Axions and Its test

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Outline

- Strong CP problem
- Axion solution to strong CP problem
- Axion Cosmology
- Axion Direct searches
- Axion Dark matter searches

Basic Transformation in the QFT

- C for charge conjugation
- P for parity: $\mathrm{P}: (t, \mathbf{x}) \mapsto (t, -\mathbf{x})$
- T for time reversal: $\mathrm{T}: (t, \mathbf{x}) \mapsto (-t, \mathbf{x})$

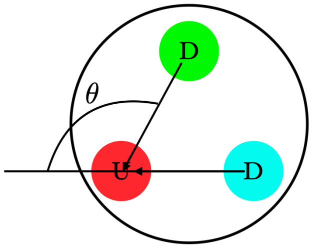
CPT Thoerem: the Lorentz invariance and unitarity requires that all the terms in the Lagrangian should be invariant under the simultanous application of C, P and T.

CP symmetry is already broken in the weak interactions. How about strong interaction?

Classical calculation of Neutron eDM

- The neutron eDM in classical formula

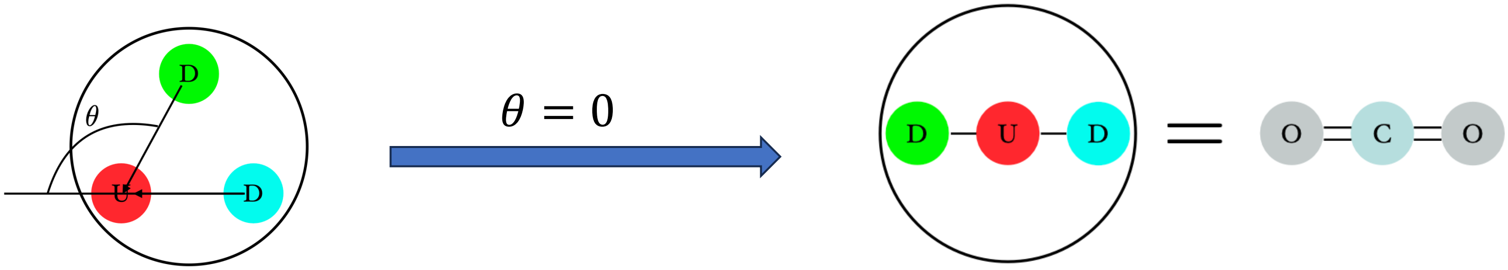
$$\vec{d} = \sum q\,\vec{r}$$



- Using the fact that the neutron has a size $r_n \sim 1/m_\pi$, the student would then arrive at the classical estimate that

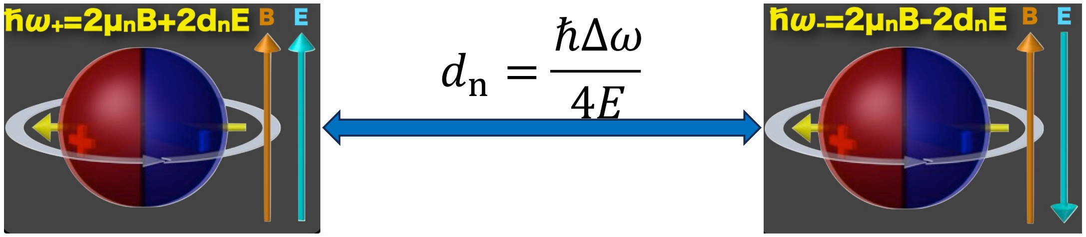
$$|d_n| \approx 10^{-13}\sqrt{1-\cos\theta}\; e\ \mathrm{cm}$$

- As a vector, the eDM will point in the same (opposite) direction as the spin



Measurement of Neutron eDM

- Precesion Measurement



$$d_{\mathrm{n}} = \frac{\hbar\Delta\omega}{4E}$$

- The current best measurement of the neutron eDM is

$$|d_n| \leq 10^{-26} e\ \mathrm{cm}$$

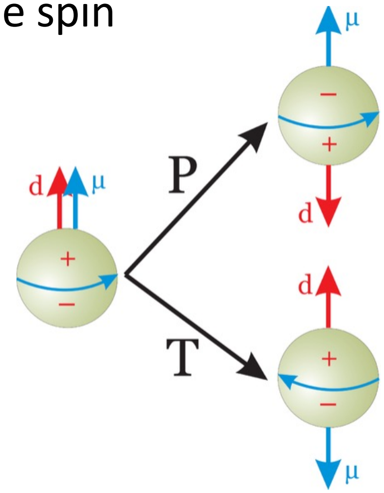
This result is much smaller than the theoretical prediction ?

Classical solution to Neutron eDM

$$\vec{d} = \sum q\,\vec{r} \qquad\qquad \vec{s} = \vec{r}\times\vec{p}$$

- Remember the eDM will point in the same (opposite) direction as the spin
- Solution 1:parity is good symmetry $P\colon\ \vec{d} \to -\vec{d},\ \vec{s} \to \vec{s}.$
- Solution 2:CP is good symmetry

$$T\!:\vec{d} \to \vec{d},\vec{s} \to -\vec{s}$$



- The only way for both $\hat{s} = \hat{d}$ and $\hat{s} = -\hat{d}$ to be true is if the dipole moment is zero.

But CP-violating phase in the CKM matrix is about π/3!

Strong CP Problem at Quantum level

- Low-energy QCD done incorrectly

$$\mathcal{L} \supset \frac{\theta g_s^2}{32\pi^2} G\tilde{G} + \mathrm{iq}^{+} D_\mu \gamma^\mu q + \mathrm{iq}^{c+} D_\mu \gamma^\mu q^c + qMq^c - \frac{1}{4} G^2 \qquad \tilde{G}^{\mu v} = \frac{1}{2} \varepsilon^{\mu v \rho \sigma} G_{\rho\sigma} \qquad M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$$

- SM contains these gauge groups:

	$SU(3)$	$SU(2)_L$	$SU(2)_R$	$U(1)_B$	$U(1)_A$
A_μ	adj				
q	$\square$	$\square$		1	1
q^c	$\overline{\square}$		$\overline{\square}$	-1	1
M		$\overline{\square}$	$\square$		-2

QCD confines Experiment result: $\langle q q^c \rangle \neq 0$

$SU(2)_L \times SU(2)_R \rightarrow SU(2)_D$,

and also breaks $U(1)_A$

- As with any spontaneous symmetry breaking, there will exist Goldstone bosons:

$$U = e^{i\frac{\Pi^a}{\sqrt{2} f\pi}\sigma^a}$$

	$SU(2)_L$	$SU(2)_R$	$U(1)_B$	$U(1)_A$
U	$\square$	$\overline{\square}$		2

Strong CP Problem at Quantum level

- In all renormalizable operators consistent with symmetries with arbitrary coefficients, we consider the leading order

$$\mathcal{L} = f_\pi^2 \mathrm{Tr}\, \partial_\mu U \partial^\mu U^\dagger + a f_\pi^3 \mathrm{Tr}\, MU + h.c.$$

- The mass matrix can be obtained via above Lagrangian

$$\mathrm{V} = a f_\pi (m_u + m_d) \pi^+ \pi^- + \frac{a f_\pi}{2} \begin{pmatrix} \pi^0 & \eta' \end{pmatrix} \begin{pmatrix} m_u + m_d & m_u - m_d \\ m_u - m_d & m_u + m_d \end{pmatrix} \begin{pmatrix} \pi^0 \\ \eta' \end{pmatrix} \qquad \pi^\pm = \frac{\pi^1 \pm i\pi^2}{\sqrt{2}}$$

- The mass relationship:

$$2 m_{\pi^+} = m_{\pi^0} + m_{\eta'}$$

- The experiment result:

$m_{\pi^+} \approx m_{\pi^0} \approx 140\mathrm{MeV}$ while $m_{\eta'} \approx 960\mathrm{MeV}$

Something went Wrong?

Anomalous symmetries

- If one rotates the quarks by

$$u \rightarrow e^{i\alpha} u, u^c \rightarrow e^{i\alpha} u^c$$

- The Lagrangian also changes as

$$\mathcal{L} \rightarrow \mathcal{L} + \alpha \frac{g^2}{16\pi^2} G\tilde{G} \qquad \mathcal{L} \supset \theta \frac{g^2}{32\pi^2} G\tilde{G}$$

- Thus, $U(1)_A$ symmetry is actually not a good symmetry of the theory. For 2-flavor QCD, the proper anomalous symmetry is

$$u \rightarrow e^{i\alpha} u,\ d \rightarrow e^{i\alpha} d,\ \theta \rightarrow \theta - 2\alpha$$

- θ is not normal spurion as M because that θ realizes the symmetry non-linearly, So we need to let θ appear in Lagrangian as $e^{i\theta}$

Because of θ , transforms under this symmetry, the corresponding pseudo-Goldstone boson, η′, obtains a mass to solve the previous problem !

The theory of pions and neutrons

- Recall the anomalous symmetry

$$U \to e^{i\alpha}U,\ \theta \to \theta - 2\alpha,\ M \to e^{-i\alpha}M$$

- Written in terms of the η′ boson, the good symmetry of the theory :

$$\eta' \to \eta' + \sqrt{2}\alpha f_{\eta'}, \theta \to \theta - 2\alpha, M \to e^{-i\alpha}M$$

- The effective Lagrangian, which is invariant under $SU(2)_L \times SU(2)_R \times U(1)_B$ but not invariant under $U(1)_A$

$$\mathcal{L} = f_\pi^2 \mathrm{Tr}\partial_\mu U \partial^\mu U^\dagger + a f_\pi^3 \mathrm{Tr}MU + b f_\pi^4 \mathrm{det}U + h.c.$$

- The phase of the complex coefficient *b* is fixed to be $b = |b|e^{i\theta}$. The mass of η′ boson is

$$\mathcal{L} = \frac{1}{2} m_{\eta'}^2 \left(\eta' - \frac{\theta f_{\eta'}}{\sqrt{2}}\right)^2 + \cdots$$

Plugging this into the matrix *U*

→

$$\mathrm{U} = e^{i\frac{\theta}{2}} e^{i\frac{\pi^a}{\sqrt{2}f_\pi}\sigma^a}$$

The theory of pions and neutrons

- The first step is to find the vacuum about which to expand.

$$\langle \pi^0 \rangle = \phi\sqrt{2} f_\pi \longrightarrow U = \begin{pmatrix} e^{i\phi+i\theta} & 0 \\ 0 & e^{-i\phi+i\theta} \end{pmatrix}$$

- The potential comes from the term

$$V = -a f_\pi^3 \mathrm{Tr}\left(\begin{pmatrix} m_u e^{i\theta_u} & 0 \\ 0 & m_d e^{i\theta_d} \end{pmatrix} U \right) + \text{h.c.} = -2a f_\pi^3 \left[m_u \cos\left(\phi + \frac{\bar{\theta}}{2}\right) + m_d \cos\left(\phi - \frac{\bar{\theta}}{2}\right) \right]$$

$\bar{\theta} = \theta + \theta_u + \theta_d$. (only physical quantity)

- The minimum of this potential can be found to be

$$\tan\phi = \frac{m_u - m_d}{m_u + m_d} \tan\frac{\bar{\theta}}{2}, \quad V = -m_\pi^2 f_\pi^2 \sqrt{1 + \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2\frac{\bar{\theta}}{2}}$$

- The pion masses are

$$m_{\pi^0}^2 = a f_\pi \sqrt{m_u^2 + m_d^2 + 2 m_u m_d \cos\bar{\theta}}\,, \quad m_{\pi^\pm}^2 = a f_\pi (m_u + m_d)$$

It is an observed fact that $m_{\pi^+} \approx m_{\pi^0}$, giving the first indication that $\bar{\theta} \approx 0$.

The theory of neutrons

- We can thus construct a nucleon field *N*

$$N = qqq = \binom{p}{n}$$

- Write down all of the leading-order terms with arbitrary coefficients

$$\mathcal{L} = -m_N N U^\dagger N^c - c_1 N M N^c - c_2 N U^\dagger M^\dagger U^\dagger N^c - \frac{i}{2}(g_A - 1)\left[N^\dagger \sigma^\mu U \partial_\mu U^\dagger N + N^{c,\dagger} \sigma^\mu U^\dagger \partial_\mu U N^c\right]$$

- Expanding these terms out to leading order in pions, the leading CP related terms are

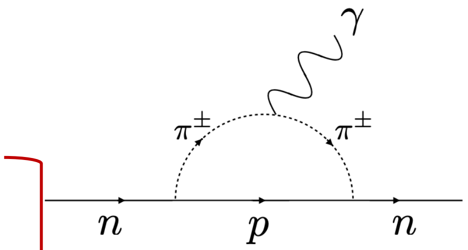
$$\mathcal{L} = -\bar{\theta}\frac{c_+\mu}{f_\pi}\pi^a N \tau^a N^c - i\frac{g_A m_N}{f_\pi}\pi^a N \tau^a N^c, \; \mu = \frac{m_u m_d}{m_u + m_d}$$

CP Violating **CP Conserving**

In Weyl notation, CP difference is that coupling is real or not. In Dirac notation, the difference will be γ^5

Experiments tell us: $c_+ \approx 1.7$, $g_A \approx 1.27$

Neutrons eDM



- The matrix element of the Feynman diagram is (Dirac Notation)

$$iM \approx \frac{e\bar{\theta} g_A c_+ \mu \log\frac{\Lambda^2}{m_\pi^2}}{4\pi^2 f_\pi^2} \varepsilon_\mu^*(q)\bar{u}(p')\gamma^{\mu v} q_v i\gamma_5 u(p)$$

- Let us now pretend that the neutron has an eDM in the Lagrangian

$$\mathcal{L} \supset d_n F_{\mu v} \bar{n} \gamma^{\mu v} i\gamma_5 n$$

- This would correspond to a diagram with the matrix element

$$iM = 2 d_n \varepsilon_\mu^*(q)\bar{u}(p')\gamma^{\mu v} q_v i\gamma_5 u(p)$$

⇒ $$d_n = \frac{e\bar{\theta} g_A c_+ \mu}{8\pi^2 f_\pi^2} \log\frac{\Lambda^2}{m_\pi^2} \sim 3\times 10^{-16}\bar{\theta}\ \mathrm{e\ cm} \qquad \Lambda = 4\pi f_\pi$$

- Comparing d_n to the bounds on the neutron eDM gives $\bar{\theta} \lesssim 10^{-10}$

The θ Vacua

- If we start with the Lagrangian $\mathcal{L} \supset -\frac{1}{4}G^2 + \frac{\theta g_s^2}{32\pi^2}G\tilde{G}$and calculate the Hamiltonian $\mathcal{H}$, we find that it is independent of θ.
- A given state evolves with time can be calculated with $e^{iHt}|x_i\rangle$ also independent of θ. This is the first puzzle
- We also can the physical quantities use the principle of lease action:

$$\langle x_f|e^{iHt}|x_i\rangle = \int_{x_i}^{x_f} d[x]e^{iS}$$

- This can be done as following relationship:

$$G\tilde{G} = \partial_\mu K^\mu,\ K^\mu = \epsilon^{\mu\nu\rho\sigma}A_\nu^a\left[F_{\rho\sigma}^a - \frac{g}{3}f^{abc}A_\rho^b A_\sigma^c\right].$$

- What is of interest is the action, which is the integral over all space of the Lagrangian

$$S = \int d^4x\mathcal{L} \supset \int d^4x\frac{\theta g_s^2}{32\pi^2}G\tilde{G} = \int d^3x\frac{\theta g_s^2}{32\pi^2}K^{\hat{r}}\Big|_{r\to\infty}$$

Thus, if K vanishes faster than $1/r^3$ at infinity, then this quantity will integrate to zero and θ cannot have any physical effect. This is the second puzzle.

The θ Vacua

The key to resolving the two puzzles: Instantons and the vacuum structure of QCD

1. **Answer to the first puzzle:**
   θ appears in the Hamiltonian formalism as a **super-selection rule**. So, although it does not explicitly appear in the Hamiltonian expression, it determines the choice of physical states.
2. **Answer to the second puzzle:**
   There exist certain finite-energy field configurations (instantons) for which $K \sim 1/r^3$, making the surface term at infinity non-zero. As a result, the θ term indeed has an effect on physical quantities.

The $\boldsymbol{\theta}$ Vacua

- We want a system with finite energy, so as r → ∞, we need the gauge field to become pure gauge so that the E and B fields vanish :

$$r \to \infty,\ A_\mu \to U\partial_\mu U^\dagger$$

- Means mappings between the gauge group SU(3), and the sphere at infinity S3 are equivalent

$$\phi_1 \to n\phi_2$$

- With n is the winding number, which of any given gauge configuration can be determined by

$$\int d^4x \frac{1}{32\pi^2} G\tilde{G} = n_1 - n_2$$

- Now we will find the eigenstates of the Hamiltonian, with the vacuum is the lowest energy eigenstate. The matrix that we are diagonalizing is called a circulant matrix and is of the form

$$\begin{pmatrix} 1 & 2 & 3 & \cdots & D-1 \end{pmatrix} \cdot \begin{pmatrix} E & \epsilon_1 & \epsilon_2 & \cdots & \epsilon_{D-1} \\ \epsilon_{D-1} & E & \epsilon_1 & \epsilon_2 & \cdots \\ . & . & . & . & . \\ \epsilon_1 & & \epsilon_2 & \cdots & \epsilon_{D-1} \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \\ \cdots \\ D-1 \end{pmatrix}$$

The θ Vacua

- The eigenvectors are

$$\left(1, w_j\,, w_j^2, \cdots w_j^{D-1}\right),\ w_j = e^{2\pi i j/D}$$

- The eigenstates of the Hamiltonian for $D \rightarrow \infty$, are

$$|\theta\rangle = N \sum e^{i\theta n} |n\rangle$$

- θ is what is called a super-selection rule. Because it is impossible to transition from one value of θ to another. We note that

$$\begin{aligned} \langle\theta|\mathcal{O}|\theta\rangle \quad &= \sum\nolimits_{m,n} e^{i\theta(m-n)} \langle m|\mathcal{O}|n\rangle = \sum\nolimits_{\Delta,n} e^{i\theta\Delta} \langle n+\Delta|\mathcal{O}|n\rangle = \sum\nolimits_{\Delta,n} e^{i\theta \int d^4x \frac{1}{32\pi^2} G\tilde{G}} \langle n+\Delta|\mathcal{O}|n\rangle \\ &= \sum\nolimits_{\Delta} \int dA e^{i\int d^4x \mathcal{L} + \frac{\theta}{32\pi^2} G\tilde{G}} \delta\left(\Delta - \int d^4x \frac{1}{32\pi^2} G\tilde{G}\right) = \int dA e^{i\int d^4x \mathcal{L} + \frac{\theta}{32\pi^2} G\tilde{G}} \end{aligned}$$

Hence choosing the θ vacua in the Hamiltonian is completely equivalent to having the θ term in the Lagrangian.

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- Axion solution to strong CP problem
- Axion Cosmology
- Axion Direct searches
- Axion Dark matter searches

Solution to strong CP problem——massless quark

$$\mathcal{L} \supset m_u e^{i\theta_u} u u^c + m_d e^{i\theta_d} d d^c + \frac{\theta g_s^2}{32\pi^2} G\tilde{G}$$

- The only physical quantity is $\bar{\theta} = \theta + \theta_u + \theta_d$

anomalous symmetries:

$$u \to e^{i\alpha} u,\ \theta_u \to \theta_u - \alpha,\ \theta \to \theta + \alpha$$

$$d \to e^{i\alpha} d,\ \theta_d \to \theta_d - \alpha,\ \theta \to \theta + \alpha$$

Up quark is massless

New anomalous symmetries:

$$u \to e^{i\alpha} u, \theta \to \theta + \alpha$$

$$d \to e^{i\alpha} d,\ \theta_d \to \theta_d - \alpha,\ \theta \to \theta + \alpha$$

No $\bar{\theta}$, So CP symmetry,
no neutron eDM,
Strong CP problem Solved

An alternate way of the massless up quark solution

$$m_u < 10^{-10} m_d$$

Experiment ruled out (Lattice)

Other solutions, such as P or CP is good symmetry to forbide neutron eDM, is not good as CKM is observed to be large.

Solution to strong CP problem

RG Running:

- If $\bar{\theta}$ is set to 0 at some high scale and the EFT to low energies is just the SM, then $\bar{\theta}$ will still be very small at low energies, thus it can be effectively ignored.
- It must also respect the $SU(3)_Q \times SU(3)_{u^c} \times SU(3)_{d^c}$ flavor symmetries of the SM. Thus,

$$\beta_{\bar{\theta}} = g^2 \arg\mathrm{Tr} Y_u^4 Y_d^4 Y_u^2 Y_d^2$$

- It is important to note that θ is a topological parameter that does not appear in perturbation theory. Instead, it the phase of the quark masses that evolves with RG.

Solution to strong CP problem—Parity

- If parity were a good symmetry of nature, then the neutron eDM would be zero.

$$P\colon SU(2)_L \leftrightarrow SU(2)_R,\ Q_L \leftrightarrow Q_R^\dagger,\ H_L \leftrightarrow H_R^\dagger,\ L_L \leftrightarrow L_R^\dagger,$$

- The θ term is P and CP odd and is forbidden by parity, while the Yukawa are of the form

$$\mathcal{L} \supset \frac{y_u Q_L H_L Q_R H_R}{\Lambda_u} + \frac{y_d Q_L H_L^\dagger Q_R H_R^\dagger}{\Lambda_d} + \text{h.c.}$$

- Under parity, the Yukawa matrices obey

$$Y_u = \frac{y_u v_R}{\Lambda_u} = Y_u^\dagger,\ Y_d = \frac{y_d v_R}{\Lambda_d} = Y_d^\dagger$$

- Hermitian matrices obey arg det Y = 0, but their individual elements can be complex.

$$\bar{\theta} = \theta + \arg\det Y_u + \arg\det Y_d = 0$$

All of the nice properties of the parity solution are preserved with this type of breaking and the Strong CP problem is solved. Thus the simplest parity solution to the Strong CP problem is on par with the axion in its simplicity.

Solution to strong CP problem—CP

- Solutions to the Strong CP problem that utilize CP symmetry are typically called Nelson-Barr models. The tree-level Lagrangian under consideration is

$$\mathcal{L} = \mu q q^c + Y^{ij} H Q_i d_j^c + A^{ia} \eta^a d_i^c q$$

- The 4 X4 mass matrix for the quarks is then

$$M = \begin{pmatrix} \mu & A\eta \\ 0 & m_d \end{pmatrix}$$

- where $m_d = Yv$ is the 3×3 down quark mass matrix. It is simple to check that at tree level arg det $M = 0$, while the CKM phase is non-zero and large if $\mu \lesssim A\eta$. By fiddling with the size of various Yukawa couplings, loop-level corrections to $\bar{\theta}$ can be made small.

Even more so than parity-based solutions, CP-based solutions are very fragile, as many coincidences of scales are needed for the CKM angle to be large.

Solution to strong CP problem——QCD Axion

Axion is typically considered to be the simplest solution to the Strong CP problem, though the minimal parity-based solution gives the axion EFT a run for its money.

- The EFT consists of a single new particle, the axion (a), and a single new coupling (f_a)

$$\mathcal{L} \supset \left(\frac{a}{f_a} + \theta\right)\frac{1}{32\pi^2} G\tilde{G}$$

- The axion obeys an anomalous symmetry

$$a \to a + \alpha f_a,\ \theta \to \theta - \alpha$$

- every non-derivative interaction of the axion can be obtained by observing that wherever we have a coupling θ, we can replace it with $\theta + a/f_a$.

- UV completions of the QCD axion will occasionally generate other couplings, (To quark coupling can be generated by RG evolution)

$$\mathcal{L} \supset \frac{a}{f_B}\frac{1}{32\pi^2} B\tilde{B} + \frac{a}{f_W}\frac{1}{32\pi^2} W\tilde{W} + \frac{\partial_\mu a}{f_Q} Q^\dagger \sigma^\mu Q$$

QCD Axion Solution to strong CP problem

- Using our trick from before, we find that the axion potential is

$$V = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2 \frac{\bar{\theta}}{2}} \quad \Longrightarrow \quad \mathrm{V} = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4 m_u m_d}{(m_u + m_d)^2} \sin^2 \left(\frac{a}{2 f_a} + \frac{\bar{\theta}}{2} \right)}$$

- To solve strong CP problem

$$d_n = \frac{e \bar{\theta} g_A c_+ \mu}{8 \pi^2 f_\pi^2} \log \frac{\Lambda^2}{m_\pi^2} \sim 3 \times 10^{-16} \bar{\theta} \text{ e cm} \quad \Longrightarrow \quad d_n \propto \frac{a}{f_a} + \bar{\theta} = 0$$

- QCD axion mass relationship, expand V at the minimal potential, with small a/fa

$$\mathrm{V} = -m_\pi^2 f_\pi^2 + \frac{1}{2} \frac{m_u m_d}{(m_u + m_d)^2} \frac{m_\pi^2 f_\pi^2}{f_a^2} a^2 \quad \Longrightarrow \quad m_a = \frac{m_\pi f_\pi}{f_a} \frac{\sqrt{m_u m_d}}{m_u + m_d} \simeq 5.7 \left(\frac{10^{12} \mathrm{GeV}}{f_a} \right) \mu \mathrm{eV}$$

Axion Quality Problem

- There is a very well-defined anomalous symmetry associated with the axion coupling, there is no symmetry associated with it. This lack of proper symmetry properties generates two issues that together are called the axion quality problem:
  1. Because the axion has no symmetry properties, thus the axion also has a host of other couplings. Thus we need UV complete model.
  2. Even if one imposes the anomalous symmetry, gravitational effects will break it and the axion will obtain a separate mass term that is not centered around a zero neutron eDM, which reintroduces the problem.

UV completion model of QCD Axion

- A complex scalar Φ with an approximate $U(1)_{PQ}$ symmetry, a is pseudo-Goldstone boson

$$\Phi = (f_a + r)e^{ia/f_a}$$

- The Lagrangian related to Φ and q is

$$V = -m^2\Phi\Phi^\dagger + \lambda\left(\Phi\Phi^\dagger\right)^2 + y\Phi q q^c + \text{h.c.}$$

- After we redefine $q' = qe^{ia/f_a}$, the axion will appear in the $G\tilde{G}$ coupling due to the anomaly

QP 1) The couplings $\varepsilon q q^c$ and $\varepsilon^2\Phi^2$will break the anomalous $U(1)_{PQ}$ symmetry. This generate

$V \sim \varepsilon^2 f_a^2 \cos\left(\frac{a}{f_a} + \phi\right)$ requires it small thus, $\varepsilon \lesssim 10^{-19}\text{GeV}$

QP2) Gravity breaks this anomalous symmetry, which induces higher dimensional operators

$V \sim \frac{\Phi^n}{M_p^{n-4}} \sim \frac{f_a^n}{M_p^{n-4}} \cos\left(\frac{a}{f_a} + \phi_n\right)$ requires $n \gtrsim 14$

Solving the Axion Quality Problem

- More plausibly, the accidental U(1$_{)\mathrm{PQ}}$ symmetry could result from chiral gauge theories in much the same way that U(1)$_{\mathrm{B-L}}$ is an accidental symmetry of the renormalizable SM.

	$SU(3)_c$	$SU(N)$	$SU(M)$	$U(1)_{PQ}$
Q_1	$\square$	$\square$	$\square$	1
$3 \times Q_2$		$\square$	$\square$	-1
$M \times Q_3$	$\overline{\square}$	$\overline{\square}$		1
$3M \times Q_4$		$\overline{\square}$		-1

- The lowest-dimensional operator we can write down that violates this accidental $U(1)_{PQ}$ symmetry is

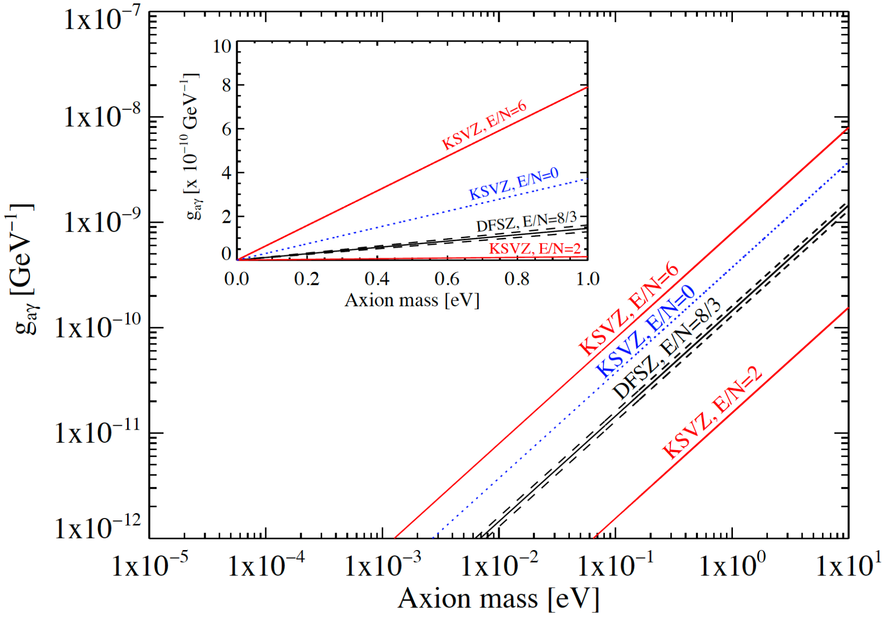
$$\mathcal{L} \supset \frac{Q_2^M Q_4^M}{M_p^{3M-4}} \sim \frac{f_a^{3M}}{M_p^{3M-4}} \cos\left(\frac{a}{f_a} + \phi\right)$$

- By taking the gauge group M ≥ 5, we can suppress higher-dimensional operators enough that the axion still solves the Strong CP problem .

Axion-full theory

- Two classes of "invisible axion" models:
  1. KSVZ: New scalar doublet + heavy quarks
  2. DFSZ: New two scalar doublet model

- Useful benchmarks:



Variations of QCD Axion- Couplings

- Recall the QCD axion Lagrangian

$$\mathcal{L} \supset \left(\frac{a}{f_a} + \theta\right)\frac{1}{32\pi^2} G\tilde{G} + \frac{a}{f_B}\frac{1}{32\pi^2} B\tilde{B} + \frac{a}{f_W}\frac{1}{32\pi^2} W\widetilde{W} + \frac{\partial_\mu a}{f_Q} Q^\dagger \sigma^\mu Q$$

$$= \left(\frac{a}{f_a} + \theta\right)\frac{1}{32\pi^2} G\tilde{G} + \frac{\partial_\mu a}{f_Q} Q^\dagger \sigma^\mu Q + \frac{g_{a\gamma\gamma} a}{4} F\tilde{F}$$

- $g_{a\gamma\gamma}$ can be much larger than$1/f_a$

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi}\left[\frac{1}{f_{\mathrm{UV}}} - \frac{1.92(4)}{f_a}\right]$$

- f_Q can be within a loop factor of f_a because of RG evolution. But it also can be larger

In principle, $g_{a\gamma\gamma}$ and f_Q can be extra free paramters as f_a

Variations of QCD Axion- Mass

- Introduce a new confining gauge group to which the axion couples

$$\left(\frac{a}{f_a}+\theta\right)\frac{g_s^2}{32\pi^2}G\tilde{G}+\left(\frac{a}{f_a}+\theta'\right)\frac{g_s'^2}{32\pi^2}G'\tilde{G}'$$

$\bar{\theta} \approx \bar{\theta}'$ up to 10^{-10} to solve the Strong CP problem. Z_2symmetry which keep these two angles same. After this symmetry breaking, axion get higher mass.

- The axion non-linearly realizes the Z_N symmetry and interacts with N copies of QCD

$$\mathcal{L} = \sum_k \left(\frac{a}{f}+\frac{2\pi k}{N}+\theta\right)G_k\tilde{G}_k \quad \Longrightarrow \quad \frac{m_a(N)}{m_a(N=1)} \sim \frac{4}{2^{N/2}}$$

makes the axion mass exponentially lighter

In principle, m_a can be free paramters as f_a

Axion Effective Lagrangian in QCD

- Axion effective Lagrangian for 2-flavor QCD

$$\mathcal{L}_a = \frac{1}{2}\left(\partial_\mu a\right)^2 + \frac{a}{f_a}\frac{g_s^2}{32\pi^2} G\tilde{G} + \frac{1}{4} g^0_{a\gamma} aF\tilde{F} + \frac{\partial_\mu a}{2f_a} c_q^0 \bar{q}\gamma^\mu\gamma_5 q - \bar{q}_L M_q q_R + \text{h.c.}\,.$$

- We could use field transformation to solve the strong CP provlem:

$$q \to e^{i\gamma_5 \frac{a}{2f_a} Q_a} q \quad \Longrightarrow \quad -g_s^2 \frac{\text{Tr}\, Q_a}{32\pi^2}\frac{a}{f_a} G\tilde{G}$$

- Then the Lagrangian becomes

$$\mathcal{L}_a = \frac{1}{2}\left(\partial_\mu a\right)^2 + \frac{1}{4} g_{a\gamma} aF\tilde{F} + \frac{\partial_\mu a}{2f_a} \bar{q} c_q \gamma^\mu\gamma_5 q - \bar{q}_L M_a q_R + \text{h.c.}$$

$$g_{a\gamma} = g^0_{a\gamma} - (2N_c)\frac{\alpha}{2\pi f_a}\text{Tr}(Q_a Q^2) \quad \xrightarrow[Q_a = M_q^{-1}/\text{Tr}M_q^{-1}]{\qquad\qquad} \quad g_{a\gamma} = g^0_{a\gamma} - \frac{\alpha}{2\pi f_a}\left(\frac{2}{3}\frac{4m_d + m_u}{m_u + m_d}\right)$$

$$c_q = c_q^0 - Q_a$$

$$M_a = e^{i\frac{a}{2f_a}Q_a} M_q e^{i\frac{a}{2f_a}Q_a},$$

Axion Effective Lagrangian to Pions

- Axion chiral Lagrangian for poins and axions

Only include iso-triplet contributions

$$\mathcal{L}_a^{\chi\mathrm{PT}} = \frac{f_\pi^2}{4}\left[\mathrm{Tr}\left((D^\mu U)^\dagger D^\mu U\right) + 2B_0\mathrm{Tr}\left(UM_a^\dagger + M_a U^\dagger\right)\right] + \frac{\partial^\mu a}{2f_a}\frac{\mathrm{i}}{4} f_\pi^2 \mathrm{Tr}\left[c_q\sigma^a\right]\mathrm{Tr}\left[\sigma^a\left(UD_\mu U^\dagger - U^\dagger D_\mu U\right)\right]$$

- Expanding the iso-triplet current terms:

$$\frac{\partial_\mu a}{2f_a}\frac{1}{2}\mathrm{Tr}\left[c_q\sigma^a\right]J_a^\mu \simeq -\frac{1}{2}\left(\frac{m_d-m_u}{m_u+m_d} + c_d^0 - c_u^0\right)\frac{f_\pi}{f_a}\,\partial_\mu a\partial^\mu\pi^0$$
$$+\frac{1}{3}\left(\frac{m_d-m_u}{m_u+m_d} + c_d^0 - c_u^0\right)\frac{1}{f_a f_\pi}\,\partial_\mu a(2\partial^\mu\pi^0\pi^+\pi^- - \pi_0\partial^\mu\pi^+\pi^- - \pi_0\pi^+\partial^\mu\pi^-)$$

- axion-pion coupling

$$C_{a\pi} = -\frac{1}{3}\left(c_u^0 - c_d^0 - \frac{m_d-m_u}{m_u+m_d}\right)$$

Axion Effective Lagrangian to nucleon

- The LO effective axion-nucleon Lagrangian reads

$$\mathcal{L}_N = \bar{N} v^\mu \partial_\mu N + 2 g_A \frac{c_u - c_d}{2} \frac{\partial_\mu a}{2 f_a} (\bar{p} S^\mu p - \bar{n} S^\mu n) + 2 g_0^{ud} \frac{c_u + c_d}{2} \frac{\partial_\mu a}{2 f_a} (\bar{p} S^\mu p + \bar{n} S^\mu n) + \cdots ,$$

- Above Lagrangian can be simplified based on $g_A = \Delta u - \Delta d$ and $g_0^{ud} = \Delta u + \Delta d$

$$\mathcal{L}_N \supset \frac{\partial_\mu a}{2 f_a} \left\{ \frac{c_u - c_d}{2} (\Delta u - \Delta d)(\bar{p} \gamma^\mu \gamma_5 p - \bar{n} \gamma^\mu \gamma_5 n) + \frac{c_u + c_d}{2} (\Delta u + \Delta d)(\bar{p} \gamma^\mu \gamma_5 p + \bar{n} \gamma^\mu \gamma_5 n) \right\}$$
$$= \frac{\partial_\mu a}{2 f_a} \bar{N} C_{aN} \gamma^\mu \gamma_5 N$$

$$C_{ap} = -\left(\frac{m_d}{m_u + m_d} \Delta u + \frac{m_u}{m_u + m_d} \Delta d \right) + c_u^0 \Delta u + c_d^0 \Delta d,$$
$$C_{an} = -\left(\frac{m_u}{m_u + m_d} \Delta u + \frac{m_d}{m_u + m_d} \Delta d \right) + c_d^0 \Delta u + c_u^0 \Delta d$$

- where $\Delta u = 0.897(27), \Delta d = -0.376(27)$ and $m_u^{\overline{\mathrm{MS}}}(2\mathrm{GeV})/m_d^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) = 0.48(3)$

Axion Effective Lagrangian to Electron

- The LO effective axion-electron Lagrangian reads

$$\mathcal{L}_e = C_{ae}\frac{\partial_\mu a}{2f_a}\,\bar{e}\gamma^\mu\gamma_5 e \qquad \text{with.} \quad C_{ae} = c_e^0 + \delta c_e$$

- The relevant one-loop diagram is logarithmically divergent, and can be understood as an RGE effect on the C_{ae} coefficient from the PQ scale down to the IR scale μ_{IR}

$$\delta c_e = \frac{3\alpha^2}{4\pi^2}\left[\frac{E}{N}\log\left(\frac{f_a}{\mu_{\mathrm{IR}}}\right) - \frac{2}{3}\frac{4m_d + m_u}{m_u + m_d}\log\left(\frac{\Lambda_\chi}{\mu_{\mathrm{IR}}}\right)\right]$$

- Where E/N is related to $g_{a\gamma}^0$, tipically, $\Lambda_\chi \simeq 1\mathrm{GeV}$, $\mu_{\mathrm{IR}} = m_e$

Summary of (QCD) Axion

- The relation between the axion mass and the axion decay constant

$$m_a = 5.691(51)\left(\frac{10^{12}\mathrm{GeV}}{f_a}\right)\mu\mathrm{eV}$$

- The axion interaction Lagrangian

$$\mathcal{L}_a^{\mathrm{int}} \supset \frac{\alpha}{8\pi}\frac{C_{a\gamma}}{f_a}aF\tilde{F} + C_{af}\frac{\partial_\mu a}{2f_a}\bar{f}\gamma^\mu\gamma_5 f + \frac{C_{a\pi}}{f_a f_\pi}\partial_\mu a[\partial\pi\pi\pi]^\mu$$

$$C_{a\gamma} = \frac{E}{N} - 1.92(4)$$

$$C_{ap} = -0.47(3) + 0.88(3)c_u^0 - 0.39(2)c_d^0 - C_{a,\,\mathrm{sea}}\,,$$

$$C_{an} = -0.02(3) + 0.88(3)c_d^0 - 0.39(2)c_u^0 - C_{a,\,\mathrm{sea}}\,,$$

$$C_{a,\,\mathrm{sea}} = 0.038(5)c_s^0 + 0.012(5)c_c^0 + 0.009(2)c_b^0 + 0.0035(4)c_t^0,$$

$$C_{ae} = c_e^0 + \frac{3\alpha^2}{4\pi^2}\left[\frac{E}{N}\log\left(\frac{f_a}{m_e}\right) - 1.92(4)\log\left(\frac{\mathrm{GeV}}{m_e}\right)\right],$$

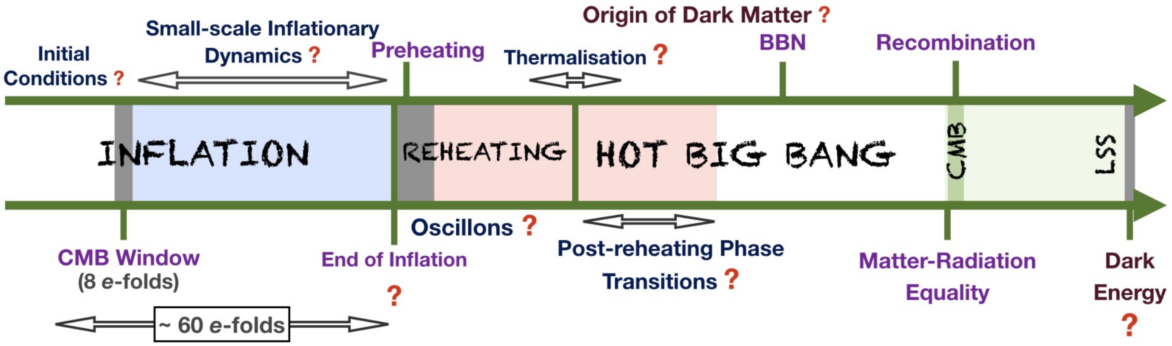
$$C_{a\pi} = 0.12(1) + \frac{1}{3}\left(c_d^0 - c_u^0\right),$$

Question: Could axion has CP violation couplings to fermions, such as $a\bar{f}f$?

Outline

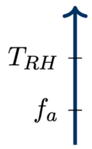
- Strong CP problem
- Axion solution to strong CP problem
- Axion Cosmology
- Axion Direct searches
- Axion Dark matter searches

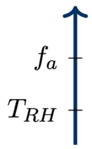
Axion Cosmology



- Radiation Doninant:$R \propto t^{1/2}$
- Matter Dominant:$R \propto t^{2/3}$
- Vacuum Energy: $R \propto e^{Ht}$

- Reheating temperature of universe?
- All we know is $T_{RH} > \text{MeV}$
- Axion physics depends on if $T_{RH} > f_a$ or $T_{RH} < f_a$





-PQ restored at high T

-PQ never restored

Axion Evolutin

- Consider scalar with mass, m_a
- Scalar action,

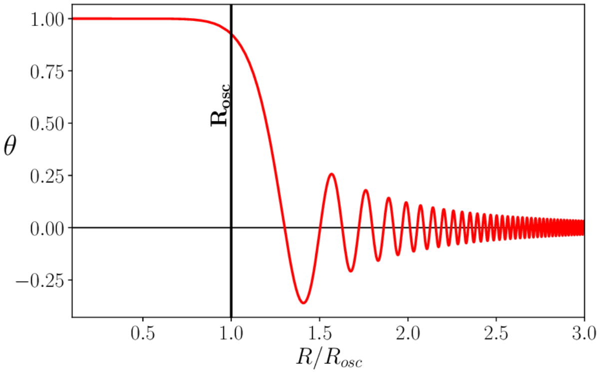
$$S \supset \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m_\phi^2\phi^2\right]$$

$$g^{\mu\nu} = \mathrm{diag}(1, -R, -R, -R)$$

- Euler-Lagrange equation of motion $\left(H = \dot{R}/R \sim T^2/M_{\mathrm{pl}}\right)$:

$$\ddot{a} + 3H\dot{a} + m_a^2(t)\mathrm{a} = 0$$

Axion Evolutin



- For $H \gg m_a \Rightarrow$ field is stuck:

$$\ddot{a} + 3H\dot{a} \simeq 0 \;\Rightarrow\; \mathrm{a} = a_0$$

- For $H = 0$: $\Rightarrow$ harmonic oscillator

$$a(t) = f_a\theta_0\cos(m_a t)$$

- For $H \propto t^{-1}$

$$\mathrm{a}(t) = f_a\theta_0(R/R_0)^{3/2}\cos(m_a t)$$

- The energy density at time of transition is:

$$\rho_0 \sim V(a_0) \sim m_a^2\theta_0^2 f_\mathrm{a}^2$$

- After field gets unstuck it undergoes (damped) oscillations

$$\rho \propto R^{-3}$$

Axion Relic Abundance

- The axion number density

$$n_a(T_R) = \frac{\rho_a(T_R)}{m_a} = n_a(T_R)\left(\frac{R(T_R)}{R(T)}\right)^3 \sim \theta_i^2 f_a^2 m_a \left(\frac{T_R}{\sqrt{m_a m_{\rm pl}}}\right)^3$$

- At T_R the entropy density is $s = \frac{2\pi^2}{45}\, g_{*S}(T_R) T_R^3$. The entropy density scales with the scale factor as $s \propto R^{-3}$

$$\left(\frac{R(T_R)}{R(T_0)}\right)^3 = \left(\frac{g_{*s}(T_0)}{g_{*s}(T_R)}\right)\left(\frac{T_0}{T_R}\right)^3$$

- where $T_0 < T_R$ is the temperature at a later epoch, for example today or matter radiation equality. The energy density at T_0 is thus

$$\rho_a(T_0) = \rho_a(T_R)\frac{g_{*s}(T_0)}{g_{*s}(T_R)}\left(\frac{T_0}{T_R}\right)^3 \sim \theta_i^2 \frac{f_a^2 m_a^2}{\left(m_a m_{\rm pl}\right)^{3/2}}\, T_0^3$$

Axion Relic Abundance

- Oscillate temperature:

$$3H(T_{\rm osc}) \approx m_a(T_{\rm osc}) \text{ , with} H(T_{\rm osc}) \sim T_{\rm osc}^2/m_{\rm pl}.$$

- Axion energy density at matter radiation equality

$$\rho_a(T_{\rm MR}) = \frac{m_a}{m_a(T_{\rm osc})}\rho_a(T_{\rm osc})\frac{g_{*s}(T_{\rm MR})}{g_{*s}(T_{\rm osc})}\left(\frac{T_{\rm MR}}{T_{\rm osc}}\right)^3$$

- With $\rho_a(T_{\rm osc}) \approx \frac{1}{2} m_a(T_{\rm osc})^2 f_a^2 \theta_i^2$, and the oscillation temperature is

$$T_{\rm osc} \sim \left(m_a m_{\rm pl} \Lambda^n\right)^{1/(n+2)} \sim {\rm GeV}\left(\frac{10^{12}{\rm GeV}}{f_a}\right)^{0.1}$$

- The Axion relic abandunce is(with $\rho_{\rm tot} = 3H_0^2 M_{\rm Pl}^2 = 8.07\times10^{-11} h^2 {\rm eV}^4$)

$$\Omega_a h^2 \sim 0.1\theta_i^2\left(\frac{f_a}{3\times10^{11}{\rm GeV}}\right)^{1+\frac{1}{n+2}} \sim 0.1\theta_i^2\left(\frac{f_a}{3\times10^{11}{\rm GeV}}\right)^{1.1}$$

Outline

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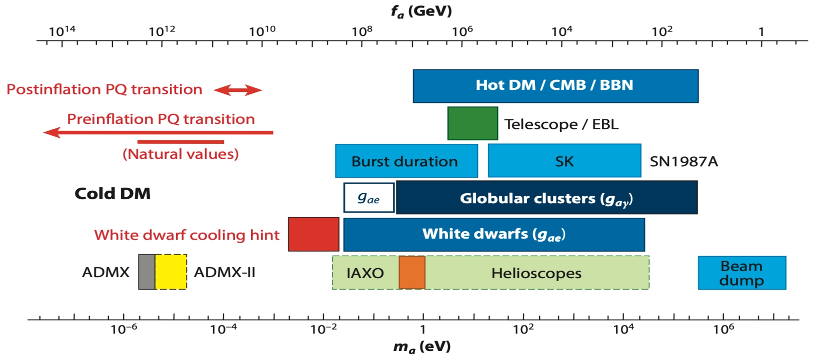
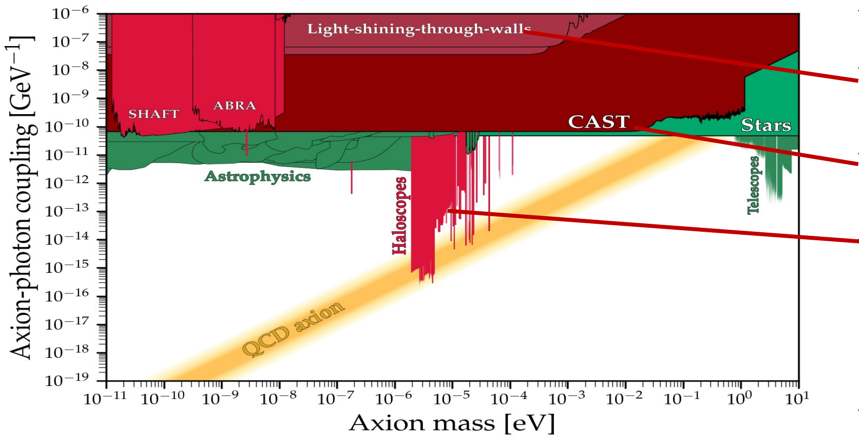
Axion search

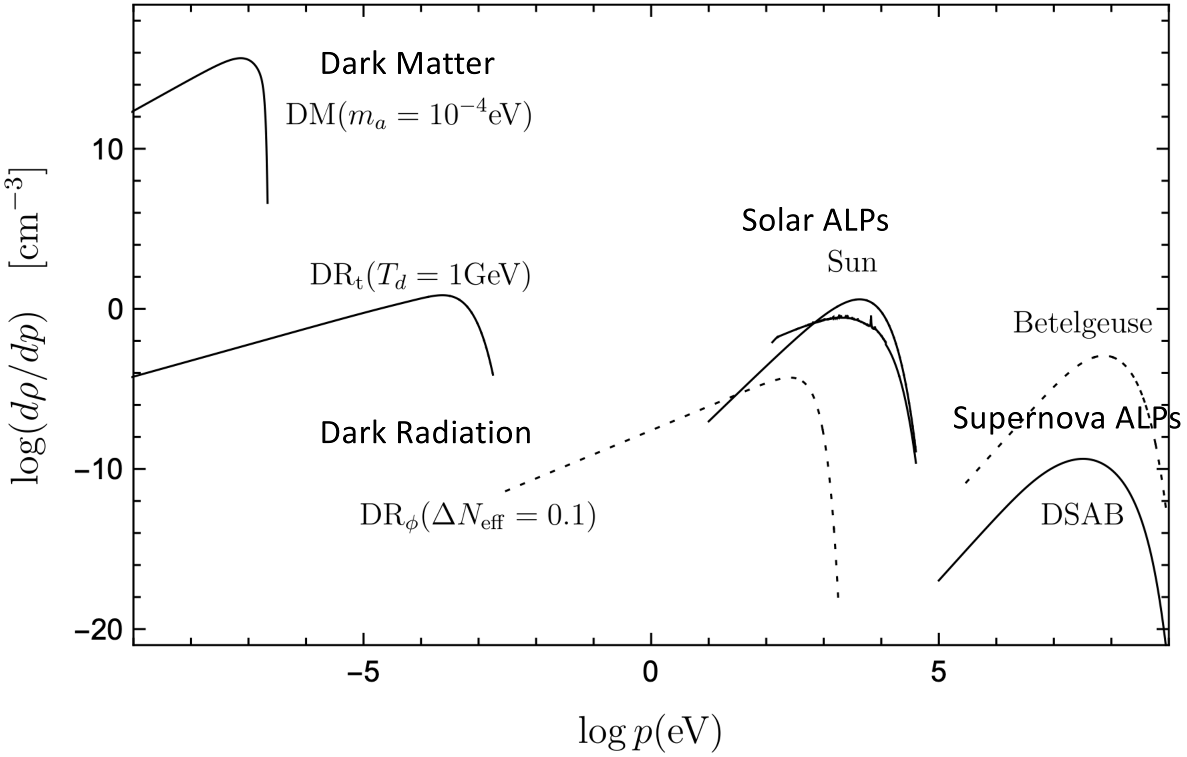


Limits on the axion established by cosmology and astrophysics.



Detection method	$g_{a\gamma}$	g_{ae}	g_{aN}	$g_{A\gamma n}$	$g_{a\gamma}g_{ae}$	$g_{a\gamma}g_{aN}$	$g_{ae}g_{aN}$	$g_N\bar{g}_N$	**Model dependency**
Light shining through wall	×								no
Polarization experiments	×								no
Spin-dependent 5th force			×				×	×	no
Helioscopes	×				×	×			Sun
Primakoff-Bragg in crystals	×				×				Sun
Underground ion. detectors	×	×	×			×	×		Sun*
Haloscopes	×								DM
Pick up coil & LC circuit	×								DM
Dish antenna & dielectric	×								DM
DM-induced EDM (NMR)			×	×					DM
Spin precession in cavity		×							DM
Atomic transitions		×	×						DM

Axion and ALP Nature source



Axion and ALP Production in Lab

- Because of the interaction between SM particles to ALPs:

$$\mathcal{L}_{\text{ALP}-\text{ int.}} = -\frac{g_{a\gamma}}{4} F_{\mu\nu}\tilde{F}^{\mu\nu} a - a \sum_{\psi} g_{a\psi} (i\bar{\psi}\gamma^5\psi) - a F_{\mu\nu} \sum_{\psi} \frac{g_{a\psi\gamma}}{2} (i\bar{\psi}\sigma^{\mu\nu}\gamma^5\psi) + \cdots$$

$$-\frac{\bar{g}_{a\gamma}}{4} F_{\mu\nu} F^{\mu\nu} a - a \sum \bar{g}_{a\psi} (\bar{\psi}\psi)$$

- Make use of the relationship

$$F_{\mu\nu}\tilde{F}^{\mu\nu} = -4\left(\vec{E}\cdot\vec{B}\right), j_{\psi}^{0} = \langle\bar{\psi}\psi\rangle, j_{\psi}^{5} = \left\langle i\bar{\psi}\gamma^5\psi\right\rangle$$

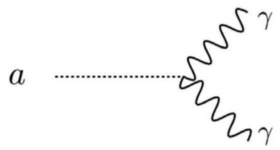
- The equations of motion for an axion/ALP field

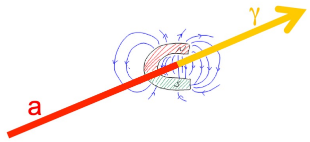
$$(\Box + m_a^2)a = g_{a\gamma}(\vec{E}\cdot\vec{B}) - \sum_{\psi}\left(g_{a\psi} j_{\psi}^{5} + \bar{g}_{a\psi} j_{\psi}^{0}\right) + ..$$

Axion search -LSW

- Because of the interaction between SM Photon to axion:

$$\mathcal{L}_a^{\text{int}} \supset \frac{\alpha}{8\pi}\frac{C_{a\gamma}}{f_a} aF\tilde{F} = g_{a\gamma} a\,\vec{E}\cdot\vec{B}$$





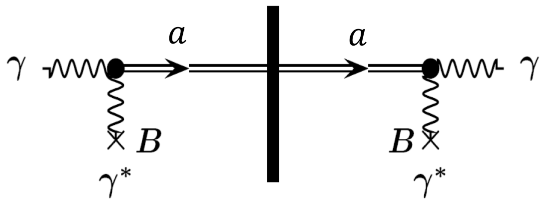
- The probability of $\gamma \to a$ is

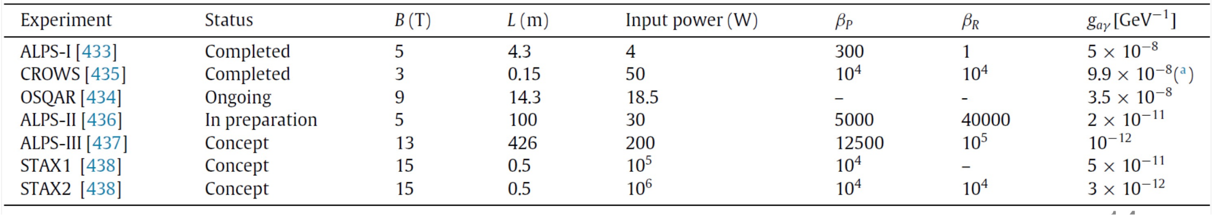
$$P_{\gamma\to a}(B,l,q\,) = \frac{1}{4}(g_{a\gamma}Bl)^2\left[\frac{\sin\left(\frac{1}{2}ql\right)}{\frac{1}{2}ql}\right]^2$$

$$\mathrm{q} = \kappa_\gamma - \kappa_a \sim m_a^2/2\,\omega$$

- The probability of LSW experiment is

$$\mathcal{P}(\gamma \to a \to \gamma) = P_p P_r$$

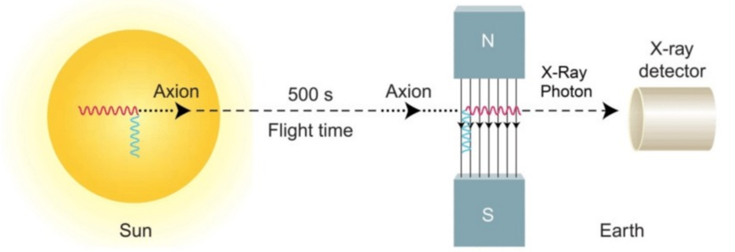




Experiment	Status	B (T)	L (m)	Input power (W)	β_P	β_R	$g_{a\gamma}$ [GeV^{-1}]
ALPS-I [433]	Completed	5	4.3	4	300	1	5×10^{-8}
CROWS [435]	Completed	3	0.15	50	10^4	10^4	9.9×10^{-8}([a])
OSQAR [434]	Ongoing	9	14.3	18.5	–	-	3.5×10^{-8}
ALPS-II [436]	In preparation	5	100	30	5000	40000	2×10^{-11}
ALPS-III [437]	Concept	13	426	200	12500	10^5	10^{-12}
STAX1 [438]	Concept	15	0.5	10^5	10^4	–	5×10^{-11}
STAX2 [438]	Concept	15	0.5	10^6	10^4	10^4	3×10^{-12}

Axion search - Helioscopes

- The interaction between SM Photon to axion:



$$\mathcal{L}_a^{\mathrm{int}} \supset \frac{\alpha}{8\pi}\frac{C_{a\gamma}}{f_a} aF\tilde{F} = g_{a\gamma} a\, \vec{E}\cdot\vec{B}$$

- The probability of $\gamma \to a$ is

$$P_{a\to\gamma} = \left(\frac{B g_{a\gamma}}{2}\right)^2 \frac{1}{q^2 + \Gamma^2/4}\left[1 + e^{-\Gamma L} - 2e^{-\Gamma L/2}\cos(qL)\right] \qquad q = \left|\frac{m_\gamma^2 - m_a^2}{2E_a}\right|, m_\gamma(\mathrm{eV}) = \sqrt{\frac{4\pi\alpha N_e}{m_e}} \approx 28.9\sqrt{\frac{Z}{A}\rho\left(\frac{\mathrm{kg}}{\mathrm{m}^3}\right)}$$

- In Vacuum ($\Gamma = 0, m_\gamma = 0$),

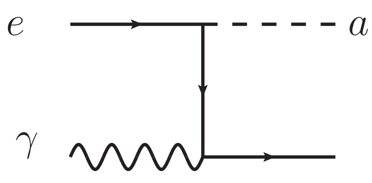
$$P_{a\to\gamma} = \frac{1}{4}(g_{a\gamma}Bl)^2\left[\frac{\sin\left(\frac{1}{2}ql\right)}{\frac{1}{2}ql}\right]^2$$

- Coherence condition: $\mathrm{qL} < \pi$ $\quad qL < \pi \Rightarrow \sqrt{m_\gamma^2 - \frac{2\pi E_a}{L}} < m_a < \sqrt{m_\gamma^2 + \frac{2\pi E_a}{L}}$

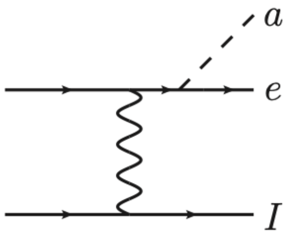
- Expected number of photons: $\mathrm{N}_\gamma = \Phi_a \cdot \mathrm{A} \cdot \mathrm{P}_{a\to\gamma}$

Helioscope: solar axion production

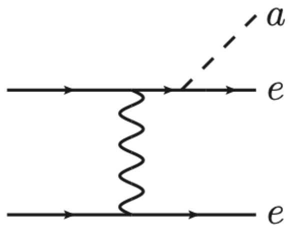
- The axion produced in the Sun via photon, electron or nucleon interactions



Compton

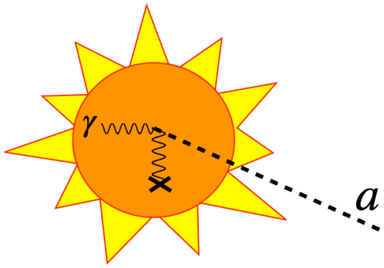


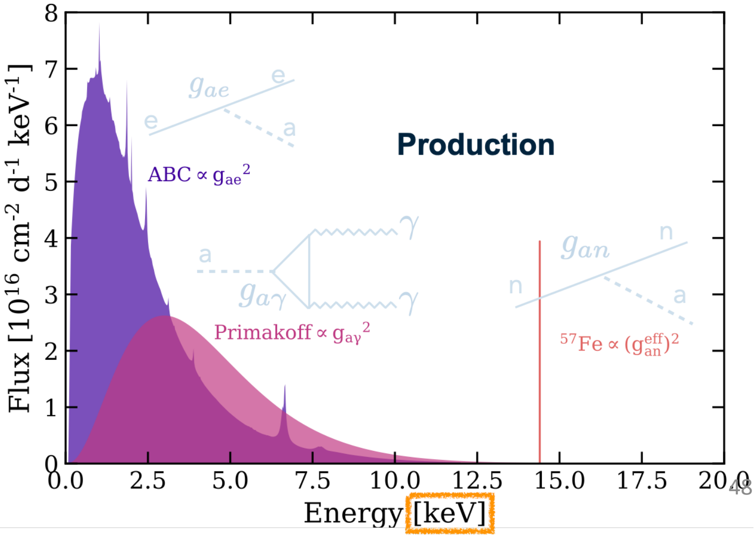
e – I bremsstrahlung



e – e bremsstrahlung

- Primakoff process:



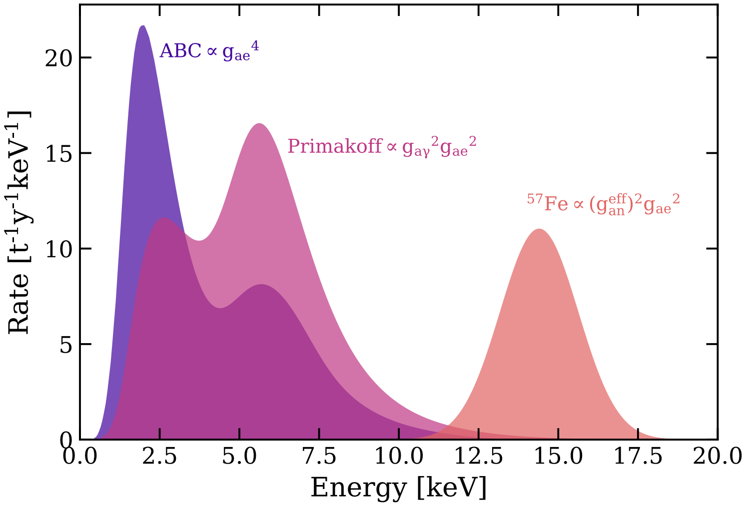


Helioscope: solar axion detection

- Axion-electric effect

$$\mathcal{L} \supset -g_{ae}\frac{\partial_\mu a}{2m_e}\bar{e}\gamma^\mu\gamma_5 e - \frac{1}{4}g_{a\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu}$$

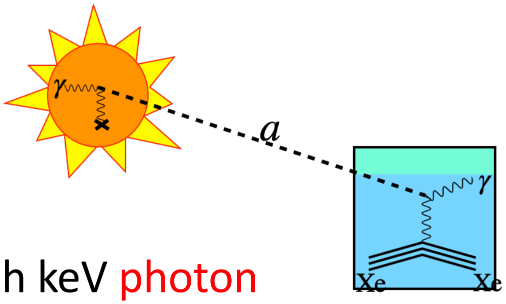
- keV axions are absorbed by electrons $\sigma_{\rm ae} = \sigma_{\rm pe}\frac{g_{\rm ae}^2}{\beta_a}\frac{3E_a^2}{16\pi\alpha m_e^2}\left(1-\frac{\beta_a^{2/3}}{3}\right)$



Inverse Primakoff effect

- Inverse Primakoff effect cross section

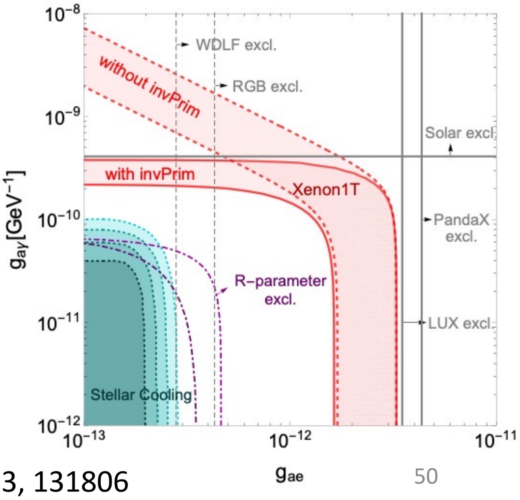
$$\frac{d\sigma^{\mathrm{invPrim}}_{a\to\gamma}}{d\Omega} = \frac{\alpha}{16\pi} g^2_{a\gamma} \frac{\boldsymbol{q}^2}{\boldsymbol{k}^2}\left(4-\boldsymbol{q}^2/\boldsymbol{k}^2\right) F_a^2(\boldsymbol{q}^2)$$



- keV photon ionizes Xe : Liquid Xenon exps can hardly distinguish keV photon signal from Electron Recoil

- The total event number

$$\frac{dR}{dE_r} = \frac{N_A}{A}\left(\frac{d\Phi_a^{\mathrm{ABC}}}{dE}(E_r) + \frac{d\Phi_a^{\mathrm{Prim}}}{dE}(E_r)\right)\times\left(\sigma_{a\to\gamma}^{\mathrm{invPrim}}(E_r)+\sigma_{ae}(E_r)\right)$$



CG, JL, L-TW, X-PW, WX, *Phys.Rev.Lett.* 125 (2020) 13, 131806

Axion-Like-Particles' Search

- The axion-like particle Lagrangian

$$\mathcal{L}_{\mathrm{eff}}^{\mathrm{D}\leq 5} = \sum_f \frac{C_{ff}}{2}\frac{\partial^\mu a}{f_a}\bar{f}\gamma_\mu\gamma_5 f + \frac{\alpha C_{\gamma\gamma}}{4\pi}\frac{a}{f_a}F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$+ \frac{\alpha C_{\gamma Z}}{2\pi s_w c_w}\frac{a}{f_a}F_{\mu\nu}\tilde{Z}^{\mu\nu} + \frac{\alpha C_{ZZ}}{4\pi s_w^2 c_w^2}\frac{a}{f_a}Z_{\mu\nu}\tilde{Z}^{\mu\nu} + \frac{\alpha C_{WW}}{\pi s_w^2}\frac{a}{f_a}\epsilon_{\mu\nu\rho\sigma}\partial^\mu W_+^\nu \partial^\rho W_-^\sigma + \cdots$$

- The relationship among the coefficients

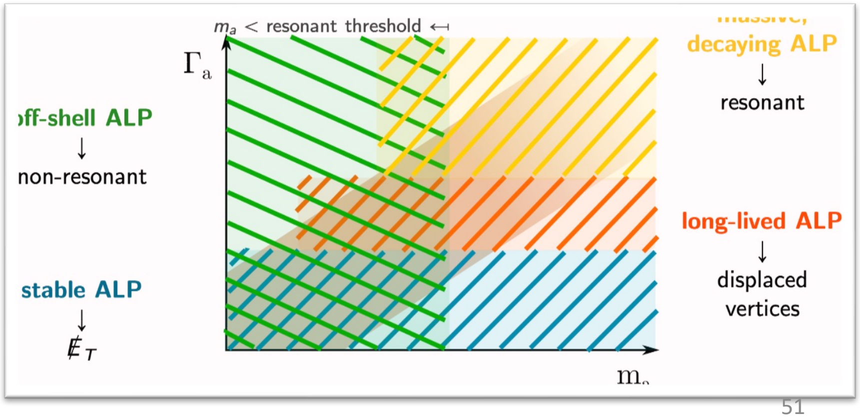
$$C_{\gamma\gamma} = C_{WW} + C_{BB} \qquad C_{\gamma Z} = c_w^2 C_{WW} - s_w^2 C_{BB} \qquad C_{ZZ} = c_w^4 C_{WW} + s_w^4 C_{BB}$$

- Extra decay widths

$$\Gamma(Z \to \gamma a) = \frac{\alpha^2(m_Z)m_Z^3}{96\pi^3 s_w^2 c_w^2 f_a^2}\left|C_{\gamma Z}^{\mathrm{eff}}\right|^2\left(1 - \frac{m_a^2}{m_Z^2}\right)^3$$

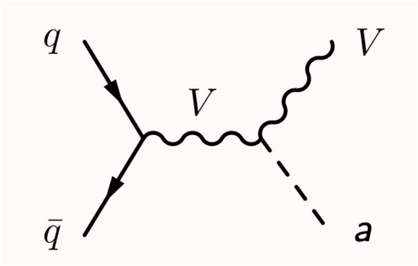
$$\Gamma(a \to l^+l^-) = \frac{m_a m_l^2}{8\pi f_a^2}\left|C_{ll}^{\mathrm{eff}}\right|^2\sqrt{1 - \frac{4m_l^2}{m_a^2}}$$

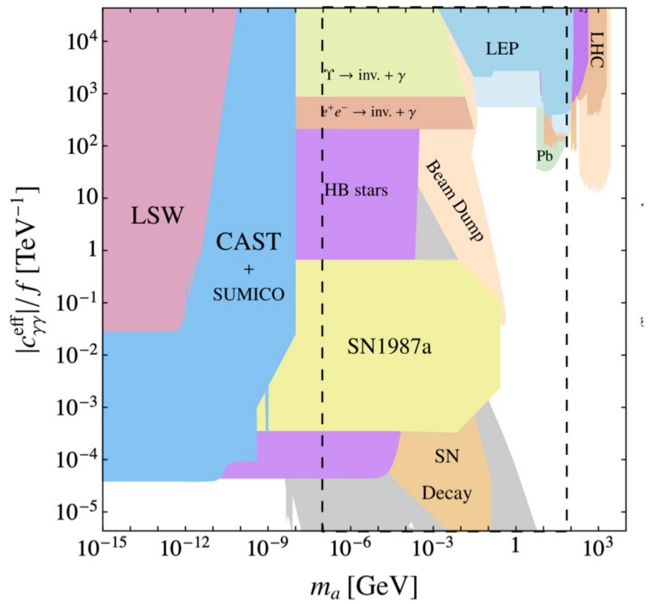
$$\Gamma(a \to \gamma\gamma) = \frac{\alpha^2 m_a^3}{64\pi^3 f_a^2}\left|C_{\gamma\gamma}^{\mathrm{eff}}\right|^2$$



ALP Search at Colliders (LHC)

- APL as MET (Mono-X search)



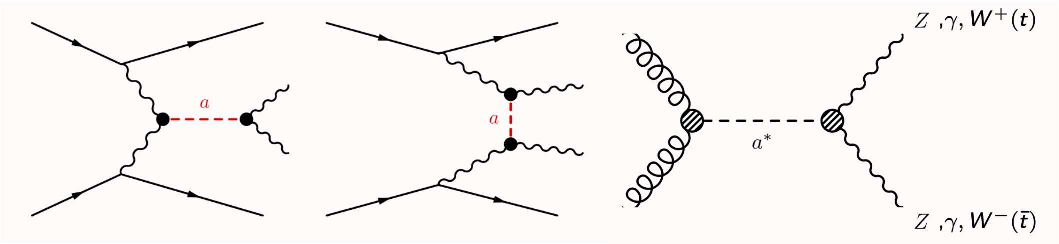


MB, MN, AT, *JHEP* 12 (2017) 044

- APL as LLP (Displaced vertex)

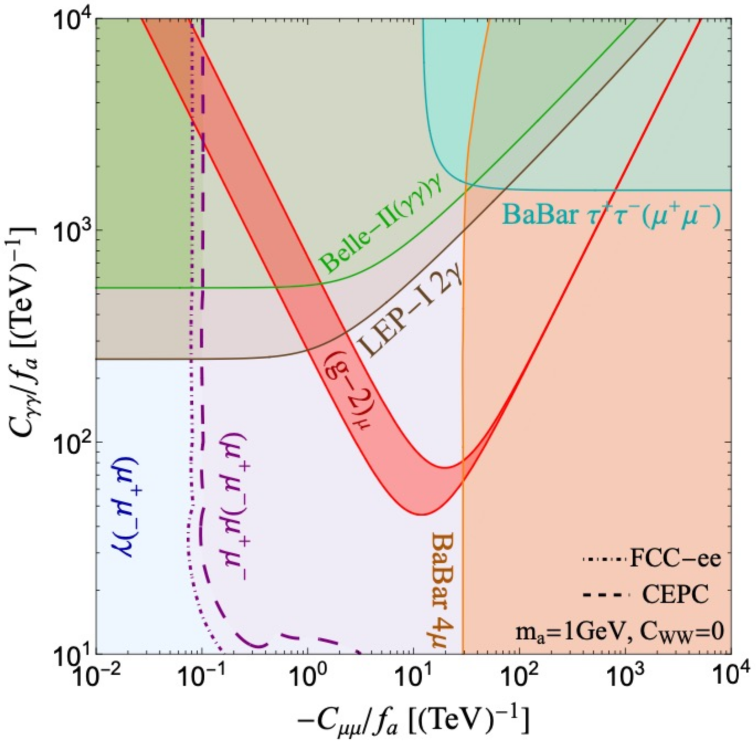
- APL as resonance ($a \to \gamma\gamma, \bar{f}f, Z\gamma$ …)

- APL as off-shell mediator (non−resonant effects)



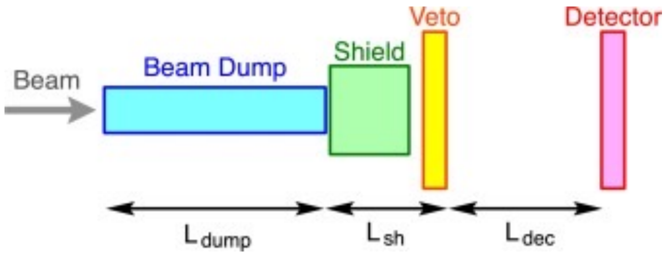
ALP Search at future collider (CEPC)

- Constraining $a\gamma$ coupling only:
  - Belle-II, LEP: $e^+e^- \to a\gamma \to (\gamma\gamma)\gamma$
  - LHC: $pp \to a\gamma \to (\gamma\gamma)\gamma$
- Constraining $a - \mu$ coupling only:
  - BaBar: recast $e^+e^- \to \mu^+\mu^- Z'$
  - CMS : (4μ): $pp \to \mu^+\mu^-$a
- Constraining both coupling
  - CMS: $(\bar{t}t + 2\mu): pp \to \bar{t}t\phi \to \bar{t}t(\mu^+\mu^-)$



Long-lived ALP Search (NA64)

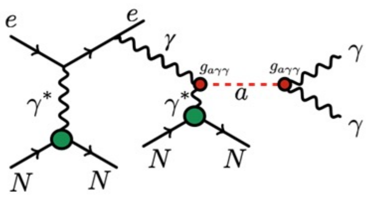
- ALP production and decat at Beam-dump experiments

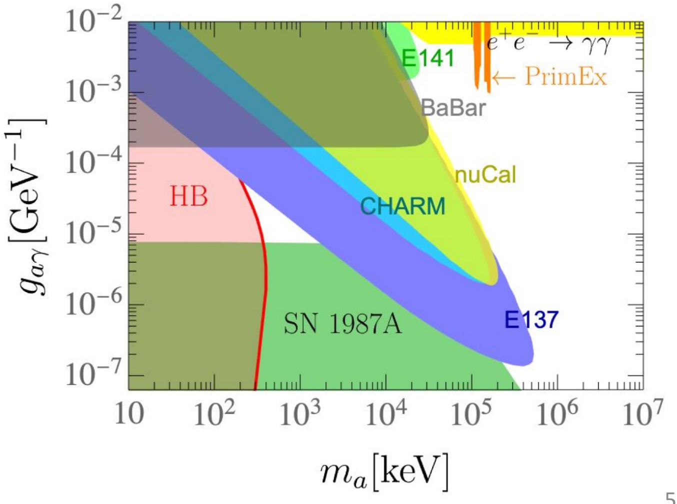


$$p_{\text{decay}} = e^{-\frac{L_{sh}m_a}{E_a\tau_a}} - e^{-\frac{L_{tot}m_a}{E_a\tau_a}}$$

- ALP life-time

$$\tau = \frac{64\pi}{g_{a\gamma}^2 m_a^3}$$





Phys.Lett.B 809 (2020) 135709

Outline

- Strong CP problem
- Axion solution to strong CP problem
- Axion Cosmology
- Axion Direct searches
- Axion Dark matter searches

Axion DM search - Haloscopes

- Axion Lagrangian:

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu a\right)^2 - \frac{1}{2} m_a{}^2 a^2 - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{4} g_{a\gamma\gamma} a F_{\mu\nu}\tilde{F}^{\mu\nu}$$
$$= \frac{1}{2}\left(\partial_\mu a\right)^2 - \frac{1}{2} m_a{}^2 a^2 + \frac{1}{2}(E^2 - B^2) - g_{a\gamma\gamma} a\, \vec{E} \cdot \vec{B}$$

- Euler-Lagrange equation

$$\frac{\partial}{\partial x^\mu}\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu a)}\right) - \frac{\partial \mathcal{L}}{\partial a} = 0$$

- Maxwell's equation with Axion

$$\nabla \cdot \mathbf{E} = \rho - g_{a\gamma} \mathbf{B} \cdot \nabla a$$
$$\nabla \times \mathbf{B} - \dot{\mathbf{E}} = \mathbf{J} + g_{a\gamma}(\mathbf{B}\dot{a} - \mathbf{E} \times \nabla a)$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} + \dot{\mathbf{B}} = 0$$
$$\ddot{a} - \nabla^2 a + m_a^2 a = g_{a\gamma} \mathbf{E} \cdot \mathbf{B}.$$

Homework

Axion search - Haloscopes

- Maxwell's equation with Axion

$$\begin{gathered}\nabla\cdot\mathbf{E}=\rho-g_{a\gamma}\mathbf{B}\cdot\nabla a\\ \nabla\times\mathbf{B}-\dot{\mathbf{E}}=\mathbf{J}+g_{a\gamma}(\mathbf{B}\dot{a}-\mathbf{E}\times\nabla a)\\ \nabla\cdot\mathbf{B}=0\\ \nabla\times\mathbf{E}+\dot{\mathbf{B}}=0\\ \ddot{a}-\nabla^2a+m_a^2a=g_{a\gamma}\mathbf{E}\cdot\mathbf{B}.\end{gathered}$$

- Obvious solution:

$$\mathbf{E}_a(t)=-g_{a\gamma}\mathbf{B}_e a(t)$$

- Detect oscillating magnetic field generated by dark matter axions in an external homogeneous magnetic field

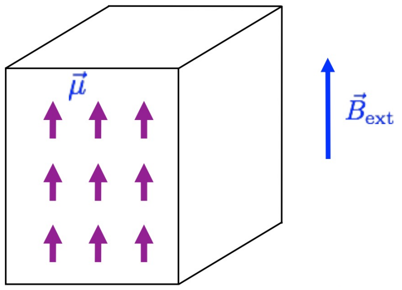
$$\mathbf{B}_a=\frac{1}{i\omega}\nabla\times\mathbf{E}_a=-g_{a\gamma}\mathbf{V}\times\mathbf{B}_e a$$

axion DM velocity (10^{-3})

Axion field: $a(t)=a_0\cos(m_a t+\delta_\tau(t))$

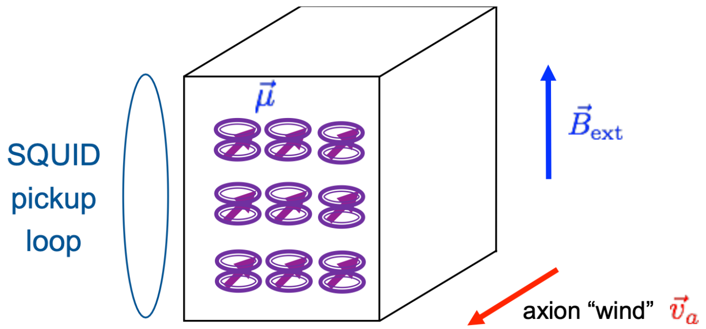
external magnetic field

Axion search - Haloscopes



CASPEr-electric

- spin to axion coupling: $H_{\mathrm{e}} \propto a\vec{\boldsymbol{\sigma}} \cdot \vec{\boldsymbol{E}}^{*} = \vec{\boldsymbol{\sigma}} \cdot \vec{\boldsymbol{B}}_{1}^{*} \cos(\omega_a t)$
- spin "feels" an effective magnetic field: $\vec{\boldsymbol{B}}_{1}^{*} \cos \omega_a t = g_d a_0 \vec{\boldsymbol{E}}^{*} \cos \omega_a t$



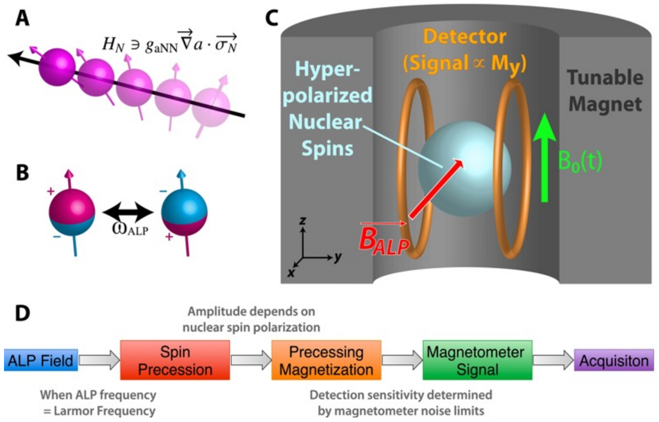
CASPEr-Wind

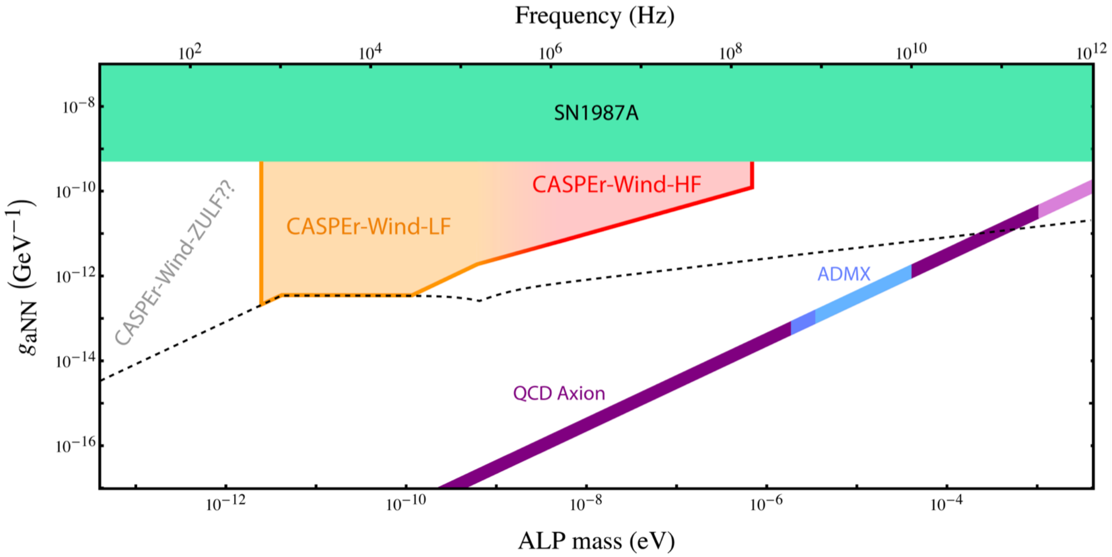
- spin to axion coupling: $H_{\text{wind}} \propto \vec{\boldsymbol{\sigma}} \cdot \vec{\nabla} \mathrm{a} = \vec{\boldsymbol{\sigma}} \cdot \vec{\boldsymbol{B}}_{1}^{*} \cos(\omega_a t)$
- spin "feels" an effective magnetic field: $\vec{\boldsymbol{B}}_{1}^{*} \cos \omega_a t = \vec{\nabla} \mathrm{a}$

Axion property

- Dark matter density $\rho_{\mathrm{DM}} \propto a_0^2$
- ALP compton frequancy $\omega_a = m_a$

Cosmic Axion Spin Precession Experiment





Phys. Rev. Lett. **116**, 190801 (2016)

Haloscope: Comagnetometer for axion DM

- Bloch Equation

$$\dot{\vec{S}} = \gamma\left(\vec{B} + \frac{\vec{b}}{\gamma}\right) \times \vec{S} - \Gamma\vec{S}$$

- Transverse EOMs ($\dot{S}_z = 0$ & $|\vec{S}| \approx |S_z|$, And care only about $S_\perp = S_x + iS_y$)

$$\dot{S}_\perp = i\gamma\left(B_z + \frac{b_z(=0)}{\gamma}\right)S_\perp - i\gamma\left(B_\perp + \frac{b_\perp}{\gamma}\right)S_z - \Gamma S_\perp$$

- If B_z is constant

Signal: Generated from ALP

$$S_\perp(\omega = m_a) = \frac{\boxed{b_\perp} + \gamma B_\perp(\omega = m_a)}{(\boxed{\gamma B_z} - m_a) + \boxed{i\Gamma}}\boxed{S_z}$$

Spin in Z direction

Controlled Resonance Frequency

Decoherence Rate

- We need a way to generate S_z, and to measure $S_\perp$.

  Both can be achieved with optical lasers for Alkali metals: "Pump" laser polarizes the spins, "Probe" laser measures them

  Problem: $SNR \propto \gamma$, and the gyromagnetic ratio of alkali metals is large.

Haloscope: Comagnetometer for axion DM

- Coupled Bloch equation

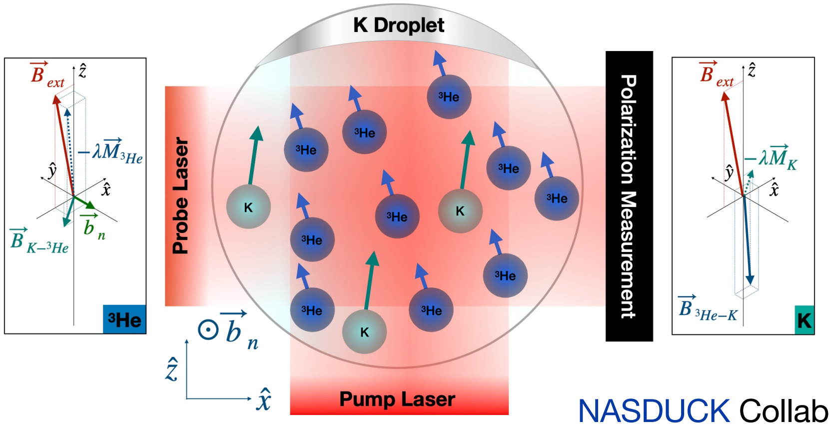
$$\frac{\delta \mathbf{P^e}}{\delta t} = \frac{\gamma_e}{Q}\left[\mathbf{B}+\mathbf{L}+\boxed{\lambda M_0^n \mathbf{P^n}}+\mathbf{b^e}\right]\times\mathbf{P^e} - \mathbf{\Omega}\times\mathbf{P^e} + \frac{R_p\mathbf{S_p}+R_m\mathbf{S_m}\boxed{+R_{\mathrm{se}}^{\mathrm{ne}}\mathbf{P^n}}}{Q} - \frac{\{R_1^e, R_2^e, R_2^e\}}{Q}\mathbf{P^e}$$

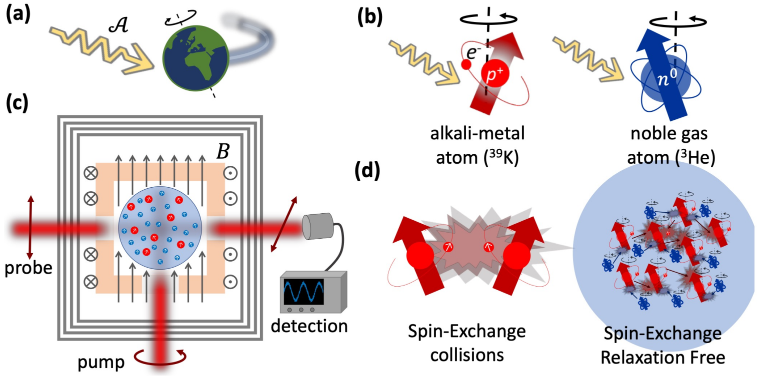
spin interaction   spin exchange collision

$$\frac{\delta \mathbf{P^n}}{\delta t} = \gamma_n\left(\mathbf{B}+\boxed{\lambda M_0^e \mathbf{P^e}}+\boxed{\mathbf{b^n}}\right)\times\mathbf{P^n} - \Omega\times\mathbf{P^n}\boxed{+R_{\mathrm{se}}^{\mathrm{en}}\mathbf{P^e}} - \{R_1^n, R_2^n, R_2^n\}\,\mathbf{P^n}$$

Exotic B field

- Alkali atoms and nobel-gas atom

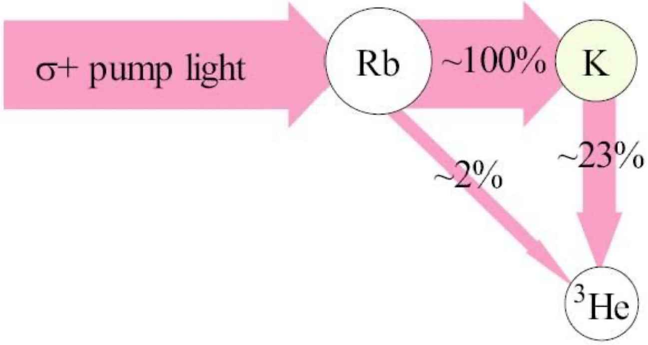




NASDUCK Collaboratiion, Published in: *Nature Commun.* 14 (2023) 1, 5784

Comagnetometer in Hybrid Spin Resonance

- Spin-exchange Optical Pumping:
  - •1960: Bouchiat/Carver/Varnum (Princeton), PRL 5, 373, P=0.01%
  - •Now:Rb-K optical pumping P>70%



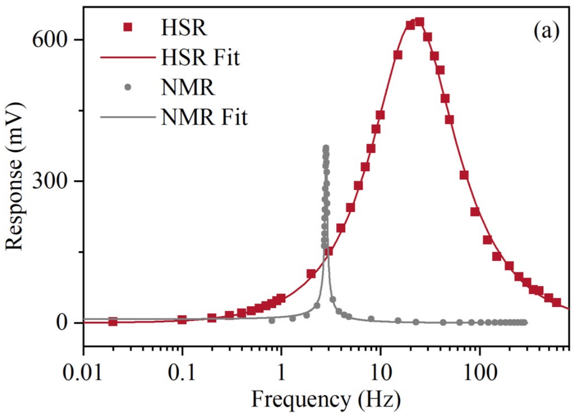
- Method: tune external B field to make Larmor frequency equal (HSR region)

$$\omega_{\text{alkali}} = \gamma_{\text{alkali}} \left(\hat{B}_{\text{ext}} + \hat{B}_{\text{noble}}\right)$$

$$\omega_{\text{noble}} = \gamma_{\text{noble}} \left(\hat{B}_{\text{ext}} + \hat{B}_{\text{alkali}}\right)$$

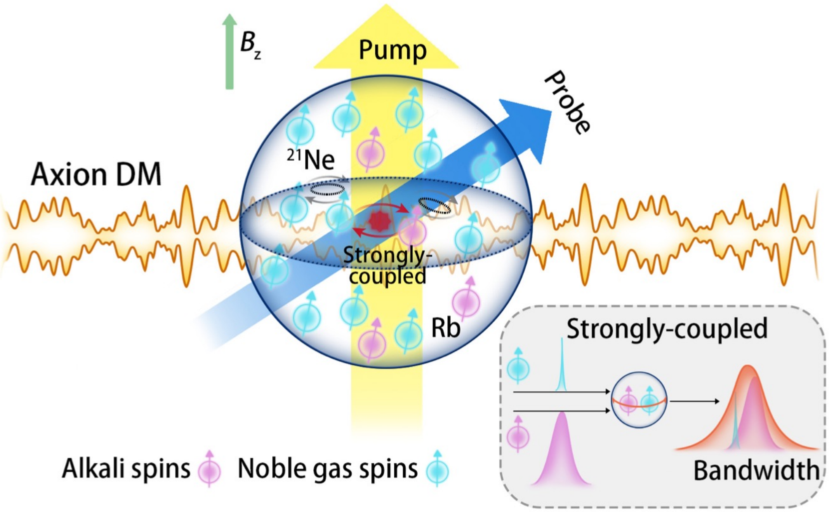
- Require $\omega_{\text{alkali}} = \omega_{\text{noble}}$

$$B_{\text{HSR}} \sim -B_{\text{noble}}$$



Comagnetometer HSR search on ALP DM

- Good control on photon-shot-noise and magnetic noise
- Sharp amplification is wasted
- Smaller amplification but with much wider resonance
  1. Do not need to scan (e.g. 35 months)
  2. Long-time measurement at single point to compensate amplification lost



Comagnetometer HSR search on ALP DM

- Random phase in different *p* mode

$$\nabla a(x) = \sum_{\boldsymbol{p}} \boxed{\sqrt{\frac{2N_{\boldsymbol{p}}}{V\omega_{\boldsymbol{p}}}}} \cos(\omega_{\boldsymbol{p}} t - \boldsymbol{p}\cdot\boldsymbol{x} + \boxed{\phi_{\boldsymbol{p}}})\,\boldsymbol{p}$$

ϕ_p: is uniform random variable in [0, 2Pi]

$N_p = \rho_{\mathrm{DM}} V f(p)(\Delta p)^3/\omega_p$: is mean occupation number of p mode

- Signal is stochastic instead of deterministic

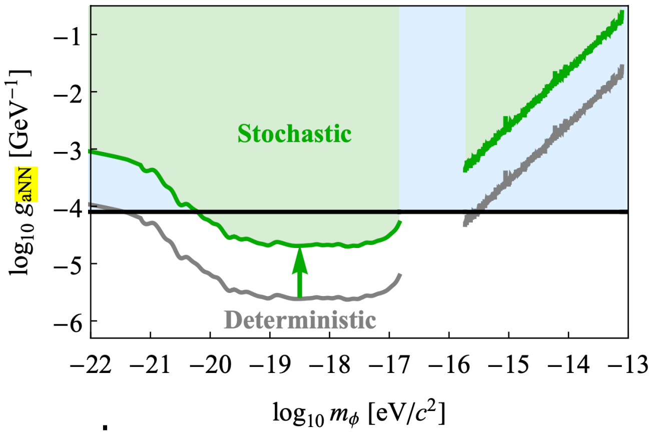
Junyi Lee et al, 2209.03289, PRX 2023

$$\beta_j = \frac{g_{aN}}{\gamma_N} \nabla a(j\Delta t)\cdot \hat{\mathbf{m}}(j\Delta t)$$

$$A_k = \frac{2}{N}\mathrm{Re}[\tilde{\beta}_k], \quad B_k = -\frac{2}{N}\mathrm{Im}[\tilde{\beta}_k].$$

$$L(\mathbf{d}|g_{aNN}, \sigma_b^2) = \frac{1}{\sqrt{(2\pi)^{2N}\det(\Sigma)}} \exp\left(-\frac{1}{2}\mathbf{d}^T\Sigma^{-1}\mathbf{d}\right)$$

$$\Sigma = \Sigma_a + \sigma_b^2 \cdot \mathbb{1}$$

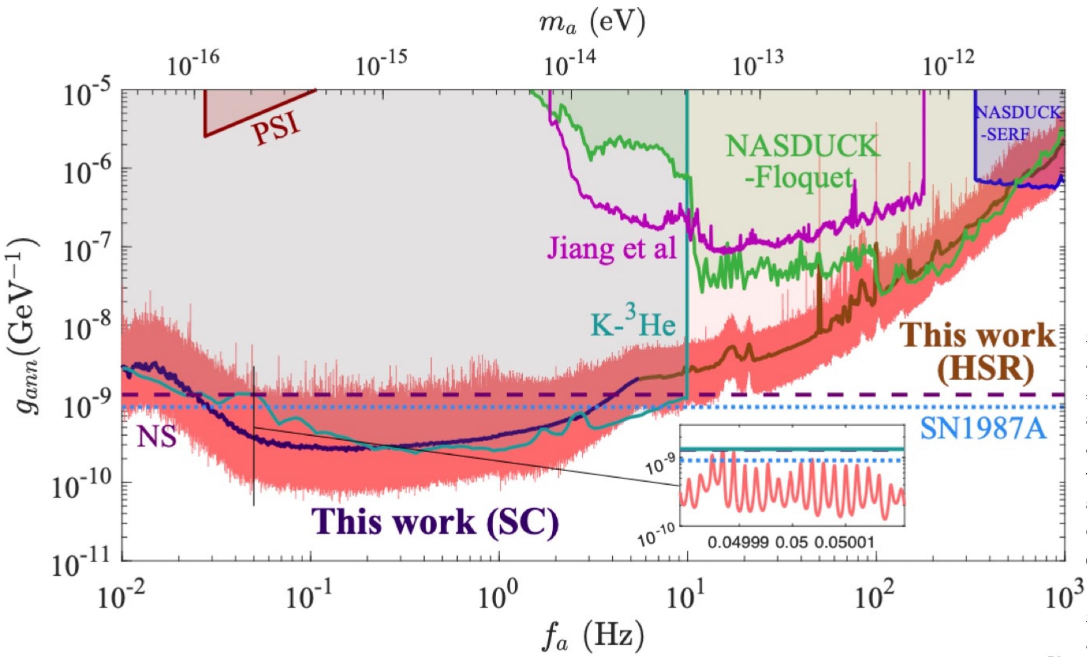


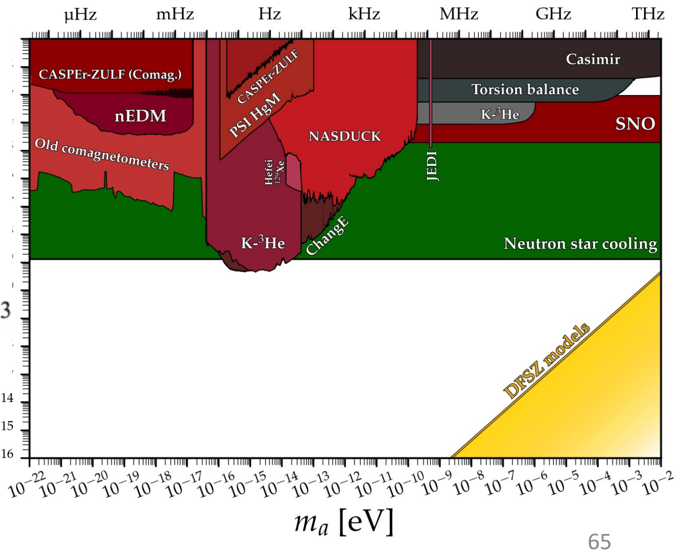
$$\boxed{\gamma_{95\%}^{\mathrm{stoch}} = 8.4\,\gamma_{95\%}^{\mathrm{det}}}$$

- Signal and white background are multivariate Gaussian distribution Signal contains non-diagonal term

Haloscope: HSR search for ALP DM

CHANGE group, arXiv: 2306.08039

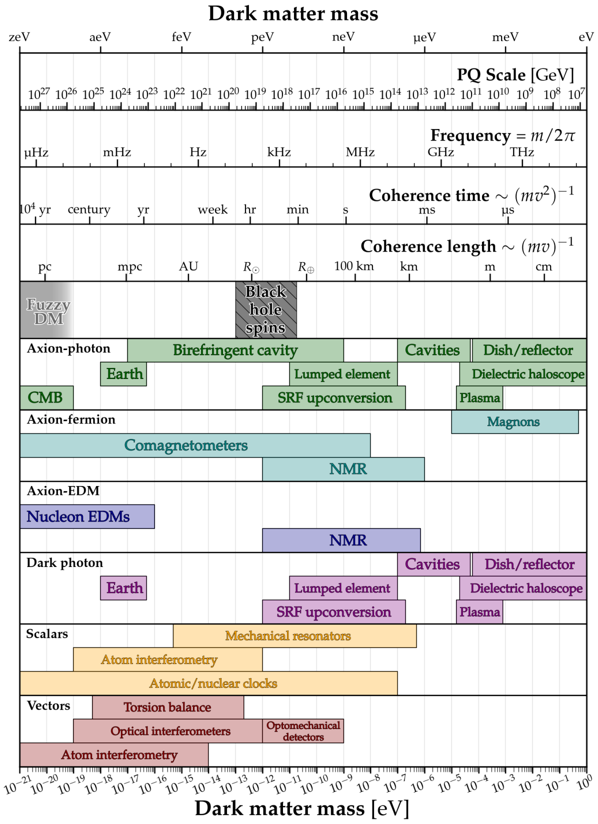


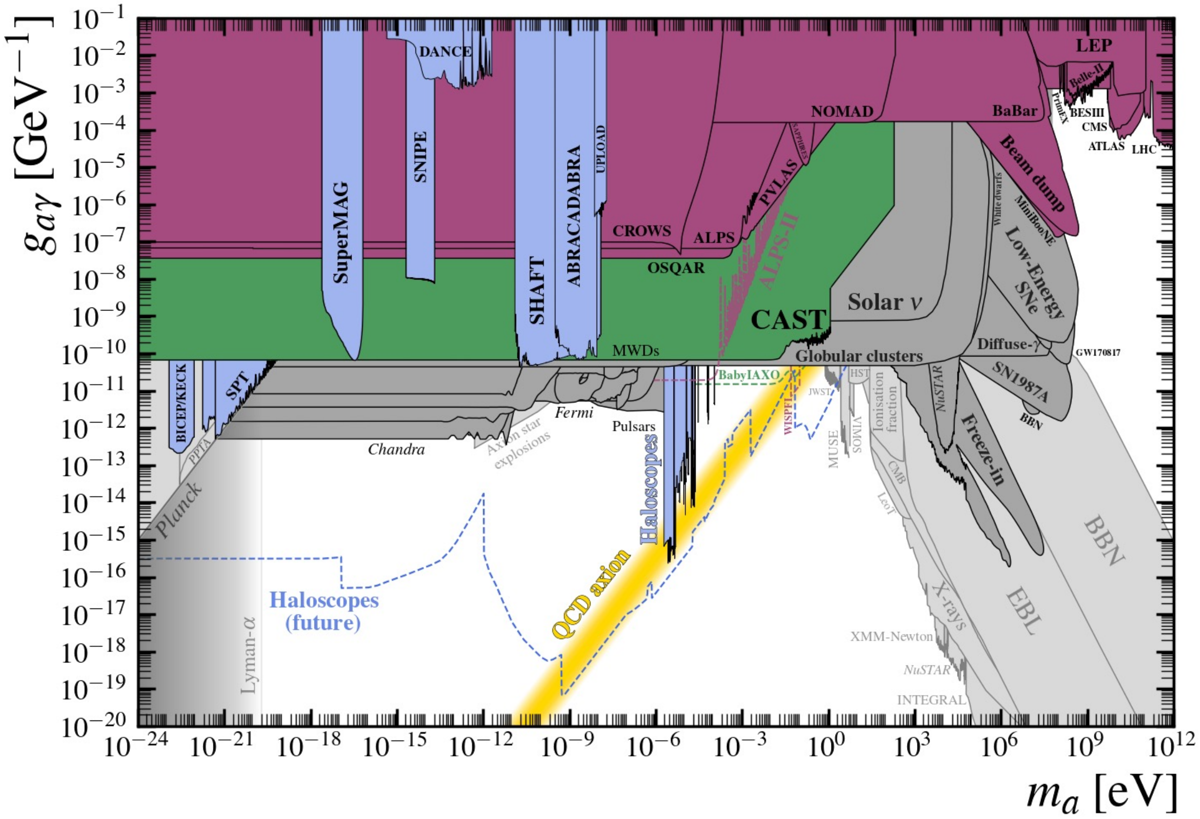


Summary

- The existence of an Axion-remains the most compelling solution to the strong CP problem
- The concomitant axions play an interesting cosmological role and arise naturally in theories beyond the Standard Model
- There are both ongoing and proposed experiments which in the next decade or so should tell us if axions exist
    1. Astrophysical Observations
    2. Particle experiments for Light Axion (LSW, Solar axion Search, Axiom DM Search)
    3. Particle experiments for heavy ALP (LHC, Beam-dump…)

Summary


Dark matter mass
zeV
aeV
feV
peV
neV
μeV
meV
eV
PQ Scale [GeV]
Frequency = $m/2\pi$
μHz
mHz
Hz
kHz
MHz
GHz
THz
Coherence time ~ $(mv^2)^{-1}$
10^4 yr
century
yr
week
hr
min
s
ms
μs
Coherence length ~ $(mv)^{-1}$
pc
mpc
AU
$R_\odot$
$R_\oplus$
100 km
km
m
cm
Fuzzy DM
Black hole spins
Axion-photon
Birefringent cavity
Cavities
Dish/reflector
Earth
Lumped element
Dielectric haloscope
CMB
SRF upconversion
Plasma
Axion-fermion
Magnons
Comagnetometers
NMR
Axion-EDM
Nucleon EDMs
NMR
Dark photon
Cavities
Dish/reflector
Earth
Lumped element
Dielectric haloscope
SRF upconversion
Plasma
Scalars
Mechanical resonators
Atom interferometry
Atomic/nuclear clocks
Vectors
Torsion balance
Optical interferometers
Optomechanical detectors
Atom interferometry
Dark matter mass [eV]


$g_{a\gamma}$ [GeV^{-1}]
m_a [eV]
10^{-1}
10^{-20}
10^{-24}
10^{12}
DANCE
SuperMAG
SNIPE
SHAFT
ABRACADABRA
UPLOAD
LEP
NOMAD
BaBar
Belle-II
BESIII
CMS
ATLAS
LHC
PrimEx
Beam dump
MiniBooNE
White dwarfs
Low-Energy SNe
CROWS
ALPS
ALPS-II
PVLAS
OSQAR
CAST
Solar ν
MWDs
Globular clusters
Diffuse-γ
GW170817
SN1987A
BBN
BabyIAXO
BICEP/KECK
SPT
PPTA
Fermi
Pulsars
Chandra
Axion star explosions
Haloscopes
WISPLC
JWST
HST
MUSE
VIMOS
Ionisation fraction
NuSTAR
Freeze-in
CMB
Leo T
Planck
Lyman-α
Haloscopes (future)
QCD axion
X-rays
XMM-Newton
NuSTAR
INTEGRAL
EBL
BBN