Relaxation Mechanism

A possible solution to electroweak hierarchy problem

Yuxiang Cao

University of Science and Technology of China

Contents

Electroweak Hierarchy Problem

The Problem:

- Enormous scale disparity: $M_H \approx 125$ GeV vs $M_{Pl} \approx 10^{19}$ GeV
- Quantum corrections to Higgs mass:

$$\delta m_H^2 = \frac{\lambda}{16\pi^2}\Lambda^2 + \mathcal{O}(\log\Lambda)$$

  where Λ is the UV cutoff scale
- Physical Higgs mass: $m_H^2 = m_H^2(\text{bare}) + \delta m_H^2$
- Requires cancellation to $\sim 10^{-34}$ precision between bare mass and radiative corrections

Traditional Solutions:

1. **Dynamical mechanisms**
   - **Supersymmetry**
   - **Composite Higgs**
   - **Extra dimensions**
2. **Anthropic explanation**

Key Challenge: Traditional dynamical solutions increasingly constrained by LHC data, requiring significant fine-tuning in their parameter spaces.

Axion

Origin:

- Proposed to solve **Strong CP Problem** (PQ mechanism, Peccei-Quinn 1977)
- QCD instantons suggest $\theta_{\text{QCD}} < 10^{-10}$, but why?
- Axion is Nambu-Goldstone boson of broken $U(1)_{\text{PQ}}$ symmetry

Key Properties:

- Pseudo-scalar (spin-0, $J^P = 0^-$)
- Extremely light ($m_a \sim \mu$eV – meV)
- Weakly interacting (couples to $\frac{a}{f_a} F\tilde{F}$)

Crucial Formulas:

$$\mathcal{L} \supset (\theta + \frac{a}{f_a})\frac{1}{32\pi^2} G\tilde{G}$$

$$m_a \approx 5.7 \left(\frac{10^{12}\ \text{GeV}}{f_a}\right) \mu\text{eV}$$

$$\theta_{\text{eff}} = \theta + \frac{a}{f_a} \to 0$$

Relaxation Mechanism

Put forward by Peter, David and Surjeet in 2015, the main idea is cosmological evolution of an axion-like field (relaxion) coupled to the Higgs during the early Universe drives the Higgs boson mass:

While classically rolling down its potential, the relaxion field ϕ scans down the Higgs mass parameter, starting from $\mathcal{O}(1)\Lambda_{\mathrm{cutoff}}$, until a stopping mechanism when the Higgs mass parameter approaches zero.

The advantage of this Model:

1. No new physics at the weak scale.
2. Can naturally raise the cutoff of the Higgs boson. 10^5TeV in this paper.

Toy Model

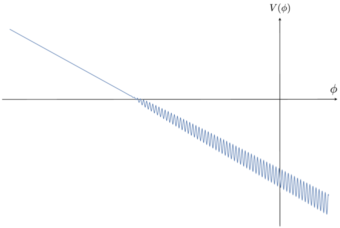
The toy model start with Standard Model + QCD axion + inflation sector.
Difference against general axion model:

1. Large field range, extend its periodic potential to much larger than the cutoff, even the Plank scale.
2. A small coupling g to the Higgs boson in order to break the shift symmetry[1]

Example of additional potential:

$$(-M^2+g\phi)|h|^2+(gM^2\phi+g^2\phi^2+\cdots)+\Lambda^4\cos(\phi/f)$$

where M is cutoff energy, ϕ is the axion and g is coupling. The initial value for ϕ is large , so m_h^2, is positive During inflation, ϕ will slow roll, thereby scanning the physical Higgs mass. At ctritical point $\phi=\frac{M^2}{g}$, the quadratic term crosses zero and the Higgs develops VEV. As $\Lambda^4 \propto f_\pi^2 m_\pi^2$ will rise with VEV, then it become barriers that stop the rolling.



[1] $(-M^2+g\phi)|h|^2+V(g\phi)+\frac{1}{32\pi^2}\frac{\phi}{f}\tilde{G}^{\mu\nu}G_{\mu\nu}$ is symmetry under $\phi\to\phi+2\pi f$ if g=0

Key point

There are several constraints should be satisfied for this model:

1. Inflation should last long enough for ϕ to scan the entire range. During N e-folds of inflation, ϕ changes by an amount $\Delta\phi \sim (\dot{\phi}/H_\mathsf{i})N \sim (gM^2/H_\mathsf{i}^2)N$. ($H_\mathsf{i}$ is the Hubble scale during inflation) Requiring $\Delta\phi \gtrsim (M^2/g)$ gives the requirement:
$$N \gtrsim \frac{H_\mathsf{i}^2}{g^2}.$$
2. The vacuum energy during inflation $\sim H_\mathsf{i}^2 M_\mathsf{pl}^2$ should be greater than the vacuum energy change along the ϕ potential $\sim M^4$:
$$H_\mathsf{i} > \frac{M^2}{M_\mathsf{pl}}$$
3. Hubble scale during inflation should be lower than the QCD scale:
$$H_\mathsf{i} < \Lambda_{\mathrm{QCD}}$$
4. Classical rolling dominated other than quantum fluctuations
$$H_\mathsf{i} < \frac{V'_\phi}{H_\mathsf{i}^2} \to H_\mathsf{i} < (gM^2)^{\frac{1}{3}}$$

With the requirement and the rolling stop condition $gM^2 \sim \Lambda^4/f$, we get a constrain on cutoff on M:
$$M < \left(\frac{\Lambda^4 M_\mathsf{pl}^3}{f}\right)^{\frac{1}{6}} \sim 10^7\ \mathrm{GeV} \times \left(\frac{10^9\ \mathrm{GeV}}{f}\right)^{\frac{1}{6}}$$

Model with dynamical slope of potential

The toy above is ruled out by the strong CP problem.

The relation $gM^2 \sim \Lambda^4/f$ predicts that the local minimum for ϕ is displaced from the minimum of the QCD part by $\mathcal{O}(f)$ which generates[2] $\theta \sim 1$.One solution is the slope of the ϕ potential decreases dynamically after inflation.
We can make it by tying the slope to the value of the inflation σ with new term $\kappa\sigma^2\phi^2$. By setting the additional slope, $\kappa\sigma^2 M^2/g$ to be larger than gM^2.

- During inflation: a new effective coupling $\tilde{g}^2 = \kappa\sigma^2$
- After inflation: inflation field drops to 0 and slope back to $\sim gM^2$

In order to solve the strong CP problem as well, we need the slope of the potential to drop by a factor of $\theta \lesssim 10^{-10}$ after inflation so that the axion is only displaced by this amount from its (local) minimum. Thus we require $gM^2 \sim \theta\tilde{g}^2(M^2/g)$, or $g \sim \tilde{g}\sqrt{\theta}$.

And now the requirement in the previous slide become:

$$N \gtrsim \theta\frac{H_{\rm i}^2}{g^2}, \quad H_i > \frac{M^2}{M_{\rm pl}\sqrt{\theta}}, \quad H_i < \Lambda_{\rm QCD}, \quad H_i < \left(\frac{gM^2}{\theta}\right)^{1/3}, \quad gM^2 f \sim \Lambda^4\theta$$

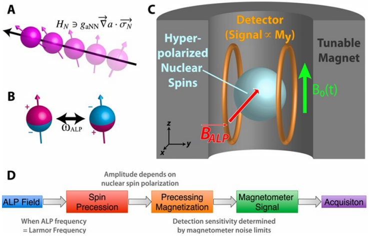
The cutoff now is $M < \left(\frac{\Lambda^4 M_{\rm pl}^3}{f}\right)^{1/6}\theta^{1/4} \sim 30\ \text{TeV} \times \left(\frac{10^9\ \text{GeV}}{f}\right)^{1/6}\left(\frac{\theta}{10^{-10}}\right)^{1/4}$

[2] $\theta_{\rm eff} = \theta + \frac{\phi}{f}$

Possible observables

The relaxation mechanism predicts a light, weakly-coupled scalar field with unique properties. Despite the absence of new TeV-scale physics, several experimental approaches could detect its presence:

Laboratory Experiments:
E.g. Cosmic Axion Spin Precession Experiment (CASPEr)



Cosmological Signatures:
In this model the axion potential now has an overall slope and acquires an initial velocity in the early Universe after reheating which would change the final axion dark matter abundance.

Conclusion

- The relaxation mechanism provides an elegant solution to the Electroweak hierarchy problem without requiring new physics at the TeV scale
- Key features:
  - Cosmological evolution of an axion-like field coupled to the Higgs
  - Dynamically scans Higgs mass from cutoff scale down to observed value
  - Allows cutoff scales up to $\sim 10^5$ TeV, significantly higher than conventional models
- Combines solutions to Strong CP problem and hierarchy problem in a unified framework
- Experimental prospects:
  - Low-energy precision experiments (e.g., CASPEr)
  - Cosmological observations related to axion dark matter
- Provides a versatile framework for future theoretical development:
  - Alternative stopping mechanisms
  - Applications to other scalar fields
  - Further refinement of cosmological constraints

1. Graham P W, Kaplan D E, Rajendran S. Cosmological relaxation of the electroweak scale[J]. Physical review letters, 2015, 115(22): 221801.
2. Fonseca N, Morgante E, Sato R, et al. Relaxion fluctuations (self-stopping relaxion) and overview of relaxion stopping mechanisms[J]. Journal of high energy physics, 2020, 2020(5): 1-63.

Thank you!