Detection of Stochastic Gravitational Wave Background

郭怀珂

4.25, 2025

Basics about Gravitational Waves

- Ripples of spacetime
- Propagates at the speed of light
- Squeezes and stretches as it passes by
- Carries energy and momentum

$$ds^2 = -g_{\mu\nu} dx^\mu dx^\nu$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad |h_{\mu\nu}| \ll 1$$

$$\Box^2 h_{\mu\nu} = -16\pi G S_{\mu\nu}$$

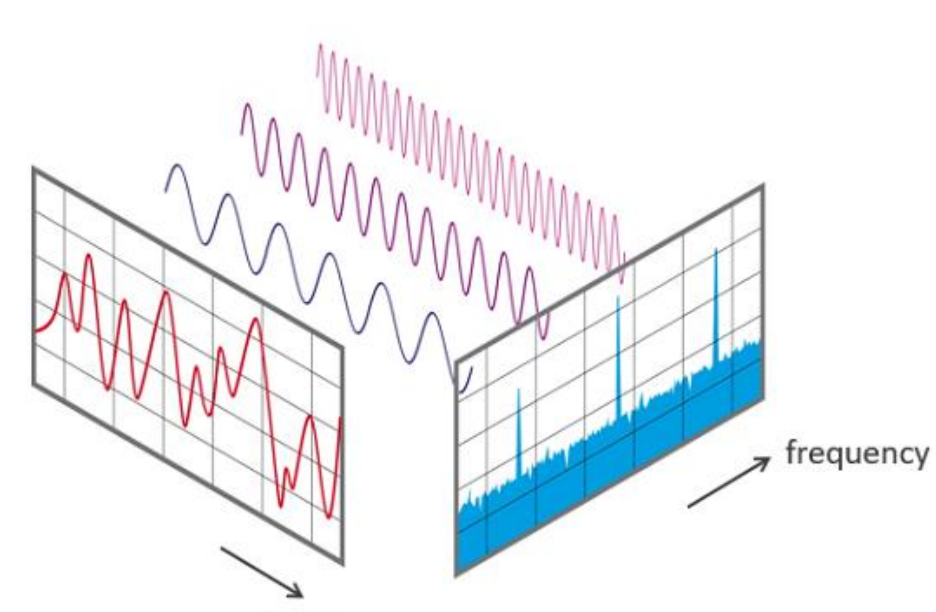
$S_{\mu\nu}$ → source

See, e.g., Weinberg, Gravitation and Cosmology (1972)
Maggiore, Gravitational Waves, Vol I and II

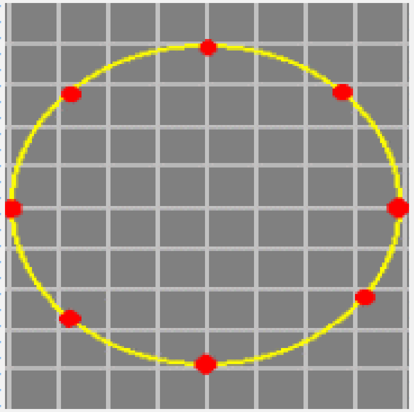
Plane Wave

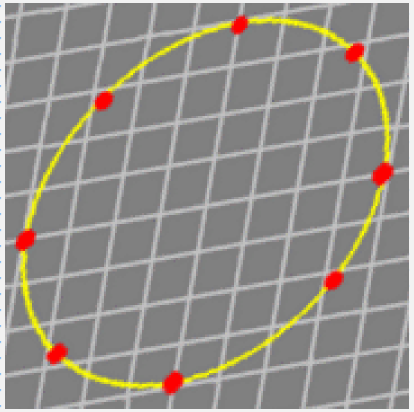
- A special gauge, the TT gauge, can be chosen in vacuum (caution: extended sources).
- Only 2 degrees of freedom (+, x).

$$h_{ij}^{\mathrm{TT}}(t,z) = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}_{ij} \cos[\omega(t - z/c)]$$



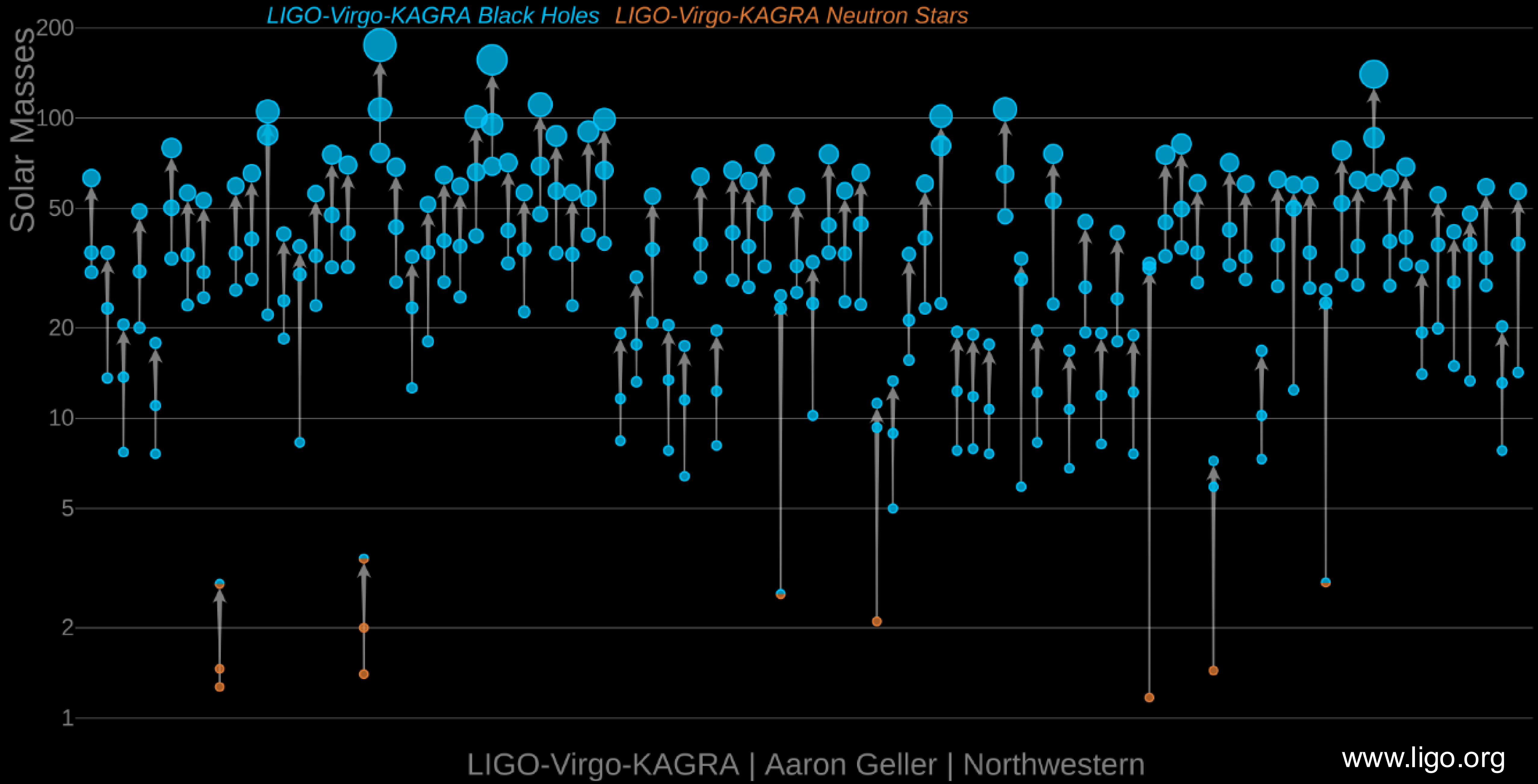
www.nti-audio.com





https://www.johnstonsarchive.net/relativity/pictures.html

Masses in the Stellar Graveyard



www.ligo.org

Searches at LIGO

CBC

short within LIGO band

BH-BH, BH-NS, NS-NS

Bursts

short, non-inspiral

Transient ↑

Persistent ↓

Continuous Waves

const frequency or very slowly varying, long lasting

Stochastic

random, from all directions, like a noise

Searches at LIGO

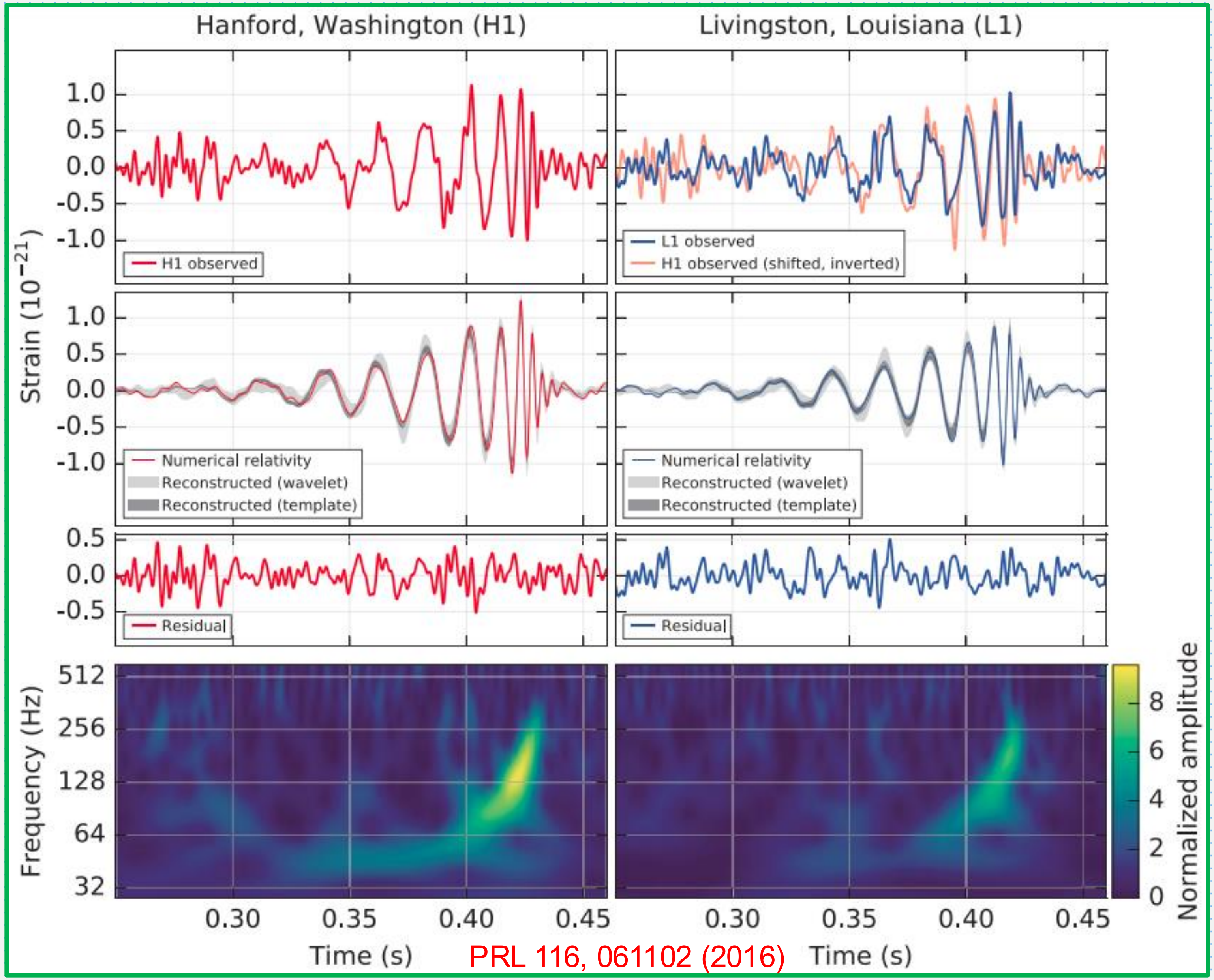
CBC

short within LIGO band

BH-BH, BH-NS, NS-NS

Transient

Persistent



PRL 116, 061102 (2016)

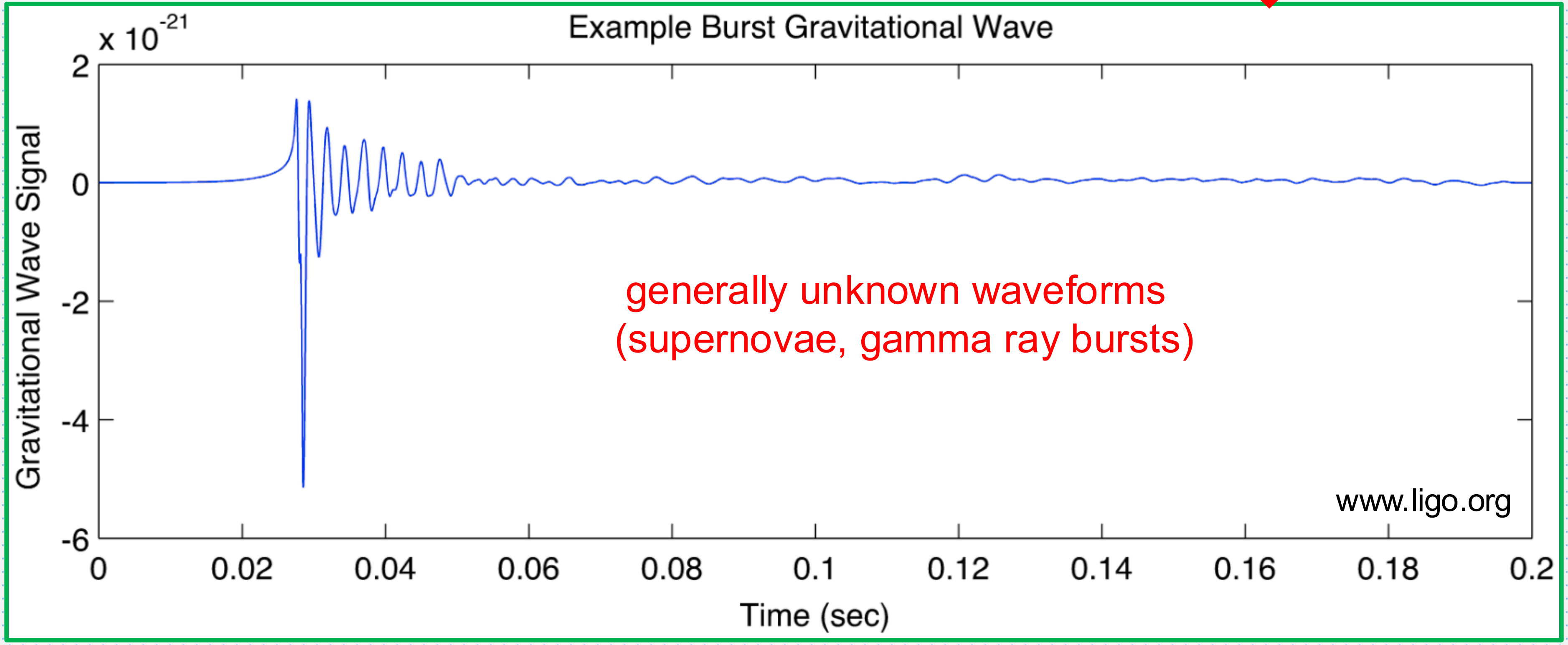
Searches at LIGO

Transient

Persistent

Bursts

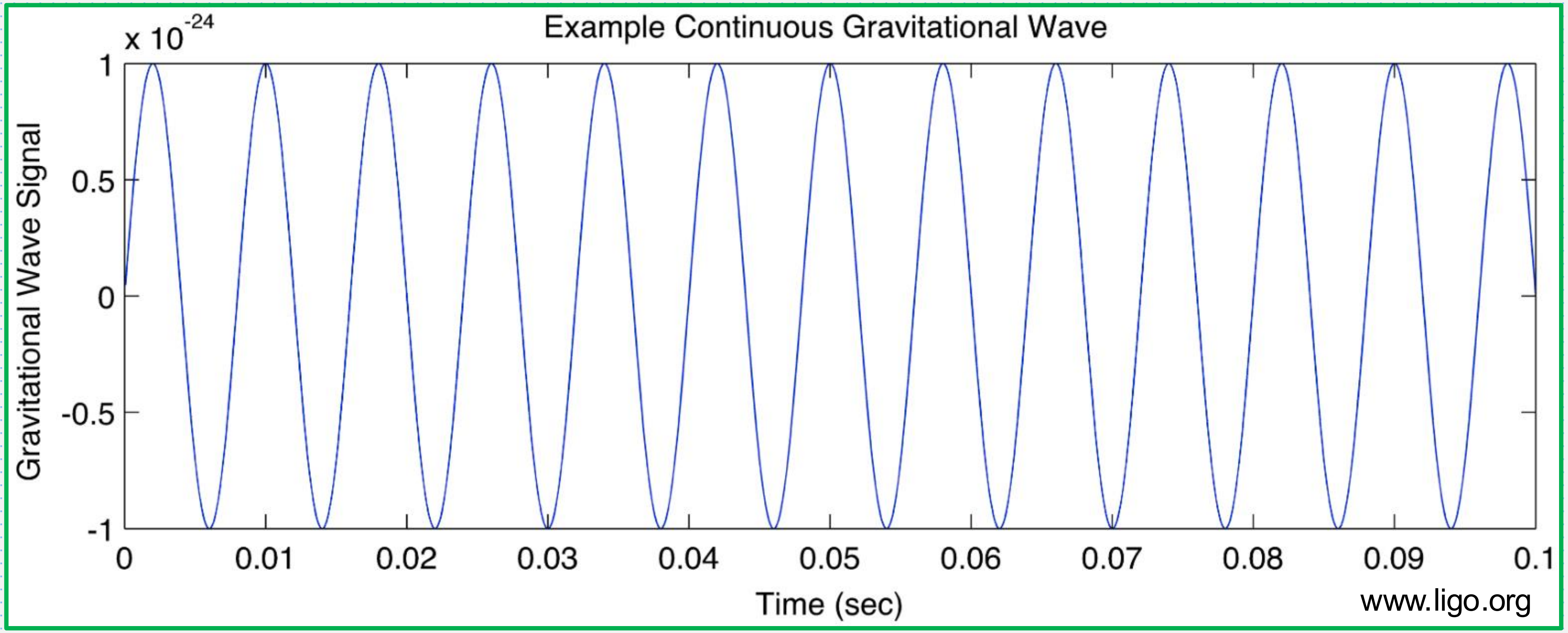
short, non-inspiral



Searches at LIGO

Transient

Persistent

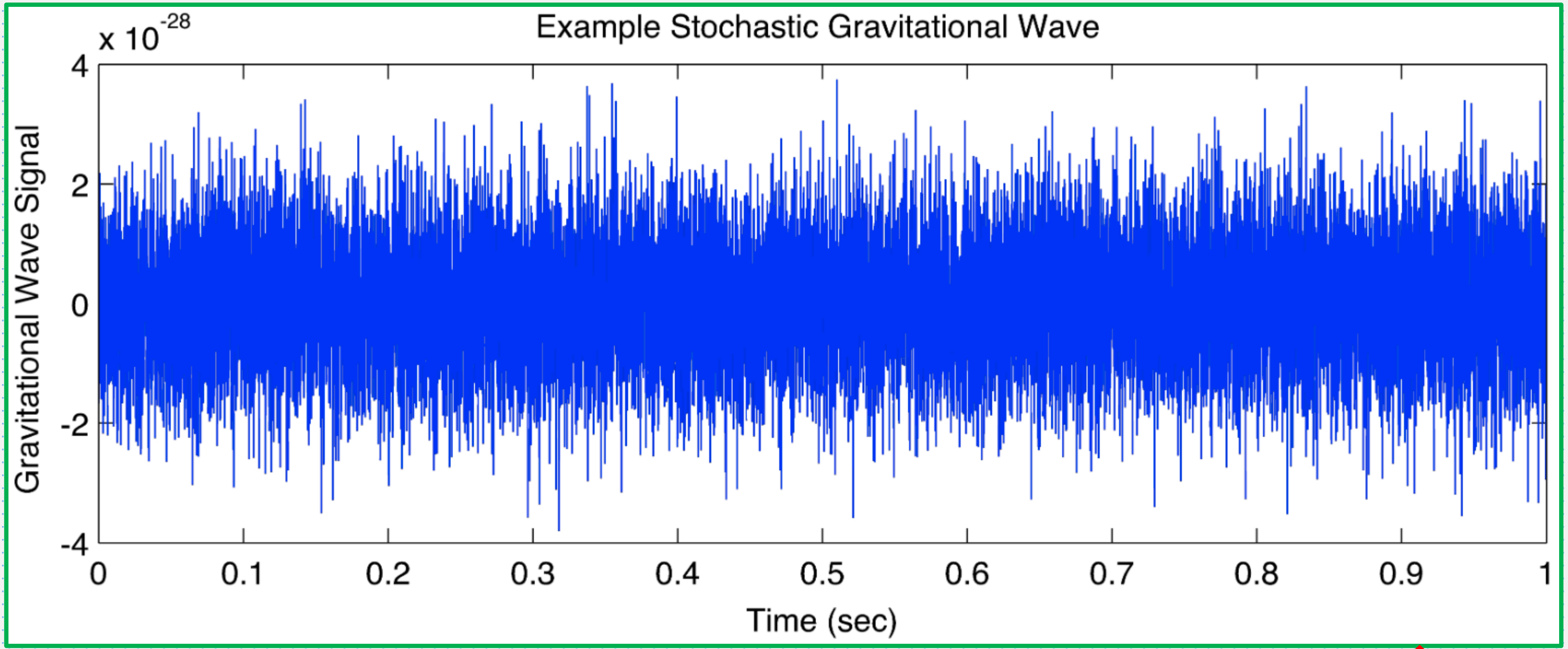


Continuous Waves

const frequency or very slowly varying, long lasting

main target: neutron stars (also dark photon etc)

Searches at LIGO



Transient

Persistent

astrophysical, or cosmological origins

Stochastic

random, from all directions, like a noise

Stochastic GWs

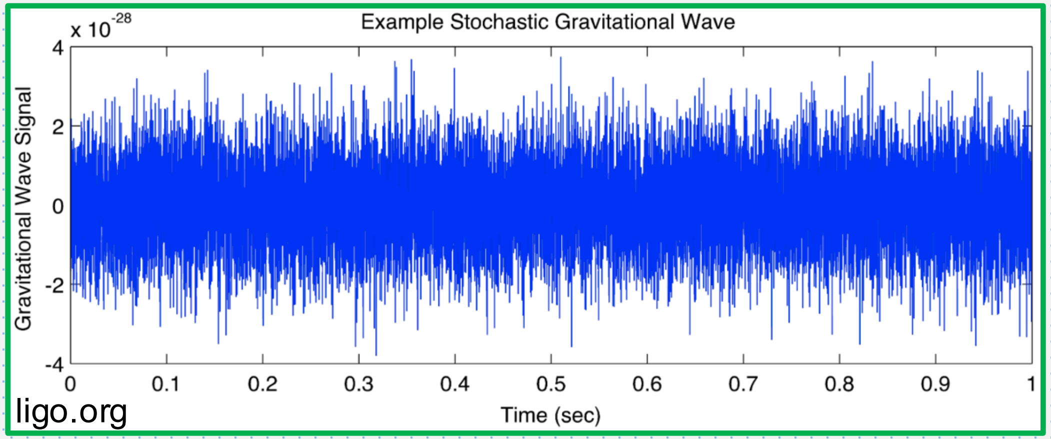
✓ Gaussian, Stationary, Isotropic, Unpolarized

$$\langle h^*_A(f,\Omega) h_{A'}(f',\Omega')\rangle = \frac{3H_0^2}{32\pi^3}\delta^2(\Omega,\Omega')\delta_{AA'}\delta(f-f')f^{-3}\Omega_{\rm GW}(f)$$

Energy density Spectrum

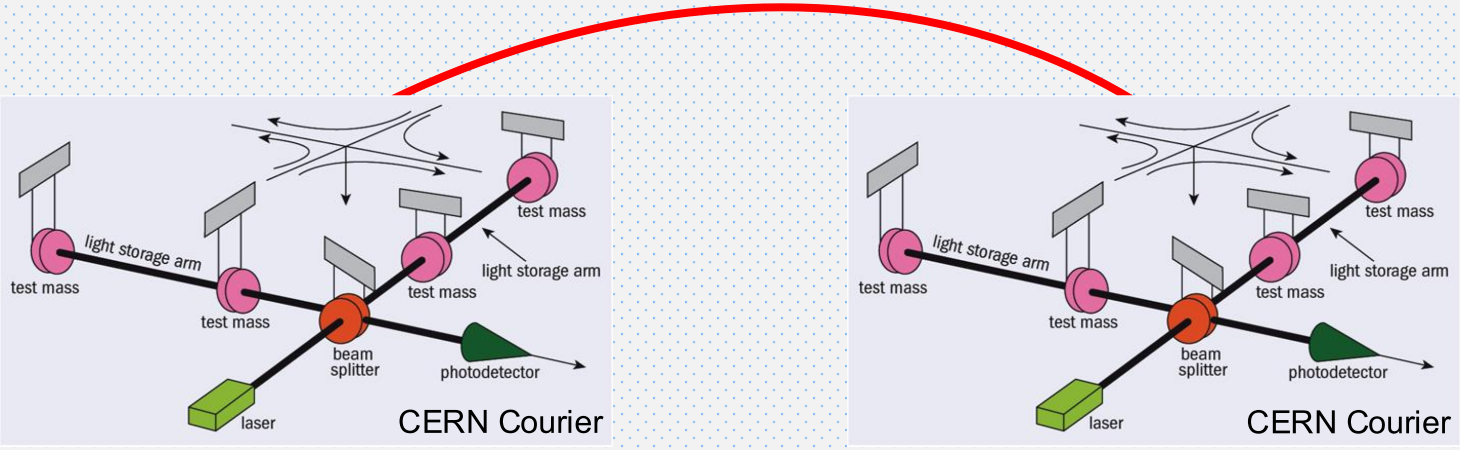
$$\Omega_{\rm GW}(f) = \frac{d\rho_{\rm GW}}{\rho_c d\log f}$$

stochastic GWs: noise-like



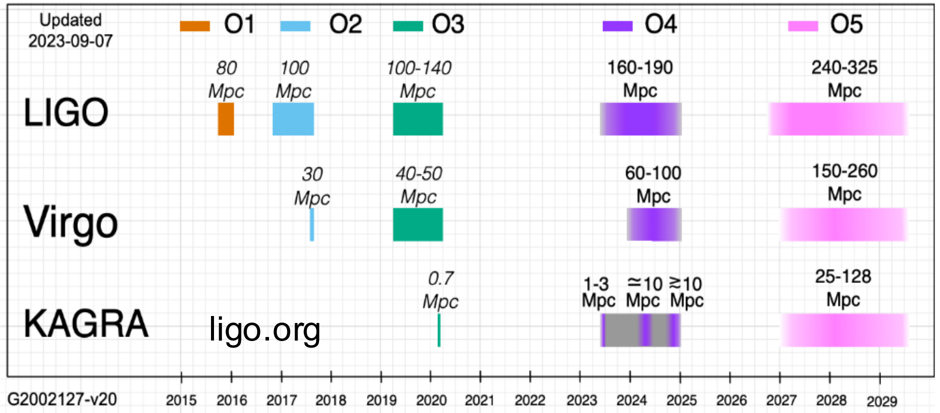
ligo.org

solution: cross-correlation



CERN Courier

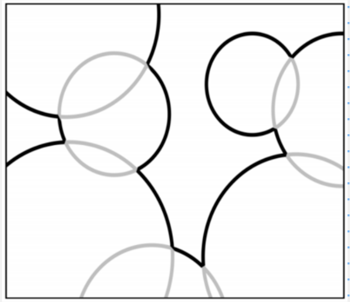
CERN Courier



ligo.org

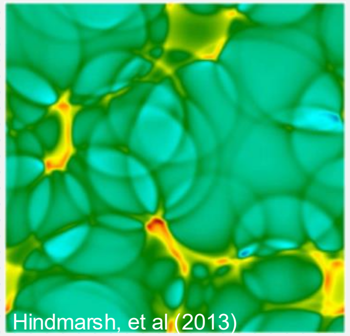
PT Spectra

bubble collision



$$\Omega_{\rm coll}(f)h^2 = 1.67\times10^{-5}\Delta\left(\frac{H_{\rm pt}}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2 \times\left(\frac{100}{g_*}\right)^{1/3} S_{\rm env}(f),$$

sound waves



$$\Omega_{\rm sw}(f)h^2 = 2.65\times10^{-6}\left(\frac{H_{\rm pt}}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3} \times v_w\left(\frac{f}{f_{\rm sw}}\right)^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}\Upsilon(\tau_{\rm sw}),$$

$$\Upsilon = 1-(1+2\tau_{\rm sw}H_{\rm pt})^{-1/2}$$ (RD)

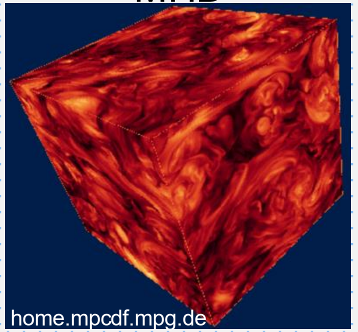
HG, Sinha, Vagie, White, JCAP [2007.08537]

$$\Upsilon = \frac{2[1-y^{3(w-1)/2}]}{3(1-w)}$$

HG, Yang Xiao, ... [2410.23666]

Reduces to Ellis, et al, JCAP [2003.07360]

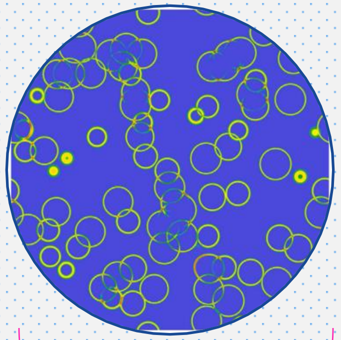
MHD



$$h^2\Omega_{\rm turb}(f) = 3.35\times10^{-4}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm turb}\,\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_*}\right)^{1/3} v_w\,S_{\rm turb}(f)$$

Chiara Caprini et al JCAP [1512.06239]

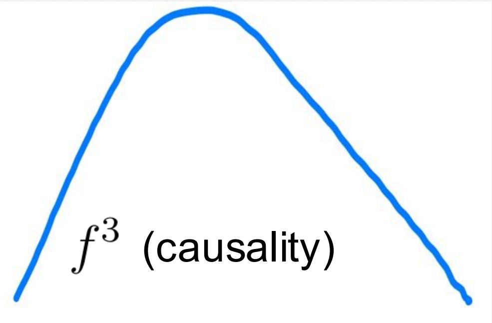
Basic Properties



Hubble size: 1/H*

$$f_{\rm now} = 1.65 \times 10^{-5} \left(\frac{f_{\rm PT}}{\beta}\right)\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6} {\rm Hz}$$

~100-1000



Cai, Pi, Sasak, PRD [1909.13728]

nHz (~100MeV) QCD scale

EPTA

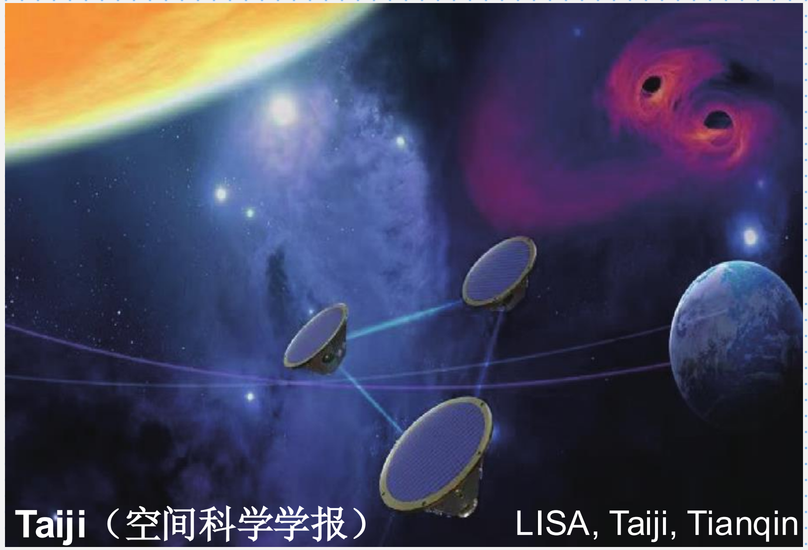
NANOGrav Physics Frontiers Center

PPTA

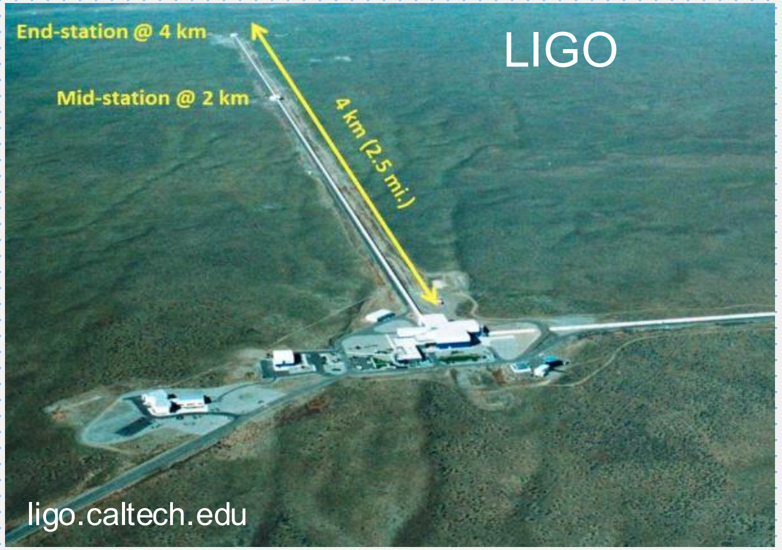
InPTA Indian Pulsar Timing Array

中国脉冲星测时阵列（CPTA）

~mHz : (~100GeV) weak scale



~100Hz (~PeV - EeV) high scale



Multiband Searches

NANOGrav, ApjL [2306.16219]

EPTA [2306.16227]

Xue,Bian,Shu,Yuan,Zhu, et al, PRL [2110.03096]

Bian et al [2307.02376]

Wu, Chen, Huang [2307.03141]

...

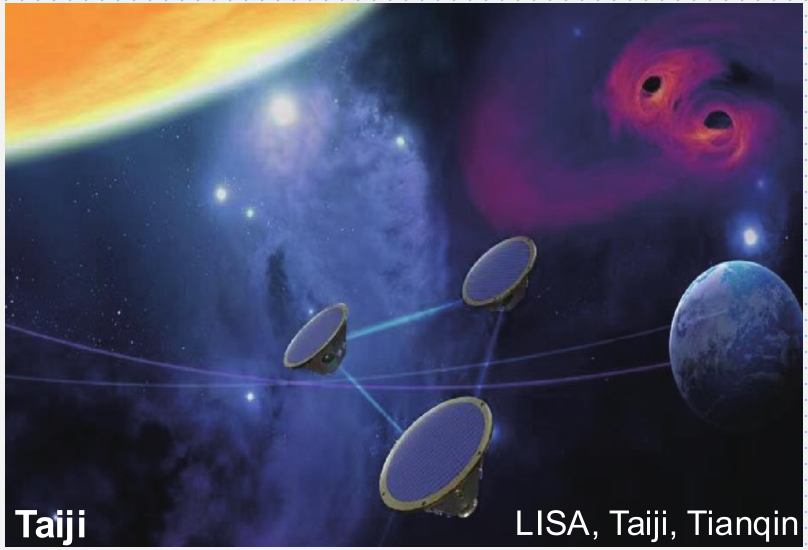
Boileau et al, MNRAS [2105.04283]
LISA: Caprini et al [2403.03723]
Network: Wang, Han, PRD [2108.11151] ...
TDI optimization: Wang, Li, Xu, Fan, PRD [2201.10902]
...

Romero,Martinovic,Callister,HG,Martínez,Sakellariadou, Yang,Zhao, PRL [2102.01714]
Badger, ..., HG, ..., PRD [2209.14707]
Jiang, Huang, JCAP [2203.11781]
Yu, Wang, PRD [2211.13111]

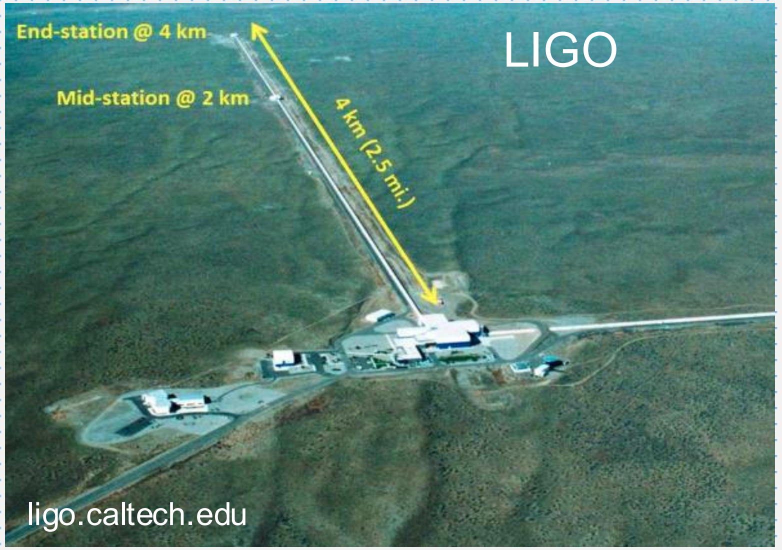
nHz (~100MeV) QCD scale

中国脉冲星测时阵列（CPTA）

~mHz : (~100GeV) weak scale



~100Hz (~PeV - EeV) high scale



Broken Power Law Model

We thus also consider a generic broken power law model.

$$\Omega_{\rm BPL}(f) = \Omega_* \left(\frac{f}{f_*}\right)^{n_1} \left[1 + \left(\frac{f}{f_*}\right)^{\Delta}\right]^{(n_2-n_1)/\Delta}$$

- n1: low f power,   fixed to be 3, (causality, Cai, Pi, Sasaki, PRD [1909.13728])
- n2: high f power, -4(SW), -1(BC), not entirely determined, will vary in the range (-8,0)
- Omega*, f*, reference amplitude and frequency.
- Δ=2 (SW), 4(BC), fixed to be 2 which gives a more conservative result

In all models (BPL, SW, BC), we also consider the non-negligible CBC contribution.

$$\Omega_{\rm CBC} = \Omega_{\rm ref}(f/f_{\rm ref})^{2/3}$$

$$f_{\rm ref} = 25 \text{ Hz}$$

The Bayesian Analysis Framework

calibration uncertainty

Likelihood

$$\log p(\hat{C}_{IJ}(f)|\boldsymbol{\theta}_{\rm gw},\lambda) \propto -\frac{1}{2}\sum_f \frac{\left[\hat{C}_{IJ}(f) - \lambda\Omega_{\rm gw}(f,\boldsymbol{\theta}_{\rm gw})\right]^2}{\sigma^2_{IJ}(f)}$$

Gaussian noise

$$\sigma^2_{IJ}(f) \approx \frac{1}{2T\Delta f}\frac{P_I(f)P_J(f)}{\gamma^2_{IJ}(f)S_0^2(f)}$$

Priors for two analysis strategies:

broken power law

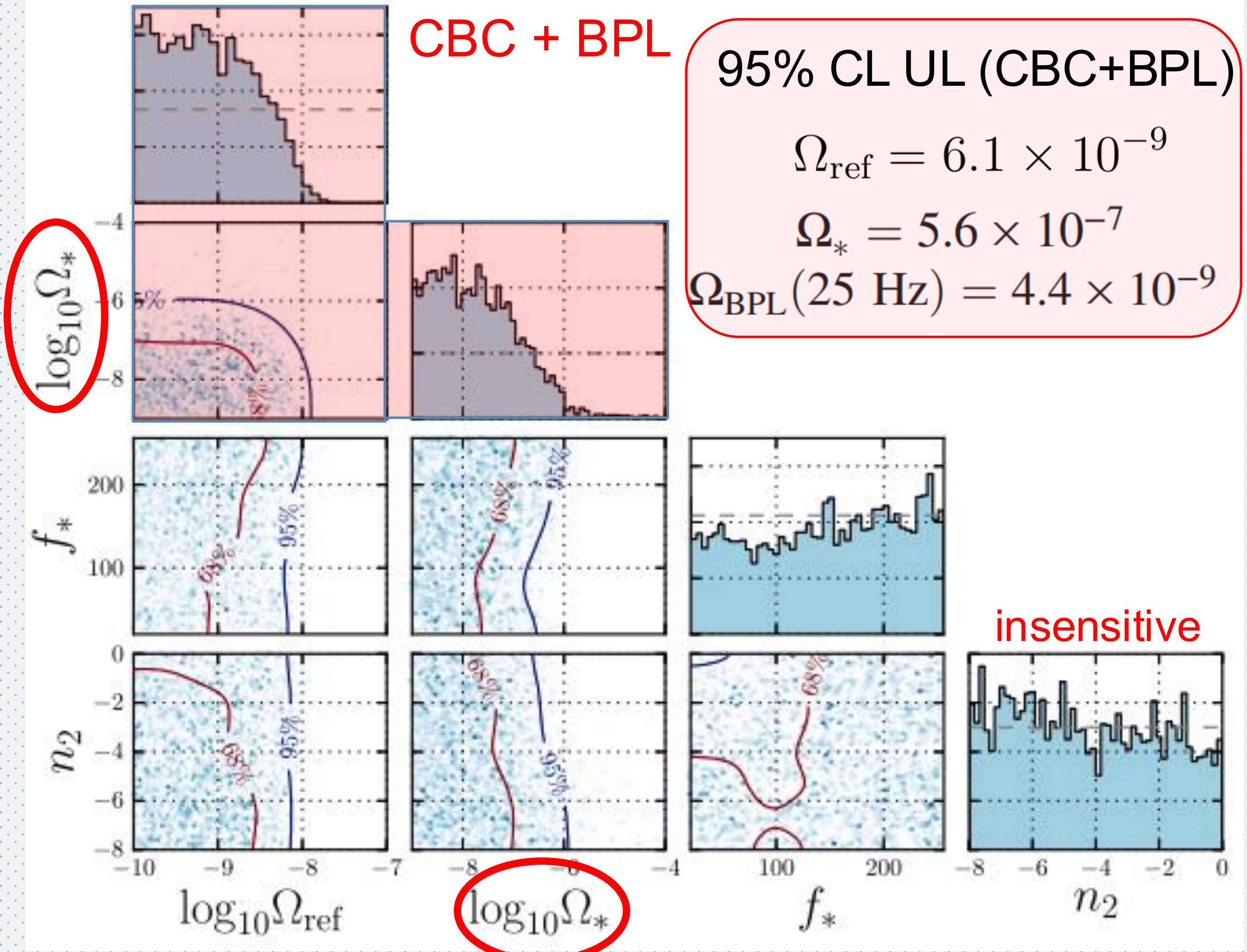
$$\Omega_{\rm bpl}(f) = \Omega_* \left(\frac{f}{f_*}\right)^{n_1}\left[1+\left(\frac{f}{f_*}\right)^{\Delta}\right]^{(n_2-n_1)/\Delta}$$

Broken power law model	
Parameter	Prior
$\Omega_{\rm ref}$	LogUniform(10^{-10}, 10^{-7})
Ω_*	LogUniform(10^{-9}, 10^{-4})
f_*	Uniform(20, 256 Hz)
n_1	3
n_2	Uniform(-8,0)
Δ	2

sound waves, or bubble collision

Phenomenological model	
Parameter	Prior
$\Omega_{\rm ref}$	LogUniform(10^{-10}, 10^{-7})
α	LogUniform (10^{-3}, 10)
$\beta/H_{\rm pt}$	LogUniform (10^{-1}, 10^{3})
$T_{\rm pt}$	LogUniform (10^{5}, 10^{9} GeV)
$v_{\rm w}$	1
κ_ϕ	1
$\kappa_{\rm sw}$	$f(\alpha, v_{\rm w}) \in [0.1-0.9]$

Broken Power Law Searches



95% CL UL (CBC+BPL)

$$\Omega_{ref} = 6.1 \times 10^{-9}$$
$$\Omega_* = 5.6 \times 10^{-7}$$
$$\Omega_{BPL}(25\ \text{Hz}) = 4.4 \times 10^{-9}$$

Posterior distributions for 2 variables (correlations)

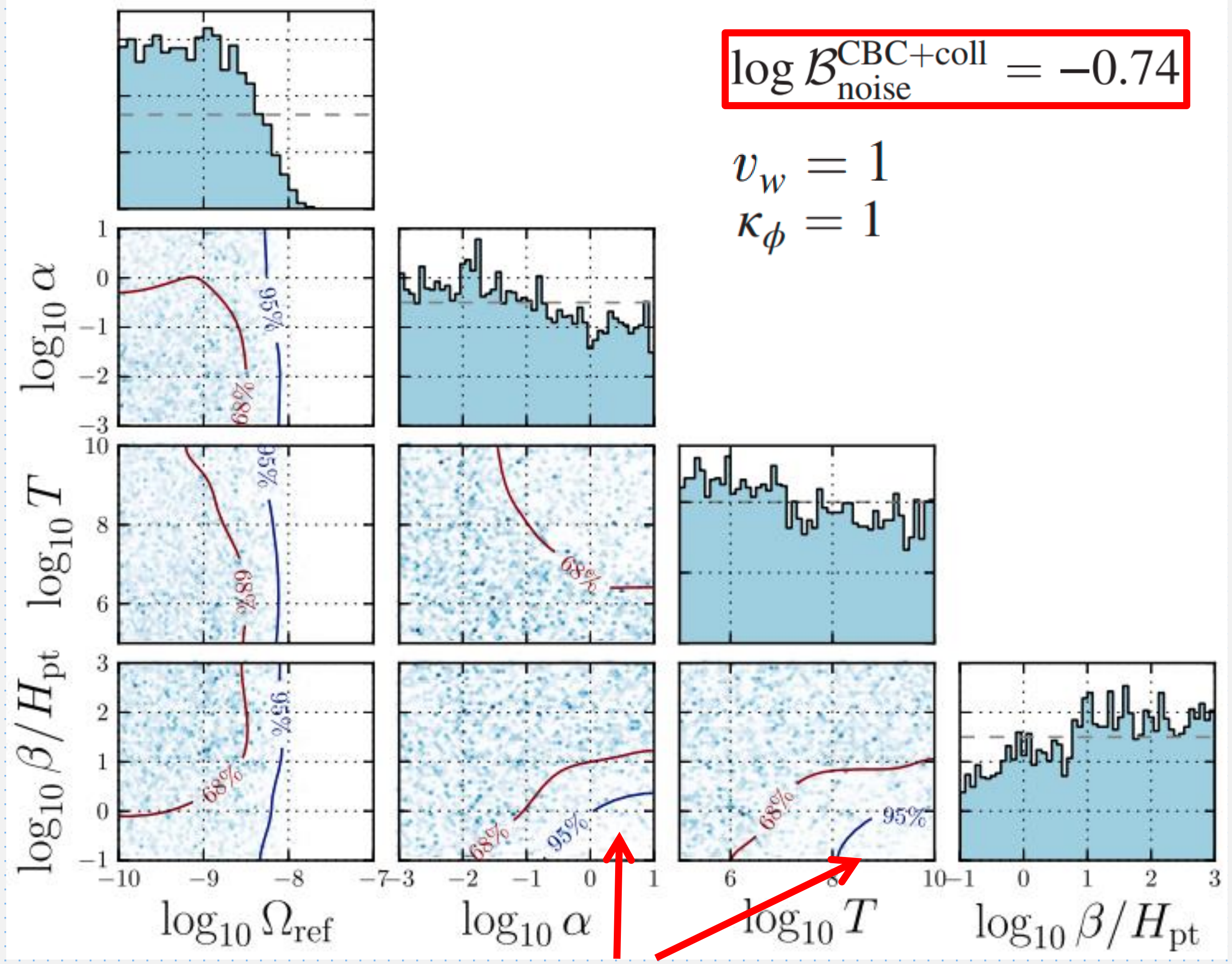
No Evidence for BPL Signal

$$\log \mathcal{B}^{CBC+BPL}_{noise} = -1.4$$
$$\log \mathcal{B}^{BPL}_{noise} = -0.78$$
$$\log \mathcal{B}^{CBC+BPL}_{CBC} = -0.81$$

95% CL UL with fixed n2

Broken power law model			
	$f_* = 1$ Hz	$f_* = 25$ Hz	$f_* = 200$ Hz
$n_2 = -1$ (BC)	3.3×10^{-7}	3.5×10^{-8}	2.8×10^{-7}
$n_2 = -2$ (BC)	8.2×10^{-6}	6.0×10^{-8}	3.7×10^{-7}
$n_2 = -4$ (SW)	5.2×10^{-5}	1.8×10^{-7}	3.7×10^{-7}

Bubble Collision + CBC



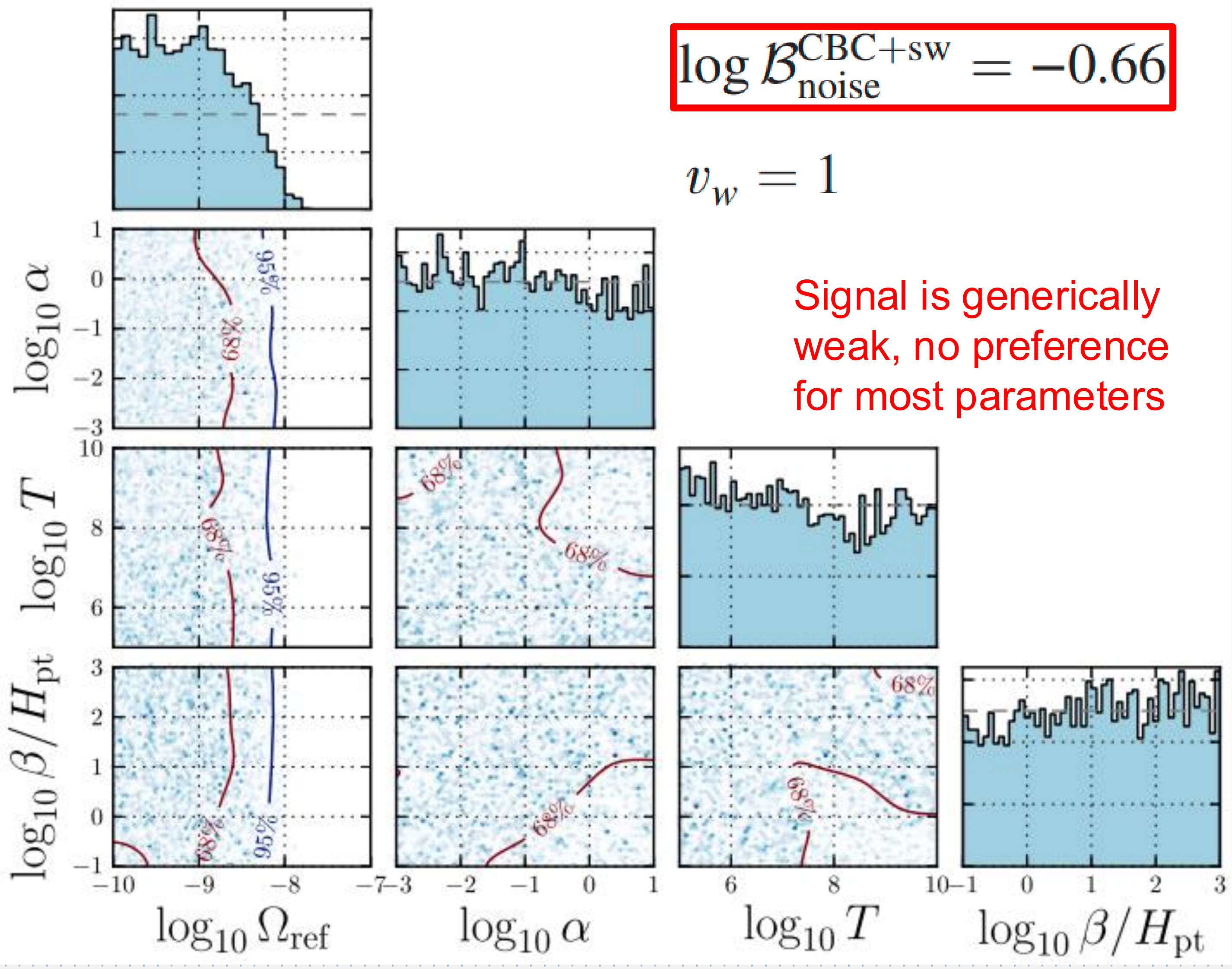
No Evidence for Bubble Collision Signal

95% CL UL with fixed Tpt and beta/Hpt

Phenomenological model (bubble collisions)				
$\Omega^{95\%}_{\mathrm{coll}}$ (25 Hz)				
$\beta / H_{\mathrm{pt}} \backslash T_{\mathrm{pt}}$	10^7 GeV	10^8 GeV	10^9 GeV	10^{10} GeV
0.1	9.2×10^{-9}	8.8×10^{-9}	1.0×10^{-8}	7.2×10^{-9}
1	1.0×10^{-8}	8.4×10^{-9}	5.0×10^{-9}	…
10	4.0×10^{-9}	6.3×10^{-9}	…	…

no sensitivity

Sound Waves + CBC



No Evidence for Sound Waves Signal

95% CL UL with fixed Tpt and beta/Hpt

$\Omega_{\mathrm{sw}}(25\ \mathrm{Hz}) \quad 5.9 \times 10^{-9}$

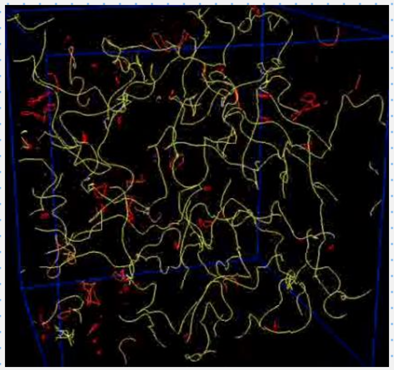
$\beta/H_{\mathrm{pt}} < 1$ and $T_{\mathrm{pt}} > 10^8$ GeV

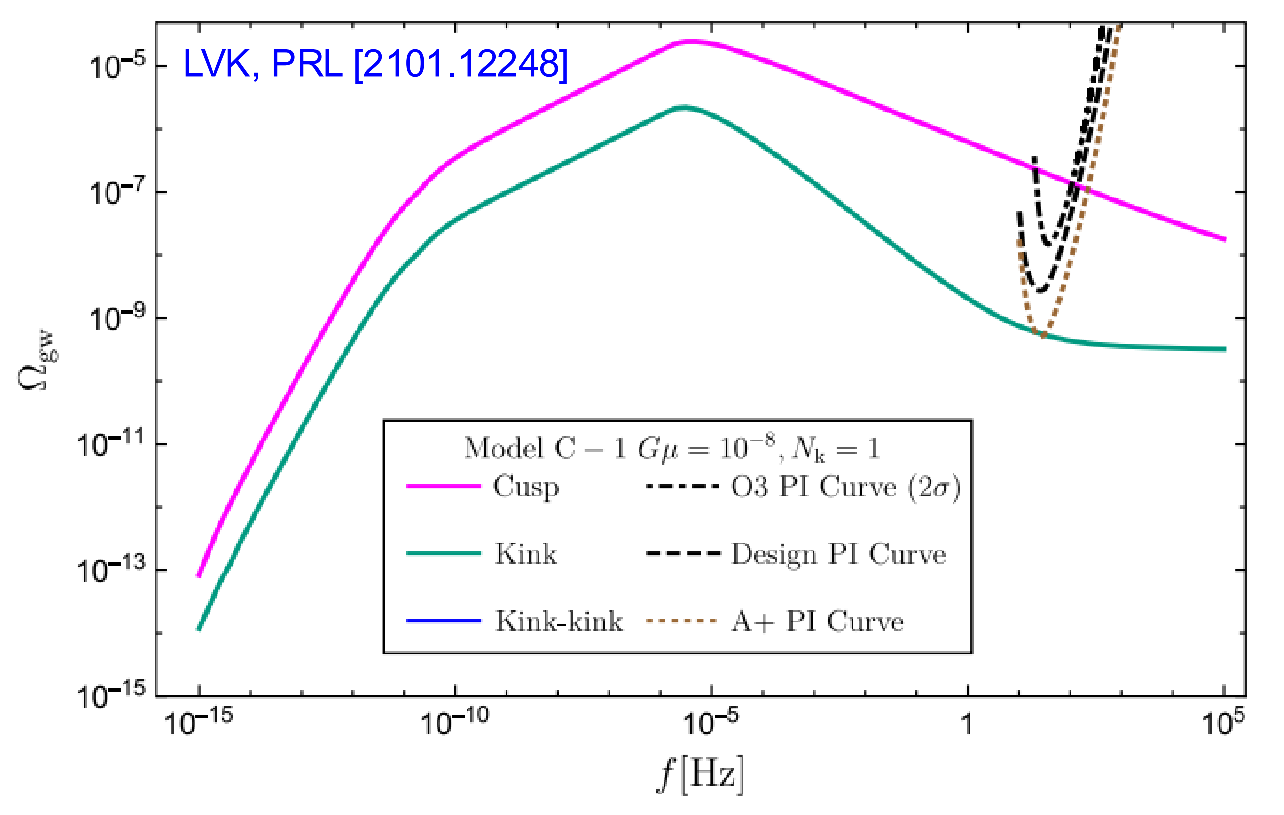
See also:

Jiang, Huang, JCAP [2203.11781]

Yu, Wang, PRD [2211.13111]

Cosmic Strings: Multiband Probes





LIGO

O1: LIGO-Virgo, PRD [1712.01168]
O2: LIGO-Virgo, PRD [1903.02886]
O3: LVK, PRL [2101.12248]

LISA/Taiji/Tianqin

Auclair et al, JCAP [1909.00819]
Chen,Huang,Liu, et al JCAP [2310.00411]
Wang,Li, PRD [2311.07116]
...

PTA

Zhu, et al (PPTA) MNRAS [2011.13490]
Blasi, Brdar, Schmitz, PRL [2009.06607]
Bian,Shu,Wang,Yuan,Zong (PPTA) PRD [2205.07293]
Chen,Huang (PPTA) ApJ [2205.07194]
NANOGrav, ApjL [2306.16219]
EPTA [2306.16227]
...

The Bayesian Analysis Framework

$$\ln \mathcal{L}(\hat{C}_a^{IJ}|G\mu, N_k) = -\frac{1}{2}\sum_{IJ,a}\frac{(\hat{C}_a^{IJ} - \Omega_{\mathrm{GW}}^{(M)}(f_a; G\mu, N_k))^2}{\sigma_{IJ}^2(f_a)}$$

model parameters(Nc=1)

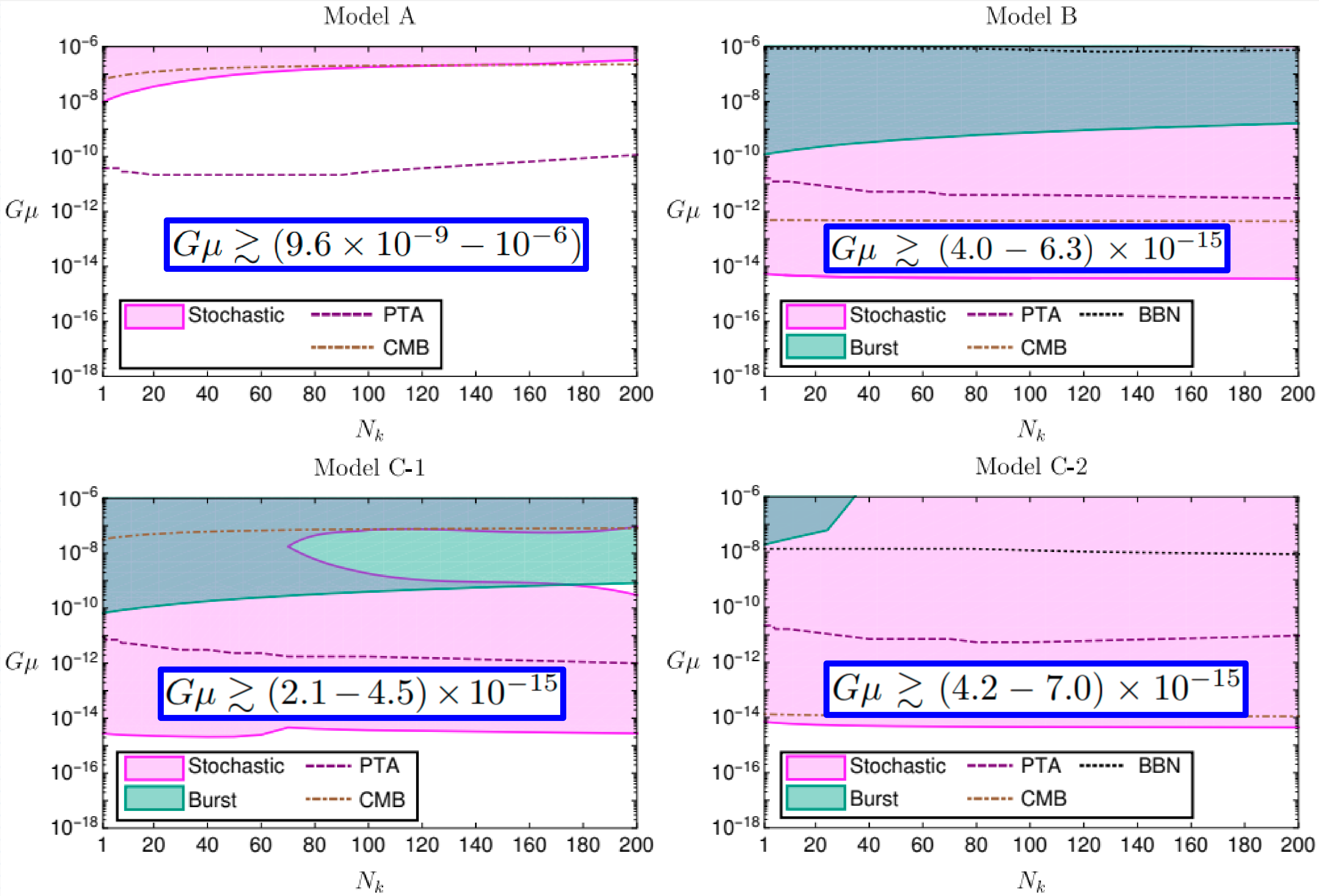
- Data sets: O1, O2, O3
- $G\mu$: $(10^{-18}, 10^{-6})$ uniform prior on the logarithmic scale
- Nk from 1 to 200, with each a separate model

Posterior: $p(G\mu|N_k) = \mathcal{L}(G\mu, N_k)p(G\mu|I_{G\mu})p(N_k|I_{N_k})$

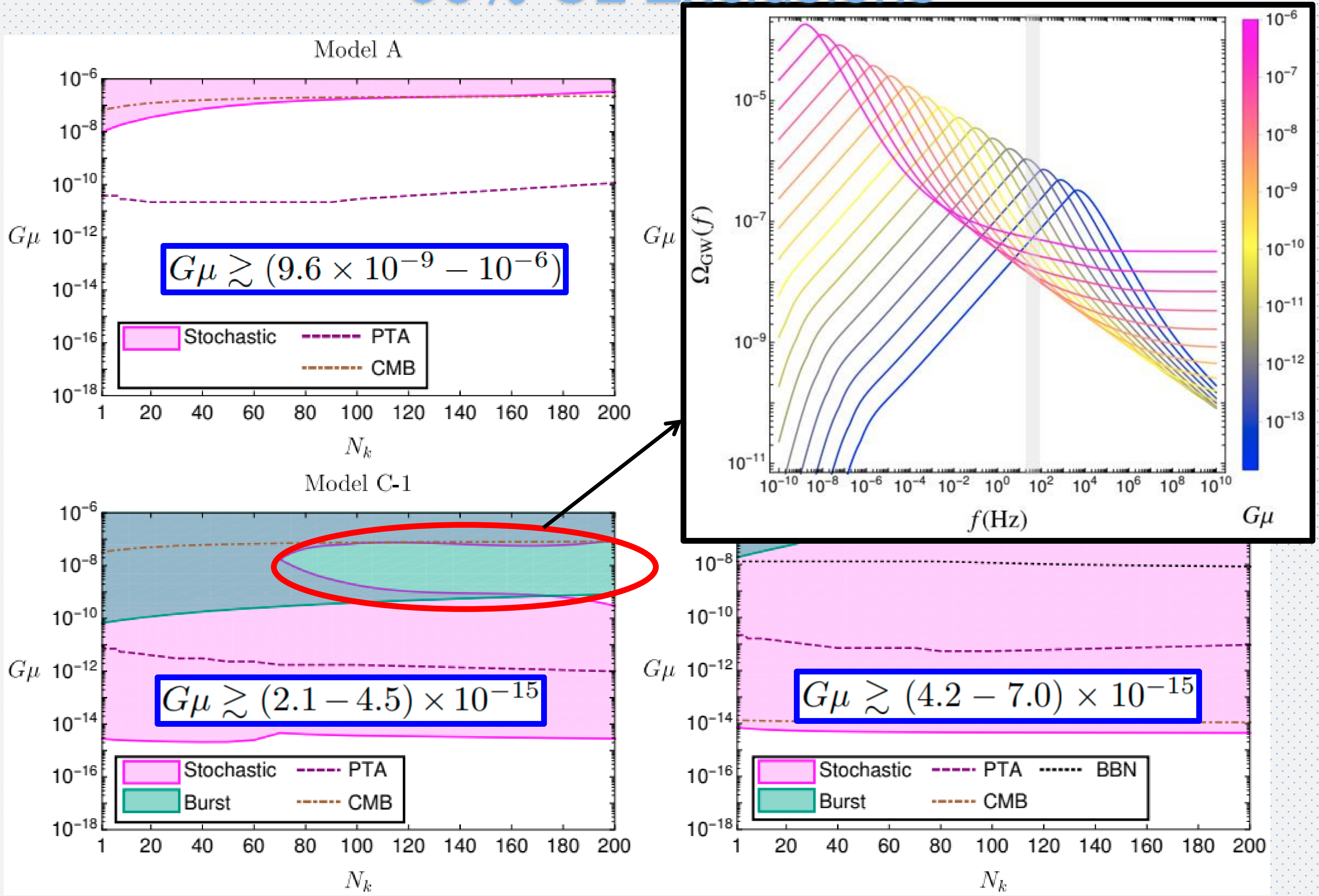
- 95% credible region boundaries determined for each Nk by:

$$\frac{1}{N}\int_{p\geqslant p_0} p(G\mu|N_k)d\ln G\mu = 0.95$$

95% CL Exclusions



95% CL Exclusions



Outline

- Why do we want to study solitons？
- Magnetic Monopoles
- Gravitational waves from cosmic phase transitions and Cosmic strings
- How to detect their gravitational waves?

Motivated by the desire to explore the new tool: gravitational waves