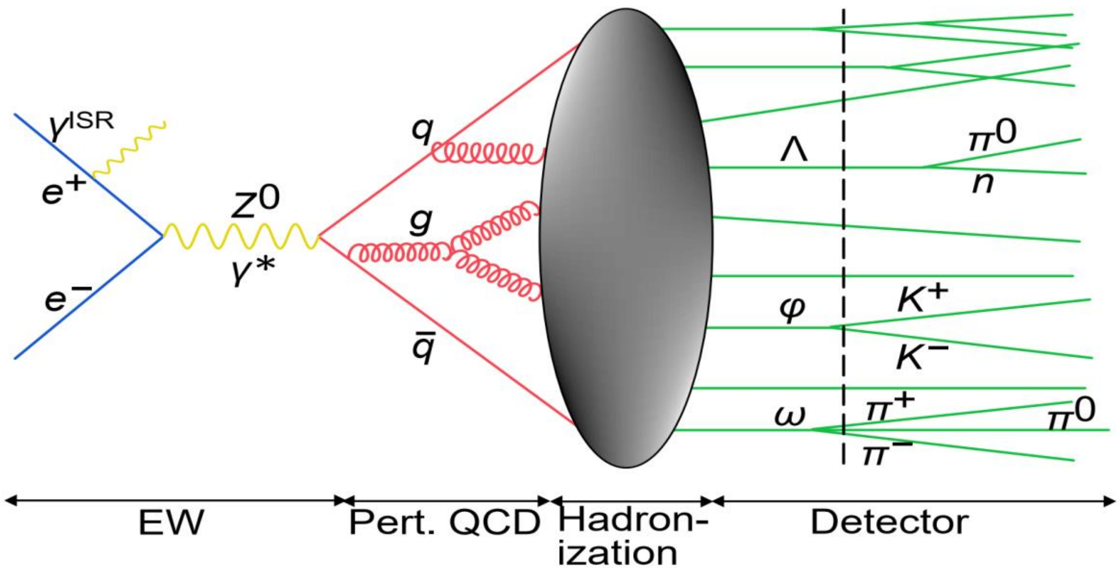


Fit


A
$\sqrt{s}$ = 2.900 GeV
MIXN sample
χ^2/ndof =157.72/96 = 1.64
BESIII data
Gaussian fit
B
C
$\sqrt{s}$ = 2.900 GeV
MIXN sample
n_{ch} = 4
χ^2/ndof =139.77/96 = 1.46
BESIII data
Gaussian fit
D
E
$\sqrt{s}$ = 2.900 GeV
MIXN sample
K_T = 0.200-0.260 GeV/c
χ^2/ndof =159.90/96 = 1.67
BESIII data
Gaussian fit
F
G
$\sqrt{s}$ = 2.900 GeV
MIXN sample
K = 0.24-0.30 GeV/c
χ^2/ndof =157.15/96 = 1.64
BESIII data
Gaussian fit
H
$R(Q^2)$
χ
Q^2 (GeV^2)

Research Status

- Experimental approaches in hadron physics:
  - Probing the internal structures of hadrons through precise measurements of cross sections and electromagnetic form factors.
  - Investigating hadron source structures by analyzing production processes, including fragmentation function measurements, BEC studies, and etc.
  - Comparing with the former, works related to the later are very rare, most of which performed in high-energy region.
  - However, experimental study in the non-perturbative energy region are very important.



Research Status

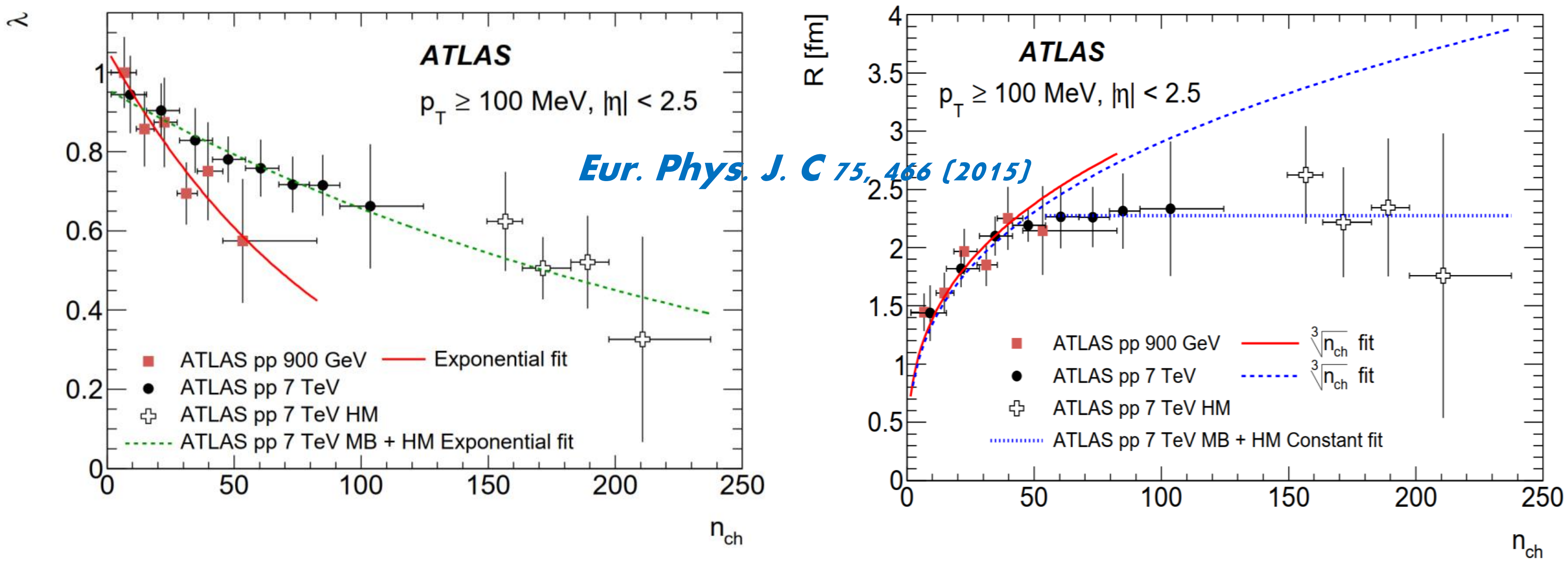
- 1-d BEC parameters were extracted by a lot of experiments, in different reactions, with most of the energies from dozens of GeV to TeV.
- Incoherent parameter λ, always determined to be less than 1, indicating a partially coherent emissions.
- Hydrodynamic-based theoretical studies attribute this to the mutual coherence of closely located emitters due to the uncertainty principle, residual nonfemtoscopic correlations (such as minijets and initial-state fluctuations), and etc.
- The effective source size r is found to be in on the femtometer scale.

Table: Summary of part previous measurement results of BEC parameters.

Experiment	$\sqrt{s}$ (GeV)	r (fm)	λ	Exp. type
MARK II	29	0.97±0.11	0.27±0.04	e^+e^-
AMY	58	0.58±0.06	0.39±0.05	e^+e^-
OPAL	91	0.79±0.02	0.85±0.01	e^+e^-
NA22	21.7	0.83±0.06	0.33±0.02	π^+p
ZEUS	10.5	0.67±0.04	0.43±0.09	ep
CMS	900	0.90±0.03	0.63±0.02	pp
CMS	2360	1.12±0.10	0.66±0.07	pp
ATLAS	13000	2.12±0.08	1.00±0.08	pp

Research Status

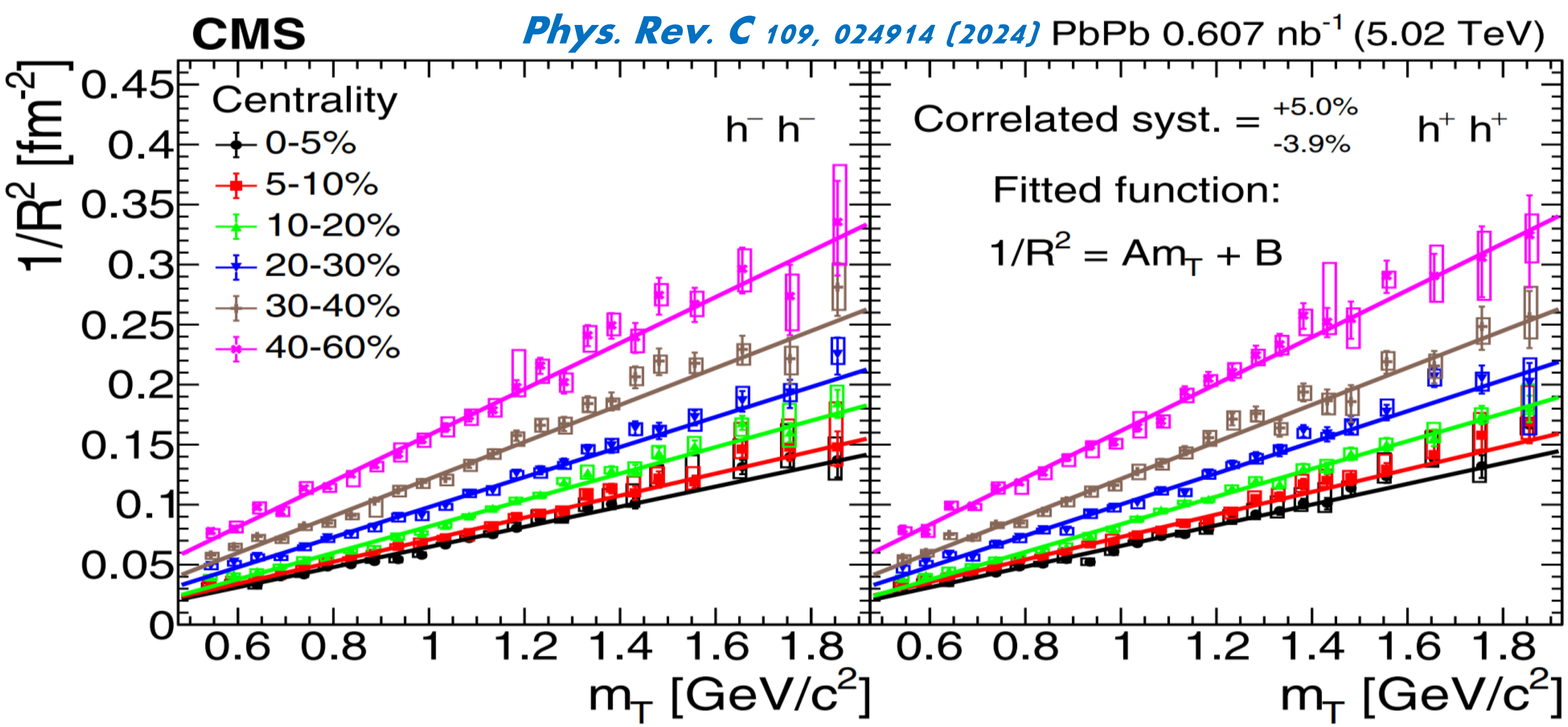
- The dependence of BEC parameters on charged particle multiplicity (n_{ch}) widely studied.
- Revealing a decreasing trend for λ and an increasing trend for r with rising n_{ch}.
- A direct proportionality to $\sqrt[3]{n_{ch}}$ and the onset of saturation in r at high multiplicities have also been observed.
- These trends are successfully reproduced by hydrodynamic/hydrokinetic and Pomeron-based models of multiparticle production.



Eur. Phys. J. C 75, 466 (2015)

Research Status

- The dependence of BEC parameters on pair average transverse momentum ($K_T = (p_{T,1} + p_{T,2})/2$) widely investigated.
- Both λ and r show a decreasing trend with rising K_T.
- This behavior for r is predicted by hydrodynamic-based theoretical work, with some studies suggesting a direct proportionality to $1/\sqrt{m_T}$, confirmed experimentally.



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- Experimental work:
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Summary

- Intensity interferometry (Hanbury-Brown/Twiss effect), originating from astrophysics, developed by Hanbury-Brown and Twiss in the 1950s, has widely used in subatomic physics field as a powerful probe to hadron source geometry.
- However, most of previous work mainly focus on the high-energy scenario, leaving a critical gap in low-energy region, where experiment works play a more important roles.
- BESIII accumulate large data samples in the energy range 1.94~4.96 GeV, offering unique opportunities to perform this kind of work.
- The first BEC study for charged pions at BESIII has been internal reviewed in BESIII Collaboration.
- In the future, more BEC analysis could be performed at BESIII, such as $\pi^0\pi^0$, $K\bar{K}$, and BEC in three particles, in 3-d source assumption, time-evolution source assumption, etc.

Summary

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Thanks for your attention!