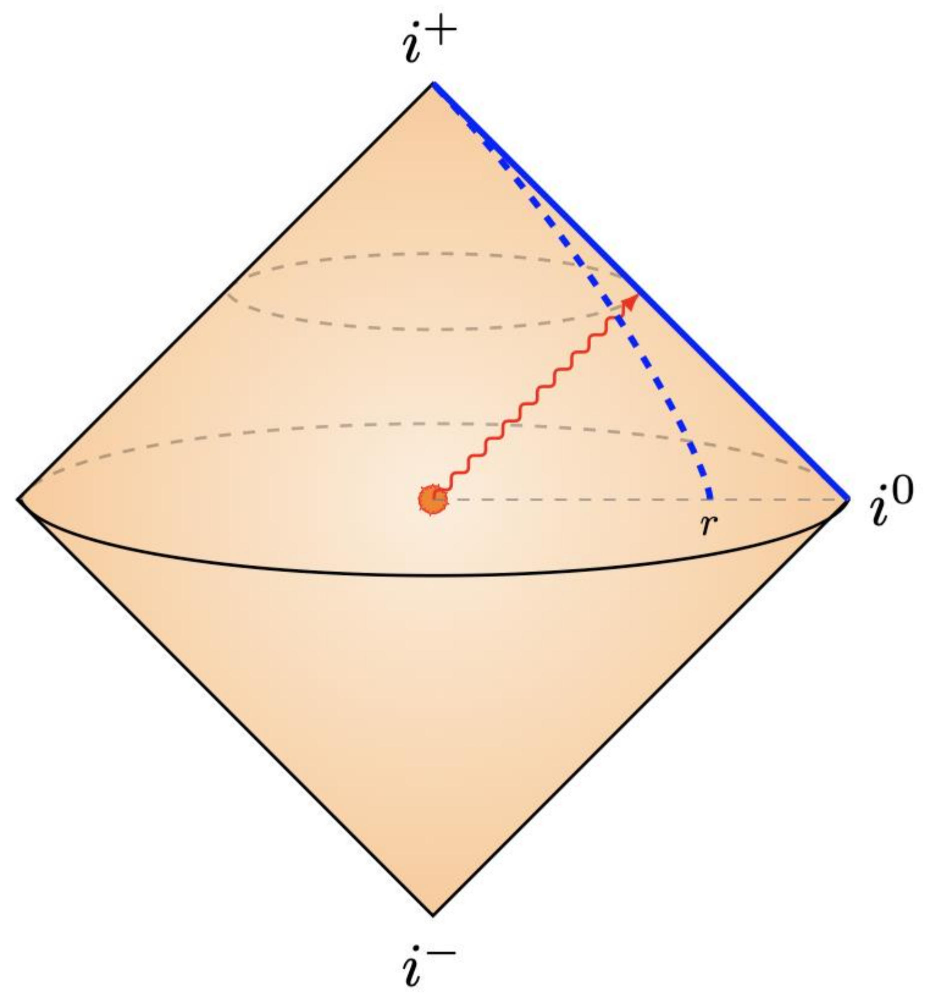
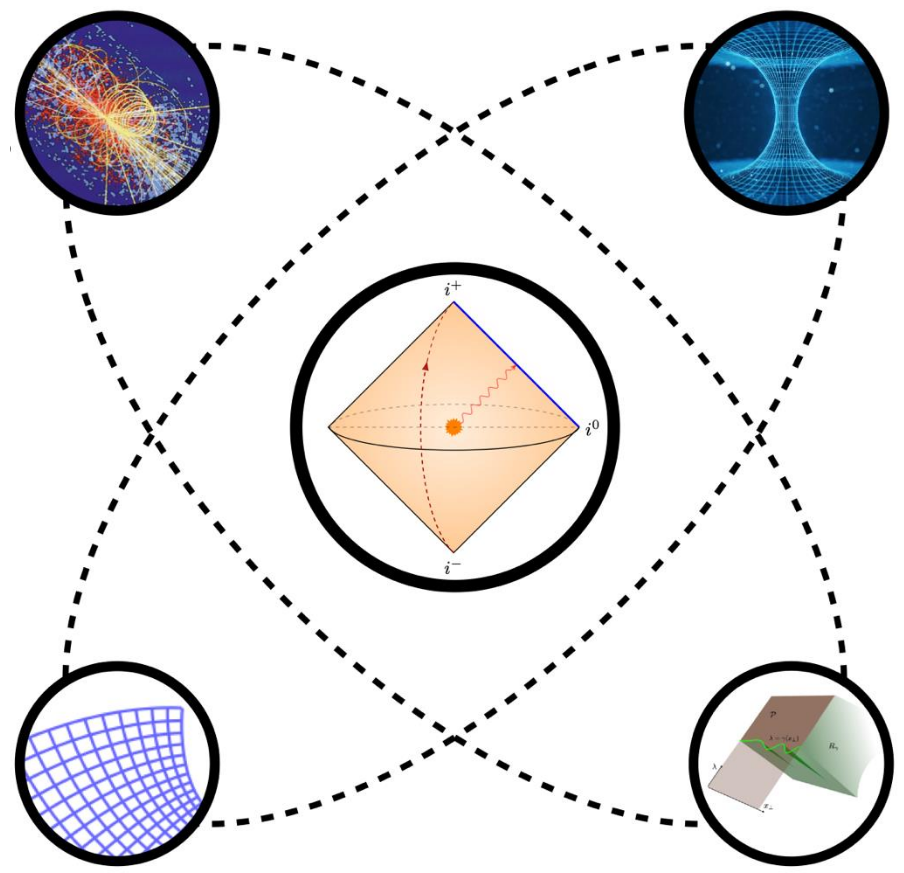
Energy Correlators:
A Journey From Theory to Experiment

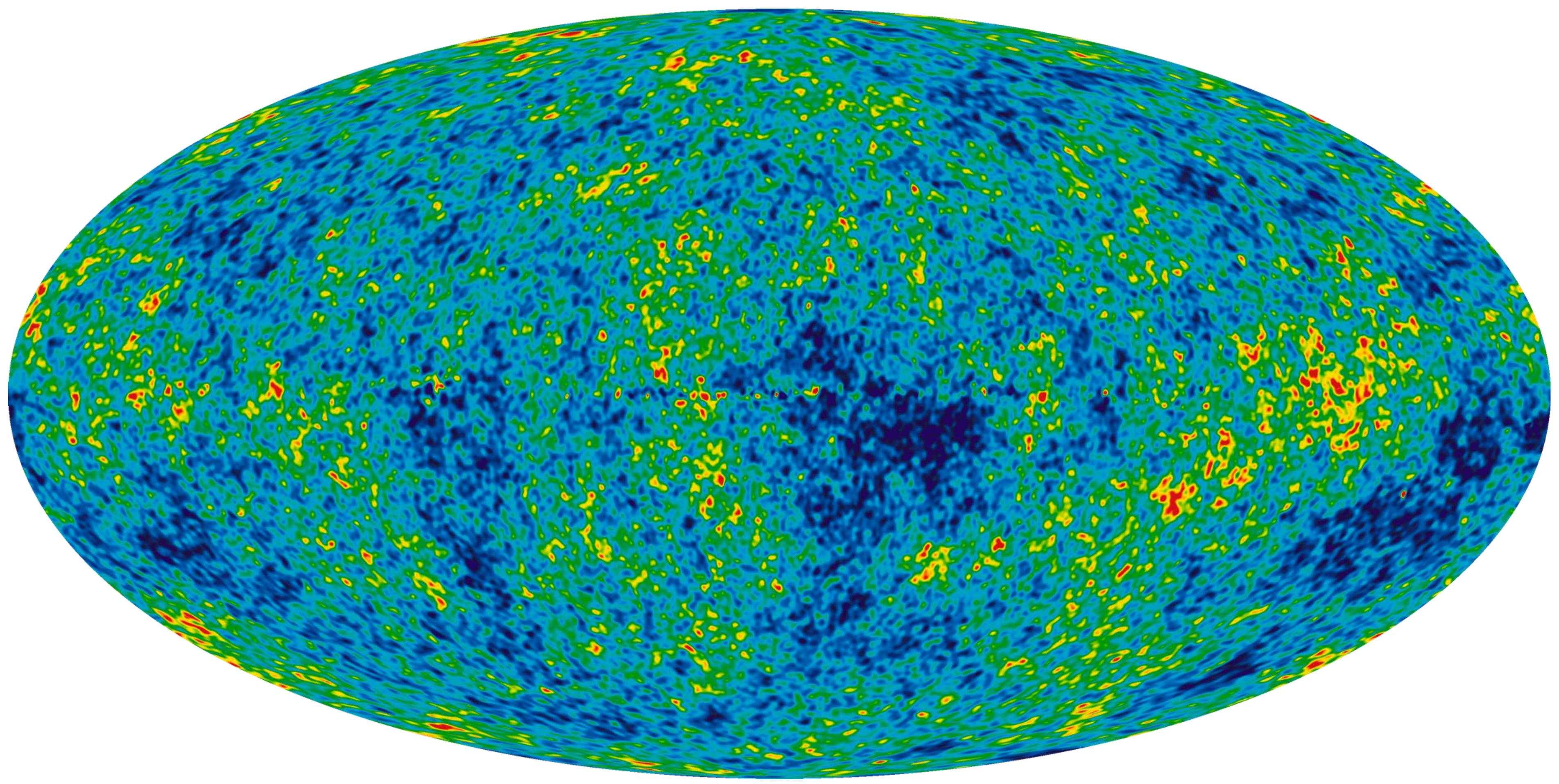


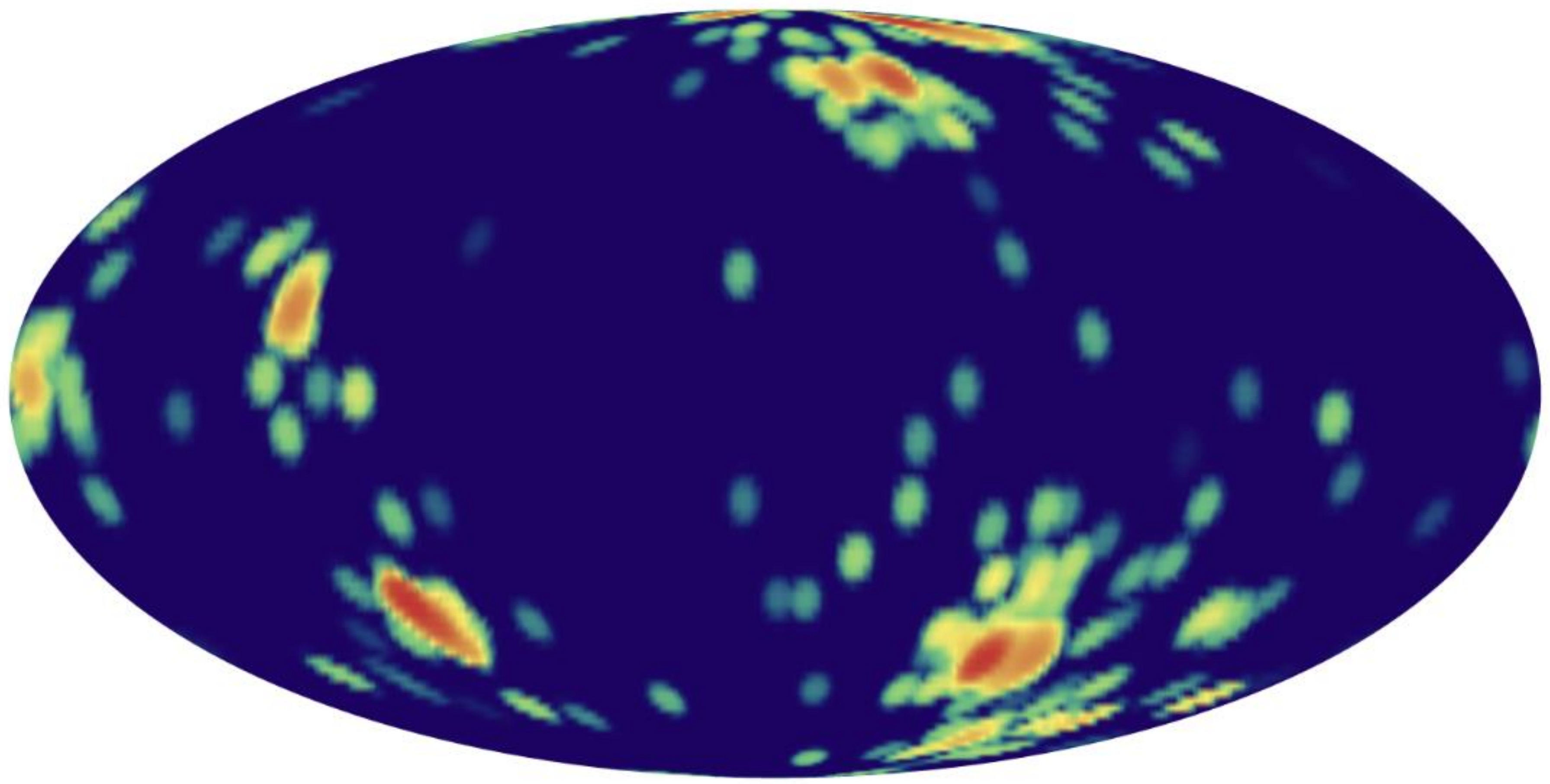
HuaXing Zhu
Peking University

Based on a review article with Ian Moult



见微沙龙，中国科学技术大学
April 18, 2025





After a particle collision, the underlying microscopic physics gets imprinted into detailed correlations in macroscopic fluxes, much in analogy to how our cosmic history is imprinted into correlations in the Cosmic Microwave Background.

Understanding how to map correlations in macroscopic fluxes to properties of the underlying quantum field theory (QFT) is therefore key to addressing a wide variety of questions in particle and nuclear physics.

Outline

- 50 years of energy flux theories of energy operators
    - QCD -> CFT -> QCD
- Energy operators in particle physics
- Energy operators in nuclear physics

The birth of energy operator

Jet Structure in e^+e^- Annihilation
with Massless Hadrons*

George Sterman

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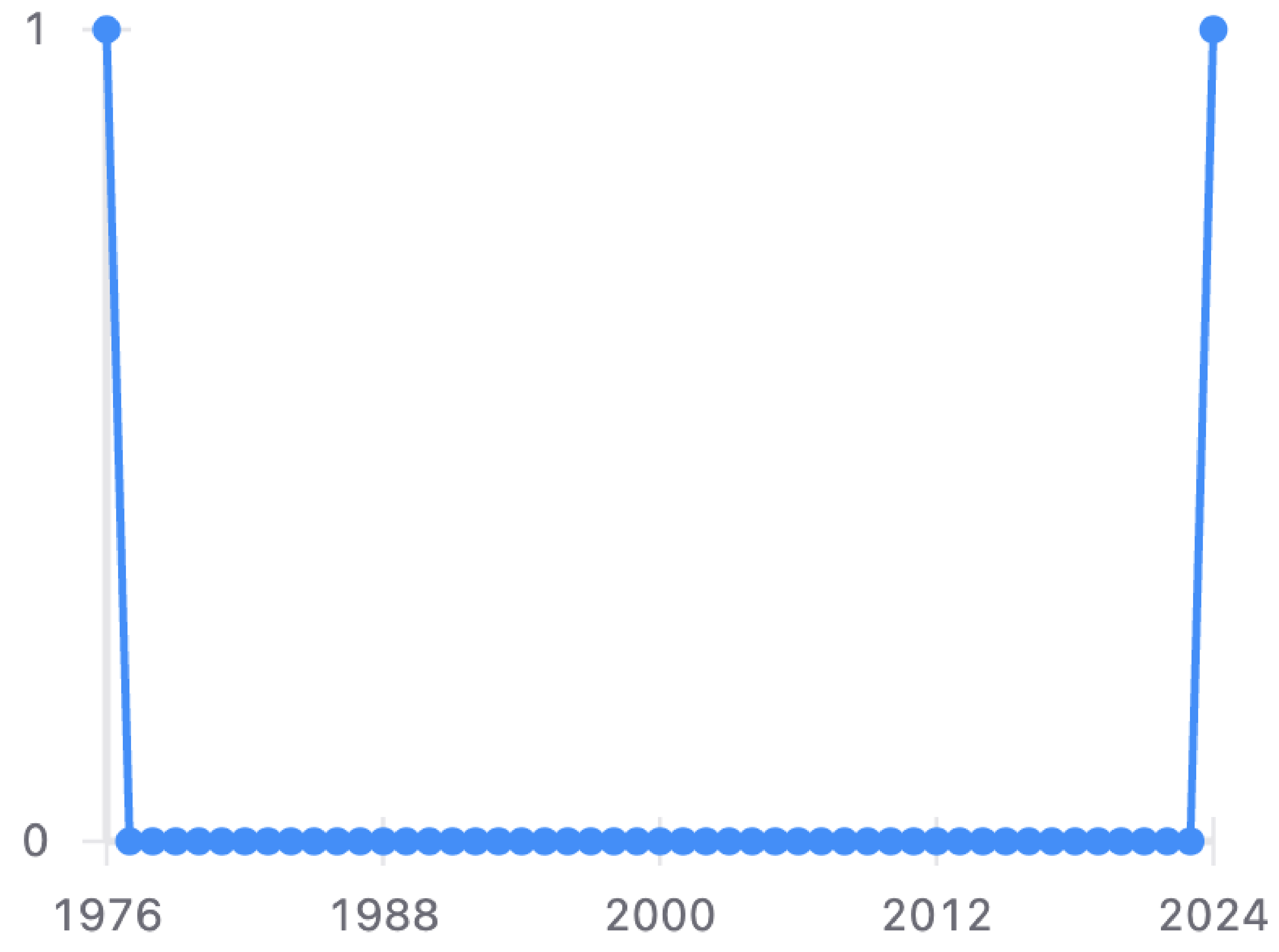
Urbana, Illinois 61801

Abstract

The cancellation of mass singularities in e^+e^- annihilation processes in the presence of directly coupled massless particles has been investigated. Ensembles of states have been identified, the summation over which produces transition probabilities which are free of mass singularities when the masses of one or more species of hadrons vanish. States included in these ensembles can be characterized as having a "jet-like" structure. Application to annihilation processes at high energies in massive theories is discussed.

Citations per year



$$\mathcal{E}(\hat{n})|X\rangle = \sum_{i \in X} k_i^0 \delta^2(\Omega_n - \Omega_k)|X\rangle$$

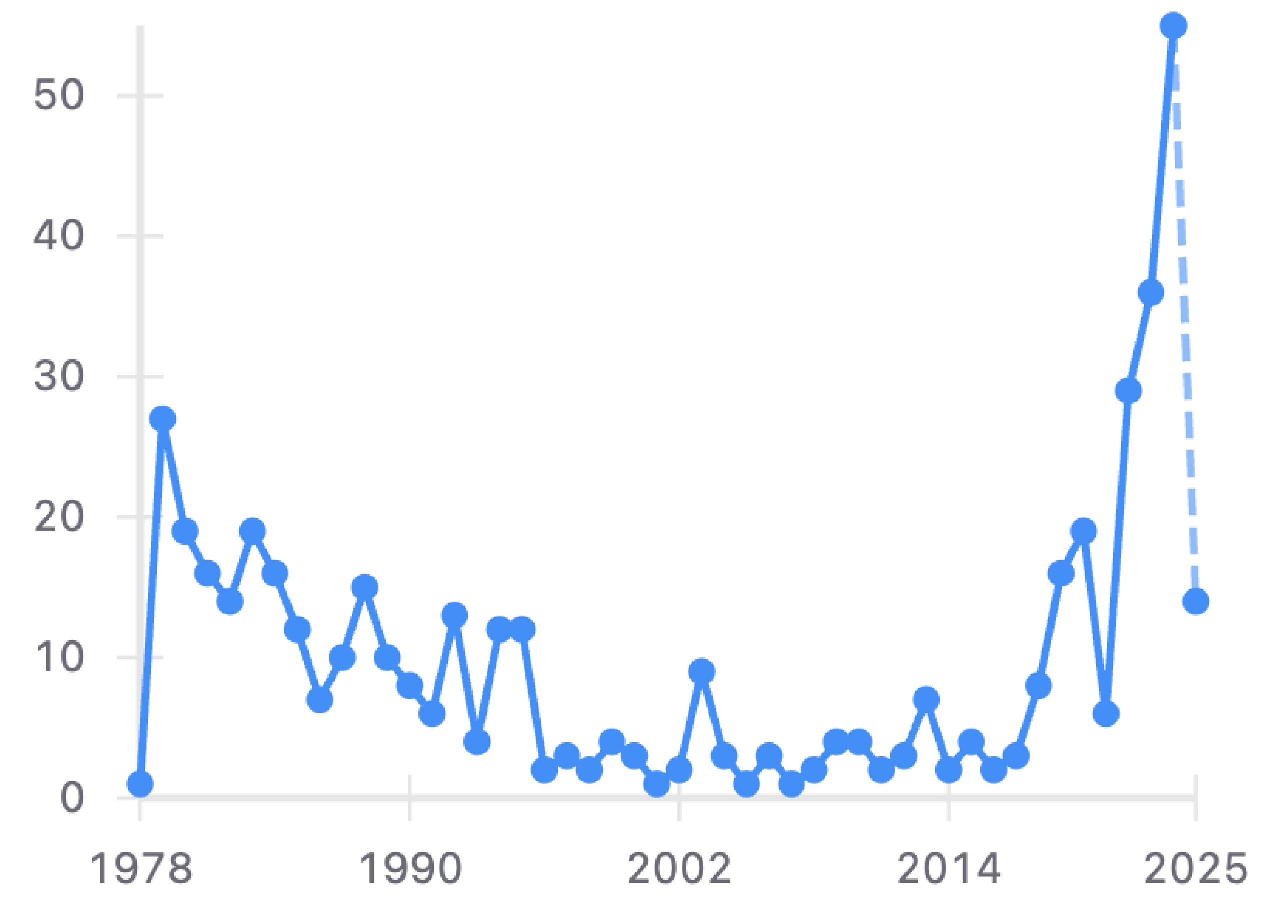
Correlation of energy operators

Energy Correlations in Electron-Positron Annihilation: Testing Quantum Chromodynamics

C. Louis Basham, Lowell S. Brown, Stephen D. Ellis, and Sherwin T. Love
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 21 August 1978)

An experimental measure is presented for a precise test of quantum chromodynamics. This measure involves the asymmetry in the energy-weighted opening angles of the jets of hadrons produced in the process $e^+e^- \rightarrow$ hadrons at energy W. It is special for several reasons: It is reliably calculable in asymptotically free perturbation theory; it has rapidly vanishing (order $1/W^2$) corrections due to nonperturbative confinement effects; and it is straightforward to determine experimentally.

Citations per year



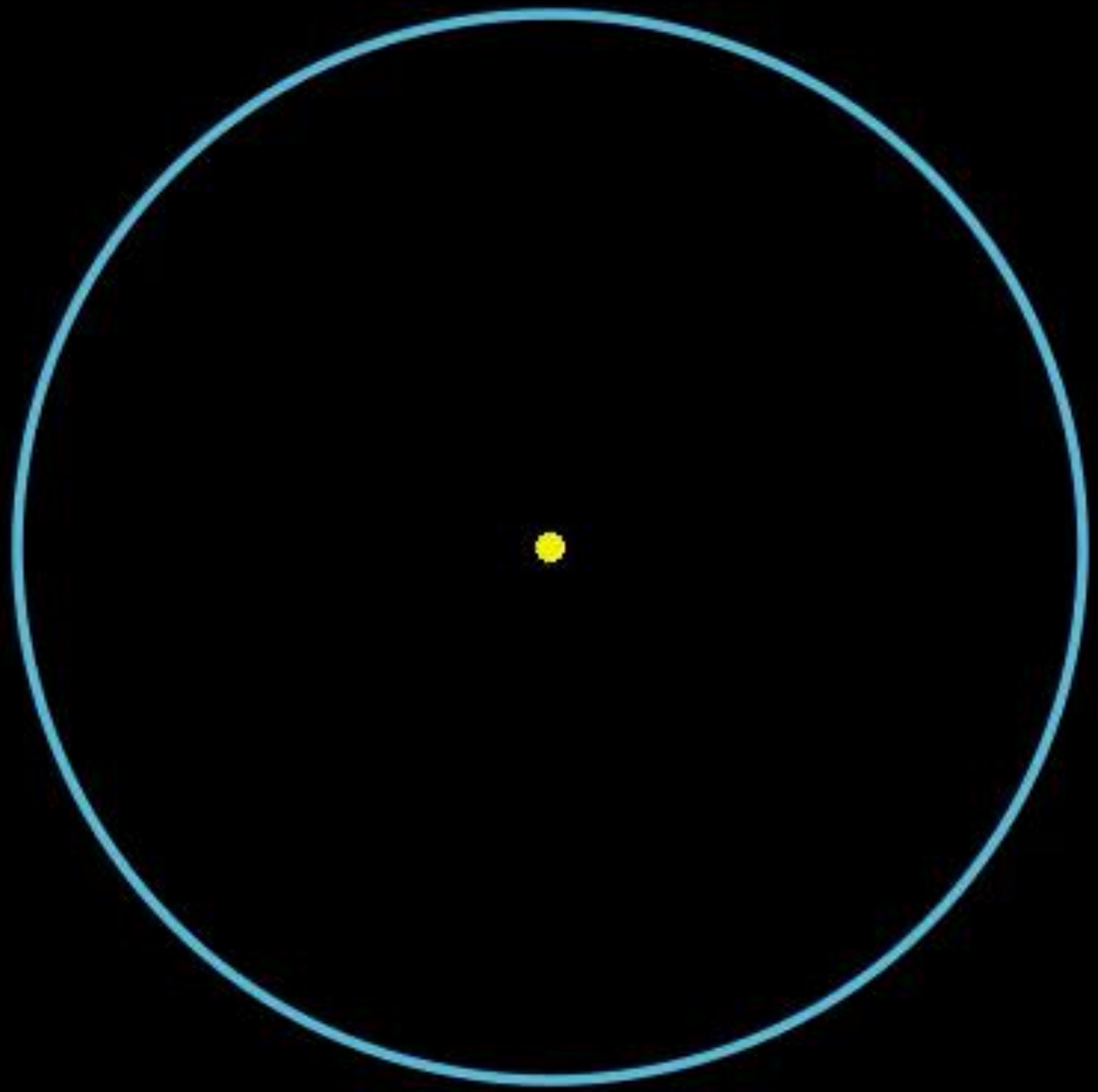
$$\sigma_0 = \int \mathrm{d}^4x \, e^{\mathrm{i}Q\cdot x} \langle 0|J(x)J(0)|0\rangle$$

Appelquist, Georgi, 1973

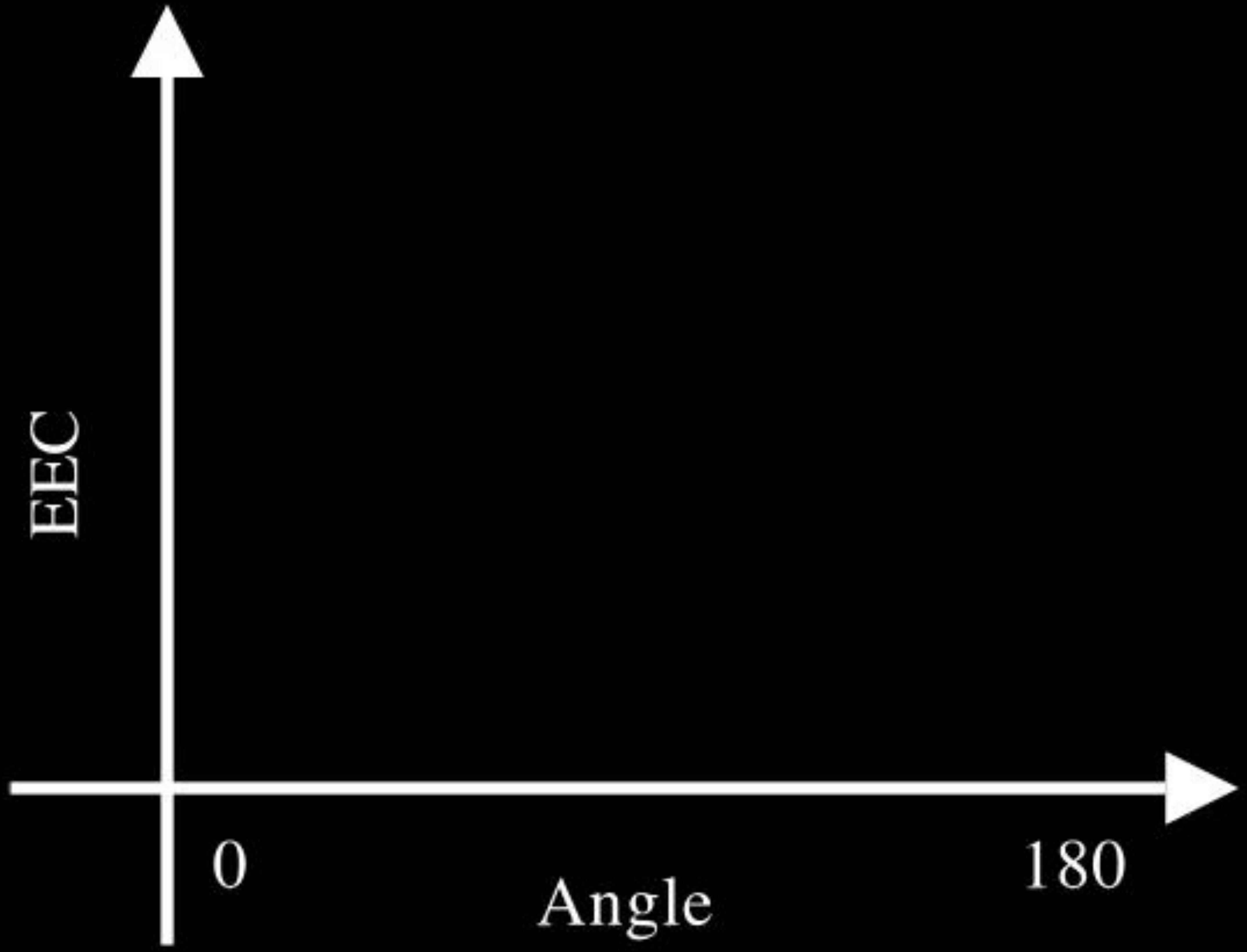
$$\int \mathrm{d}^4x \, e^{\mathrm{i}Q\cdot x} \, \langle 0|J(x)\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\cdots\mathcal{E}(\hat{n}_k)J(0)|0\rangle$$

Basham, Brown, Ellis, Love, 1978

What is Energy-Energy Correlator? Basham, Brown, Ellis, Love, 1978

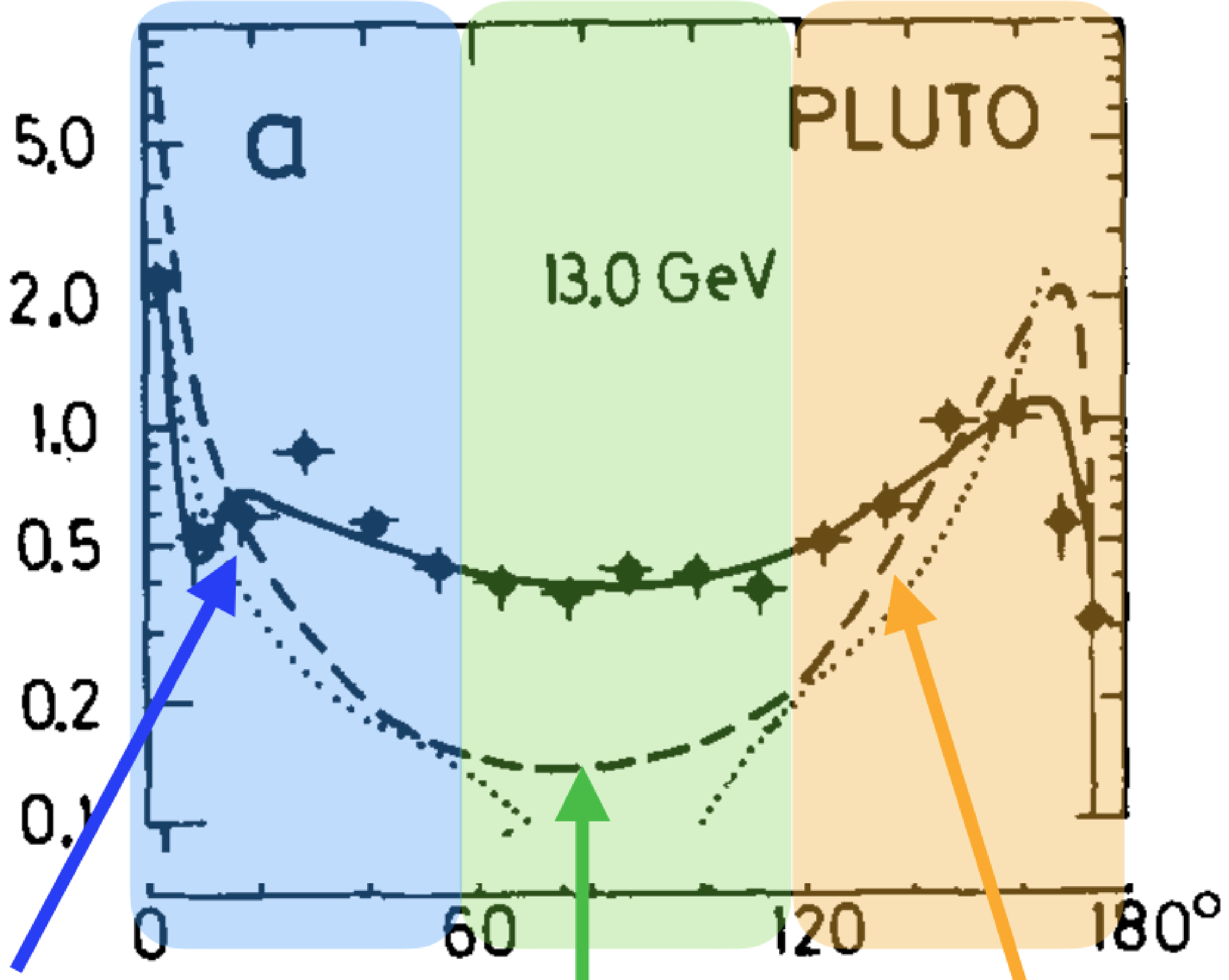


Credit: Hua Xing Zhu



$$\mathrm{EEC}(\cos\theta) = \sum_{a,b} \frac{1}{\sigma_{\mathrm{tot}}} \int d\sigma_{e^+e^- \to abX'} \frac{E_a E_b}{Q^2} \delta(\cos\theta - \hat{\boldsymbol{p}}_a \cdot \hat{\boldsymbol{p}}_b)$$

Experimentalists do not know what and how to extract from it



Collinear:

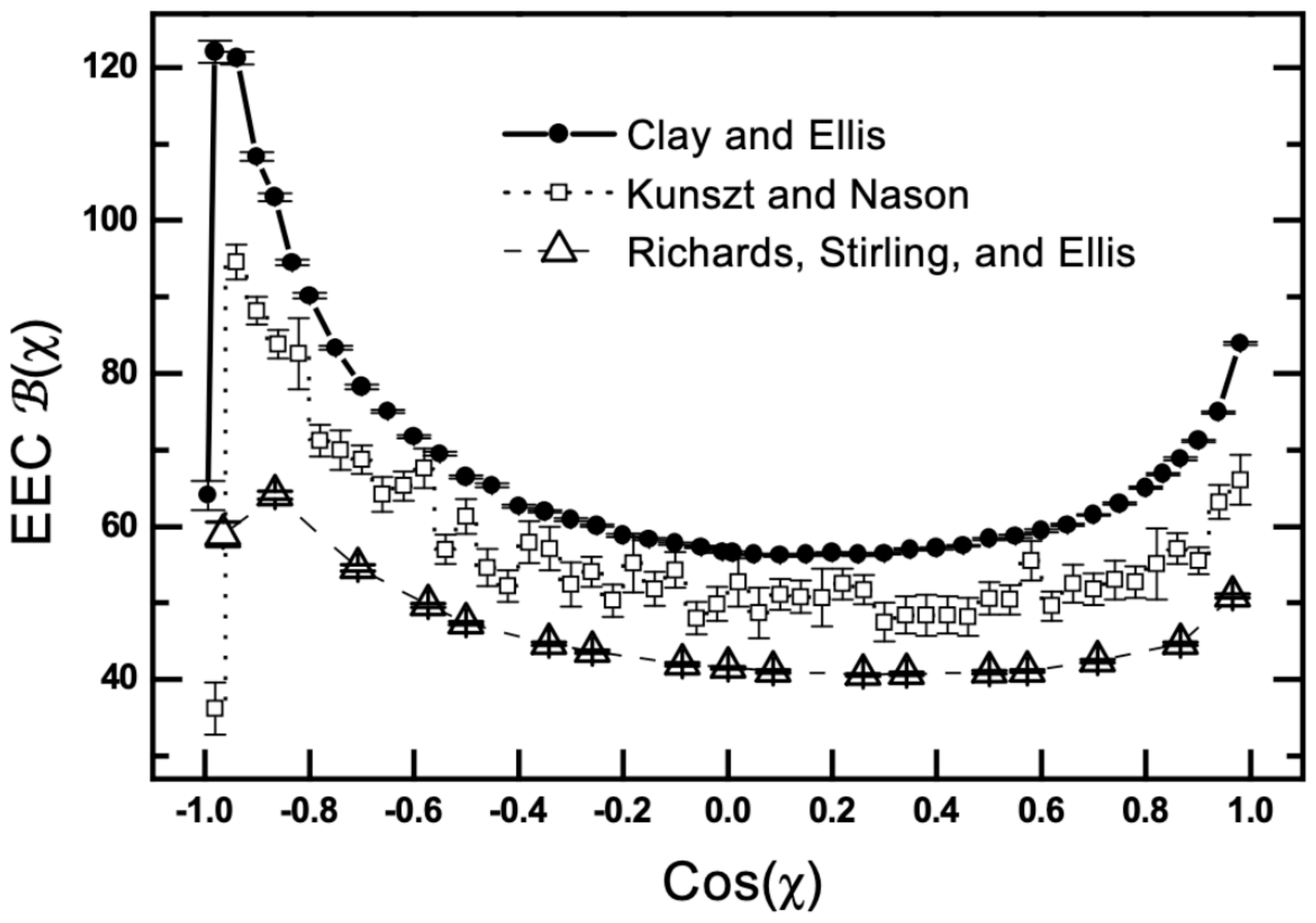
Konishi, Ukawa, Veneziano

Central:

Basham, Brown, Ellis, Love

b2b:

Parisi, Petronzio

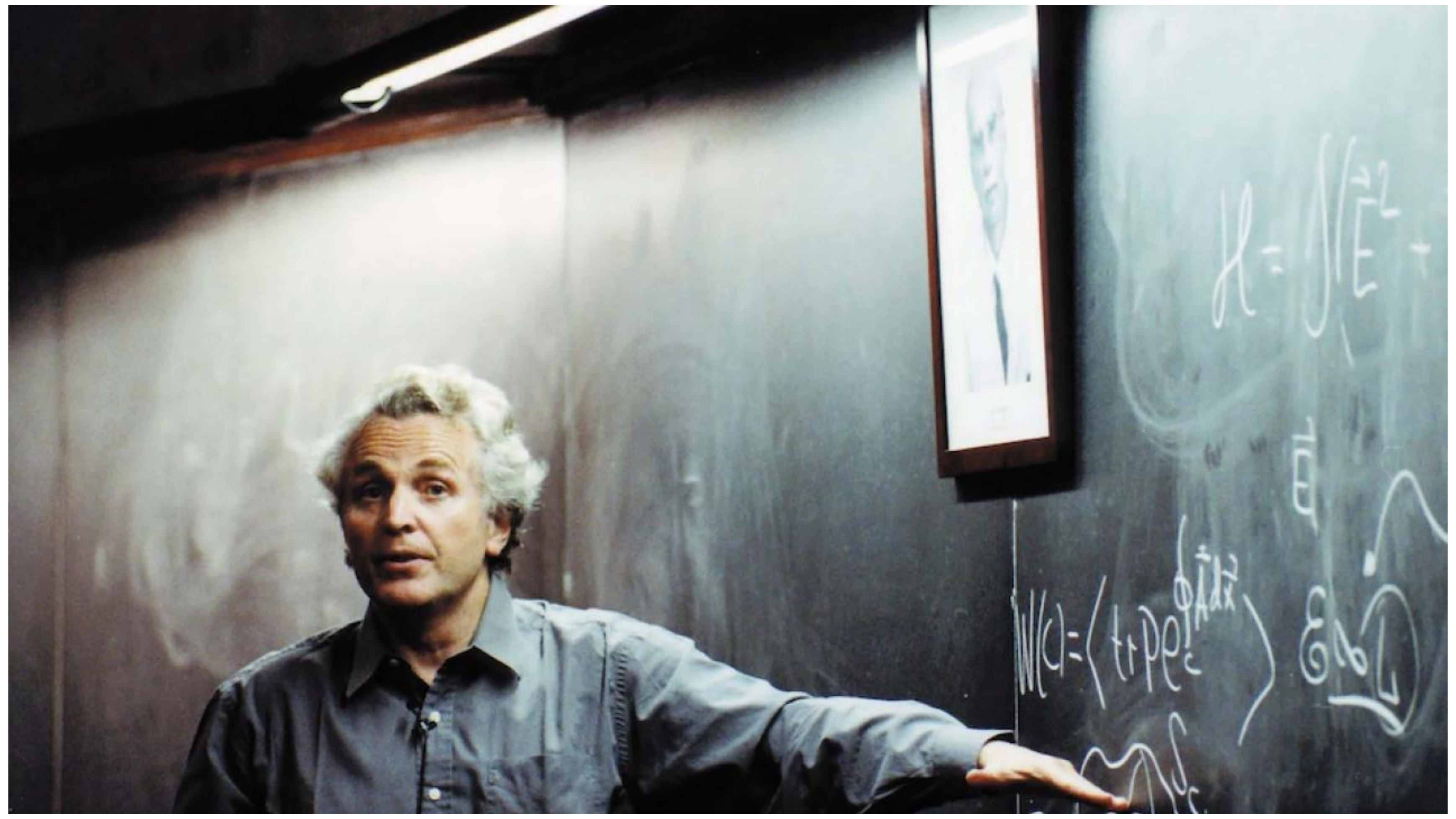


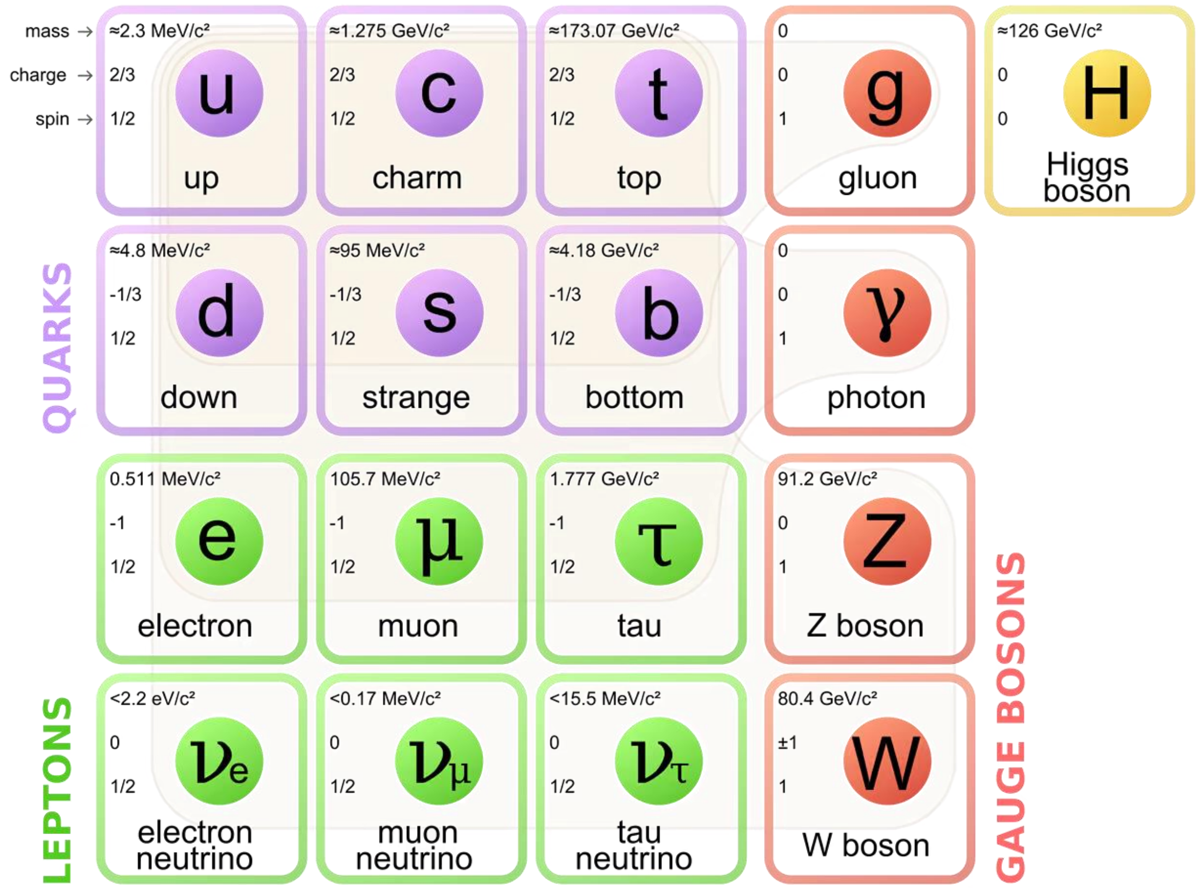
Clay, Ellis, 1995

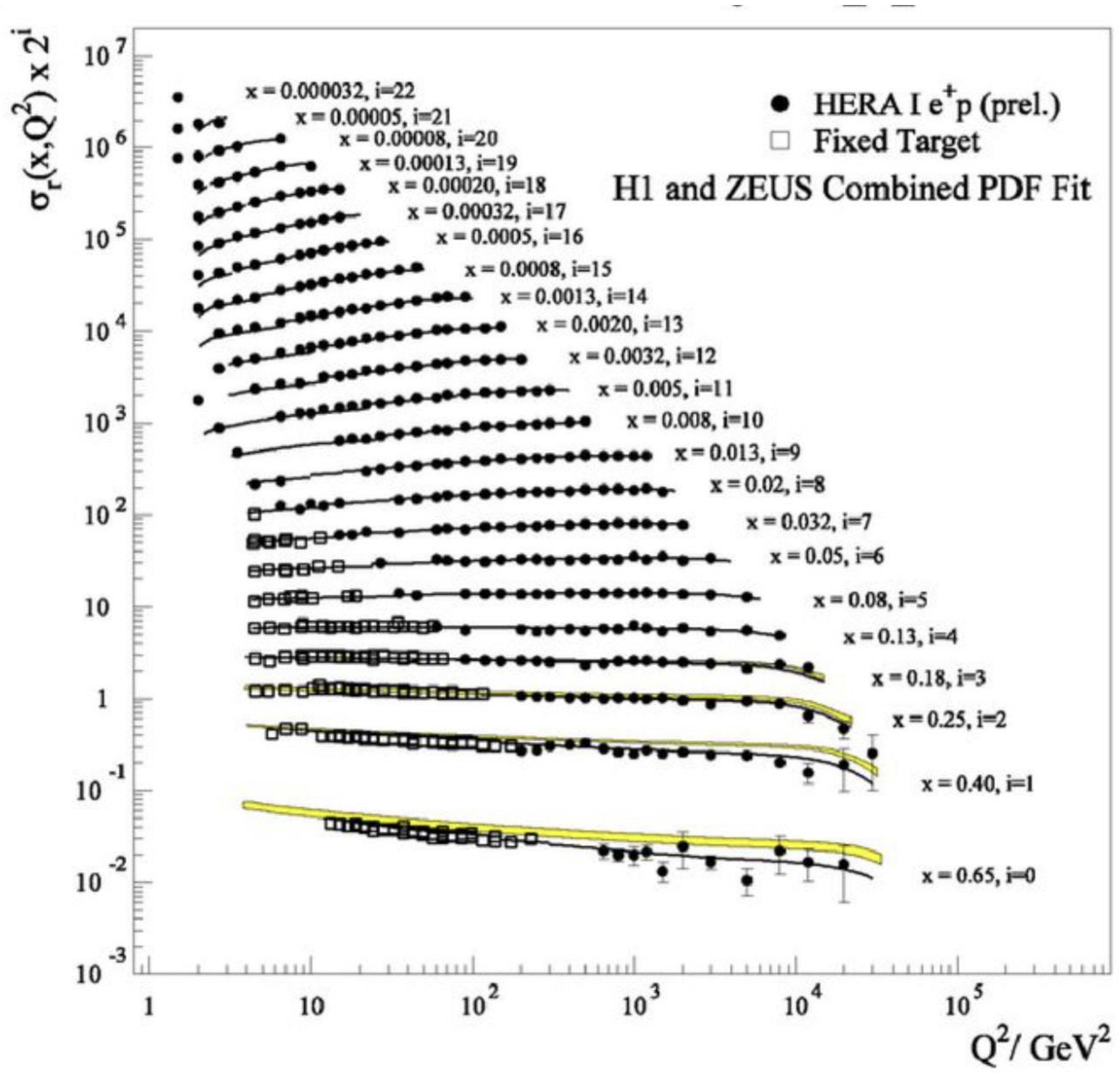
most precise and detailed experimental tests of QCD available [7, 8]. However, that potential has not been realized due to disagreement over the predicted value of the next-to-leading order correction in the strong coupling constant [9, 10, 11, 12, 13]. We report on a new calculation of the $O(\alpha_s{}^2)$

I wanted to learn about elementary particles by studying boiling water.

— Polyakov, 1970



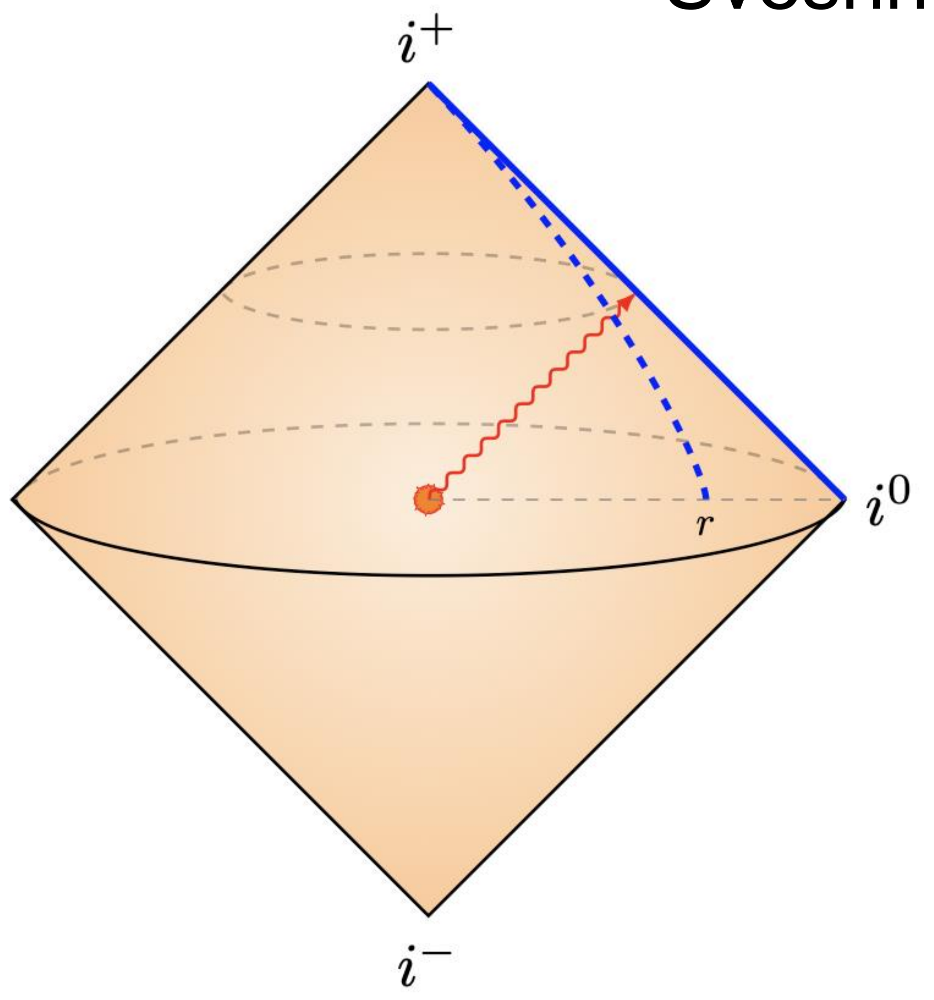


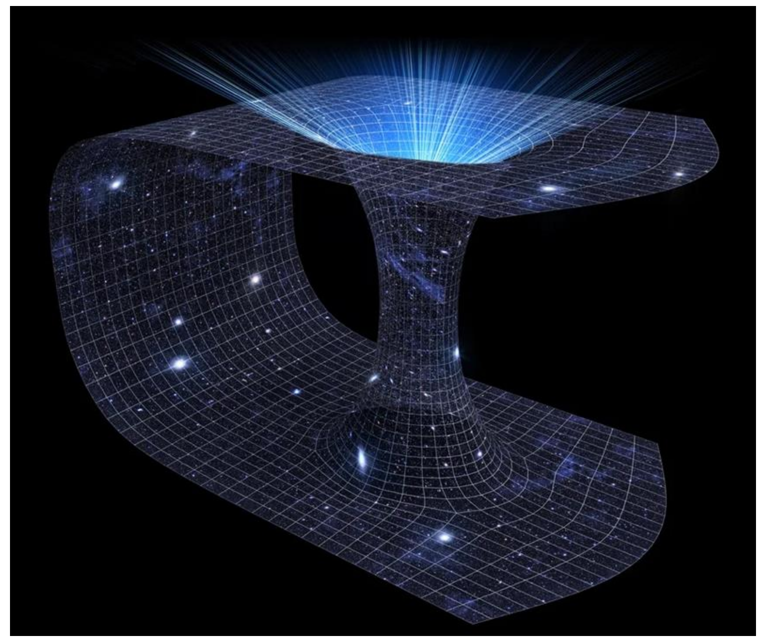


Energy operators

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, \vec{n}_i T^{0i}(t, r\vec{n})$$

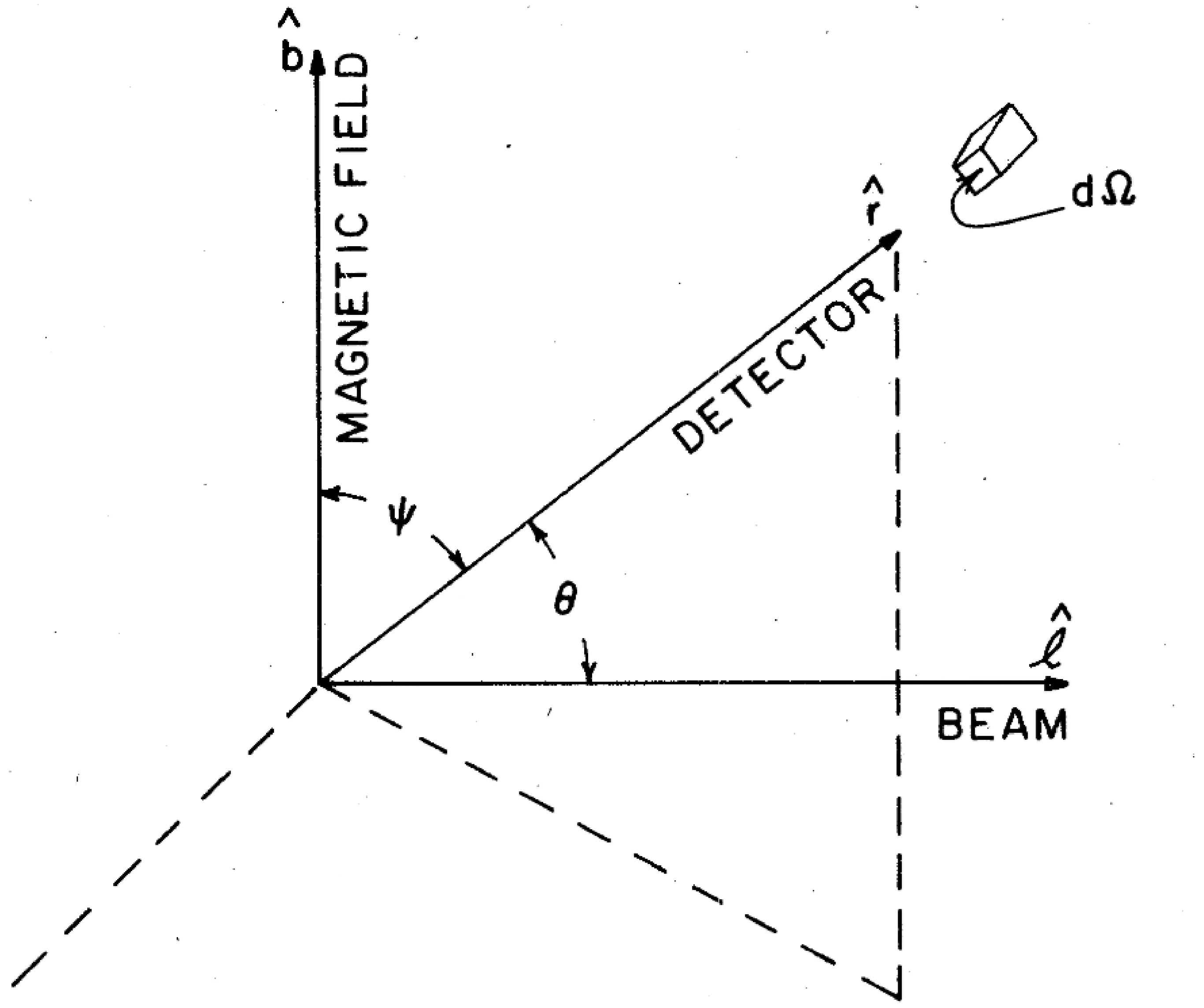
Sveshnikov, Tkachov, 1995





Morris, Thorne, 1989

Basham, Brown, Ellis, Love, 1978



One-point detector function is related to a three-point correlation in QFT!

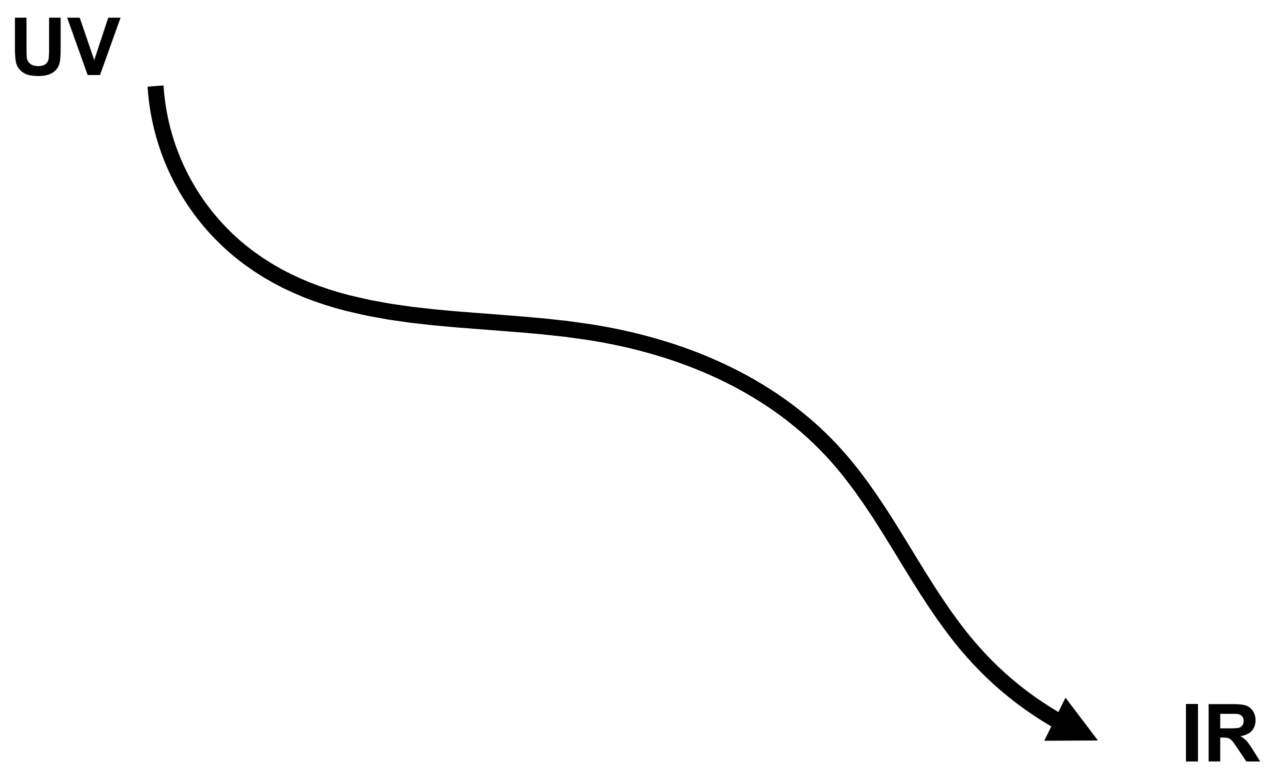
$$\langle JTJ \rangle$$

Conformal collider bound

Zamolodchikov Cardy

2024 fundamental physics breakthrough prize

$$T^{\mu}_{\mu} = \frac{c}{16\pi^2} W_{\mu\nu\delta\sigma} W^{\mu\nu\delta\sigma} - \frac{a}{16\pi^2} E$$



a and c monotonically deceases under RG flow

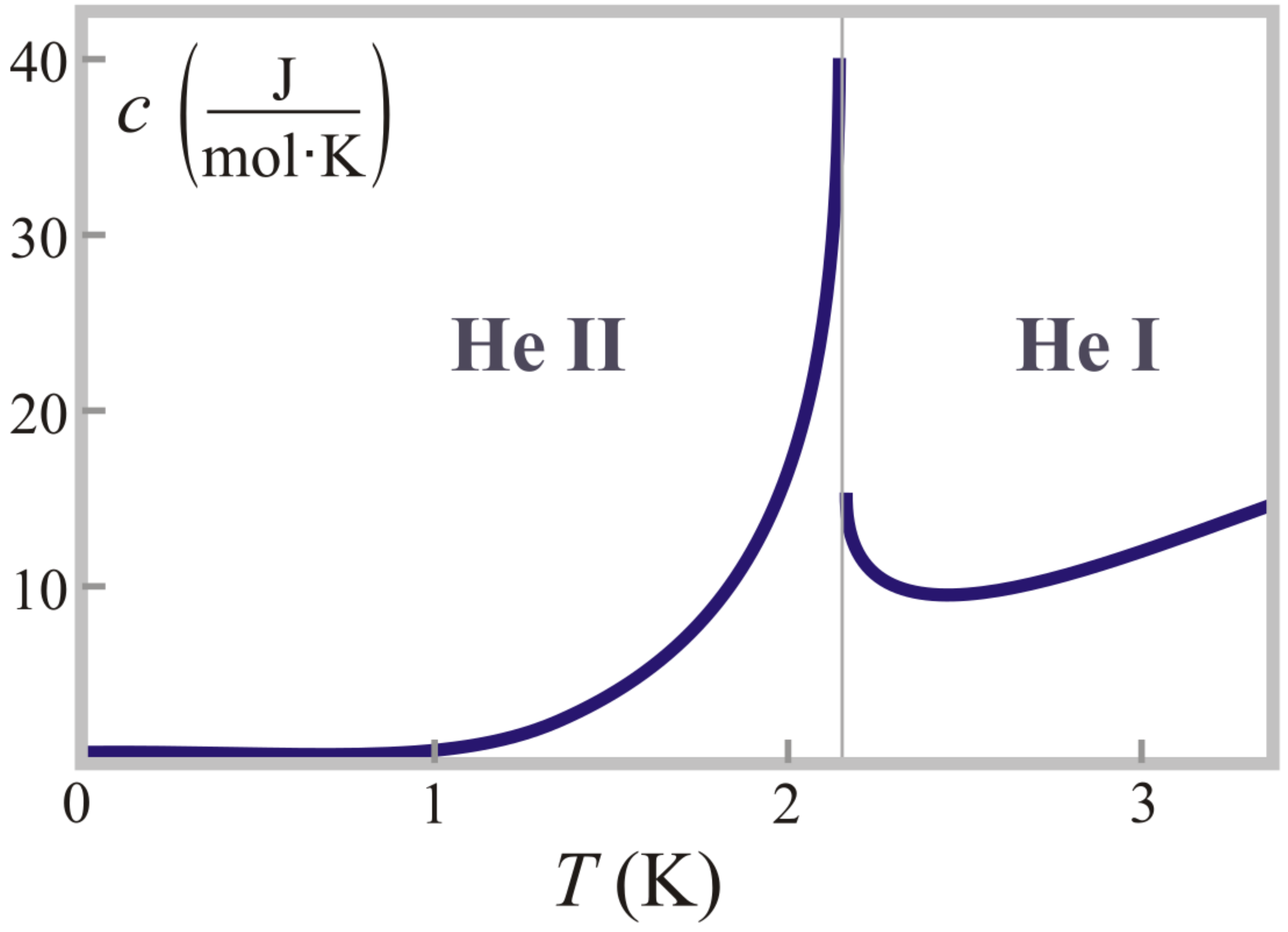
$$\langle \mathcal{E}(\theta) \rangle = 1 + 3\frac{c-a}{c}\left(\cos^2\theta - \frac{1}{3}\right)$$

$$\frac{3c}{2} \geq a \geq 0$$

Hofman, Maldacena, 2008

Lightray OPE

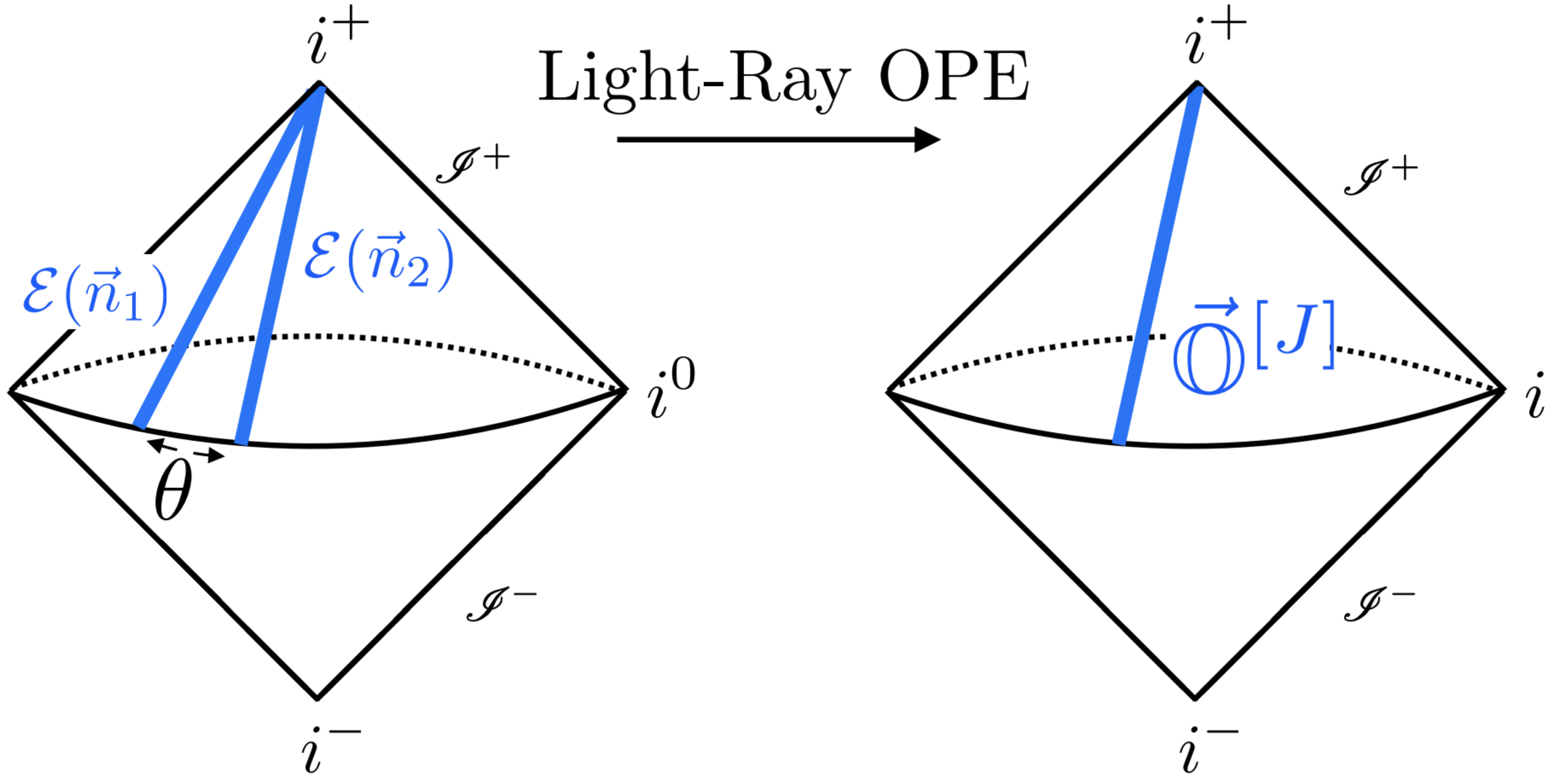
$$O_1(x_1)O_2(x_2) = \sum_3 c_{123}(x_1 - x_2)O_3(x_2)$$



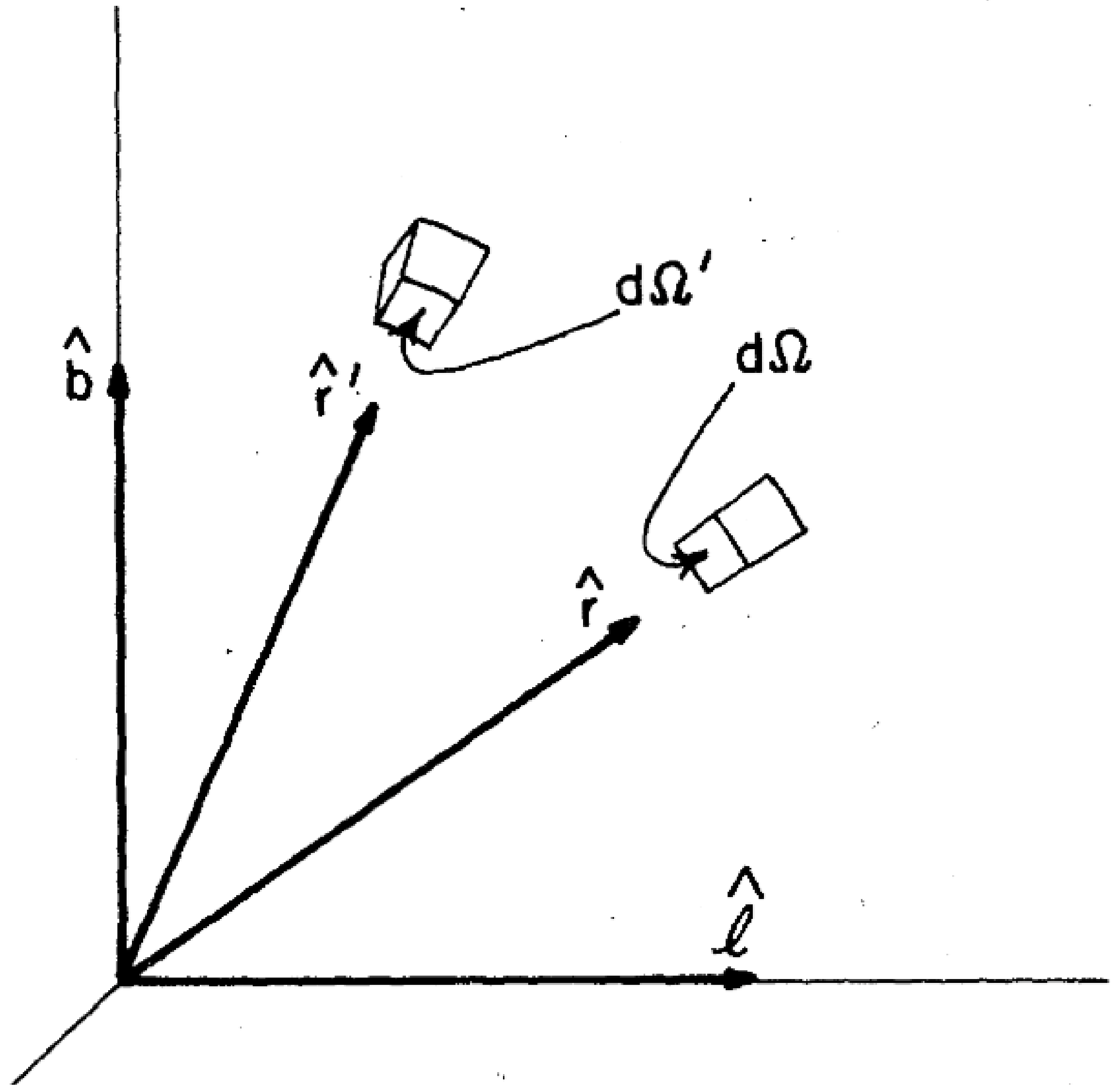
What happen if two detectors are close to each other?

Hofman, Maldacena, 2008
Kravchuk, Simmons-Dufffin, 2018
Kravchuk, Kologlu, Simmons-Dufffin, Zhiboedov, 2019

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = \sum c_i \theta^{\tau_i - 4} \mathbb{O}_i(\hat{n}_1)$$



The analytic beauty of energy correlators



$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma^{(3)}}{\mathrm{d}C} = \frac{\alpha_s}{2\pi}C_F\int_{x_2^-(C)}^{x_2^+(C)}\mathrm{d}x \times \frac{6x\left[C\left(x^3+(x-2)^2\right)-6(1-x)(1+x^2)\right]}{C(C+6)^2(x-6/(C+6))\sqrt{(6/(C+6)-x)(x_2^+-x)(x-x_2^-)x}}$$

LO result for C parameter involves elliptic function

Difficult to make progress

Ellis, Ross, Terrano, 1981

One-point detector function is related to a local four-point correlation in QFT!

$$\langle \mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\rangle \sim \langle JTTJ\rangle$$

Huge efforts in the past on computing 4-pt correlators and related integrals =>

Full analytic results of EEC up to NLO (N=4 SYM)

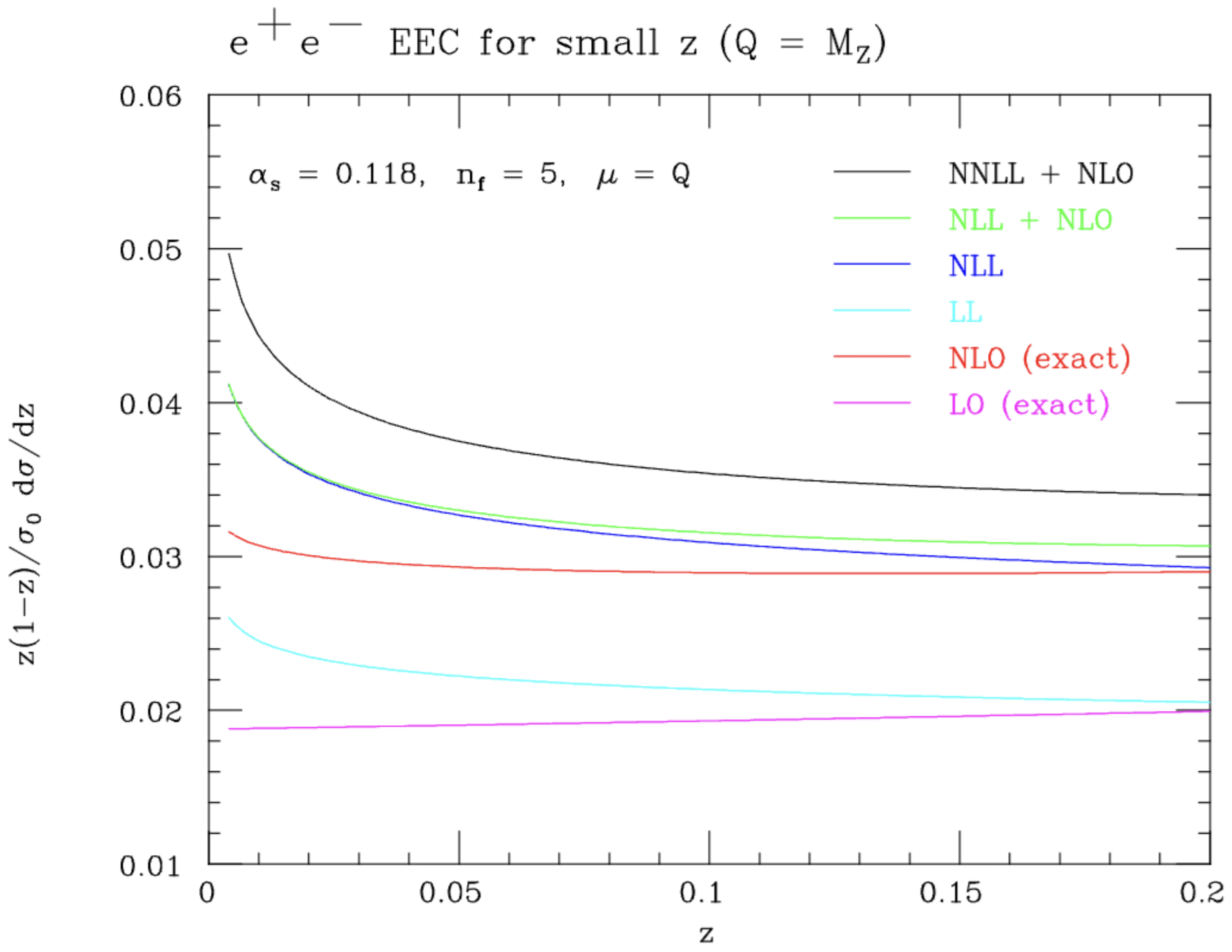
$$F_2(z) = 4\sqrt{z}\left[\mathrm{Li}_2(-\sqrt{z}) - \mathrm{Li}_2(\sqrt{z}) + \frac{\ln z}{2}\ln\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\right] + (1+z)\left[2\mathrm{Li}_2(z) + \ln^2(1-z)\right] + 2\ln(1-z)\ln\left(\frac{z}{1-z}\right) + z\frac{\pi^2}{3}, \quad (46)$$

$$F_3(z) = \frac{1}{4}\left\{(1-z)(1+2z)\left[\ln^2\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\ln\left(\frac{1-z}{z}\right) - 8\mathrm{Li}_3\left(\frac{\sqrt{z}}{\sqrt{z}-1}\right) - 8\mathrm{Li}_3\left(\frac{\sqrt{z}}{\sqrt{z}+1}\right)\right] - 4(z-4)\mathrm{Li}_3(z)\right.$$
$$+ 6\left(3+3z-4z^2\right)\mathrm{Li}_3\left(\frac{z}{z-1}\right) - 2z(1+4z)\zeta_3 + 2\left[2\left(2z^2-z-2\right)\ln(1-z) + (3-4z)z\ln z\right]\mathrm{Li}_2(z)$$
$$\left. + \frac{1}{3}\ln^2(1-z)\left[4\left(3z^2-2z-1\right)\ln(1-z) + 3(3-4z)z\ln z\right] + \frac{\pi^2}{3}\left[2z^2\ln z - \left(2z^2+z-2\right)\ln(1-z)\right]\right\}. \quad (47)$$

Balitsky et al., 2013	**N=4 SYM@NLO**
Henn, Sokatchev, Yan, Zhiboedov, 2019	**N=4 SYM@NNLO**
Dixon, Luo, Shtabovenko, Yang, HXZ, 2018	**QCD@NLO**

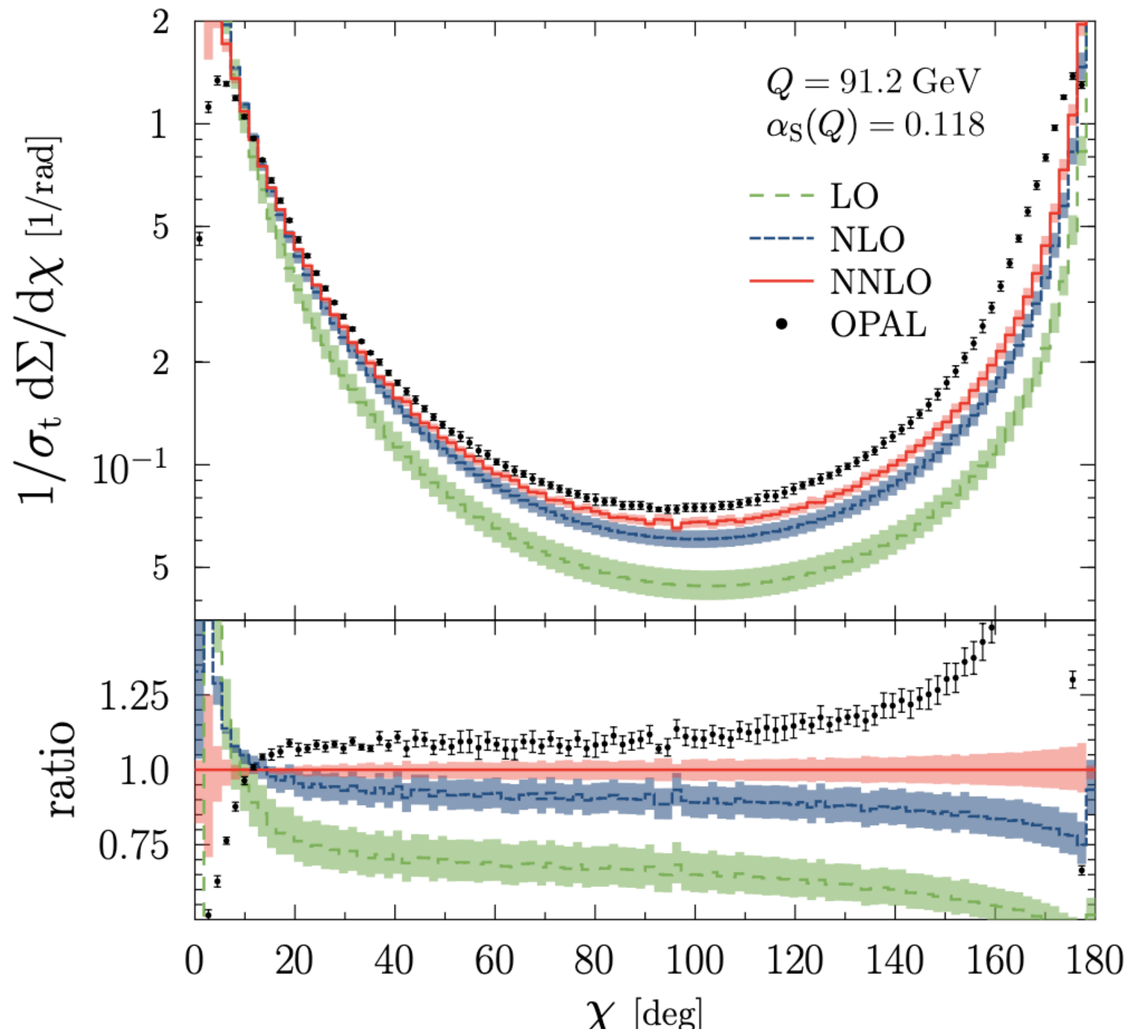
Back to QCD: pert. theory

Collinear resummation

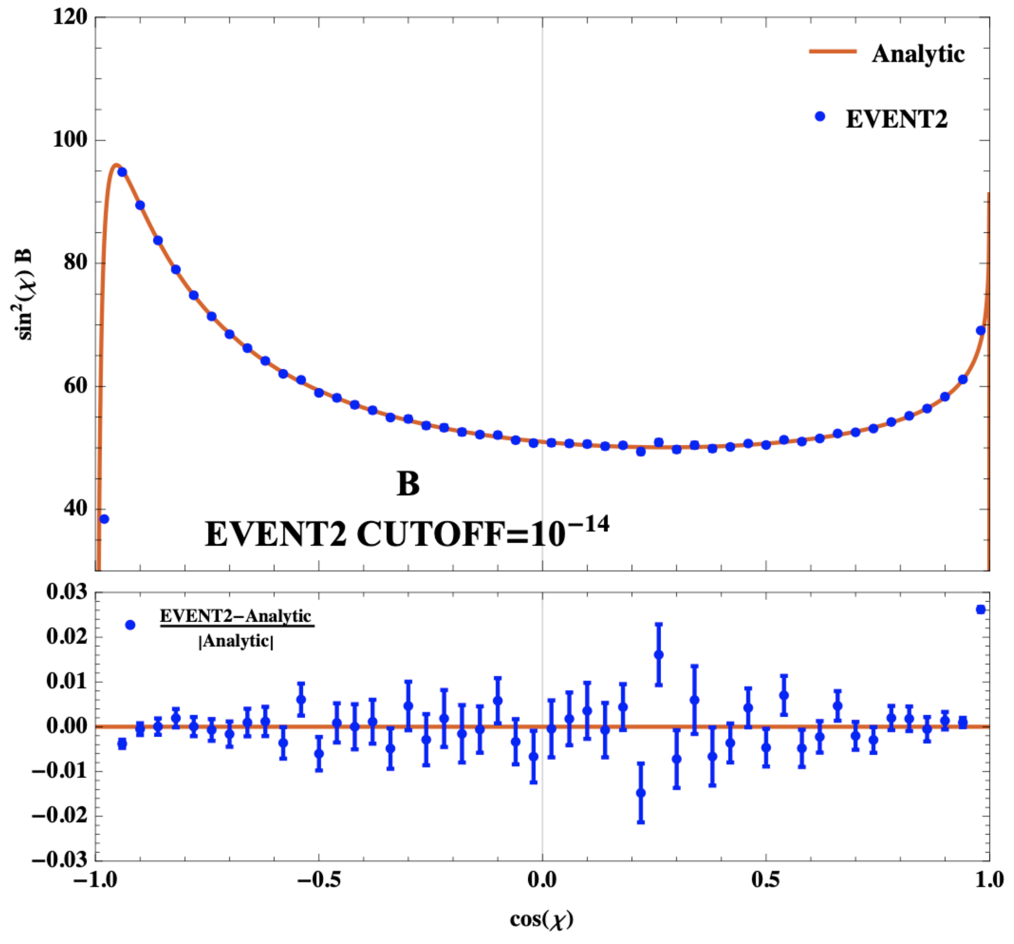


Konishi, Ukawa, Veneziano, 1979
Ellis, Stirling, 1981
Dixon, Moult, HXZ, 1999

Fixed order

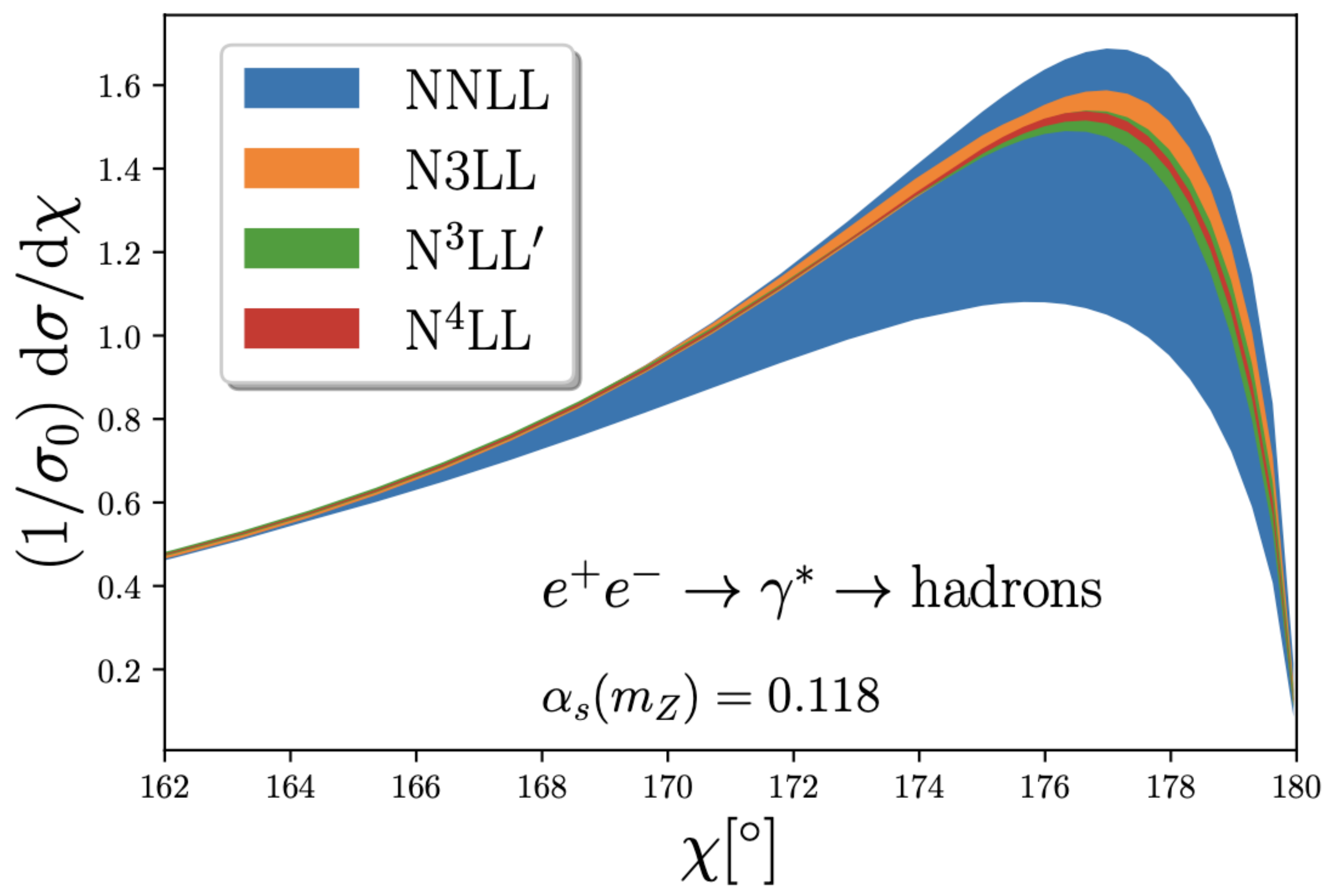


Del Duca, Duhr, Kardos, Somogyi, Trocsanyi, 2016



Dixon, Luo, Shtabovenko, Yang, HXZ, 2018

b2b resummation



Parasi, Petronzio, 1979
Chao, Soper, Collins, 1983
de Florian, Grazzini, 2004
Moult, HXZ, 2018
Mistlberger, Duhr, Vita, 2022

Back to QCD: non-pert. theory

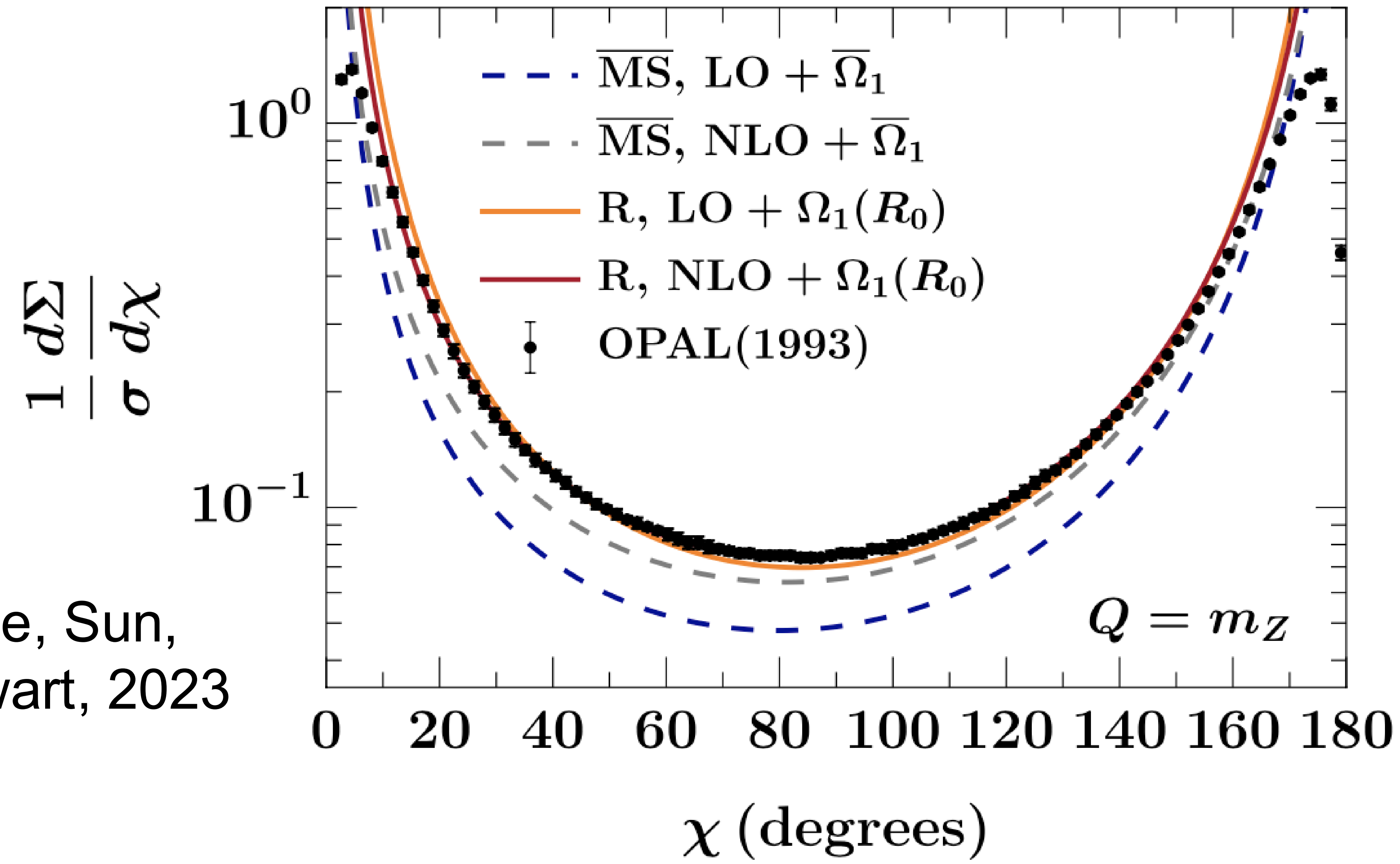
$$\langle \mathrm{EEC}(\chi)\rangle_{\mathrm{NP}} = \frac{1}{Q}\langle 0|W^{\dagger}(0)\ \mathcal{E}(\vec{n})\ W(0)\ |0\rangle = \frac{1}{Q}\mathcal{G}(\chi)$$

Korchemsky, Sterman, 1999

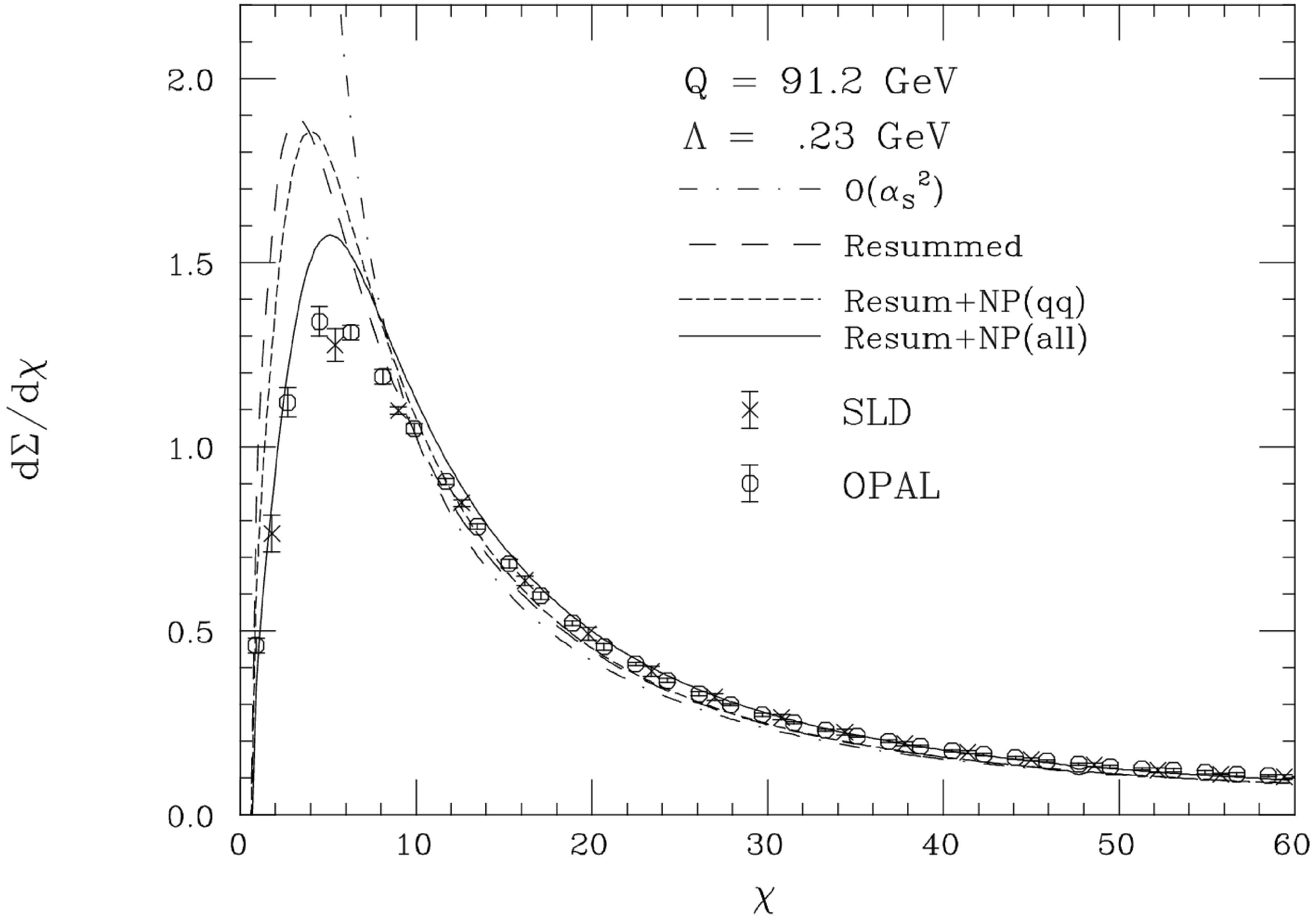
$$\mathcal{G}(\chi) = \frac{\lambda_1}{4\pi\ \sin^3\chi}$$

enhanced power corrections from symmetry analysis

Compare with perturbation: $\sim \frac{1}{\sin^2\chi}$



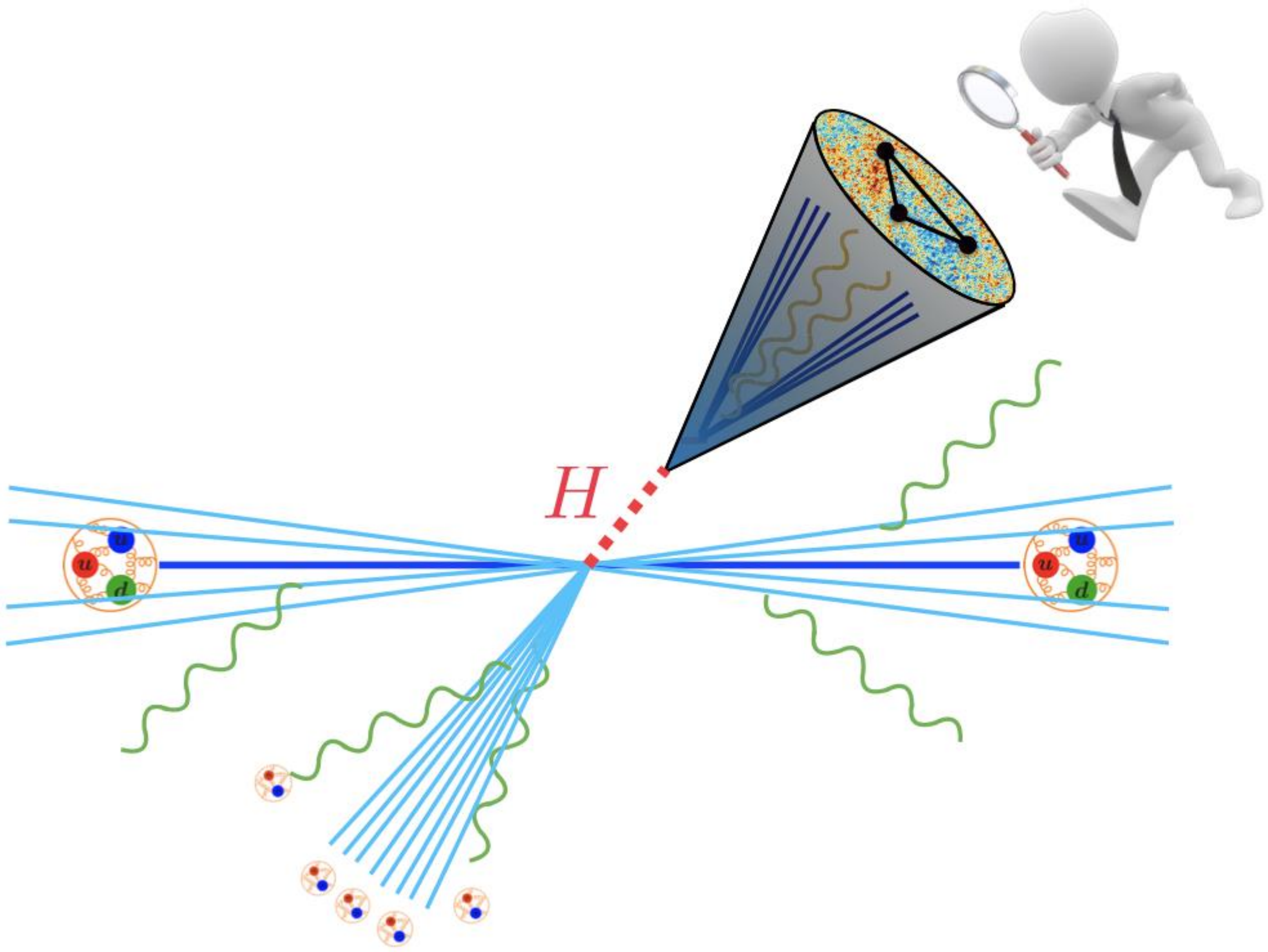
Lee, Sun, Stewart, 2023



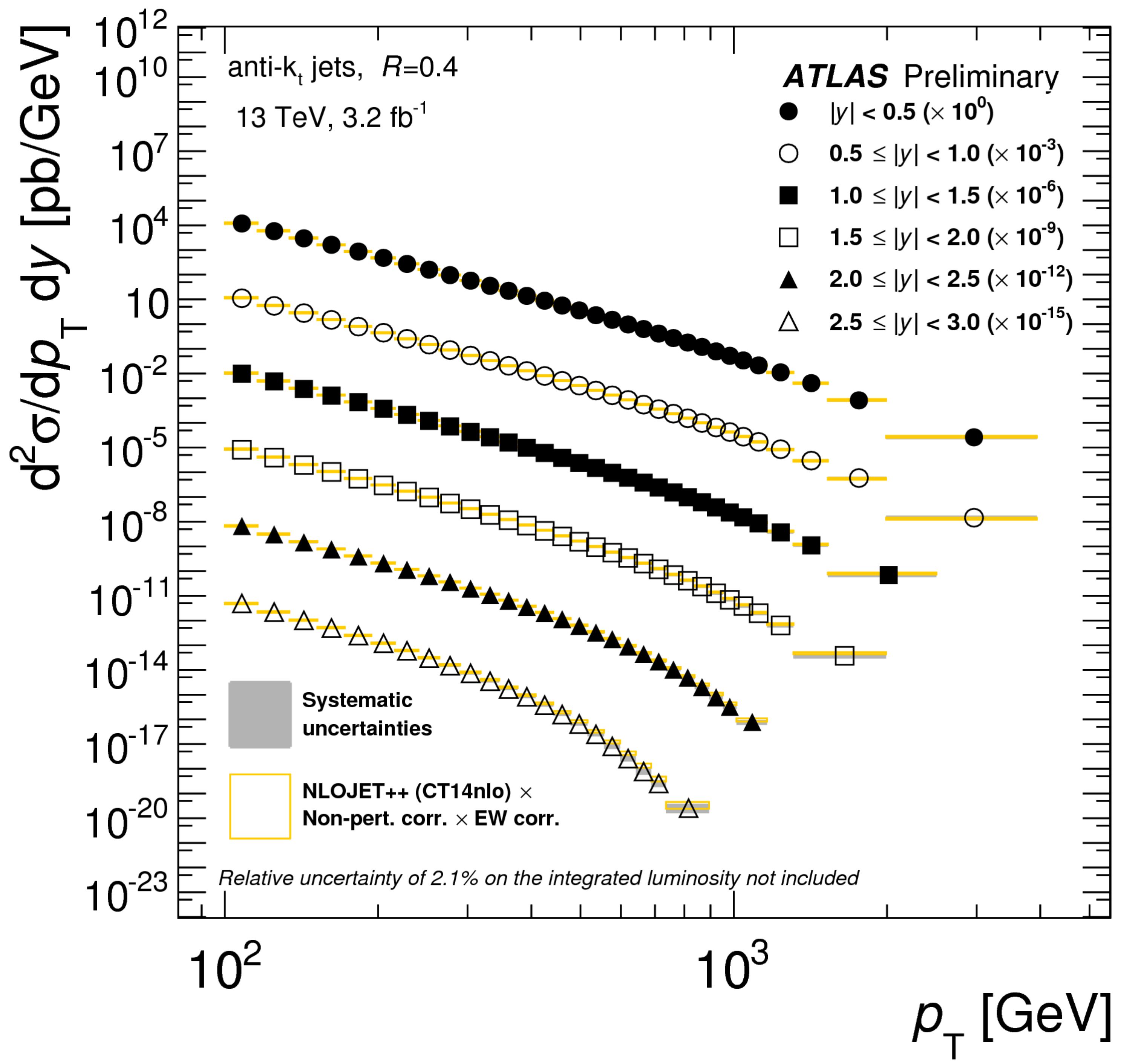
Dokshitzer, Marchesini, Webber, 1999

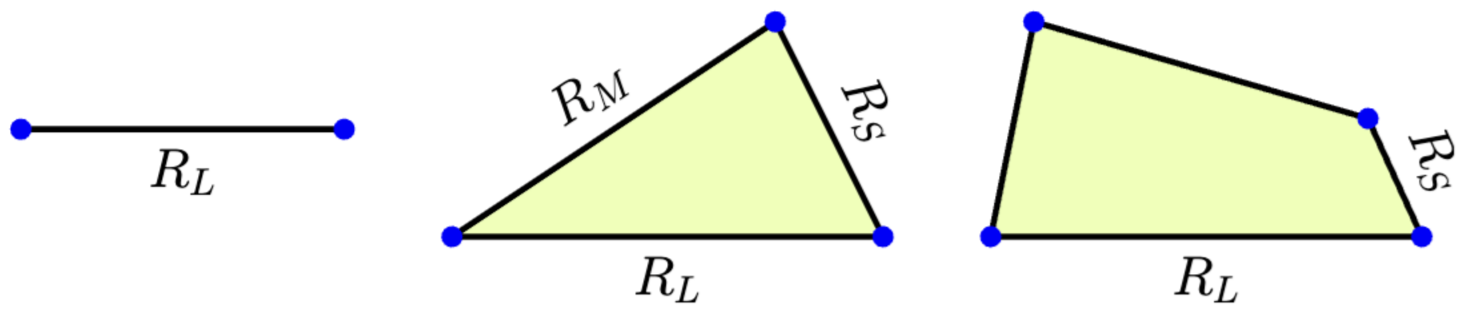
Importance of quark-gluon correlation

Looking inside a jet



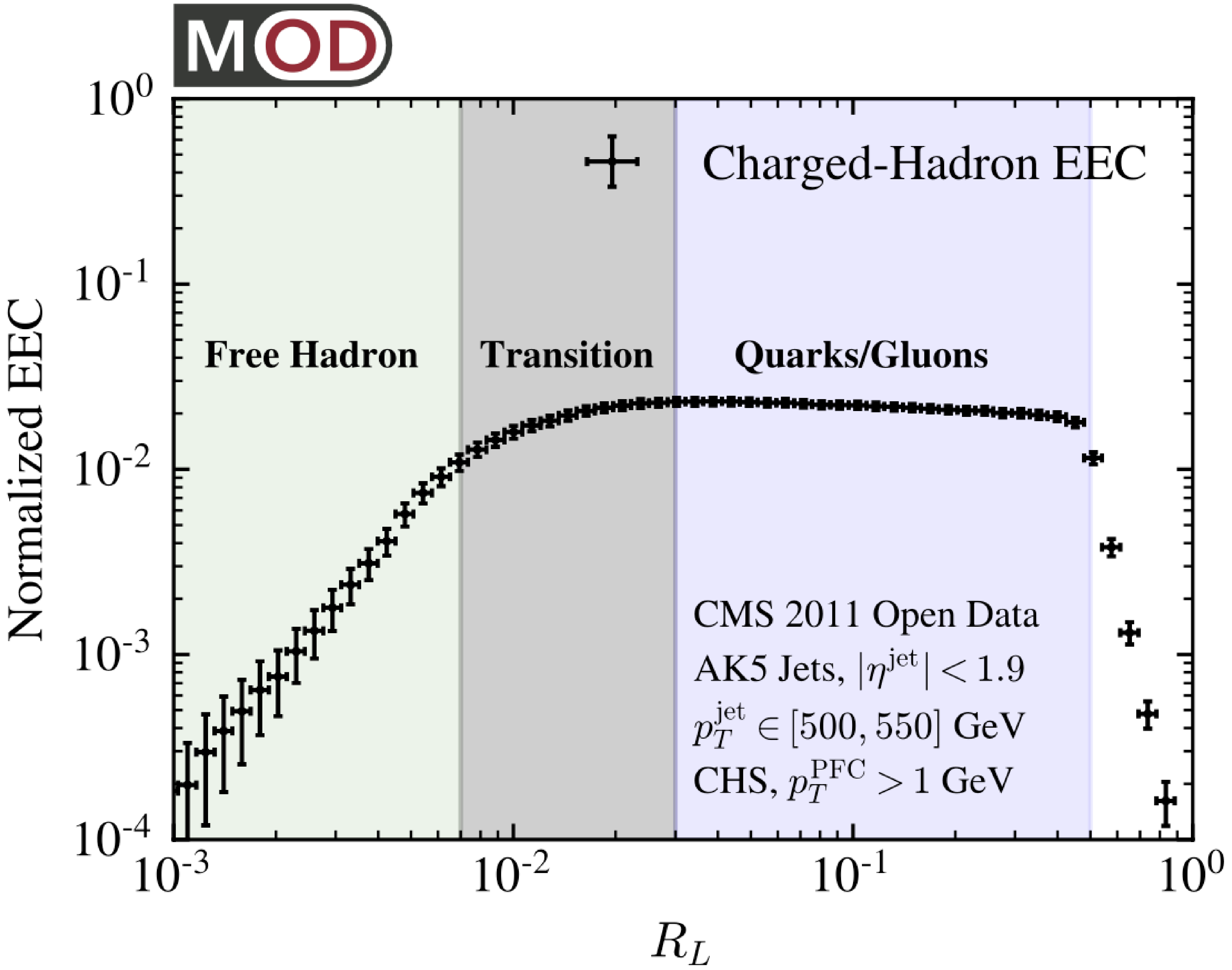
Rethinking jet substructure in terms of energy correlators



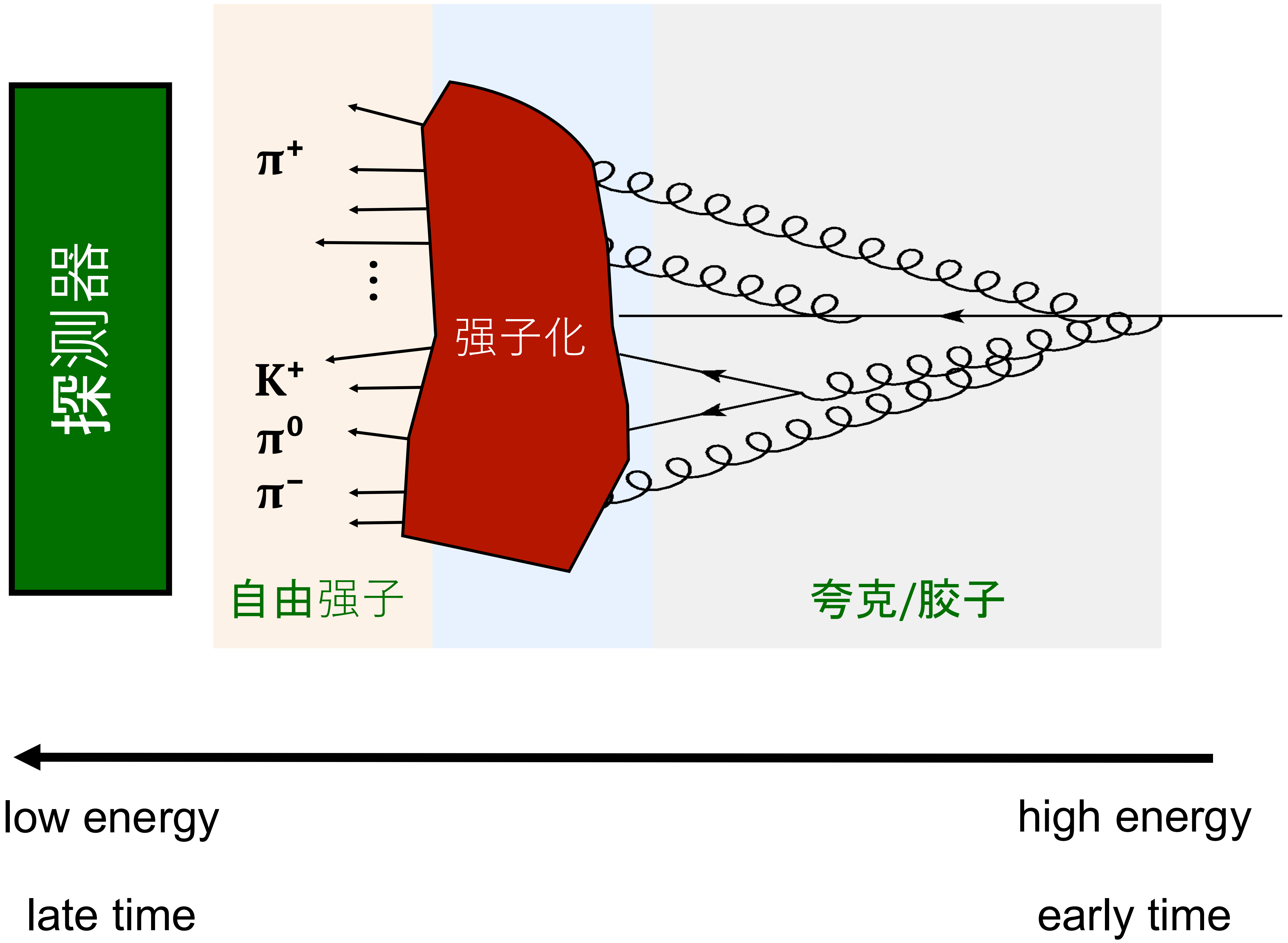


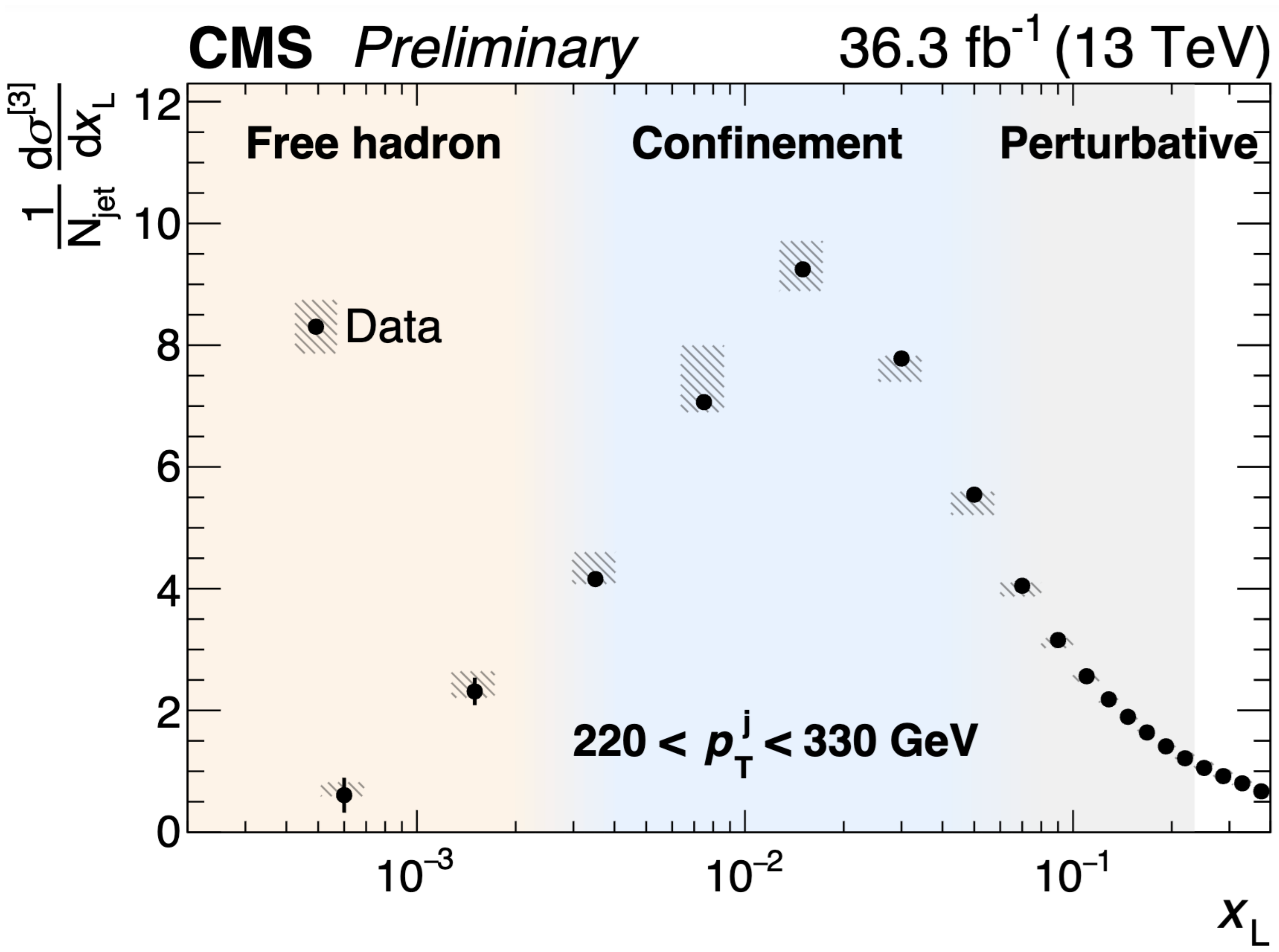
Chen, Moult, Zhang, HXZ, 2020
Chen, Luo, Moult, Yang, Zhang, HXZ, 2019

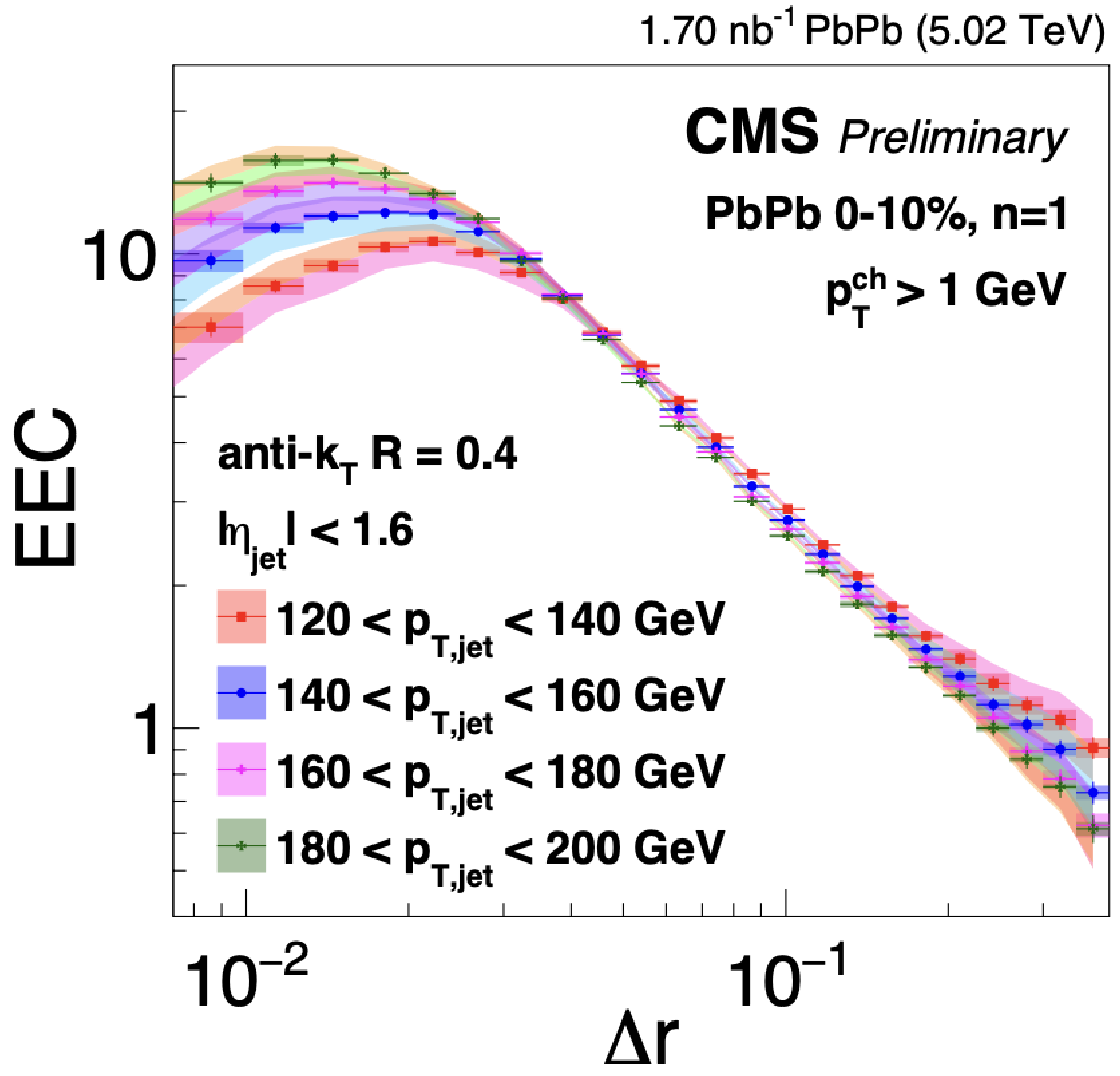
Discovery of scaling and imaging the hadronization transition

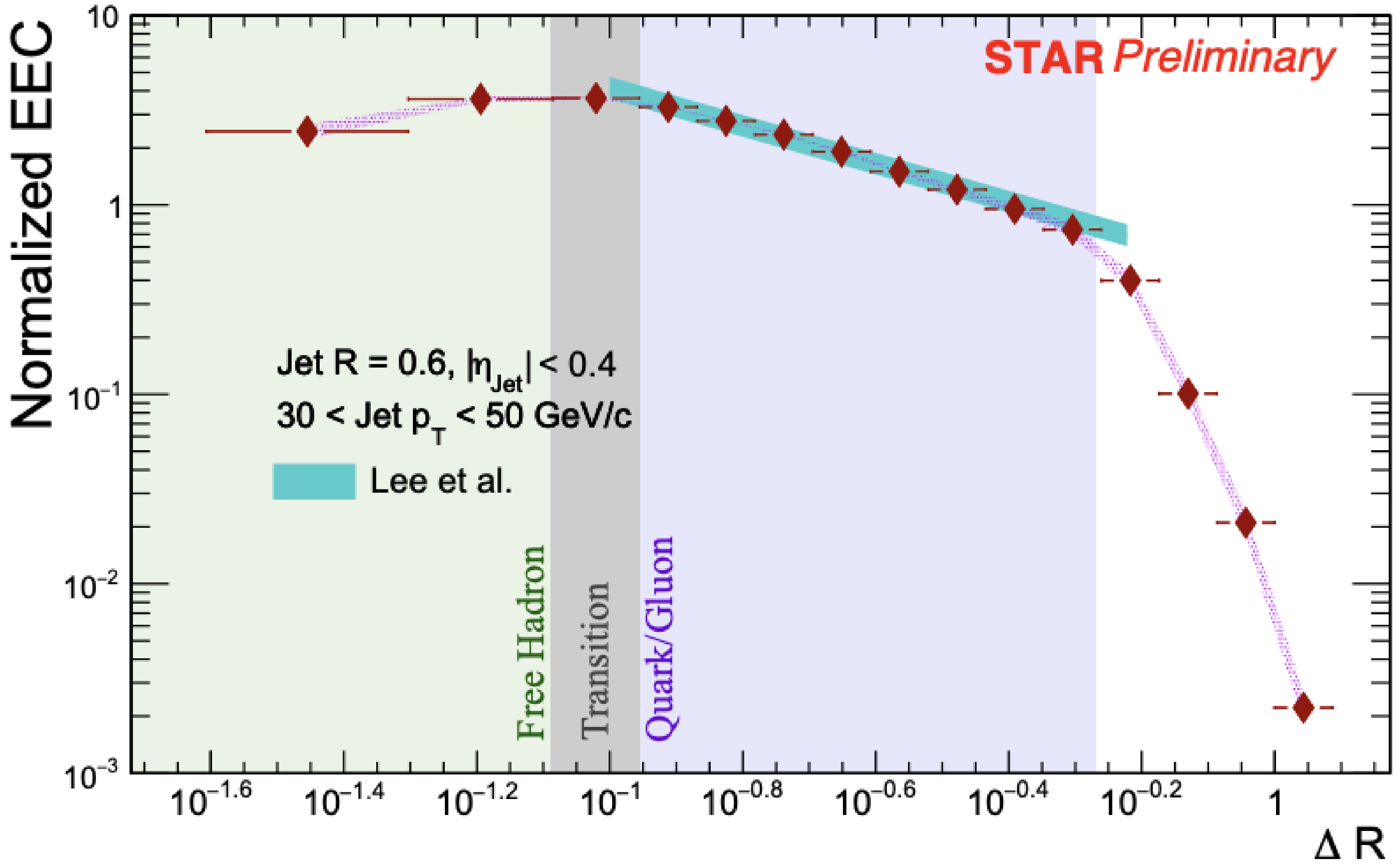


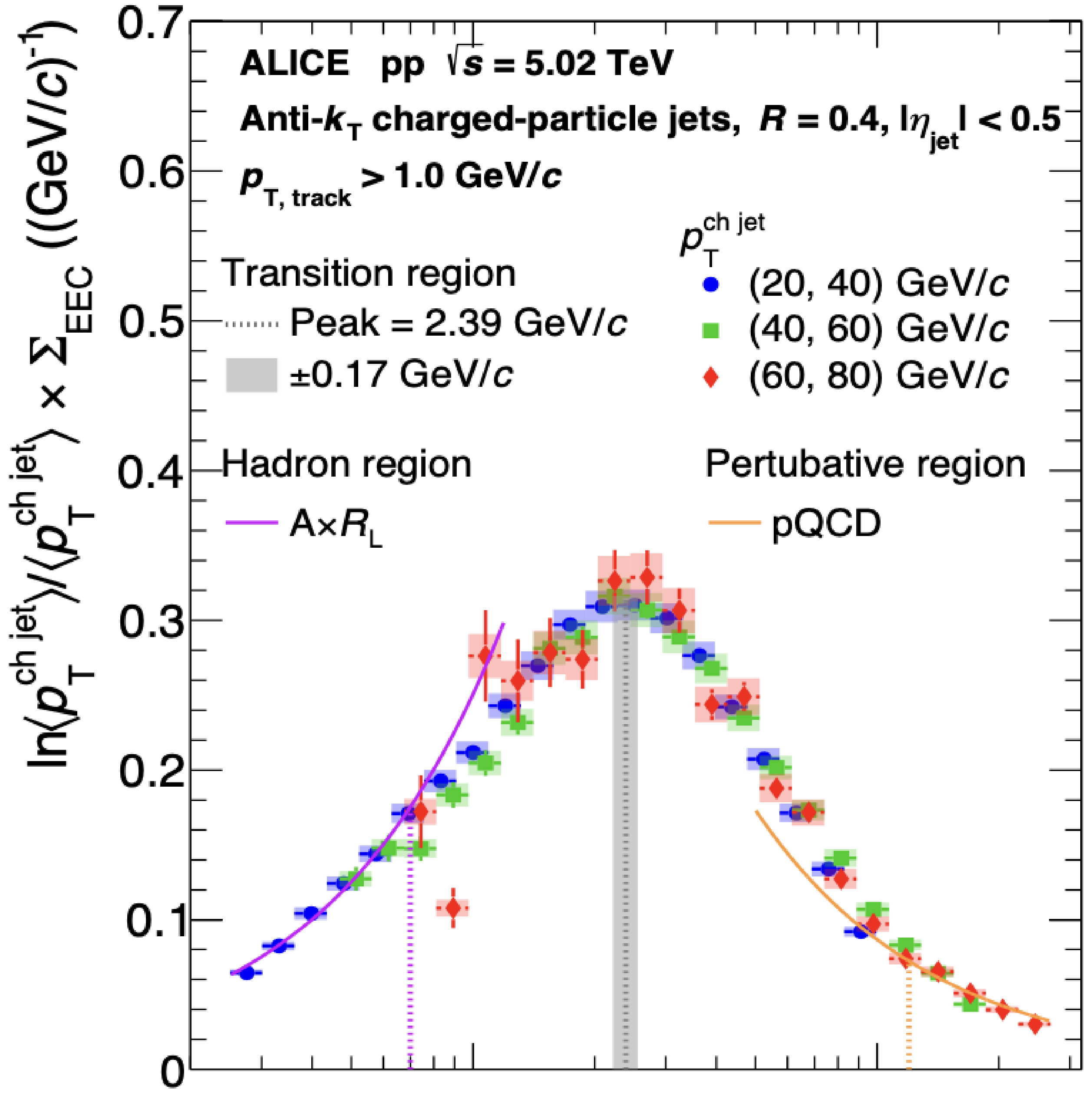
Komiske, Moult, Thaler, HXZ, 2022




CMS Preliminary
36.3 fb^{-1} (13 TeV)
Free hadron
Confinement
Perturbative
Data
$220 < p_T^j < 330$ GeV
$\frac{1}{N_{jet}} \frac{d\sigma^{[3]}}{dx_L}$
x_L

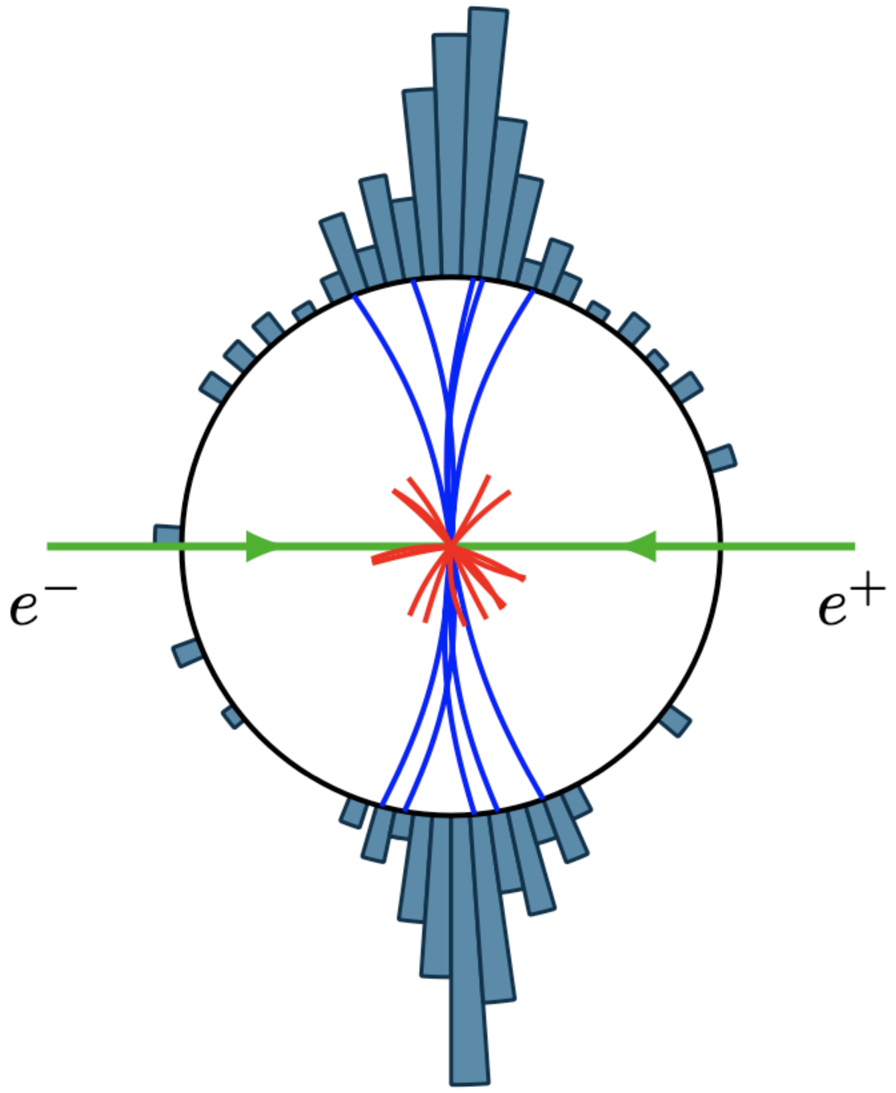

1.70 nb^{-1} PbPb (5.02 TeV)
CMS Preliminary
PbPb 0-10%, n=1
$p_T^{ch} > 1$ GeV
anti-k_T R = 0.4
$|\eta_{jet}| < 1.6$
$120 < p_{T,jet} < 140$ GeV
$140 < p_{T,jet} < 160$ GeV
$160 < p_{T,jet} < 180$ GeV
$180 < p_{T,jet} < 200$ GeV
EEC
Δr

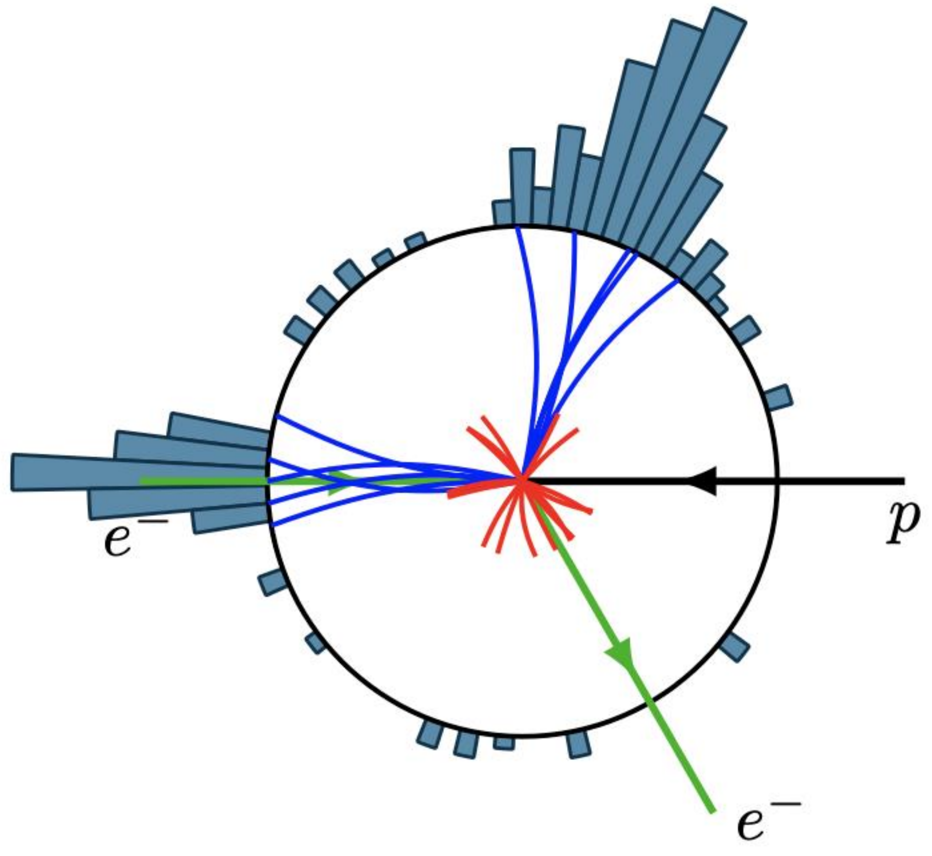

STAR Preliminary
Normalized EEC
Jet R = 0.6, $|\eta_{Jet}| < 0.4$
$30 <$ Jet $p_T < 50$ GeV/c
Lee et al.
Free Hadron
Transition
Quark/Gluon
ΔR

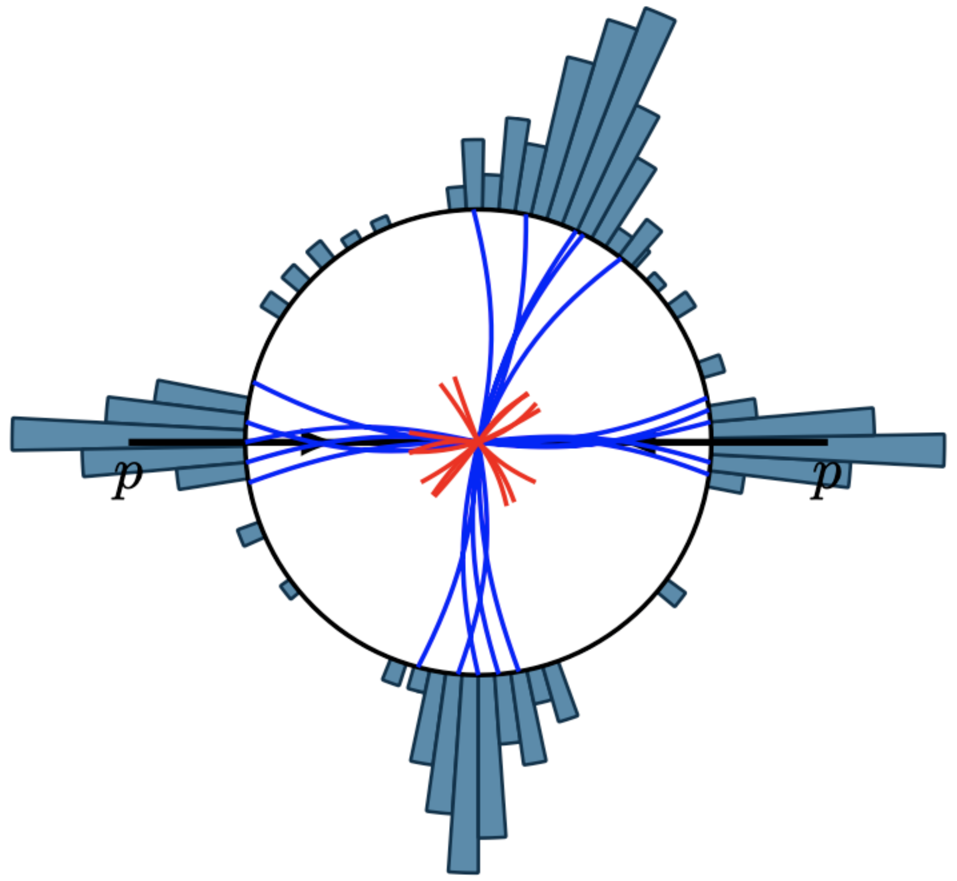

ALICE pp $\sqrt{s}$ = 5.02 TeV
Anti-k_T charged-particle jets, $R = 0.4$, $|\eta_{jet}| < 0.5$
$p_{T, track} > 1.0$ GeV/c
$p_T^{ch\ jet}$
(20, 40) GeV/c
(40, 60) GeV/c
(60, 80) GeV/c
Transition region
Peak = 2.39 GeV/c
±0.17 GeV/c
Hadron region
A×R_L
Pertubative region
pQCD
$\ln\langle p_T^{ch\ jet}\rangle/\langle p_T^{ch\ jet}\rangle \times \Sigma_{EEC}$ ((GeV/c)$^{-1}$)

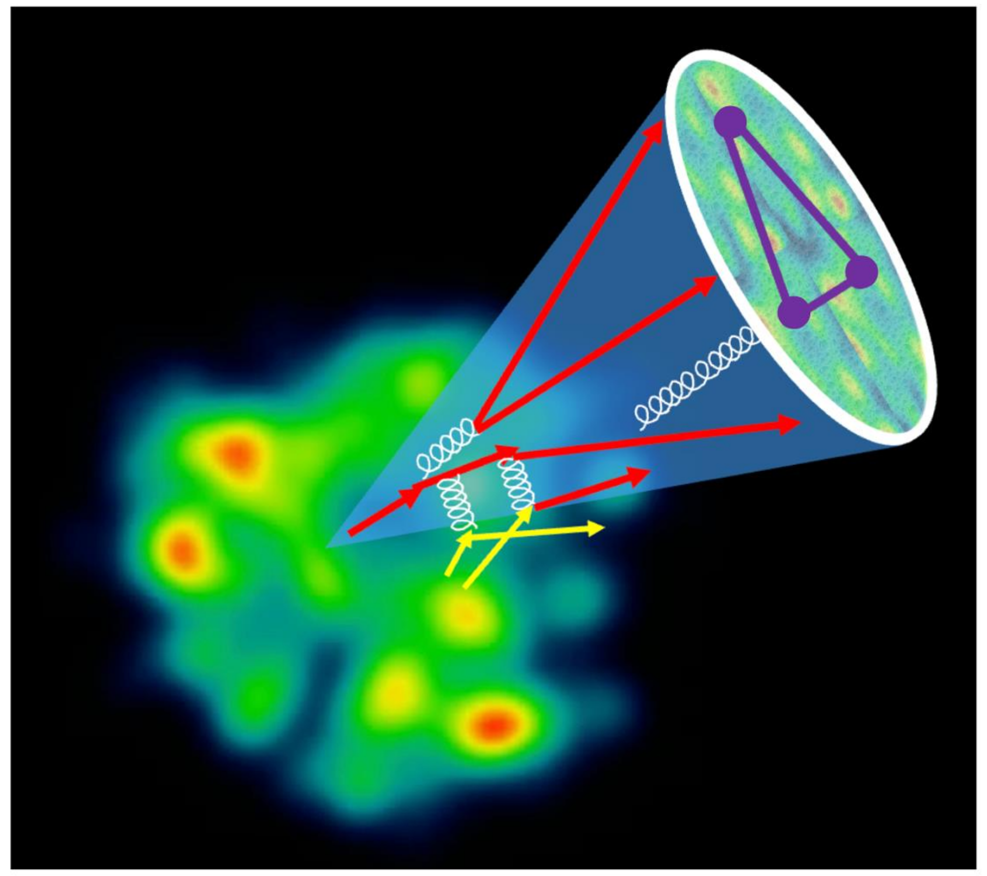
$$\langle \mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\rangle_\psi$$

ψ: different states



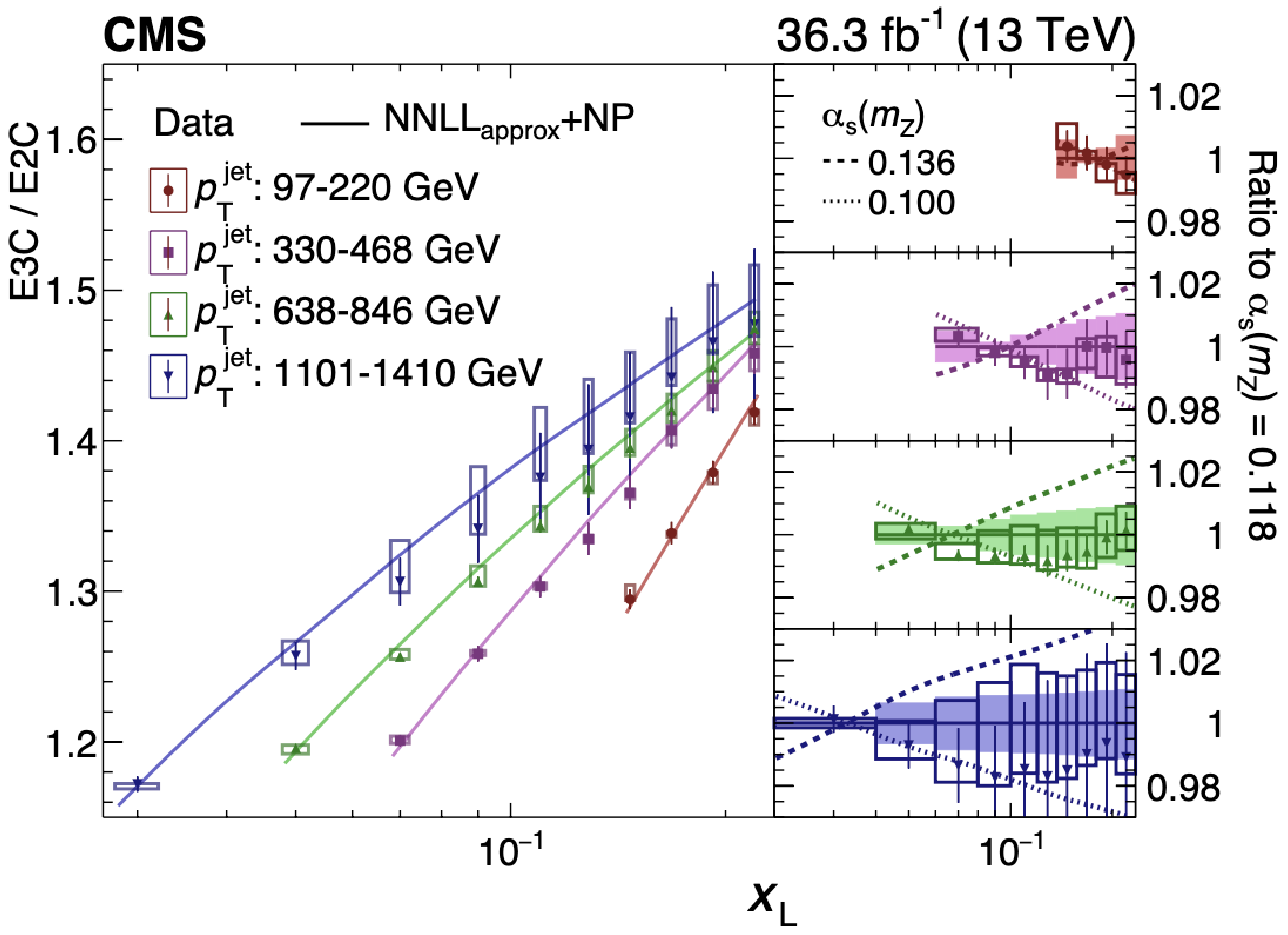


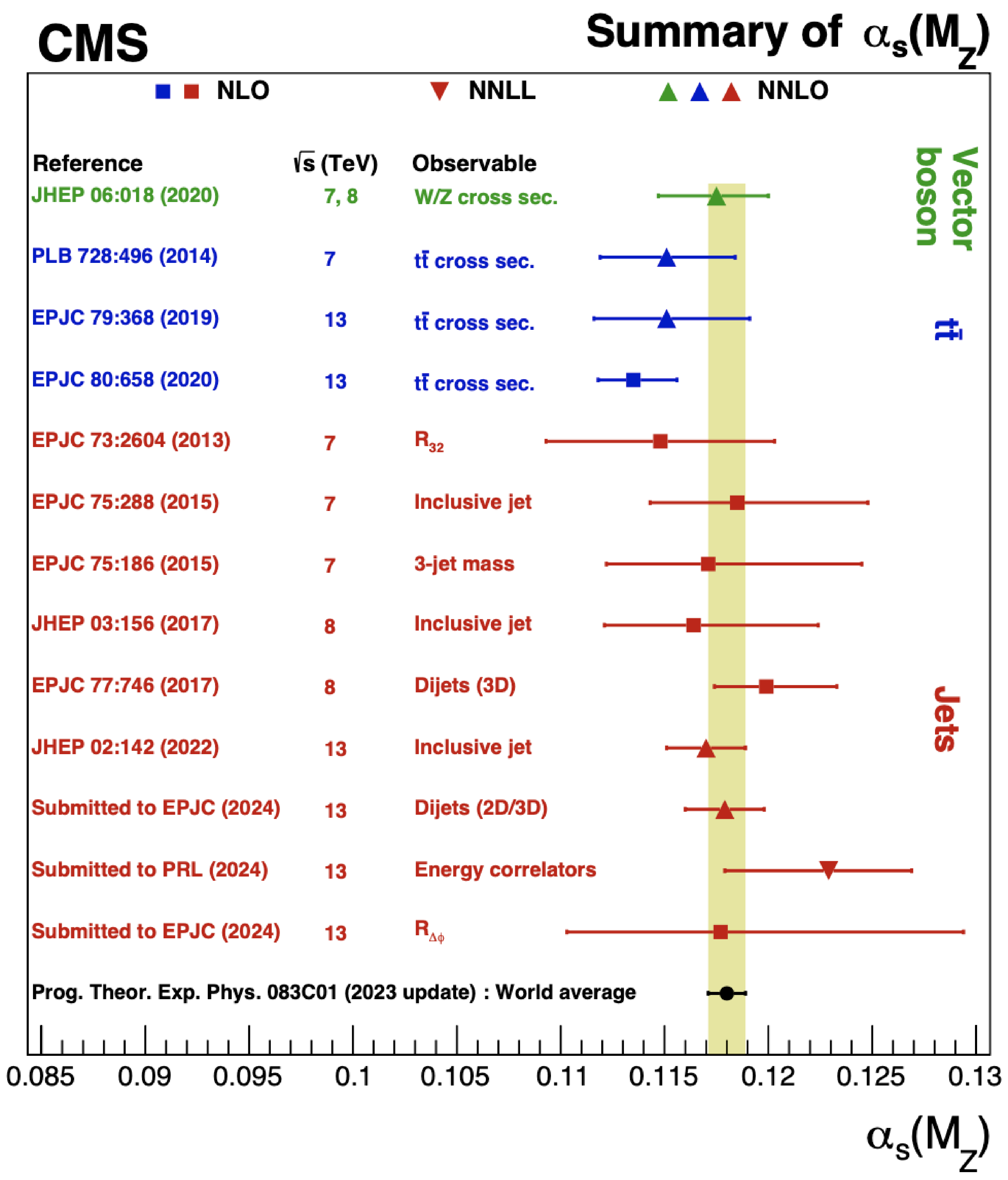




Energy operator in particle physics

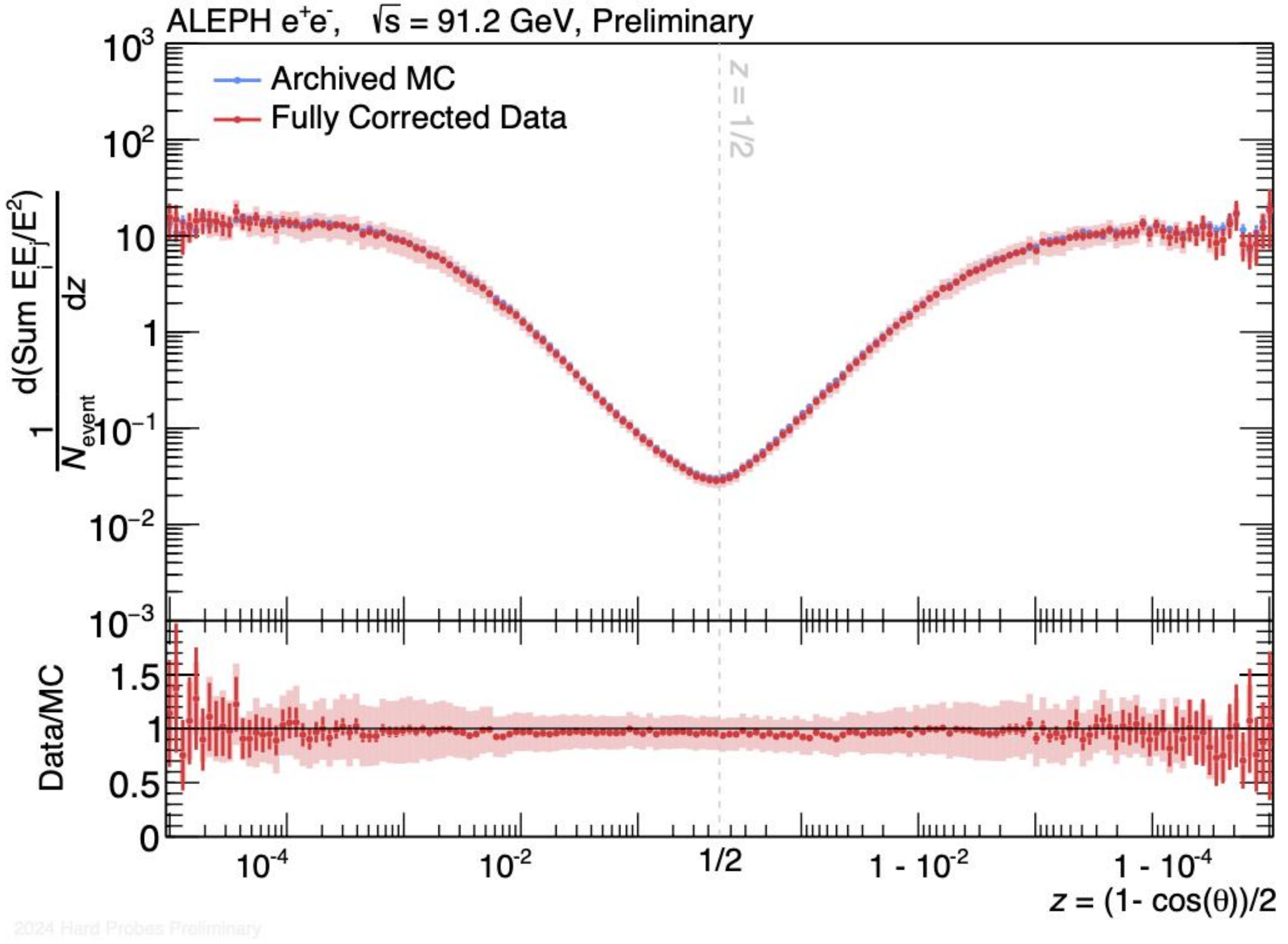
Strong coupling constant

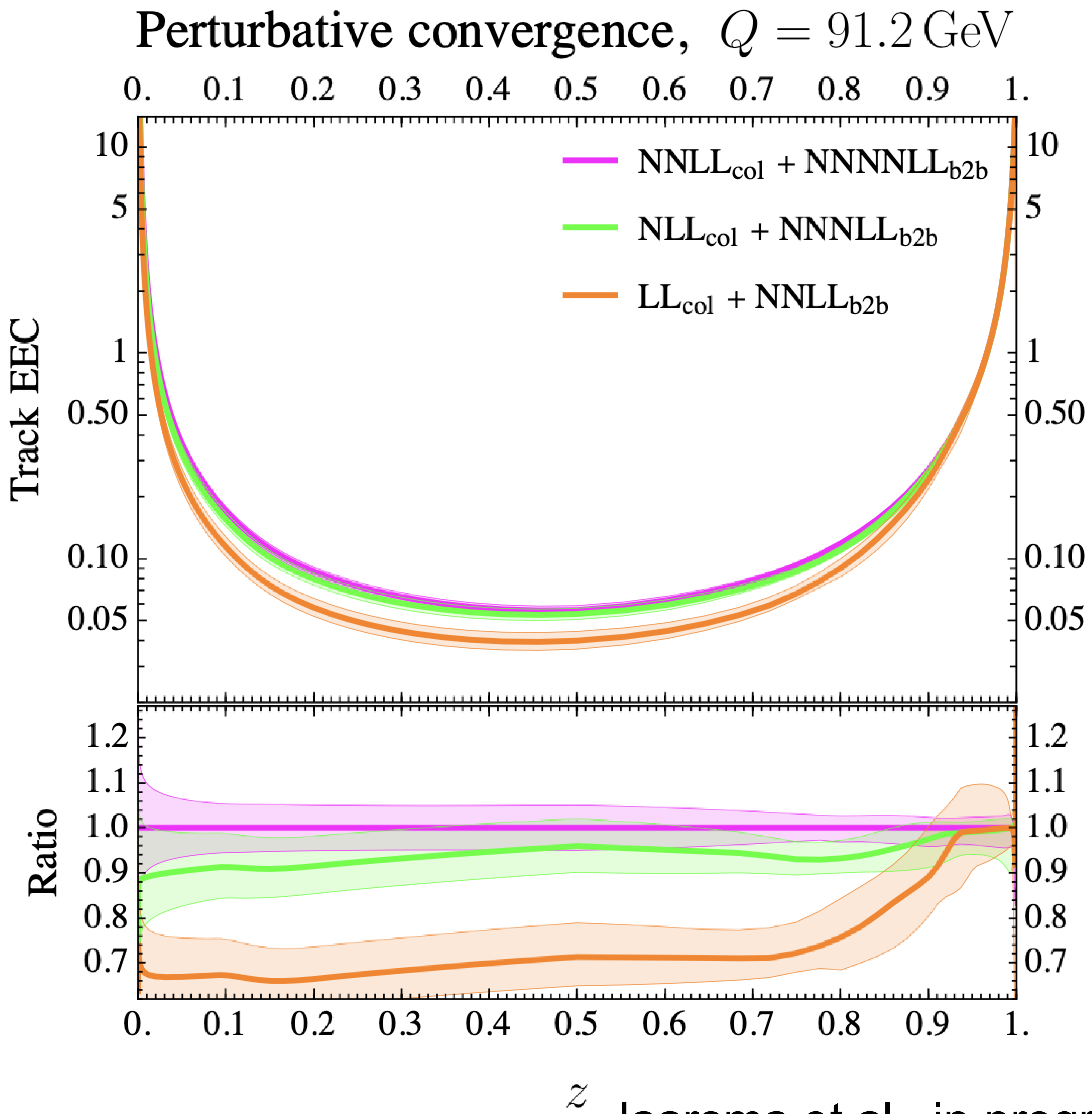




Most precise alphas from jet substructure!

LEP reloaded

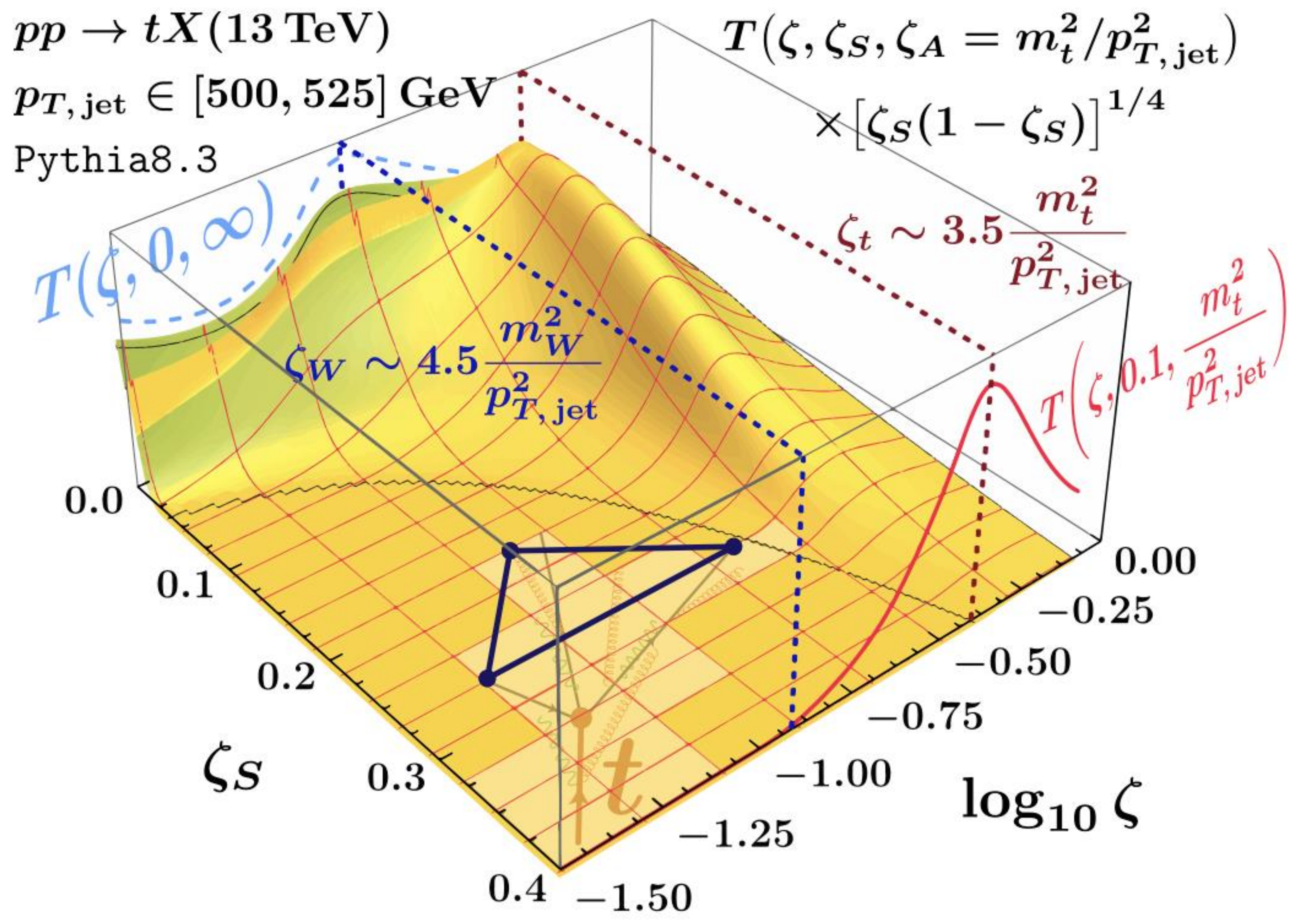


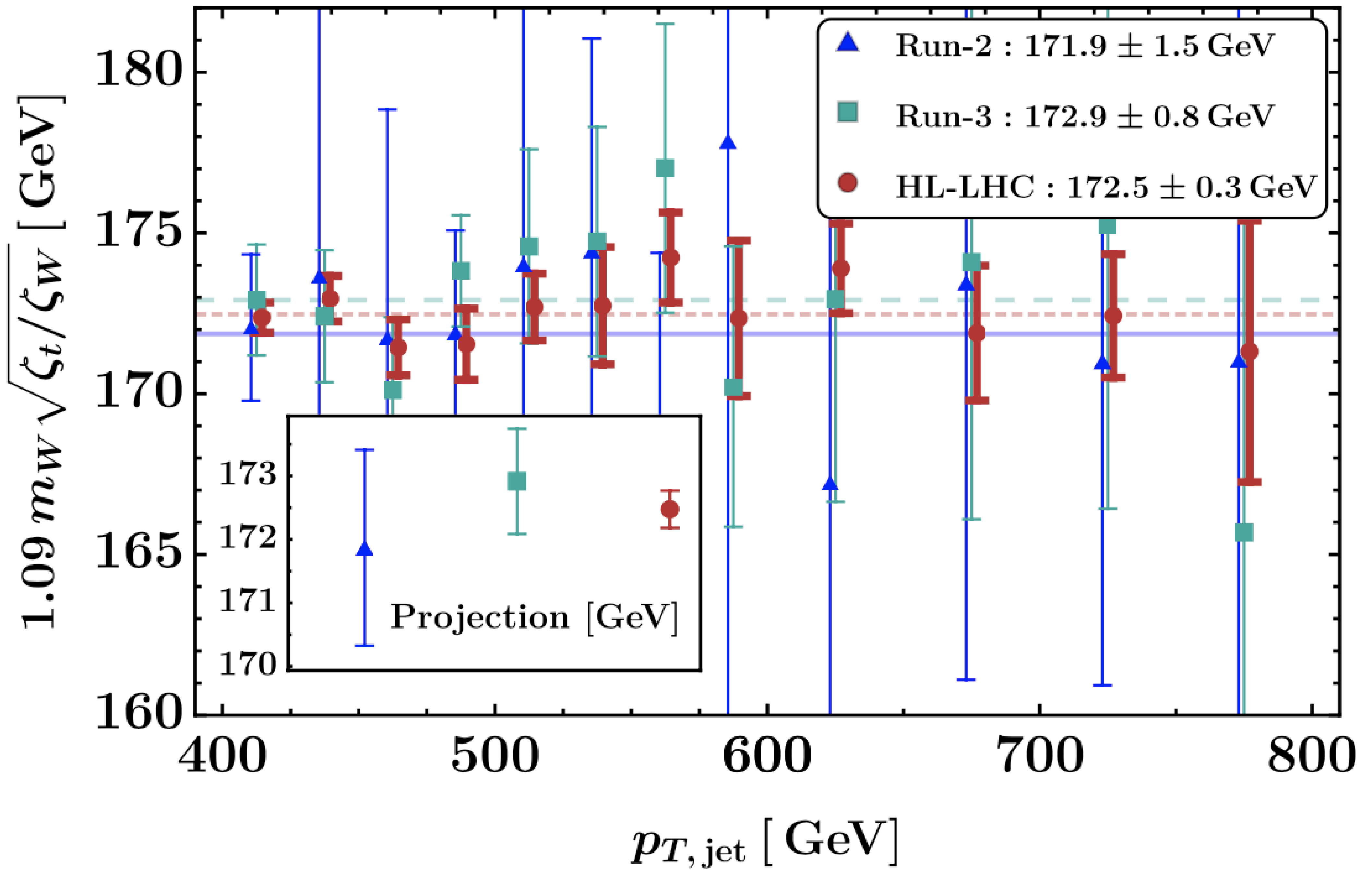


Jaarsma et al., in progress

Complete analysis of EEC in the full phase space from experiment and theory

Top quark mass

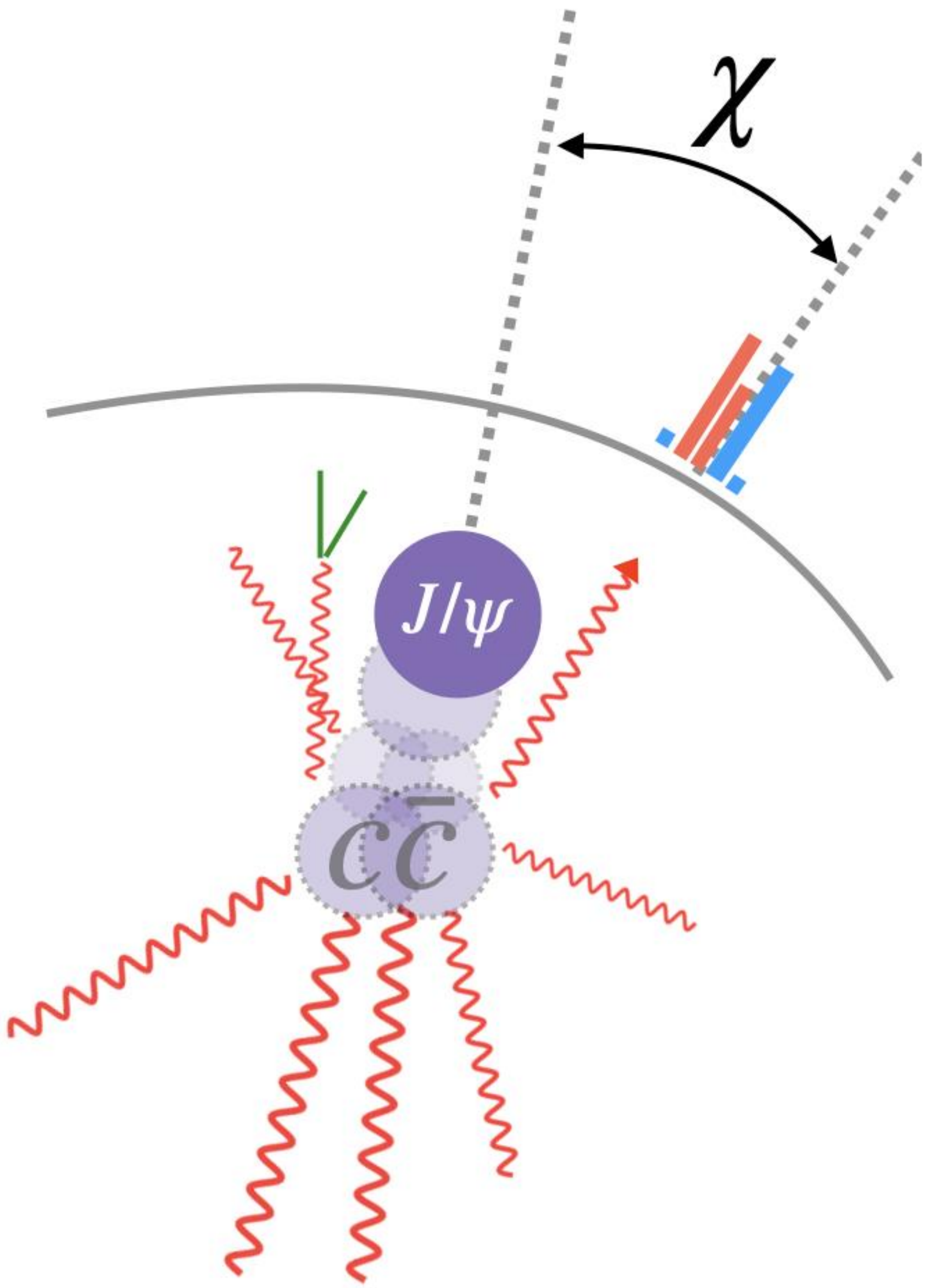


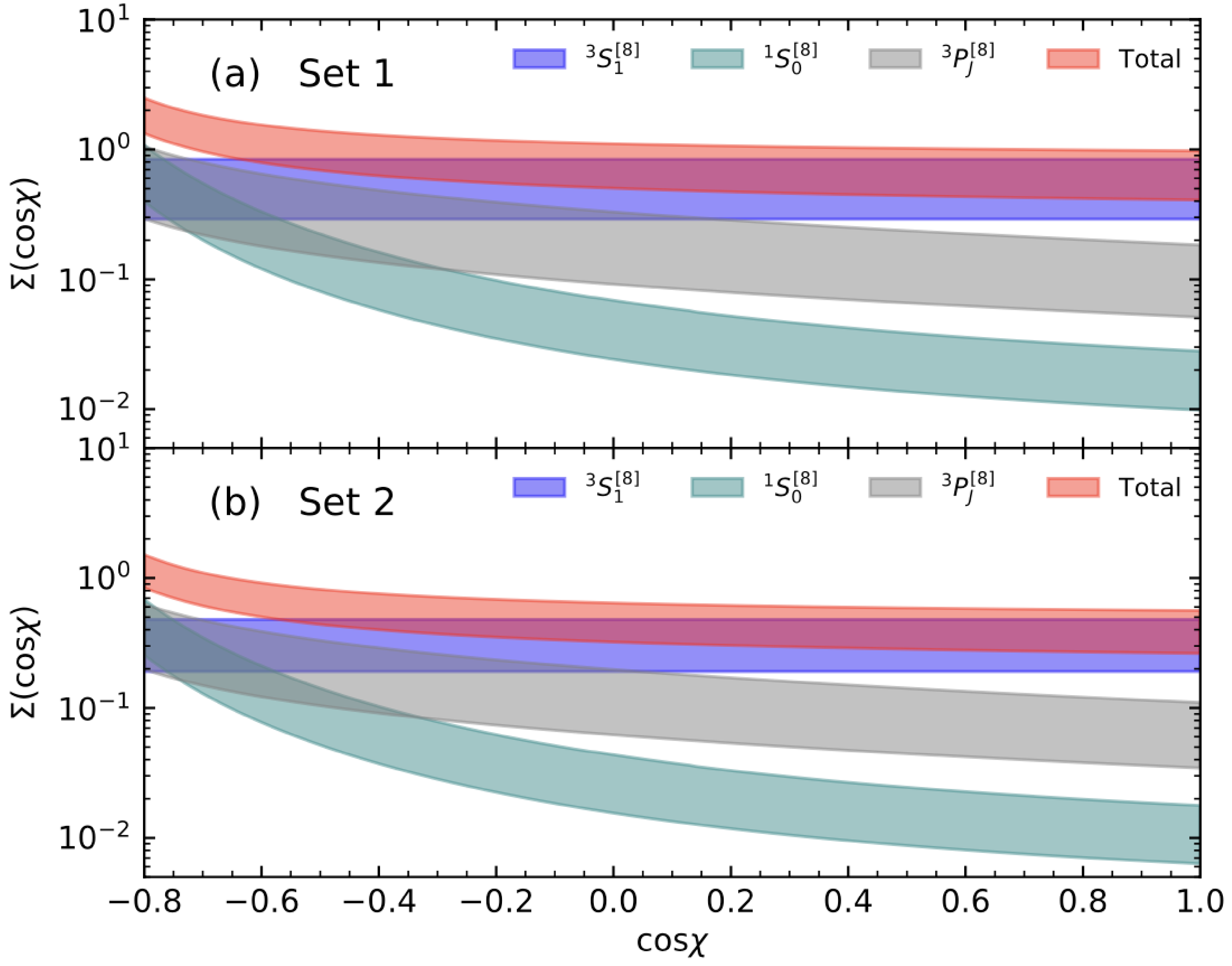


Holguin, et al., 2024

Three-point energy correlation can provide a precise measurement for top quark mass at high luminosity LHC

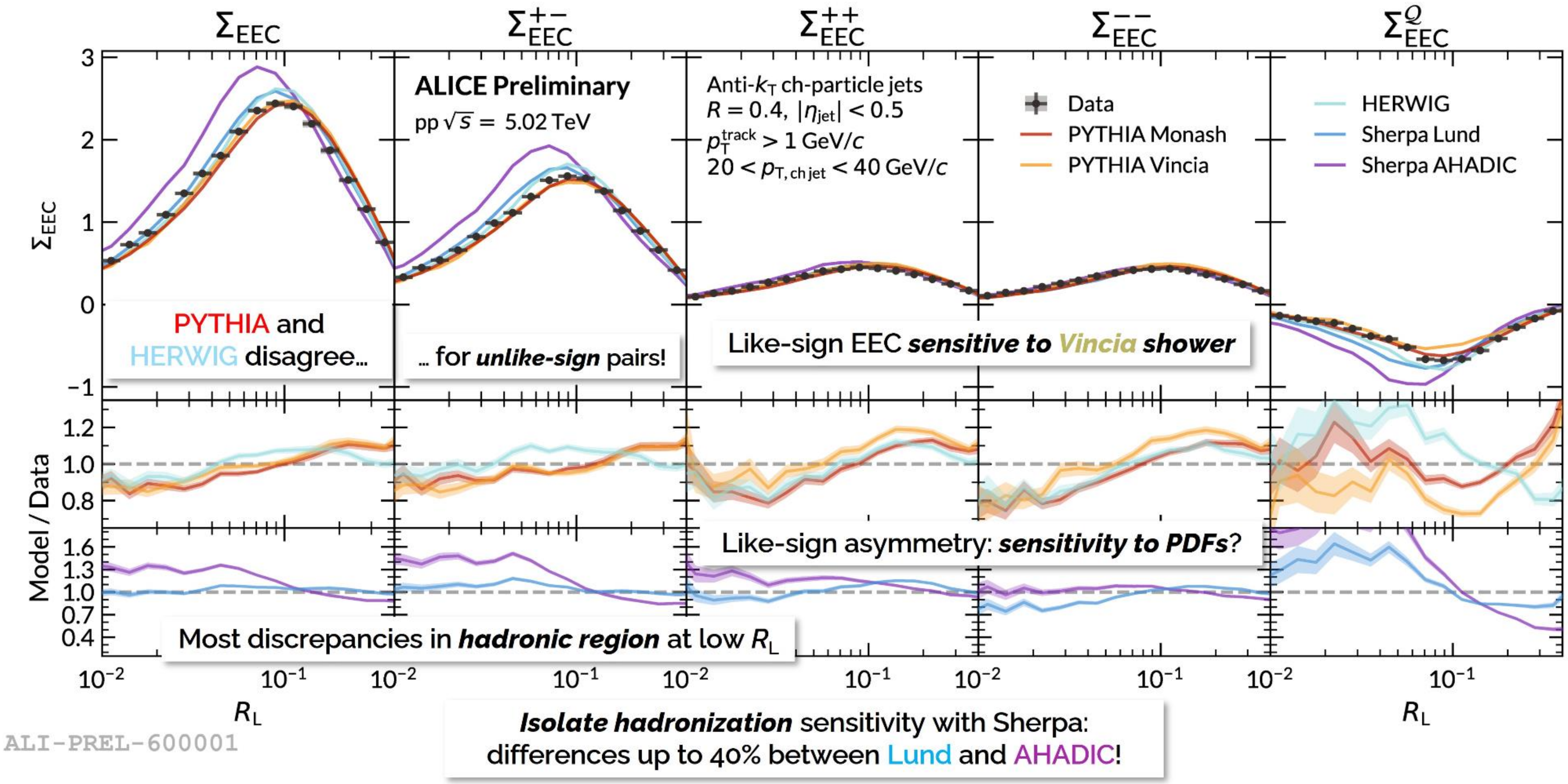
Quarkonium energy correlators





Chen, Liu, Ma, 2024

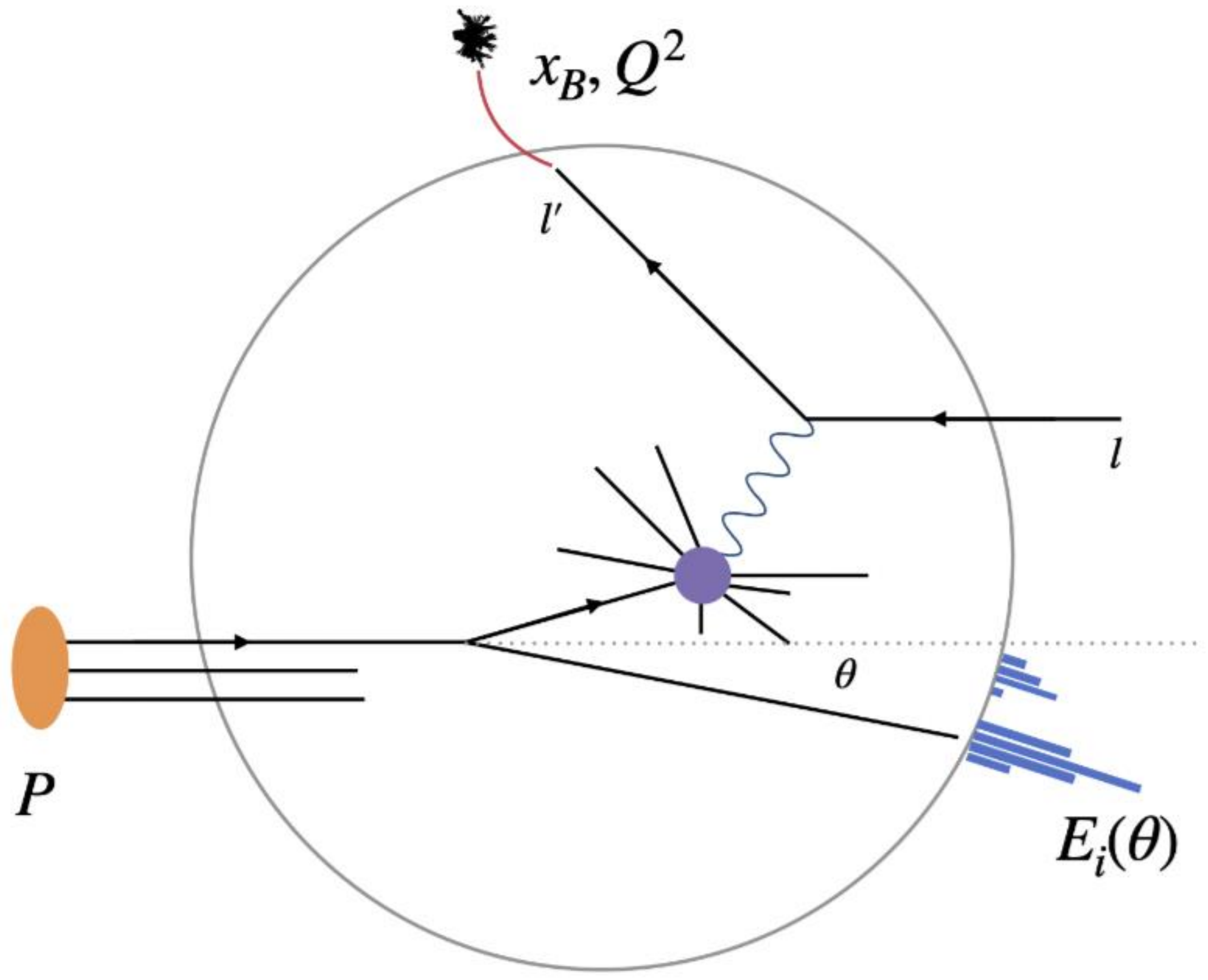
Charging the energy correlators

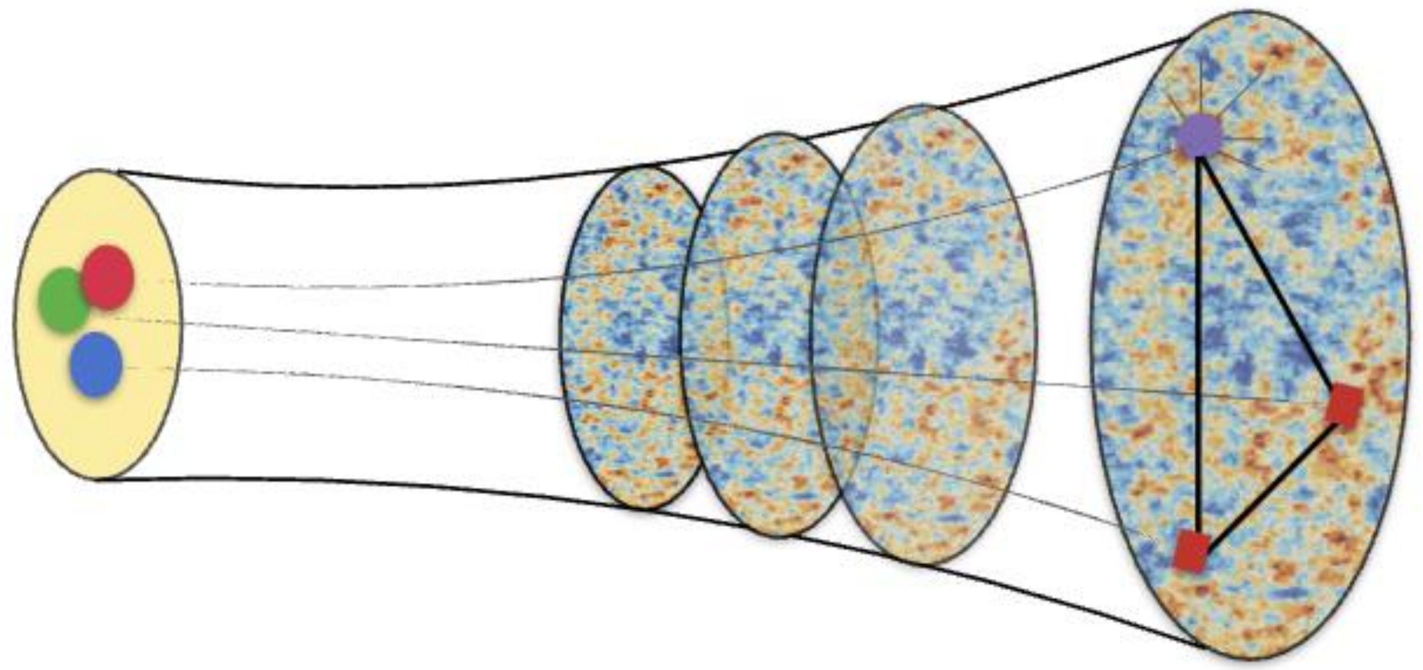


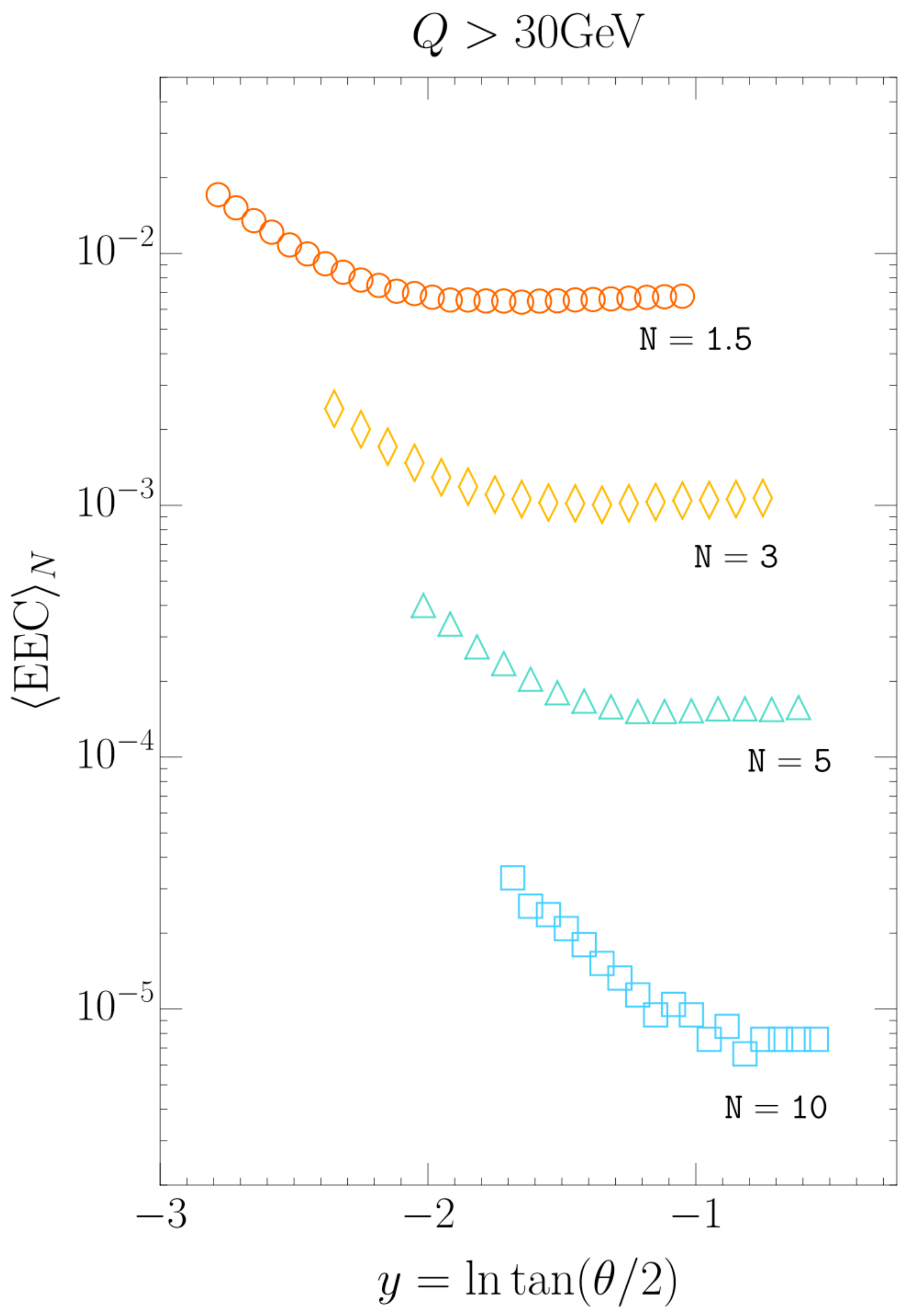
Poster by Hwang from Quark Matter 2025

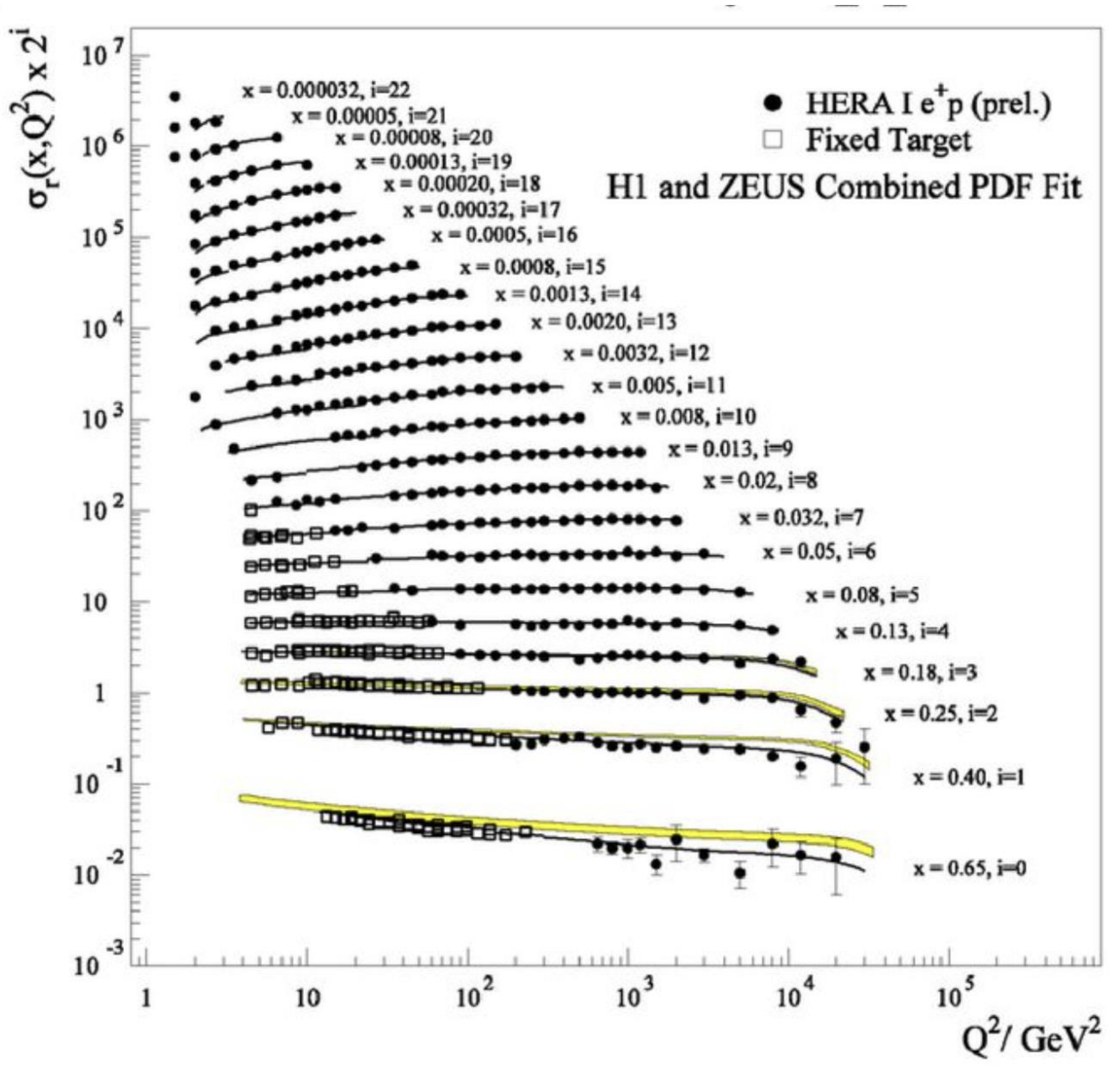
Energy operator in nuclear physics

Nucleon Energy Correlators





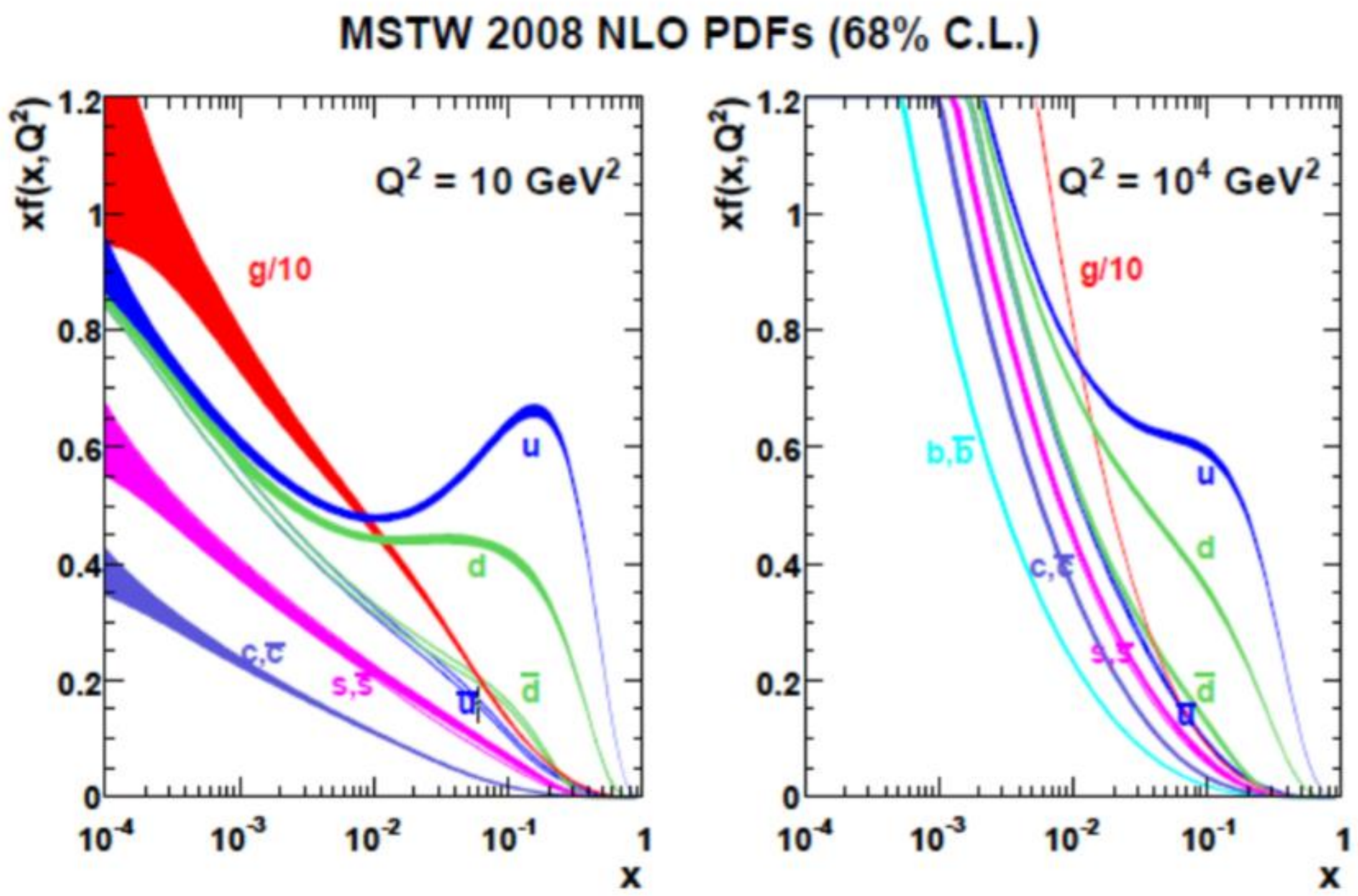


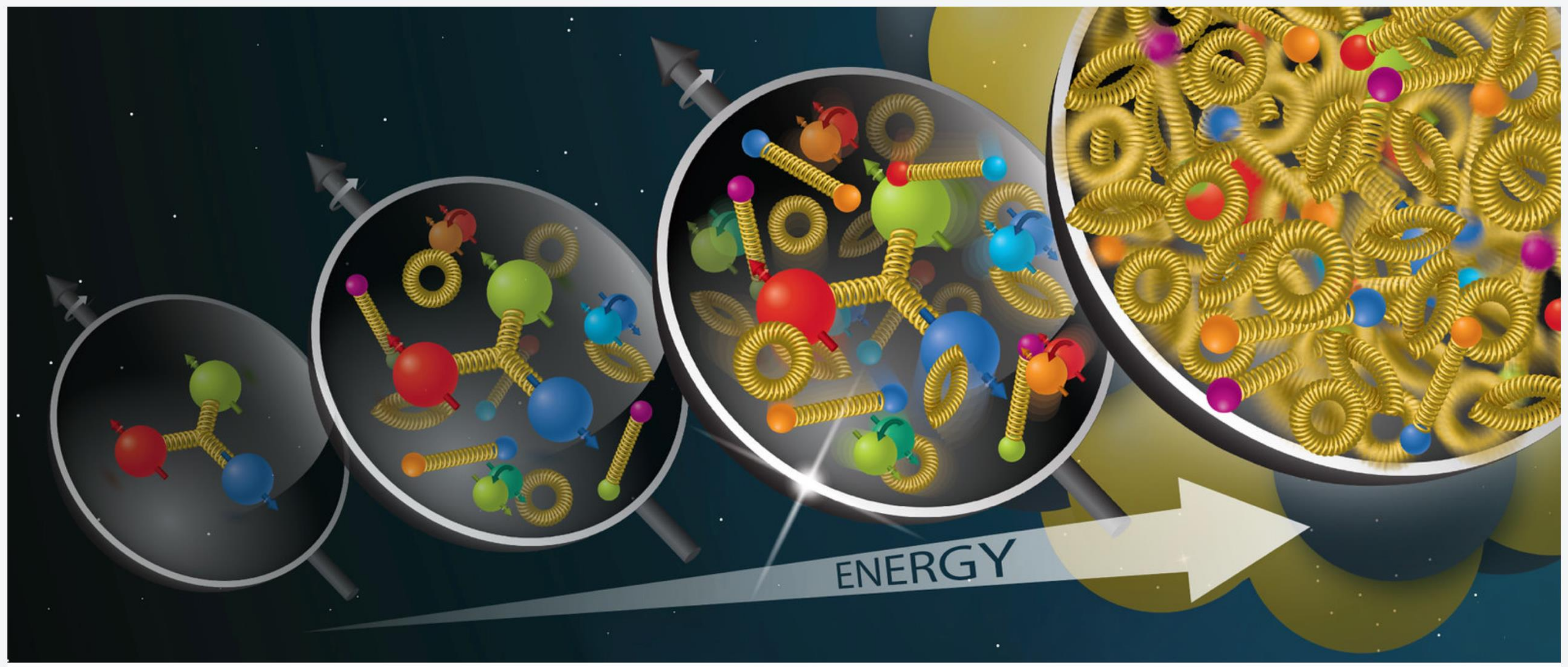


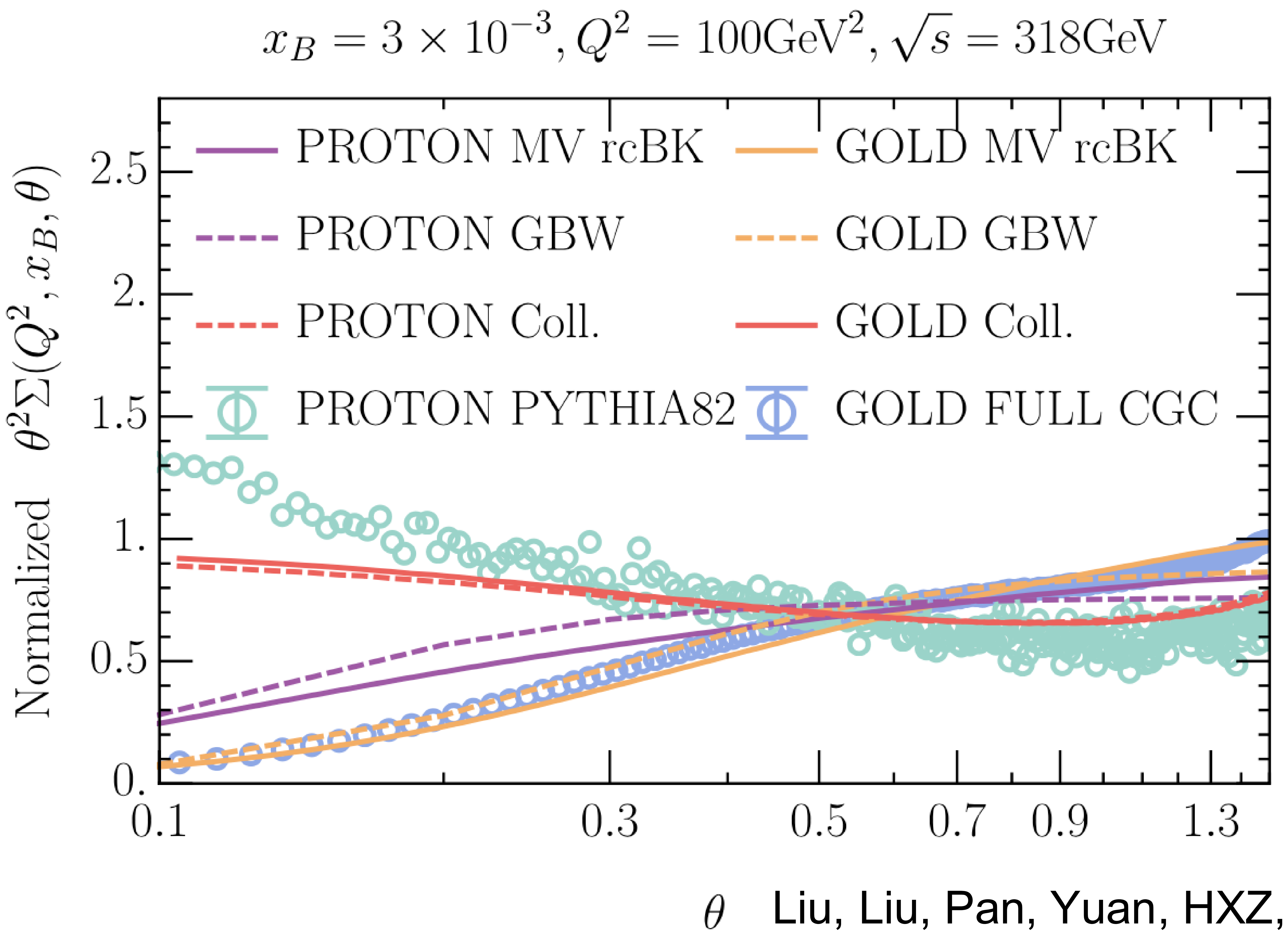
A new way to measure Bjorken scaling violation!

Liu, HXZ, 2022
Cao, Liu, HXZ, 2023

CGC and saturation scale



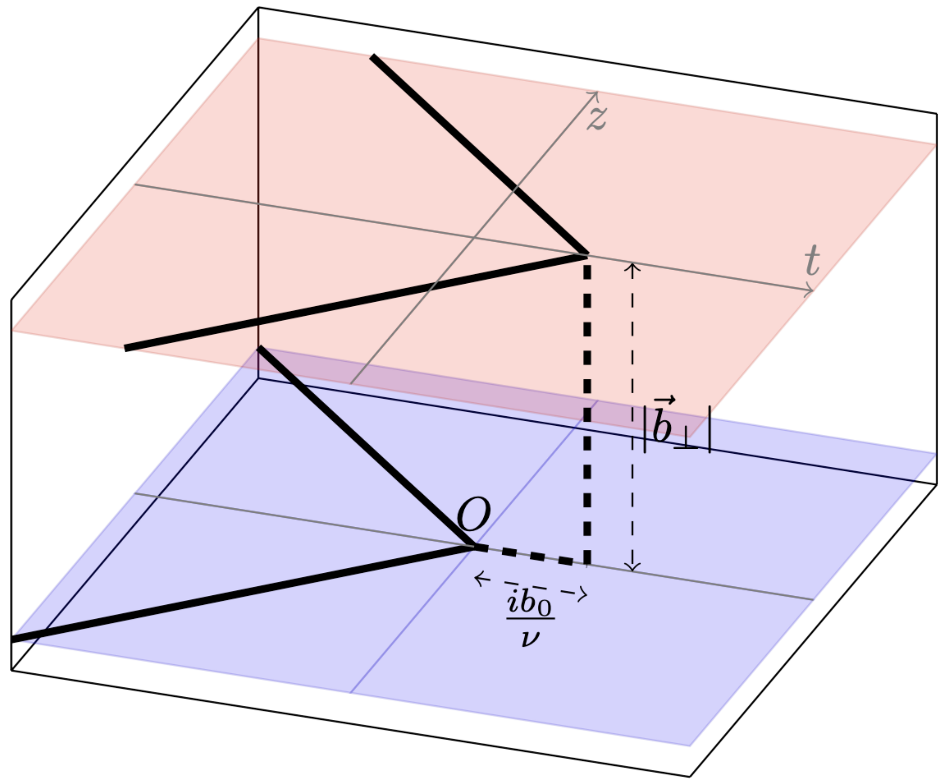




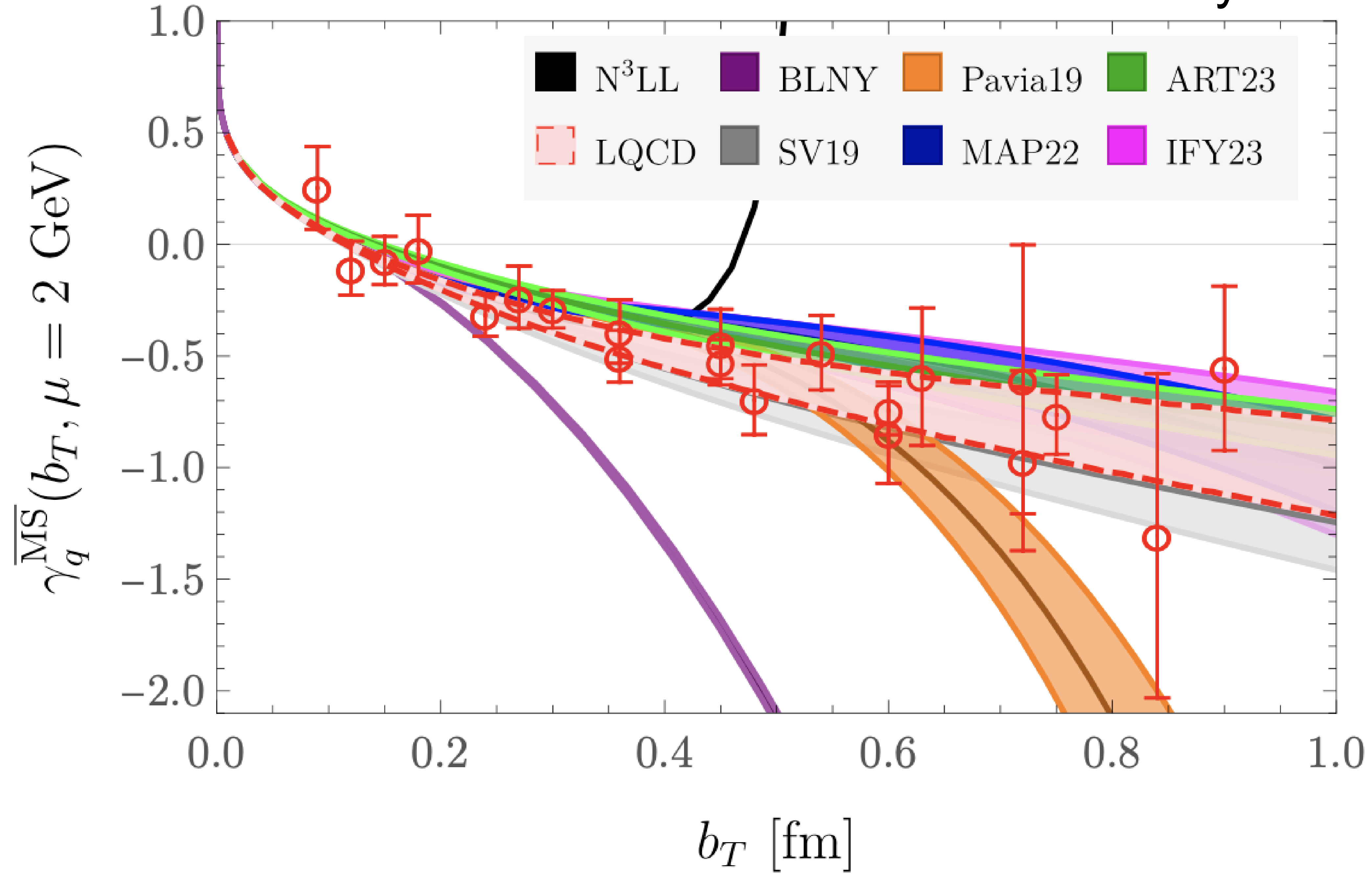
Liu, Liu, Pan, Yuan, HXZ, 2023

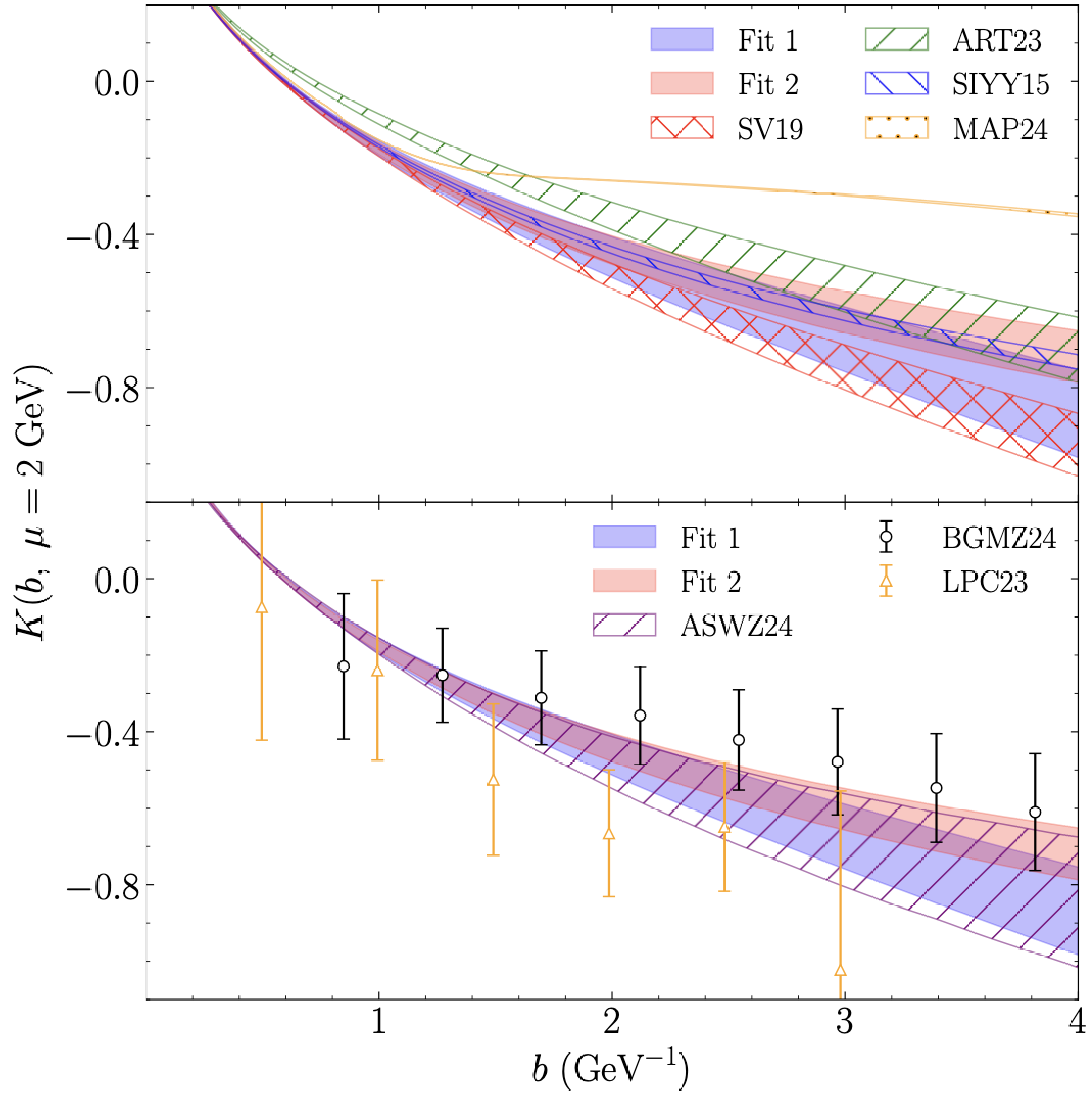
Search for the scale of gluon saturation from angle scan

Collins-Soper Kernel



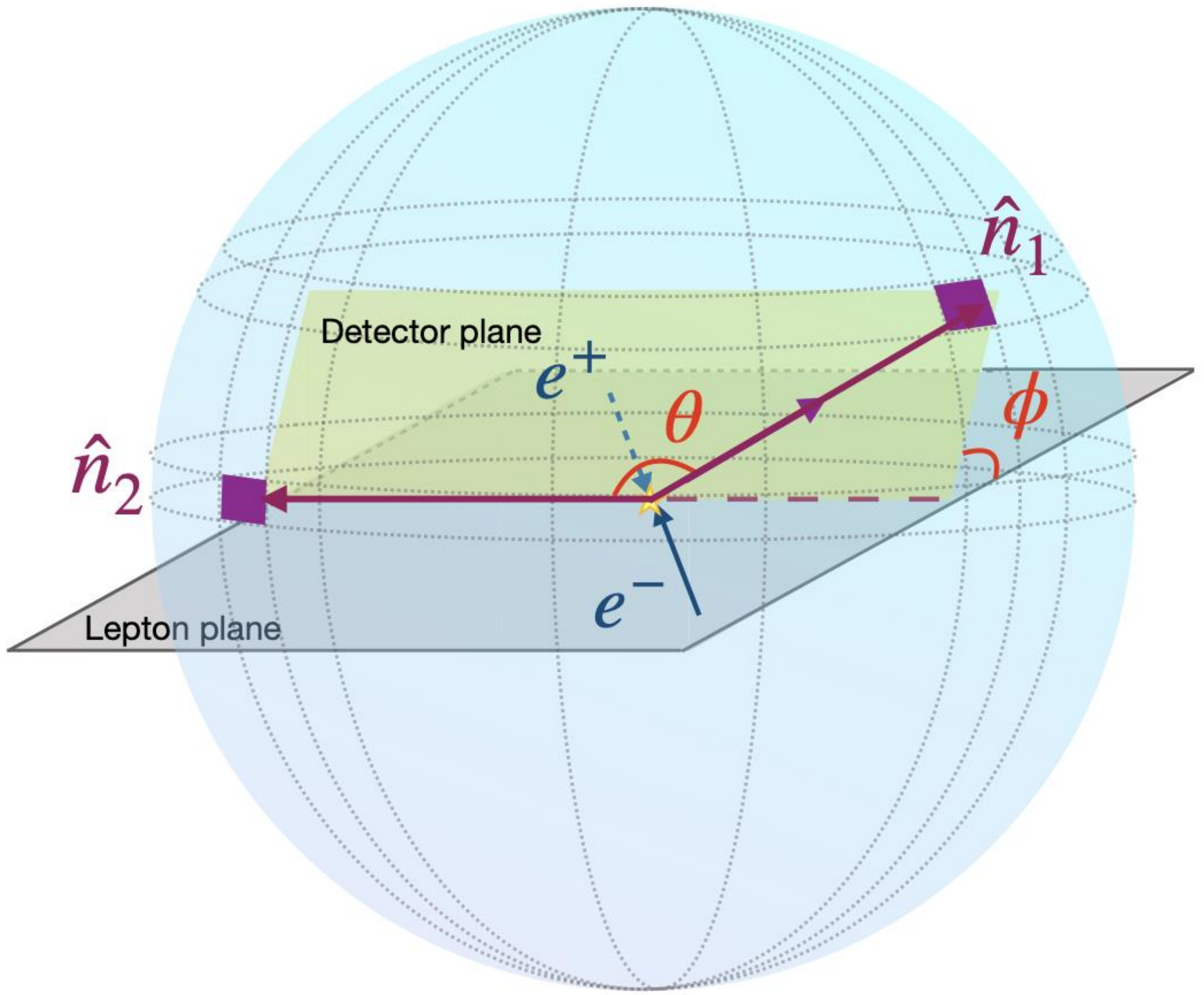
Active research from lattice community

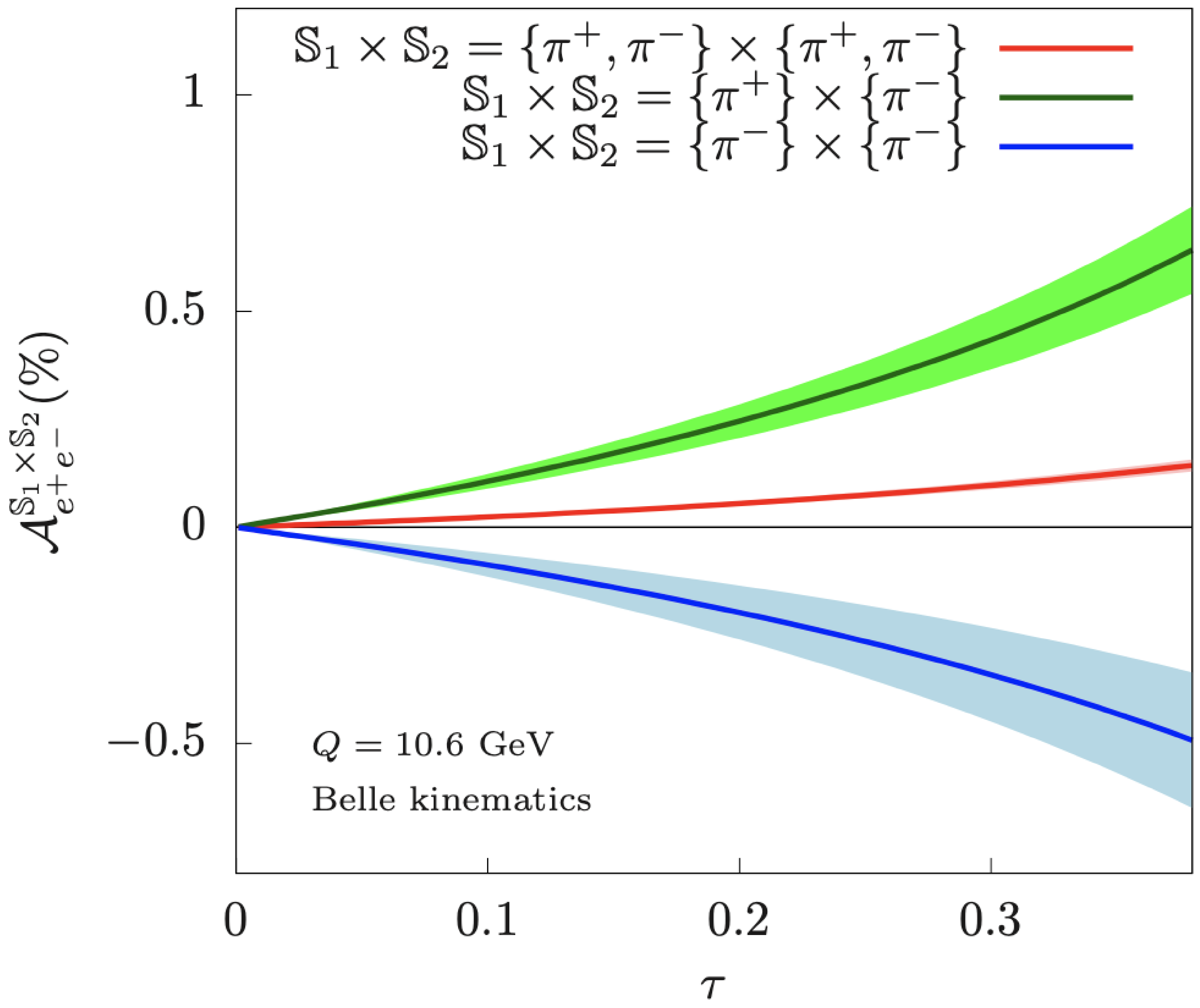




Kang, Penttala, Zhang, 2024

Non-perturbative Spin physics



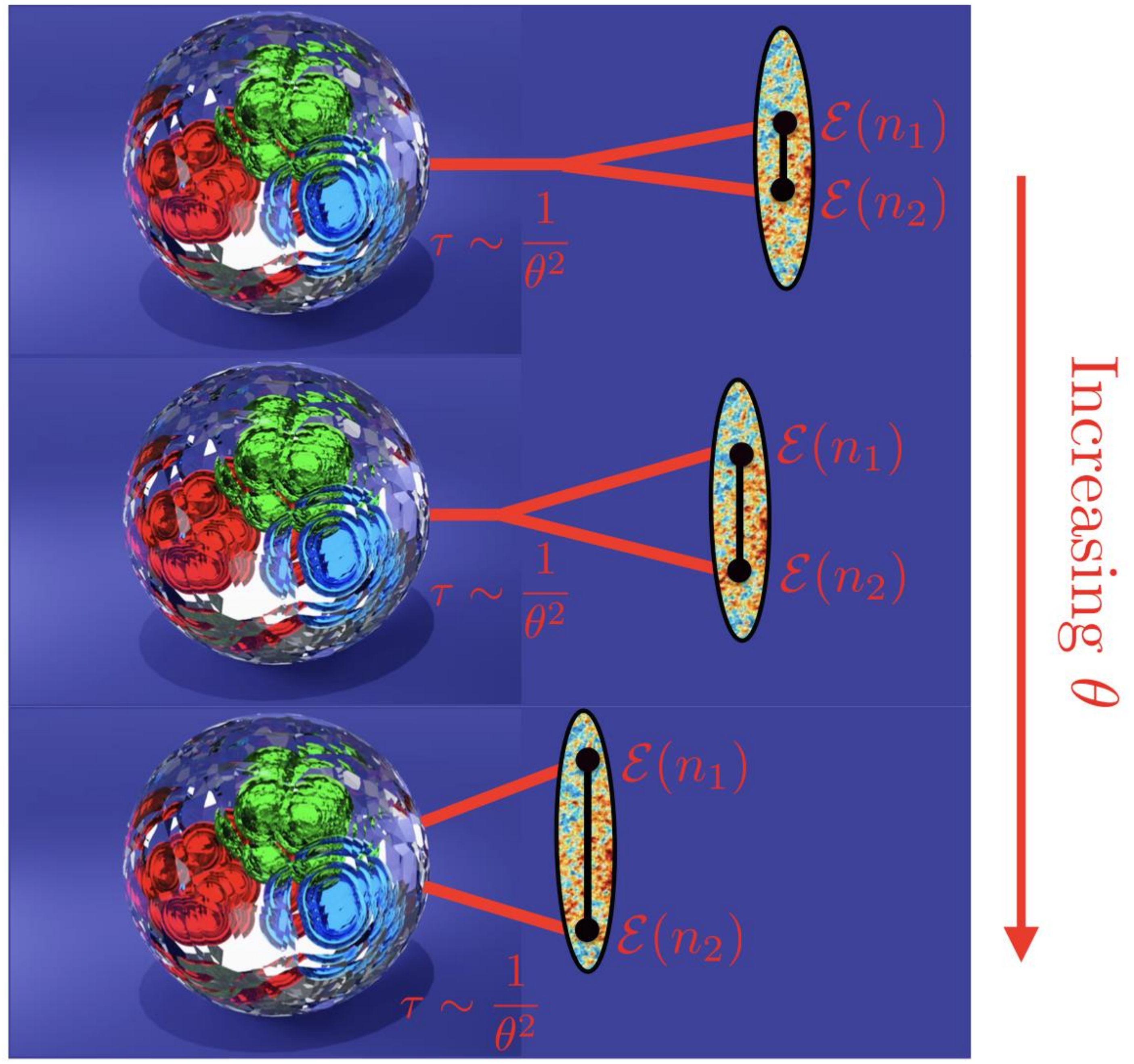


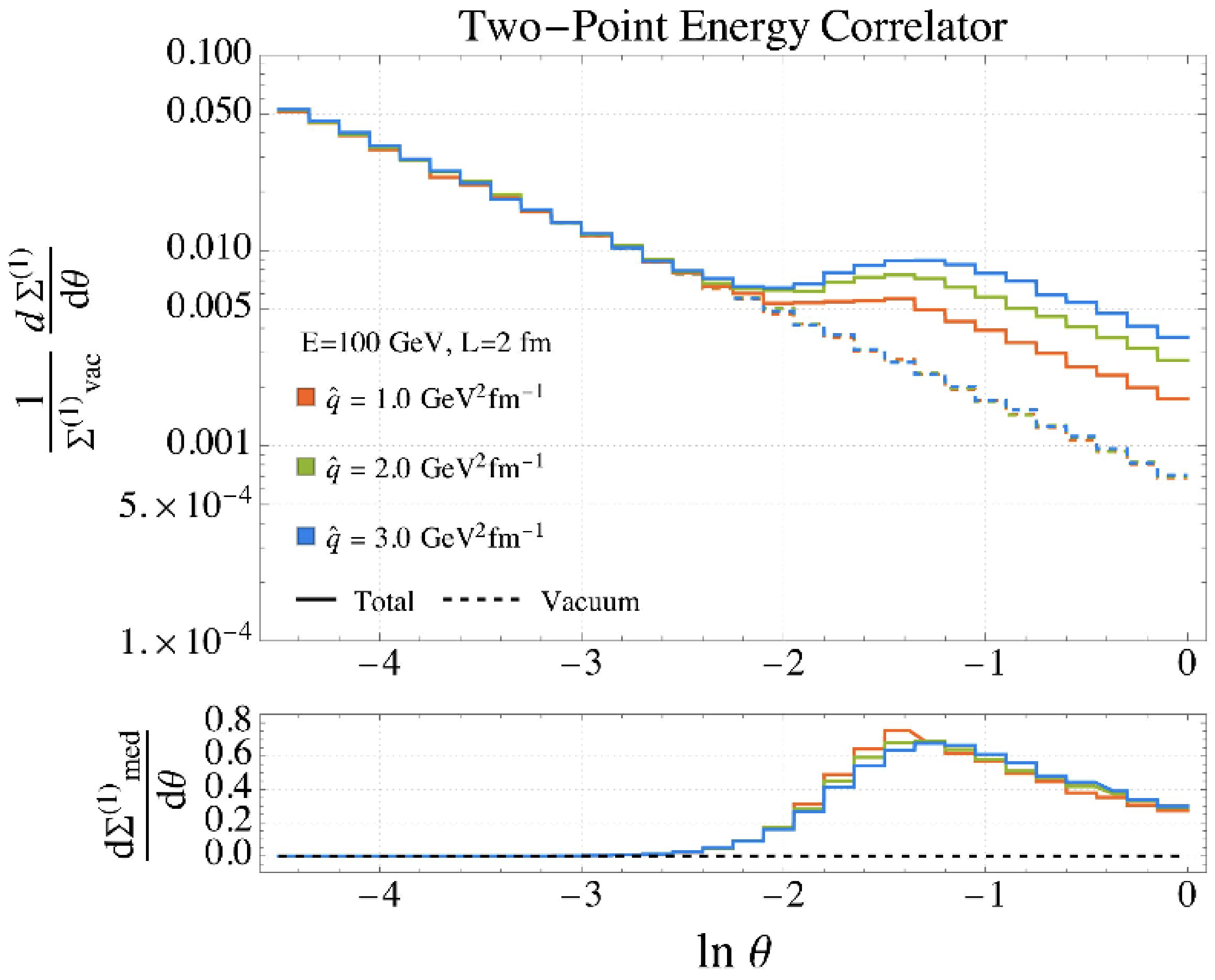
Probe Collins and Sivers function of QCD

Important for understanding QCD chiral symmetry breaking!

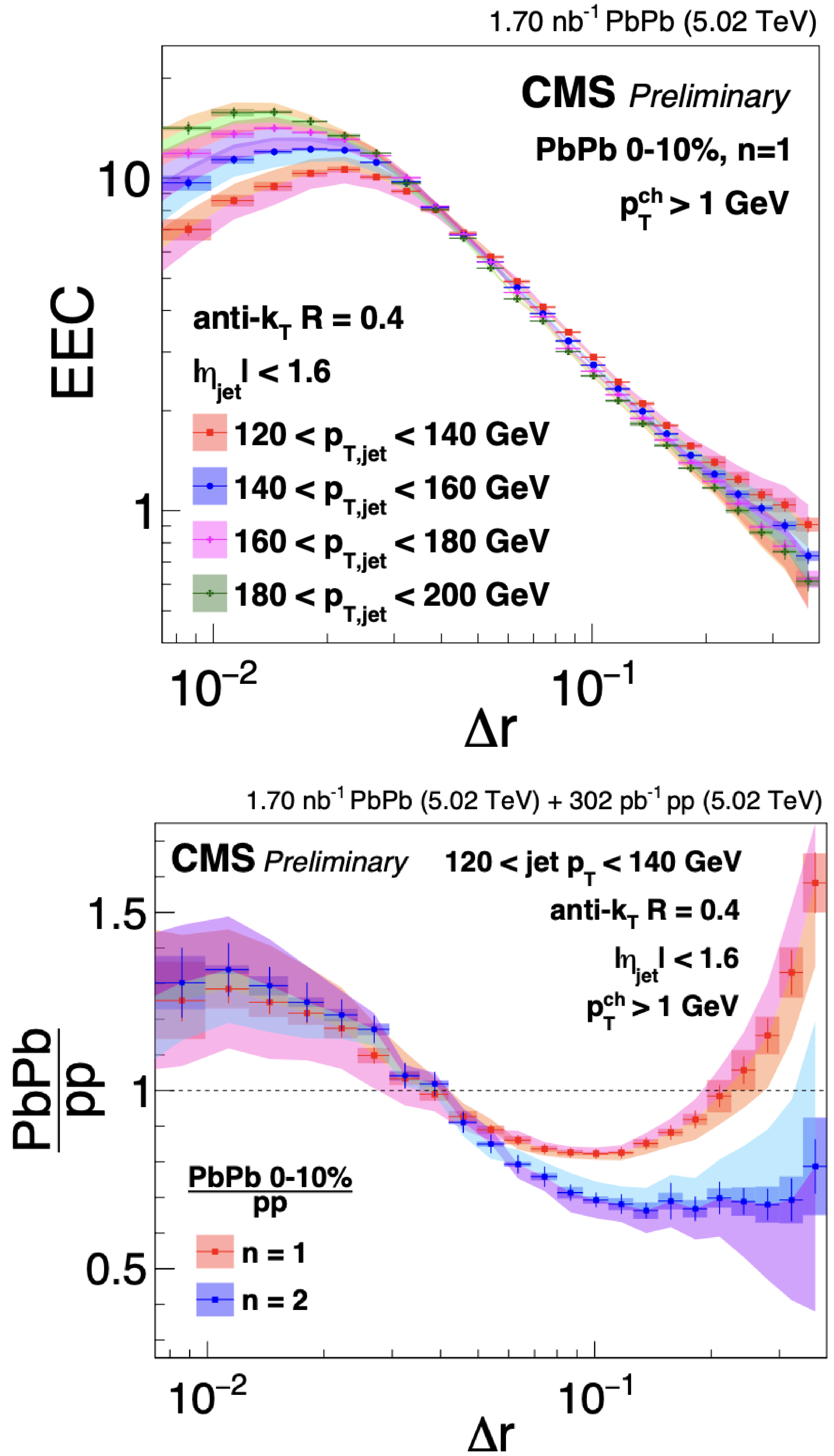
Kang, Lee, Fan, Shao, 2024

Probing properties of QGP

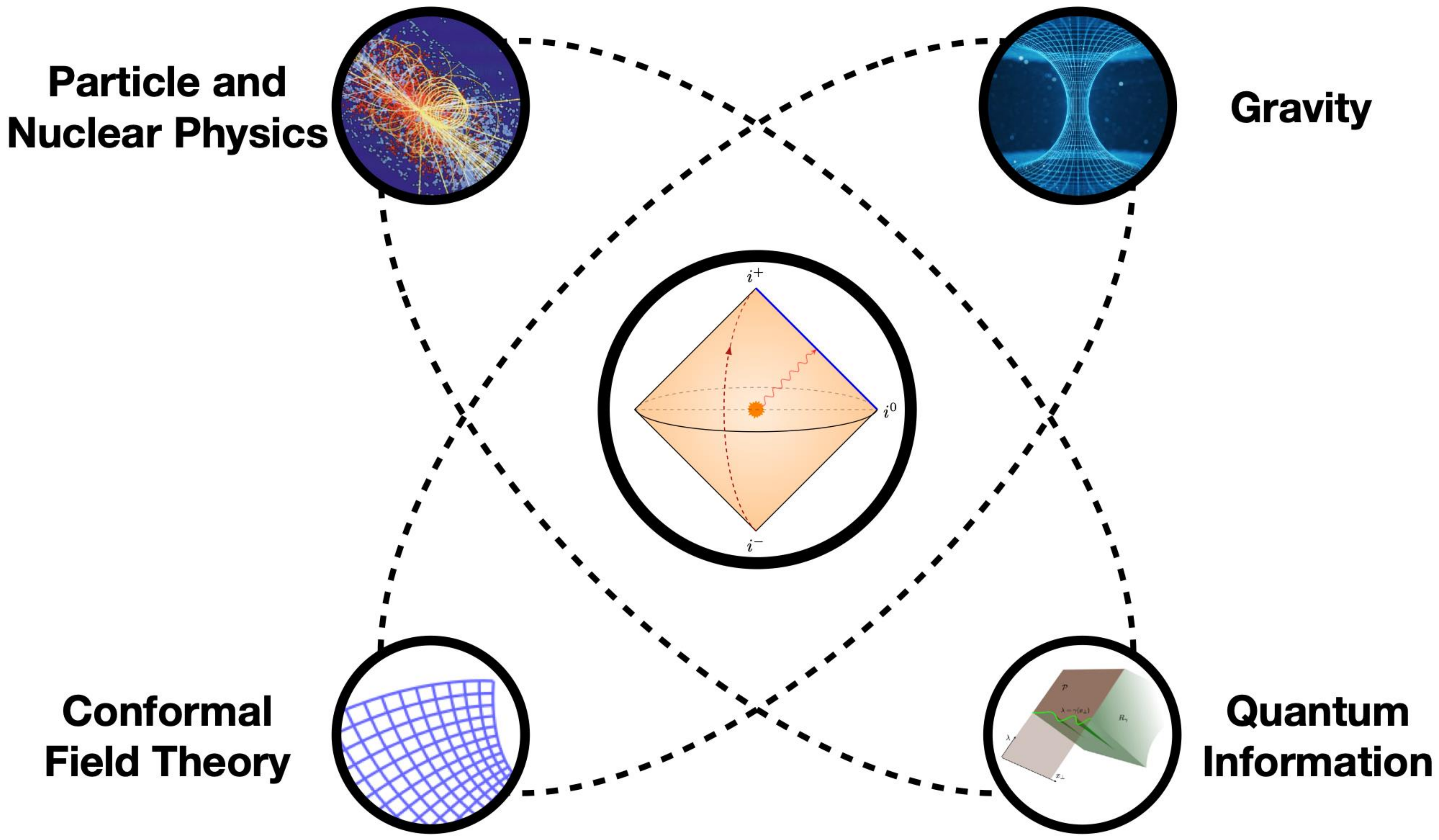




Andres et al., 2022
Yang et al., 2023
…

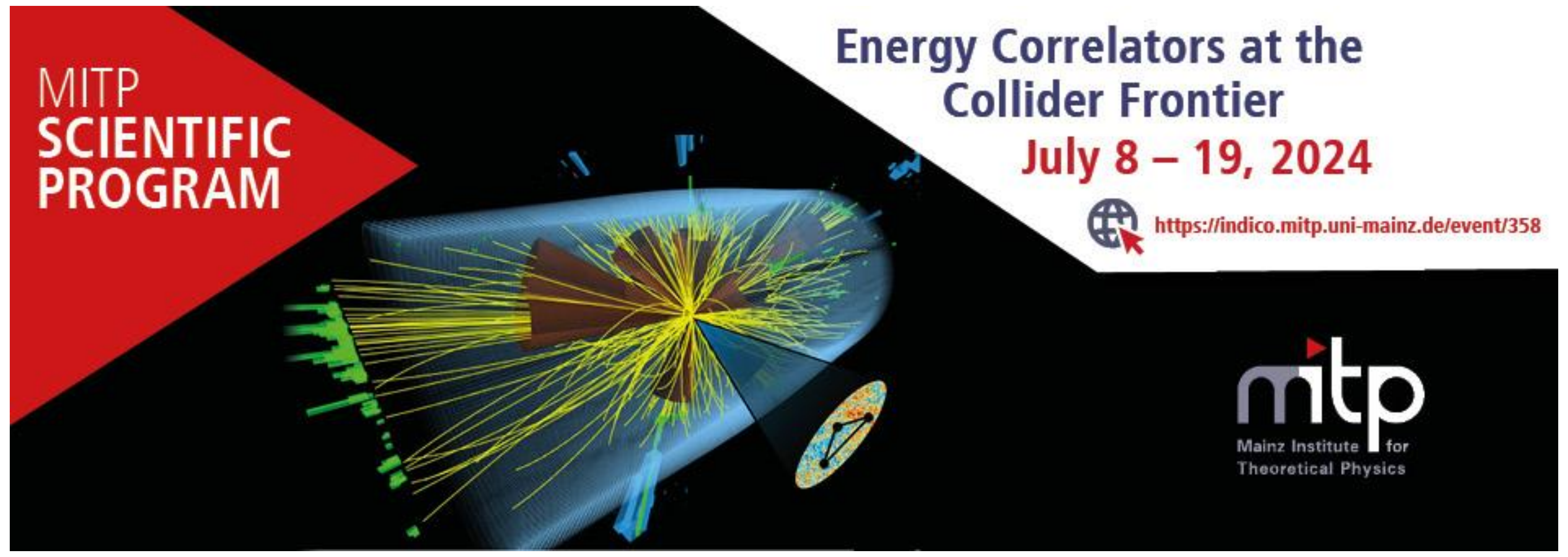


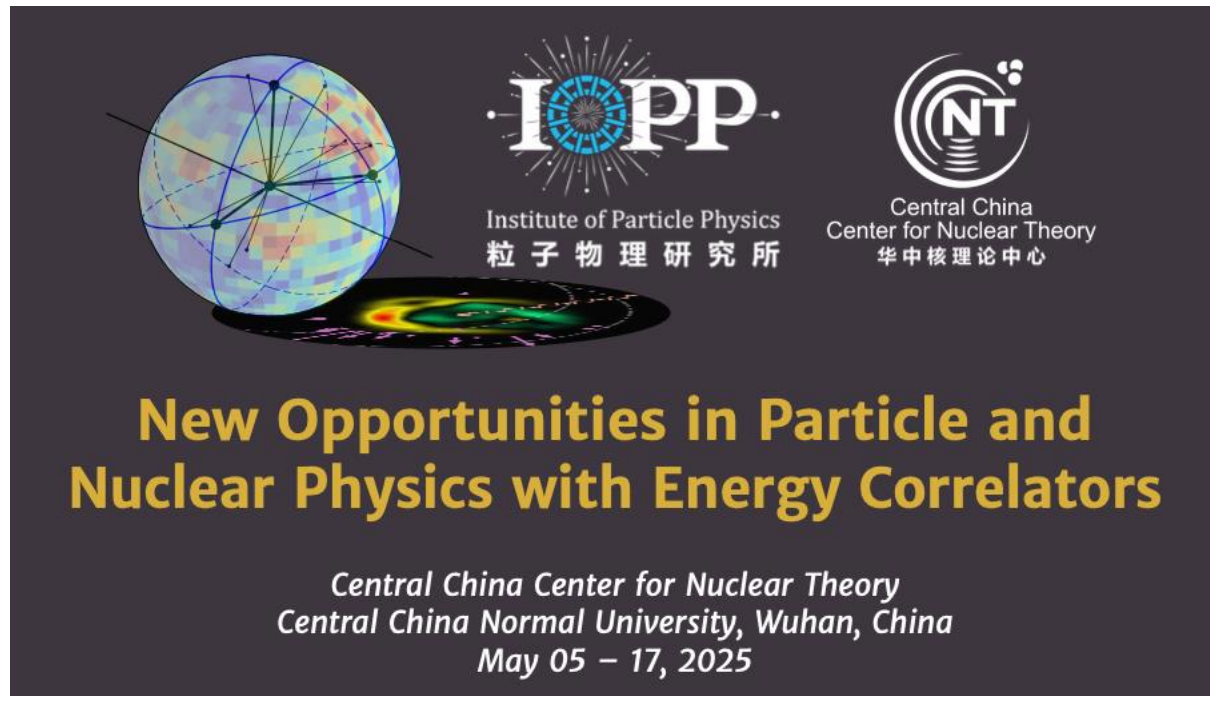
Summary

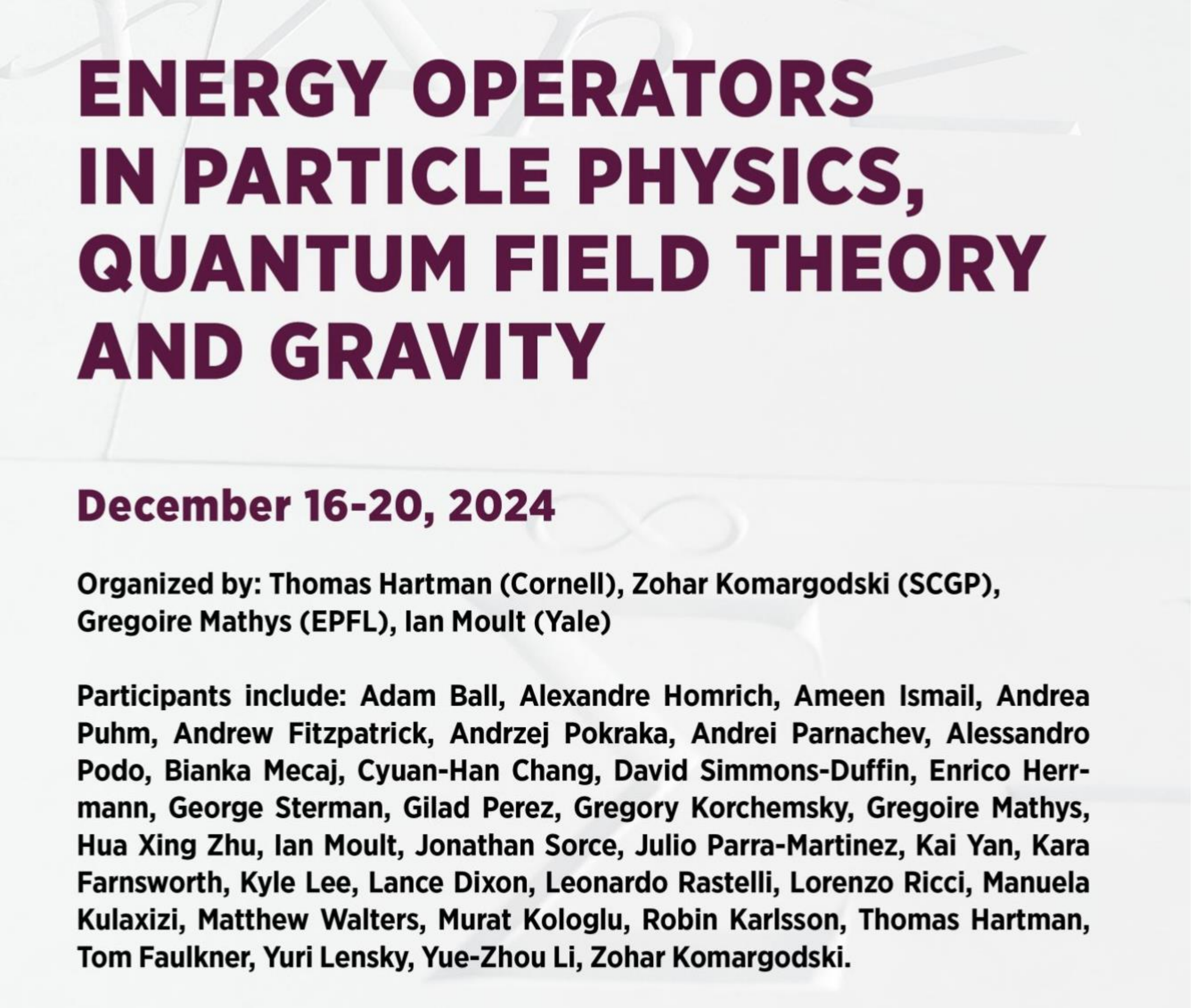


Energy correlators physics provides a window connecting particles and nuclear physics, conformal field theory, gravity and quantum information

1. Wuhan, Huazhong normal university EEC one-day workshop, 2023
2. Mainz MITP two-week program, 2024
3. Simons Center for Geometry and Physics, 2024
4. Wuhan C3NT inauguration two-week program, 2025
5. Lanzhou fragmentation function and EEC workshop, 2025
6. **KITP 3 month program, 2027**







Expecting more surprises in the coming years. Stay tuned!