The $\phi(2170)$ state in $e^+e^- \to \phi\pi^+\pi^-$ reaction as a triangle singularity

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Outline

Introduction: the $\phi(2170)$ state

Formalism: the triangle singularity

Fitted results: the peak stucture of $\phi(2170)$ can be reproduced by the $K_1 - K - \bar{K}$ triangle loop

Summary

Discover of the $\phi(2170)$

$e^+e^- \to \gamma_{ISR}\phi f_0(980)$
BABAR 2006

$e^+e^- \to \phi\pi^+\pi^-$
Belle 2009

$e^+e^- \to K^+K^-$
BESIII 2019

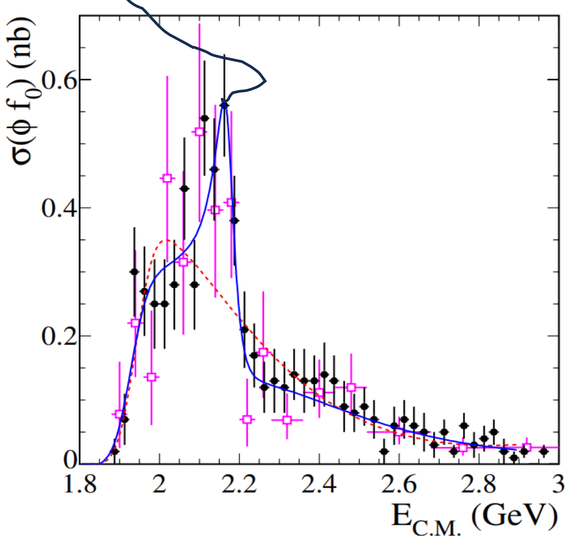
$e^+e^- \to K^+K^-\pi^0\pi^0$
BESIII 2020

$e^+e^- \to \phi\eta'$
BESIII 2020

$e^+e^- \to \phi\eta$
BESIII 2021

$e^+e^- \to K_S^0K_L^0$
BESIII 2021

$e^+e^- \to \phi\pi^+\pi^-$
BESIII 2023



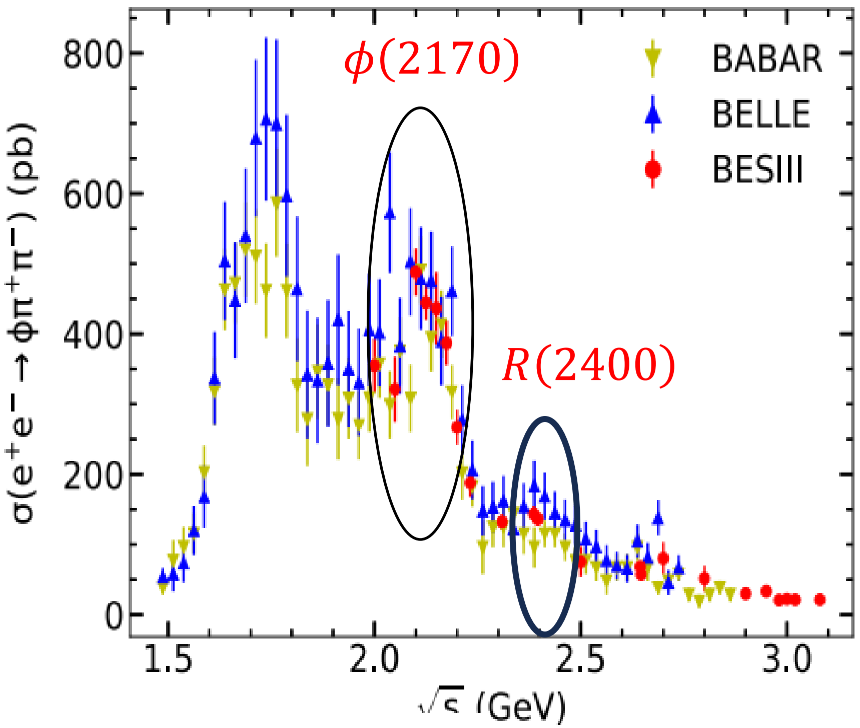
PHYSICAL REVIEW D 74, 091103(R) (2006)

Structure at 2175 MeV in $e^+e^- \to \phi f_0(980)$ observed via initial-state radiation

We study the initial-state-radiation processes $e^+e^- \to K^+K^-\pi^+\pi^-\gamma$ and $e^+e^- \to K^+K^-\pi^0\pi^0\gamma$ using an integrated luminosity of 232 fb^{-1} collected at the $\Upsilon(4S)$ mass with the *BABAR* detector at SLAC. Even though these reactions are dominated by intermediate states with excited kaons, we are able to study for the first time the cross section for $e^+e^- \to \phi(1020)f_0(980)$ as a function of center-of-mass energy. We observe a structure near threshold consistent with a 1^{--} resonance with mass $m = 2.175 \pm 0.010 \pm 0.015$ GeV/c^2 and width $\Gamma = 58 \pm 16 \pm 20$ MeV. We observe no $Y(4260)$ signal and set a limit of $\mathcal{B}_{Y\to\phi\pi^+\pi^-} \cdot \Gamma_{ee}^{Y} < 0.4$ eV (90% confidence level), which excludes some models.

PHYSICAL REVIEW D **108,** 032011 (2023)

Measurement of $e^+e^- \to \phi\pi^+\pi^-$ cross sections at center-of-mass energies from 2.00 to 3.08 GeV



Using data corresponding to an integrated luminosity of 651 pb^{-1} accumulated at 22 center-of-mass energies from 2.00 to 3.08 GeV by the BESIII experiment, the process $e^+e^- \to \phi\pi^+\pi^-$ is studied. The cross sections for $e^+e^- \to \phi\pi^+\pi^-$ are consistent with previous results, but with improved precision. To measure the mass and width of the structure observed in the cross section line shape, a combine fit is performed after enhancing the contribution from $\phi f_0(980)$. The fit reveals a structure with the mass of $M = 2178 \pm 20 \pm 5$ MeV$/c^2$ and the width of $\Gamma = 140 \pm 36 \pm 16$ MeV, where the first uncertainties are statistical and the second ones are systematic.

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In addition, no clear structure around 2.4 GeV has been found in this analysis. Since this structure at the same energy has been seen in the $K^+K^-f_0(980)$ mode with $f_0(980) \to \pi^+\pi^-$ and $\pi^0\pi^0$ [2,3], a future study of this channel with an amplitude analysis will be helpful to improve knowledge of the R(2400) state.

Resonance parameter of $\phi(2170)$

	BABAR		BESIII		Belle	
	M(MeV)	Γ(MeV)	M(MeV)	Γ(MeV)	M(MeV)	Γ(MeV)
$e^+e^- \to \gamma_{ISR}\phi f_0(980)$	$2175 \pm 10 \pm 15$	$58 \pm 16 \pm 20$	$2135 \pm 8 \pm 9$	$104 \pm 24 \pm 12$	2163 ± 32	125 ± 40
$e^+e^- \to K^+K^-$	2201 ± 19	70 ± 38	$2239.2 \pm 7.1 \pm 11.3$	$139.8 \pm 12.3 \pm 20.6$		
$e^+e^- \to K^+K^-\pi^0\pi^0$	2169 ± 20	102 ± 27	$2126.5 \pm 16.8 \pm 12.4$	$106.9 \pm 32.1 \pm 28.1$		
$e^+e^- \to \phi\eta'$			$2177.5 \pm 4.8 \pm 19.5$	$149.0 \pm 15.6 \pm 8.9$		
$e^+e^- \to \phi\eta$	$2125 \pm 22 \pm 10$	$61 \pm 50 \pm 13$	$2163.5 \pm 6.2 \pm 3.0$	$31.1^{+21.1}_{-11.6}$		
$e^+e^- \to K_S^0K_L^0$			$2273.7 \pm 5.7 \pm 19.3$	$86 \pm 44 \pm 51$		
$e^+e^- \to \phi\pi^+\pi^-$	$2180 \pm 8 \pm 8$	$77 \pm 15 \pm 10$	$2178 \pm 20 \pm 5$	$140 \pm 36 \pm 16$	2079 ± 13	192 ± 23

Citation: S. Navas et al. (Particle Data Group), Phys. Rev. D 110, 030001 (2024) and 2025 update

$\phi(2170)$

$I^G(J^{PC}) = 0^-(1^{--})$

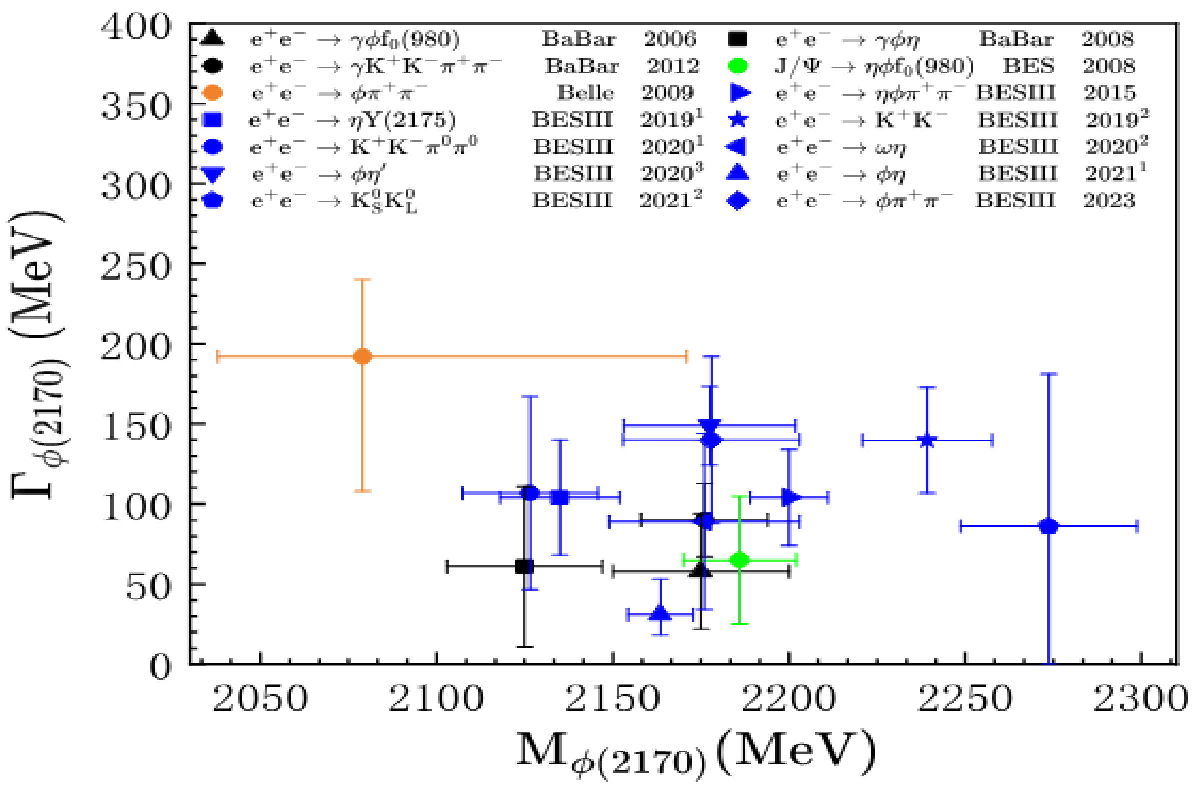
$\phi(2170)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
2164 ± 5 OUR AVERAGE					
$2164.7 \pm 9.1 \pm 3.1$		[1] ABLIKIM	24AN	BES3	$e^+e^- \to K_S^0K_L^0\pi^0$
$2178 \pm 20 \pm 5$		[2] ABLIKIM	23AX	BES3	$e^+e^- \to \phi\pi^+\pi^-$
$2190 \pm 19 \pm 37$		[3] ABLIKIM	22L	BES3	2.0–3.08 $e^+e^- \to K^+K^-\pi^0$
$2163.5 \pm 6.2 \pm 3.0$		[4] ABLIKIM	21T	BES3	$e^+e^- \to \phi\eta$
$2177.5 \pm 4.8 \pm 19.5$		[5] ABLIKIM	20M	BES3	$e^+e^- \to \eta'\phi$
$2126.5 \pm 16.8 \pm 12.4$		[6] ABLIKIM	20S	BES3	$e^+e^- \to K^+K^-\pi^0\pi^0$

$\phi(2170)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID		TECN	COMMENT
88^{+26}_{-21} OUR AVERAGE		Error includes scale factor of 2.5. See the ideogram below.			
$32.4 \pm 21.0 \pm 1.8$		[1] ABLIKIM	24AN	BES3	$e^+e^- \to K_S^0K_L^0\pi^0$
$140 \pm 36 \pm 16$		[2] ABLIKIM	23AX	BES3	$e^+e^- \to \phi\pi^+\pi^-$
$191 \pm 28 \pm 60$		[3] ABLIKIM	22L	BES3	2.0–3.08 $e^+e^- \to K^+K^-\pi^0$
$31.1^{+21.1}_{-11.6} \pm 1.1$		[4] ABLIKIM	21T	BES3	$e^+e^- \to \phi\eta$
$149.0 \pm 15.6 \pm 8.9$		[5] ABLIKIM	20M	BES3	$e^+e^- \to \eta'\phi$
$106.9 \pm 32.1 \pm 28.1$		[6] ABLIKIM	20S	BES3	$e^+e^- \to K^+K^-\pi^0\pi^0$

The current situation of $\phi(2170)$



Published experimental information

- ✓ **Limited decay modes**
- ✓ **Inconsistence on mass & width**

From Wen-Biao Yan's PPT

Theoretical perspectives on the $\phi(2170)$

$s\bar{s}$

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$ss\bar{s}\bar{s}$

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$\phi K\bar{K}$ or $\phi f_0(980)$ molecular state

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NEW explanation:
triangle singularity

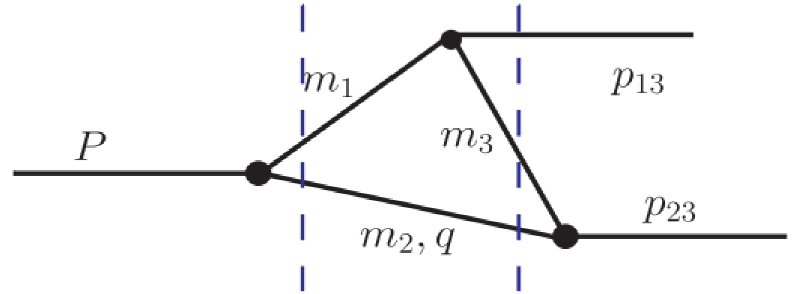


FIG. 2. A triangle diagram showing the notations used in the general discussion of triangle singularities, where m_i's denote the masses of the intermediate particles and P, p_{13}, and p_{23} correspond to the four-momenta of the external particles. In the text, we have defined $\vec{k} \equiv \vec{p}_{13}$ for simplicity. The two dashed vertical lines correspond to the two relevant cuts.

$$I = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - m_2^2 + i\varepsilon)[(P-q)^2 - m_1^2 + i\varepsilon][(p - q - p_{13})^2 - m_3^2 + i\varepsilon]}$$

$$I(m_{23}) = \int d^3 q \frac{1}{[P^0 - \omega_1(\vec{q}) - \omega_2(\vec{q}) + i\epsilon][E_{23} - \omega_2(\vec{q}) - \omega_3(\vec{k}+\vec{q}) + i\epsilon]} = 2\pi \int_0^\infty dq \frac{q^2}{P^0 - \omega_1(q) - \omega_2(q) + i\epsilon} f(q),$$

$\omega_3(\vec{k} + \vec{q}) = \sqrt{m_3^2 + (\vec{k} + \vec{q})^2}$, $E_{23} = P^0 - k^0$, and

$$f(q) = \int_{-1}^{1} dz \frac{1}{E_{23} - \omega_2(q) - \sqrt{m_3^2 + q^2 + k^2 + 2qkz} + i\epsilon}.$$

Melahat Bayar, Francesca Aceti, Feng-Kun Guo, and Eulogio Oset, Phys. Rev. D 94, 074039 (2016)

$$q_{on+} = \frac{\lambda^{1/2}(M^2, m_1^2, m_2^2)}{2M} + i\varepsilon$$

$$q_{a-} = \gamma(\nu E_2^* - p_2^*) - i\varepsilon$$

$$v = \frac{k}{E_{23}},$$

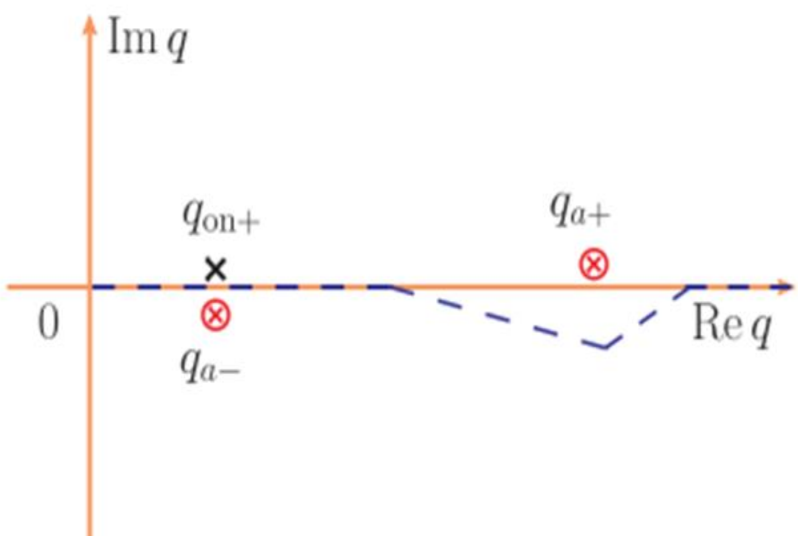
$$\gamma = \frac{1}{\sqrt{1-v^2}} = \frac{E_{23}}{m_{23}},$$

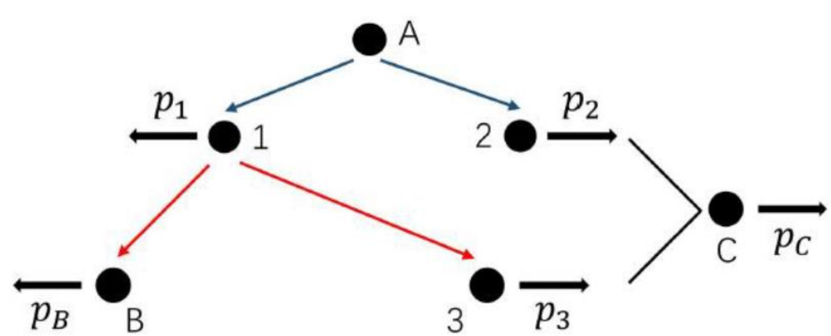
$$E_2^* = \frac{1}{2m_{23}}(m_{23}^2 + m_2^2 - m_3^2),$$

$$p_2^* = \frac{1}{2m_{23}}\sqrt{\lambda(m_{23}^2, m_2^2, m_3^2)}.$$

The condition for a triangle singularity to emerge is given mathematically by

$$\lim_{\epsilon \to 0}(q_{\text{on}+} - q_{a-}) = 0.$$





Particle 3 catch up particle 2.

An example: the $a_1(1420)$

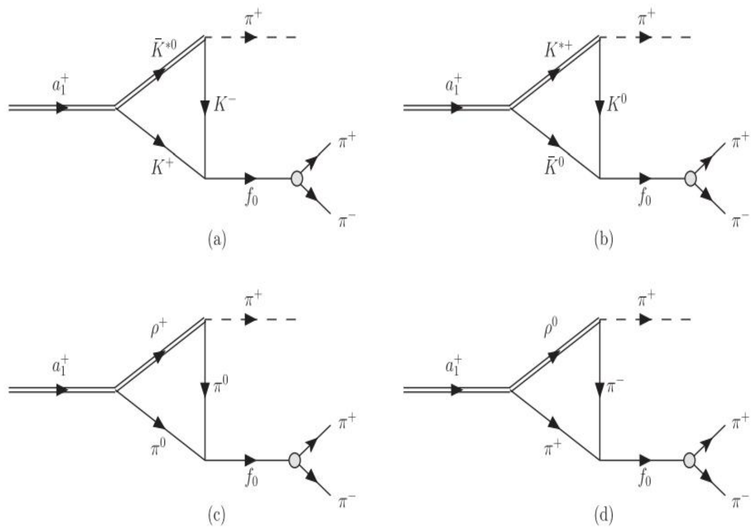


FIG. 1. Feynman diagrams for the process $a_1^+(1260) \to \pi^+\pi^+\pi^-$, where the $a_1^+(1260)$ decays to $\bar{K}^*K$ [diagrams (a) and (b)] and to $\rho\pi$ [diagrams (c) and (d)].

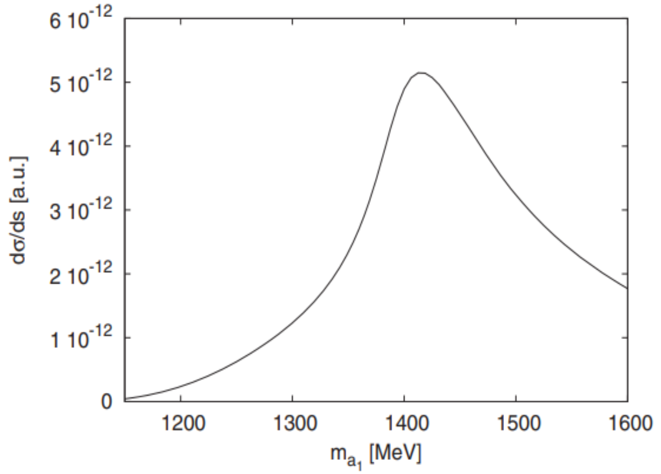


FIG. 7. Cross section for the decay $a_1^+(1260) \to \pi^+ f_0(980)$ as a function of the center of mass energy.

F. Aceti, L.R. Dai, and E. Oset

PHYSICAL REVIEW D **94,** 096015 (2016)

$a_1(1420)$ peak as the $\pi f_0(980)$ decay mode of the $a_1(1260)$

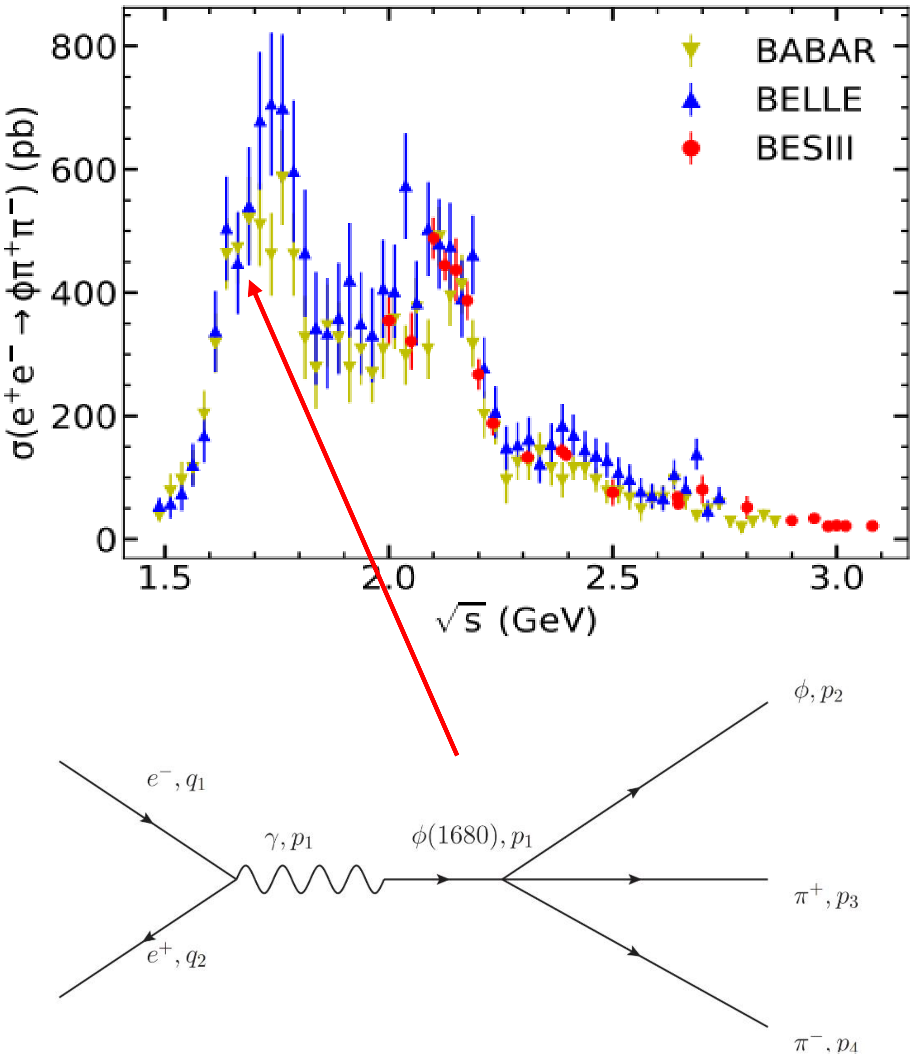
PHYSICAL REVIEW LETTERS **127,** 082501 (2021)

Triangle Singularity as the Origin of the $a_1(1420)$

The COMPASS Collaboration experiment recently discovered a new isovector resonancelike signal with axial-vector quantum numbers, the $a_1(1420)$, decaying to $f_0(980)\pi$. With a mass too close to and a width smaller than the axial-vector ground state $a_1(1260)$, it was immediately interpreted as a new light exotic meson, similar to the X, Y, Z states in the hidden-charm sector. We show that a resonancelike signal fully matching the experimental data is produced by the decay of the $a_1(1260)$ resonance into $K^*(\to K\pi)\bar{K}$ and subsequent rescattering through a triangle singularity into the coupled $f_0(980)\pi$ channel. The amplitude for this process is calculated using a new approach based on dispersion relations. The triangle-singularity model is fitted to the partial-wave data of the COMPASS experiment. Despite having fewer parameters, this fit shows a slightly better quality than the one using a resonance hypothesis and thus eliminates the need for an additional resonance in order to describe the data. We thereby demonstrate for the first time in the light-meson sector that a resonancelike structure in the experimental data can be described by rescattering through a triangle singularity, providing evidence for a genuine three-body effect.

$e^+e^- \to \phi\pi^+\pi^-$ reaction

Contribution of $\phi(1680)$



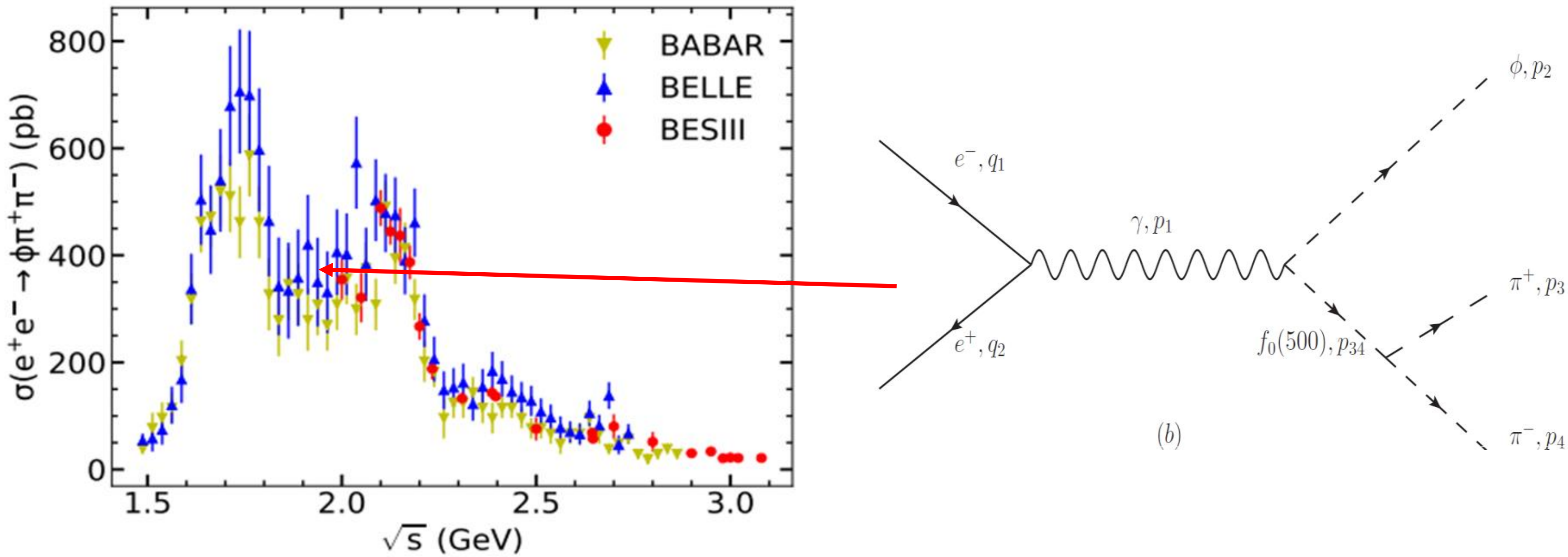
Citation: S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024) and 2025 update

ϕ(1680) DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}^*(892)+$ c.c.	seen
Γ_2	$K^0_S K\pi$	seen
Γ_3	$K\overline{K}$	seen
Γ_4	$K^0_L K^0_S$	
Γ_5	e^+e^-	seen
Γ_6	$\omega\pi\pi$	not seen
Γ_7	$\phi\pi\pi$	
Γ_8	$K^+K^-\pi^+\pi^-$	seen
Γ_9	$\eta\phi$	seen
Γ_{10}	$\eta\gamma$	seen
Γ_{11}	$K^+K^-\pi^0$	
Γ_{12}	$f_2'(1525)\gamma$	not seen

$$\mathcal{M}_1 = \frac{ieg_{\phi(1680)}M^2_{\phi(1680)}}{s[s - M^2_{\phi(1680)} + iM_{\phi(1680)}\Gamma_{\phi(1680)}]}\bar{\nu}_e(q_2,s_2)(\gamma^\alpha - \not{p}_1 p_1^\alpha/s)u_e(q_1,s_1)\varepsilon_\alpha(p_2,s_\phi)$$

Contribution from ππ final state interaction



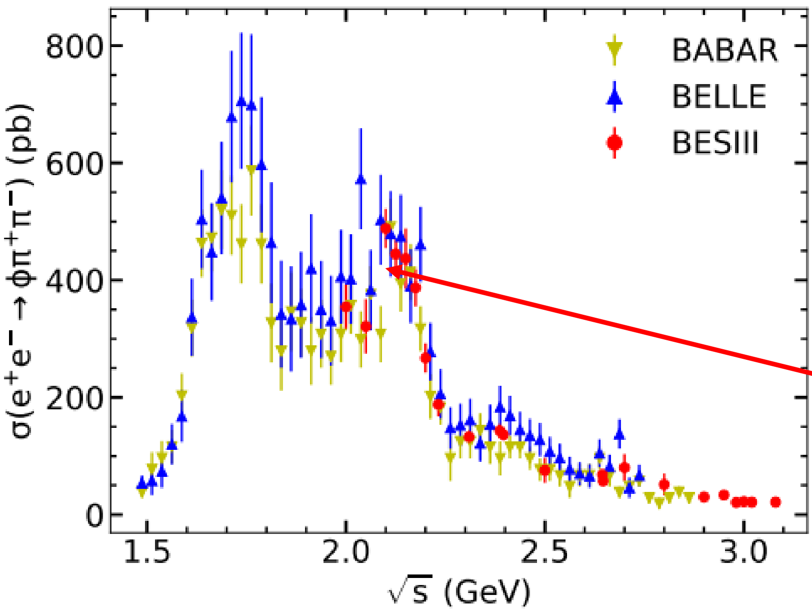
$$\mathcal{M}_2 = \frac{ie\mathcal{F}(s)g_{\gamma\phi f_0(500)}g_{f_0(500)\pi\pi}M^2_{f_0(500)}\mathcal{F}_{f_0(500)}(p^2_{34})}{s[p^2_{34} - M^2_{f_0(500)} + iM_{f_0(500)}\Gamma_{f_0(500)}]}\bar{\nu}_e(q_2, s_2)\gamma^\alpha u_e(q_1, s_1)\varepsilon_\alpha(p_2, s_\phi)$$

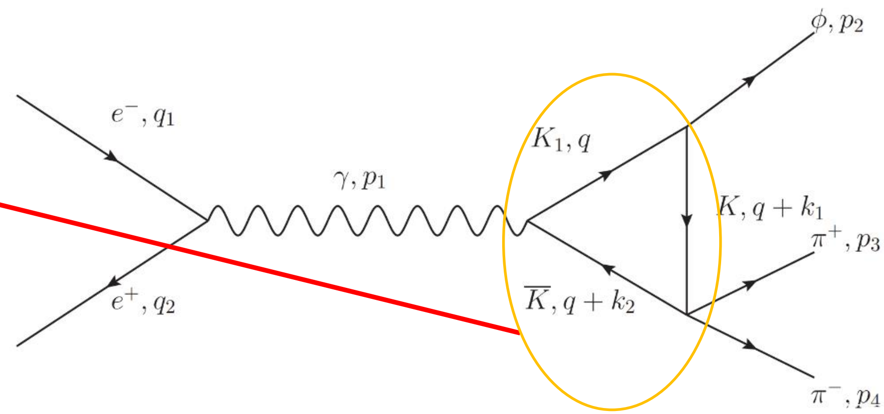
NEW explanation of ϕ(2170) in the $e^+e^- \to \phi\pi^+\pi^-$ reaction: triangle singularity

M. Ablikim et al. (BESIII Collaboration), Phys. Rev. D 108, 032011(2023)

J. P. Lees et al. (BABAR Collaboration), Phys. Rev. D 86,012008 (2012).

C. P. Shen et al. (Belle Collaboration), Phys. Rev. D 80, 031101 (2009).





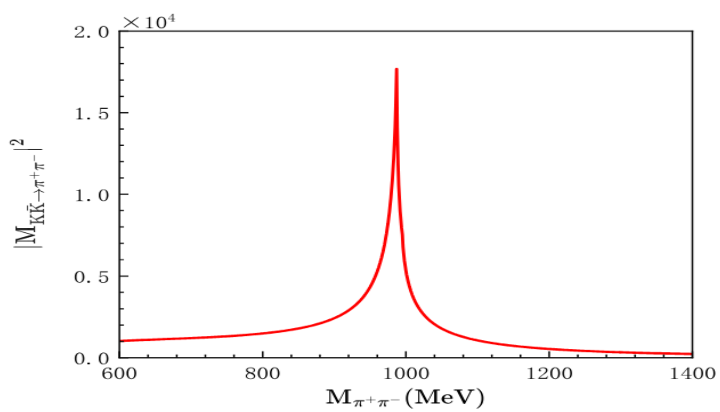
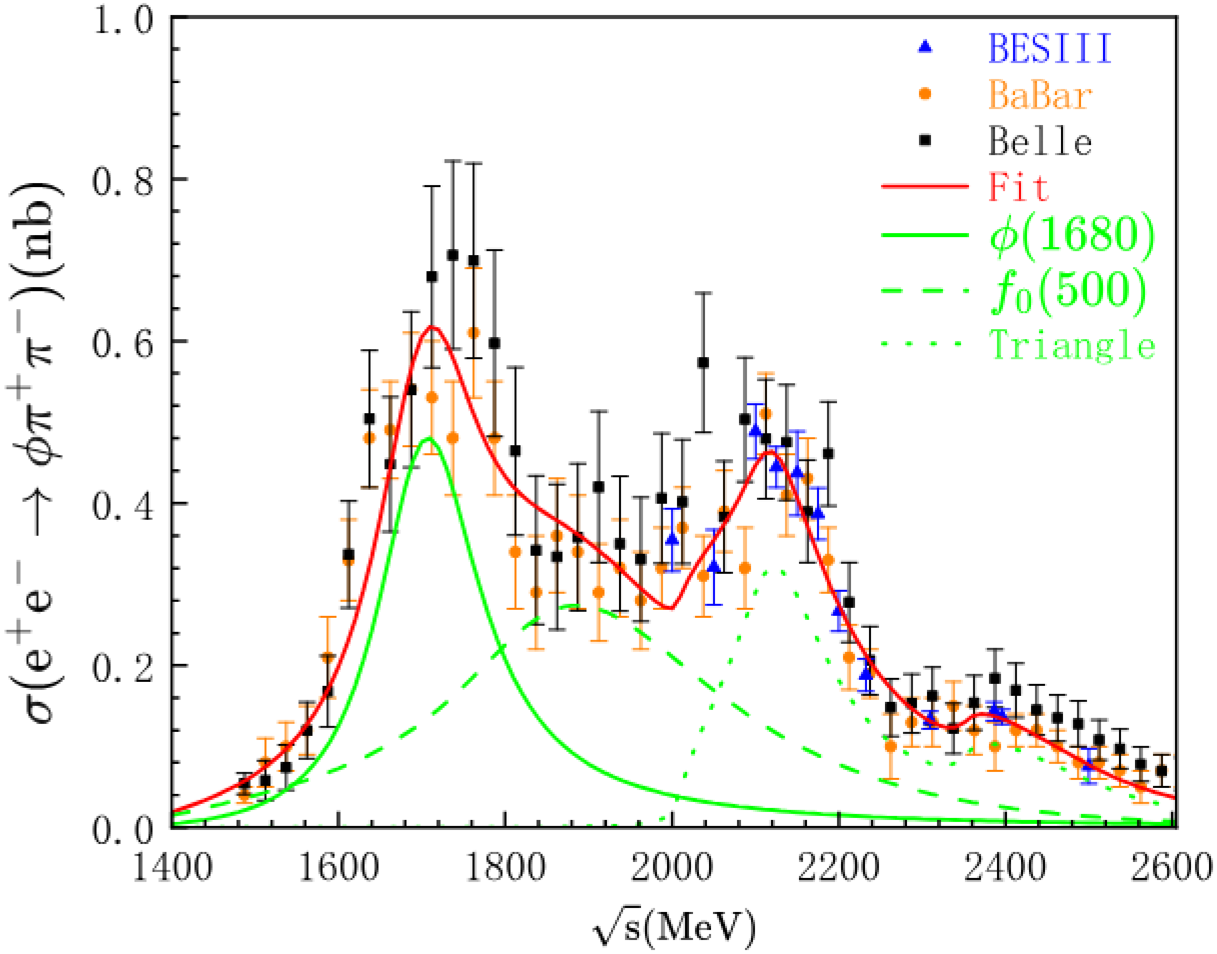


FIG. 3. The two-body scattering amplitude of $K\bar{K} \to \pi^+\pi^-$ transition.

$$M_3 = \frac{ie\mathcal{F}(s) g_{\gamma K_1 \bar{K}} g_{K_1 \phi K} M_{K_1}}{s} \int d^4q \frac{\bar{\nu}_e(q_2,s_2)\gamma_\beta u_e(q_1,s_1)(g^{\beta\alpha} - q^\beta q^\alpha/q^2)\varepsilon_\alpha(p_2,s_\phi) M_{K\bar{K}\to\pi^+\pi^-}}{(q^2 - M_{K_1}^2 + iM_{K_1}\Gamma_{K_1})[(q-p_2)^2 - m_K^2][(q-p_1)^2 - m_K^2]}$$

$$F(s) = e^{-b\sqrt{s}}$$

Fitted results



Fitted model parameters

TABLE I. The values of the model parameters determined by fitting them to the total cross sections of the $e^+e^- \to \phi\pi^+\pi^-$ reaction.

Parameter	Fitted results	Parameter	Fitted results
$g_{\phi(1680)}$	0.25 ± 0.01	$g^a_{K_1}$ $(\times 10^{-4})$	1.34 ± 0.20
$g_{\gamma\phi f_0(500)}$ $(\times 10^{-3})$	1.40 ± 0.08	$g^b_{K_1}$ $(\times 10^{-4})$	2.57 ± 0.52
$M^a_{K_1}$ (MeV)	1612 ± 23	$\Gamma^a_{K_1}$ (MeV)	60 ± 22
$M^b_{K_1}$ (MeV)	1855 ± 88	$\Gamma^b_{K_1}$ (MeV)	21 ± 20
b $(\times 10^{-3}\mathrm{MeV}^{-1})$	5.18 ± 0.40		

Citation: S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024) and 2025 update

$K_1(1650)$

$I(J^P) = \frac{1}{2}(1^+)$

This entry contains various peaks in strange meson systems ($K^+\phi$, $K\pi\pi$) reported in partial-wave analysis in the 1600–1900 mass region.

$K_1(1650)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
1650±50		FRAME	86	OMEG	+	13 $K^+p \to \phi K^+p$

$K_1(1650)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*		*TECN*	*CHG*	*COMMENT*
150± 50		FRAME	86	OMEG	+	13 $K^+p \to \phi K^+p$

TABLE I. Fit results from the default amplitude model. The significances are evaluated accounting for total (statistical) uncertainties. The listed masses and widths without uncertainties are taken from PDG [14] and are fixed in the fit. The listed world averages of the two K_2 and $K^*(1680)$ resonances do not contain the contributions from the previous LHCb run 1 results.

J^P	Contribution		Significance (σ)	M_0 (MeV)	Γ_0 (MeV)	FF (%)
1^+	2^1P_1	$K(1^+)$	4.5 (4.5)	$1861 \pm 10^{+16}_{-46}$	$149 \pm 41^{+231}_{-23}$	
	2^3P_1	$K'(1^+)$	4.5 (4.5)	$1911 \pm 37^{+124}_{-48}$	$276 \pm 50^{+319}_{-159}$	

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Observation of New Resonances Decaying to $J/\psi K^+$ and $J/\psi\phi$

We should select ππ events outside the $f_0(980)$ mass region to avoid the large contribution from the triangle singularity effects

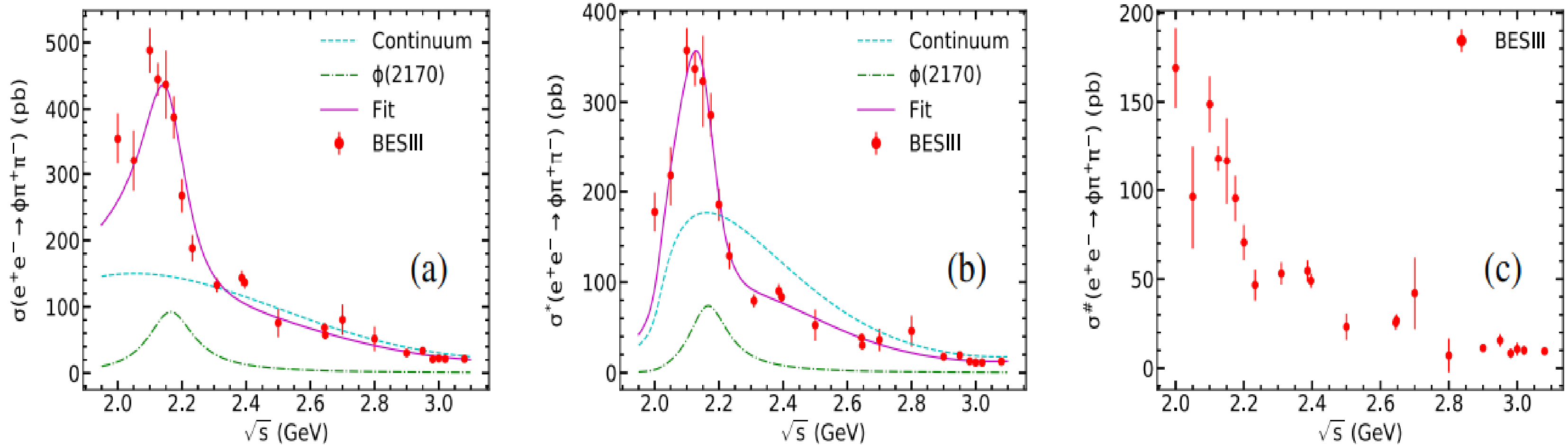
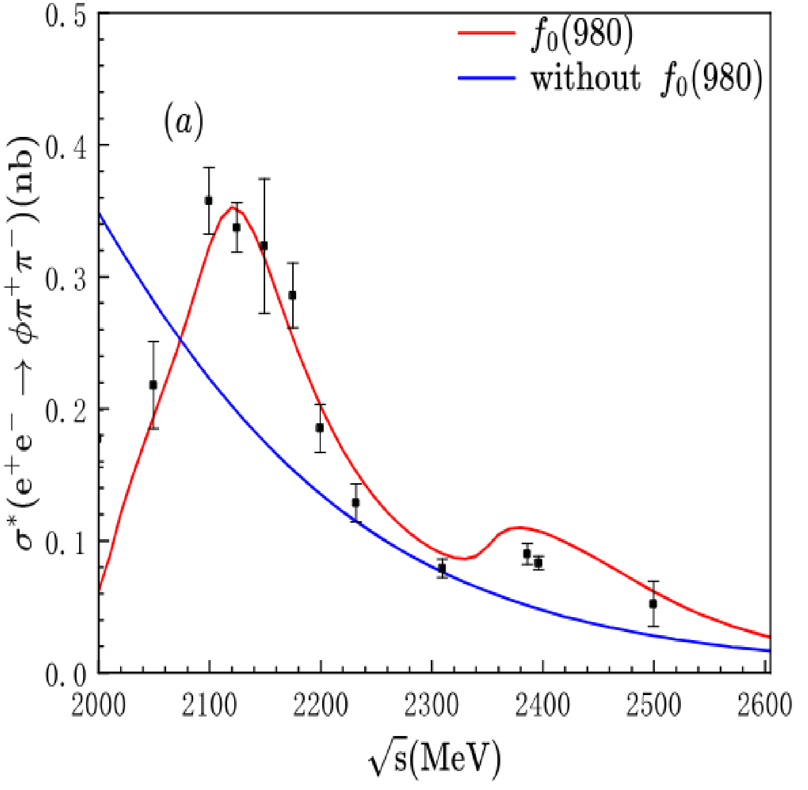
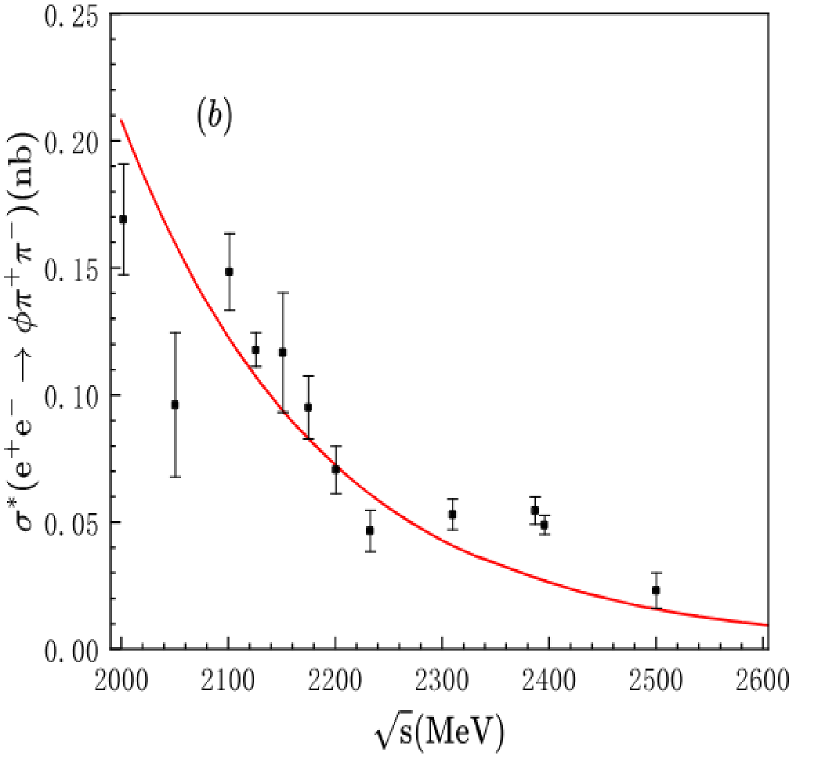


FIG. 5. The fit results of the cross sections of the process $e^+e^- \to \phi\pi^+\pi^-$ with events in (a) full $M_{\pi^+\pi^-}$ range, (b) $M_{\pi^+\pi^-} \in [0.85, 1.1]$ GeV/c^2, and (c) $M_{\pi^+\pi^-} \notin [0.85, 1.1]$ GeV/c^2 GeV/c^2. The red dots with error bars represent the obtained cross sections and the uncertainties incorporate statistical and uncorrelated systematic uncertainties. In (a) and (b), the purple solid line represents the fit function and the green dotted line represents the contribution from $\phi(2170)$. The fits in (a) and (b) show the constructive interference results within two parallel solutions.

Role of $f_0(980)$



$M_{\pi^+\pi^-} \in [850,1100]$ MeV



$M_{\pi^+\pi^-} \notin [850,1100]$ MeV

Summary

We conclude that

The $\phi(2170)$ state in $e^+e^- \to \phi\pi^+\pi^-$ reaction can be explained as a triangle singularity.
Existence of a previously unobserved K_1 state.

However

We need more efforts, both on theoretical and experimental sides.

Thank you very much for your attention!