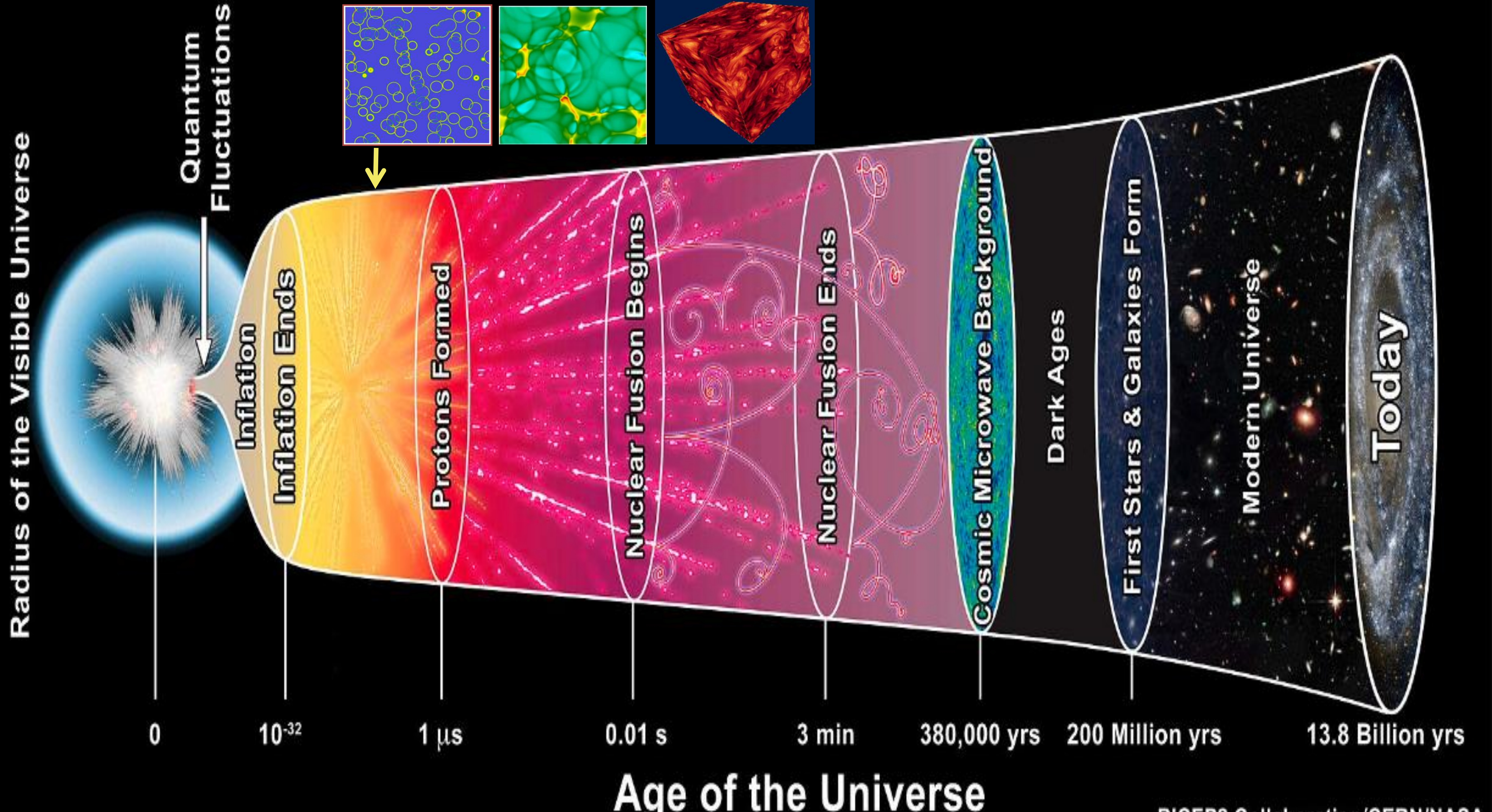
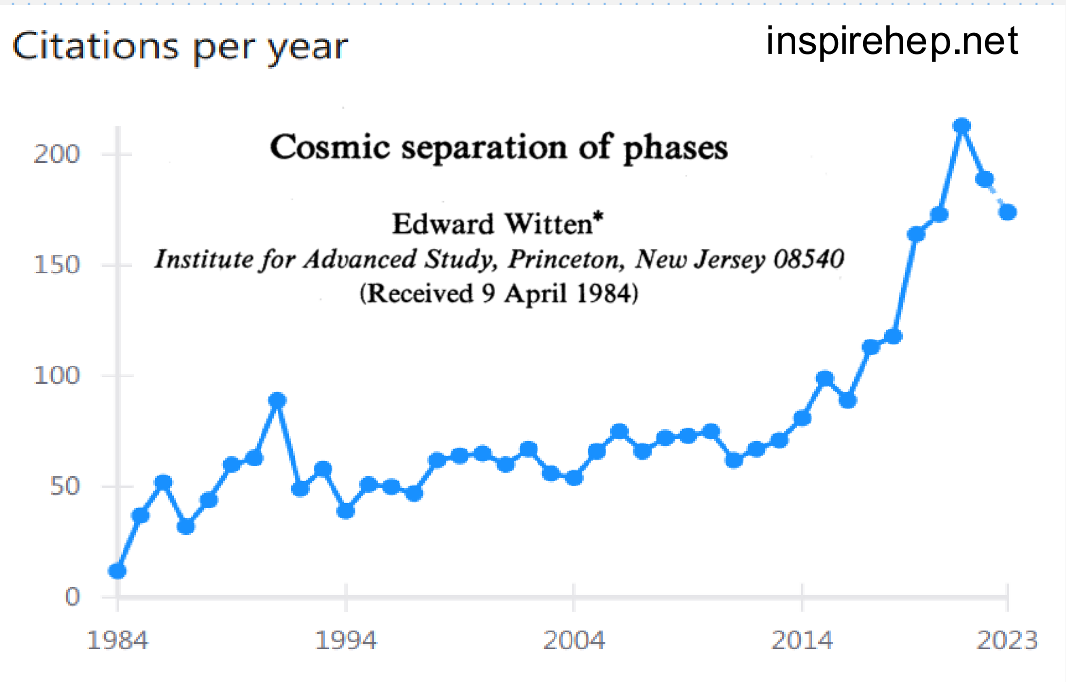
Gravitational Waves from Cosmic Phase Transitions and Cosmic Strings

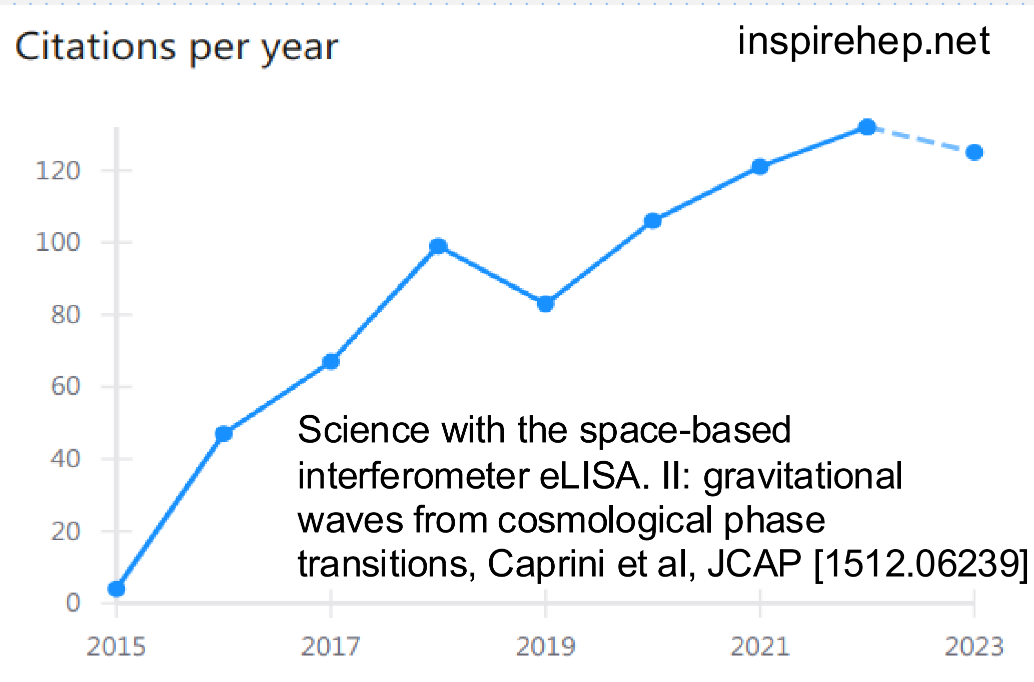
郭怀珂

4.25, 2025



BICEP2 Collaboration/CERN/NASA





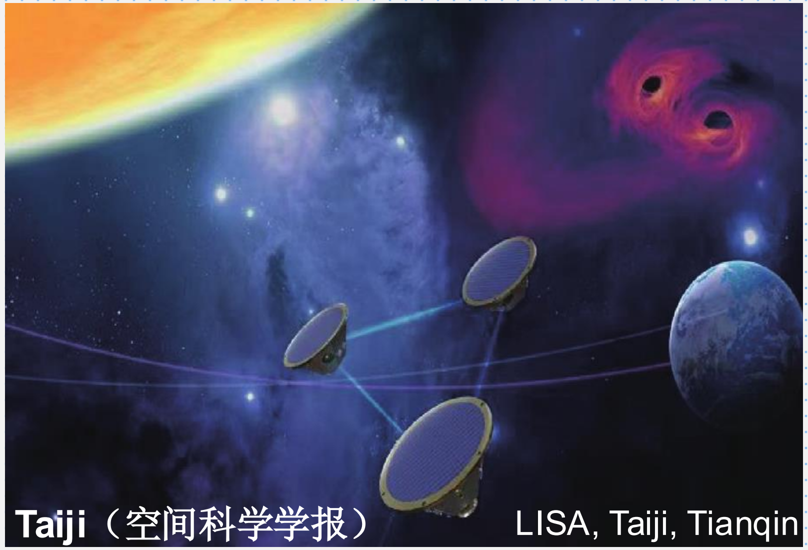
EPTA

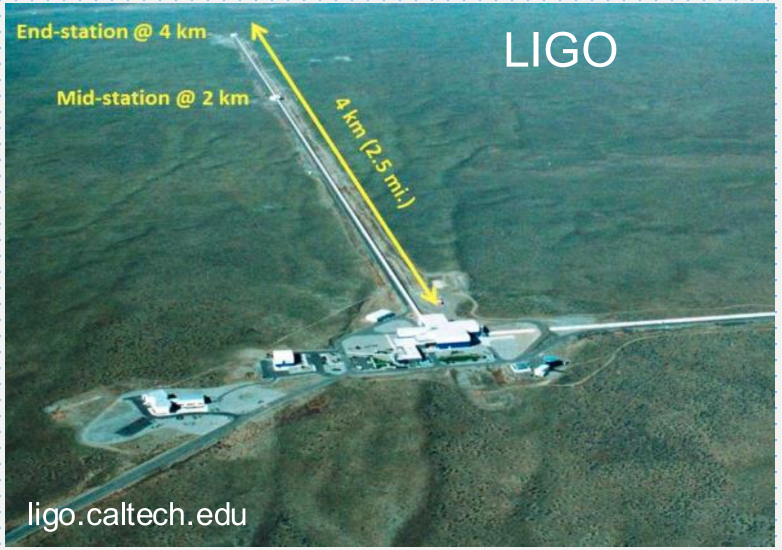
NANOGrav Physics Frontiers Center

PPTA

InPTA Indian Pulsar Timing Array

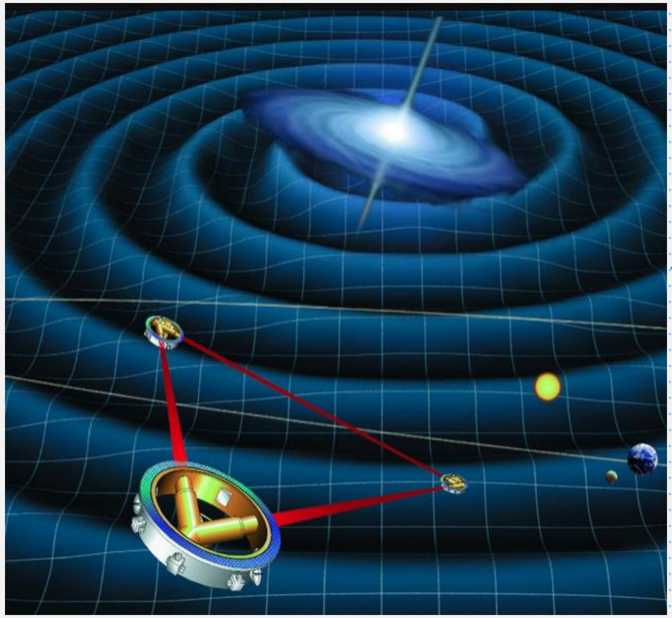
中国脉冲星测时阵列（CPTA）



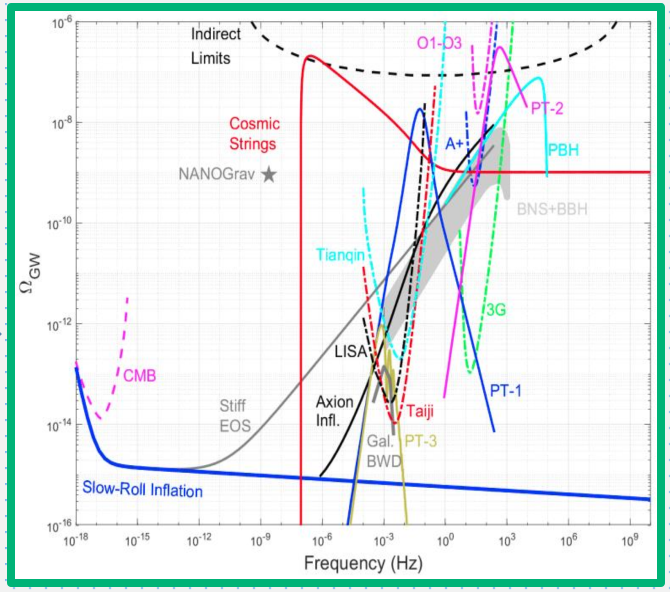


From Theory to Experiment

theorist



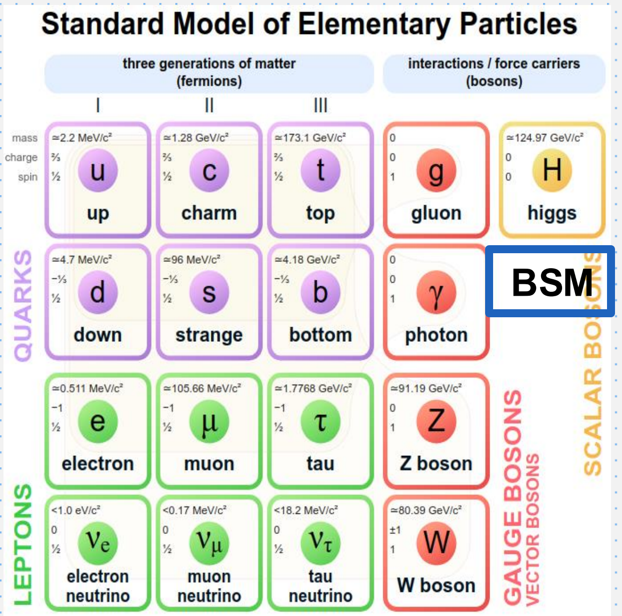
LIGO, LISA/Taiji/Tianqin, PTA, ...



Gravitational Wave Spectrum

α
β
v_w
T_*
g_s
...

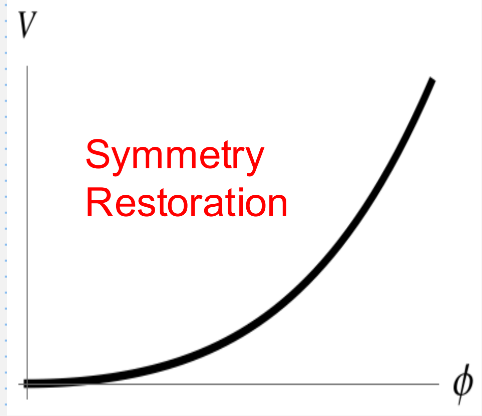
Phase Transition Parameters

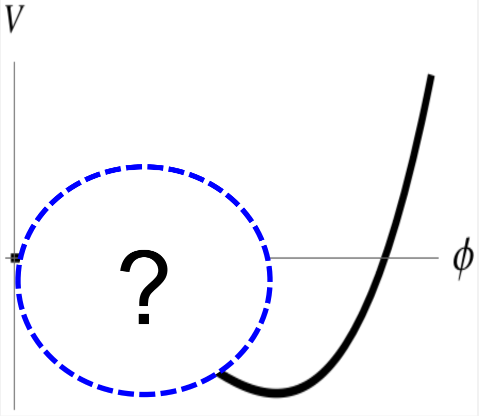


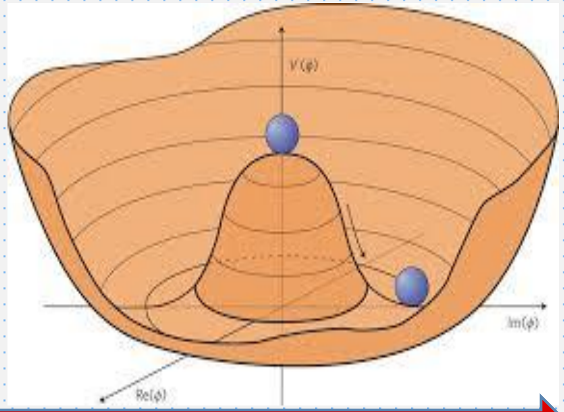
Particle Physics Model

experimentalist

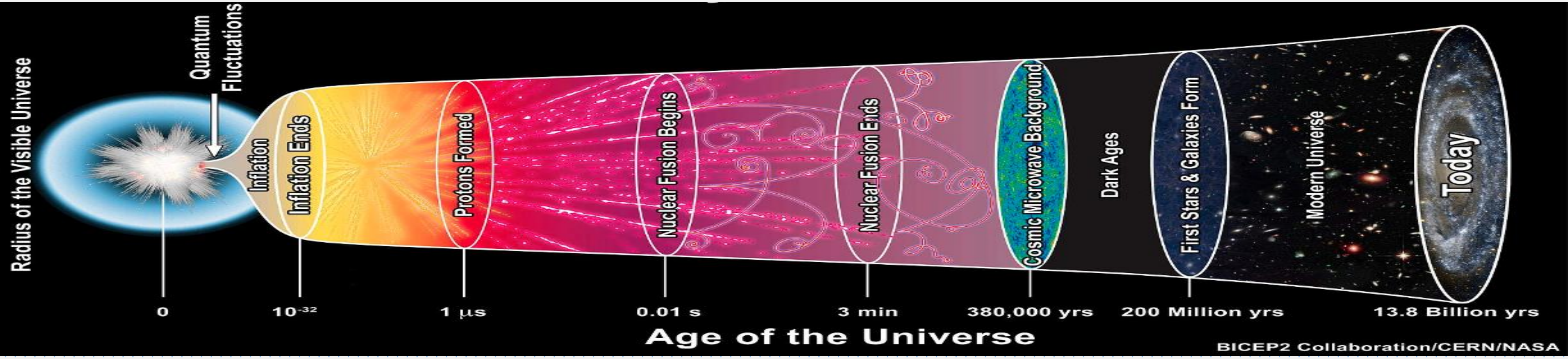
Electroweak Phase Transition



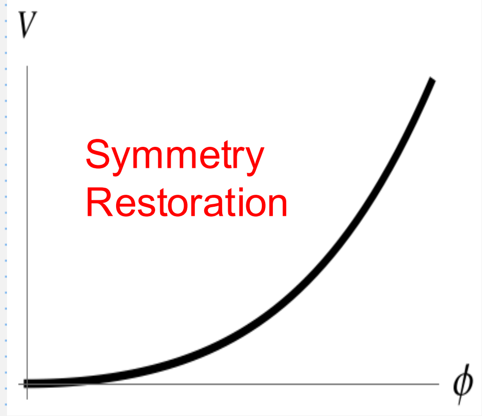


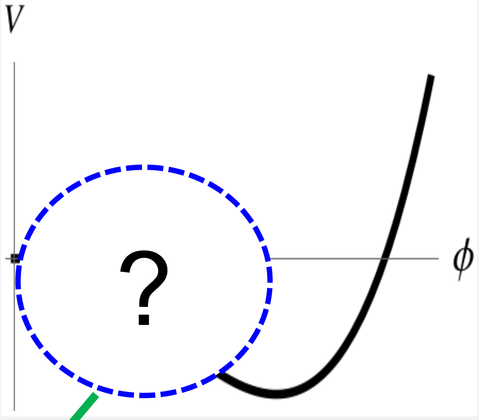


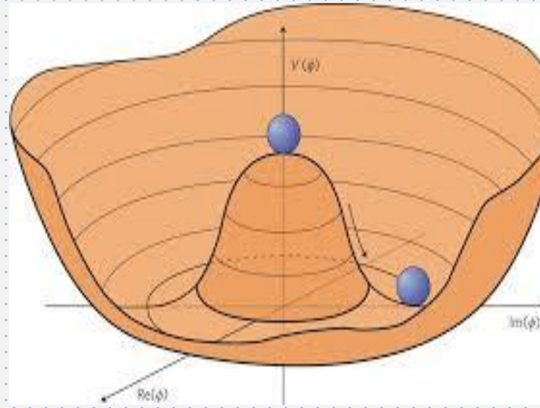
Temperature drops



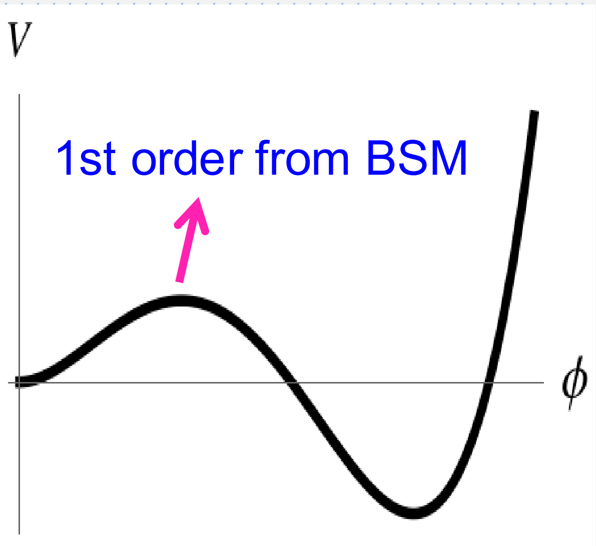
Electroweak Phase Transition

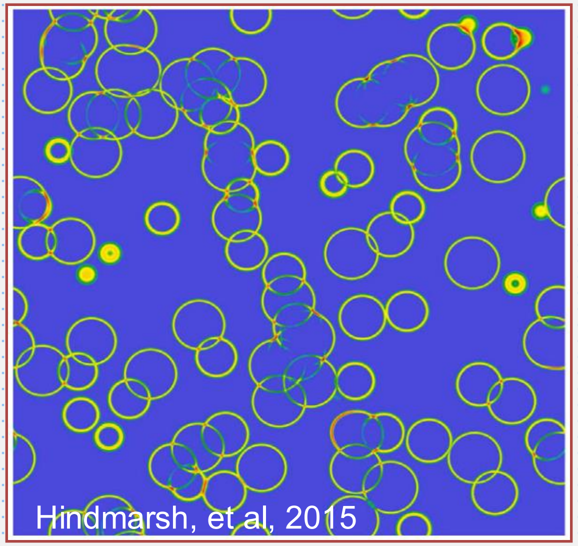






Temperature drops





Electroweak Baryogenesis

- Modified Higgs potential (Higgs physics, GW)
- Extra CP-violation (EDM, LHC)
- New particles, symmetries (LHC, GW)

Trodden, Rev.Mod.Phys [9803479]

Phenomenological Studies

Detection of early-universe gravitational-wave signatures and fundamental physics

Robert Caldwell, Yanou Cui, Huai-Ke Guo, Vuk Mandic, Alberto Mariotti, Jose Miguel No, Michael J. Ramsey-Musolf, Mairi Sakellariadou, Kuver Sinha, Lian-Tao Wang, Graham White, Yue Zhao, Haipeng An, Ligong Bian, Chiara Caprini, Sebastien Clesse, James M. Cline, Giulia Cusin, Bartosz Fornal, Ryusuke Jinno, Benoit Laurent, Noam Levi, Kun-Feng Lyu, Mario Martinez, Andrew L. Miller, Diego Redigolo, Claudia Scarlata, Alexander Sevrin, Barmak Shams Es Haghi, Jing Shu, Xavier Siemens, Danièle A. Steer, Raman Sundrum, Carlos Tamarit, David J. Weir, Ke-Pan Xie, Feng-Wei Yang & Siyi Zhou — Show fewer authors

General Relativity and Gravitation **54**, Article number: 156 (2022) | Cite this article

arXiv > hep-ph > arXiv:2203.08206

High Energy Physics - Phenomenology

[Submitted on 15 Mar 2022]

Probing the Electroweak Phase Transition with Exotic Higgs Decays

Marcela Carena, Jonathan Kozaczuk, Zhen Liu, Tong Ou, Michael J. Ramsey-Musolf, Jessie Shelton, Yikun Wang, Ke-Pan Xie

arXiv > hep-ph > arXiv:2203.10046

High Energy Physics - Phenomenology

[Submitted on 18 Mar 2022]

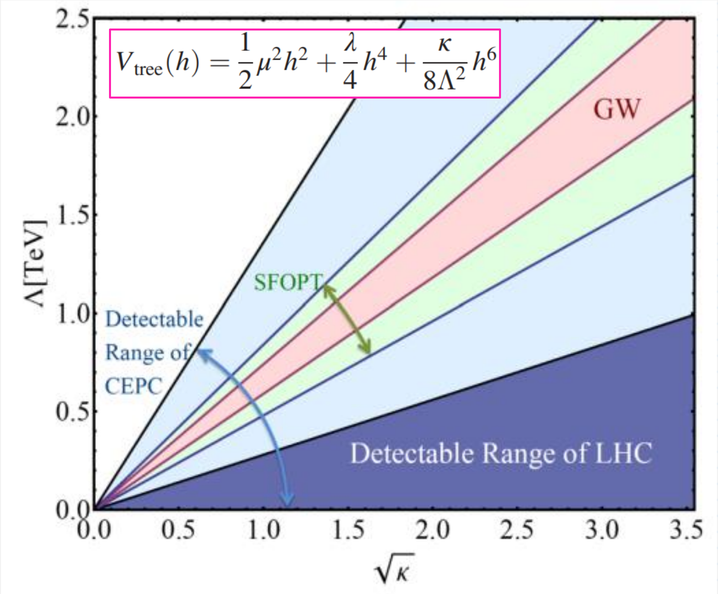
Scalar-mediated dark matter model at colliders and gravitational wave detectors -- A White paper for Snowmass 2021

Jia Liu, Xiao-Ping Wang, Ke-Pan Xie

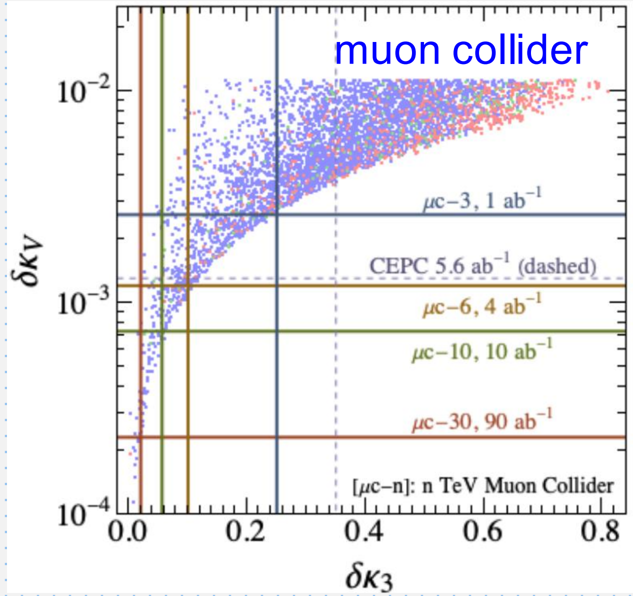
Snowmass 2021 White papers

Models	Strong 1^{st} order phase transition	GW signal	Cold DM	Dark Radiation and small scale structure
SM charged				
Triplet [20–22]	✓	✓	✓	✗
complex and real Triplet [23] (Georgi-Machacek model)	✓	✓	✓	✗
Multiplet [24]	✓	✓	✓	
2HDM [25–30]	✓	✓		✗
MLRSM [31]	✓	✓	✗	✗
NMSSM [32–36]	✓	✓	✓	✗
SM uncharged				
S_r (xSM) [37–49]	✓	✓	✗	✗
2 S_r's [50]	✓	✓	✓	✗
S_c (cxSM) [49, 51–54]	✓	✓	✓	✗
$U(1)_D$ (no interaction with SM) [55]	✓	✓	✓	✗
$U(1)_D$ (Higgs Portal) [56]	✓	✓	✓	
$U(1)_D$ (Kinetic Mixing) [57]	✓	✓	✓	
Composite $SU(7)/SU(6)$ [58]	✓	✓	✓	
$U(1)_L$ [59]	✓	✓	✓	✗
$SU(2)_D \to$ global $SO(3)$ by a doublet [60–62]			✓	✗
$SU(2)_D \to U(1)_D$ by a triplet [63–65]			✓	✓
$SU(2)_D \to Z_2$ by two triplets [66]			✓	✗
$SU(2)_D \to Z_3$ by a quadruplet [67, 68]			✓	✗
$SU(2)_D \times U(1)_{B-L} \to Z_2 \times Z_2$ by a quintuplet and a S_c [69]			✓	✗
$SU(2)_D$ with two dark Higgs doublets [70]	✓	✓	✗	✗
$SU(3)_D \to Z_2 \times Z_2$ by two triplets [62, 71]			✓	✗
$SU(3)_D$ (dark QCD) (Higgs Portal) [72, 73]	✓	✓	✓	
$G_{SM} \times G_{D,SM} \times Z_2$ [74]	✓	✓	✓	
$G_{SM} \times G_{D,SM} \times G_{D,SM} \cdots$ [75]	✓	✓	✓	
Current work				
$SU(2)_D \to U(1)_D$ (see the text)	✓	✓	✓	✓

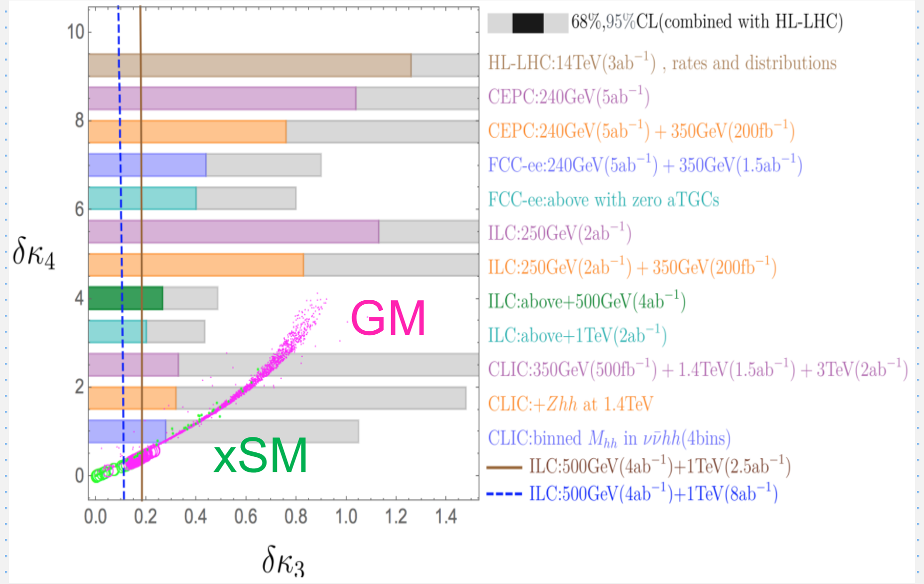
Ghosh, HG, Han, Liu, JHEP [2012.09758]



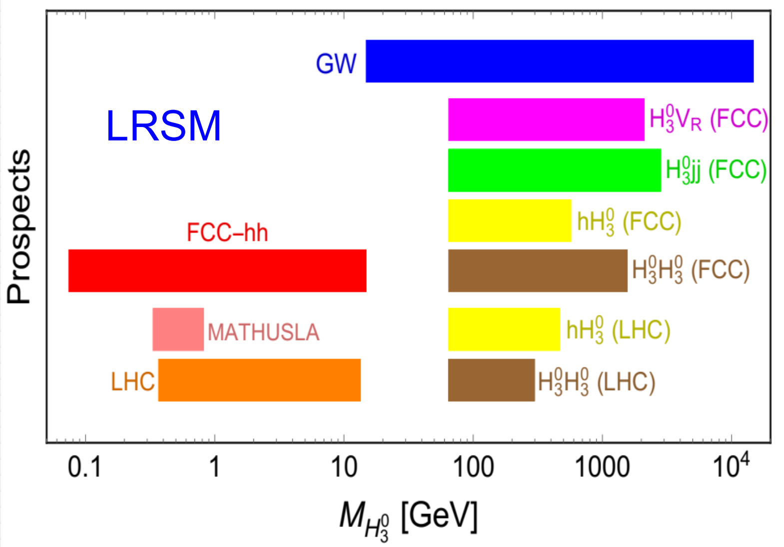
Fapeng Huang, Yi-Fu Cai, Xinmin Zhang, et al, PRD [1601.01640]



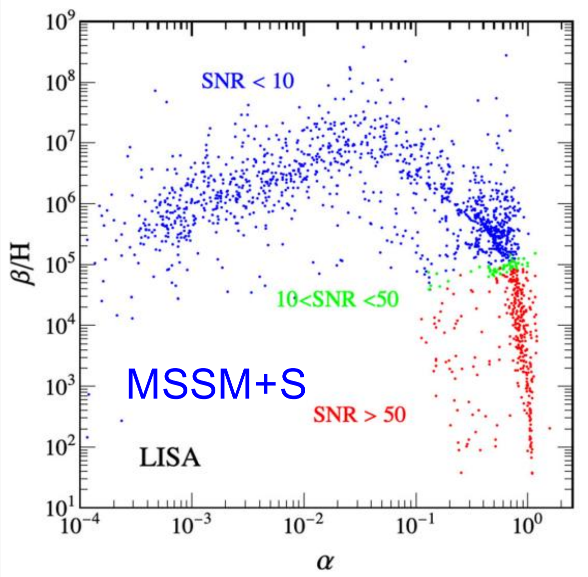
Wei Liu, Ke-Pan Xie, JHEP [2101.10469]



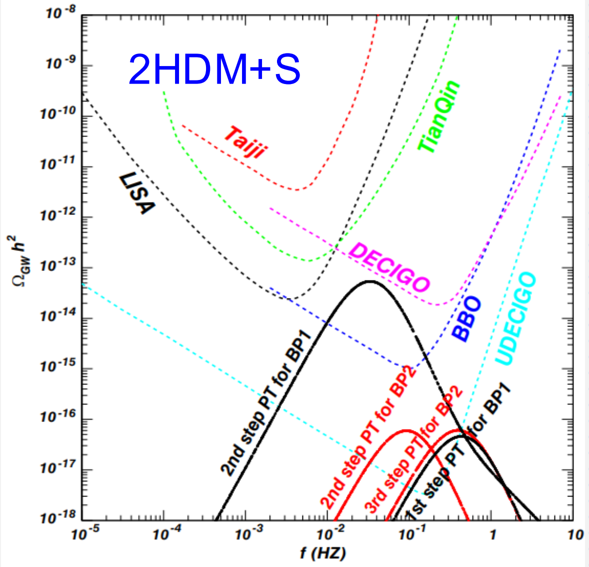
Ligong Bian, HG, Yongcheng Wu, Ruiyu Zhou, PRD [1906.11664]



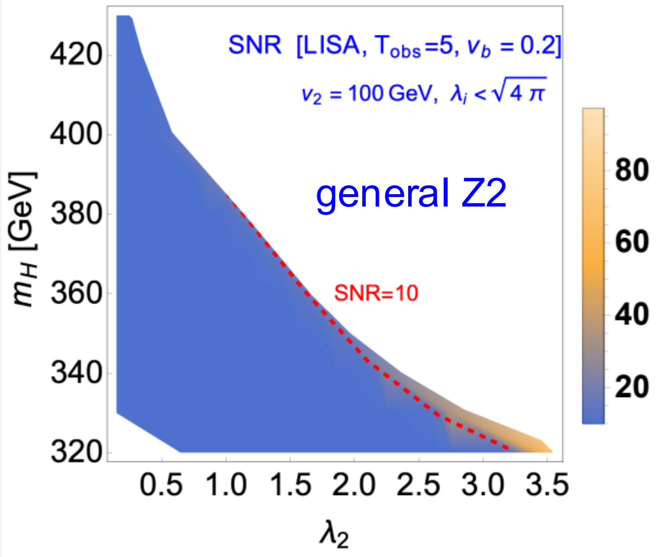
Mingqiu Li, Qi-Shu Yan, Yongchao Zhang, Zhijie Zhao, JHEP [2012.13686]



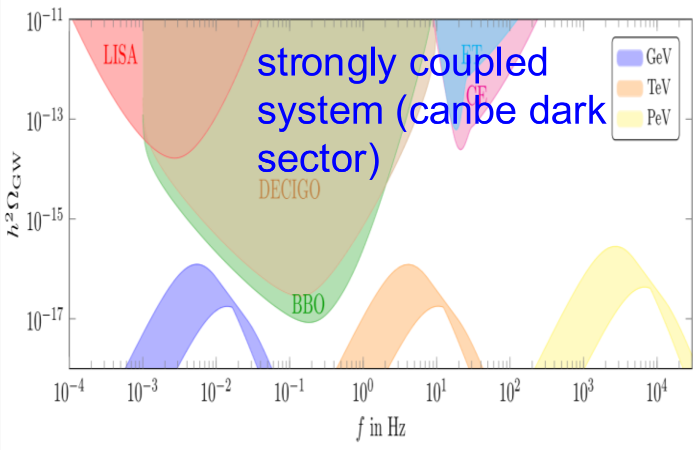
Wenyu Wang, Ke-Pan Xie, Wu-Long Xu, Jin Min Yang, EPJC [2204.01928]



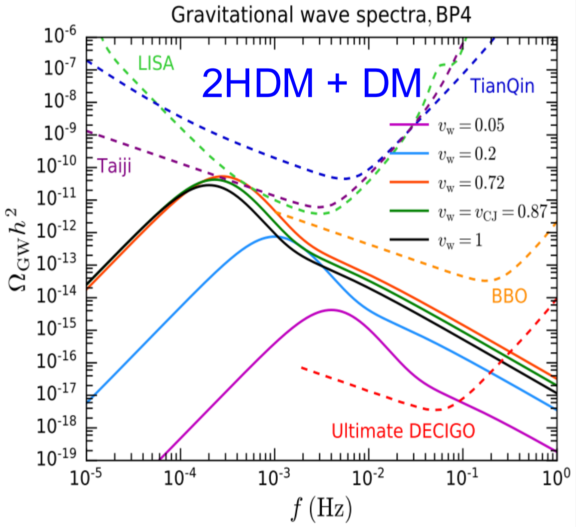
Songtao Liu, Lei Wang, PRD [2302.04639]



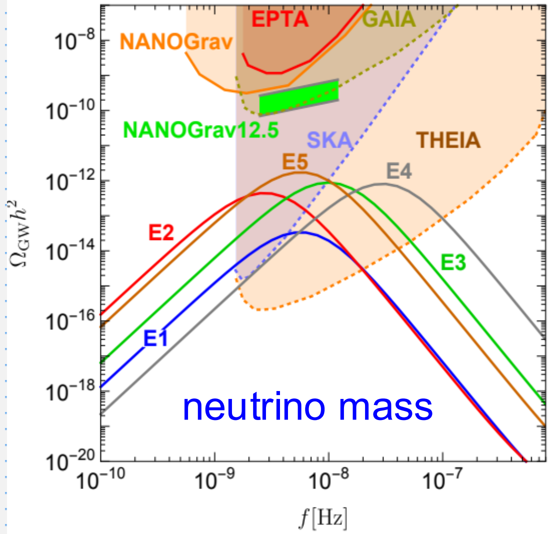
Qing-Hong Cao, Katsuya Hashino, Xu-Xiang Li, Jiang-Hao Yu [2212.07756]



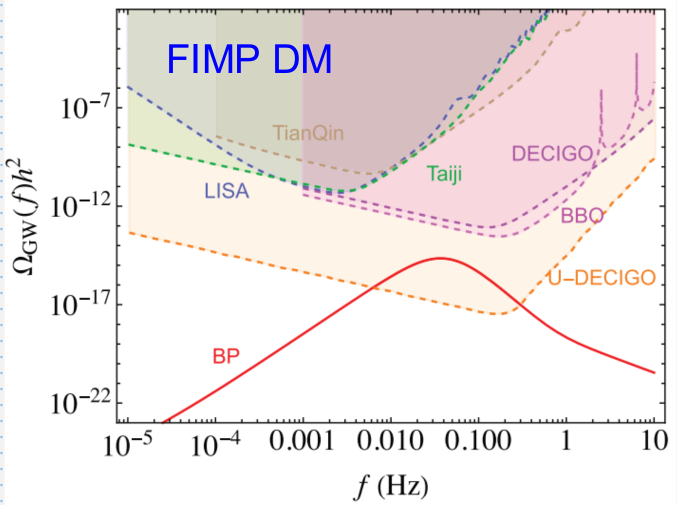
Zhi-Wei Wang, et al, PRD [2012.11614]



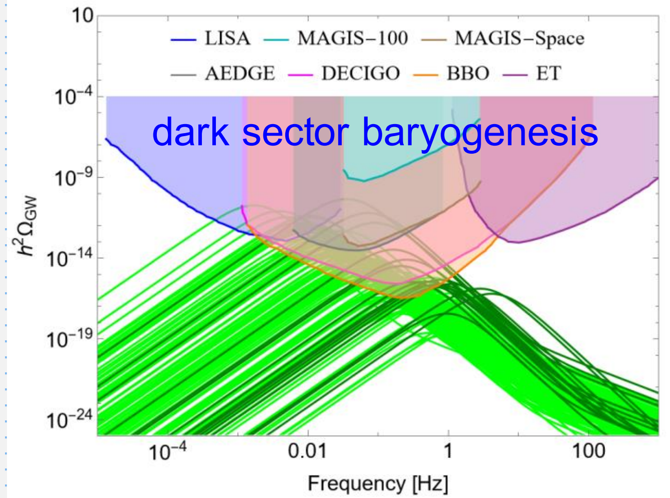
Zhao Zhang, Chengfeng Cai, Xue-Min Jiang, Yi-Lei Tang, Zhao-Huan Yu, and Hong-Hao Zhang, JHEP [2102.01588]



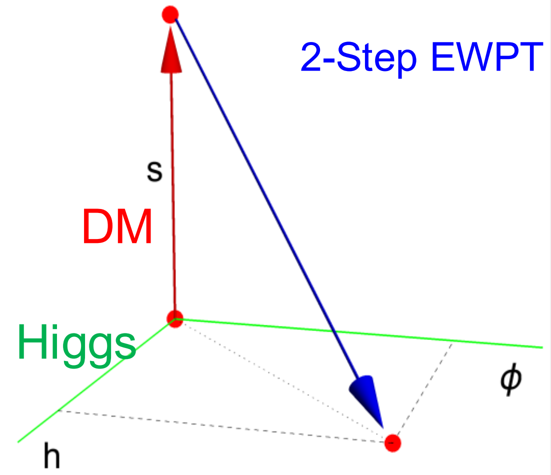
Pasquale Di Bari, Danny Marfatia, Ye-Ling Zhou, JHEP [2106.00025]



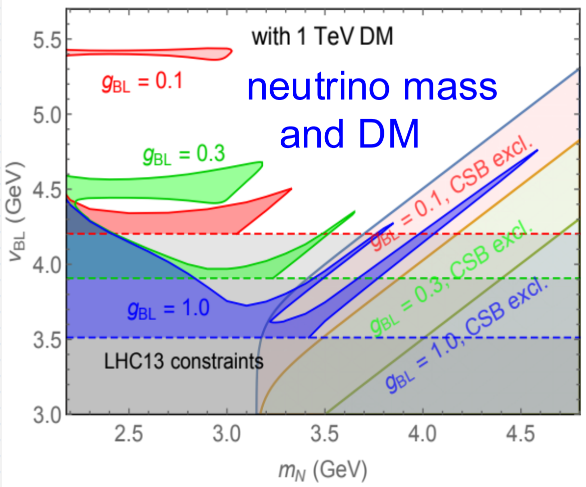
Xuewen Liu, Shu-Yuan Guo, Bin Zhu, Ying Li, Sci.Bull. [2022.06.011]



Marcela Carena, Ying-Ying Li, Tong Ou, Yikun Wang, JHEP [2210.14352]



Wei Chao, HG, Jing Shu, JCAP [1702.02698]

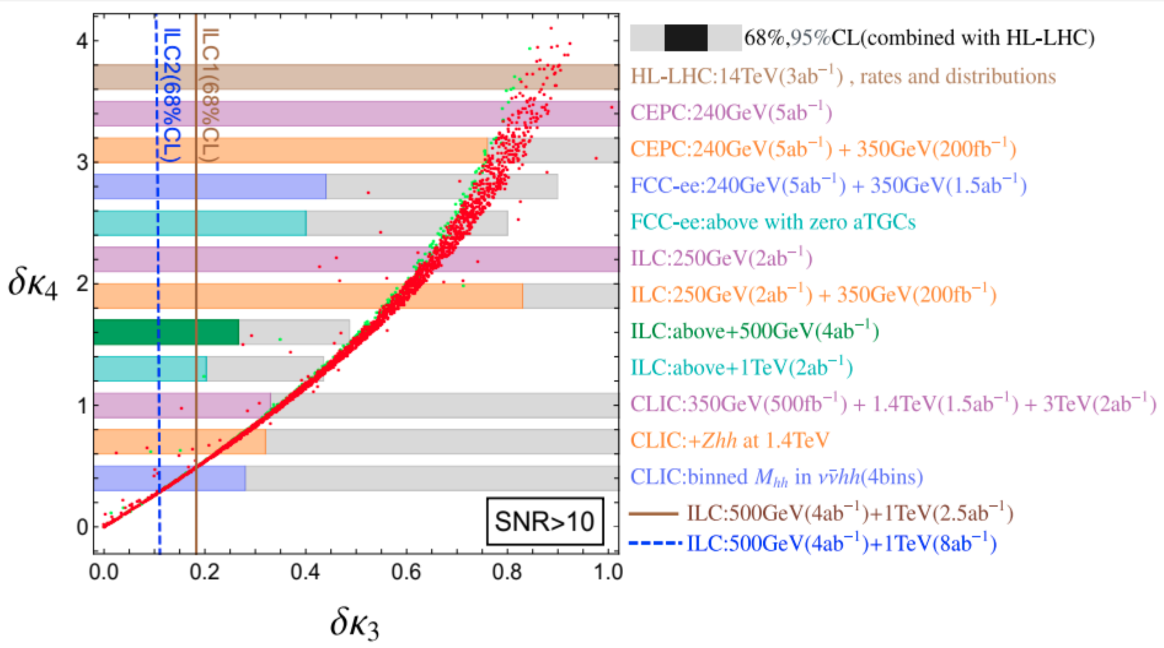


Ligong Bian, Wei Cheng, HG, Yongchao Zhang, CPC [1907.13589]

Collider and GW Complementarity

Contents

Snowmass 2021 Whitepaper, GRG [2203.07972]



Alves, Ghosh, HG, Sinha, Vagie, JHEP [1812.09333]

Simplest model: xSM (SM + S)

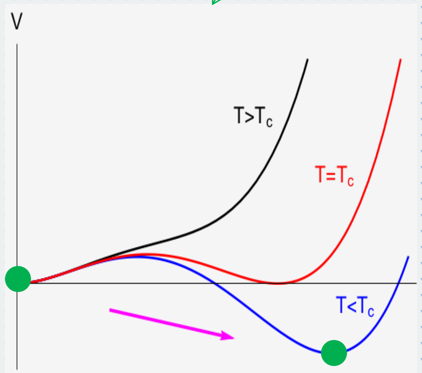
$$\Delta\mathcal{L} = -\frac{1}{2}\frac{m_{h_1}^2}{v}(1+\delta\kappa_3)h_1^3 - \frac{1}{8}\frac{m_{h_1}^2}{v^2}(1+\delta\kappa_4)h_1^4$$

Nucleation Rate: Perturbative Method

The Usual Method
(Tree + Coleman-Weinberg + Daisy Resummation)

$$V_T = V_{\rm tree} + V_{\rm CW} + \frac{T^2}{24}\sum_i c_i M_i^2(\phi) - \frac{T}{12\pi}\sum_j d_j [M_j^2(\phi)]^{3/2}$$

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 - ET\phi^3 + \frac{\lambda}{4}\phi^4$$



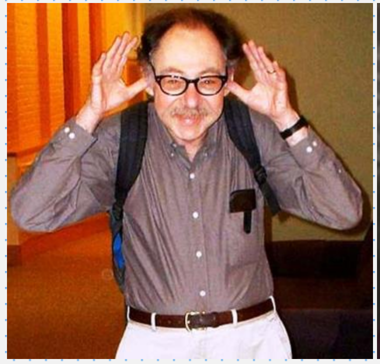
- Infrared problem (Linde, 1980)
- Gauge invariance (ok for high-T expansion) (Patel,Ramsey-Musolf, JHEP [1101.4665])

Nucleation rate

$$p = p_0 \exp\left[-\frac{S_{3,b}(T)}{T}\right]$$

fluctuations

critical bubble



Sidney Coleman

Andrei Linde

Gauge-invariant method proposed

Löfgren, Ramsey-Musolf, et al, PRL [2112.05472], JHEP [2112.08912]

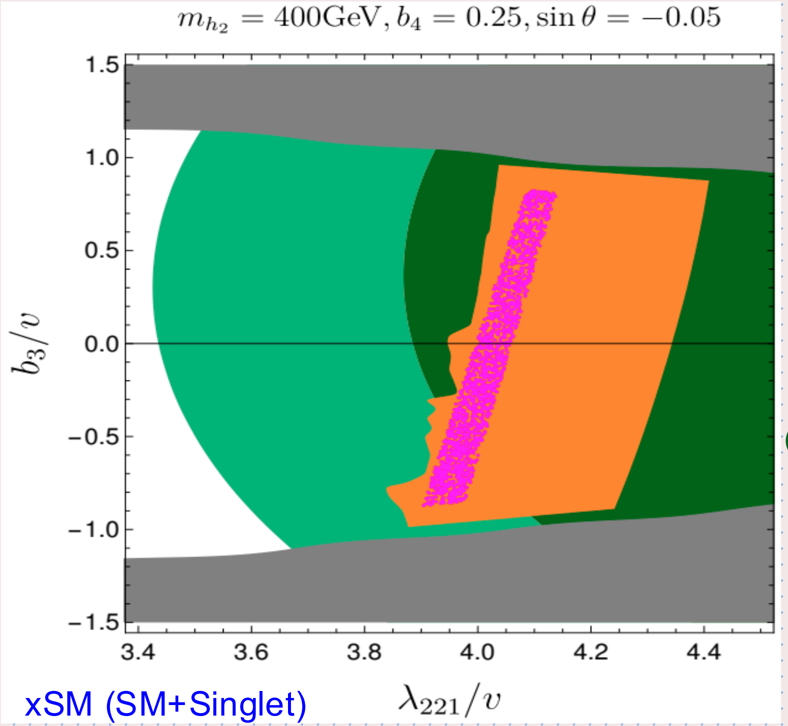
$$S_3 = a_0 g^{-3/2} + a_1 g^{-1/2} + \Delta$$

$$-\frac{1}{T}S_3[\phi_b] = -\frac{1}{T}\int d^3x \left[V_{\rm LO}^{\rm eff}(\phi_b) + V_{\rm NLO}^{\rm eff}(\phi_b) + \frac{1}{2}\left[1 + Z_{\rm NLO}(\phi_b, T)\right](\partial_i\phi_b)^2 + \cdots\right]$$

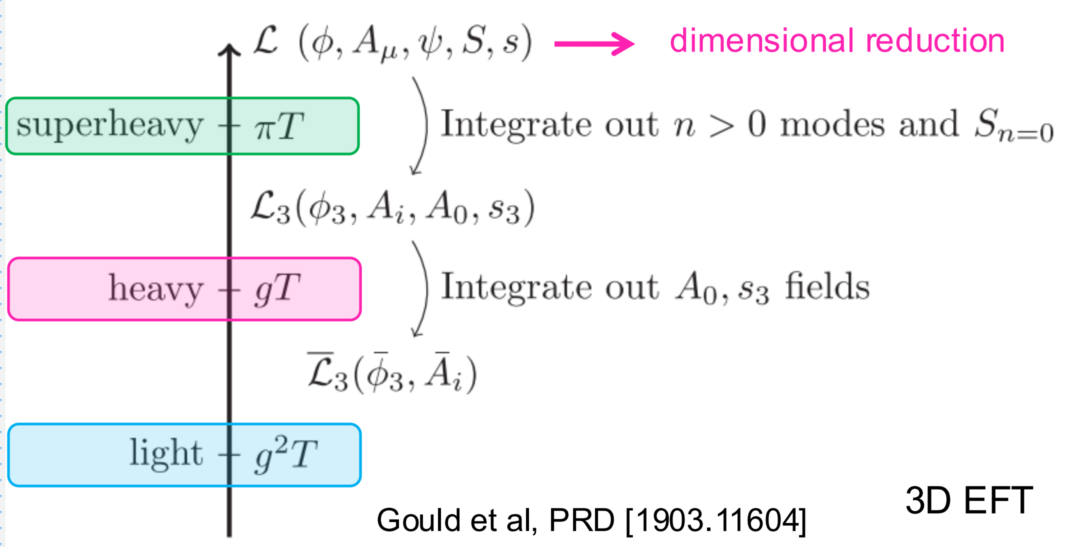
Nucleation Rate: Non-Perturbative Method

- Non-perturbative method overcomes these problems
- But yet quite limited in BSM studies

perturbative vs non-perturbative

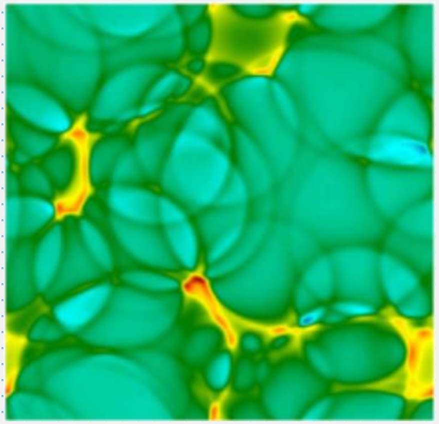


Alves,Goncalves,Ghosh,HG,Sinha, JHEP [1909.05268]



Dimensional Reduction (Status)		
SM	√	Farakos,Kajantie,Rummukainen,Shaposhnikov (1994)
MSSM	√	Cline,Kainulainen(1996), Losada(1996), Laine (1996)
xSM (SM + Singlet)	√	Brauner,Tenkanen,Tranberg,Vuorinen,Weir, JHEP [1609.06230]
ΣSM (SM + Triplet)	√	Niemi,Patel,Ramsey-Musolf,Tenkanen,Weir, PRD [1802.10500]
2HDM	√	Gorda,Helset,Niemi,Tenkanena,Weir, JHEP [1802.05056]

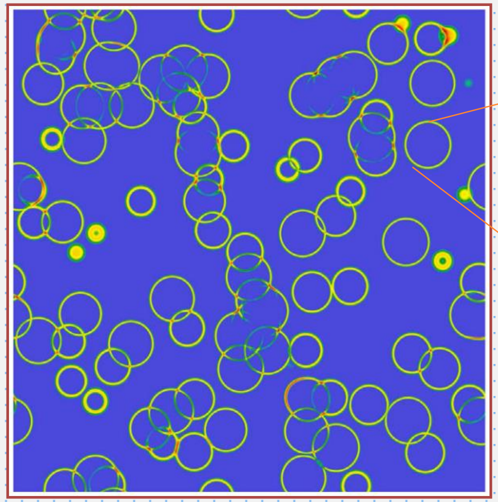
The Picture



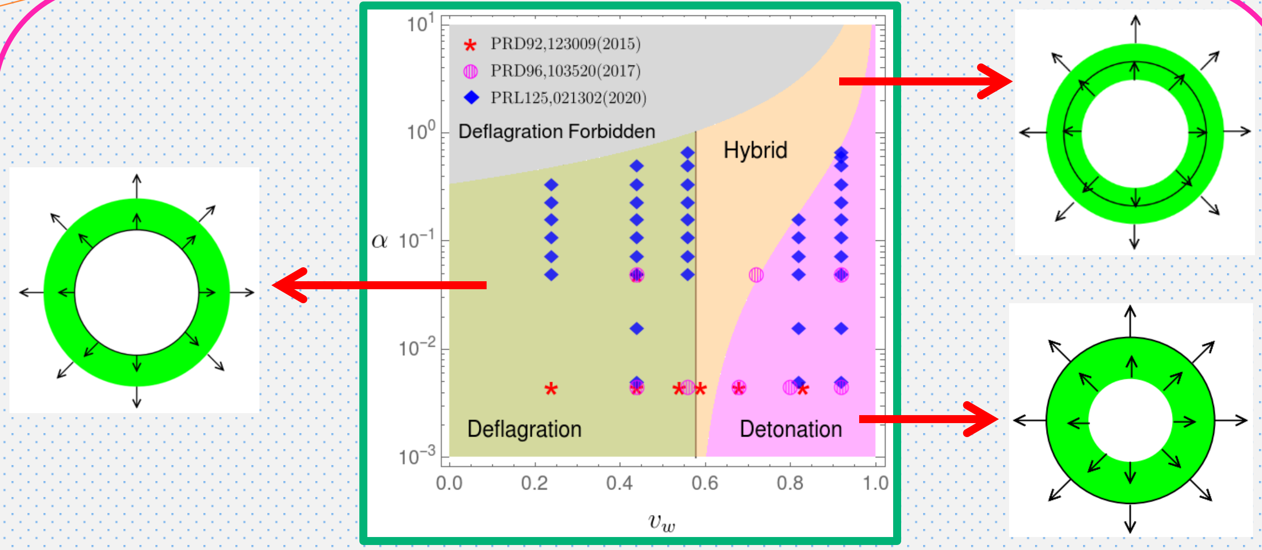
Precise calculation of PT parameters:

Minkowski spacetime: Hindmarsh,Hijazi, JCAP [1909.10040]
Expanding universe: HG,Sinha,Vagie,White, JCAP [2007.08537], JHEP [2103.06933]

- False Vacuum Fraction
- Unbroken Wall Area
- Bubble Lifetime Distribution
- Bubble Number Density



Hindmarsh et al, 2015



Espinosa,Konstandin,No,Servant JCAP [1004.4187]

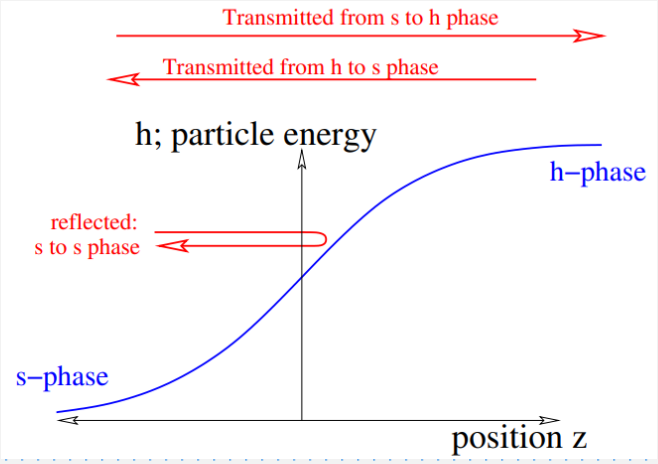
Wall Velocity: vw

Usually chosen as fixed value in baryon asymmetry and GW studies

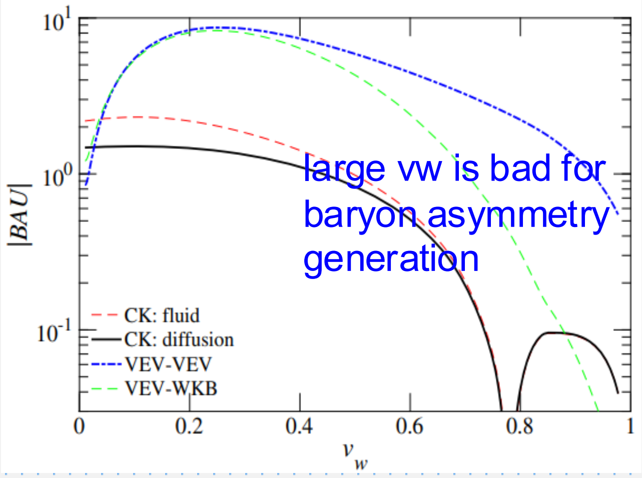
But, significant advances in recent years (driven by GW studies)

$$\Box\phi + V_T'(\phi) + \sum \frac{dm^2}{d\phi} \int \frac{d^3p}{(2\pi)^3 2E} \delta f(p,x) = 0$$

- Friction from out-of-equilibrium (Moore,Prokopec, PRL [9503296]; PRD [9506475])
- Transition radiation (Bodeker,Moore, JCAP [1703.08215])
- All orders resummation (Höche et al, JCAP [2007.10343])
- Lineared distribution or not (Laurent,Cline, PRD [2007.10935]; PRD[2204.13120])
- Singularity or not (Dorsch,Huberb,Konstandin, JCAP [2112.12548], Laurent,Cline)
- Hydrodynamic (Cai, Wang, JCAP [2011.11451], Wang, Yuwen, PRD [2205.02492], 2310.07691)



Bodeker,Moore, JCAP [1703.08215]



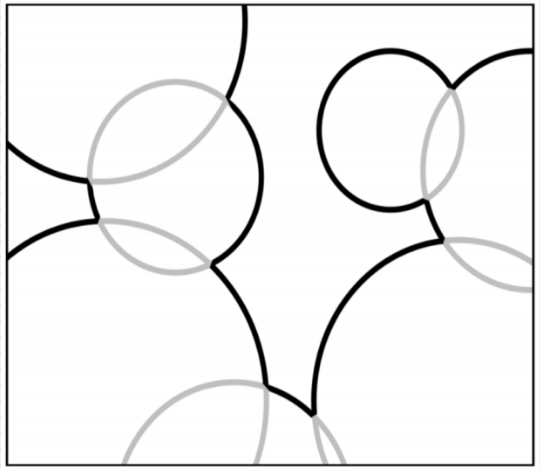
Cline,Kainulainen, PRD [2001.00568]

Gravitational Wave Sources

The current understanding:

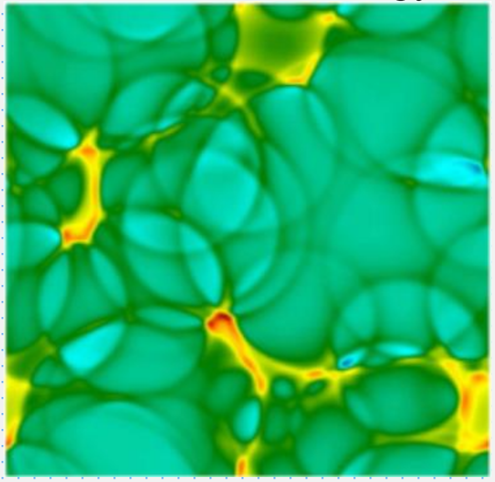
$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

energy near the wall



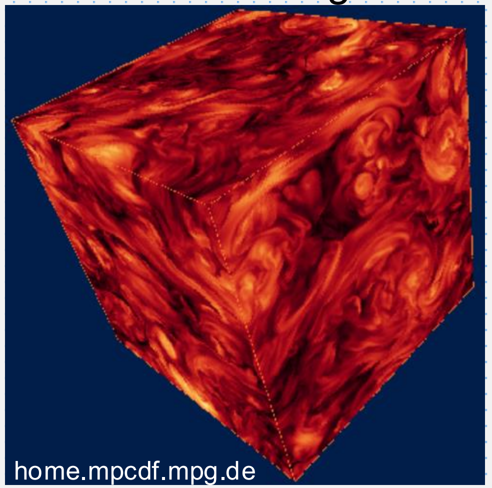
Bubble Collisions

fluid kinetic energy



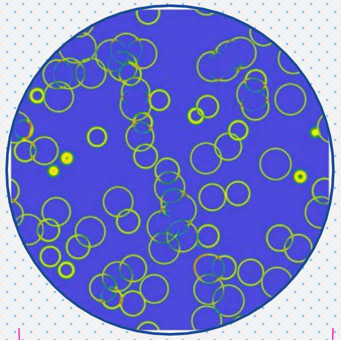
Sound Waves

turbulent fluid + magnetic field



Magnetohydrodynamic Turbulence

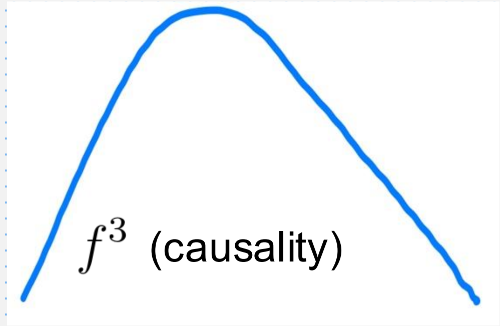
Basic Properties



Hubble size: 1/H*

$$f_{\rm now} = 1.65 \times 10^{-5} \left(\frac{f_{\rm PT}}{\beta}\right)\left(\frac{\beta}{H_*}\right)\left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6} {\rm Hz}$$

~100-1000



Cai, Pi, Sasak, PRD [1909.13728]

nHz (~100MeV) QCD scale

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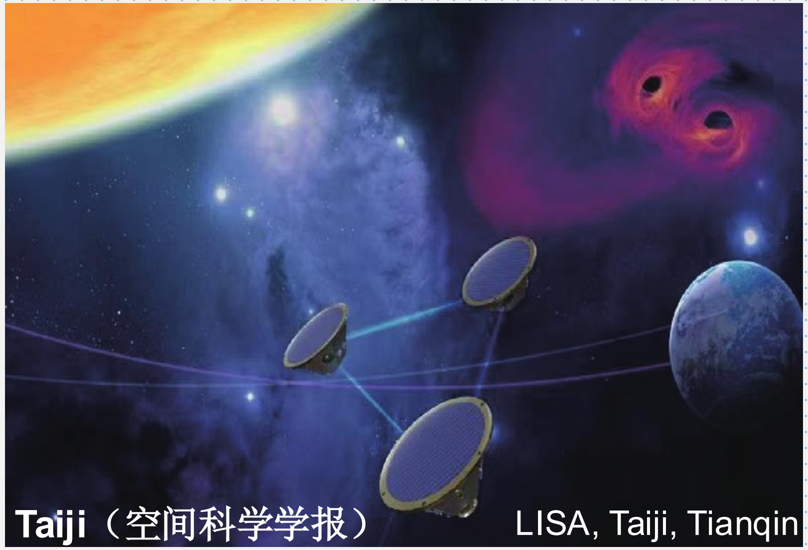
PPTA

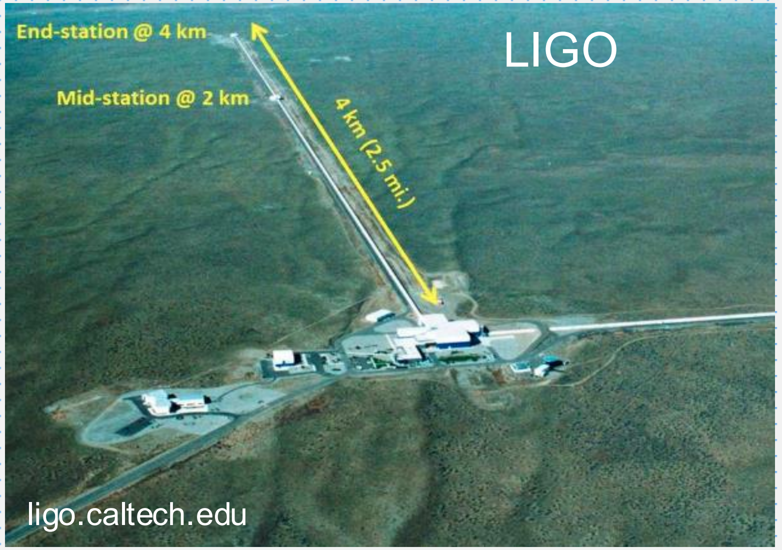
InPTA Indian Pulsar Timing Array

中国脉冲星测时阵列（CPTA）

~mHz : (~100GeV) weak scale



~100Hz (~PeV - EeV) high scale



Multiband Searches

NANOGrav, ApjL [2306.16219]

EPTA [2306.16227]

Xue,Bian,Shu,Yuan,Zhu, et al, PRL [2110.03096]

Bian et al [2307.02376]

Wu, Chen, Huang [2307.03141]

...

Boileau et al, MNRAS [2105.04283]
LISA: Caprini et al [2403.03723]
Network: Wang, Han, PRD [2108.11151] ...
TDI optimization: Wang, Li, Xu, Fan, PRD [2201.10902]
...

Romero,Martinovic,Callister,HG,Martínez,Sakellariadou, Yang,Zhao, PRL [2102.01714]
Badger, ..., HG, ..., PRD [2209.14707]
Jiang, Huang, JCAP [2203.11781]
Yu, Wang, PRD [2211.13111]

nHz (~100MeV) QCD scale

EPTA

NANOGrav Physics Frontiers Center

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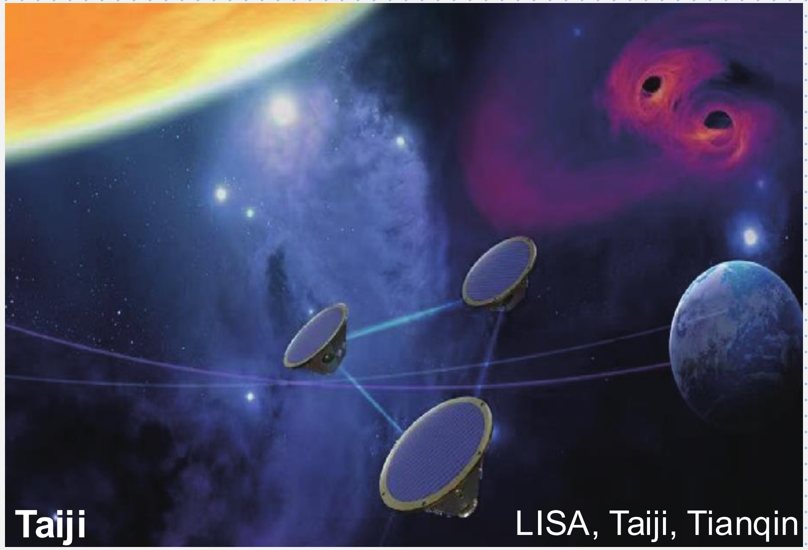
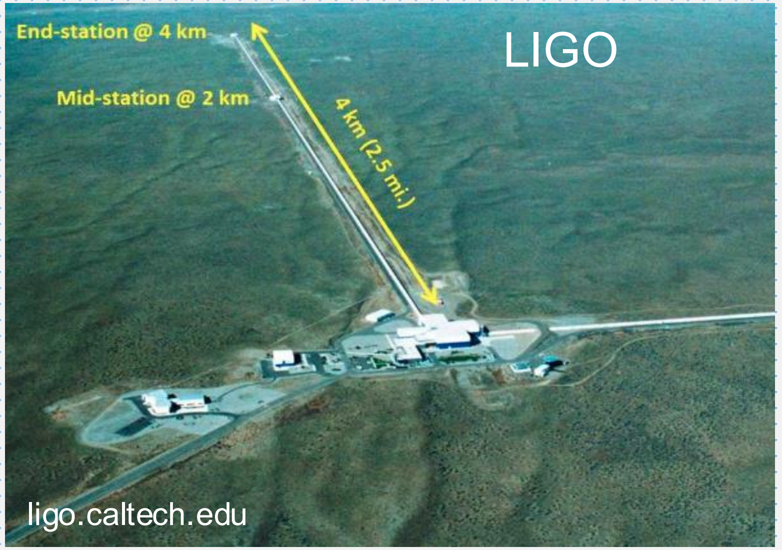
~mHz : (~100GeV) weak scale

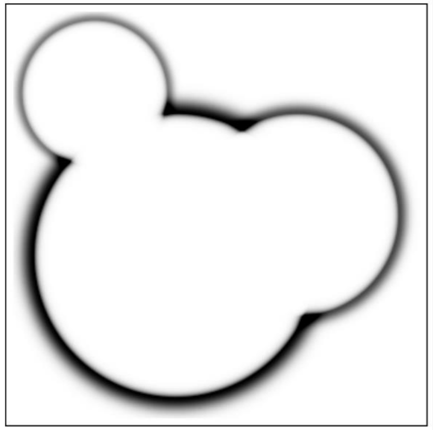


~100Hz (~PeV - EeV) high scale



Bubble Collisions

Envelope approximation:
Kosowsky, Turner, Watkins, Kamionkowski,
PRL69,2026(1992), PRD45,4514(1992), PRD47,4372(1993), PRD [9310044]



thin shell of uncollided walls

$$h^2\Omega_{\rm BC}(f) = 1.67\times10^{-5}\left(\frac{100}{g_*}\right)^{1/3}\Delta(v_w)\left(\frac{H_n}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2 S_{\rm env}(f)$$

simulation

$$\Delta = \frac{0.11 v_w^3}{0.42+v_w^2},$$

$$\frac{f_*}{\beta} = \frac{0.62}{1.8-0.1v_w+v_w^2},$$

$$S_{\rm env} = \left[\frac{3.8(f/f_{\rm env})^{2.8}}{1+2.8(f/f_{\rm env})^{3.8}}\right]$$

Huber, Konstandin, JCAP [0806.1828]

Chiara Caprini et al JCAP [1512.06239]

analytical

$$\Delta = \frac{0.48 v_w^3}{1+5.3v_w^2+5v_w^4},$$

$$\frac{f_*}{\beta} = \frac{0.35}{1+0.069v_w+0.69v_w^4},$$

$$S_{\rm env} = \left[c_l\left(\frac{f}{f_{\rm env}}\right)^{-3} + (1-c_l-c_h)\left(\frac{f}{f_{\rm env}}\right)^{-1} + c_h\left(\frac{f}{f_{\rm env}}\right)\right]^{-1}$$

$$(c_l = 0.064, \quad c_h = 0.48)$$

Jinno, Takimoto, PRD [1605.01403]

$$\Omega_{\rm BC}(f \gtrsim f_{\rm peak}) \propto f^{-1}$$

$$\Omega_{\rm BC}(f \lesssim f_{\rm peak}) \propto f^{3}$$

Bubble Collisions: Recent Development

- Wall thickness (probe effective potential)

Cutting et al, PRD [2005.13537], Gould et al, PRD [2107.05657], Mégevand,Membiela, JCAP [2302.13349]

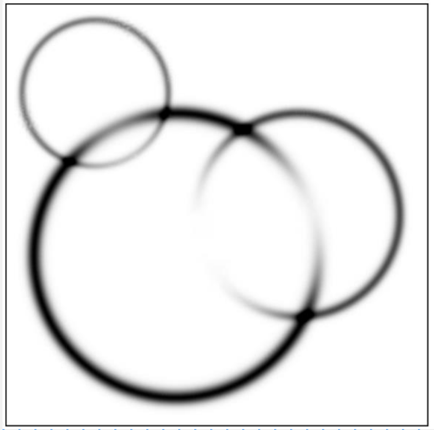
- Duration and Expanding Universe

Zhong, Gong, Qiu, JHEP [2107.01845]

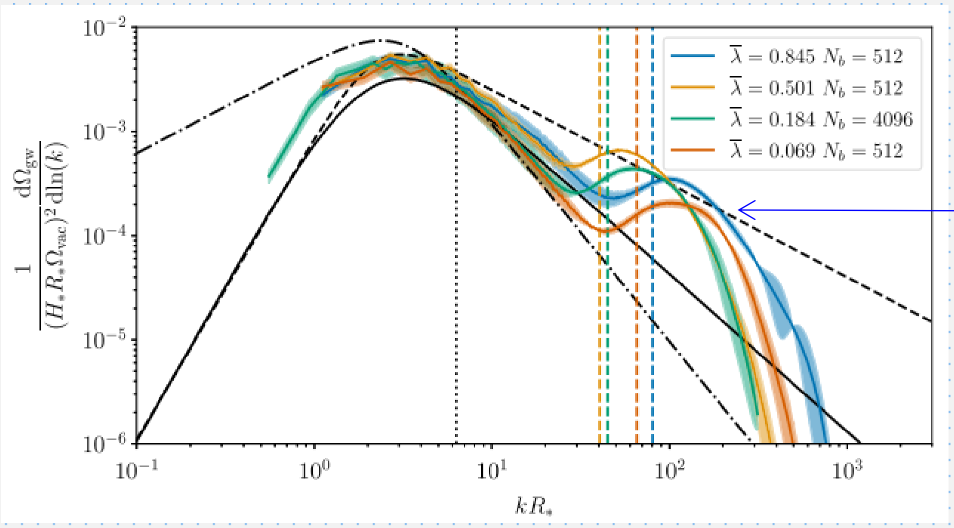
- Scalar + Gauge

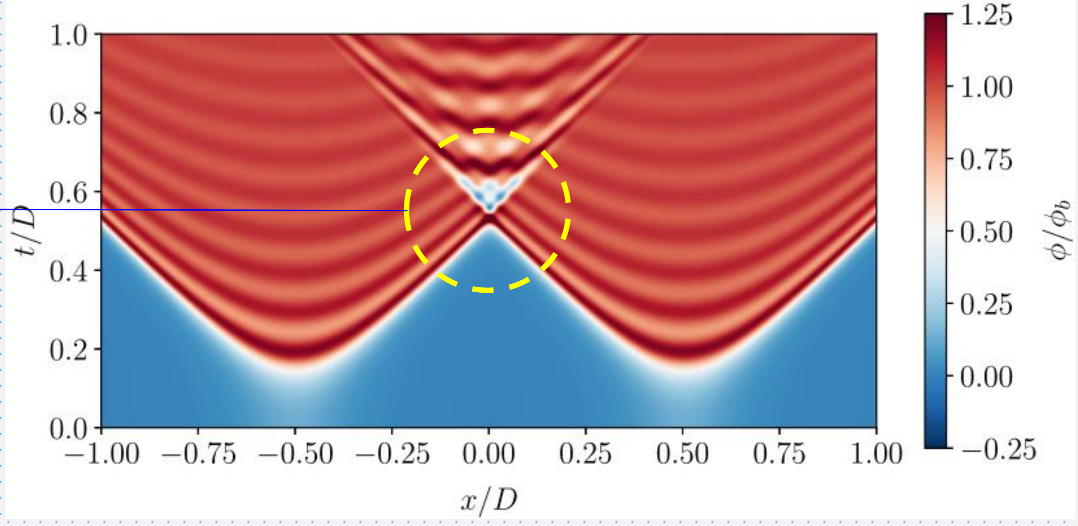
Di, Wang, Zhou, Bian, Cai, Liu, PRL [2012.15625], Yang, Bian, PRD [2102.01398], Lewicki, Vaskonen, EPJC [2007.04967]

Bulk flow model



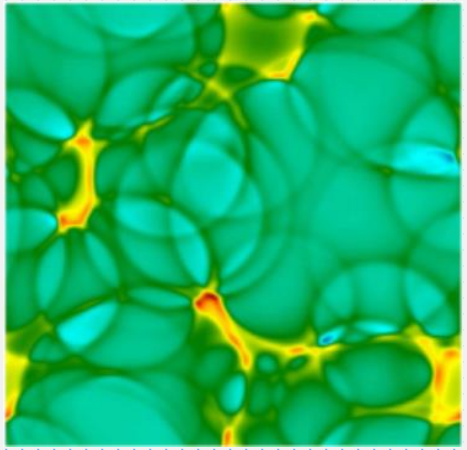
Jinno,Takimoto, JCAP [1707.03111], Konstandin, JCAP [1712.06869]





Sound Waves

Hindmarsh,Huber,Rummukainen,Weir, PRL [1304.2433]



$$T^{ij} \propto (p+e)v^i v^j$$

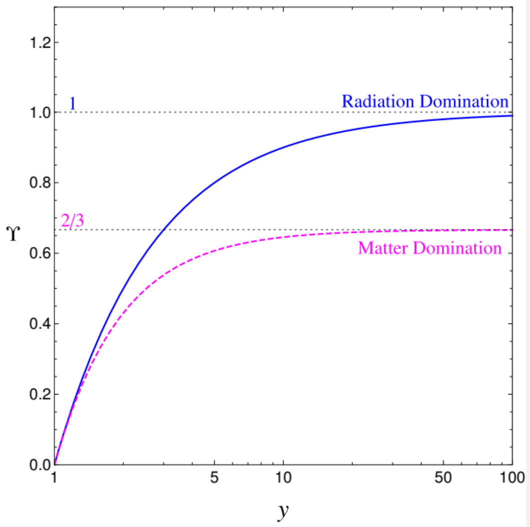
$$h^2\Omega_{\rm sw}(f) = 2.65\times 10^{-6}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 v_w S_{\rm sw}(f)\Upsilon(\tau_{\rm sw})$$

$$S_{\rm sw}(f) = \left(\frac{f}{f_{\rm sw}}\right)^3\left[\frac{7}{4+3(f/f_{\rm sw})^2}\right]^{7/2} \qquad f_* = \frac{2\beta}{\sqrt{3}v_w} \approx \frac{3.4}{R_*}$$

Hindmarsh,Huber,Rummukainen,Weir, PRD [1504.03291]

$$\Omega_{\rm SW}(f \gtrsim f_{\rm peak}) \propto f^{-4}$$

$$\Omega_{\rm SW}(f \lesssim f_{\rm peak}) \propto f^{3}$$



Slight different fit obtained by the same group, PRD [1704.05871]

$$\Upsilon = 1-(1+2\tau_{\rm sw}H_{\rm pt})^{-1/2}$$ (radiation domination)

HG, Sinha, Vagie, White, JCAP [2007.08537]

$$\Upsilon = \frac{2[1-y^{3(w-1)/2}]}{3(1-w)}$$

HG, Yang Xiao, ... [2410.23666]

Sound Waves: Recent Development

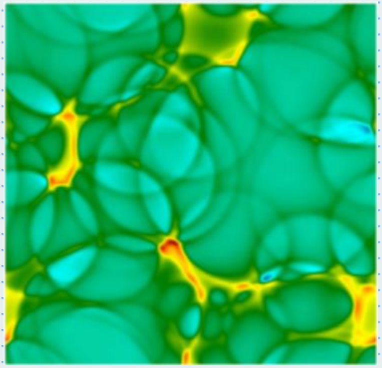
Analytical Modelling

- Refine the sound shell model
- Synergy with simulations

Sound Shell Model

Hindmarsh, PRL [1608.04735]
Hindmarsh, Hijazi, JCAP [1909.10040]
HG, Sinha, Vagie, White, JCAP [2007.08537]
Cai, Wang, Yuwen, PRD Letter [2305.00074]
Pol, Procacci, Caprini [2308.12943]

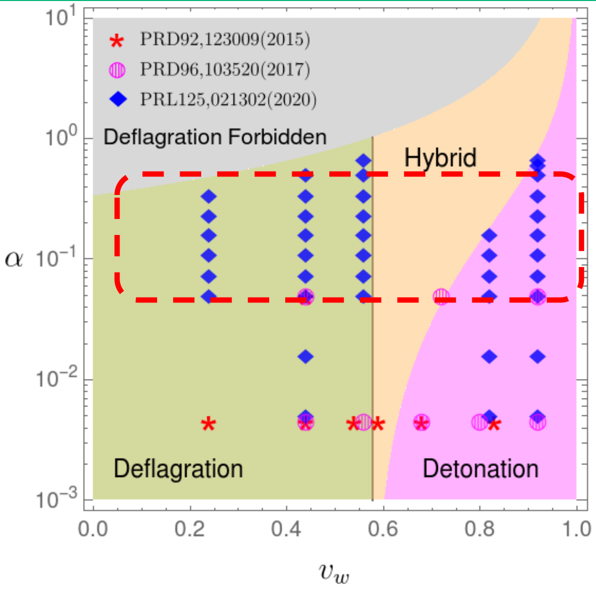
$$v^i_{\mathbf{q}} = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}}$$

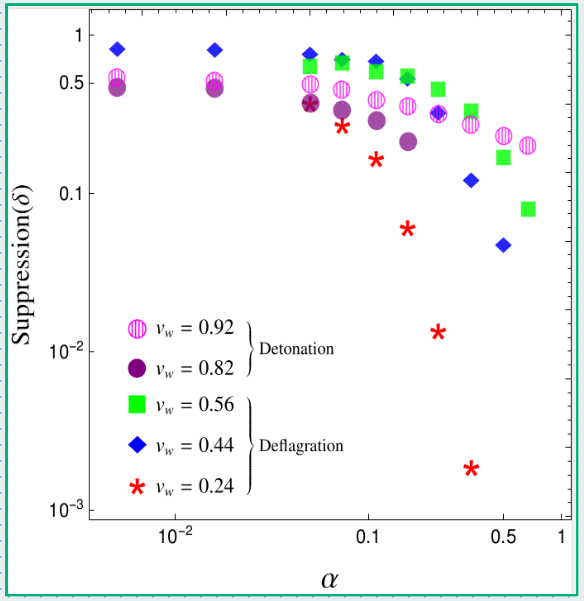


Numerical Simulation

- Suppression found for strong transitions with small vw
- Need to cover more parameter space (very strong PT)

$$h^2\Omega_{\rm sw}(f) = 2.65\times 10^{-6}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 v_w S_{\rm sw}(f)\Upsilon(\tau_{\rm sw})$$





Cutting, Hindmarsh, Weir, PRL [1906.00480]

Magnetohydrodynamic Turbulence

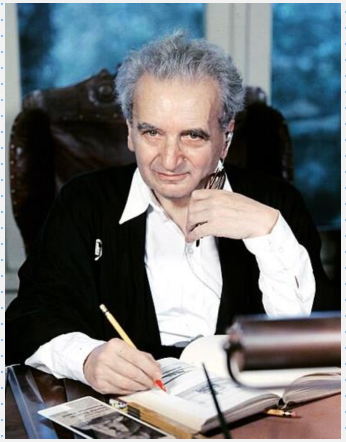
Earlier studies based on Kolmogorov spectrum:
Kamionkowski,Kosowsky,Turner, PRD [9310044]
Kosowsky,Mack,Kahniashvili, PRD [0111483]
Gogoberidze,Kahniashvili,Kosowsky, PRD [0705.1733]

$$T^{ij} \sim (p+e)v^i v^j - B_i B_j$$

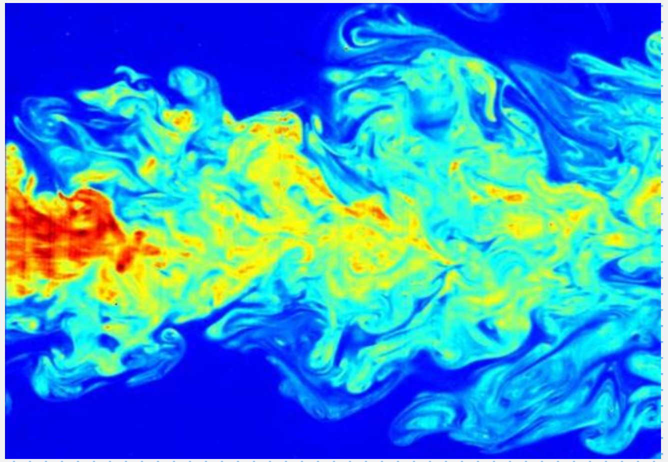
$$h^2\Omega_{\rm turb}(f) = 3.35\times 10^{-4}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm turb}\,\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{100}{g_*}\right)^{1/3} v_w\, S_{\rm turb}(f)$$

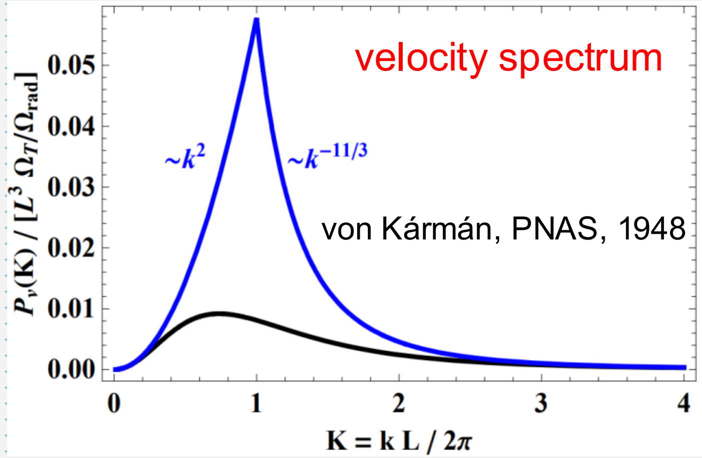
Caprini,Durrer,Servant, JCAP [0909.0622] (used von Kármán's spectrum)

Andrey Nikolaevich Kolmogorov



Theodore von Kármán



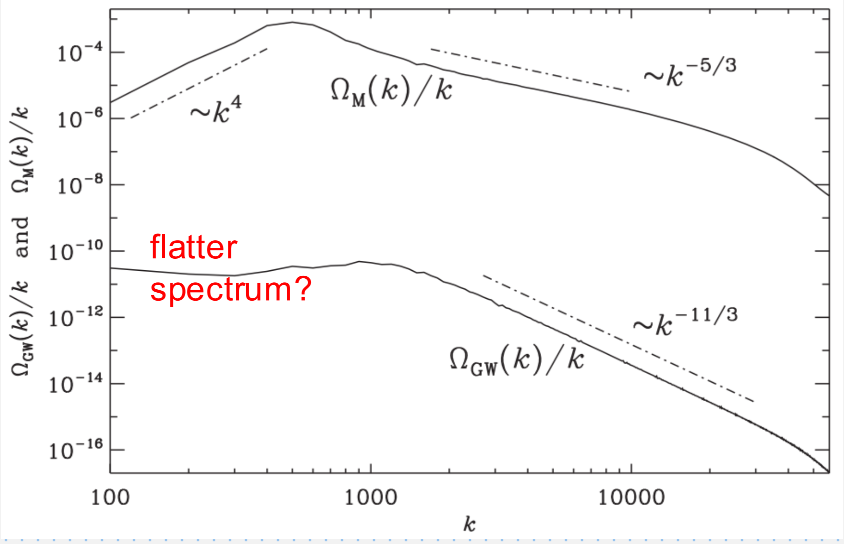


Magnetohydrodynamic Turbulence: Recent Development

Progress on numerical simulations, and analytical modellings

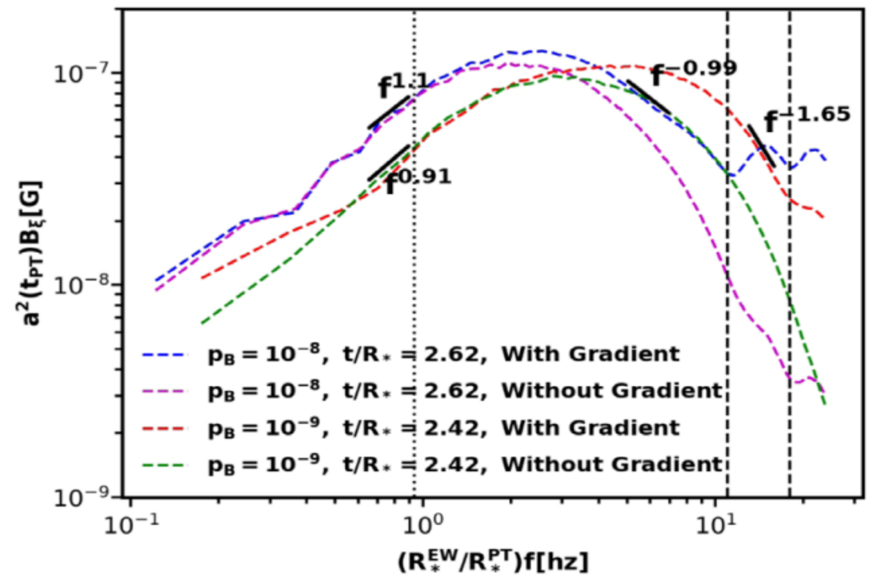
- Strong dependence on initial conditions
- Flatter spectrum at low frequency (violate causality?)

Magnetic Field Generation (simulation)



Pol et al, PRD [1903.08585]

as initial conditions?

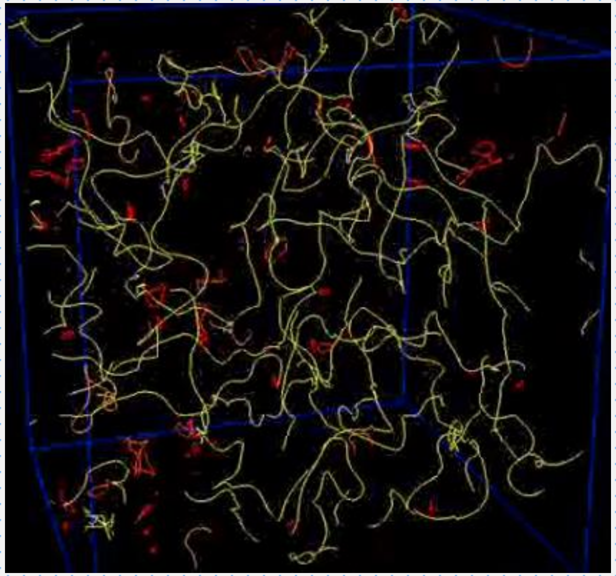


Di,Wang,Zhou,Bian,Cai, PRL [2012.15625]
Yang,Bian,PRD [2102.01398]

Modelling: Sharma, Brandenburg, PRD [2206.00055]
Time decorrelation: Auclair et al, JCAP [2205.02588]
Decay, viscosity: Dahl et al, PRD [2112.12013]
Polarization: Pol et al, JCAP [2107.05356]

Cosmic Strings

- Formed in the early unvierse (GUT, etc)
- Topological defects in 1D
- Formed in the early unvierse (GUT, etc)
- Thin, described by the Nambu-Goto action

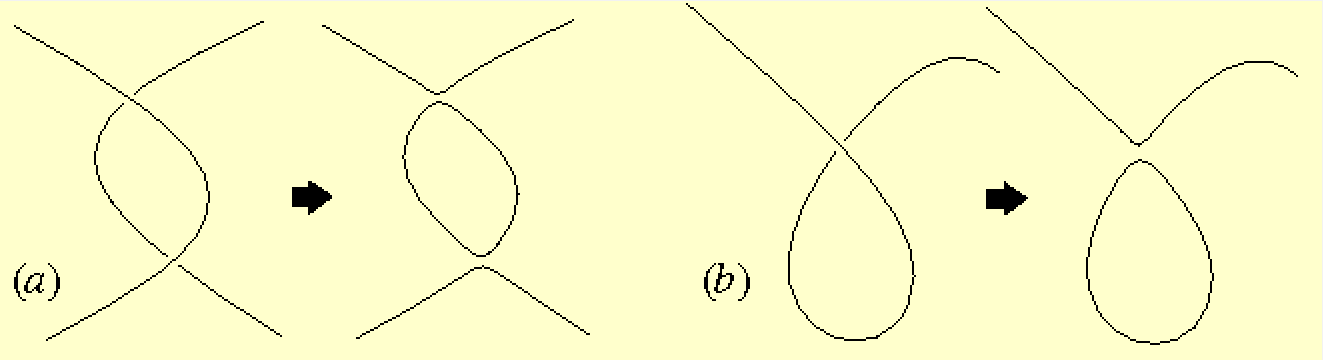


Simulations

Yellow: long strings
Red: loops

loop distribution

- Long strings interconnect to form loops
- Loops oscillate and radiate GWs (and shrink)
- Scaling loop distributions reached in RD and MD



https://www.ligo.org/science/Publication-S5S6CosmicStrings

Gravitational Wave Production

Burst types: cusps, kinks and kink-kink collisions (Damour, Vilenkin, PRL 85,3671, PRD 64, 064008).

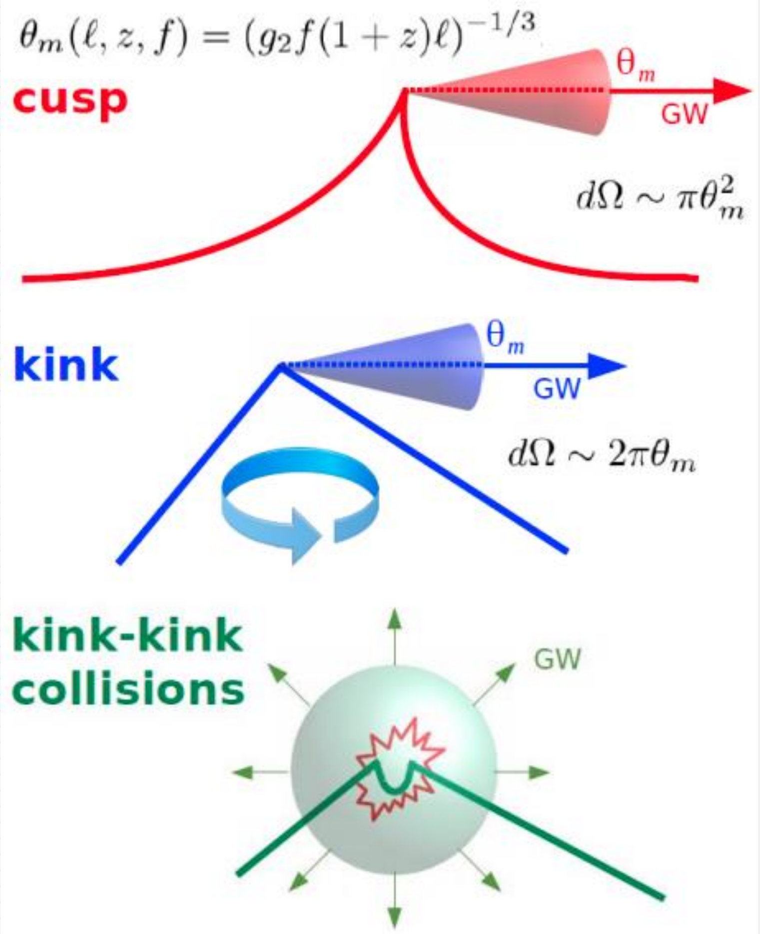


Image credit: Florent Robinet

simple waveforms in frequency domain

$$h_i(\ell, z, f) = A_i(\ell, z) f^{-q_i}$$

$$A_i(\ell, z) = g_{1,i} \frac{G\mu\, \ell^{2-q_i}}{(1+z)^{q_i-1} r(z)}$$

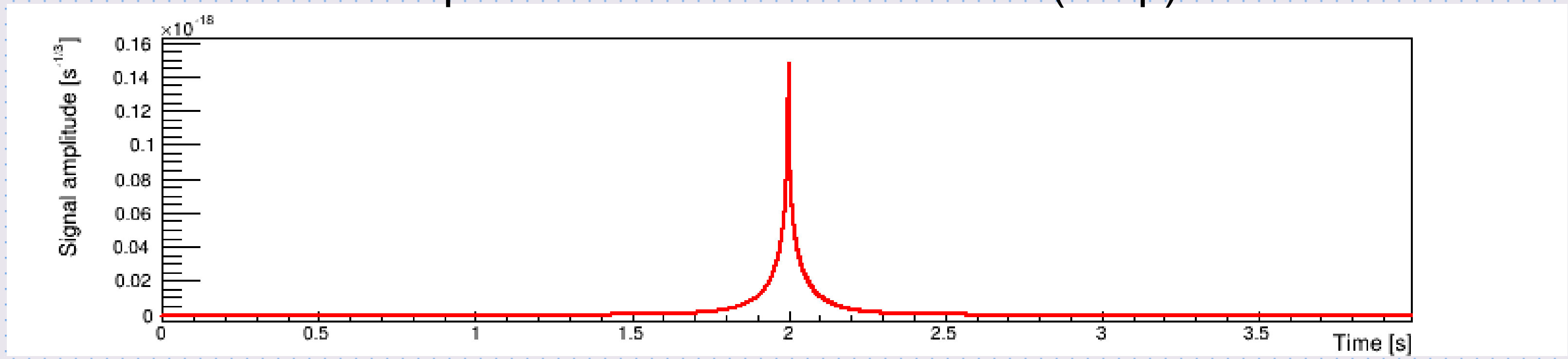
$q = 4/3, 5/3, 2$ for cusp, kink, and kk

scale

$$G\mu \sim (\eta/M_{\mathrm{Pl}})^2$$

particle physics model dependence

example waveform in time domain (cusp)



https://www.ligo.org/science/Publication-S5S6CosmicStrings

Gravitational Waves

1. Stochastic gravitational wave background (stochastic GW search)

$$\Omega_{\rm GW}(f) = \frac{4\pi^2}{3H_0^2} f^3 \sum_i \int {\rm d}z \int {\rm d}\ell \; h_i^2 \times \frac{{\rm d}^2 R_i}{{\rm d}z{\rm d}\ell}$$

cusp, kink, kk

Burst Rate

$$\frac{{\rm d}R_i}{{\rm d}z} = \frac{\varphi_V(z)}{H_0^3(1+z)} \int_{\ell_{\rm min}}^{\ell_{\rm max}} {\rm d}\ell \frac{2N_i}{\ell} n(\ell, t) \Delta_i$$

smaller than horizon size

loop distribution function

f larger than fundamental frequency (2/l)

parameters: $G\mu, N_c = 1, N_k, N_{kk} \approx N_k^2/4$

2. Individual burst, if strong enough, can also be detected (burst search).

Loop Distribution Function

3 models considered

- Model A: (Blanco-Pillado et al., PRD 89,023512) (simulations)
- Model B: Lorentz et al., JCAP 10 (2010) 003 (analytical modelling, matched onto simulation result)
- Model C: Auclair et al., JCAP 06 (2019) 015 (interpolation between above 2)

C-1 (C-2) reproduces LDF of Model A (B) in the radiation era
and LDF of Model B (A) in the matter era

modelling loop distribution (F, dimensionless)

$$\frac{\partial}{\partial t}\left[a^3 F(l,t)\right] = a^3 \mathcal{P}(l,t) + a^3 \gamma_d \frac{\partial}{\partial t} F(l,t)$$

$a^3 \mathcal{P}(l,t)$: production from long strings

$a^3 \gamma_d \frac{\partial}{\partial t} F(l,t)$: decay due to GW radiation

Large Nc or Nk does not necessarily lead to large signal (loops decays faster)

$$\gamma_d = \Gamma_d G\mu$$

$$\Gamma_{\rm d} \equiv \frac{P_{\rm gw}}{G\mu^2} = \sum_i \frac{P_{{\rm gw},i}}{G\mu^2}$$

dimensionless decay constant

$$= N_{\rm c} \frac{3\pi^2 g_{1,{\rm c}}^2}{(2\delta)^{1/3} g_2^{2/3}} + N_{\rm k} \frac{3\pi^2 g_{1,{\rm k}}^2}{(2\delta)^{2/3} g_2^{1/3}} + N_{\rm kk} 2\pi^2 g_{1,{\rm kk}}^2$$

cusp (first term), kink (second term), kink-kink collision (third term)