电弱统一理论与Higgs

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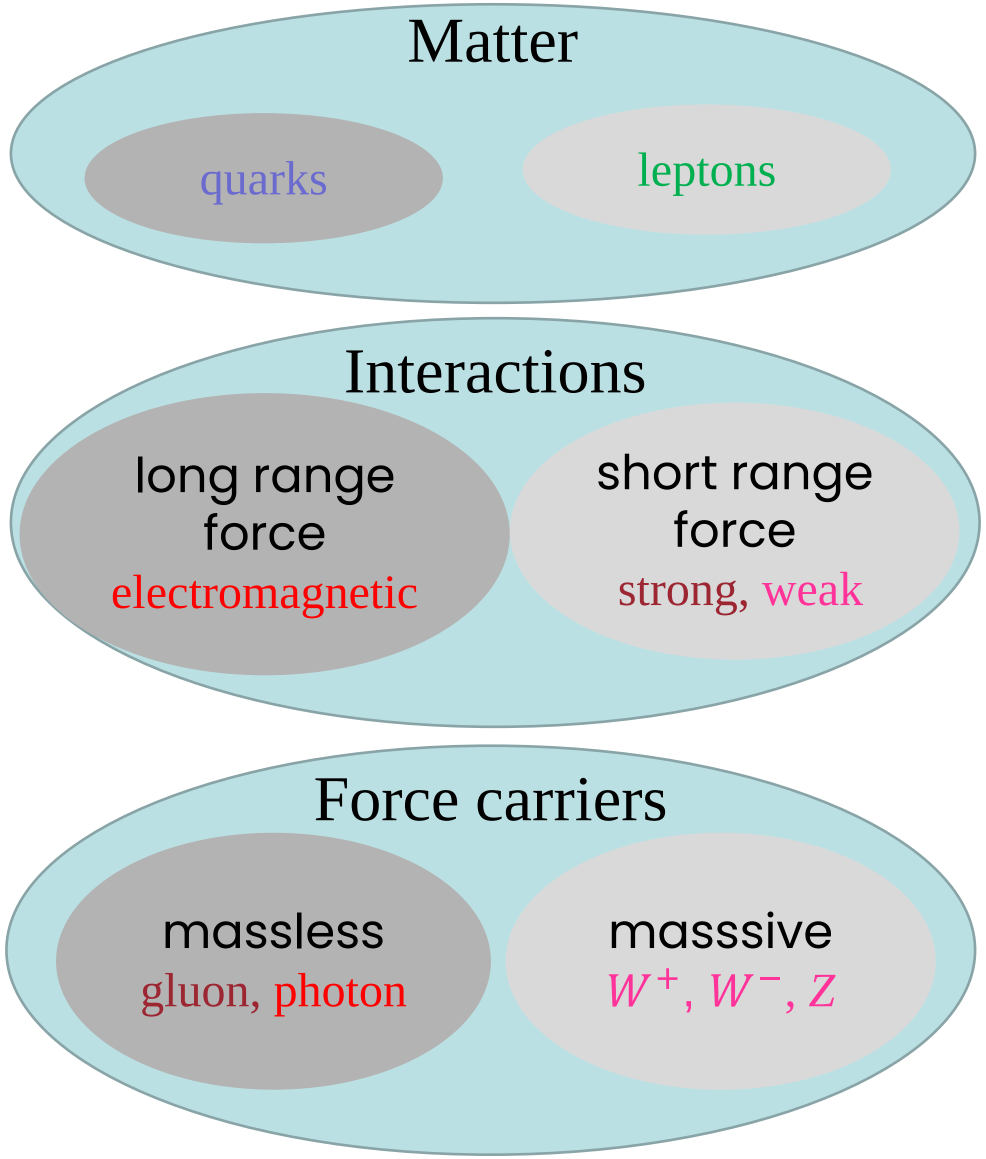
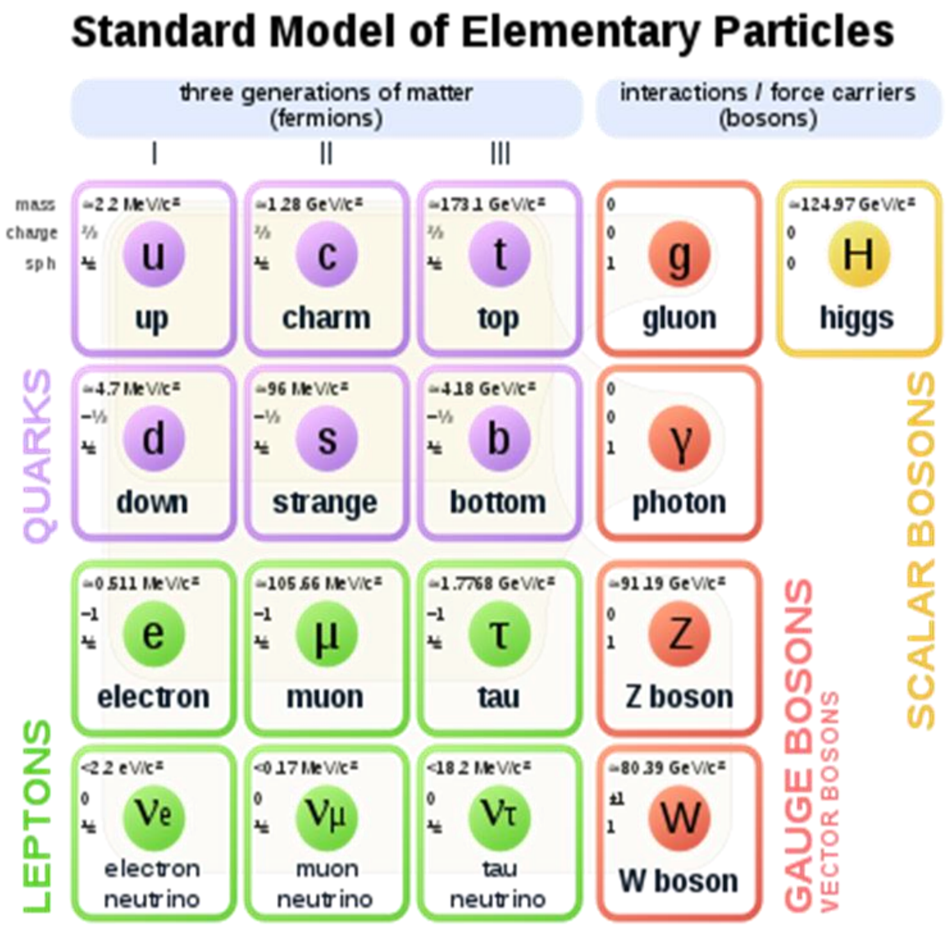
Contents

Introduction to The Standard Model

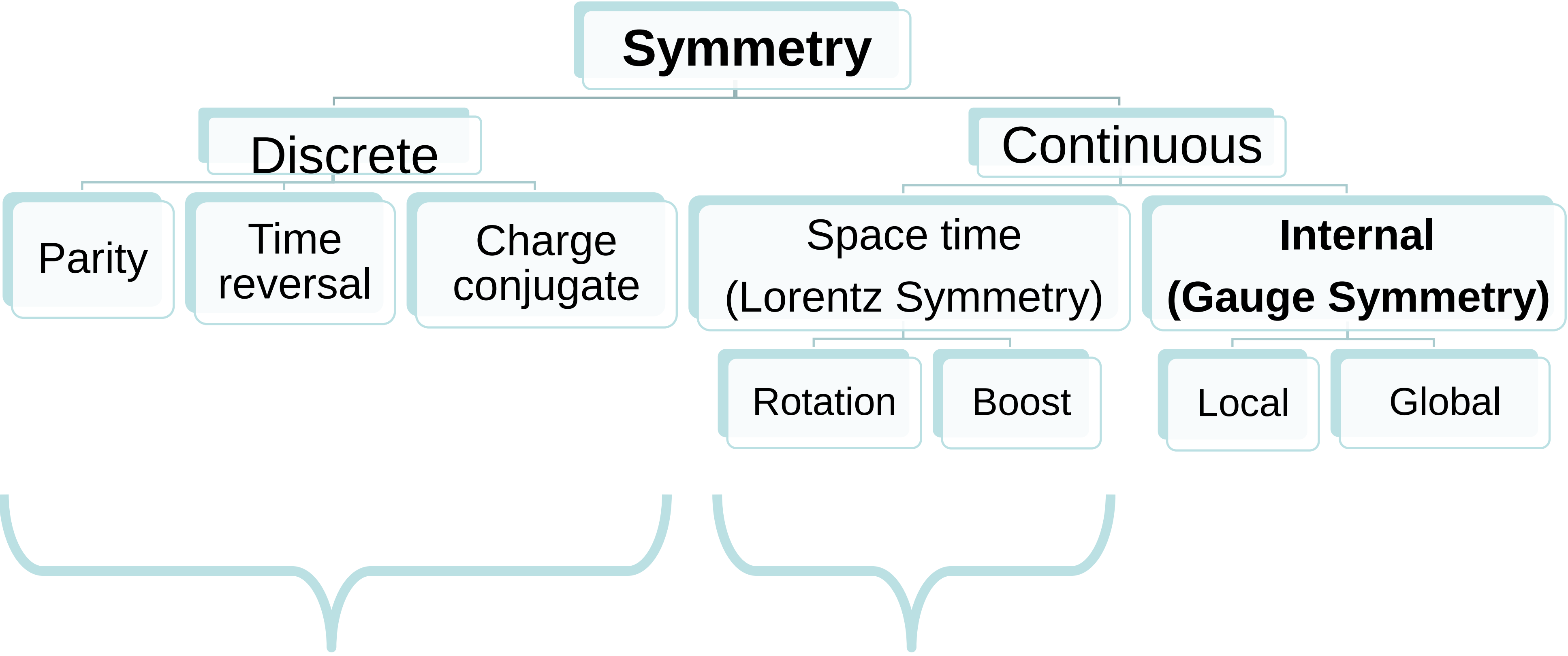




massless gluon carries short range force due to **quark confinement**

Contents

Gauge invariance



Can be violated
(weak interaction violate CP)

Strict symmetry

Gauge invariance

Global Symmetry

- Equation of motion for a field is determined by Euler Lagrange equation

$$\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}\right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0.$$

- The theory has a **symmetry** if Lagrangian is invariant under some special type of variation
  - example

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - m^2 \phi\phi^*,$$

$$\phi \to e^{-i\alpha}\phi, \phi^* \to e^{i\alpha}\phi^*$$

$$\phi = \phi_1 + i\phi_2$$

$$\mathcal{L} = (\partial_\mu \phi)(\partial^\mu \phi^*) - m^2 \phi\phi^*.$$

- The equation of motion is

$$(\partial_\mu\partial^\mu + m^2)\phi = 0, \quad (\partial_\mu\partial^\mu + m^2)\phi^* = 0.$$

Gauge invariance

Global Symmetry

Noether's theorem: *If a Lagrangian has a continuous symmetry then there exists a current associated with that symmetry that is conserved when the equations of motion are satisfied*

- Consider the case that α can be infinitesimal thus the symmetry is **continuous**

$$0=\frac{\delta\mathcal{L}}{\delta\alpha}=\sum_n\left\{\underbrace{\left[\frac{\partial\mathcal{L}}{\partial\phi_n}-\partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_n)}\right]}_{=0}\frac{\delta\phi_n}{\delta\alpha}+\partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_n)}\frac{\delta\phi_n}{\delta\alpha}\right]\right\},$$

$$\partial_\mu J^\mu=0,\quad J^\mu=\sum_n\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_n)}\frac{\delta\phi_n}{\delta\alpha}\qquad \frac{\delta\phi}{\delta\alpha}=-i\phi,\qquad \frac{\delta\phi^*}{\delta\alpha}=i\phi^*,$$

$$J_\mu=-i(\phi\partial_\mu\phi^*-\phi^*\partial_\mu\phi)\text{ and }\partial_\mu J^\mu=-i(\phi\partial_\nu\partial^\nu\phi^*-\phi^*\partial_\nu\partial^\nu\phi)=0$$

- J_μ is known as **Noether current** and $\partial_\mu J^\mu=0$ is called the **conserved current**

$$Q=\int d^3x\,J^0\text{ and }\partial_t Q=0$$

- Q is the total charge and is **conserved**

Gauge invariance

Global Symmetry

- Free electric Lagrangian

$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi,$$

- Lagrangian is invariant under U(1) transformation

$$\psi(x) \to \psi'(x) = e^{i\theta}\psi(x), \quad \bar{\psi}(x) \to \overline{\psi'}(x) = e^{-i\theta}\bar{\psi}(x)$$

- Noether current

$$J^{\mu} = -\bar{\psi}\gamma^{\mu}\psi$$

- Conserved charge
  - $\psi^{\dagger}\psi$ is particle density

$$Q = \int d^3x J^0 = -\int d^3x\, \psi^{\dagger}\psi$$

Gauge invariance

Global Symmetry

Symmetry	Group	Conserved Current	Conserved Charge	Status
Baryon Number	$U(1)_B$	$j_B^\mu = \frac{1}{3} \sum_{q=u,d,c,s,t,b} \bar{q}\,\gamma^\mu q$	$\frac{1}{3}\int \sum_{q=u,d,c,s,t,b} q^\dagger\, q\, d^3x$	☑ Strict (No observed violation)
Lepton Number	$U(1)_L$	$j_{l_i}^\mu = \bar{l}_i \gamma^\mu l_i + \bar{\nu}_i \gamma^\mu \nu_i \quad (i = e, \mu, \tau)$	$\int (l_i^\dagger l_i + \nu_i^\dagger \nu_i) d^3\,x$	⚠ Approximate (Flavor mixing via neutrino oscillations)
Isospin	$SU(2)_I$	$j_{I,a}^\mu = \bar{q}\gamma^\mu \frac{\tau_a}{2} q \quad (a = 1,2,3)$	$\int q^\dagger \frac{\tau_a}{2} q\, d^3x$	⚠ Approximate (Broken by $m_u \neq m_d$)
Flavor Symmetry	$SU(3)_F$	$Q_{F,a} = \int q^\dagger \frac{\lambda_a}{2} q\, d^3x$	$\int q^\dagger \frac{\lambda_a}{2} q\, d^3x$	✕ Broken ($m_s \gg m_{u,d}$)

Gauge invariance

Local Symmetry, Abelian case

- Free electric Lagrangian is not invariant under local U(1) transformation

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi,$$

$$\psi(x) \to \psi'(x) = e^{i\theta(x)}\psi(x),$$
$$\bar{\psi}(x) \to \bar{\psi'}(x) = e^{-i\theta(x)}\bar{\psi}(x)$$

$$\downarrow$$

$$\mathcal{L}' = \mathcal{L} + e(\partial_{\mu}\,\theta(x))\bar{\psi}\gamma^{\mu}\psi$$

- Introduce covariant derivative D_{μ} and gauge field A_{μ} (known as photon), so that $\mathcal{L}$ keeps invariant under local U(1) transformation

$$\partial_{\mu}\psi \to D_{\mu}\psi = (\partial_{\mu} + ieA_{\mu})\psi$$

$$\psi(x) \to \psi'(x) = e^{i\theta(x)}\psi(x),$$
$$\bar{\psi}(x) \to \bar{\psi'}(x) = e^{-i\theta(x)}\bar{\psi}(x)$$
$$A_{\mu}(x) \to A'_{\mu}(x) = A_{\mu}(x) - \frac{1}{e}\partial_{\mu}\theta(x)$$

$$\downarrow$$

$$e^{i\theta(x)}D_{\mu}\psi$$

Gauge invariance

Local Symmetry, Abelian case

- The new introduced gauge field A_μ need a kinetic energy term which must be invariant under local U(1) transformation

$$F_{\mu\nu} \to F'_{\mu\nu} = \partial_\mu\left(A_\nu - \partial_\nu\theta(x)\right) - \partial_\nu\left(A_\mu - \partial_\mu\theta(x)\right) = F_{\mu\nu},\ \ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- QED Lagrangian is invariant under local U(1) transformation

$$\mathcal{L}_{QED} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} = \bar{\psi}\,(i\gamma^\mu\partial_\mu + ieA_\mu)\psi - \bar{\psi}m\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

  - $\bar{\psi}ieA_\mu\psi$ is the interaction term of photon and electric.
  - Mass term of photon violate the gauge invariance.

$$\frac{1}{2}m_A^2 A_\mu A^\mu \to \frac{1}{2}m_A^2\left(A_\mu A^\mu - 2A_\mu\partial^\mu\theta(x) + \partial_\mu\theta(x)\partial^\mu\theta(x)\right)$$

  - In gauge field theory, gauge boson must be massless.
  - Photon has No self-coupling.

Gauge invariance

Local Symmetry, non-Abelian SU(N) case

- Lagrangian for Dirac multiplet

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi - \frac{1}{2}Tr(F^{a}_{\mu\nu}T^{a}\,F^{b,\mu\nu}T^{b}) = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - \gamma^{\mu}gA^{a}_{\mu}T^{a} - m)\psi - \frac{1}{4}F^{a}_{\mu\nu}F^{a,\mu\nu},$$

$$F^{a}_{\mu\nu} = \partial_{\mu}A^{a}_{\nu} - \partial_{\nu}A^{a}_{\mu} + gf^{abc}A^{b}_{\mu}A^{c}_{\nu}$$

- Lagrangian is invariant under SU(N) transformation

$$U(x) = e^{i\theta^{a}(x)T^{a}} \in SU(n)$$

$$\psi(x) \rightarrow U(x)\psi(x)$$

$$D_{\mu} \rightarrow U(x)D_{\mu}U^{\dagger}(x)$$

$$F^{a}_{\mu\nu}T^{a} \rightarrow U(x)F^{a}_{\mu\nu}T^{a}U^{\dagger}(x)$$

$$A^{a}_{\mu}(x)T^{a} \rightarrow U(x)A^{a}_{\mu}(x)T^{a}U^{\dagger}(x) - \frac{i}{g}(\partial_{\mu}U(x))U^{\dagger}(x)$$

- Gauge boson is massless.
- Lagrangian contains cubic and quartic terms in A^{a}_{ν}.
- In Abelian case f^{abc}=0.

	generator	structure constants
SU(2)	$T^{a}=\tau^{a}$(a=1..3)	$f^{abc} \rightarrow \epsilon^{abc}$
SU(3)	$T^{a}=\lambda^{a}$(a=1..8)	f^{abc}

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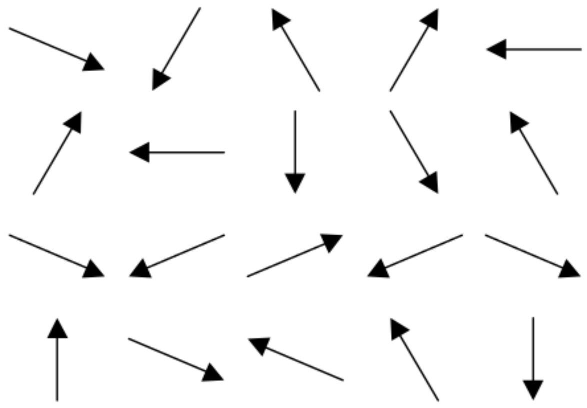
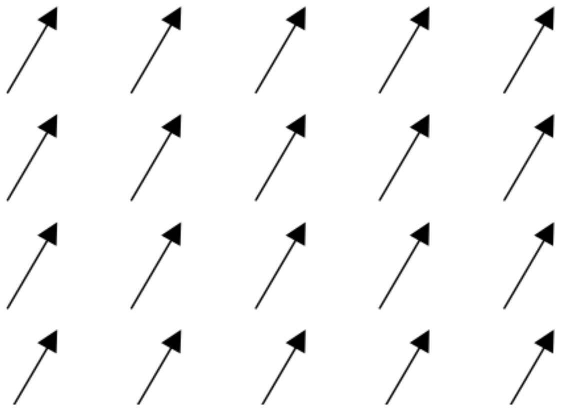
Spontaneous symmetry breaking

overview

- Spontaneous symmetry breaking in magnet



Condition: $T>T_C$(Curie point)
Symmetry: SO(3)



Condition: $T<T_C$
Symmetry: SO(2)

- Spontaneous symmetry breaking in QFT
  - Vacuum (ground state) is invariant (unique): system is symmetric
  - Vacuum (ground state) is not invariant (unique):
    1. Lagrangian is invariant: spontaneous symmetry breaking
    2. Lagrangian is not invariant: explicit symmetry breaking

Spontaneous symmetry breaking

discrete symmetry

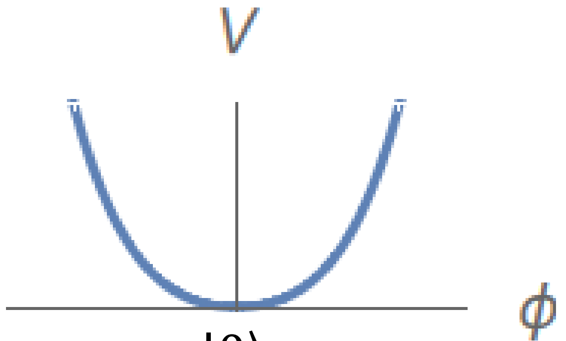
- **Lagrangian of scalar ϕ^4 theory**

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \xrightarrow{m^2 = -\mu^2 > 0} \mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

  - Lagrangian has a discrete symmetry: $Z^{-1}\phi(x)Z = -\phi(x)$, $Z^\dagger = Z^{-1} = Z$

- **Minima of potential**

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$



$|0\rangle$

$\langle 0|\phi|0\rangle = 0$

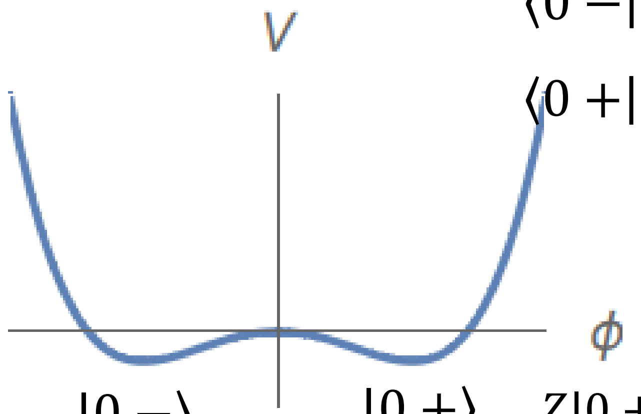
$m^2 > 0$

Vacuum is invariant

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

$$\langle 0-|\phi|0-\rangle = -v = -\mu\sqrt{6/\lambda}$$

$$\langle 0+|\phi|0+\rangle = +v = \mu\sqrt{6/\lambda}$$



$|0-\rangle$ $\quad$ $|0+\rangle$

$m^2 < 0$

$Z|0+\rangle = |0-\rangle, Z|0-\rangle = |0+\rangle$

Spontaneous symmetry breaking

discrete symmetry

- $V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4$ has two vacuums
  - Vacuums are degenerate
  - Physical state are the excitation near the vacuum

$$\phi(x) = v + \sigma(x), \qquad \langle 0+|\sigma(x)|0+\rangle = 0$$

$$\begin{aligned} V(\phi) &= \frac{\lambda}{24}\left((v+\sigma)^2 - v^2\right)^2 - \frac{\lambda}{24}v^4 \\ &= \frac{\lambda}{24}(\sigma^2 + 2\sigma v)^2 - \frac{\lambda}{24}v^4 \\ &= \frac{\lambda}{24}(\sigma^4 + 4\sigma^3 v + 4\sigma^2 v^2) - \frac{\lambda}{24}v^4 \\ &= \frac{1}{6}\lambda v^2\sigma^2 + \frac{1}{6}\lambda v\sigma^3 + \frac{\lambda}{24}\sigma^4 - \frac{\lambda}{24}v^4 \end{aligned}$$

  - $m_\sigma = \frac{1}{3}\lambda v^2$
  - $\phi(x) \to -\phi(x)$ symmetry is broken due to σ^3 term

Spontaneous symmetry breaking

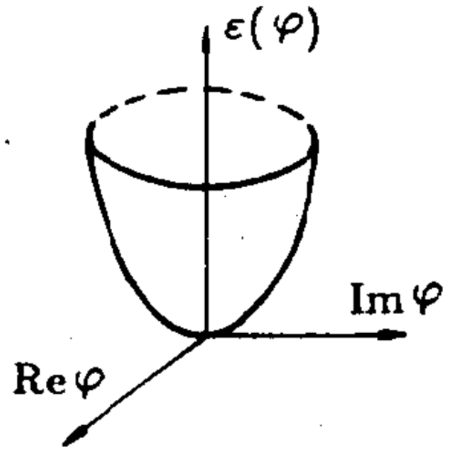
continuous symmetry, Abelian U(1)

- Lagrangian of complex ϕ^4 theory

$$\mathcal{L} = \partial^\mu\phi^\dagger\partial_\mu\phi - m^2\phi^\dagger\phi - \frac{\lambda}{4}(\phi^\dagger\phi)^2$$

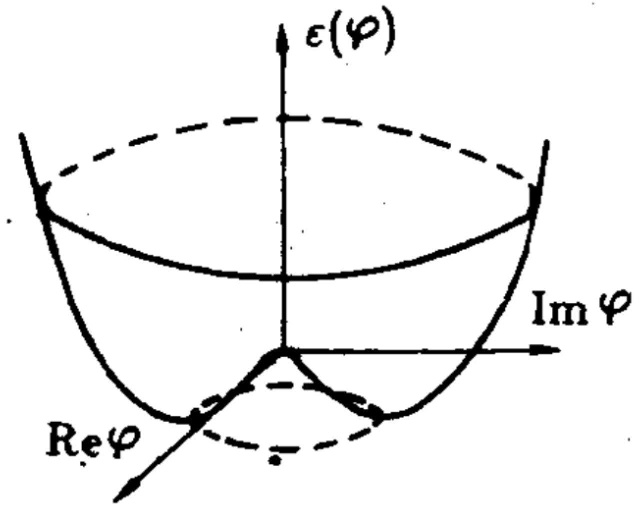
  - Lagrangian is invariant under global U(1) transformation: $\phi \to e^{-i\alpha}\phi$

- Minima of potential



$$m^2 > 0$$
$$\phi^\dagger = \phi = 0$$
$$\langle 0|\phi|0\rangle = 0$$

Vacuum is invariant



$$m^2 < 0$$
$$\phi(x) = \frac{v}{\sqrt{2}}e^{i\theta}$$
$$v = \sqrt{-4m^2/\lambda}$$
$$\mu^2 = \frac{\lambda v^2}{4}$$
$$\langle\theta|\phi|\theta\rangle = \frac{v}{\sqrt{2}}e^{i\theta}$$

Vacuums $|\theta\rangle$ are degenerate

Spontaneous symmetry breaking

continuous symmetry, Abelian U(1)

- $V(\phi) = m^2\phi^\dagger\phi + \frac{\lambda}{4}\left(\phi^\dagger\phi\right)^2$ has infinite number of vacuums
  - select $\theta = 0$ as the physical vacuum

1. parameterization: $\phi(x) = \frac{1}{\sqrt{2}}\big(v + a(x) + ib(x)\big)$

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\left(\partial_\mu a\right)^2 + \frac{1}{2}\left(\partial_\mu b\right)^2 - \frac{1}{2}\mu^2|v + a(x) + ib(x)|^2 - \frac{\lambda}{16}|v + a(x) + ib(x)|^4 \\ &= \frac{1}{2}\left(\partial_\mu a\right)^2 + \frac{1}{2}\left(\partial_\mu b\right)^2 - \frac{\mu^2}{2}[(v+a)^2 + b^2] - \frac{\lambda}{16}[(v+a)^2 + b^2]^2 \\ &= \frac{1}{2}\left(\partial_\mu a\right)^2 + \frac{1}{2}\left(\partial_\mu b\right)^2 - \frac{\lambda}{16}(4v^2a^2) - \frac{\lambda}{16}\big(4va(a^2+b^2)\big) - \frac{\lambda}{16}(a^2+b^2)^2 \\ &= \frac{1}{2}\left(\partial_\mu a\right)^2 + \frac{1}{2}\left(\partial_\mu b\right)^2 - \mu^2 a^2 - \frac{\sqrt{\lambda}\mu}{2}a(a^2+b^2) - \frac{\lambda}{16}(a^2+b^2)^2\end{aligned}$$

Physical particle a: $m_a = 2\mu^2$, unphysical goldstone b: $m_b = 0$

2. parameterization: $\phi(x) = \frac{1}{\sqrt{2}}\big(v + \sigma(x)\big)e^{-i\pi(x)/v}$

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \sigma\right)^2 + \frac{1}{2}\left(1 + \frac{\sigma}{v}\right)^2\left(\partial_\mu \pi\right)^2 - \mu^2\sigma^2 - \frac{\sqrt{\lambda}\mu}{2}\sigma^3 - \frac{\lambda}{16}\sigma^4$$

Physical particle a: $m_\sigma = 2\mu^2$, unphysical goldstone π: $m_\pi = 0$

Spontaneous symmetry breaking

continuous symmetry, non-Abelian

- Lagrangian the Linear Sigma Model (real scalar)

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \phi_i\right)^2 - \frac{1}{2} m^2 \phi_i^2 - \frac{\lambda}{4}(\phi_i \phi_i)^2$$

  - Real scalar field $\vec{\phi} = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_N \end{pmatrix}$
  - Lagrangian is invariant under transformation $\vec{\phi} \to R\vec{\phi}$

$$R = e^{i\theta^a t^a}, \quad a = 1, \ldots \frac{N(N-1)}{2}, \quad R^T R = 1, |R| = 1$$

$$t^1 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, t^2 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, t^3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$ for N=3. $t^{a\dagger} = t^a$ and $t^{aT} = -t^a$

  - If $m^2 = -\mu^2 < 0$, Minima of potential located at $\vec{\phi}(x) = \vec{v}, \ |\vec{v}| = v = \frac{\mu}{\sqrt{\lambda}}$. Vacuum is not invariant

Spontaneous symmetry breaking

continuous symmetry, non-Abelian

- Select $\vec{v} = (0, \ldots, 0, v)^T$ as the physical vacuum
  - For the infinitesimal transformation $\theta^a \ll 1$

$$\vec{v} \to \vec{v} + i\theta^a t^a \vec{v} \quad \text{or} \quad v_i \to v_i + i\theta^a t^a_{ij} v_j = v\delta_{iN} + i\theta^a t^a_{iN} v$$

  - There are $N-1$ t^a lead to $t^a\vec{v} \neq 0$ ($t^a_{iN} \neq 0$).
    1. Vacuum is shifted by these t^a
    2. $N-1$ symmetry is broken
  - There are $(N-1)(N-2)/2$ t^a lead to $t^a\vec{v} = 0$ ($t^a_{iN} = 0$).
    1. Vacuum is invariant under transformation t^a
    2. The remaining symmetry is SO(N-1)
- Define $\vec{\phi}(x) = \left(\pi_1(x), \ldots, \pi_{N-1}(x), v + \sigma(x)\right)^T$

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \pi_k\right)^2 + \frac{1}{2}\left(\partial_\mu \sigma\right)^2 - \mu^2\sigma^2 - \sqrt{\lambda}\mu\sigma^3 - \frac{\lambda}{4}\sigma^4 - \sqrt{\lambda}\mu\pi_k\pi_k\sigma - \frac{\lambda}{2}\pi_k\pi_k\sigma^2 - \frac{\lambda}{4}(\pi_k\pi_k)^2$$

  - Goldstone particle π, $m_\pi = 0$ and $m_\sigma = 2\mu^2$
  - Number of π = Number of broken symmetry
  - Goldstone respect SO(N-1) symmetry

Spontaneous symmetry breaking

Goldstone's Theorem

Goldstone's theorem: *Spontaneous breaking of continuous global symmetries implies the existence of massless particles*

- Prove 1:
  - Potential is invariant under transformation

$$\phi_i' = \left(e^{i\theta^a T^a}\right)_{ij}\phi_j \approx \phi_i + i\theta^a t^a_{ij}\phi_j$$

$$V(\phi_i) = V(\phi_i') = V\left(\left(e^{i\theta^a T^a}\right)_{ij}\phi_j\right) = V(\phi_i) + i\theta^a t^a_{ij}\phi_j \frac{\partial V}{\partial \phi_i}$$

θ^a is arbitrary

$$t^a_{ij}\phi_j \frac{\partial V}{\partial \phi_i} = 0$$

$$\frac{\partial}{\partial \phi_j}\left(t^a_{ik}\phi_k \frac{\partial V}{\partial \phi_i}\right)\bigg|_{\vec{\phi}(x)=\vec{v}} = \frac{\partial}{\partial \phi_j}\left(t^a_{ik}\phi_k\right)\frac{\partial V}{\partial \phi_i}\bigg|_{\vec{\phi}(x)=\vec{v}} + t^a_{ik}\phi_k\left(\frac{\partial^2 V}{\partial \phi_j \partial \phi_i}\right)\bigg|_{\vec{\phi}(x)=\vec{v}} = 0$$

$\frac{\partial V}{\partial \phi_i}\Big|_{\vec{\phi}(x)=\vec{v}} = 0$ since $\vec{v}$ minimizes V

Spontaneous symmetry breaking

Goldstone's Theorem

- Prove 1:

$$t_{ik}^{a}\phi_k\left(\frac{\partial^2 V}{\partial\phi_j\partial\phi_i}\right)\Bigg|_{\vec{\phi}(x)=\vec{v}} = t_{ik}^{a}v_k M_{ij}^2 = 0$$

- ➢ Broken symmetry $t^a\vec{v} \neq 0\ (t_{iN}^{a} \neq 0) \rightarrow M_{ij}^2 = 0$
- ➢ Remaining symmetry $t^a\vec{v} = 0\ (t_{iN}^{a} = 0) \rightarrow$ trivial case
- ➢ Expanding V respect to $\vec{v}$

$$V\left(\vec{\phi}\right) \approx V(\vec{v}) + \frac{1}{2}(\phi_i - v_i)(\phi_j - v_j)\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\Bigg|_{\vec{\phi}(x)=\vec{v}}$$

- ➢ $M_{ij}^2 = \frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\Big|_{\vec{\phi}(x)=\vec{v}}$ is a symmetric matrix whose eigenvalues give the masses of the fields

Spontaneous symmetry breaking

Goldstone's Theorem

- Prove 2:
  - Lagrangian is invariant under transformation

$$\phi_i \to \phi_i' = \left(e^{i\theta^a t^a}\right)_{ij}\phi_j$$

  - Corresponding **Noether current** and **conserved charge** is

$$J^{\mu a} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} t^a \phi, \quad Q^a = \int d^3x\, J^{0a}; \quad \partial_\mu J^{\mu a} = 0, \quad \frac{dQ^a}{dt} = 0$$

  - Spontaneous symmetry breaking case

$$\langle 0|\vec{\phi}|0\rangle = \vec{v}$$

  - Equal time commutation relation

$$\left[Q^a, \vec{\phi}\right] = -t^a\, \vec{\phi}; \quad \langle 0|\left[Q^a, \vec{\phi}\right]|0\rangle = -t^a \vec{v}$$

1. Broken symmetry $\langle 0|\left[Q^a, \vec{\phi}\right]|0\rangle = -t^a \vec{v} \neq 0$, which is the case we concern
2. Remaining symmetry $\langle 0|\left[Q^a, \vec{\phi}\right]|0\rangle = -t^a \vec{v} = 0$. Trival case.

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- Prove 2:

$$\langle 0|\left[Q^a,\vec{\phi}\right]|0\rangle = \sum_n \int d^3 y[\langle 0|J^{0a}(y)|n\rangle\langle n|\vec{\phi}(x)|0\rangle - \langle 0|\vec{\phi}(x)|n\rangle\langle n|J^{0a}(y)|0\rangle]$$

translation invariance: $J^{0a}(y) = e^{-iP\cdot y}J^{0a}(0)e^{iP\cdot y}$

$e^{iP\cdot y}\ |0\rangle = |0\rangle$ and $e^{iP\cdot y}\ |n\rangle = e^{ip_n\cdot y}\ |n\rangle$

$$= \sum_n \int d^3 y\left[\langle 0|J^{0a}(0)|n\rangle\left\langle n\middle|\vec{\phi}(x)\middle|0\right\rangle e^{ip_n\cdot y} - \left\langle 0\middle|\vec{\phi}(x)\middle|n\right\rangle\langle n|J^{0a}(0)|0\rangle e^{-ip_n\cdot y}\right]$$

$$\int d^3 y\, e^{i\overrightarrow{p_n}\cdot(\vec{y}-\vec{x})} = (2\pi)^3\delta^3(\overrightarrow{p_n})$$

Equal time $x_0 = y_0$

$p_n^{\mu} = (M_n,\overrightarrow{p_n})$

$$= \sum_n \delta^3(\overrightarrow{p_n})\left[\langle 0|J^{0a}(0)|n\rangle\left\langle n\middle|\vec{\phi}(0)\middle|0\right\rangle e^{iM_n y^0} - \text{conj}\right] \neq 0$$

1. $\delta^3(\overrightarrow{p_n})$: Momentum is conserved and the momentum of the intermediate state |n⟩ is zero
2. $e^{iM_n y^0}$: The evolution of the intermediate state depends on its mass M_n
3. Exist at least one intermediate state $|n\rangle$ satisfied $\langle 0|J^{0a}(0)|n\rangle\left\langle n\middle|\vec{\phi}(0)\middle|0\right\rangle \neq 0$

Spontaneous symmetry breaking

Goldstone's Theorem

- Prove 2:

$$\partial_{y^0}\langle 0|\left[Q^a,\vec{\phi}(x)\right]|0\rangle = \int \mathrm{d}^3y\,\partial_{y^0}\left\langle 0\,\middle|\,\left[\mathrm{J}^{0\mathrm{a}}(\mathrm{y}),\,\vec{\phi}(\mathrm{x})\right]\middle|\,0\right\rangle$$

$$\partial_0 J^{0a} = \nabla\cdot\vec{J}^a$$

$$= \int \mathrm{d}^3y\,\left\langle 0\,\middle|\,\left[\nabla\cdot\vec{J}^a(\mathrm{y}),\,\vec{\phi}(\mathrm{x})\right]\middle|\,0\right\rangle$$

$$= \int d^3y\,\nabla_y\cdot\left\langle 0\middle|\left[\vec{J}^a(\mathrm{y}),\,\vec{\phi}(\mathrm{x})\right]\middle|0\right\rangle = \oint_\infty dS\cdot\left\langle 0\middle|\left[\vec{J}^a(\mathrm{y}),\,\vec{\phi}(\mathrm{x})\right]\middle|0\right\rangle = 0$$

$$\partial_{y^0}\langle 0|\left[Q^a,\vec{\phi}(x)\right]|0\rangle = \partial_{y^0}\left(\sum_n \delta^3(\vec{p_n})\left[\langle 0|J^{0a}(0)|n\rangle\left\langle n\middle|\vec{\phi}(0)\middle|0\right\rangle e^{iM_n y^0} - \mathrm{conj}\right]\right)$$

$$= \sum_n \delta^3(\vec{p_n})\,iM_n\left[\langle 0|J^{0a}(0)|n\rangle\left\langle n\middle|\vec{\phi}(0)\middle|0\right\rangle e^{iM_n y^0} + \left\langle 0\middle|\vec{\phi}(0)\middle|n\right\rangle\langle n|J^{0a}(0)|0\rangle e^{-iM_n y^0}\right] = 0$$

- Remember that there exist $|n\rangle$ satisfied $\langle 0|J^{0a}(0)|n\rangle\left\langle n\middle|\vec{\phi}(0)\middle|0\right\rangle \neq 0$. Consider the case $\vec{p_n}$=0, we have $M_n = 0$. Thus the particle associate to state $|n\rangle$ must be massless.

Contents

Gauge theories with spontaneous symmetry breaking

The Higgs Mechanism-An Abelian Example

recall

$$\mathcal{L} = -\frac{1}{4}\left(F_{\mu\nu}\right)^2 + \left(D_\mu\phi^\dagger\right)(D^\mu\phi) - V(\phi); \quad V(\phi) = m^2\phi^\dagger\phi + \frac{\lambda}{4}\left(\phi^\dagger\phi\right)^2; \quad D_\mu = \partial_\mu - igA_\mu$$

- Lagrangian is invariant under local U(1) transformation

$$\phi(x) \to e^{i\alpha(x)}\phi(x), \quad A_\mu(x) \to A_\mu(x) + \frac{1}{g}\partial_\mu\alpha(x)$$

- Consider the case $m^2 = -\mu^2 < 0$, ϕ has nonzero vacuum expectation value

$$\langle 0|\phi(x)|0\rangle = \frac{v}{\sqrt{2}}, \quad v = \sqrt{\frac{4\mu^2}{\lambda}}$$

Gauge theories with spontaneous symmetry breaking

The Higgs Mechanism-An Abelian Example

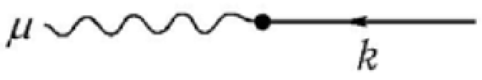
- Parameterization 1

$$\phi(x) = \frac{1}{\sqrt{2}}(v + H(x) + iG(x))$$

$$V(\phi) = \frac{1}{4}\lambda v^2 H^2 + \frac{1}{4}\lambda v H(H^2 + G^2) + \frac{\lambda}{16}(H^2 + G^2)^2$$

$$(D_\mu \phi^\dagger)(D^\mu \phi) = \frac{1}{2}\left[(\partial_\mu H)^2 + (\partial_\mu G)^2 + g^2 A_\mu A^\mu (v + H)^2 + g^2 A_\mu A^\mu G^2\right] + gA^\mu(\partial_\mu G)(v + H) - gA^\mu(\partial_\mu H)G$$

1. Higgs is massive $m_H = \sqrt{\lambda/2}v$
2. Goldstone is massless
3. Term $gvA^\mu(\partial_\mu G)$ means the transformation (interaction) between field *A* and *G*.
4. The Goldstone boson supplies exactly the right pole to make the vacuum polarization amplitude properly transverse



$$= igv(-ik^\mu) = m_A k^\mu$$

$$= im_A^2 g^{\mu\nu} + (m_A k^\mu)\frac{i}{k^2}(-m_A k^\nu)$$

$$= im_A^2\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right).$$

Gauge theories with spontaneous symmetry breaking

The Higgs Mechanism-An Abelian Example

- Parameterization 2

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + H(x)\right)e^{-i\pi(x)/v}$$

- Unitary gauge $\alpha(x) = \frac{\pi(x)}{v}$

$$V(\phi) = \frac{1}{4}\lambda v^2 H^2 + \frac{1}{4}\lambda v H^3 + \frac{\lambda}{16} H^4$$

$$(D_\mu \phi^\dagger)(D^\mu \phi) = \frac{1}{2}\left(\partial_\mu H\right)^2 + \frac{1}{2} g^2 v^2 A_\mu A^\mu + g^2 v H A_\mu A^\mu + \frac{1}{2} g^2 H^2 A_\mu A^\mu$$

1. Goldstone disappeared, unphysical!
2. Gauge boson obtain mass $m_A = gv$

Gauge theories with spontaneous symmetry breaking

The Higgs Mechanism-An Abelian Example

Higgs mechanism: *spontaneous symmetry breaking generates a mass for a gauge boson*

- Goldstone is massless and unphysical, thus can be eliminated by Unitary gauge
- Massless boson has two transvers degree of freedom
- Massive boson has two transvers degree of freedom and one longitudinal degree of freedom
- gauge boson acquired its extra degree of freedom by “eating” the Goldstone boson
- degree of freedom in the theorem remains the same (2+1 →3)

Gauge theories with spontaneous symmetry breaking

The Higgs Mechanism-non Abelian Example

- Complex scale field ϕ under the Gauge group G, covariant derivative is

$$D_\mu\phi = \partial_\mu\phi - igA_\mu^a t_R^a \phi$$

- vacuum expectation value

$$\langle 0|\phi(x)|0\rangle = \frac{v}{\sqrt{2}}$$

- Kinetic term

$$\mathcal{L}_{\text{kin}} = (D_\mu\phi)^\dagger (D^\mu\phi)$$

keep only v's contributions

$$\sim \left(-igA_\mu^a t_R^a \frac{v}{\sqrt{2}}\right)^\dagger \left(-igA^{\mu b} t_R^b \frac{v}{\sqrt{2}}\right)$$

$$= \frac{g^2}{2} A_\mu^a A^{\mu b} (v^\dagger t_R^a t_R^b v)$$

$$= \frac{g^2}{4} A_\mu^a A^{\mu b} \left(v^\dagger \{t_R^a, t_R^b\} v\right)$$

$$= \frac{1}{4} g^2 v_i \{t_R^a, t_R^b\}_{ij} v_j \; A_\mu^a A^{\mu b}$$

$$t_R^{a\dagger} = t_R^a, v^\dagger = v^T$$

Gauge theories with spontaneous symmetry breaking

The Higgs Mechanism-non Abelian Example

- Mass of gauge boson

$$(m^2)_{ab} = \frac{1}{2} g^2 v_i \left\{t_R^a, t_R^b\right\}_{ij} v_j$$

1. Broken symmetry $t^a \vec{v} \neq 0$ ($t_{iN}^a \neq 0$) (chosen a special direction)
2. Remaining symmetry $t^a \vec{v} = 0$ ($t_{iN}^a = 0$)
3. Only for Broken generator, the gauge boson obtain mass

Gauge theories with spontaneous symmetry breaking

The Higgs Mechanism： $G_1 \times G_2$

- ϕ belongs to the $R_1 \times R_2$representation in $G_1 \times G_2$ group

$$D_\mu \phi = \partial_\mu \phi - i g_1 A_\mu^{a_1} t_{R_1}^{a_1} \phi - i g_2 B_\mu^{a_2} t_{R_2}^{a_2} \phi$$

  - g_1 and g_2 are the coupling constants of $G_1$and G_2
  - $A_\mu^{a_1}$ and $A_\mu^{a_2}$ are the gauge field of $G_1$and G_2 $(a_1 = 1, \ldots, d_{R_1})$ $(a_2 = 1, \ldots, d_{R_2})$
  - $t_{R_1}^{a_1}$ and $t_{R_2}^{a_2}$ are the generator in the $R_1 \times R_2$ representation of $G_1 \times G_2$ group
  - ϕ_{i_1,i_2}: $d_{R_1} \times d_{R_2}$

- VEV

$$\langle 0 | \phi_{i_1,i_2} | 0 \rangle = v_{i_1,i_2}$$

- Gauge boson mass

$$\mathcal{L}_{\text{mass}} = v^\dagger \left(g_1 A_\mu^{a_1} t_{R_1}^{a_1} + g_2 B_\mu^{a_2} t_{R_2}^{a_2} \right)^2 \mathrm{v}$$

$$(m^2)_{ab} = \begin{pmatrix} \frac{1}{2} g_1^2 v^\dagger \{t_{R_1}, t_{R_1}\} v & \frac{1}{2} g_1 g_2 v^\dagger \{t_{R_1}, t_{R_2}\} v \\ \frac{1}{2} g_1 g_2 v^\dagger \{t_{R_2}, t_{R_1}\} v & \frac{1}{2} g_2^2 v^\dagger \{t_{R_2}, t_{R_2}\} v \end{pmatrix}$$

Gauge theories with spontaneous symmetry breaking

The Higgs Mechanism-non Abelian Example

- SU(2) example
  - ϕ belongs to the fundamental representation (complex spinor representation) in SU(2) Group

$$\langle 0|\phi_0|0\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$(m^2)_{ab} = \frac{g^2}{2}(0, v/\sqrt{2})\left(\frac{\tau^a}{2}\frac{\tau^b}{2} + \frac{\tau^b}{2}\frac{\tau^a}{2}\right)\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$= \frac{g^2 v^2}{4}\delta^{ab} \rightarrow \text{Three massive gauge boson}$$

Gauge theories with spontaneous symmetry breaking

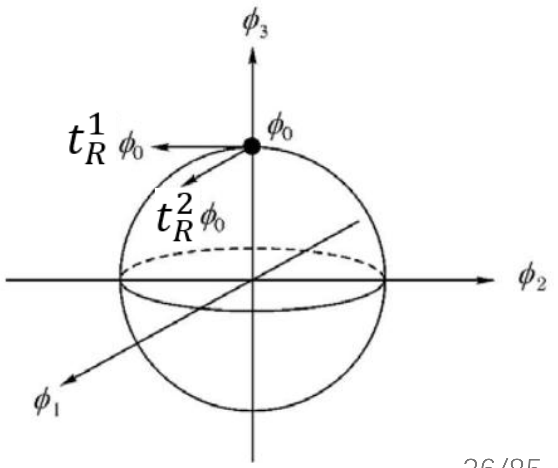
The Higgs Mechanism-non Abelian Example

- SU(2) example
  - ➢ ϕ belongs to the adjoint representation (real vector representation) in SU(2) Group

$$(t_R{}^{a})_{ij} = -i\varepsilon^{aij};\quad \left(D_\mu\phi\right)_a = \partial_\mu\phi_a + g\varepsilon_{aij}A_\mu^i\phi_j$$

$$\epsilon^{1ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}_{ij},\quad \epsilon^{2ij} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}_{ij},\quad \epsilon^{3ij} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij}$$

$$\langle 0|\phi_0|0\rangle = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \Rightarrow m^2 = \begin{pmatrix} g^2v^2 & & \\ & g^2v^2 & \\ & & 0 \end{pmatrix}$$



1. Broken symmetry ϵ^{1ij} and ϵ^{2ij}, two gauge boson obtain mass
2. Remaining symmetry $U(1)$ ϵ^{3ij}, one gauge boson remain massless

Gauge theories with spontaneous symmetry breaking

The Higgs Mechanism-non Abelian Example

- SU(N) example
  - ϕ belongs to the fundamental representation (complex spinor representation) in SU(N) Group

$$\langle 0|\phi_0|0\rangle = \begin{pmatrix} 0 \\ \dots \\ v \end{pmatrix}$$

$$t_F^a = \frac{1}{2}\begin{pmatrix} 0 & 1 & \\ 1 & \dots & \\ & & 0 \end{pmatrix},\quad \frac{1}{2}\begin{pmatrix} 0 & -i & \\ i & \dots & \\ & & 0 \end{pmatrix},\quad \frac{1}{\sqrt{2n(n+1)}}\begin{pmatrix} 1 & & & & \\ & \dots & & & \\ & & n & & \\ & & & \dots & \\ & & & & 0 \end{pmatrix}$$

$t_F^{a\dagger} = t_F^a$
$\mathrm{Tr}(t_F^a)$=0

Number of generators	$N(N-1)/2$	$N(N-1)/2$	$(N-1)$
Number of broken generators	$(N-1)$	$(N-1)$	1
Mass of gauge boson m_A	$\frac{gv}{2}$	$\frac{gv}{2}$	$\sqrt{\frac{N-1}{2N}}gv$

Gauge theories with spontaneous symmetry breaking

The Higgs Mechanism-non Abelian Example

- SU(N) example
  - ϕ belongs to the adjoint representation (real vector representation) in SU(N) Group

$$D_\mu \phi_a = \partial_\mu \phi_a + g f_{abc} A_\mu^b \phi_c$$

$$\downarrow$$

$$D_\mu \boldsymbol{\phi} = \partial_\mu \boldsymbol{\phi} - i g A_\mu^a [t_F^a, \boldsymbol{\phi}]$$

$$\phi^a \to \left(e^{i\theta^b t_A^b}\right)^{ac} \phi^c; \quad (t_A^a)_{bc} = -i f^{abc}; \quad \boldsymbol{\phi} = \phi^a t_F^a$$

Example SU2: $\left(e^{i\theta^3 t_A^3}\right)^{ac} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$

  - VEV is a matrix,

$$\langle 0|\boldsymbol{\phi}|0\rangle = \langle 0|\boldsymbol{\phi}|0\rangle^\dagger = v; \qquad \mathrm{Tr}\langle 0|\boldsymbol{\phi}|0\rangle = 0$$

$$\downarrow \quad \text{SU(N) transformation}$$

$$v = \mathrm{diag}\{\underbrace{v_1, \ldots, v_1}_{\#=N1}, \underbrace{v_2, \ldots, v_2}_{\#=N2}, \ldots, v_n\}, \quad \sum v_i N_i = 0$$

#=N1 #=N2 ⋯

Gauge theories with spontaneous symmetry breaking

The Higgs Mechanism-non Abelian Example

- SU(N) example
  - Gauge boson Mass

$$\mathcal{L}_{\text{kin}} = \frac{1}{2}\text{Tr}\left(\left(D_\mu v\right)^\dagger (D^\mu v)\right)$$
$$= \frac{1}{2} g^2 \text{Tr}\left([t_F^a, v][t_F^b, v]\right) A_\mu^a A^{\mu b}$$

⬇

$$m_{ab}^2 = -g^2 \text{Tr}\left([t_F^a, v][t_F^b, v]\right)$$

1. unbroken symmetry: $[t_F^a, v] = 0$
2. Broken symmetry: $[t_F^a, v] \neq 0$, the masses of gauge boson are determined by v

v =diag$\{v_1,\dots, v_1 , v_2 ,\dots, v_2 ,\dots, v_n\}$, $\sum v_i N_i = 0$

Gauge theories with spontaneous symmetry breaking

The Higgs Mechanism-non Abelian Example

- SU(N) example
  - Remaining symmetry
    1. $SU(N_1)\times SU(N_2)\times\cdots\times SU(N_n)$

$$t_F^{i,a} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & SU(N_i) & \vdots \\ 0 & \cdots & 0 \end{bmatrix}; \qquad [t_F^{i,a}, v] = 0$$

    2. $U(1)^{n-1}$

$$t_F^{diag,1} \sim \text{diag}\left\{1, \dots, 1, -\frac{N_1}{N_2}, \dots, -\frac{N_1}{N_2}, \dots, 0, \dots, 0\right\}$$
$$t_F^{diag,2} \sim \text{diag}\left\{0, \dots, 0, 1, \dots, 1, -\frac{N_2}{N_3}, \dots, -\frac{N_2}{N_3}, \dots\right\}$$
$$\dots$$
$$[t_F^{diag,i}, v] = 0$$

Gauge theories with spontaneous symmetry breaking

The Higgs Mechanism-non Abelian Example

- SU(5) example

$$v = \text{diag}\{-\frac{v}{3}, -\frac{v}{3}, -\frac{v}{3}, \frac{v}{2}, \frac{v}{2}\}$$

➢ SU(5)→ SU(3)×SU(2)×U(1) (SM)

$$t_F^{1,a} = \frac{1}{2}\begin{pmatrix} \lambda_a & 0_{3\times2} \\ 0_{2\times3} & 0_{2\times2} \end{pmatrix}, \quad (a = 1,2,\ldots,8)$$

$$t_F^{2,a} = \frac{1}{2}\begin{pmatrix} 0_{3\times3} & 0_{3\times2} \\ 0_{2\times3} & \tau_a \end{pmatrix}, \quad (a = 1,2,3)$$

$$t_F^{diag,1} = \frac{1}{\sqrt{60}}\,\text{diag}\{2,2,2,-3,-3\}$$

Contents

- Introduction to The Standard Model
- Gauge invariance
- Spontaneous symmetry breaking
- Gauge theories with spontaneous symmetry breaking
- Quantization of spontaneously broken gauge theories
- The Glashow-Weinberg-Salam electroweak theory

Quantization of spontaneously broken gauge theories

- Unitary gauge
  - Without un-physical goldstone ☑
  - Renormalizability ✗
- R_ξ gauge
  - With un-physical goldstone ✗
  - Renormalizability ☑

Quantization of spontaneously broken gauge theories Abelian Example，Unitary gauge

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- Lagrangian

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |D_\mu \phi|^2 - V(\phi); \quad \phi = \frac{1}{\sqrt{2}}(v + H)e^{-i\pi/v}$$

- In Unitary gauge: $\pi(x) = 0$

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}(F_\mu v)^2 + |\partial_\mu \phi + igA_\mu \phi|^2 - V(\phi) \\ &= -\frac{1}{4}(F_\mu v)^2 + \frac{1}{2}|\partial_\mu (v + H) + igA_\mu (v + H)|^2 + \cdots \\ &= -\frac{1}{4}(F_\mu v)^2 + \frac{1}{2} m_A^2 A_\mu A^\mu + \cdots\end{aligned}$$

$$m_A = gv$$

- Path Integral with gauge fixing term $\delta(\pi(x))$

$$\int \mathrm{D}\,A\, \mathcal{D}\phi\, \mathcal{D}\phi^\dagger\, \delta(\pi(x))\, e^{iS}$$

Quantization of spontaneously broken gauge theories Abelian Example，Unitary gauge

- variable substitution: $\phi,\ \phi^\dagger \to H, \pi$ and ghost field c

$$\mathcal{D}X = \mathcal{D}Y\ \det\left(\frac{\delta X(Y)}{\delta Y}\right)$$

$$J = \det\left(\frac{\delta\phi}{\delta H}\frac{\delta\phi}{\delta\pi}\right) \propto \det\left(1+\frac{H}{v}\right) \propto \int Dc\ \mathcal{D}\bar{c}\ e^{-im_{gh}^2 \int d^4x\bar{c}\left(1+\frac{H}{v}\right)c}$$

1. Ghost field is an un-physical field introduced to compensate for gauge multiplicity degrees of freedom
2. Ghost-ghost-Higgs vertex: $-\frac{m_{gh}^2}{v}\bar{c}cH$
3. Gauge boson propagator: $D^{\mu\nu}(k) = \frac{-i}{k^2-m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_A^2}\right)$
4. Ghost propagator: $D^{\text{ghost}} = \frac{-i}{m_{gh}^2}$
5. Ghost mass m_{gh} is arbitrary

- Renormalizability
  1. For $k\to\infty(UV)$, $D^{\mu\nu}(\infty)\to\frac{1}{m_A^2}$, $D^{\text{ghost}}\to\frac{1}{m_{gh}^2}$. Loop integral contains $\int dk$
  2. UV divergence for any external legs

Quantization of spontaneously broken gauge theories Abelian Example， R_ξ gauge

- Lagrangian

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 + |\mathcal{D}_\mu\phi|^2 - V(\phi)$$

- In R_ξ gauge gauge: $\phi(x) = \frac{1}{\sqrt{2}}\left(v + H(x) + iG(x)\right)$

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}(\partial_\mu - igA_\mu)(v + H + iG)(\partial^\mu + igA^\mu)(v + H - iG) \\ &= -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}\left(\partial_\mu H + i\partial_\mu G - igvA_\mu - igA_\mu H + gA_\mu G\right)(\partial^\mu H - i\partial^\mu G + igvA^\mu + igA^\mu H + gA^\mu G) \\ &= -\frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}(\partial_\mu H)^2 + \frac{1}{2}(\partial_\mu G)^2 + \frac{1}{2}g^2v^2A_\mu A^\mu - gvA^\mu\partial_\mu G \\ &\quad + gA^\mu\left(G\partial_\mu H - H\partial_\mu G\right) + g^2vHA_\mu A^\mu + \frac{1}{2}g^2(H^2 + G^2)A_\mu A^\mu - V(\phi)\end{aligned}$$

- Gauge fixing condition

$$\tilde{G}(A) = \partial^\mu A_\mu + \xi gvG = 0$$

1. ξ is unphysical and arbitrary

Quantization of spontaneously broken gauge theories Abelian Example， R_ξ gauge

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- Gauge fixing Lagrangian

$$\mathcal{L}_{gf} = -\frac{1}{2\xi}\left(\tilde{G}(A)\right)^2 = -\frac{1}{2\xi}\left(\partial^\mu A_\mu\right)^2 - gvG\partial^\mu A_\mu - \frac{\xi}{2}g^2v^2G^2$$

1. $-gvG\partial^\mu A_\mu$ in $\mathcal{L}_{gf}$ and $-gvA^\mu\partial_\mu G$ in $\mathcal{L}$ can be written as total derivative, thus can be discarded. No gauge boson-gold stone mixing.
2. Goldstone mass $m_G^2 = \xi g^2v^2 = \xi m_A^2$

- Faddeev-Popov ghost Lagrangian

$$\mathcal{L}_{gh} = \bar{c}\frac{\delta\tilde{G}(A^\alpha)}{\delta\alpha}c$$

$$A_\mu \to A_\mu - \partial_\mu\alpha, \quad \phi \to \phi - ig\alpha$$
$$H \to H + g\,\alpha\,G, \quad G \to G - g\,\alpha\,(v + H)$$
$$\tilde{G}(A^\alpha) = \partial^\mu\left(A_\mu - \partial_\mu\alpha\right) + \xi gv\left(G - g\alpha(v+H)\right)$$

$$= \bar{c}\left(-\partial^2 - \xi g^2v^2\left(1 + \frac{H}{v}\right)\right)c = -\bar{c}\partial^2 c - \xi g^2v^2\bar{c}c - \xi g^2 vH\bar{c}c$$

1. Ghost mass $m_{gh}^2 = \xi g^2v^2 = \xi m_A^2 = m_G^2$ (not $2\xi g^2v^2$)

Quantization of spontaneously broken gauge theories Abelian Example， R_ξ gauge

- Feyman rules
  - Ghost-ghost-Higgs vetex

$$-i\xi g^2 v = -i\frac{m_{gh}^2}{v}$$

  - Gauge boson propagator

$$\frac{-i}{k^2 - m_A^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) + \frac{-i\xi}{k^2 - \xi m_A^2}\frac{k^\mu k^\nu}{k^2}$$

1. The first term corresponds to physical transvers polarization
2. The second item is the non-physical item which depends on gauge
3. The second item + goldstone contribution → physical result

  - Goldstone and Ghost propagator

$$\frac{i}{k^2 - \xi m_A^2}$$

  - $\xi = 0$ →Landau gauge; $\xi = 1$ →Feyman gauge; $\xi = \infty$ → Unitary gauge (goldstone, ghost decouple)

Quantization of spontaneously broken gauge theories non-Abelian Example， R_ξ gauge

- SU(N) ⊂ SO(2N)

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \cdots \\ \phi_N \end{pmatrix} \in C^N$$

$$\boldsymbol{\Phi} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \\ b_1 \\ b_2 \\ \vdots \\ b_N \end{pmatrix} \in R^{2N}$$

$$\phi_j = a_j + i b_j, \quad a_j, b_j \in R$$

$$\phi \to U\,\phi,$$
$$U = A + iB \in SU(N),$$
$$A, B \in R^{N*N}$$
$$U^\dagger U = I \Rightarrow A^\top A + B^\top B = I, A^\top B = B^\top A$$

$$\boldsymbol{\Phi} \to M_U \boldsymbol{\Phi},$$
$$M_U = \begin{pmatrix} A & -B \\ B & A \end{pmatrix} \in R^{2N\times 2N}$$

$$M_U^\top M_U = \begin{pmatrix} A^\top & B^\top \\ -B^\top & A^\top \end{pmatrix}\begin{pmatrix} A & -B \\ B & A \end{pmatrix} = \begin{pmatrix} A^\top A + B^\top B & -A^\top B + B^\top A \\ -B^\top A + A^\top B & B^\top B + A^\top A \end{pmatrix} = I$$

$$M_U \in SO(2N)$$

Quantization of spontaneously broken gauge theories non-Abelian Example， R_ξ gauge

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- Lagrangian

$$\mathcal{L} = -\frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{1}{2}\left(D_\mu\phi\right)^2 - V(\phi); \qquad D_\mu\phi = \partial_\mu\phi - ig_a A^a_\mu t^a\phi\,;$$

  - t^a is the generator of SO(N), antisymmetric hermitian matrix (pure imaginary)
  - g_a is the coupling constant corresponding to the different generators

- VEV

$$\langle 0|\phi_i|0\rangle = v_i; \quad \phi_i(x) = v_i + \chi_i(x)$$

- Higgs mass
  - Mass of χ_i (Higgs or goldstone) given by $V(\phi)$

$$(m_H^2)_{ij} = \left.\frac{\partial^2 V}{\partial\phi_i\partial\phi_j}\right|_{\phi=v}$$

  - Broken generator: $t^a_{ij}v_j \neq 0 \rightarrow$ massless goldstone
  - Un-broken generator: $t^a_{ij}v_j = 0 \rightarrow (m_H^2)_{ij}(t^a\, v)_j$

Quantization of spontaneously broken gauge theories non-Abelian Example, R_ξ gauge

- Gauge boson mass
  - Define: $T^a = i g_a t^a$; $F_i^a = T_{ij}^a v_j$; $T^a{}_{ij} = -T^a{}_{ji} \in R$

$$\left(D_\mu \phi\right)_i = \partial_\mu \chi_i - i g_a A_\mu^a t_{ij}^a (v+\chi)_j = \partial_\mu \chi_i - A_\mu^a (F^a + T^a \chi)_i$$

$$\frac{1}{2}\left(D_\mu \phi\right)^2 \rightarrow \frac{1}{2}\left(F_i^a F_i^b\right) A_\mu^a A^{\mu b} +$$

$$(m_A^2)^{ab} = F_i^a F_i^b = (F F^\top)^{ab}$$

Quantization of spontaneously broken gauge theories non-Abelian Example， R_ξ gauge

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- Goldstone mass
  - The mass of a Goldstone particle is derived from the gauge fixing term

$$\widetilde{G^a}(A) = \partial^\mu A^a_\mu + \xi F^a_i \chi_i$$

$$\mathcal{L}_{gf} = -\frac{1}{2\xi}\left(\partial^\mu A^a_\mu\right)^2 - \left(\partial^\mu A^a_\mu\right) F^a_i \chi_i - \boxed{\frac{\xi}{2} F^a_i F^a_j \chi_i \chi_j}$$

$$\Downarrow$$

$$(m_G^2)_{ij} = \xi F^a_i F^a_j = \xi (F^\top F)_{ij}$$

  - Mass of χ_i is given by

$$m^2 = m_H^2 + m_G^2$$

$$(m_H^2)_{ij}(m_G^2)_{jk} = (m_H^2)_{ij}(T^a v)_j \xi F^a_k = (m_H^2)_{ij}(i g_a t^a v)_j \xi F^a_k = 0$$

1. Two mass matrix can be orthogonalized separately, and the result after orthogonalization is

   $m_H^2 \rightarrow$ Higgs mass (physical)

   $m_G^2 \rightarrow$ Goldstone mass (unphysical)

2. $m_G^2 = \xi\,(F^\top F)$; $m_A^2 = (F^\top F)$, similar to U(1)

Quantization of spontaneously broken gauge theories non-Abelian Example，R_ξ gauge

- **Ghost mass**
  - Gauge transformation

$$\phi \to \phi - \alpha^a T^a \phi, \qquad A_\mu^a \to A_\mu^a - D_\mu^{ab}\alpha^b$$

  - Ghost Lagrangian

$$\delta G^a/\delta\alpha^b = -\partial^\mu D_\mu^{ab} - \xi F_i^a T_{ij}^b (v+\chi)_j = -\partial^\mu D_\mu^{ab} - \xi F_i^a F_i^b - \xi F_i^a T_{ij}^b \chi_j$$

$$\Downarrow$$

$$L_{gh} = \bar{c}^a\left(-\partial^\mu D_\mu^{ab}\right)c^b - \xi (m_A^2)^{ab}\bar{c}^a c^b - \xi F_i^a T_{ij}^b \chi_j \bar{c}^a c^b$$

$$m_{gh}^2 = \xi m_A^2$$

Quantization of spontaneously broken gauge theories The Goldstone Equivalence Theorem

- Gauge boson at its rest frame $k^\mu = (m_A,\ 0,\ 0,\ 0)$

$\epsilon_0^\mu(k) = (1,\ 0,\ 0,\ 0)$ scalar polarization, unphysical

$\epsilon_1^\mu(k) = \frac{1}{\sqrt{2}}(0,\ 1,\ i,\ 0)$ transverse polarization, physical

$\epsilon_2^\mu(k) = \frac{1}{\sqrt{2}}(0,\ 1,\ -i,\ 0)$ transverse polarization, physical

$\epsilon_3^\mu(k) = (0,\ 0,\ 0,\ 1)$ longitudinal polarization, physical

- For physical polarization

$$k_\mu \epsilon_\lambda^\mu(k) = 0, \qquad \lambda = 1,2,3, \qquad \epsilon_i(k) \cdot \epsilon_j^*(k) = -\delta_{ij}$$

$$\sum_{\lambda=1}^{3} \epsilon_\lambda^\mu \epsilon_\lambda^\nu(k) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{m_A^2}$$

- For unphysical polarization

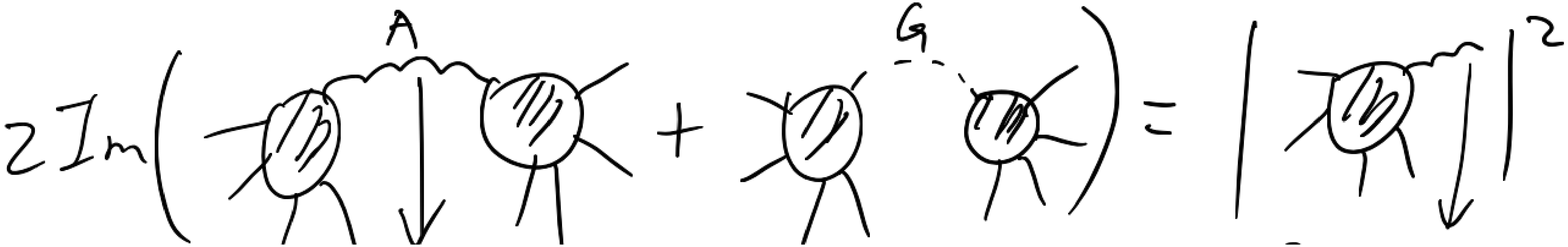
$$\epsilon_0^\mu \epsilon_0^\nu = (1,\ 0,\ 0,\ 0)^\mu\ (1,\ 0,\ 0,\ 0)^\nu = \frac{(m_A, 0, 0, 0)^\mu}{m_A} \frac{(m_A, 0, 0, 0)^\nu}{m_A} = \frac{k^\mu k^\nu}{m_A^2}$$

Quantization of spontaneously broken gauge theories The Goldstone Equivalence Theorem

- Gauge boson at its rest frame $k^{\mu} = (m_A, 0, 0, 0)$
  - Cancelation between scalar polarization and goldstone

$$-g^{\mu\nu} = \sum_{\text{physical pol}} \epsilon_{\lambda}^{\mu}\epsilon_{\lambda}^{\nu}(k) - \text{unphysical pol}$$

  - In Feynman-'t Hooft gauge



$-g^{\mu\nu}$

$$\sum_{\text{physical pol}} \epsilon_{\lambda}^{\mu}\epsilon_{\lambda}^{\nu}(k)$$

Quantization of spontaneously broken gauge theories The Goldstone Equivalence Theorem

- **Gauge boson at high energy limit**

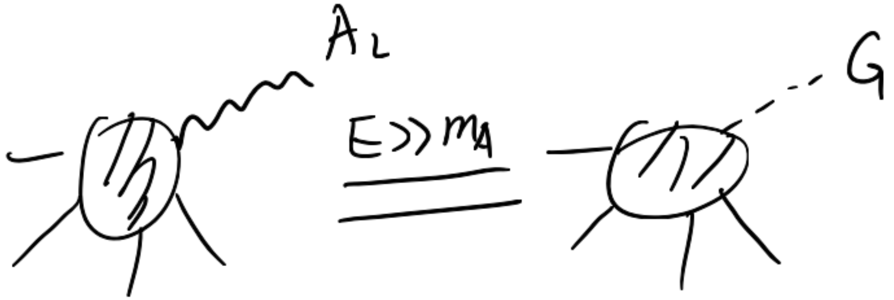
  - Boost $k^{\mu} = (m_A, 0, 0, 0)$ to $k^{\mu} = (E, 0, 0, k_z)$

$$\epsilon_0^{\mu} = \left(\frac{E}{m_A}, 0, 0, \frac{k_z}{m_A}\right); \ \epsilon_3^{\mu} = \left(\frac{k_z}{m_A}, 0, 0, \frac{E}{m_A}\right)$$

  - high energy limit: $E \gg m_A$

$$E^2 - k_z^2 = m_A^2 \Rightarrow E \approx k_z$$

$$\epsilon_3^{\mu} \xRightarrow[E \gg m_A]{} \epsilon_0^{\mu} = \frac{k^{\mu}}{m_A}$$



- longitudinal gauge bosons become precisely those of their associated Goldstone bosons

Contents

The Glashow-Weinberg-Salam electroweak theory History

- 1896: Spontaneous radioactive decay (Becquerel).
- 1897: β decay (Thomson): $N_i \to N_f + e^-$
- 1914: Discovery of the continuous β energy spectrum (Chadwick). (Energy not conserved? Non-two-body decay?)
- 1930: Neutrino hypothesis (Pauli):$N_i \to N_f + e^- + \bar{\nu}$
- 1932: Discovery of the neutron (Chadwick):$n \to p + e^- + \bar{\nu}$
- 1934: Four-fermion interaction theory:

$$\mathcal{L} = -G_F/\sqrt{2}\ (\overline{\psi_p}\,\gamma_\mu\,\psi_n)(\overline{\psi_e}\gamma^\mu\psi_\nu) + h.c.$$

  - ➢ G_F is the Fermi constant. Successful experimentally but non-renormalizable theoretically. This theory violates unitarity: at √s ≥ 500 GeV, $P(\nu e^- \to \nu e^-) > 1$
  - ➢ Modern view: low-energy effective theory
- 1950: $\theta - \tau$ puzzle
  - ➢ $\tau^+ \to \pi^+ + \pi^+ + \pi^-, \theta^+ \to \pi^+ + \pi^0$
  - ➢ τ^+ *and* θ^+ have identical physical properties except for parity
  - ➢ θ and τ are same particle, but with different decay mode

The Glashow-Weinberg-Salam electroweak theory History

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- 1956: Parity violation in weak interactions (Lee-Yang)
- 1957: Experimental verification of parity violation (Wu)
- 1957: Intermediate vector boson theory (IVB, Schwinger),
  - still non-renormalizable and violates unitarity: at √s > 500 GeV, $P(e^-e^+ \to W_L^+ W_L^-) > 1$
- Glashow model: SU(2) × U(1) gauge theory with four bosons $W^{\pm}, Z^0, \gamma$
  - cannot explain mass origin
- Higgs mechanism: Explains the origin of gauge boson masses
- Weinberg-Salam model: SU(2) × U(1) + Higgs
- 1971: EW renormalizability proved
- 1973: Discovery of weak neutral currents
- 1983: Discovery of $W^{\pm}, Z^0$ bosons of EW theory (CERN, SPS)
- 2012: Discovery of Higgs (CERN, LHC)

The Glashow-Weinberg-Salam electroweak theory Gauge field of SM

- Gauge group of SM: $SU_L(2) \times U_Y(1) \xrightarrow{\langle 0|\phi_0|0\rangle \neq 0} U_{em}(1)$
  - $SU_L(2)$ is the Weak Isospin group, and “L” indicates that this symmetry applies to the left-handed particle (fermion)
  - $U_Y(1)$ is the hypercharge group
  - $U_{em}(1)$ is the electromagnetic gauge group
- The representation of Higgs field in $SU_L(2) \times U_Y(1)$ is $\left(2, \frac{1}{2}\right)$
  - 2 means the Higgs is a doublet in $SU_L(2)$
  - $\frac{1}{2}$ means the Higgs has hypercharge $Y = \frac{1}{2}$

$\phi(x) \rightarrow e^{i\alpha^a(x)t^a} e^{i\beta(x)/2} \phi(x)$; $t^a = \tau^a/2$ is Pauli matrix divide by 2

$$D_\mu \phi = \left(\partial_\mu - igA_\mu^a t^a - i\frac{1}{2}g' B_\mu\right)$$

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

$$\langle 0|\phi_0|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}, \quad v = \sqrt{\frac{\mu^2}{\lambda}}$$

The Glashow-Weinberg-Salam electroweak theory Gauge field of SM

- Gauge boson mass

$$\mathcal{L}_{mass} = \frac{1}{2}(0, v)\left(gA_\mu^a\tau^a + \frac{1}{2}g'B_\mu\right)^2\begin{pmatrix}0\\ v\end{pmatrix}$$

$$gA_\mu^a\tau^a + \frac{1}{2}g'B_\mu = \frac{1}{2}\begin{pmatrix} gA_\mu^3 + g'B_\mu & g\left(A_\mu^1 - iA_\mu^2\right) \\ g\left(A_\mu^1 + iA_\mu^2\right) & -gA_\mu^3 + g'B_\mu \end{pmatrix}$$

$$= \frac{1}{2}\frac{v^2}{4}\left[g^2\left(A_\mu^1\right)^2 + g^2\left(A_\mu^2\right)^2 + \left(gA_\mu^3 - g'B_\mu\right)^2\right]$$

- There is mixing between A_μ^3 and B_μ

The Glashow-Weinberg-Salam electroweak theory Gauge field of SM

- Redefine gauge field A_μ^a and B_μ

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2);\ Z_\mu = \frac{gA_\mu^3 - g'B_\mu}{\sqrt{g^2+g'^2}};\ A_\mu = \frac{g'A_\mu^3 + gB_\mu}{\sqrt{g^2+g'^2}}$$

$$\theta_w = \tan^{-1}\left(\frac{g'}{g}\right) s_w = \sin\theta_w\,,\quad c_w = \cos\theta_w \sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}},\quad \cos\theta_w = \frac{g}{\sqrt{g^2+g'^2}}$$

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix}\begin{pmatrix} A^3 \\ B \end{pmatrix}$$

$$gA_\mu^a\tau^a + \frac{1}{2}g'B_\mu = \frac{1}{2}\begin{pmatrix} gA_\mu^3 + g'B_\mu & g(A_\mu^1 - iA_\mu^2) \\ g(A_\mu^1 + iA_\mu^2) & -gA_\mu^3 + g'B_\mu \end{pmatrix} = \frac{1}{2}g\begin{pmatrix} \frac{Z_\mu c_{2w} + A_\mu s_{2w}}{c_w} & \sqrt{2}W_\mu^+ \\ \sqrt{2}W_\mu^- & -Z_\mu c_{2w} - A_\mu s_{2w} \end{pmatrix}$$

- $\mathcal{L}_{mass} = \frac{g^2v^2}{4}W^{+\mu}W_\mu^- + \frac{g^2v^2}{8c_w^2}Z^\mu Z_\mu$
- A is the remining $U_{em}(1)$ gauge field, massless photon
- W boson: $\mathrm{m}_w = gv/2$
- Z boson: $\mathrm{m}_z = \mathrm{m}_w/c_w$

The Glashow-Weinberg-Salam electroweak theory Gauge field of SM

- Parameters in SM
  - The generator for any field under $SU_L(2) \times U_Y(1)$ is (t^a, Y)

$$D_\mu = \partial_\mu - igA_\mu^a T^a - ig' B_\mu Y$$

$$= \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{Z_\mu}{\sqrt{g^2+g'^2}}(g^2T^3 - g'^2 Y) - i\frac{gg'}{\sqrt{g^2+g'^2}}A_\mu(T^3+Y)$$

=Q

unbroken generator

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} = g\sin\theta_w\,; \quad \alpha = e^2/(4\pi) = \sqrt{2}G_F m_w^2 sw^2/\pi$$

- The parameters of the standard model are conventionally chosen as $G_F,\ m_w,\ m_z$

$$m_Z = 91.1876 \pm 0.0021\ \text{GeV}, \quad m_W = 80.379 \pm 0.012\ \text{GeV}, \quad G_F = (1.1663787 \pm 0.0000006) \times 10^{-5}\ \text{GeV}^{-2}$$

- θ_w depends on renormalization scheme

$$\sin^2\theta_w^{OS} \equiv 1 - \frac{m_W^2}{m_Z^2} = 0.223 \pm 0.001; \qquad \sin^2\theta_w^{\overline{MS}}(m_Z) = 0.231 \pm 0.0001$$

The Glashow-Weinberg-Salam electroweak theory Gauge field of SM

- Gauge boson self coupling
  - $SU_L(2)$ Gauge boson tensor: $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc} A^b_\mu A^c_\nu$
  - $U_Y(1)$ Gauge boson tensor: $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$
  - $U_{em}(1)$ Gauge boson tensor: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$\mathcal{L}_{kin} = -\frac{1}{4} F^{a\mu\nu} F^a_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}$$

$$= -\frac{1}{4}\left[2 \times \frac{1}{2}\left(F^1_{\mu\nu} - iF^2_{\mu\nu}\right)\left(F^{1\mu\nu} + iF^{2\mu\nu}\right) + F^{3\mu\nu} F^3_{\mu\nu}\right] - \frac{1}{4}\left(\partial^\mu B^\nu - \partial^\nu B^\mu\right)\left(\partial_\mu B_\nu - \partial_\nu B_\mu\right)$$

$$\frac{1}{\sqrt{2}}\left(F^1_{\mu\nu} - iF^2_{\mu\nu}\right) = \frac{1}{\sqrt{2}}\left[\partial_\mu\left(A^1_\nu - iA^2_\nu\right) - \partial_\nu\left(A^1_\mu - iA^2_\mu\right) - ig\left(A^3_\mu\left(A^1_\nu - iA^2_\nu\right) - A^3_\nu\left(A^1_\mu - iA^2_\mu\right)\right)\right] = D_\mu W^+_\nu - D_\nu W^+_\mu$$

$$\frac{1}{\sqrt{2}}\left(F^1_{\mu\nu} + iF^2_{\mu\nu}\right) = D_\mu W^-_\nu - D_\nu W^-_\mu$$

$$D_\mu = \partial_\mu - igA^3_\mu = \partial_\mu - ieA_\mu - ig\cos\theta_w Z_\mu$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}}\left(A^1_\mu \mp iA^2_\mu\right)$$

$$= -\frac{1}{2}\left(D_\mu W^+_\nu - D_\nu W^+_\mu\right)\left(D^\mu W^{-\nu} - D^\nu W^{-\mu}\right) - \frac{1}{4}\left(F^{3\mu\nu} F^3_{\mu\nu} + B^{\mu\nu} B_{\mu\nu}\right)$$

The Glashow-Weinberg-Salam electroweak theory Gauge field of SM

$$A_\mu^3 = \cos\theta_W Z_\mu + \sin\theta_W A_\mu ; \quad B_\mu = -\sin\theta_W Z_\mu + \cos\theta_W A_\mu$$

$$B^{\mu\nu}B_{\mu\nu} = \sin^2\theta_w Z_{\mu\nu}Z^{\mu\nu} - 2\sin\theta_w\cos\theta_w Z_{\mu\nu}F^{\mu\nu} + \cos^2\theta_w F_{\mu\nu}F^{\mu\nu}$$

$$F^{3,\mu\nu}F^3_{\mu\nu} = cos^2\theta_w Z^{\mu\nu}Z_{\mu\nu} + sin^2\theta_w F^{\mu\nu}F_{\mu\nu} + 2cos\theta_w sin\theta_w Z^{\mu\nu}F_{\mu\nu}$$
$$+(2g/i)(cos\theta_w Z^{\mu\nu} + sin\theta_w F^{\mu\nu})\left(W_\mu^- W_\nu^+ - W_\mu^+ W_\nu^-\right) - 2g^2\left(W_\mu^+ W^{-\mu} W_\nu^+ W^{-\nu} - W_\mu^+ W^{-\nu} W_\nu^+ W^{-\mu}\right)$$

$$= -1/4\, F^{\mu\nu}F_{\mu\nu} - 1/4\, Z^{\mu\nu}Z_{\mu\nu} - D_\mu W_\nu^+ D^\mu W^{-\nu} + D_\mu W_\nu^+ D^\nu W^{-\mu} + ieF^{\mu\nu}W_\mu^+ W_\nu^- + igcos\theta_w Z^{\mu\nu}W_\mu^+ W_\nu^-$$
$$- 1/2\, g^2\left(W^{+\mu}W_\mu^- W^{+\nu}W_\nu^- - W^{+\mu}W_\mu^+ W^{-\nu}W_\nu^-\right)$$

- 3 gauge boson coupling: $AW^+W^-; ZW^+W^-$
- 4 gauge boson coupling: $W^+W^-W^+W^-; ZZW^+W^-; AAW^+W^-; AZW^+W^-$
- No *AZ* coupling

The Glashow-Weinberg-Salam electroweak theory Higgs sector

- Higgs potential term

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}G^{+} \\ v + H + iG^{0} \end{pmatrix}$$

Unitary gauge

$$\phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H \end{pmatrix}; \qquad V(\phi) = \lambda v^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4$$

- Higgs mass: $\mathrm{m_H^2} = 2\lambda \mathrm{v}^2$
- Feynman rules
  1. HHH coupling: $-i\frac{3m_h^2}{v}$
  2. HHHH coupling: $-i\frac{3m_h^2}{v^2}$

The Glashow-Weinberg-Salam electroweak theory Higgs sector

- Kinetic term

$$D_\mu\phi^+ D^\mu\phi \Rightarrow \frac{1}{2}\partial^\mu H\partial_\mu H + \left(m_W^2 W^{+\mu}W_\mu^- + \frac{1}{2}m_Z^2 Z^\mu Z_\mu\right)\left(\frac{2H}{v} + \frac{H^2}{v^2}\right)$$

➢ Vertices Feynman rules

1. *WWH* coupling: $\frac{2im_W^2}{v}g_{\mu\nu} = igm_W g_{\mu\nu}$
2. *ZZH* coupling: $\frac{2im_Z^2}{v}g_{\mu\nu} = i\frac{g}{\cos\theta_W}m_Z g_{\mu\nu}$
3. *WWHH* coupling: $i\frac{g^2}{2}g_{\mu\nu}$
4. *ZZHH* coupling: $i\frac{g^2}{2\,cos^2\theta_W}g_{\mu\nu}$

- In R_ξ gauge: goldstones have interaction with *W,Z,A* and *H*

The Glashow-Weinberg-Salam electroweak theory Leptons

- Representation of leptons, electron as an example
  - Left handed electron and neutrino $l_L = \begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix}$ is the $SU_L(2)$ doublet and with $U_Y(1)$ hypercharge $Y = \frac{1}{2}$
  - Right handed electron e_R is the $SU_L(2)$ singlet and with hypercharge $U_Y(1)$ $Y = 1$
  - There is no right handed neutrino in SM

- Mass of electron
  - $m_e(\bar{e}_L e_R + \bar{e}_R e_L)$ violate gauge invariance
  - Introduce Yukawa coupling: $-y_e \bar{l}_L \phi e_R + h.c.$ is gauge invariant

spontaneous symmetry breaking

$$\frac{1}{\sqrt{2}} y_e v \bar{e}_L e_R + \frac{1}{\sqrt{2}} y_e h \bar{e}_L e_R + h.c.$$

  - $m_e = \frac{1}{\sqrt{2}} y_e v$
  - H-e-e vertex Feynman rule: $\frac{i}{v} m_e \sim m_e$
  - no right handed neutrino→ neutrino is massless

The Glashow-Weinberg-Salam electroweak theory Leptons

- Interaction between leptons and gauge bosons

$$D_\mu l_L = \partial_\mu l_L - igA_\mu^a T^a l_L - ig'\left(-\frac{1}{2}\right)B_\mu l_L$$

$$D_\mu e_R = \partial_\mu e_R - ig'(-1)B_\mu e_R$$

$$\mathcal{L}_{\text{lep}} = \overline{l_L}(i\gamma^\mu D_\mu)l_L + \overline{e_R}(i\gamma^\mu D_\mu)e_R - (y_e \overline{l_L}\phi e_R + \text{h.c.})$$

$$= \overline{l_L}(i\gamma^\mu\partial_\mu)l_L + \overline{e_R}(i\gamma^\mu\partial_\mu)e_R + \cdots + g(W_\mu^+ J_W^{\mu+} + W_\mu^- J_W^{\mu-} + Z_\mu J_Z^\mu) + eA_\mu J_{EM}^\mu$$

- $J_W^{\mu+} = \frac{1}{\sqrt{2}}\overline{\nu_{e,L}}\gamma^\mu e_L,\quad J_W^{\mu-} = \frac{1}{\sqrt{2}}\overline{e_L}\gamma^\mu \nu_{e,L}$
- $J_Z^\mu = \frac{1}{\cos\theta_W}\left[\overline{\nu_{e,L}}\gamma^\mu\left(\frac{1}{2}\right)\overline{\nu_{e,L}} + \overline{e_L}\gamma^\mu\left(-\frac{1}{2} + \sin^2\theta_W\right)e_L + \overline{e_R}\gamma^\mu \sin^2\theta_W\, e_R\right]$
- $J_{EM}^\mu = \bar{e}\gamma^\mu(-1)e = \overline{e_L}\gamma^\mu(-1)e_L + \overline{e_R}\gamma^\mu(-1)e_R$

The Glashow-Weinberg-Salam electroweak theory Leptons

- Vertices Feynman rules
  - e_L- e_L-Z and e_R- e_R-Z : $\frac{ie\gamma^\mu}{\sin\theta_w\cos\theta_w}\left(-\frac{1}{2}+\sin^2\theta_w\right)$ and $\frac{ie\gamma^\mu}{\cos\theta_w\sin\theta_w}\sin^2\theta_w$
  - $\nu_{e,L}$-$\nu_{e,L}$-Z : $\frac{ie\gamma^\mu}{\sin\theta_w\cos\theta_w}\cdot\frac{1}{2}$
  - $\nu_{e,L}$-e_L-W : $\frac{ie\gamma^\mu}{\sqrt{2}\sin\theta_w}$

    1. Electroweak interaction do not mix leptons of different generation
    2. Low energy limit $s \ll m_w$, W propagator ~ constant

$$\frac{e^2}{8\sin^2\theta_w M_W^2} \longrightarrow \frac{G_F}{\sqrt{2}}$$

The Glashow-Weinberg-Salam electroweak theory Quarks

- Representation of quarks under $SU_c(3) \times SU_L(2) \times U_Y(1)$
  - Left handed quarks $q_L = \begin{pmatrix} u_{i,L} \\ d_{i,L} \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}; \begin{pmatrix} c_L \\ s_L \end{pmatrix}; \begin{pmatrix} t_L \\ b_L \end{pmatrix}: \left(3, 2, \frac{1}{6}\right)$
    - has 3 colors; doublet of $SU_L(2)$; hypercharge $\frac{1}{6}$ is in order to satisfy the charge of quarks
  - Right handed quarks $q_R = u_{i,R};\ d_{i,R}$ $\quad u_{i,R} = u_R; c_R; t_R : \left(3, 1, \frac{2}{3}\right)$ $\quad d_{i,R} = d_R; s_R; b_R: \left(3, 1, -\frac{1}{3}\right)$
    - has 3 colors; singlet of $SU_L(2)$; hypercharge $\frac{2}{3}$ and $-\frac{1}{3}$ is in order to satisfy the charge of quarks
- Charge of quarks: $Q = T^3 + Y$

$$T^3 \begin{pmatrix} u_{i,L} \\ d_{i,L} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} u_{i,L} \\ d_{i,L} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} u_{i,L} \\ -\frac{1}{2} d_{i,L} \end{pmatrix}; \quad T^3 u_{i,R} = T^3 d_{i,R} = 0$$

$$Y u_{i,L} = Y d_{i,L} = \frac{1}{6}; \quad Y u_{i,R} = \frac{2}{3}; \quad Y d_{i,R} = -\frac{1}{3}$$

  - $Q u_{i,L} = Q u_{i,R} = \frac{2}{3}$
  - $Q d_{i,L} = Q d_{i,R} = -\frac{1}{3}$

The Glashow-Weinberg-Salam electroweak theory Quarks

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- Quark masses for first generation $q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}; u_R; d_R$

$$\mathcal{L} = \overline{q_L}(i\gamma^\mu D_\mu)q_L + \overline{u_R}(i\gamma^\mu D_\mu)u_R + \overline{d_R}(i\gamma^\mu D_\mu)d_R - (y_u\bar{q}_L\tilde{\phi}u_R + y_d\bar{q}_L\phi d_R + h.c.)$$

  - $\epsilon^{\alpha\beta}$: $SU_L(2)$ antisymmetric tensor
  - $\bar{q}_L\tilde{\phi} = \bar{q}_L i\sigma^2\phi^* = \epsilon^{\alpha\beta}\bar{q}_{L,\alpha}\phi^\dagger_\beta$ is $SU_L(2)$ invariant
  - $m_u = y_u\frac{v}{\sqrt{2}}; \quad m_d = y_d\frac{v}{\sqrt{2}}$

The Glashow-Weinberg-Salam electroweak theory Quarks

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- Quark masses for 3 generation $q_L = \begin{pmatrix} u_{i,L} \\ d_{i,L} \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}; \begin{pmatrix} c_L \\ s_L \end{pmatrix}; \begin{pmatrix} t_L \\ b_L \end{pmatrix}$ $u_{i,R} = u_R; c_R; t_R$ $d_{i,R} = d_R; s_R; b_R$

$$\begin{aligned}\mathcal{L} = &\overline{u_{iL}}(i\gamma^\mu\partial_\mu)u_{iL} + \overline{u_{iR}}(i\gamma^\mu\partial_\mu)u_{iR} + \overline{d_{iL}}(i\gamma^\mu\partial_\mu)d_{iL} + \overline{d_{iR}}(i\gamma^\mu\partial_\mu)d_{iR} \\ &+g\left(W_\mu^+ J_W^{\mu+} + W_\mu^- J_W^{\mu-} + Z_\mu J_Z^\mu\right) + eA_\mu J_{EM}^\mu \\ &-\left[(y_u)_{ij}\frac{v}{\sqrt{2}}\overline{u_{iL}}u_{jR} + (y_d)_{ij}\frac{v}{\sqrt{2}}\overline{d_{iL}}d_{jR} + h.c.\right]\end{aligned}$$

- $J_W^{\mu+} = \frac{1}{\sqrt{2}}\overline{u_{iL}}\gamma^\mu d_{iL}$
- $J_W^{\mu-} = \frac{1}{\sqrt{2}}\overline{d_{iL}}\gamma^\mu u_{iL}$
- $J_Z^\mu = \frac{1}{\cos\theta_W}\left[\overline{u_{iL}}\gamma^\mu\left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)u_{iL} + \overline{u_{iR}}\gamma^\mu\left(-\frac{2}{3}\sin^2\theta_W\right)u_{iR} + \overline{d_{iL}}\gamma^\mu\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)d_{iL} + \overline{d_{iR}}\gamma^\mu\left(\frac{1}{3}\sin^2\theta_W\right)d_{iR}\right]$
- $J_{EM}^\mu = \bar{u}_i\gamma^\mu\left(+\frac{2}{3}\right)u_i + \bar{d}_i\gamma^\mu\left(-\frac{1}{3}\right)d_i$

The Glashow-Weinberg-Salam electroweak theory Quarks

- Diagonalization of mass matrix

$$y_u = U_{u,L} y_u^{diag} U_{u,R}^\dagger, \quad y_u^{diag} = \frac{\sqrt{2}}{v}\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

$$y_d = U_{d,L} y_d^{diag} U_{d,R}^\dagger, \quad y_d^{diag} = \frac{\sqrt{2}}{v}\begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

$u_{iL} = (U_{u,L})_{ij} u'_{jL}, \quad d_{iL} = (U_{d,L})_{ij} d'_{jL}, \quad u_{iR} = (U_{u,R})_{ij} u'_{jR}, \quad d_{iR} = (U_{d,R})_{ij} d'_{jR}, \quad U$ is unitary matrix

- Lagrangian of mass term

$$\mathcal{L}_{mass} = \left[\frac{v}{\sqrt{2}} \overline{u'_{jL}} (U_u^\dagger U_u)_{jk} \left(y_u^{\text{diag}}\right)_{kk} (W_u^\dagger W_u)_{kl} u'_{lR} + \frac{v}{\sqrt{2}} \overline{d'_{jL}} (U_d^\dagger U_d)_{jk} \left(y_d^{\text{diag}}\right)_{kk} (W_d^\dagger W_d)_{kl} d'_{lR}\right]$$

$$= \left[\frac{v}{\sqrt{2}} \left(y_u^{\text{diag}}\right)_{ii} \overline{u'_{iL}} u'_{iR} + \frac{v}{\sqrt{2}} \left(y_d^{\text{diag}}\right)_{ii} \overline{d'_{iL}} d'_{iR}\right]$$

The Glashow-Weinberg-Salam electroweak theory Quarks

$$J_W^{\mu+} = \frac{1}{\sqrt{2}} \overline{u'_{iL}} \gamma^\mu V_{ij} d'_{jL}; \quad J_W^{\mu-} = \frac{1}{\sqrt{2}} \overline{d'_{iL}} \gamma^\mu V_{ij}^\dagger u'_{jL}$$

$$V_{ij} = \left(U_{u,L}^\dagger U_{d,L}\right)_{ij}$$

1. Right handed quark q_R do not participate in charged current weak interactions
2. neutral current remains the same

- Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- ➢ Unitary matrix with 9 degree of freedom
- ➢ Absorb unphysical degree of freedom into definition of quark field

$$u'_{iL} = e^{i\alpha_{iL}} u_{iL}, \quad d'_{iL} = e^{i\beta_{iL}} d_{iL}$$

- ➢ Degree of freedom becomes 9-6+1=4
- ➢ In lepton sector, $\begin{pmatrix} u_{i,L} \\ d_{i,L} \end{pmatrix} \rightarrow \begin{pmatrix} \nu_{e,L} \\ e_L \end{pmatrix}$, $U_{\nu,L}$ can be arbitrary due to massless $\nu_{e,L}$ ⇒ $V_{ij}^{lepton\ mixing} = 1$

The Glashow-Weinberg-Salam electroweak theory Quarks

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}$$

$$\begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -c_2 s_1 & c_1 c_2 c_3 + s_2 s_3 e^{i\delta} & c_1 c_2 s_3 - s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix}$$

1. $c_i = \cos\theta_i\,, \quad s_i = \sin\theta_i$
2. δ is the source of CP violation
3. θ_1 is the Cabibbo Angle

➢ Other format of parametrization

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Standard: $\theta_{13} \ll \theta_{23} \ll \theta_{12} \ll 1$ by experiment

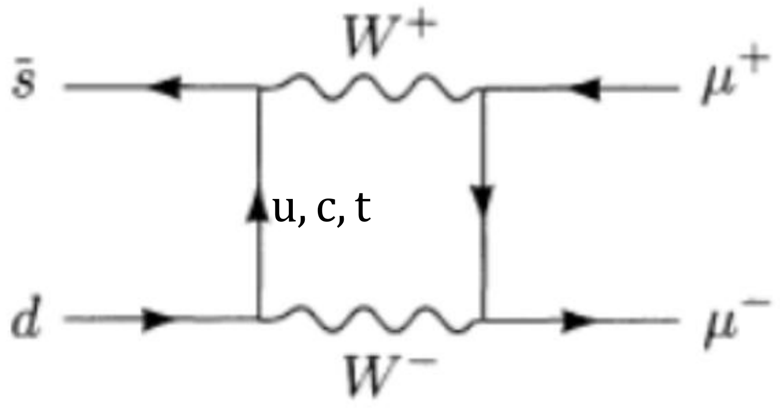
Wolfenstein parametrization

The Glashow-Weinberg-Salam electroweak theory Quarks

- Flavor-changing neutral current(FCNC)

$$J_Z^{\mu} = \overline{e_i}(\cdots)e_i + \overline{\nu_i}(\cdots)\nu_i + \overline{u_i}(\cdots)u_i + \overline{d_i}(\cdots)d_i$$

  - FCNC is forbidden at tree level
  - FCNC is suppressed by loop: $K^0(\bar{s}\,d) \rightarrow \mu^+\mu^-$

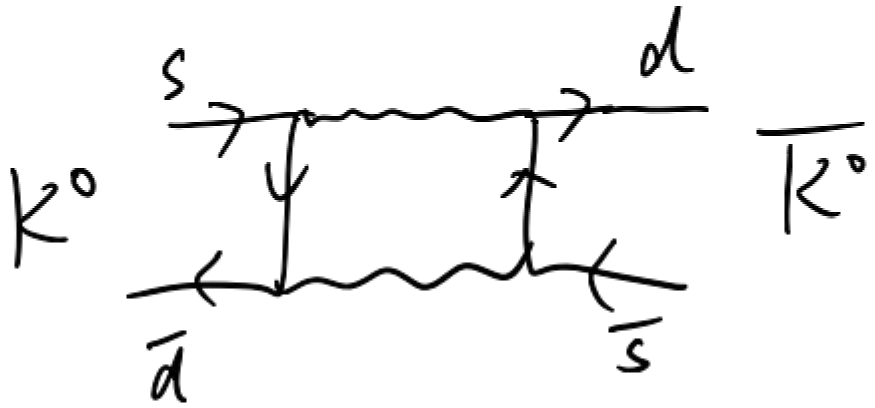


1. Glashow-Iliopoulos-Maiani suppressing: Decay width is small $V_{us}V_{ud}^* + V_{cs}V_{cd}^* = \lambda\left(1 - \frac{\lambda^2}{2}\right) - \lambda\left(1 - \frac{\lambda^2}{2}\right) = \mathcal{O}(\lambda^4) \approx 0$

must introduce c quark

The Glashow-Weinberg-Salam electroweak theory Summary

- The interaction of Z-bosons violate C and P but keeps CP
- The interaction of Z-bosons violate C, P and CP



$$P(\overline{K^0} \to K^0) \neq P(K^0 \to \overline{K^0})$$

- Yukawa coupling, f-f-Higgs: $-i\frac{1}{v}m_f$
- Flavor puzzle

$$m_e \ll m_\mu \ll m_\tau$$

$$m_u \ll m_c \ll m_t; \qquad m_d \ll m_s \ll m_b$$

$$\theta_{13} \ll \theta_{23} \ll \theta_{12} \ll 1$$

The Glashow-Weinberg-Salam electroweak theory Extension：Neutrino mass

- Solar Neutrino Problem
  - Detected Solar Neutrino is less than expected
  - Explanation： Neutrino Oscillation， $\nu_e \to \nu_\mu$ ..., require neutrino mass
  - Right handed neutrino $\nu_{i,R}$ with representation (1,1,0)
    - $\nu_{i,R}$ does not participate in the strong/weak/electromatic interaction

- Dirac Neutrino

$$\mathcal{L}_{\mathrm{Dirac}} = -\left[y_\nu \epsilon^{\alpha\beta} \overline{l_\alpha} \phi_\beta^\dagger \nu_R + \mathrm{h.c.}\right]$$

  - $m_D = y_\nu \frac{v}{\sqrt{2}}$
  - $y_\nu \sim 10^{-11}$: unnatural

- Majorana Neutrino

$$\mathcal{L}_{\mathrm{Majorana}} = -\frac{1}{2} M \overline{\nu_R^c} \nu_R + \mathrm{h.c.}$$

  - ν_R is in the representation (1,1,0) , M can be very big

The Glashow-Weinberg-Salam electroweak theory Extension: Neutrino mass

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- Seesaw mechanism: combine Dirac mass and Majorana mass

$$\mathcal{L}_{\text{mass}} = -\frac{1}{2}\begin{pmatrix}\overline{\nu_L} & \overline{\nu_R^c}\end{pmatrix}\begin{pmatrix}0 & m_D \\ m_D^T & M\end{pmatrix}\begin{pmatrix}\nu_L^c & \nu_R\end{pmatrix} + h.c.$$

  - diagonalize mass matrix: $m_D \ll M$

$$\begin{pmatrix}0 & m_D \\ m_D & M\end{pmatrix} \to \begin{pmatrix}m_{\nu,light} & 0 \\ 0 & m_{\nu,heavy}\end{pmatrix}$$

  - $m_{\nu,heavy} \approx M \approx \text{TeV}$; $\quad m_{\nu,light} \approx \frac{m_D^2}{M} < 1\text{e}V$;
  - Mass eigenstate: $\nu_{light} \approx \nu_L + \frac{m_D}{M}\nu_R^c$, $\quad \nu_{heavy} \approx \nu_R + \frac{m_D}{M}\nu_L^c$

- Generalization to 3 generation of Neutrino

$$\Psi = \begin{pmatrix}\nu_L \\ (\nu_R)^c\end{pmatrix} \qquad \nu_\text{L} = \begin{pmatrix}\nu_\text{eL} \\ \nu_{\mu\text{L}} \\ \nu_{\tau\text{L}}\end{pmatrix}, \quad (\nu_\text{R})^\text{c} = \begin{pmatrix}(\nu_\text{R1})^\text{c} \\ (\nu_\text{R2})^\text{c} \\ (\nu_\text{R3})^\text{c}\end{pmatrix} \qquad \mathcal{L}_{\text{mass}} = -\frac{1}{2}\overline{\Psi^T}\begin{pmatrix}0 & m_D \\ m_D^T & M_R\end{pmatrix}_{6\times 6}\Psi + \text{h.c.}$$

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- Optical theorem

$$\langle f|T^\dagger T|\mathrm{i}\rangle = \sum_x \int dPS_x \langle f|T^\dagger|\mathrm{x}\rangle\langle x|\mathrm{T}|\mathrm{i}\rangle = \sum_{\mathrm{x}} \int \mathrm{dPS_x}(2\pi)^4\delta^{(4)}(\mathrm{p_f}-\mathrm{p_x})(2\pi)^4\delta^{(4)}(\mathrm{p_i}-\mathrm{p_x})\mathcal{M}^*(\mathrm{f}\to\mathrm{x})\mathcal{M}(\mathrm{i}\to\mathrm{x})$$

$$1 = S^\dagger S = (1-iT^\dagger)(1+iT) \to i(T^\dagger - T) = T^\dagger T$$
$$\langle f|T|i\rangle = (2\pi)^4\delta^{(4)}(p_i-p_f)\mathcal{M}(i\to f)$$
$$\langle f|i(T^\dagger - T)|i\rangle = \mathrm{i}(2\pi)^4\delta^{(4)}(p_i-p_f)[\mathcal{M}^*(f\to i)-\mathcal{M}(i\to f)]$$

$$-i[\mathcal{M}(i\to f)-\mathcal{M}^*(f\to i)] = \sum_x \int dPS_x(2\pi)^4\delta^{(4)}(p_i-p_x)\mathcal{M}^*(f\to x)\mathcal{M}(i\to x)$$

$|\mathrm{i}\rangle = |\mathrm{f}\rangle$

$$2\mathrm{Im}\,\mathcal{M}(\mathrm{i}\to\mathrm{i}) = \sum_{\mathrm{x}} \int \mathrm{dPS_x}\,(2\pi)^4\delta^{(4)}(\mathrm{p_i}-\mathrm{p_x})|\mathcal{M}(\mathrm{i}\to\mathrm{x})|^2$$

The Glashow-Weinberg-Salam electroweak theory Unitary

- Restriction on cross section

$$\sigma(i \to x) = \frac{1}{2s}\int dPS_x |\mathcal{M}(i \to x)|^2 (2\pi)^4 \delta^{(4)}(p_i - p_x)$$

$$\mathrm{Im}\,\mathcal{M}(i \to i) = s\sum_x \sigma(i \to x)$$

- i are two massless particles

$$\mathrm{Im}\,\mathcal{M}(p_1p_2 \to p_1p_2) \geq s\sum_{k_1k_2} \sigma(p_1p_2 \to k_1k_2)$$

$$\begin{aligned}
&\sigma(p_1p_2 \to k_1k_2) \\
&= \frac{1}{2s}\int \frac{d^3\overrightarrow{k_1}}{(2\pi)^3 2E_1}\frac{d^3\overrightarrow{k_2}}{(2\pi)^3 2E_2}(2\pi)^4\delta^{(4)}(p_1+p_2-k_1-k_2)|\mathcal{M}(p_1p_2 \to k_1k_2)|^2 \\
&= \frac{1}{64\pi^2 s}\int d\cos\theta\, d\phi\, |\mathcal{M}(p_1p_2 \to k_1k_2)|^2 > \frac{C}{s}|\mathcal{M}(p_1p_2 \to p_1p_2)|^2
\end{aligned}$$

$$\mathrm{Im}\,\mathcal{M}(p_1p_2 \to p_1p_2) \geq C|\mathcal{M}(p_1p_2 \to p_1p_2)|^2$$

The Glashow-Weinberg-Salam electroweak theory Unitary

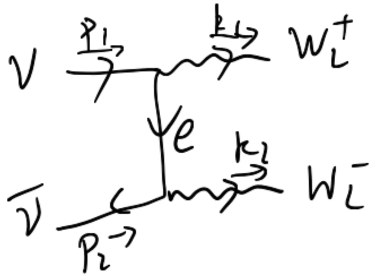
$$|\mathcal{M}| \geq \mathrm{Im}\mathcal{M}$$

$$|\mathcal{M}(p_1p_2 \to p_1p_2)| < \frac{1}{C}; \quad \sum_x \sigma(i \to x) < \frac{1}{C\,s}$$

- Unitary require: $\sigma(i \to f) < \frac{\text{constant}}{s}$

The Glashow-Weinberg-Salam electroweak theory Unitary

- Example 1: $\nu\bar{\nu} \rightarrow W_L^+ W_L^-$
  - Diagram 1



$$\mathcal{M}_t = \bar{v}(p_2)\frac{ig}{\sqrt{2}}\gamma^\nu \frac{1-\gamma^5}{2}\frac{i\gamma^\alpha(p_1-k_1)_\alpha + m_e}{(p_1-k_1)^2 - m_e^2}\frac{ig}{\sqrt{2}}\gamma^\mu\frac{1-\gamma^5}{2}u(p_1)\epsilon_\mu^*(k_1)\epsilon_\nu^*(k_2)$$

High energy limit: $\epsilon_3^\mu(k_1) \approx \frac{k_1^\mu}{m_W}, \quad \epsilon_3^\mu(k_2) \approx \frac{k_2^\mu}{m_W}$

$$\approx -\frac{ig^2}{8m_W^2}\frac{1}{(p_1-k_1)^2}\bar{v}(p_2)\gamma.k_2(1-\gamma^5)\big(\gamma\cdot(p_1-k_1)\big)\gamma.k_1(1-\gamma^5)u(p_1)$$

$$(\gamma^\alpha(p_1-k_1)_\alpha)(\gamma^\beta k_{1\beta})u(p_1)$$
$$= -(\gamma^\alpha(p_1-k_1)_\alpha)^2 u(p_1)$$
$$= -(p_1-k_1)^2 u(p_1)$$

$$= \frac{ig^2}{2m_W^2}\bar{v}(p_2)\gamma^\mu k_{2\mu}\frac{1-\gamma^5}{2}u(p_1)$$

The Glashow-Weinberg-Salam electroweak theory Unitary

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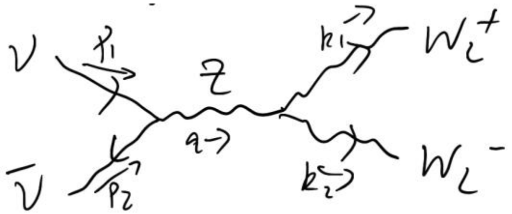
- Diagram 2



$$\mathcal{M}_s = \bar{v}(p_2)\frac{ig\gamma_\rho}{2\cos\theta_w}\frac{1-\gamma^5}{2}u(p_1)\frac{-i}{s-m_Z^2}\epsilon_\mu^*(k_1)\epsilon_\nu^*(k_2)\times ig\cos\theta_w\left[g^{\mu\nu}(k_2-k_1)^\rho+\cdots\right]$$

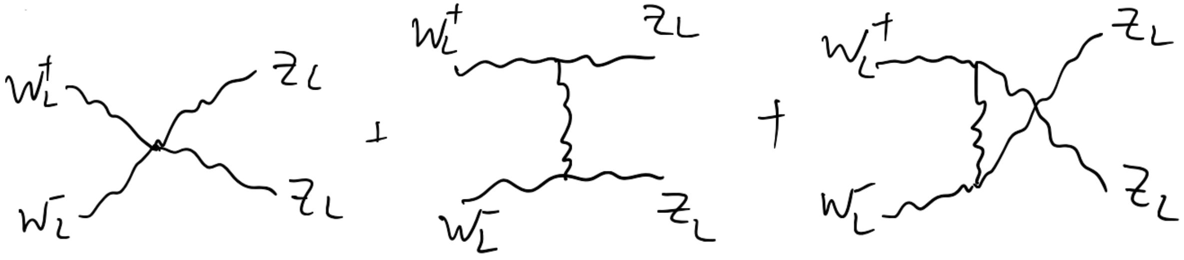
$$\xrightarrow{s\gg m}\frac{ig^2}{2m_W^2}\bar{v}(p_2)\gamma_\rho\frac{1-\gamma^5}{2}u(p_1)\frac{1}{s}\left[\frac{s}{2}(k_1-k_2)^\rho\right]$$

$$= -\frac{ig^2}{2m_W^2}\bar{v}(p_2)\gamma^\mu k_{2\mu}\frac{1-\gamma^5}{2}u(p_1)$$

- Diagram 1 + Diagram 2 $\xrightarrow{s\gg m} 0$

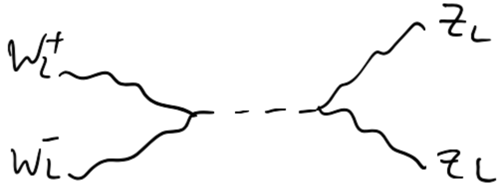
The Glashow-Weinberg-Salam electroweak theory Unitary

- Example 2: $W_L^+ W_L^- \to Z_L\ Z_L$



$$\xrightarrow{s \gg m} \frac{s}{v^2}$$

$$+$$



$$\xrightarrow{s \gg m} -\frac{s}{v^2} \cdot \frac{s}{s - m_H^2}$$

$$\approx$$

$$\frac{1}{sv^2}$$