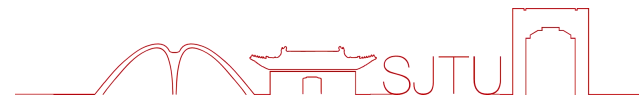




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基于卡尔曼滤波的带电粒子动能重建及其应用

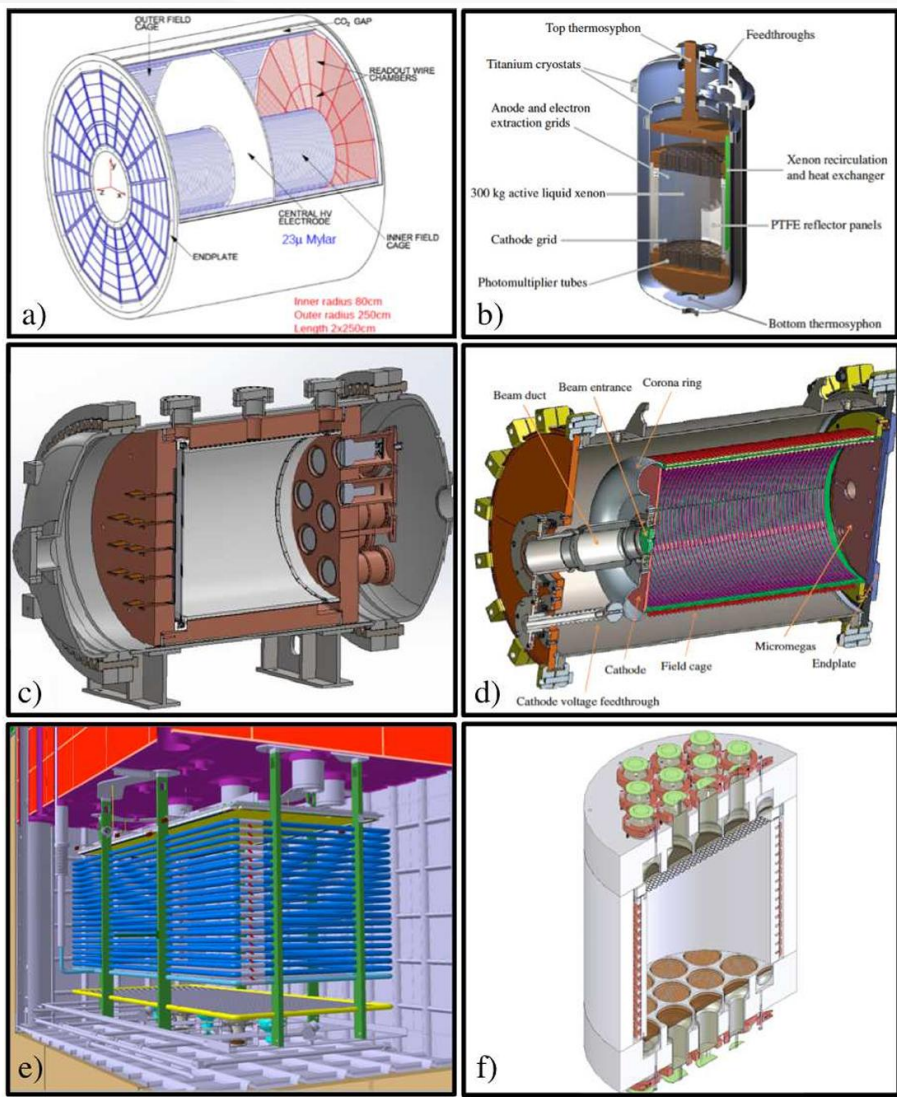
荆湛 仇晓升

2025年8月22日

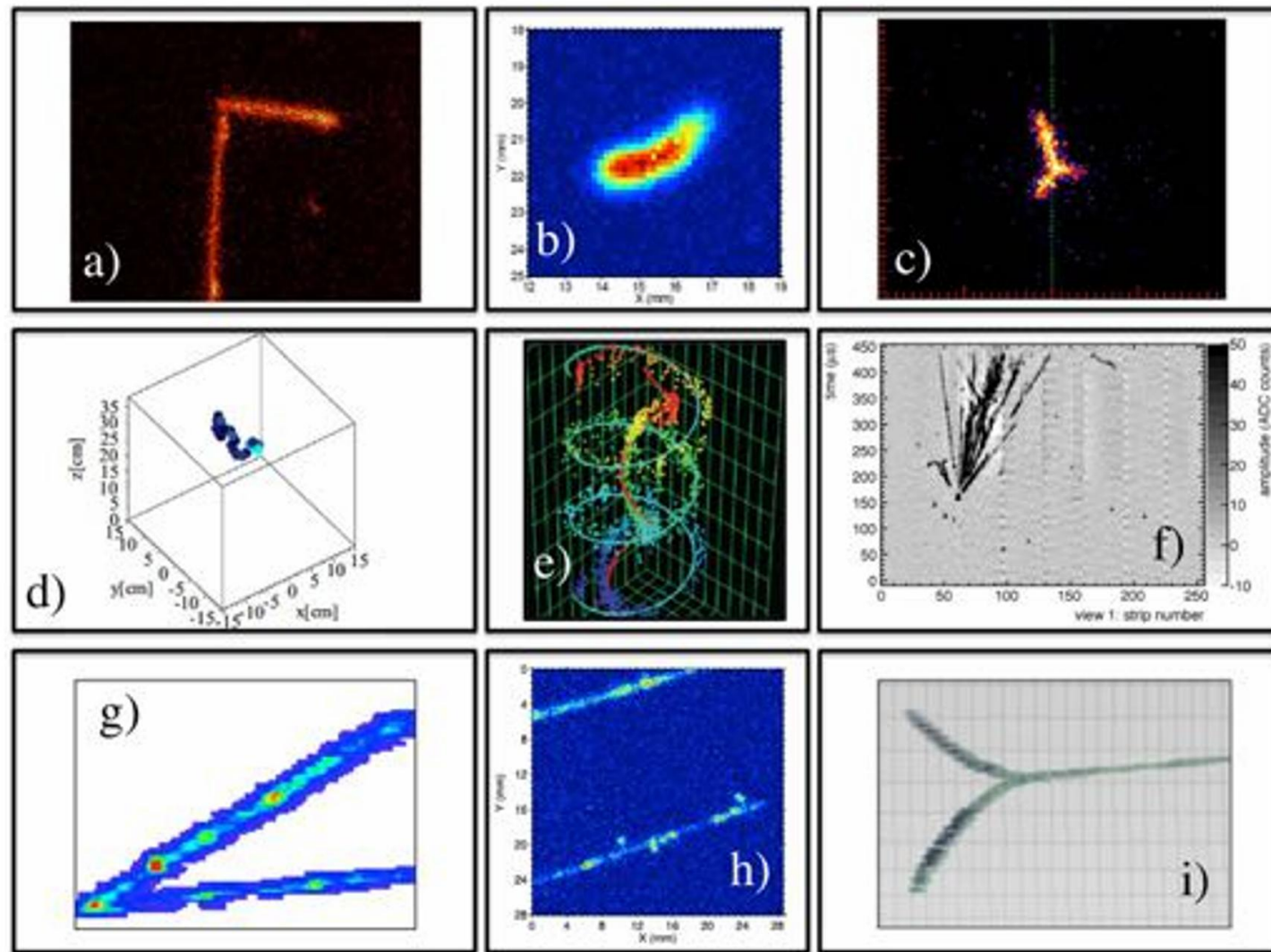
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时间投影室的径迹成像能力



一些典型的时间投影室



时间投影室中粒子径迹成像图示





基于卡尔曼滤波的粒子径迹重建

APPLICATION OF KALMAN FILTERING TO TRACK AND VERTEX FITTING

R. FRÜHWIRTH

Institut für Hochenergiephysik der Österreichischen Akademie der Wissenschaften, Vienna, Austria

Received 30 June 1987

卡尔曼滤波首次正式应用于粒子径迹重建和物理参数拟合

Track reconstruction in high density environment

M. Ivanov*, I. Belikov, P. Hristov, K. Šafařík

CERN, Geneva, Switzerland

Available online 30 May 2006

Abstract

This article presents **the reconstruction algorithm for the ALICE barrel detectors based on Kalman-filtering**. The algorithm is able to cope with ambiguous measurements in high-density environments. It consists of the following parts: space point localization, track finding and track fitting, performed in parallel with secondary vertex (V0) and kink topology finding.

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DETECTORS AND EXPERIMENTAL METHODS

Software alignment of the BESIII main drift chamber using the **Kalman Filter method**

To cite this article: Wang Ji-Ke *et al* 2009 *Chinese Phys. C* 33 210

3. 超级陶瓏装置上的RICH探测器研究

慧斌 张 (Chinese Academy of Sciences)

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46. 面向CEPC的高粒度像素型读出粒子鉴别模拟研究

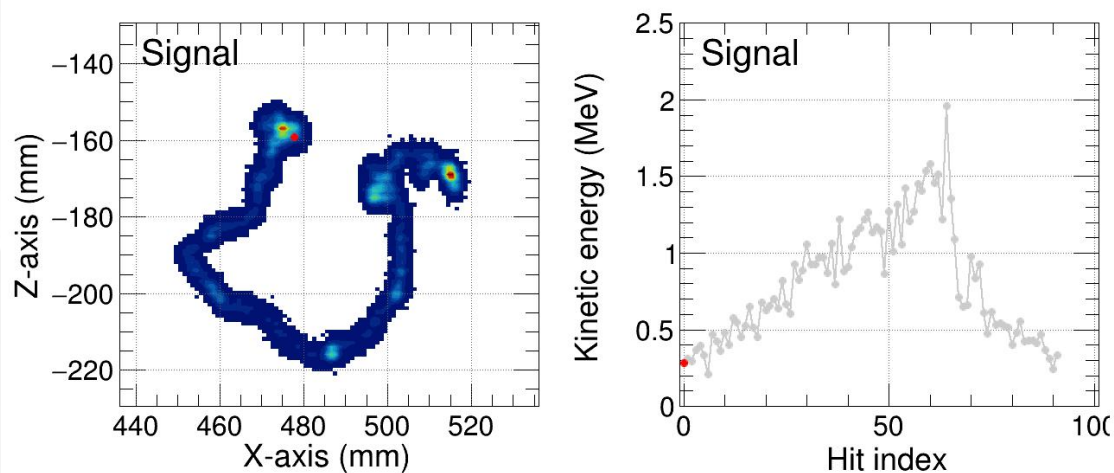
锦闲 张

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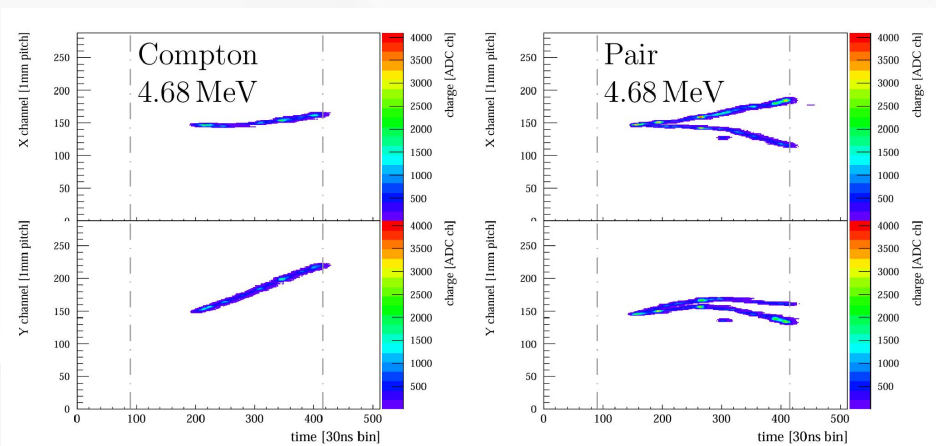
卡尔曼滤波至今应用于ALICE, BESIII, CEPC, STCF等实验中



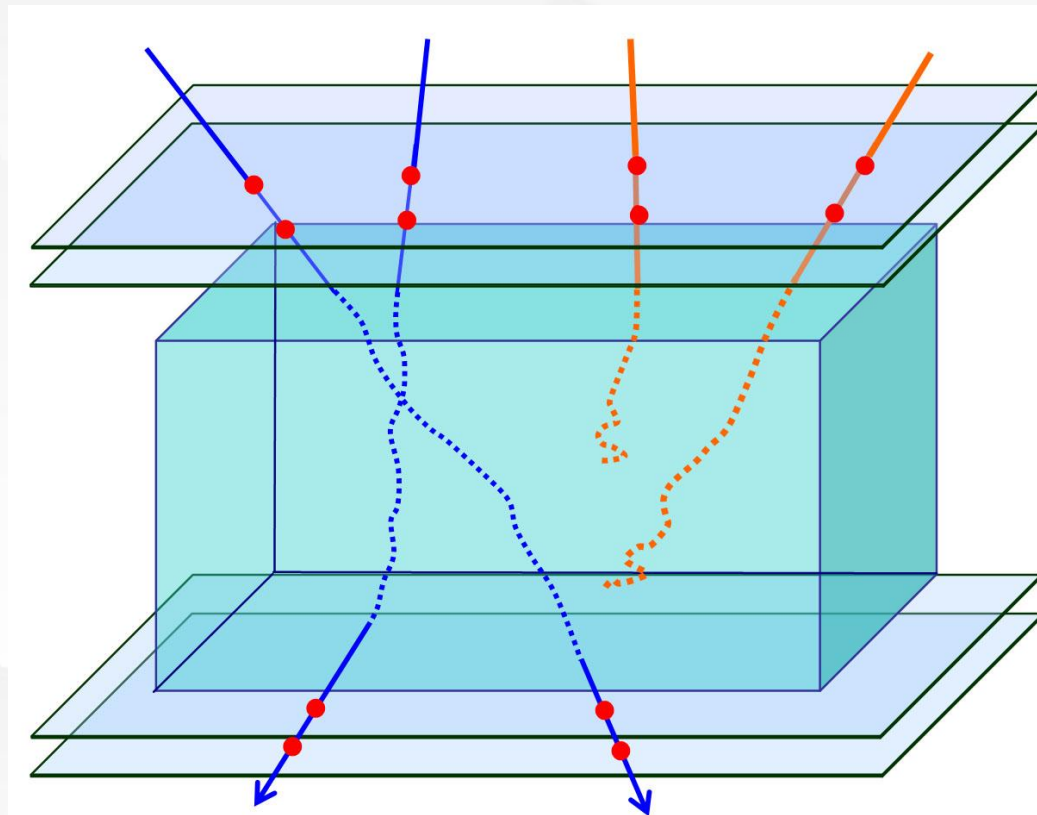
径迹动能重建的需求及应用



PANDAX-III实验：通过径迹重建进行无中微子双贝塔现象信号--本底鉴别



HARPO实验关注 γ 粒子的初始入射方向



对于致密材料，高能缪子穿透性强，低能缪子发生大角度散射/全沉积



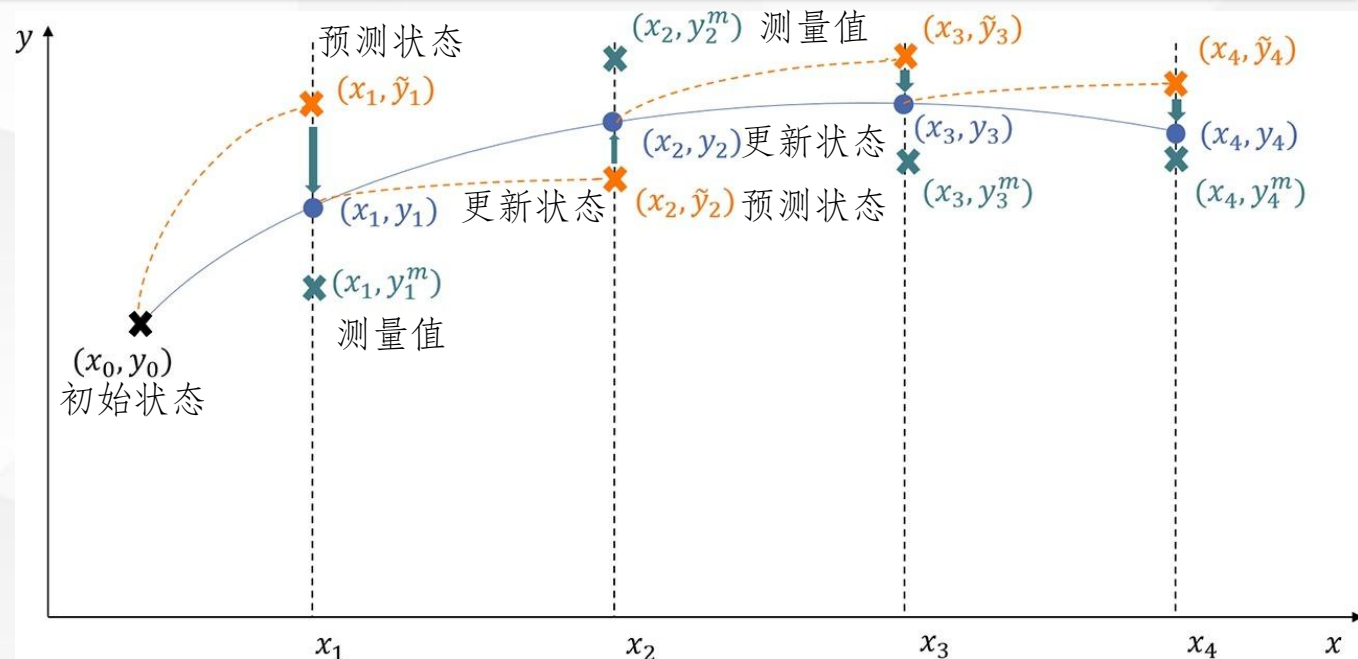
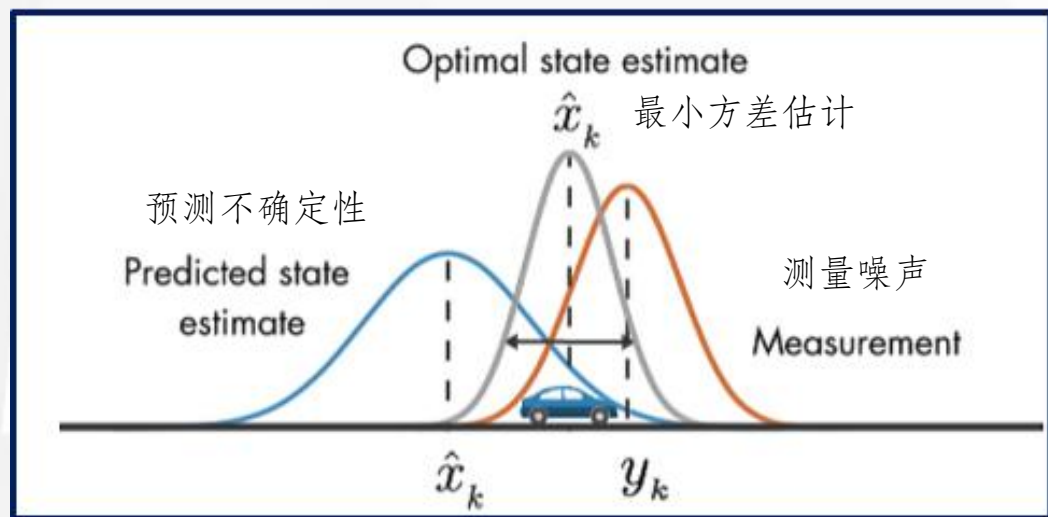
卡尔曼滤波

卡尔曼滤波：针对时变系统的最优状态估计方法

$$\begin{cases} \overset{\text{状态向量}}{\mathbf{x}_k} = \overset{\text{过程不确定性}}{F_k \mathbf{x}_{k-1}} + \overset{\text{过程不确定性}}{\omega_k} & (\text{系统动态变化}) \\ \underset{\text{观测向量}}{\mathbf{z}_k} = H_k \mathbf{x}_k + \underset{\text{测量噪声}}{V_k} & (\text{系统观测}) \end{cases}$$



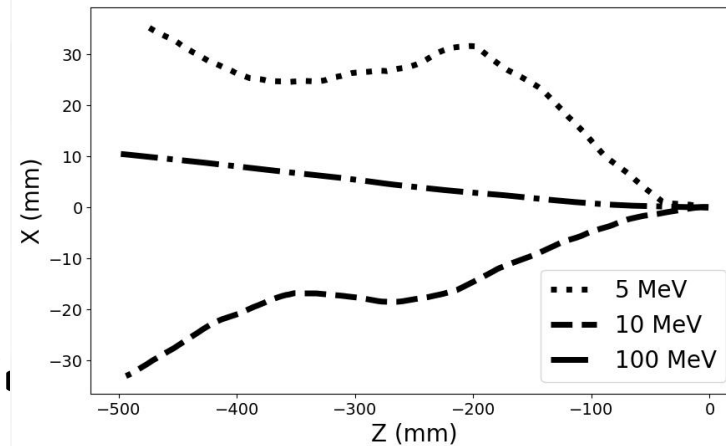
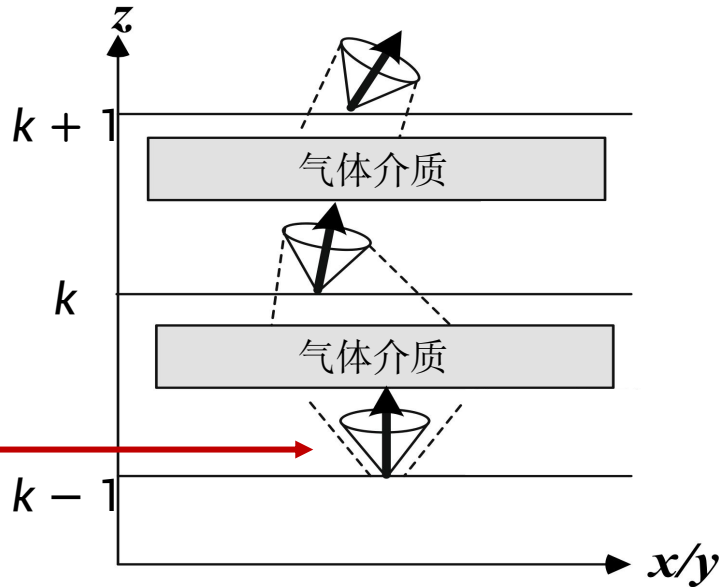
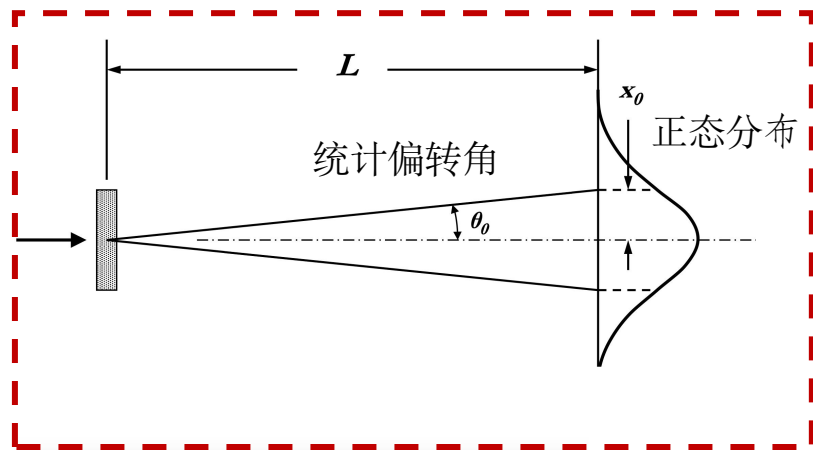
Rudolf E. KALMAN





粒子径迹模型

多次库伦散射



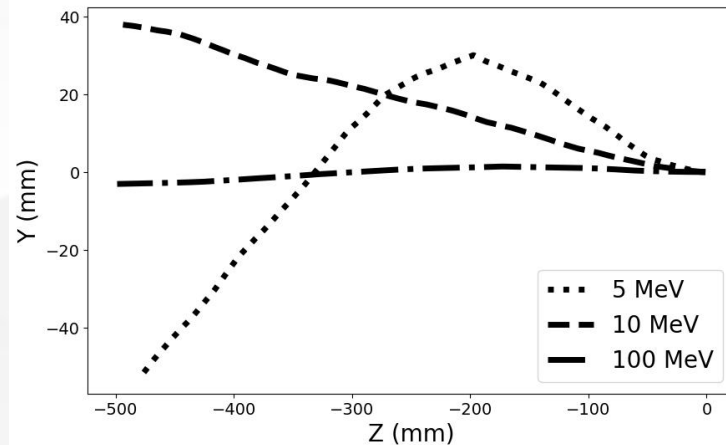
能量越低，径迹弯曲程度越大

动能——统计偏转角关系

电荷数 z 电子质量 E_e 气体介质厚度 L

$$\theta_{stat}^{rms} \approx \frac{z E_e}{p v} \sqrt{\frac{L}{L_r}} \quad (\text{莫里哀公式})$$

粒子动量 $p v$ 气体辐射长度 L_r

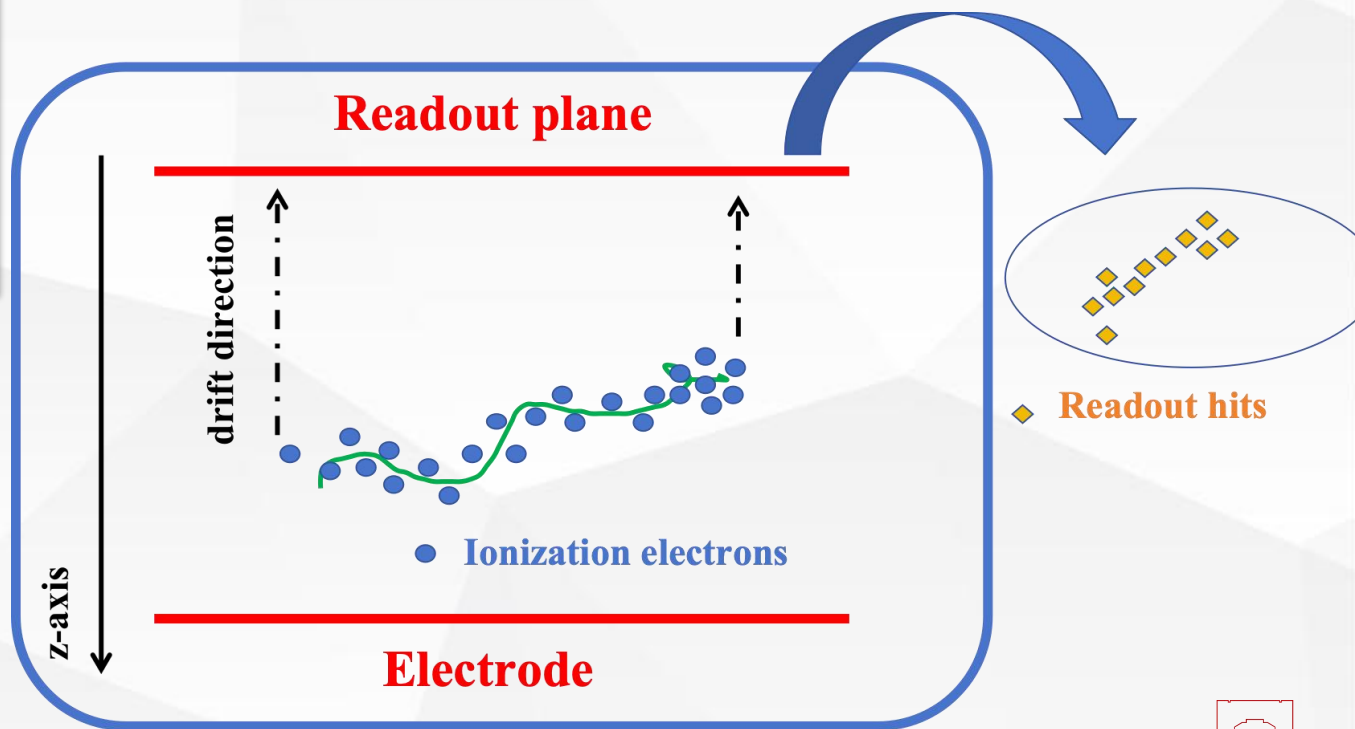


模拟参数

- 探测器几何: 0.5 m³
- 填充气体: Ar (1.784 g/cm³, 300 K, 5 个大气压)
- 初始状态: 位置 (0,0,0) mm, 速度方向(0,0,-1)
- 径迹采样长度: 1 mm
- 电子扩散+电子学读出噪声: 1mm 标准差白噪声

— 真实径迹
● 电离电子
■ 读出信号

气体时间投影室内粒子事件成像原理示意图





径迹重建的卡尔曼模型

卡尔曼滤波参数设置

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \omega_k$$

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k$$

$$\mathbf{x} = (c_x, c_y, c_z, u_x, u_y, u_z) \quad (\text{六维状态向量})$$

$$\mathbf{x}_0 = (0, 0, 0, 0, 0, -1) \quad (\text{初始状态})$$

$$\begin{cases} \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix}_k = \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix}_{k-1} + \Delta t \cdot \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}_{k-1} \\ \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}_k = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}_{k-1} + \omega_k \end{cases} \quad \Delta t = \frac{\Delta x}{\sqrt{u_x^2 + u_y^2 + u_z^2}}$$

采样距离 Δx

$$\omega_k \sim N(0, (\theta_{\text{stat}}^{\text{RMS}})^2 \cdot \mathbf{I}_3)$$

统计偏转角 $\theta_{\text{stat}}^{\text{RMS}}$

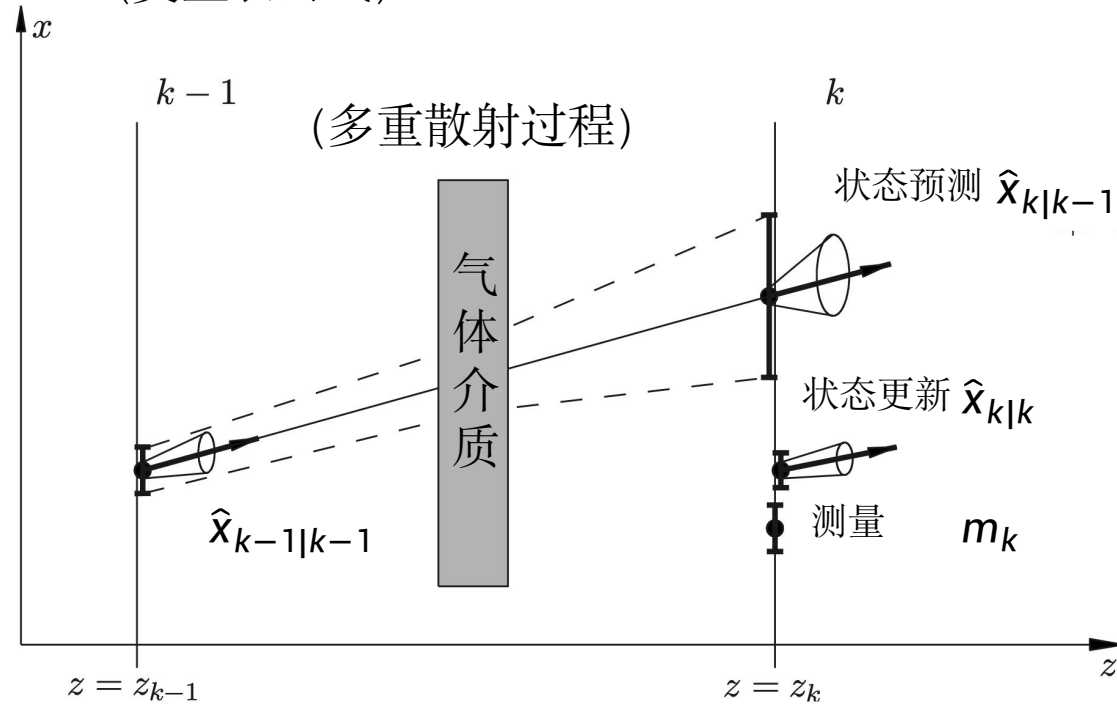
$$\begin{pmatrix} z_x \\ z_y \\ z_z \end{pmatrix}_k = \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix}_k + \mathbf{v}_k \quad \mathbf{v}_k \sim N(0, \sigma^2 \cdot \mathbf{I}_3)$$

测量误差 σ^2

统计偏转角-动量关系式

$$\theta_{\text{stat}}^{\text{rms}} \approx \frac{z E_e}{p \nu} \sqrt{\frac{L}{L_r}} \rightarrow \theta_{\text{stat}}^{\text{RMS}} \propto \frac{1}{p}$$

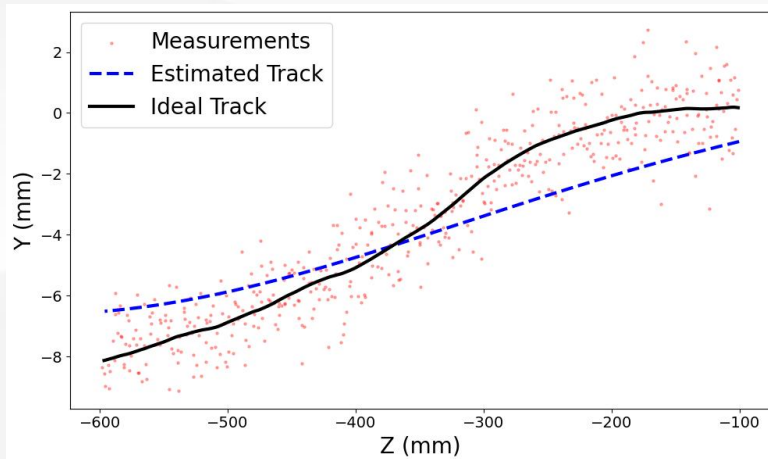
(莫里哀公式)



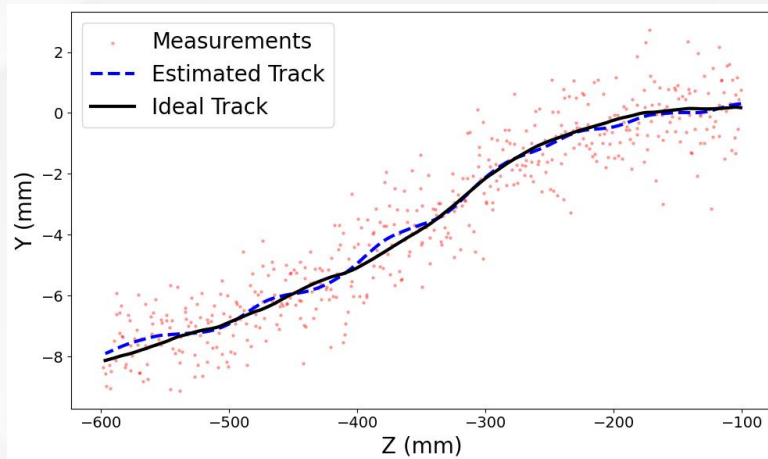
粒子径迹的卡尔曼滤波图示



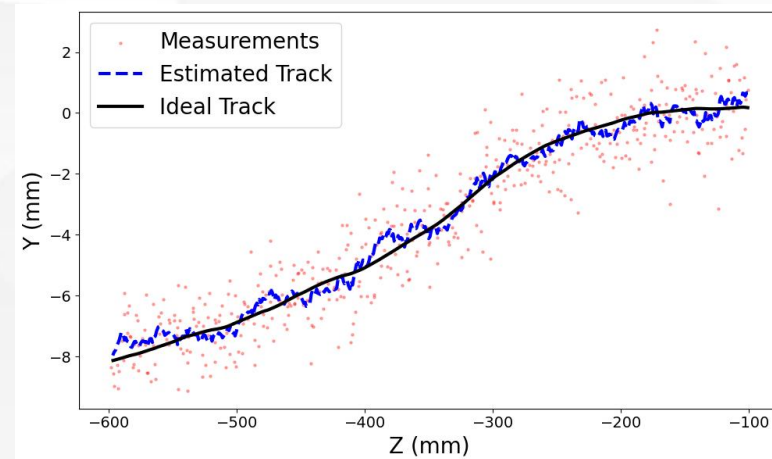
重建效果与动量的相关性



$$\theta_0 < \theta_{stat}^{rms}$$



$$\theta_1 \approx \theta_{stat}^{rms}$$



$$\theta_2 > \theta_{stat}^{rms}$$

统计偏转角 θ 对卡尔曼滤波的重建效果影响显著，以50 MeV事件为例

动量对径迹重建效果的影响

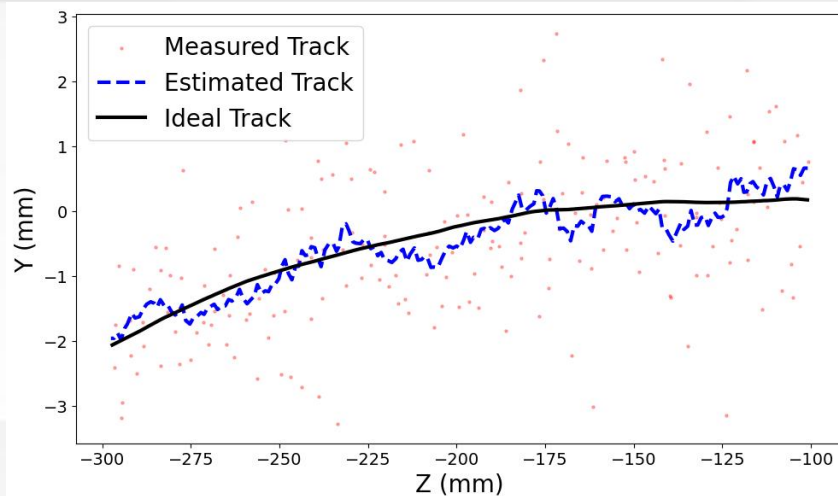
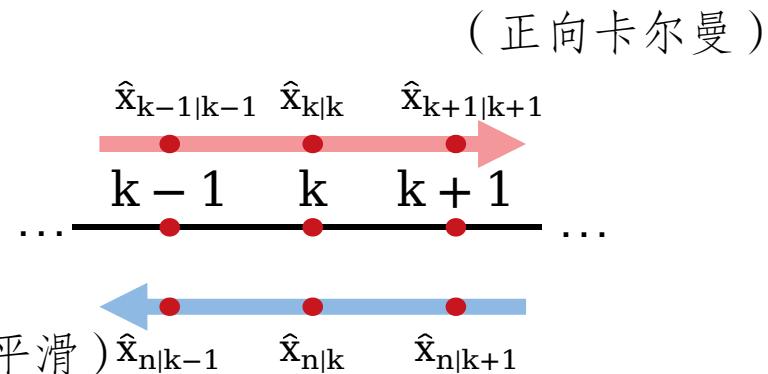
$$L(\theta_{stat}^{RMS} | \{m_1, m_2, \dots, m_T\}) \rightarrow p \propto \frac{1}{\theta_{stat}^{rms}}$$



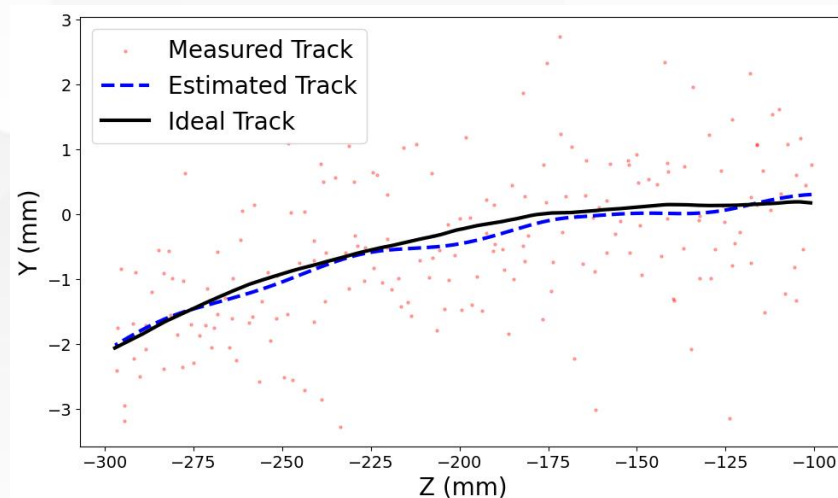
Rauch-Tung-Striebel 平滑

$(m_1, m_2, \dots, m_k) \Rightarrow \hat{x}_{k|k}$ (正向卡尔曼实时更新)

$(m_1, m_2, \dots, m_n) \Rightarrow \hat{x}_{n|k}$ (反向平滑融合所有时间步测量)



正向卡尔曼滤波 $\theta_0 \approx \theta_{stat}^{rms}$



反向平滑 $\theta_0 \approx \theta_{stat}^{rms}$





基于残差的滤波质量评估

卡尔曼-卡方滤波质量评估

重建质量因子:

$$L(\theta_0) = \left\| \frac{1}{n+1} \sum_{k=0}^n (\chi_{k|n}^2) - \dim(y_k) \right\|$$

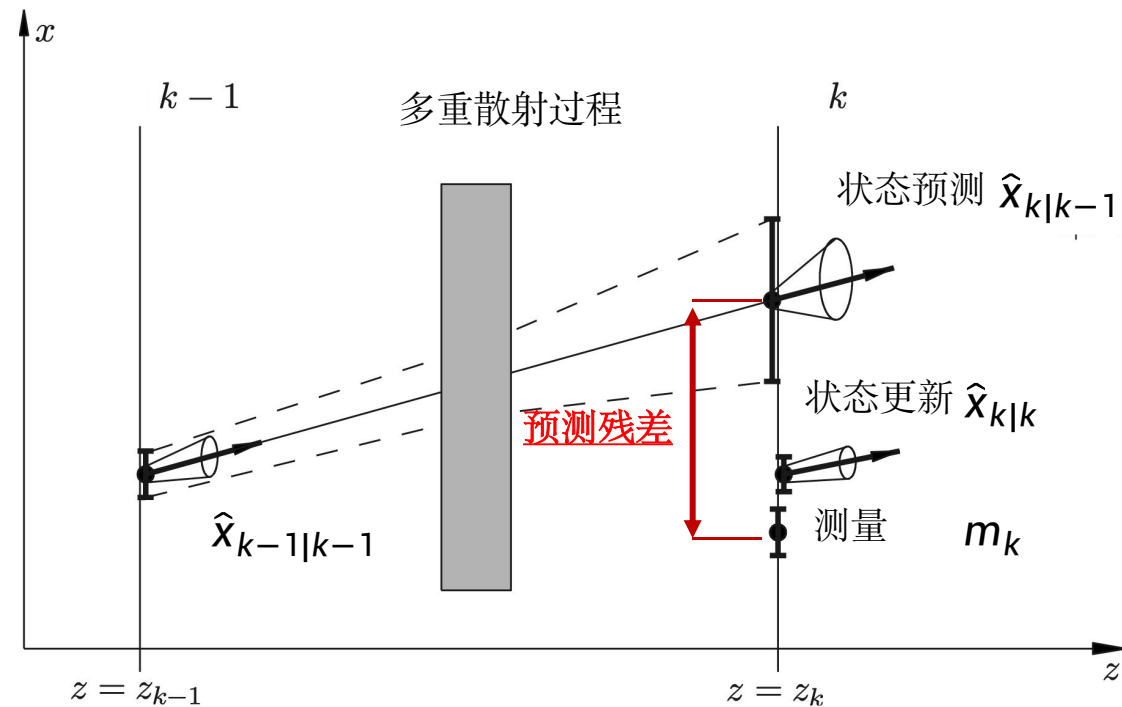
$$s_k = z_k - H_k \hat{x}_{k|k-1} \quad (\text{预测残差})$$

$$\chi_k^2 = (z_k - H_k \hat{x}_{k|k-1})^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} (z_k - H_k \hat{x}_{k|k-1})$$

$$P_{k|k-1}(Q, R) \quad (\text{预测残差卡方值})$$

$$\chi_{k|n}^2 = (z_k - H_k \hat{x}_{k|n})^T (R_k - H_k P_{k|n} H_k^T)^{-1} (z_k - H_k \hat{x}_{k|n})$$

$$P_{k|n}(Q, R) \quad (\text{平滑残差卡方值})$$



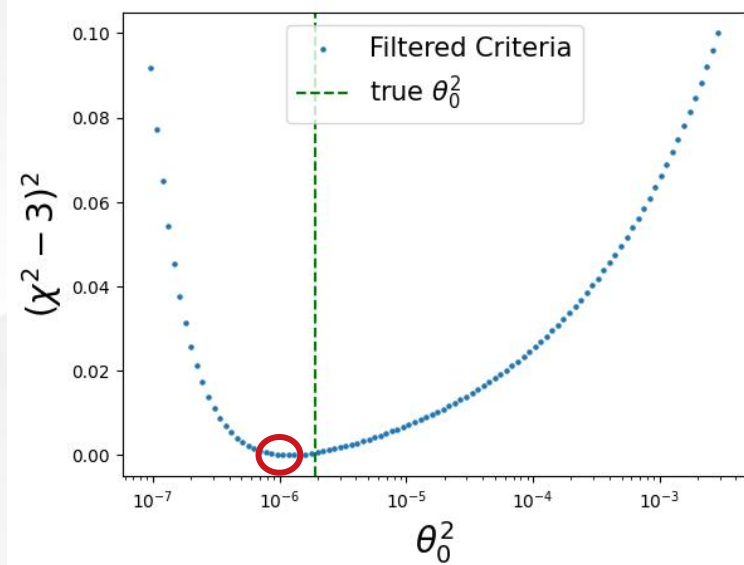
粒子径迹的卡尔曼滤波图示



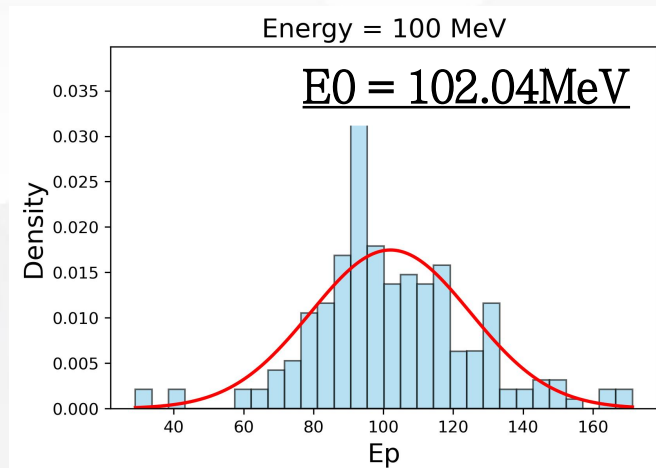
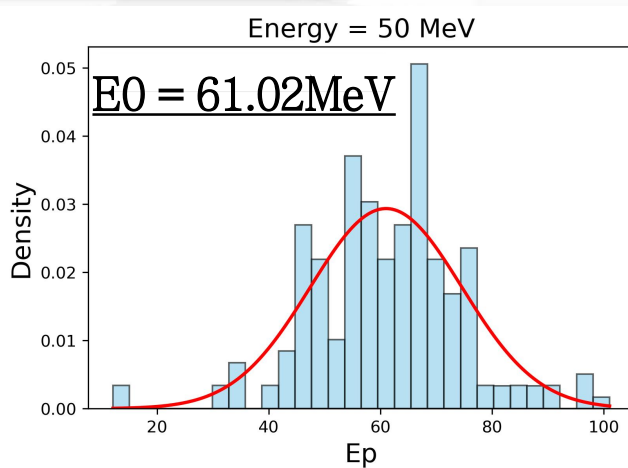
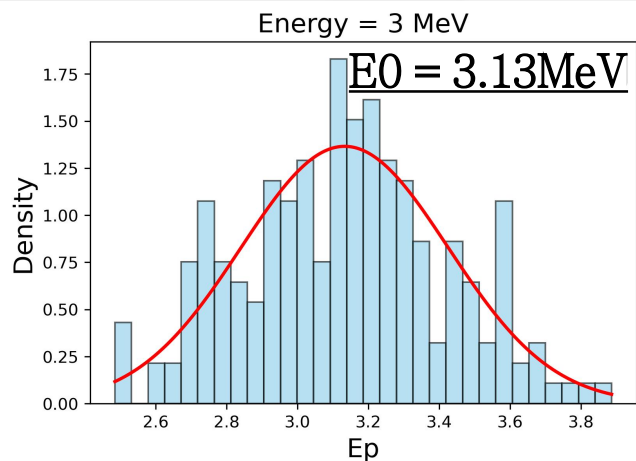


算法流程

1. 对于任意粒子事件径迹数据，采样 $\theta_0 \in [\theta_{\min}, \theta_{\max}]$
2. 执行卡尔曼滤波，记录预测残差；
计算质量评估 $L(\theta_0)$
3. 选取 $\underset{\theta_0}{\operatorname{argmin}} L(\theta_0)$ (最小重建质量因子)
4. 计算事件动能

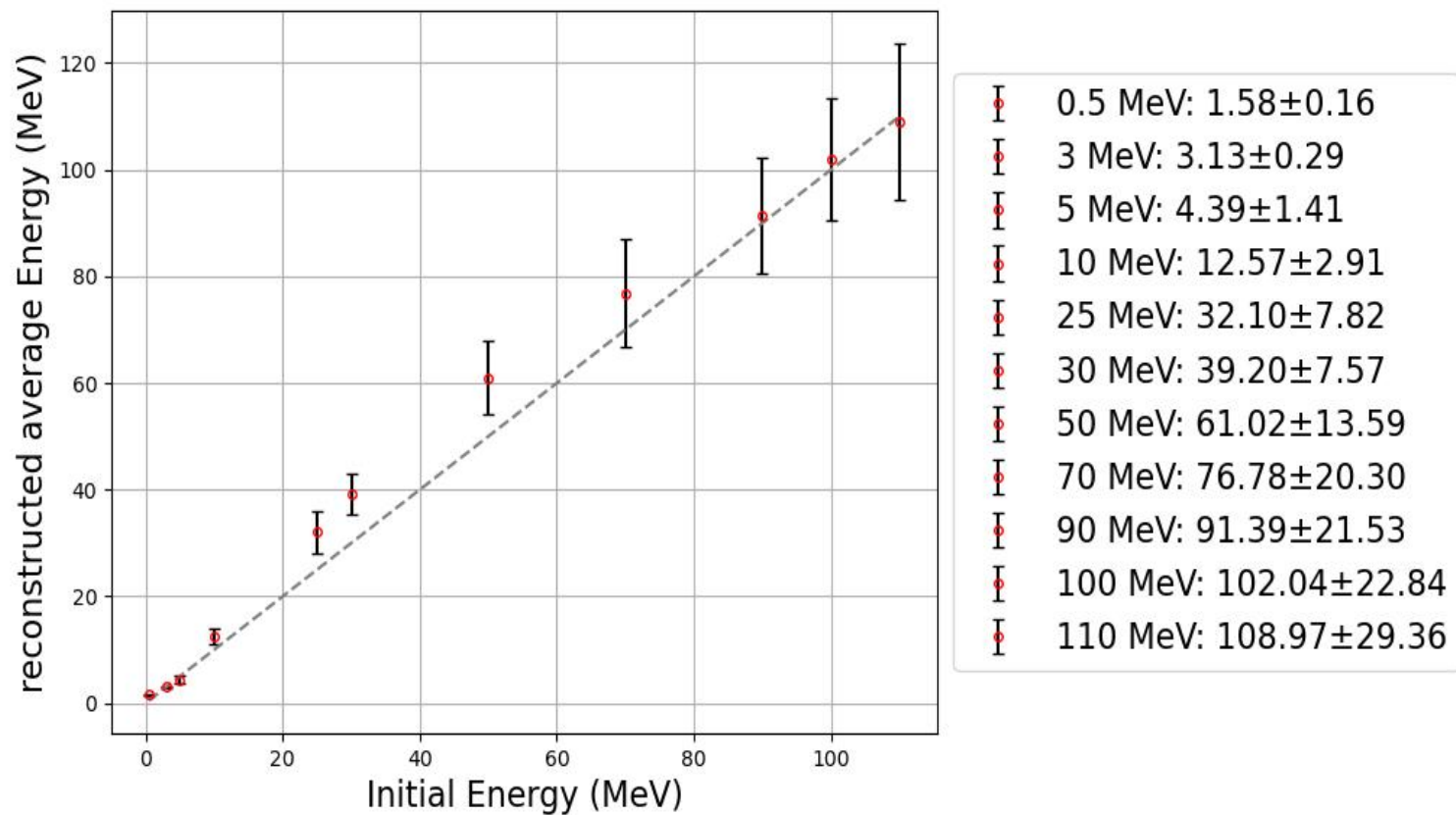


动能拟合效果，以25 MeV事件为例



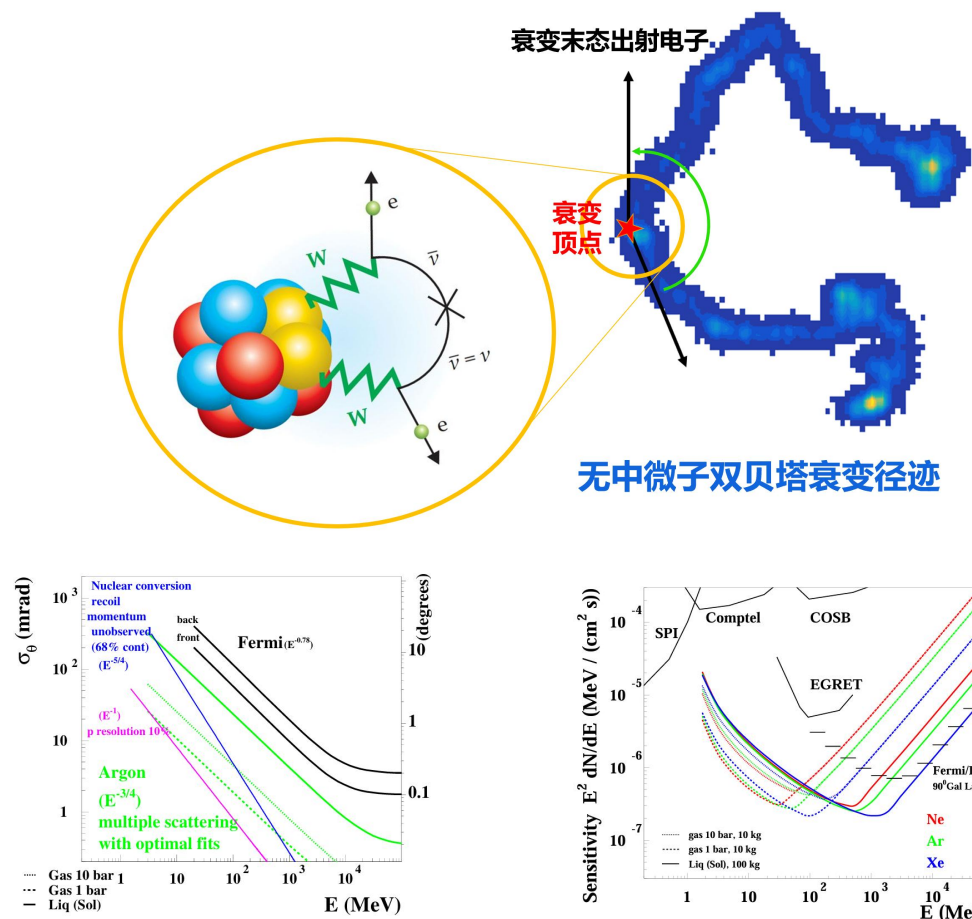


电子动能重建效果



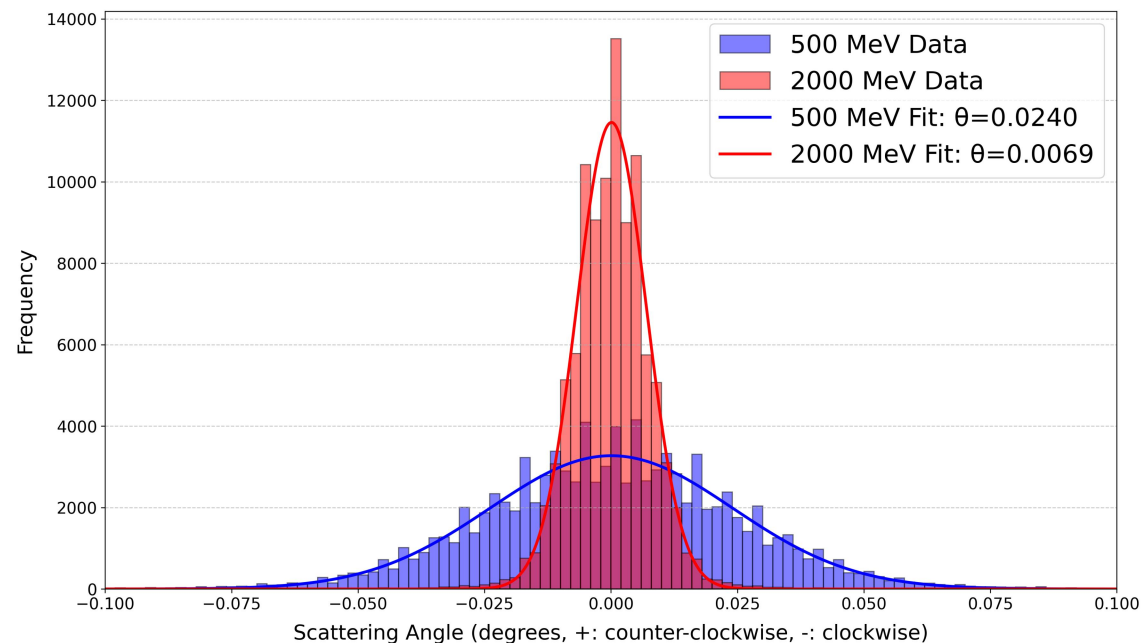
完成100 MeV能量以下电子的动能重建

高能电子重建



缪子成像

$$\theta_{stat}^{rms} \approx \frac{z E_e}{p\nu} \sqrt{\frac{L}{L_r}} \quad (m_\mu \gg m_e)$$



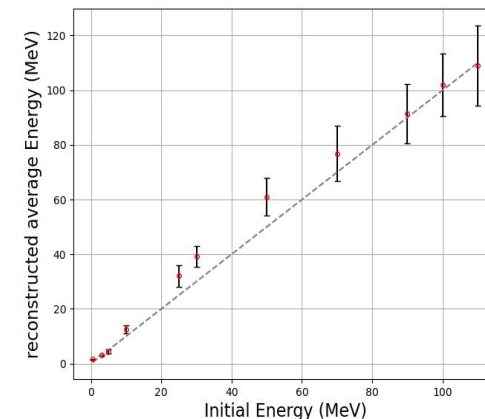
不同能量缪子的统计散射角分布 σ 展宽



基于卡尔曼滤波的能量重建

- 基于卡尔曼滤波的径迹统计偏转角拟合和动能拟合
- 定义卡尔曼--卡方的计算质量因子
- 验证在<100MeV区间电子动能重建的有效性

$$L(\theta_{\text{stat}}^{\text{RMS}} | \{m_1, m_2, \dots, m_T\})$$



讨论与未来工作

- 应用于缪子成像, 无中微子双贝塔衰变的事件顶点重建等
- 进一步扩展至扩散噪声 σ 和初始入射方向 x_0 等参数拟合, 融合能损数据 dE/dx 重建



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感谢聆听

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