第十届BESIII R值与QCD强子结构研讨会
2025.7.26-30, 乌鲁木齐

Unified study of two-boson tau decays within resonance chiral theory

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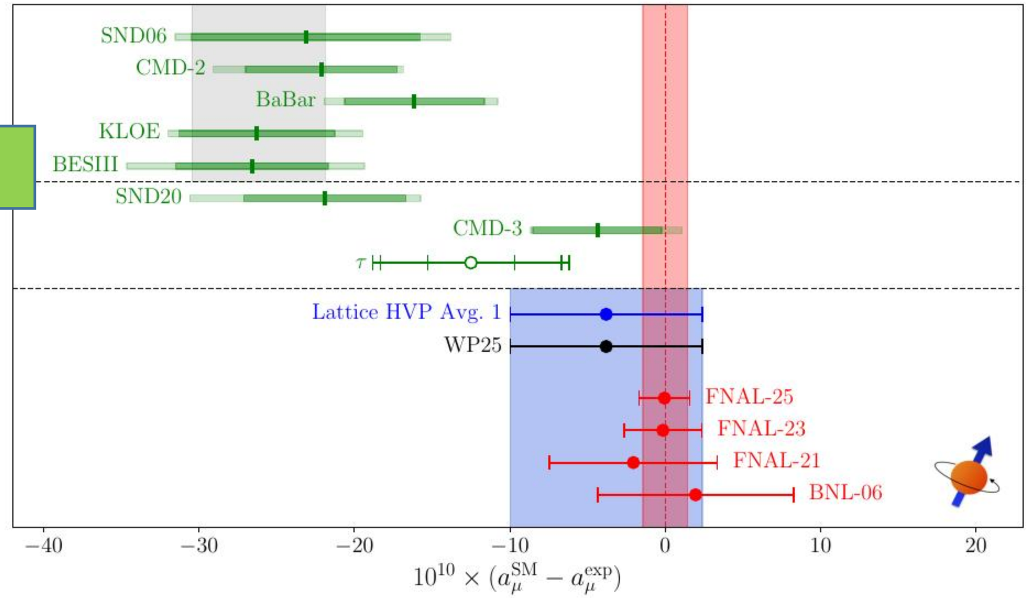
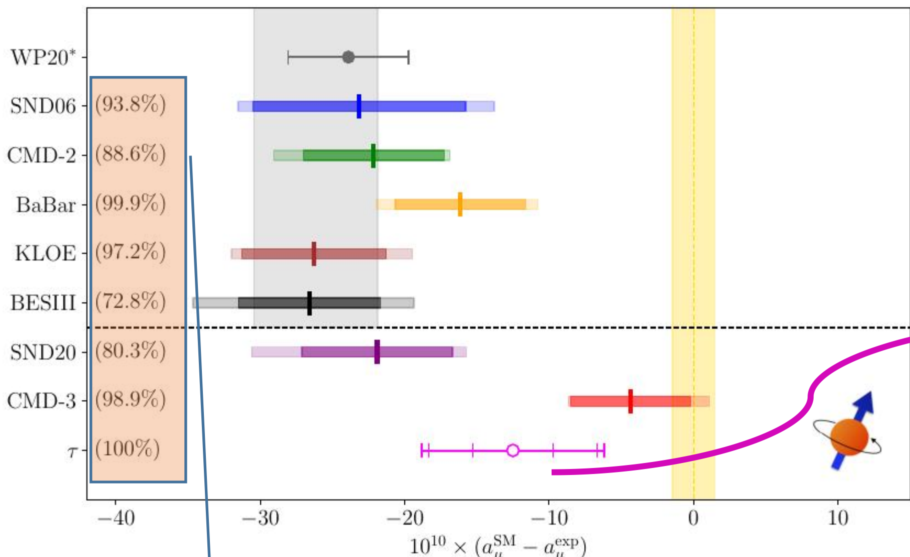
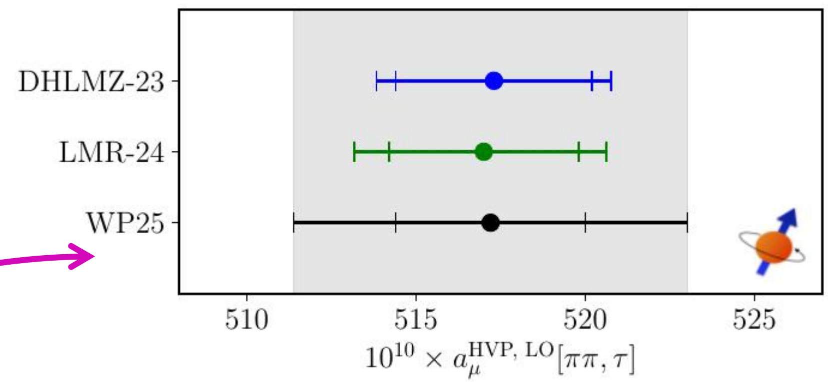
Current status on muon g-2

[2505.21476, White Paper 25]

- $e^+e^- \to \pi^+\pi^-$ from each indicated Exp
- HVP-LO beyond $\pi\pi$ from WP20
- Others, such as HLbL, EW, QED, from WP25







$\pi\pi$ contribution from each Exp to HVP integral

Alternative way to address HVP from $\pi\pi$

$$a_\mu^{\mathrm{HVP,LO}} = \frac{1}{4\pi^3}\int_{4M_2^2}^{t_{max}} dt K(t)\sigma^0_{e^+e^-\to\mathrm{hadrons}}(t)$$

Known kernel function (enhanced contribution from energy below 1GeV)

$$\sigma^0_{e^+e^-\to\pi^+\pi^-} = \frac{\pi\alpha^2}{3t}\beta_{\pi^+\pi^-}\left|F^{(0)}_{\pi\pi}(t)\right|^2 \qquad \frac{d\Gamma(\tau_{2\pi})}{dt} = \frac{G_F^2\left|V_{ud}\right|^2 m_\tau^3 S_{\mathrm{EW}}}{384\pi^3}\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+\frac{2t}{m_\tau^2}\right)\beta_{\pi^-\pi^0}\left|F^{(-)}_{\pi\pi}(t)\right|^2$$

$\tau^- \to \pi^-\pi^0\nu_\tau$: $\langle\pi^-\pi^0|\bar{d}\gamma_\mu u|0\rangle \sim F^{(-)}_{\pi\pi}(t)$ [I=1, I_3=-1]

$e^+e^- \to \pi^+\pi^-$: $\langle\pi^+\pi^-|\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d|0\rangle \sim F^{(0)}_{\pi\pi}(t)$ [I=1, I_3=0]

Isospin limit

Conserved vector current (CVC)

$$F^{(0)}_{\pi\pi}(t) = F^{(-)}_{\pi\pi}(t)$$

$$\sigma^0_{e^+e^-\to\pi^+\pi^-} = \frac{K_\sigma(t)}{K_\Gamma(t)}\frac{\beta_{\pi^+\pi^-}}{S_{\mathrm{EW}}\beta_{\pi^-\pi^0}}\frac{d\Gamma(\tau_{2\pi})}{dt}$$

$$K_\sigma(t) = \frac{\pi\alpha^2}{3t},\ K_\Gamma(t) = \frac{G_F^2\left|V_{ud}\right|^2 m_\tau^3}{384\pi^3}\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+2\frac{t}{m_\tau^2}\right)$$

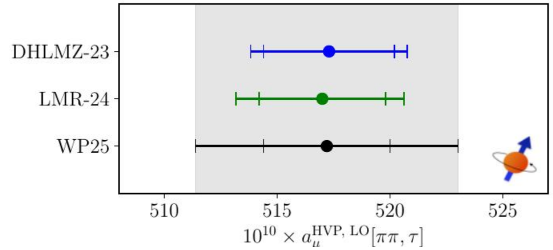
- **Isospin breaking (IB) effects become CRUCIAL at the sub-percent level.**
- **Full control of all the IB terms is yet to be reached.**

Results on the estimation of a_μ based on the tau data in WP25 are based on:

[Davier, et al., (DHLMZ), EPJC'24] [Lopez Castro, et al., (LMR) PRD'25]

+

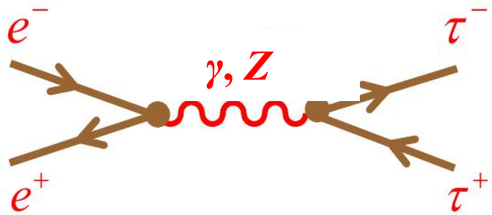
$\tau \to \nu_\tau\pi\pi$ data (Belle, ALEPH, CLEO, OPAL)



$$a_\mu^{\mathrm{HVP,\,LO}}[\pi\pi,\tau] = 517.2(2.8)_{\mathrm{exp}}(5.1)_{\mathrm{th}} \times 10^{-10}$$

Overview of tau lepton

tau产生



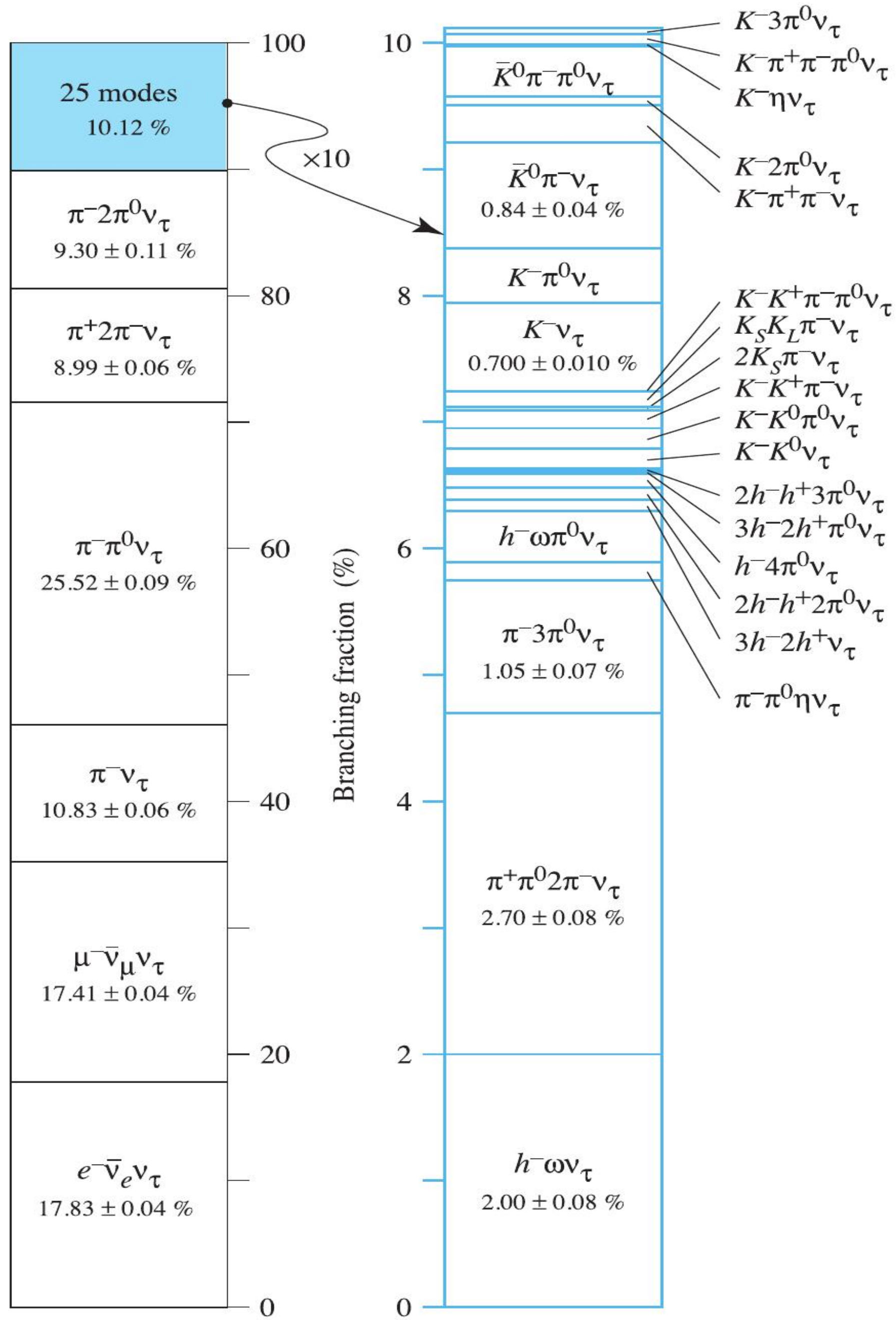
- **Tau-charm 工厂上 γ 交换主导 .**
- **CEPC等高能加速器上 Z 交换主导 .**

分支比概览

- **Br($\tau \to e\nu_\tau\overline{\nu_e}$) : 17.8%**

  Br($\tau \to \mu\nu_\tau\overline{\nu_\mu}$) : 17.4%
- **Br($\tau \to \nu$+Cabbibo allowed hadrons) ~ 62%**
- **Br($\tau \to \nu$+Cabbibo suppressed hadrons) : ~3%**

- **Br($\tau \to \nu\pi\pi$) ~ 25%, 单举衰变中分支比最大**
- **tau的衰变末态只有轻味强子，不涉及重味粒子 ($m_\tau < m_D$)**
- **在重子数守恒的假设下，tau不能衰变至含有重子的末态($m_\tau < 2m_N$)**



强衰变过程

tau的强衰变可以给我们提供什么信息？

- **inclusive衰变：（某类）所有的强子末态**

$$\tau^- \to \nu_\tau (\bar{u}d, \bar{u}s)$$

⟹ 可以用来研究标准模型的基本参数：α_S, V_{us}, ...

- **exclusive衰变：衰变至特定的强子末态**

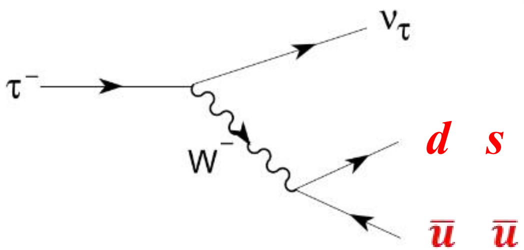
$$\tau^- \to \nu_\tau (P,\ PP',\ P_1P_2P_3,\ \ldots)$$

⟹
- 强作用形状因子，强子共振态，手征对称性，...
- CPV、轻子味道破坏、... ...

基础理论回顾

标准模型的带电流

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W_\mu^\dagger \left(V_{uD}\, \bar{u}_L \gamma^\mu D_L + \bar{\nu}_L \gamma^\mu \tau_L \right) + h.c.$$



- $$M_{\tau\to H\nu_\tau} = \frac{g^2 V_{uD}^*}{2} \frac{L_\mu H^\mu}{s - M_W^2} \approx -\frac{g^2 V_{uD}^*}{2M_W^2} L_\mu H^\mu$$

$$H_\mu = \langle n | \bar{d}_L \gamma_\mu u | 0 \rangle, L_\mu = \bar{u}_{L,\nu} \gamma_\mu u_{L,\tau}$$

因为$m_\tau \ll m_W$，另外如果在电弱能标下没有质量轻的新粒子， 四费米子等价相互作用可以用来很好地描述tau衰变（标准模型或者新物理模型均可）。

tau强衰变相关的标准模型下的四费米子作用可总结为:

$$\mathcal{L}_{CC}^{SM} = -\frac{G_F}{\sqrt{2}} V_{uD}\, \underline{\bar{l}_{\nu_\tau} \gamma^\mu (1-\gamma_5) l_\tau}\, \underline{\bar{u}\gamma_\mu (1-\gamma_5) D}, \quad [D = d, s]$$

CKM: V_{ud}/V_{us} ↓ 轻子流 ↓ 夸克流 ↓

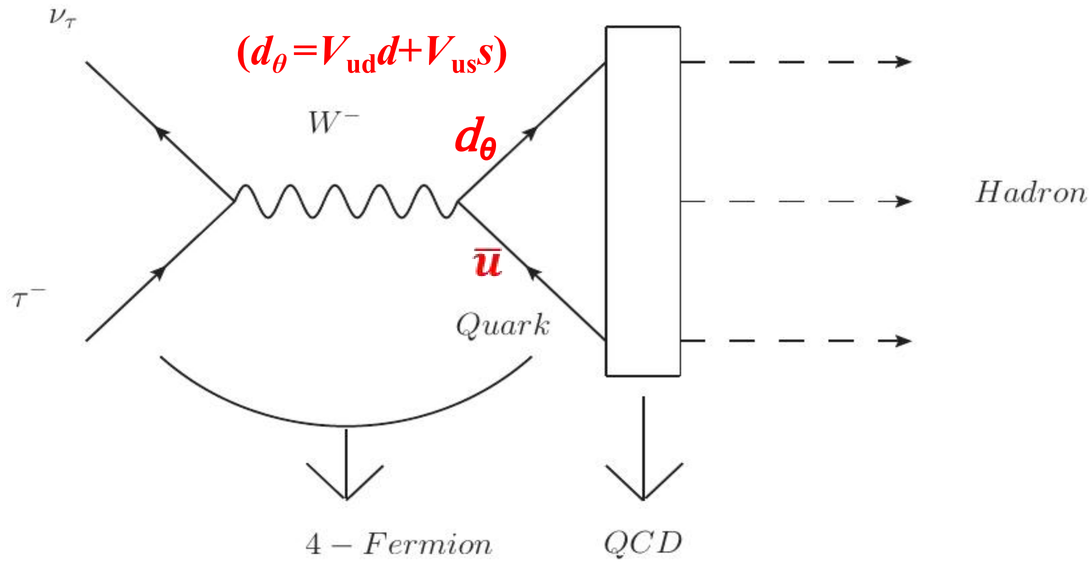
$$G_F = \frac{g^2}{4\sqrt{2}M_W^2}$$

- SMEFT → LEFT [Cirigliano et al, '10] [Y.Liao et al., '21] [J.H.Yu et al., '21][F.Z.Chen et al, '22] ...

$$\mathcal{L}_{\text{eff}} = -\frac{G_\mu V_{uD}}{\sqrt{2}} \Bigg[\left(1+\epsilon_L^{D\ell}\right) \bar{\ell}\gamma_\mu (1-\gamma_5)\nu_\ell \cdot \bar{u}\gamma^\mu (1-\gamma_5) D + \epsilon_R^{D\ell} \bar{\ell}\gamma_\mu (1-\gamma_5)\nu_\ell \cdot \bar{u}\gamma^\mu (1+\gamma_5) D$$
$$+ \bar{\ell}(1-\gamma_5)\nu_\ell \cdot \bar{u}\left[\epsilon_S^{D\ell} - \epsilon_P^{D\ell}\gamma_5\right] D + \frac{1}{4}\hat{\epsilon}_T^{D\ell} \bar{\ell}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell \cdot \bar{u}\sigma^{\mu\nu}(1-\gamma_5) D \Bigg] + \text{h.c.},$$

ε_X parameterize various new physics at high energy scale

Hadronic decays: a unique feature for tau lepton



Hadronic V-A currents

$$\mathrm{H}_\mu = \langle H^- | \bar{u} \gamma_\mu (1 - \gamma_5) d_\theta \, e^{i L_{QCD}} | 0 \rangle$$

Chiral EFT is the low energy realization of QCD:

$$e^{i\mathcal{Z}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}q \mathcal{D}\bar{q} \mathcal{D}G_\mu \; e^{i \int d^4x \mathcal{L}_{QCD}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}u \; e^{i \int d^4x \mathcal{L}_{EFT}(v_\mu, a_\mu, s, p)}$$

$$\mathcal{L}^{QCD} = \mathcal{L}_0^{QCD} + \bar{q}\gamma^\mu (v_\mu + a_\mu \gamma_5) q - \bar{q}(s - i\gamma_5 p) q$$

***v_μ, a_μ, s, p* are the external source fields .**

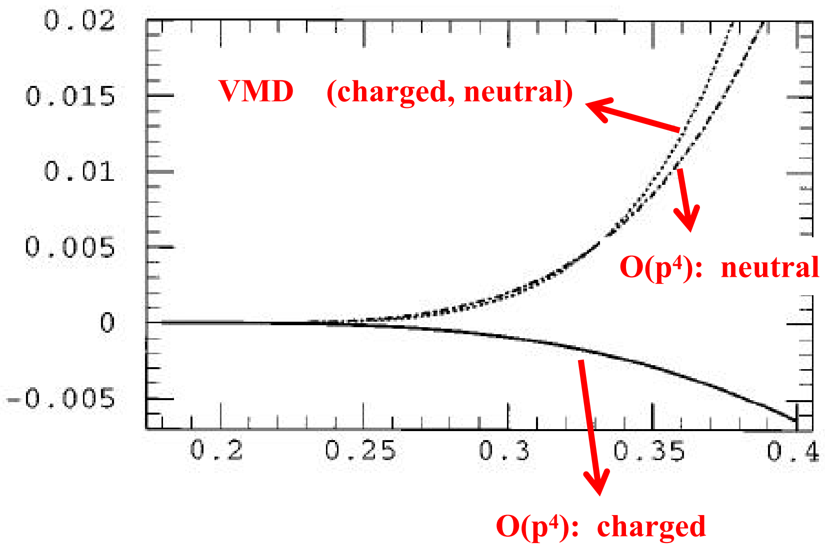
Chiral symmetry is RELEVANT to tau decays

Example: $\tau \rightarrow \nu_\tau \pi\pi\pi$ **transition amplitudes in the low energy region**
VMD models do not automatically respect chiral symmetry.

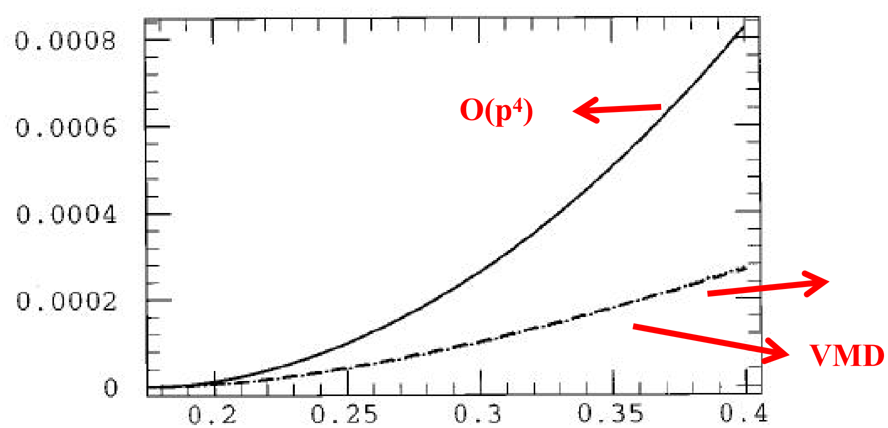
$$J_\alpha = -i\frac{2\sqrt{2}}{3f_\pi}\mathrm{BW}_a(Q^2)(B_\rho(s_2)V_{1\alpha} + B_\rho(s_1)V_{2\alpha})$$

[Kuhn,Santamaria, ZPC'90]

W_D structure function



W_{SA} structure function (neutral channel)



[Colangelo, et al., PRD'96]

➢ **Resonance chiral theory** **implements the constraint of chiral symmetry from the very beginning in the construction of the Lagrangians.**

Resonance chiral theory (RχT)

Chiral group: $G = SU(3)_L \times SU(3)_R$, $H = SU(3)_V$, $u(\phi) = G/H$

Resonances : $R \xrightarrow{G} h\,R\,h^\dagger , \quad h \in H$

pNGB and external sources : $X = u_\mu,\ \chi_\pm,\ f_\pm^{\mu\nu},\ h_{\mu\nu}$

Operators	P	C	h.c.	chiral order
u	$u^\dagger$	u^T	$u^\dagger$	1
Γ_μ	Γ^μ	$-{\Gamma_\mu}^T$	$-\Gamma_\mu$	p
u_μ	$-u^\mu$	u_μ^T	u_μ	p
$\chi_\pm$	$\pm\chi_\pm$	$\chi_\pm^T$	$\pm\chi_\pm$	p^2
$f_{\mu\nu\,\pm}$	$\pm f_\pm^{\mu\nu}$	$\mp f_{\mu\nu\,\pm}^T$	$f_{\mu\nu\,\pm}$	p^2
$h_{\mu\nu}$	$-h^{\mu\nu}$	$h_{\mu\nu}^T$	$h_{\mu\nu}$	p^2

Operators	P	C	h.c.
$V_{\mu\nu}$	$V^{\mu\nu}$	$-V_{\mu\nu}^T$	$V_{\mu\nu}$
$A_{\mu\nu}$	$-A^{\mu\nu}$	$A_{\mu\nu}^T$	$A_{\mu\nu}$
S	S	S^T	S
P	$-P$	P^T	P

Minimal RχT Lagrangian [Ecker, *et al.*, '89]

$$
\begin{aligned}
\mathcal{L}_{2V} &= \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle \\
\mathcal{L}_{2A} &= \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle , \\
\mathcal{L}_{2S} &= c_d\langle\, S u_\mu u^\mu\,\rangle + c_m\langle\, S\chi_+\,\rangle , \\
\mathcal{L}_{2P} &= i d_m\langle\, P\chi_-\,\rangle .
\end{aligned}
$$

Operators beyond minimal [Cirigliano, *et al.*, '04]:

$$\mathcal{L}_{VAP} = \lambda_1^{VA}\langle [V^{\mu\nu}, A_{\mu\nu}]\chi_-\rangle + \ldots ,$$

[Ruiz-Femenia, Pich and Portolés, '03]

$$
\begin{aligned}
\mathcal{L}_{VVP} &= d_1\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \ldots , \\
\mathcal{L}_{VJP} &= \frac{c_1}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, f_+^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \ldots ,
\end{aligned}
$$

$$d_4\varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, V^{\rho\alpha}\}u_\alpha\rangle$$

QCD dynamics in RχT

- Low energy QCD: implemented from the construction of RχT
- Intermediate energy: explicit resonance states
- High energy information: to match the same physical objects in RχT and QCD, $\langle\, J(x_n)\cdot\cdot J(0)\,\rangle^{\mathrm{R}\chi\mathrm{T}} = \langle\, J(x_n)\cdot\cdot J(0)\,\rangle^{\mathrm{QCD}}$.

For example: $\pi\pi$ vector form factor

$$\left[\mathcal{F}^{v}_{\pi\pi}(q^2)\right]^{\mathrm{R}\chi\mathrm{T}} = 1 + \frac{F_V G_V}{F^2}\frac{q^2}{M_V^2 - q^2}\,,$$

$$\left[\mathcal{F}^{v}_{\pi\pi}(q^2)\right]^{\mathrm{QCD}} \to 0, \qquad \text{for } q^2 \to \infty$$

This leads to

$$[\mathcal{F}^{v}_{\pi\pi}(q^2)]^{\mathrm{R}\chi\mathrm{T}} = [\mathcal{F}^{v}_{\pi\pi}(q^2)]^{\mathrm{QCD}} \Longrightarrow F_V G_V = F^2$$

Unified study of $\tau \to P_1P_2 + \nu_\tau$ [Hao, Duan, ZHG, 2507.00383]

$$\langle P_1P_2|\bar{D}\gamma^\mu u|0\rangle = \left[(p_2-p_1)^\mu - \frac{\Delta_{P_2P_1}}{s}q^\mu\right] F_+^{P_1P_2}(s) + \frac{\Delta_{Du}}{s} q^\mu \widehat{F}_0^{P_1P_2}(s)$$

(vector FF) (scalar FF)

$$\Delta_{P_2P_1} = m_{P_2}^2 - m_{P_1}^2\,, \qquad \Delta_{Du} = B_0(m_D - m_u)\,, \qquad q_\mu = (p_1+p_2)_\mu, \qquad s = q^2.$$

- **Invariant-mass distribution of P_1P_2**

$$\frac{d\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{48\pi^3 s} S_{\rm EW} |V_{uD}|^2 \left(1-\frac{s}{M_\tau^2}\right)\left\{\left(1+\frac{2s}{M_\tau^2}\right) q^3_{P_1P_2}(s)\left|F_+^{P_1P_2}(s)\right|^2 + \frac{3\Delta_{Du}^2}{4s} q_{P_1P_2}(s)\left|\widehat{F}_0^{P_1P_2}(s)\right|^2\right\}$$

- **Forward-Backward (FB) asymmetry distribution**

$$A_{FB}(s) = \frac{\int_0^1 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha} - \int_{-1}^0 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha}}{\int_0^1 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha} + \int_{-1}^0 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha}} = \frac{\Delta_{Du}\, q_{P_1P_2}(s)\, \Re\left[F_+^{P_1P_2}(s)\,\widehat{F}_0^{P_1P_2*}(s)\right]}{\frac{2\sqrt{s}}{3}\left(1+\frac{2s}{M_\tau^2}\right) q^2_{P_1P_2}(s)\left|F_+^{P_1P_2}(s)\right|^2 + \frac{\Delta_{Du}^2}{2\sqrt{s}}\left|\widehat{F}_0^{P_1P_2}(s)\right|^2}$$

α: angle between the momenta of P_1 and τ in the P_1P_2 rest frame

$\tau \to \pi^-\pi^0\nu_\tau$: $\Delta_{PP'}\to 0$ (同位旋破坏项), 标量F_0项可忽略，矢量F_+绝对主导!

$\tau \to K\pi\nu_\tau$: $\Delta_{PP'}\neq 0$, 标量形状因子F_0项以及矢量形状因子F_+项都有贡献!

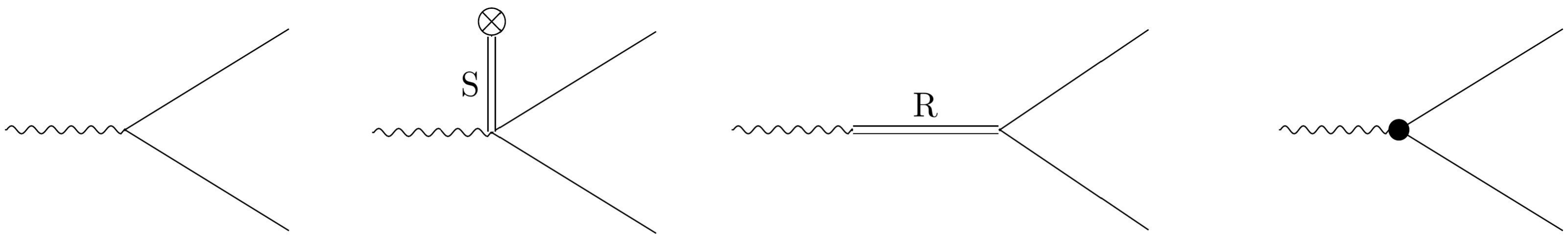
Calculation of P_1P_2 form factors

$$\mathcal{L}^{\rm LO} = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \frac{F^2}{12}M_0^2 X^2$$

$$\mathcal{L}_V = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu} u^\mu u^\nu\rangle \,,$$

$$\mathcal{L}_\Lambda^{\rm NLO,U(3)} = -\frac{F^2\Lambda_1}{12} D^\mu X D_\mu X - \frac{F^2\Lambda_2}{12} X\langle\chi_-\rangle$$

$$\mathcal{L}_S = c_d\langle S u_\mu u^\mu\rangle + c_m\langle S\chi_+\rangle \,,$$



(a) (b) (c) (d)

(a) (b)

Mixing relations of π^0-η-η'-a (axion)

[Gao,ZHG,Oller,Zhou, JHEP'23]
[Gao,Hao,ZHG,Oller,Zhou, EPJC'25]

$$\begin{pmatrix} \hat\pi^0 \\ \hat\eta \\ \hat\eta' \\ \hat a \end{pmatrix} = \begin{pmatrix} 1+z_{11} & c_\theta(-v_{12}+z_{12})+s_\theta(-v_{13}+z_{13}) & -s_\theta(-v_{12}+z_{12})+c_\theta(-v_{13}+z_{13}) & -v_{14}+z_{14} \\ v_{12}+z_{21} & c_\theta(1+z_{22})+s_\theta(z_{23}-v_{23}) & -s_\theta(1+z_{22})+c_\theta(z_{23}-v_{23}) & -v_{24}+z_{24} \\ v_{13}+z_{31} & c_\theta(z_{32}+v_{23})+s_\theta(1+z_{33}) & -s_\theta(z_{32}+v_{23})+c_\theta(1+z_{33}) & -v_{34}+z_{34} \\ v_{41}+z_{41} & c_\theta(v_{42}+z_{42})+s_\theta(v_{43}+z_{43}) & -s_\theta(v_{42}+z_{42})+c_\theta(v_{43}+z_{43}) & 1+v_{44}+z_{44} \end{pmatrix} \begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix}$$

v_{ij}: LO terms

z_{ij}: NLO terms (L_5, L_8, Λ_1, Λ_2) or [(L_5, L_8)~ (c_dc_m, c_mc_m)/M^2_S , Λ_1, Λ_2]

➢ **Scalar form factor** $\langle P_1 P_2|(m_D - m_u)\bar{D}u|0\rangle$

$$\Delta_{du} = B_0(m_d - m_u) = \Delta_{du}^{\text{Phy}} \left\{1 + \frac{m_K^2}{F^2}\left[\frac{16c_m(c_d - c_m)}{M_S^2} + \frac{16c'_m(c'_d - c'_m)}{M_{S'}^2}\right]\right\}$$

$$\Delta_{du}^{\text{Phy}} = m_{K^0}^2 - m_{K^+}^2 - (m_{\pi^0}^2 - m_{\pi^+}^2)$$

$$(m_d - m_u)\langle \pi^- P|\bar{d}u|0\rangle = \Delta_{du}^{\text{Phy}} F_0^{\pi^- P}(s) \qquad \widehat{F}_0^{\pi^- P}(s) = \frac{\Delta_{du}^{\text{Phy}}}{\Delta_{du}} F_0^{\pi^- P}(s)$$

$$\langle \pi^- P|\bar{d}\gamma^\mu u|0\rangle = \left[(p_P - p_\pi)^\mu - \frac{\Delta_{P\pi}}{s}q^\mu\right] F_+^{\pi^- P}(s) + \frac{\Delta_{du}^{\text{Phy}}}{s} q^\mu F_0^{\pi^- P}(s)$$

➢ **Finiteness of the above expression at *s* = 0 requires**

$$F_+^{\pi^-(\eta,\eta',a)}(0) = \frac{\Delta_{du}^{\text{Phy}}}{\Delta_{(\eta,\eta',a)\pi}} F_0^{\pi^-(\eta,\eta',a)}(0)$$

✓ **Non-trivial confirmation is verified in chiral EFT!**

Some explicit expressions for Form Factors

- **VFF-ππ**

$$F_+^{\pi^-\pi^0}(s) = -\frac{\sqrt{2}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s)$$

$$G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) = \frac{G_V F_V s + F^2(M_\rho^2 - s)}{M_\rho^2 - s - iM_\rho\Gamma_\rho(s)} - \frac{G'_V F'_V s}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} - \frac{G''_V F''_V s}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)},$$

$$\Gamma_\rho(s) = \frac{M_\rho s}{96\pi F_\pi^2}\left[\sigma_{\pi\pi}^3(s) + \frac{1}{2}\sigma_{KK}^3(s)\right], \quad \Gamma_{\rho',\rho''}(s) = \Gamma_{\rho',\rho''}\frac{s}{M_{\rho',\rho''}^2}\frac{\sigma_{\pi\pi}^3(s)}{\sigma_{\pi\pi}^3(M_{\rho',\rho''}^2)}, \quad \sigma_{P_1P_2}(s) = \frac{2q_{P_1P_2}(s)}{\sqrt{s}}\theta[s - (m_{P_1} + m_{P_2})^2]$$

- **VFF-πη/πη’/πa**

$$F_+^{\pi^-\eta}(s) = -\frac{\sqrt{2}v_{12}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) - \sqrt{2}\left(y_{12} - v_{13}y_{23}^{(0)}\right),$$

$$F_+^{\pi^-\eta'}(s) = -\frac{\sqrt{2}v_{13}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) - \sqrt{2}\left(y_{13} + v_{12}y_{23}^{(0)}\right),$$

$$F_+^{\pi^-a}(s) = -\frac{\sqrt{2}v_{41}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) - \sqrt{2}\left(y_{14} + v_{12}\,y_{24}^{(0)} + v_{13}\,y_{34}^{(0)}\right)$$

- **SFF-πη/πη'/πa**

$$F_0^{\pi^-\eta}(s) = \sqrt{\frac{2}{3}}(c_\theta - \sqrt{2}s_\theta) + \frac{1}{\sqrt{3}}(\Lambda_1 - 2\Lambda_2)s_\theta - \frac{1}{\sqrt{3}}y_{23}(2c_\theta + \sqrt{2}s_\theta) + 4\sqrt{\frac{2}{3}}\frac{c_\theta - \sqrt{2}s_\theta}{F^2}\Bigg\{ \left[\frac{c_m(c_m - c_d)2m_\pi^2 + c_m c_d(s + m_\pi^2 - m_\eta^2)}{M_{a_0}^2 - s - iM_{a_0}\Gamma_{a_0}(s)} - \frac{2c_m(c_m - c_d)(2m_K^2 - m_\pi^2)}{M_S^2}\right] + \left[c_{m,d}, M_{a_0}, \Gamma_{a_0}, M_S \to c'_{m,d}, M_{a'_0}, \Gamma_{a'_0}, M_{S'}\right]\Bigg\},$$

$$F_0^{\pi^-\eta}(s) = \sqrt{\frac{2}{3}}(c_\theta - \sqrt{2}s_\theta) + \frac{1}{\sqrt{3}}(\Lambda_1 - 2\Lambda_2)s_\theta - \frac{1}{\sqrt{3}}y_{23}(2c_\theta + \sqrt{2}s_\theta) + 4\sqrt{\frac{2}{3}}\frac{c_\theta - \sqrt{2}s_\theta}{F^2}\Bigg\{ \left[\frac{c_m(c_m - c_d)2m_\pi^2 + c_m c_d(s + m_\pi^2 - m_\eta^2)}{M_{a_0}^2 - s - iM_{a_0}\Gamma_{a_0}(s)} - \frac{2c_m(c_m - c_d)(2m_K^2 - m_\pi^2)}{M_S^2}\right] + \left[c_{m,d}, M_{a_0}, \Gamma_{a_0}, M_S \to c'_{m,d}, M_{a'_0}, \Gamma_{a'_0}, M_{S'}\right]\Bigg\},$$

$$F_0^{\pi^-a}(s) = \frac{(\sqrt{2}c_\theta - 2s_\theta)}{\sqrt{3}}v_{24}^{(0)} + \frac{(2c_\theta + \sqrt{2}s_\theta)}{\sqrt{3}}v_{34}^{(0)} - \frac{2}{\sqrt{3}}\left(s_\theta\, v_{24}^{(0)} - c_\theta\, v_{34}^{(0)} + \frac{F}{\sqrt{6}f_a}\right)(\Lambda_2 - \frac{1}{2}\Lambda_1) + \frac{(\sqrt{2}c_\theta - 2s_\theta)}{\sqrt{3}}y_{24}^{(0)} + \frac{(2c_\theta + \sqrt{2}s_\theta)}{\sqrt{3}}y_{34}^{(0)} + \frac{4(\sqrt{2}c_\theta - 2s_\theta)v_{24}^{(0)} + 4(2c_\theta + \sqrt{2}s_\theta)v_{34}^{(0)}}{\sqrt{3}F^2}\Bigg\{ \left[\frac{2c_m^2\, m_\pi^2 + c_m c_d\left(s - m_a^2 - m_\pi^2\right)}{M_{a_0}^2 - s - iM_{a_0}\Gamma_{a_0}(s)} + \frac{2c_m(c_d - c_m)\left(2m_K^2 - m_\pi^2\right)}{M_S^2}\right] + \left[c_{m,d}, M_{a_0}, \Gamma_{a_0}, M_S \to c'_{m,d}, M_{a'_0}, \Gamma_{a'_0}, M_{S'}\right]\Bigg\}.$$

Fits to experimental spectra and BRs

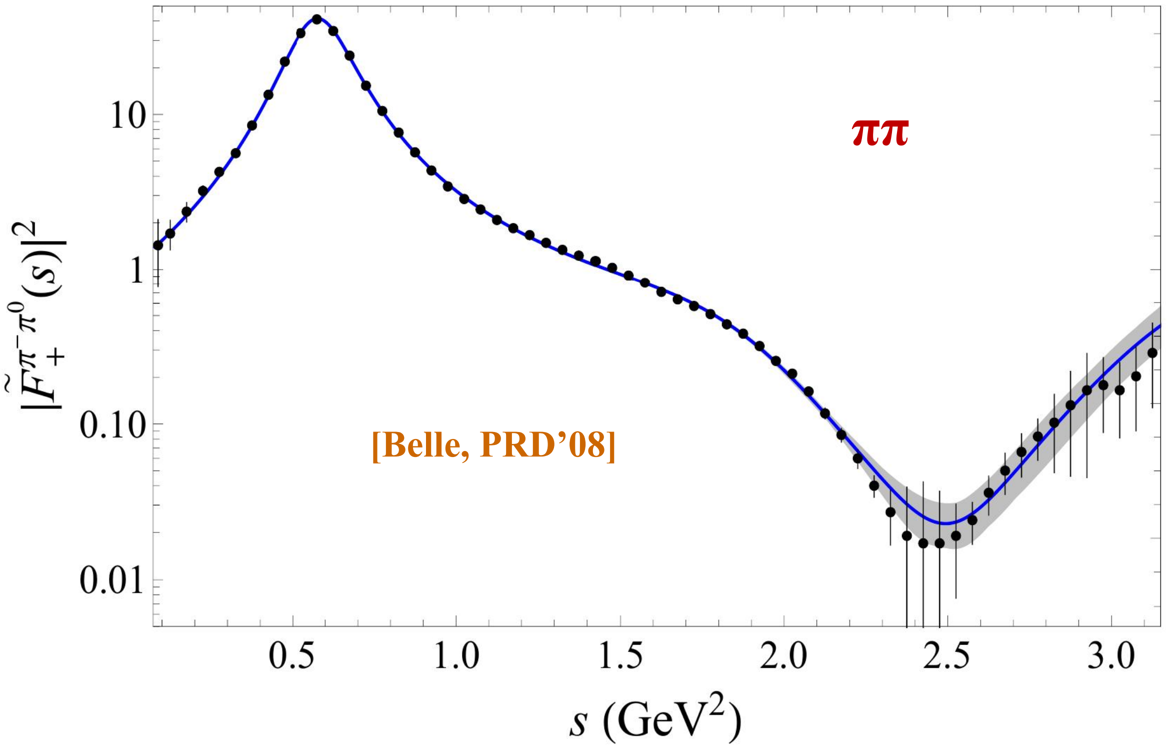
$G_V\, F_V(\mathrm{GeV}^2) \times 10^3$	$10.26^{+0.01}_{-0.01}$	$G'_V\, F'_V(\mathrm{GeV}^2) \times 10^3$	$0.64^{+0.03}_{-0.02}$
$G''_V\, F''_V(\mathrm{GeV}^2) \times 10^3$	$-0.94^{+0.05}_{-0.05}$	$M_\rho(\mathrm{GeV})$	$0.7738^{+0.0003}_{-0.0003}$
$M_{\rho'}(\mathrm{GeV})$	$1.409^{+0.004}_{-0.004}$	$\Gamma_{\rho'}(\mathrm{GeV})$	$0.338^{+0.012}_{-0.010}$
$M_{\rho''}(\mathrm{GeV})$	$1.842^{+0.012}_{-0.013}$	$\Gamma_{\rho''}(\mathrm{GeV})$	$0.268^{+0.025}_{-0.026}$
$c'_m(\mathrm{GeV})$	$0.053^{+0.007}_{-0.009}$	$M_{K^*}(\mathrm{GeV})$	$0.8956^{+0.0002}_{-0.0002}$
$\Gamma_{K^*}(\mathrm{GeV})$	$0.0477^{+0.0005}_{-0.0005}$	$M_{K^{*\prime}}(\mathrm{GeV})$	$1.339^{+0.009}_{-0.009}$
$\overline{B}_{K_S\pi^-} \times 10^3$	$3.98^{+0.04}_{-0.04}$	$\overline{B}_{K^-\eta} \times 10^4$	$1.34^{+0.04}_{-0.04}$
$\chi^2/\mathrm{d.o.f}$	$271.5/(182-14) = 1.61$		

$$c_m c_d + c'_m c'_d = \frac{F^2}{4}$$

c_m= 27 MeV, c_d=15 MeV

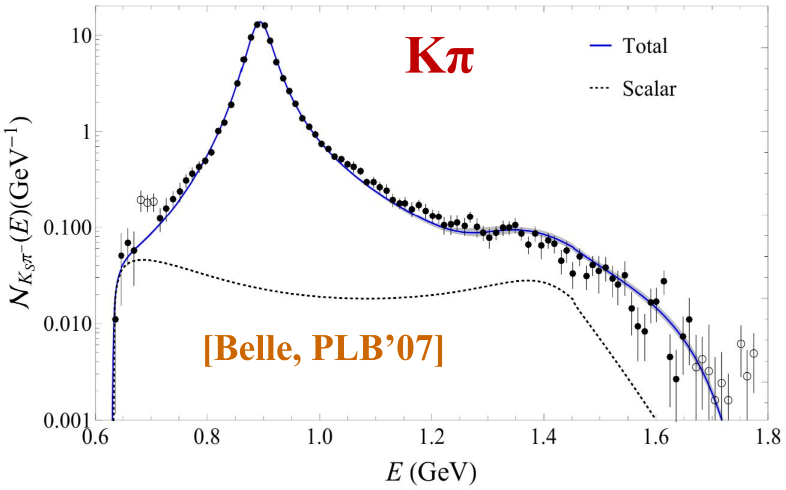
[ZHG,Oller, PRD'11]

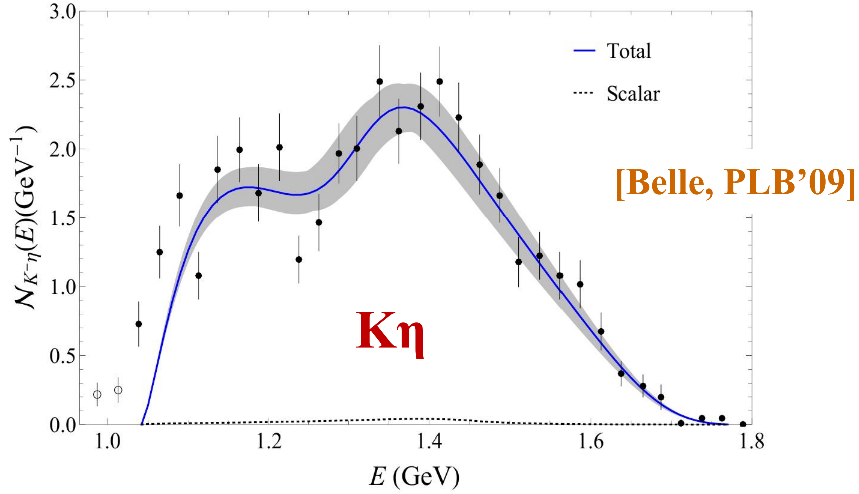
Parameters for a_0(980)/a_0(1450)/K_0*(700)//K_0*(1430) are fixed to their pole positions .



- **Crucial inputs to address muon g-2**
- **Most precise spectra is from Belle;**

  but most precise BR is from ALEPH: 25.47(13)%
- **Coherent precise measurements of both spectra and BR from one Exp would be invaluable!**

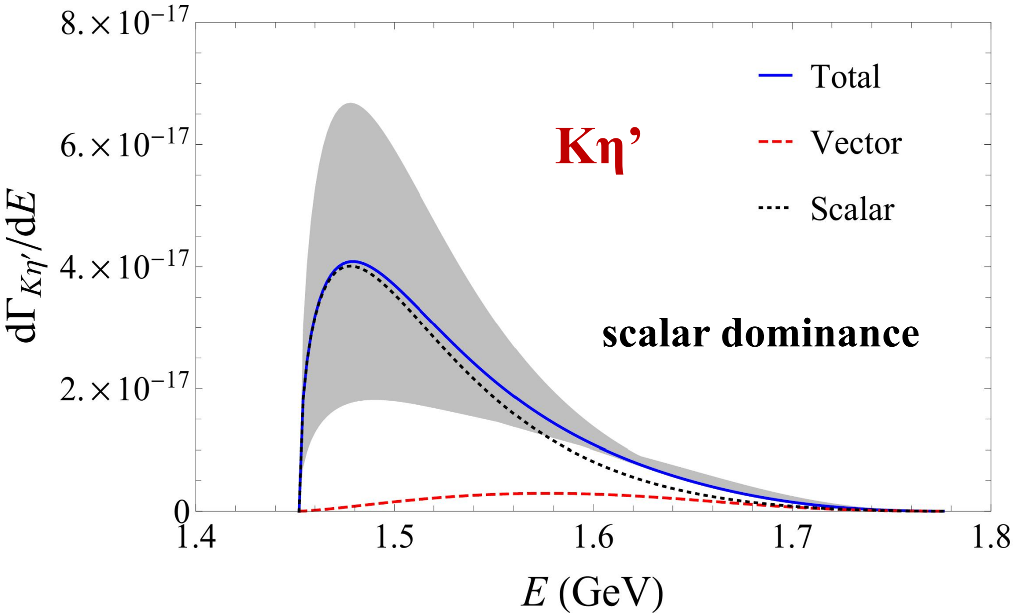




F_+^{KP}: K*, K*(1410)

F_0^{KP}: κ, K^*_0(1430)

prediction to Kη'



BR(K⁻ η')^{Theo} = (2.0±1.0)×10^{-6}

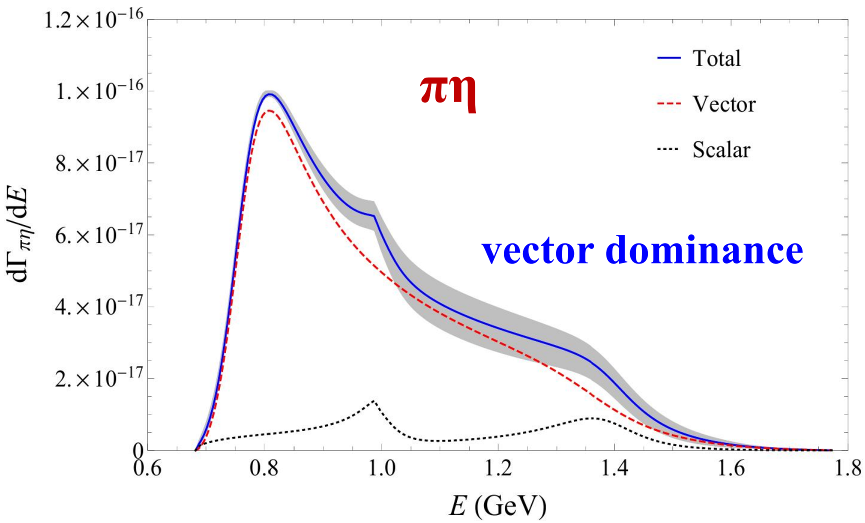
BR(K⁻ η')$^{Exp, BaBar}$ < 2.4×10^{-6}

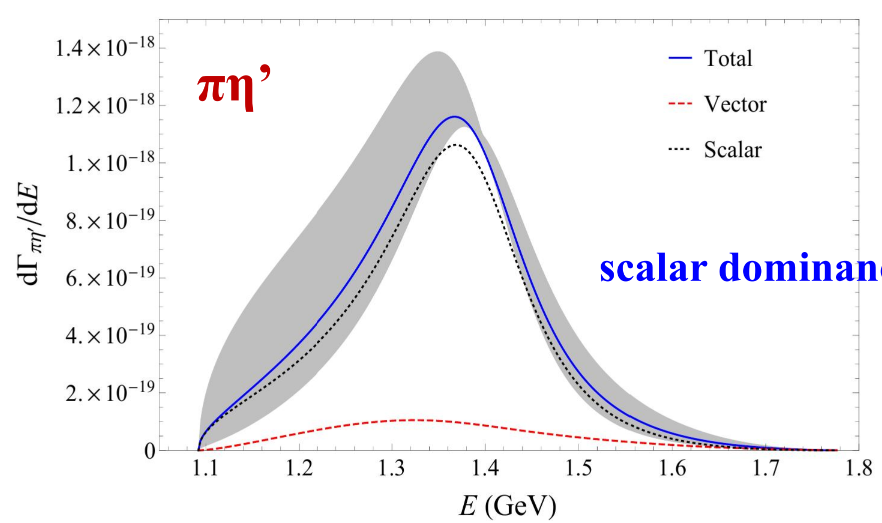
Predictions to $\tau \to \pi^-\eta^{(\prime)}\nu_\tau$ (Cabibbo allowed)：second-class currents

若$\pi^-\eta$ 的J=0，则其P=+1 （V-A型的流不允许）； 若J=1，则P=-1 （矢量流）

但是对于SM来讲，1-矢量流对应的G宇称为正，而$\pi^-\eta$的G宇称为负，表明这是一个破坏G宇称的过程（second-class current），因此可能是寻找新物理的一个有效途径。

$$\langle \pi^- P|\bar{d}\gamma^\mu u|0\rangle = \left[(p_P - p_\pi)^\mu - \frac{\Delta_{P\pi}}{s}q^\mu\right] F_+^{\pi^- P}(s) + \frac{\Delta_{du}^{\rm Phy}}{s} q^\mu F_0^{\pi^- P}(s)$$



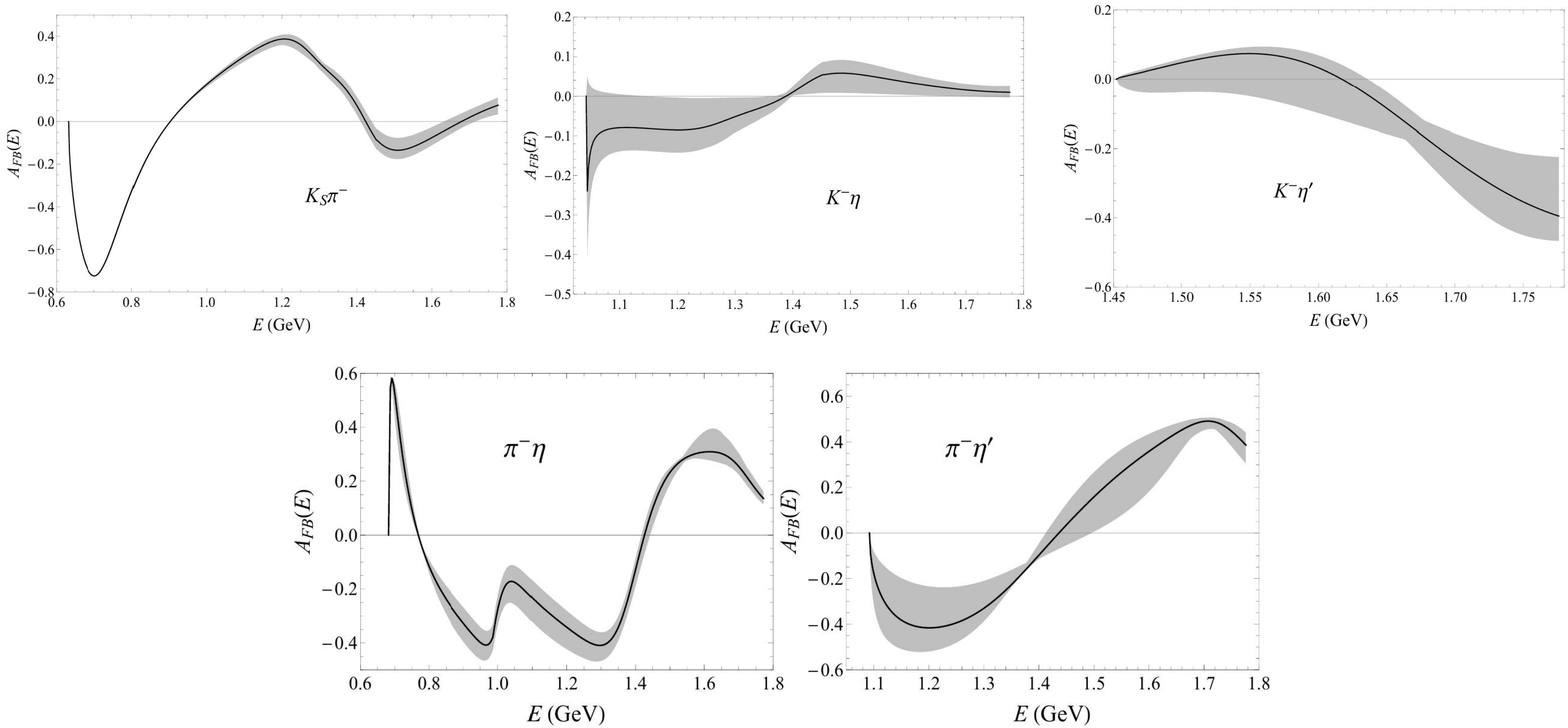


Channel	Total	Vector	Scalar	Exp Limits
$\tau^- \to \pi^-\eta\nu_\tau$ $(\times 10^5)$	$1.63^{+0.14}_{-0.14}$	$1.43^{+0.18}_{-0.21}$	$0.20^{+0.07}_{-0.04}$	< 9.9 (BaBar) [69] < 7.3 (Belle) [70]
$\tau^- \to \pi^-\eta'\nu_\tau$ $(\times 10^7)$	$1.17^{+0.36}_{-0.07}$	$0.14^{+0.09}_{-0.08}$	$1.03^{+0.44}_{-0.16}$	< 40 (BaBar) [71]

[Hao, Duan, ZHG, 2507.00383]

Predictions to Forward-Backward asymmetries

$$\frac{\Delta_{Du}\, q_{P_1P_2}(s)\, \Re\left[F_+^{P_1P_2}(s)\, \widehat{F}_0^{P_1P_2*}(s)\right]}{\frac{2\sqrt{s}}{3}\left(1+\frac{2s}{M_\tau^2}\right) q_{P_1P_2}^2(s)\left|F_+^{P_1P_2}(s)\right|^2 + \frac{\Delta_{Du}^2}{2\sqrt{s}}\left|\widehat{F}_0^{P_1P_2}(s)\right|^2}$$

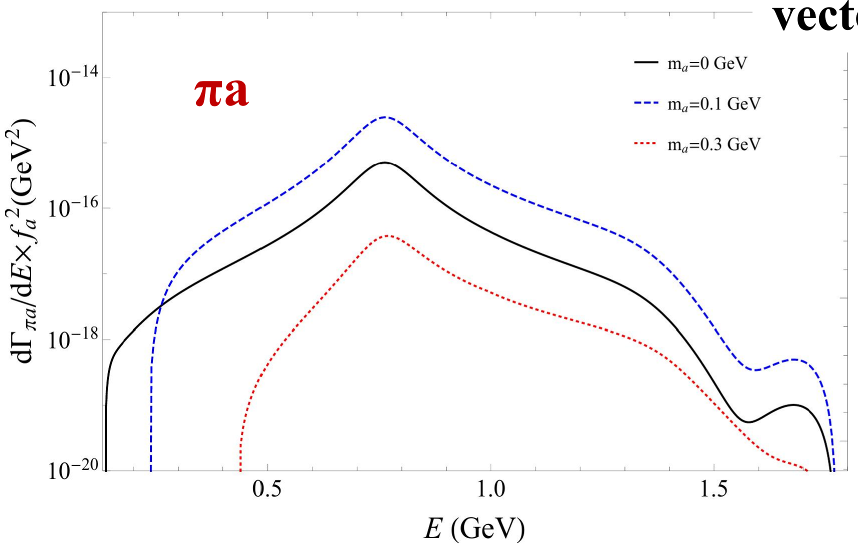


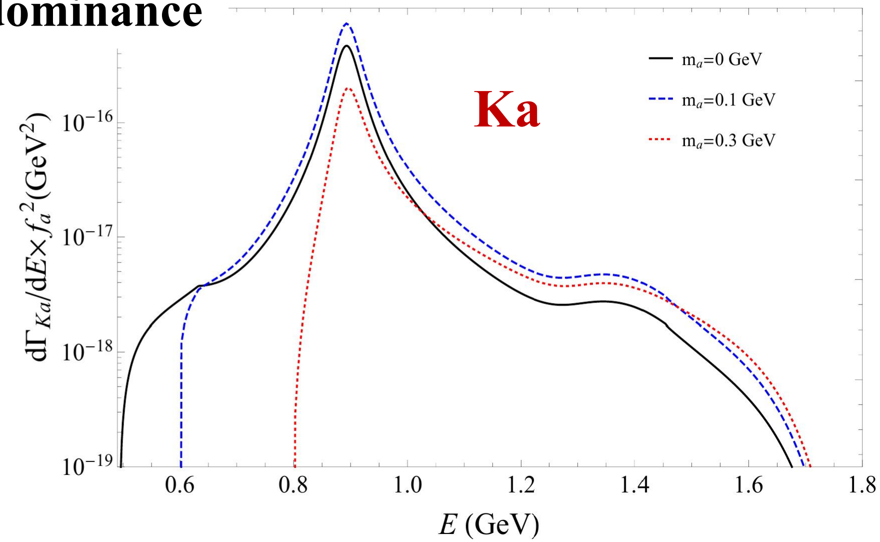
- ➢ **Measurement on A_{FB} can determine the curcial inputs for CPV**

Predictions to axion-meson production in tau decays

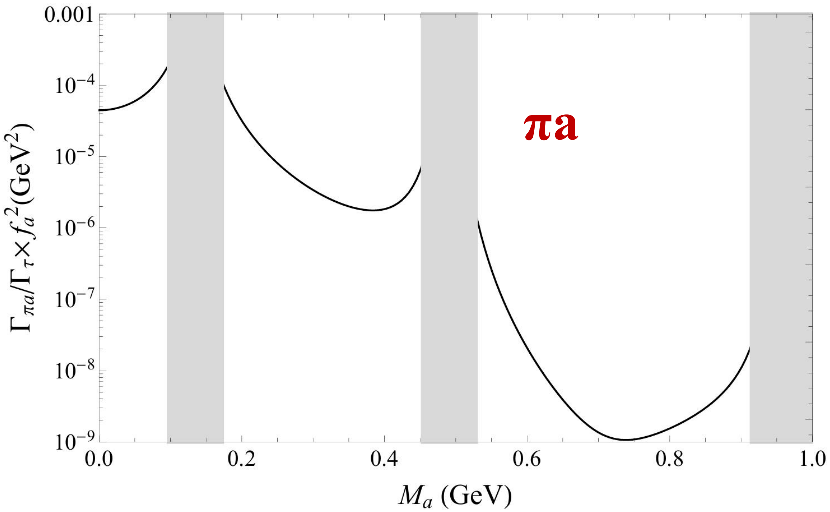
Spectra

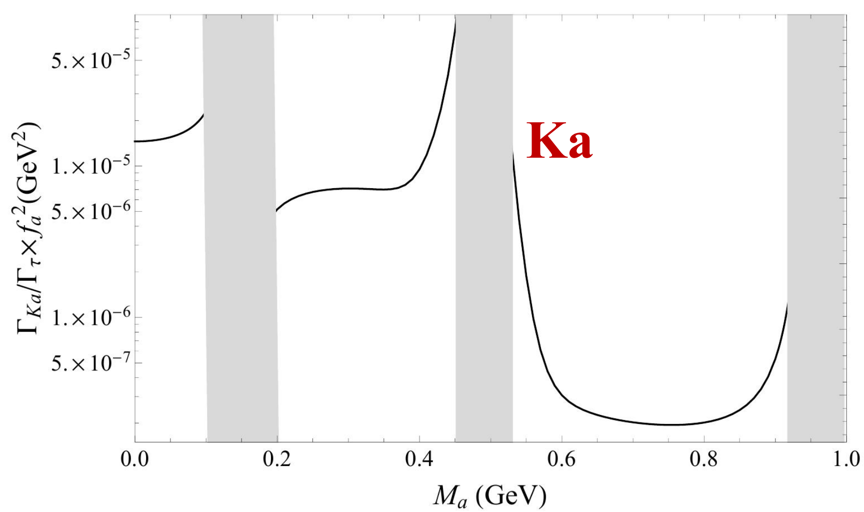
vector dominance





BRs





结语　　Tau包含丰富有趣的物理：

- 充满了烟火气息 --亟需提升精度/澄清的SM允许的过程--：
  - $\tau^- \to \nu_\tau \pi^0 \pi^-$　[为muon g-2提供关键输入]
  - $\tau^- \to \nu_\tau (K\pi, K\eta, K\eta')^-$　[V_{us}, 奇异轻强子态, CPV]
  - 第二类流主导过程的发现：$\tau^- \to \nu_\tau \pi^- \eta/\eta'$
  - τ衰变中的前后不对称性的测量
  - … …

- 同时也有诗和远方 --诱人的新物理现象--：

  轻子味道破坏，轻子数破坏，新的CP破坏，… …

谢谢大家！