

Mustafa A. Amin

A Spin on Solitons (in wave dark matter) REARICE

lots of fun high energy physics

after inflation

IΣ.

or
\n
$$
n \lambda_{dB}^3 \sim 10^{33} \left(\frac{10^{-5}eV}{m}\right)^{4} \sim 10^{33} \left(\frac{10^{-20}eV}{m}\right)^{4}
$$
\n
$$
\lambda_{dB} \sim 10^{3} cm \left(\frac{10^{-5}eV}{m}\right) \left(\frac{10^{-3}eV}{m}\right)^{4}
$$
\n
$$
\frac{n}{m}
$$
\n
$$
\frac{10^{-20}eV}{m}
$$
\n
$$
\frac{10^{-22}eV}{m}
$$
\n
$$
\frac{10^{-20}eV}{m}
$$
\n
$$
\frac{10^{-20}eV}{m
$$

some relevant literature from our group at Rice U + coll.

Spin of wave dark matter from astrophysics?

2109.04892 2111.08700 2203.11935 2211.08433 2301.11470 2309.04345 2211.08433

2304.01985 Jain, Thomas, Waniswecharungruang with Jain Jain, Zhang Jain, Karur, Mocz Jain Long, Schiappacasse Helfer, Wang Jain, Pu

Jain Wisha Thomas Zhang Schiappacasse Long Karur Mocz Helfer Wang

spin and dark matter sub-structure

Phenomenology

- reduced interference

- growth of structure, nucleation time-scales

bolla rassaballer

- polarized solitons, with macroscopic spin - polarized solitons, with macroscopic spin $\prod_{i=1}^{n}$ |
|
| and with - polarized solitons, with macroscopic spin O
O $\frac{1}{1}$

regime. We found the focus of subhorizon dynamics, and hence ignore \mathcal{L} is an operator \mathcal{L} in the substitution. In the substitution of \mathcal{L} is an operator of \mathcal{L} in the substitution. In the substitution. non-relativistic limit

Spin - 2s+1 Jain & MA (2021) [2109.04892] Adshead & Lozanov (2021)

p*^g ^g^µ*↵*g*⌫ *^Gµ*⌫*G*↵ ⁺

$$
S = \int d^4x \sqrt{-g} \left[-\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \frac{1}{2} \frac{m^2 c^2}{\hbar^2} X \right]
$$

$$
X_{\mu\nu} = \nabla_{\mu} X_{\nu} - \nabla_{\nu} X_{\mu}
$$

 ad "slow" parts

$$
\mathbf{X}(t,\mathbf{x}) \equiv \frac{\hbar}{2mc} \Re \left[\mathbf{\Psi}(t,\mathbf{x}) e^{-imc^2 t/\hbar} \right]
$$
split in "fast" and "slow" parts

$$
\mathcal{S}_{nr} = \int \mathrm{d}t \mathrm{d}^3 x \left[\frac{i\hbar}{2} \mathbf{\Psi}^\dagger \mathbf{\Psi} + \mathrm{c.c.} - \frac{\hbar^2}{2m} \nabla \mathbf{\Psi}^\dagger \cdot \nabla \mathbf{\Psi} + \frac{1}{8\pi G} \Phi \nabla^2 \Phi - m \, \Phi \, \mathbf{\Psi}^\dagger \mathbf{\Psi} \right]
$$

$$
[\mathbf{\Psi}]_i = \psi_i \text{ with } i = 1, 2, 3 \quad \text{vector case}
$$

$$
i\hbar \frac{\partial}{\partial t} \mathbf{\Psi} = -\frac{\hbar^2}{2m} \nabla^2 \mathbf{\Psi} + m \Phi \mathbf{\Psi}, \qquad \nabla^2 \Phi = 4\pi \mathbf{U}
$$

r2

⁺ *^m ,* ^r²

= 4⇡*Gm †*

. (2.4)

$$
[\mathbf{\Psi}]_i = \psi_i \text{ with } i = 1 \quad \text{scalar case}
$$

$\nabla^2 \Phi = 4 \pi G m \Psi^{\dagger} \Psi$

at this level this is just
but not when non-gravi $d = 1$ dark matter, we have a single component field (which leads to the \mathcal{U} usual \mathcal{U} is the \mathcal{U} at this level this is just 2*s*+1 equal-mass scalar felds but not when non-gravitational interactions are included!

⁼ [~]² **non-relativistic limit = multicomponent Schr**ö**dinger-Poisson**

Jain & MA (2021) [2109.04892]

$$
[\mathbf{\Psi}]_i = \psi_i \text{ with } i=1,2,3
$$

This is our master equation that we work with the term in the term in the α re-iterate that a re-iterate that a

$$
N = \int d^{3}x \Psi^{\dagger} \Psi, \text{ and } M = mN,
$$
 (particle number and rest mass)
\n
$$
E = \int d^{3}x \left[\frac{\hbar^{2}}{2m} \nabla \Psi^{\dagger} \cdot \nabla \Psi - \frac{Gm^{2}}{2} \Psi^{\dagger} \Psi \int \frac{d^{3}y}{4\pi |\mathbf{x} - \mathbf{y}|} \Psi^{\dagger} (\mathbf{y}) \Psi(\mathbf{y}) \right], \text{ (energy)}
$$

\n
$$
\mathbf{S} = \hbar \int d^{3}x \, i \Psi \times \Psi^{\dagger}, \text{ (spin angular momentum)}
$$

\n
$$
\mathbf{L} = \hbar \int d^{3}x \, \Re \left(i \Psi^{\dagger} \nabla \Psi \times \mathbf{x} \right).
$$
 (orbital angular momentum)

spin angular momentum

$S=\hbar$ z
Zanada
Zanada d^3 $x i \mathbf{V} \times \mathbf{V}^{\dagger}$

 $0 \leq |\mathcal{S}| \leq \hbar N$ $0 \leq |\mathcal{S}| \leq \hbar N$
 $0 \leq |s| \leq \hbar \mathbf{\Psi}^{\dagger} \mathbf{\Psi}$

Jain & MA (2021) [2109.04892]

Z

$[**W**]$ _{*i*} = ψ_i with $i = 1, 2, 3$

This is our master equation that we work with the term in the term in the α re-iterate that a re-iterate that a

$\alpha = i\hbar$ lli $\dot{\mathbf{r}} \times \mathbf{u}$ $s = i\hbar \mathbf{\Psi}^{\dagger} \times \mathbf{\Psi}$

$\mathbf{U} \leq |\mathcal{S}| \leq \mathbf{U}\mathbf{\Psi}^{\top}\mathbf{\Psi}$ $0<|s|<\hbar \mathbf{\Psi}^{\dagger} \mathbf{\Psi}$

condensation in the kinetic regime Jain, MA, Wanischarunarung, Thomas (2023)

din, MA, Wanischarunarung, Thomas (2023)
 distrated by Condensation in the kinetic regime

condensation in the kinetic regime

 $\hat{\rho}_{\rm max}$

$$
- nucleation time scale
$$

$$
\tau_s \sim (2s+1)\tau_{s=0}
$$

$$
\tau_{s=0} = \left[n \sigma_{\rm gr} v \mathcal{N} \right]^{-1}
$$

$$
\sigma_{\rm gr} \sim (Gm/v^2)^2, \qquad \mathcal{N} \sim n \lambda_{\rm dB}^3
$$

see Levkov et. al (2018) for scalar case with Jain, Thomas, Wanichwecharungruang (2023)

$$
\Gamma_{(s)} = \Gamma_0 / (2s + 1)
$$

of such waves becomes capable of balancing their own

kinetic relaxation — multi-component case the dynamic rale vetion is m described by the following non-relativistic Schröder-Einsteinger-Einstelle of Bose-Einsteinger-Einstelle of Bo number function for the condensing species develops and the condensing species develops and the condensing species develops and c relaxation — multi-component case an eikonal approximation where the change in relation where the change in relation where the change in relatio
The change in relation where the change in relation where the change in relationship with the change in relati where *gb*(*t*) carries all the time-dependence of the distribution function near small velocities, with *gb*(*t* = 0) = 1.

being the velocity vector for the incoming *b* species,

 P is system of equations: P is sympathy \mathcal{L} system of equations: \mathcal{L} system of equations:

$$
i\frac{\partial}{\partial t}\psi_a = -\frac{1}{2m_a}\nabla^2\psi_a + m_a\Phi\,\psi_a
$$

$$
\nabla^2\Phi = 4\pi G \sum_a m_a \psi_a^*\psi_a.
$$

$$
\frac{\partial f_{v_a}^a}{\partial t} = \sum_b m_b^3 \frac{\Lambda}{2\pi} \frac{(4\pi m_a m_b G)^2}{m_a} \nabla_{v_a^i} \left[\frac{\mathcal{D}_{ij}^{ab}}{2m_a} \nabla_{v_a^j} f_{v_a}^a + \frac{\mathcal{F}_i^{ab}}{m_b} f_v^a \right]
$$

where
$$
\mathcal{D}_{ij}^{ab} = \int \frac{d\tilde{v}_b}{(2\pi)^3} f_{\tilde{v}_b}^b \frac{\delta_{ij} - \hat{u}_i \hat{u}_j}{u} f_{\tilde{v}_b}^b \quad \text{and} \quad \mathcal{F}_i^{ab} = 0.
$$

Lain MA Thomas Wanichwecharungruang. 6 multicomponent wave kinetic equation (with a set of 6 multicomponent with array 6 multicomponent with array 6 multicomponent with a set of 6 multipliers with a set of 6 multipliers with a set of bitrary 2 body interaction) using a random phase approx- $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ the Boltzmann equation in Formula (2023)
Formula Formula Formul Planck form at leading order perturbation theory: ⇢¯*b^a* Jain, MA, Thomas, Wanichwecharungruang (2023)

With the set r replacements, we finally arrive at the follow-set r arrive at the follow-set α arrive at the follow-set α

where the state is a parameters are simply informed by the initial state α

$$
\frac{\partial f_{\mathbf{v}_a}^a}{\partial t} = \sum_b m_b^3 \frac{\Lambda}{2\pi} \frac{(4\pi m_a m_b G)^2}{m_a} \nabla_{v_a^i} \left[\frac{\mathcal{D}_{ij}^{ab}}{2m_a} \nabla_{v_a^j} f_{\mathbf{v}_a}^a + \frac{\mathcal{F}_i^{ab}}{m_b} f_{\mathbf{v}_a}^a \right]
$$
\nwhere\n
$$
\mathcal{D}_{ij}^{ab} = \int \frac{d\tilde{\mathbf{v}}_b}{(2\pi)^3} f_{\tilde{\mathbf{v}}_b}^b \frac{\delta_{ij} - \hat{u}_i \hat{u}_j}{u} f_{\tilde{\mathbf{v}}_b}^b \quad \text{and} \quad \mathcal{F}_i^{ab} = f_{\mathbf{v}_a}^a \int \frac{d\tilde{\mathbf{v}}_b}{(2\pi)^3} \frac{\hat{u}_i}{u^2} f_{\tilde{\mathbf{v}}_b}^b
$$
\n
$$
\sum_{\mathbf{v}_a = \sum_b} \frac{\Lambda (4\pi G)^2 \bar{\rho}_b}{\sigma_a^3 \sigma_b^3} \left[2 \frac{\bar{\rho}_a}{m_a^3} - \beta_{ab} \frac{\bar{\rho}_b \sigma_a}{m_b^3 \sigma_b} \right]
$$

condensation in the kinetic regime may be obtained by the kinetic regime may be obtained by the kinetic regime
The kinetic regime may be obtained by the kinetic regime may be obtained by the kinetic regime may be obtained

[2304.01985]

rightmost column show the radial profile of the magnitude of

 $\mathcal{C}(\mathcal{C})$ and $\mathcal{C}(\mathcal{C})$ and $\mathcal{C}(\mathcal{C})$ are calculate the rate of change of ch

disting Mat are these "blobs" ? Jain, MA, Wanischarunarung, Thomas (2023)

[2304.01985]

solitons in *s* **>** 0 **fields** spatially localized, coherently oscillating, long-lived

spatially localized

coherently oscillating (components)

exceptionally long-lived

$$
-\mu f = -\frac{1}{2m}\nabla^2 f + m\Phi f \qquad \nabla^2 \Phi = 4\pi G m f^2
$$

 $\nabla^2 \Phi = 4 \pi G m \, \mathbf{\Psi}^{\dagger}$

$$
i\hbar \frac{\partial}{\partial t} \Psi = -\frac{1}{2m} \nabla^2 \Psi + m \, \Phi \, \Psi , \qquad \nabla^2 \Phi = 4\pi G m \, \Psi^{\dagger} \Psi
$$

$$
\hbar = c = 1
$$

vector solitons ? the debroglie scale (*mu*)1 for the same boson mass α and velocity dispersion. This provides in the same boson mass α long time scales.⁹ Typically some nonlinear interaction balances the tendency of the fields to disperse. For our purposes here, the nonlinear interaction is provided by gravity. In the non-linear interaction is provided by gravitation is provided by gravity. In the non-linear interaction is provided by gravity. In t

1.0

0

There are a number of di⊄erent ways about thinking about thinking about thinking about such solitons. In the n
In this case of diverse about this case of diverse about such solitons. In this case of the non-electrons. In

 $\Psi(t, r) = f(r)e^{i\mu t}$ **c** with $\boldsymbol{\epsilon}^{\dagger}$ **c** = 1*,* $\mathbf{\Psi}(t, r) = f(r)e^{i\mu t} \boldsymbol{\epsilon}$ with $\boldsymbol{\epsilon}^{\dagger} \boldsymbol{\epsilon} = 1$ ϵ

clearly in upper right panels in the simulation of the simulation with a collection of the simulation with a c
In Fig. 12, where we started the simulation with a collection of the simulation of the simulation of the simul

 = 0 = *±*1 tensor $f(r) \cos[(m - \mu)t] \hat{\mathbf{z}}$ $f(r)[\hat{\mathbf{x}} \cos[(m - \mu)t]$ *X* / *f*(*r*)[*x X* / *f*(*r*)[*x* ˆ cos[(*m µ*)*t*] + *y* $f(r) \cos[(m-\mu)t]$ $\sum_{i=1}^{n}$

x

clearly in the simulation in the simulation of the simulation with a collection with a collection with a colle
The simulation with a collection with a collection with a collection of the simulation of the simulation of th

*S*tot =

~

0 *S*tot =

 = 0 = *±*1 $(n - \mu)t]\hat{\mathbf{z}}$ $f(r)[\hat{\mathbf{x}} \cos[(m - \mu)t] + \hat{\mathbf{y}} \sin[(m - \mu)t]]$ ˆ cos[(*m µ*)*t*] + *y X* / *f*(*r*)[*x* ˆ cos[(*m µ*)*t*] + *y X* / *f*(*r*) cos[(*mµ*)*t*]*z* ˆ. Similarly, ✏*[±]* will lead to coherent, circularly polarized configurations: $f(r)[\hat{\boldsymbol{x}} \cos[(m-\mu)t] + \hat{\boldsymbol{y}} \sin[(m-\mu)t]]$.

 $\frac{1}{2}$ $\$ Jain & MA (2021) [2109.04892] $T_{\rm eff}$ spin for the total spin for the these solitons sol Jain & MA (2021) [2109.048 energy soliton depends on the sign of the interaction! That is, the polarization of the ground $\text{jan } \& \text{PIA (2021)} \quad \text{[2109.04892]}$ $\hbar = c = 1$ Jain & MA (2021) [2109.04892]

N = # of particles in soliton

*S*tot =

~

0 *S*tot =

because of the vector nature, we now have a polarization. A basis for the polarization vectors

" polarized" vector solitons **idealized** i spin i and j are a robust feature of wave dynamics and so is the so clearly in the simulation in the simulation of the simulation with a collection with a collection with a collection of the simulation of the simulation with a collection of the simulation of the simulation of the simulatio suppression with the number of components!

$$
\boldsymbol{\epsilon}_0 = \hat{\boldsymbol{z}}
$$

 $\bm{\epsilon}_\pm = \hat{\bm{x}} \pm i \bm{y}/\sqrt{2}$ $\sqrt{2}$ $\frac{1}{2}$ i.e. $\sqrt{2}$ $\boldsymbol{\epsilon}_\pm = \boldsymbol{x} \pm \imath \boldsymbol{y}/\sqrt{2}$

$$
\Psi(t,r) = f(r)e^{i\mu t} \epsilon \quad \text{with} \quad \epsilon^{\dagger} \epsilon = 1.
$$

$$
\boldsymbol{X}(t,\boldsymbol{x}) = \sqrt{\frac{2}{m}} \Re \left[\boldsymbol{\Psi}(t,\boldsymbol{x}) e^{-imt} \right]
$$

*S*tot =

~

0 *S*tot =

 = 0 = *±*1 = *±*2 Jain & MA (2021) [2109.04892] Jain & MA (2021) $\sin R \text{ MA} (2021)$ $\text{I} 2109048921$ The total spin for the total spin Jain & MA (2021) [2109.04892]

N = # of particles in soliton

*S*tot =

~

0 *S*tot =

$$
\pmb{S}_{\rm tot}=0
$$

"polarized" vector solitons (with macroscopic spin) ~ itone (with r *N* = # of particles in soliton **Macroscopi** the deBroglie scale (*mv*)¹ for the same boson mass *m* and velocity dispersion. This provides us macroscopic spin (or number of components). See Fig. 9. You can also see this electronic see this electronic clearly in upper right panels in the simulation of the simulation with a collection with a collection with a c
The simulation with a collection with a collection with a collection of the simulation of the simulation of th components (2*s* + 1). One important aspect is that while ⇢*/*⇢ changes, there is no change in the deBroglie scale (*mv*)¹ for the same boson mass *m* and velocity dispersion. This provides

$$
\boldsymbol{\epsilon}_0 = \hat{\boldsymbol{z}}
$$

 $\epsilon_{\pm} = \hat{x} \pm i y/\sqrt{2}$

$$
S_{\text{tot}} = 0 \qquad \qquad S_{\text{tot}} = \hbar \frac{M_{\text{sol}}}{m} \hat{z}
$$

$$
S_{\rm tot}=i(\boldsymbol{\epsilon}\times\boldsymbol{\epsilon}^\dagger)\frac{M_{\rm sol}}{m}\hbar
$$

macroscopic intrinsic spin!

x

*S*tot =

0 *S*tot =

*S*tot = ~0 *S*tot =

N = # of particles in soliton

"polarized" vector solitons (with macroscopic spin) *s* (with macroscop) ~ 0 *S*tot = **Macroscophy** *N* = # of particles in soliton *S*tot*/*~ = *Nz* ˆ

*S*tot =

0 *S*tot =

 $\boldsymbol{S}_{\text{tot}} = 0$

 $\frac{1}{2}$ $\boldsymbol{S}_{\text{tot}}=\hbar$ *m* $\hbar \frac{1}{2}$

- bases for partially-polarized solitons Einstein

Einstein

$$
\bm{S}_{\rm tot}=i(\bm{\epsilon}\times\bm{\epsilon}^\dagger)\frac{M_{\rm sol}}{m}\hbar
$$

*^S*tot ⁼ [~]⁰ *^S*tot ⁼ *^S*tot ⁼ $R_{\rm A}$ $M_{\rm A}$ (2021) **Stock = 3** Stock = $\frac{1}{2}$ Stock = $\frac{$ **macroscopic** spin *N* $\sin 2 M \wedge (2021)$ *S*tot*/*~ = *Nz*ˆ Also see: Adshead & Lozanov (2021), Jain & MA (2021)

arxiv: 2109.04892. Mudit Jain & Mustafa A. Amin = 0 = *±*1 = *±*2 \uparrow 0 \downarrow \down = 0 = *±*1 = *±*2 *s* =0 *s* =0 = 0 = 1 = 2 \circledcirc *^S*tot ⁼ [~]⁰ *^S*tot ⁼ *^S*tot ⁼ 0 *S*tot = *S*tot = 0 *S*tot = *S*tot = 0 *S*tot = *S*tot = $M_{\rm sol}$

$$
0 \leq |\mathcal{S}_{\text{tot}}| \leq \frac{M_{\text{sol}}}{m} \hbar
$$
\n
$$
C_0 \underset{i=1, \ldots, l}{\overset{i=1, \ldots, l}{\underset{i=1, \ldots, l}{\overset{i=1, \ldots, l}{\underset{1, \ldots, l}{\overset{i=1, \ldots, l}{\overset{i=1, \ldots, l}{\overset{1, \ldots, l}}{\overset{1, \ldots, l}{\overset{2, \ldots, \ldots, l}{\overset{2, \ldots, \ldots, l}{\overset{2, \ldots, \ldots, l}{\overset{2, \ldots, l}}{\overset{2, \ldots, l}{\overset{2, \ldots, l}}{\overset{2, \ldots, l}{\overset{2, \ldots
$$

- all lowest energy for fxed *M*

 $S_{\rm core} \sim \hbar$ *M*core *m* \hbar

even when initial total spin is negligible

1*.*0

MA, Jain, Karur & Mocz(2022) $\sum_{n=1}^{\infty}$ f_{en} , the bottom paral shows the ensemble mean and $\left(1-\frac{1}{2}\right)$ Jain, MA, Thomas, Wanichwecharungruang (2023) boson in the core (within 2*rc*) is shown in the middle, along with its typical precession around the mean over a de-Broglie time scale. We take the small near the small new have the sign that we have a sign that we h

born to spin $F_{\rm eff}$ simulations, we take the condensations, we take the condensation time to be $F_{\rm eff}$ the time when the time when the time \mathcal{L}_{max}

spin density

11

a different higher energy soliton: the "hedgehogs" Sphere symmetric symmetric state in the result of the soliton set of the solitons for the solitons for the solitons of the solitons of the solitons of the solitons for the solitons for the soliton symmetric symmetric state spin-1 and spin-2 fields are the 'hedgehog'-like configura-Spherically symmetric, single node solitons for the a different higher energy soliton: the 'head tions, with C is the components \mathcal{I}_1 , \mathcal{I}_2 , \mathcal{I}_3 , \mathcal{I}_4 , \mathcal{I}_5 , \mathcal{I}_6 , \mathcal{I}_7 , \mathcal{I}_8 , \mathcal{I}_9 , $\mathcal{I}_$ 11 de janvier de la problème de la
Viene de la problème nt higher energy soliton: the "hedgehogs" Fe
I rent higher energy soliton: the "hedgeho el
L nedgehogs" Einstein

$$
W_j(\mathbf{x},t) = f(r) \frac{x^j}{r} \cos \omega t,
$$

 $_{\rm hh}^{\rm (s)}>E$

Hot ground states solutions, we belief the phenomenologies some of the phenomenologies some of the phenomenologies of the phe *z s* + 1 extremally *S*tot*/*~ = *Nz* 0 *S*tot =

POLARIZED SOLITONS (POLARIZED SOLITONS)
POLARIZED SOLITONS (POLARIZED SOLITONS) Hugungs hedgehogs not ground states **macroscopic** spin

V. DISTINGUISHABILITY ARE DESCRIPTION OF PROBES OF
The probes of probes

at least when non-relativistic ical implications. Lozanov & Adshead (2021) Lozanov & Adshead (2021) ~ Lozanov & Adshead (2021) at least when non-relativistic

POLARIZED SOLITONS

Fierz-Pauli (*s* = 2) earlier literature

Fierz-Pauli (*s* = 2)

attractive non-gravitational self-interactions

Fierz-Pauli (*s* = 2)

+

macroscopic spin

*S*tot*/*~ = *Nz*ˆ

macroscopic spin

 $\frac{1}{2}$

*S*tot*/*~ = *Nz*ˆ

x

Zhang, Jain & MA (2022)

$$
\boldsymbol{S}_{\text{tot}}=0
$$

Also see Jain (2021), Zhang & Ling (2023)

 W begin with a 3-component SC \sim component SC \sim component SO(3) system with S

at three instants *t* = 0*,* 13, and 40 (upper panel). Lower panel are snapshots of magnitude of

Schrodinger-Poisson systems with self-interactions and polarized soliton soliton soliton soliton soliton solit gravitational and non-gravitational interactions.

i-SPin: An algorithm (and publicly available code) to numerically evolve multicomponent Schrodinger-Poisson (SP) systems, including attractive/repulsive self-interactions + gravity **on the Schroen of SS-P1** our worker work work worker worker and the non- α numerically α or α multicomponent α /repulsive self-interactions \pm

problem: If SP system represents the non-relativistic limit of a massive vector field, nongravitational self-interactions (in particular, *spin-spin* type interactions) introduce new challenges related to mass and spin conservation which are not present in purely gravitational systems. On account of this, we have the following general action that includes both Newtonian gravity S_{2} is the model in purply and state the action of S_{2} *x x x x x x x* \therefore *i* \there 2° Spin-1 \circ Spin-1 \circ system W begin with a 3-component Schrüodinger-Poisson system with SO(3) system with \mathcal{A} ι type interactions) introduce new challenges \blacksquare present in purely gravitational systems. and spin conservations, and polarized soliton soliton soliton soliton soliton soliton soliton soliton soliton s

solution: Above challenges addressed with a novel analytical solution for the non-trivial 'kick' step in the algorithm (sec 4.3.2) **i**), while the third and fourth terms account for the Gauss' law for the Gauss' law for Newtonian gravity where $\overline{}$ tical solution for the non-trivial 'l *m † · V*nrel(*, †* gravitational and non-gravitational interactions. d with a novel analytical solution for the non-trivial 'kick' and spin conservations, and polarized soliton soliton soliton soliton soliton soliton soliton soliton soliton \mathcal{L}

features: (i) second order accurate evolution (ii) spin and mass conserved to machine precision (iii) reversible *FRACE SPIN-2 vector fields* α is the Schröde spin-1 vector α If and mass conserved to machine precision *ⁱ ⁱ* contributes to the Newtonian potential . Finally, the last term accounts for point interactions of the vector field , and takes the following specific relativistic and mass conserved to machine precision (¹*,* ²*,* ³) transforms as *ⁱ* ! *Rij ^j* with *R* 2 SO(3), but leaves the action unchanged.¹ On account of this, we have the following general action that includes both Newtonian gravity 2 Spin-1 Schr¨odinger-Poisson system M_{\odot} and M_{\odot} and M_{\odot} system with S_{\odot} system with S_{\odot} system with S_{\odot} volution (ii) spin-and mass conserved to W begin with a 3-component SC μ \geq S (3) symmetry with SO(3) system with S

generalizations: *n*-component fields with $SO(n)$ symmetry, an expanding universe relevant for cosmology, and the inclusion of external potentials relevant for laboratory settings and spin density *S* = *i*~ ⇥ *†* al potentials relevant for laboratory settings an exp panding universe relevant for *m † · V*nrel(*, †* (¹*,* ²*,* ³) transforms as *ⁱ* ! *Rij ^j* with *R* 2 SO(3), but leaves the action unchanged.¹ an expanding universe relevant for this includes both Newtonian general action that includes both Newtonian gra relativistic and mass in mind. The Schrödens in mind. The Schrödens in mind. That is, the Schrödens in mind. The Schrödens in min \mathcal{S} with $\mathcal{S}O(n)$ symmetry, an expanding universe relevant for \mathbf{a} interactions self-interactions s

i-SPin 2: An integrator for general spin-s (¹*,* ²*,* ³) transforms as *ⁱ* ! *Rij ^j* with *R* 2 SO(3), but leaves the action unchanged.¹

arXiv: 2305.01675 Mudit Jain, Mustafa Amin & H. Pu

systems carrying a variety of interactions, where the $2s+1$ components of the 'spinor' field # our work. A collection of a collection of appendices provide a derivation of the non-

> *ⁱ ⁱ* contributes to the Newtonian potential . Finally, the field (both short and long-range, and spin-dependent and spin-independent interactions).

(*·*) *†* +2(*† ·*) i *, m † · V*nrel(*, †*) *.* $V(T)$

(= 0*.*01). Lower panel include snapshots from the backward evolution at the same instants.

1 8⇡*G*r²

(*·*) (*† · †*

)+2(*† ·*)

³⇢² (*^S · ^S*)

Applications: (a) Laboratory systems such as spinor Bose-Einstein condensates (BECs). (b) *.* (2.3) #

(2.1)

 L

i-SPin: An integrator for multicomponent Mudit Jain & Mustafa Amin

. (2.2)

, the spin-spin

i

Here, the first two terms dictate the usual free field evolution (of each of the field component

 $1\leq k\leq n$ we use the Einstein summation through the paper of p

 $\mathbf{f} = \mathbf{f} \cdot \mathbf{f} + \mathbf{$

our work. A collection of appendices provide a derivation of the nonrelativistic action, fluid action, fluid α

$$
S_{ac} = \int dt d^3x \left[\frac{i}{2} \psi_n^{\dagger} \psi_n + c.c. - \frac{1}{2\mu} \nabla \psi_n^{\dagger} \cdot \nabla \psi_n - \mu \rho V(x) - \gamma \mathcal{S} \cdot \mathcal{B}(x, t) - V_{\alpha\sigma} - \frac{\xi}{2} \frac{1}{(2s+1)} |\psi_n \hat{A}_{nn'} \psi_{n'}|^2 + i g_{ij} \psi_n^{\dagger} [\hat{S}_i]_{nn'} \nabla_j \psi_n \right],
$$

with $B(x,t) = f(t)B(x)$, and

$$
V_{\text{nrel}}(\rho, \mathcal{S}) = -\frac{1}{2\mu^2} \left[\lambda \rho^2 + \alpha \left(\mathcal{S} \cdot \mathcal{S} \right) \right]
$$

number density $\rho = \psi_n^{\dagger} \psi_n$ $\mathcal{S} = \psi_n^*\,\hat{S}_{nn'}\,\psi_{n'}$ spin density.

8⇡*G*r²

³⇢² (*^S · ^S*)

r2

probing intrinsic spin of solitons *^s*= 2

Schrödinger non-relativistic limit

Poisson

 $\frac{1}{2}$

 $\mathcal{C}=GM/Rc^2$ $-p$

compactness of polarized solitons

observed in Settling in Settling in Settling in Sector, the gravity sector, $3c^2$ ster include include a superign with Thomas Helfer & Zipeng Wang (2023)

2*s*+1 component

Poisson

 \rightarrow

more resistance to collapse to BH for circularly polarized stars

*R*¹ and *R*2. The spin of the solitons are *S*¹ and *S*² respectively. The e↵ective potential governing

gravitational waves and spin and their contractions of mass a distance results of mass α (for example phase of gravitational waves in the weak field limit. Let us assume that we have two have two have
The weak field limit of the weak field limit. Let us assume two have two have two have two have two have two h

2

$$
\begin{aligned}\nJ_{y_{\mu}} &= -\frac{GM_1M_2}{r} \left[1 + \mathcal{O}(v^2/c^2) - \frac{2}{rc} [\hat{\mathbf{r}} \times (\mathbf{v}_1 - \mathbf{v}_2)] \cdot \sum_{a=1}^2 \frac{S_a}{M_a} \right. \\
&\left. \frac{s_{\nu}}{s_{\nu}} \frac{1}{s_{\nu}} \left\{ \frac{S_1}{M_s} \cdot \frac{S_2}{\varepsilon_0^2} - 3 \left(\frac{S_1}{M_1} \cdot \hat{\mathbf{r}} \right) \left(\frac{S_2}{M_2} \cdot \hat{\mathbf{r}} \right) + \sum_{a=1}^2 \frac{C_{ES^2}^{(a)}}{2M_1M_2} \left[S_a^2 - 3(S_a \cdot \hat{\mathbf{r}})^2 \right] \right\} + \dots \right]\n\end{aligned}
$$

Proca (*^s*= 1)

Fierz-Pauli (*^s* = 2)

Photons from Dark Photon Solitons via Parametric Resonance

with Schiappacasse & Long (2022)

MA & Mou (2019) 2009.11337 2301.11470

$\mathcal{L}_{\text{int}} \sim g^2 X X F F$

spin of soliton & polarization of photons 3*.*0 \mathcal{O}_1 and \mathcal{O}_2 , linear pol. The third, fourth, and fifth operators involve only one factor of the electromagnetic field *Aµ*(*x*). In the presence of a background data background data photon field \mathcal{O}_3 and \mathcal{O}_4 , these points and \mathcal{O}_3 and \mathcal{O}_4 radiation such such suppressed from suppressed for long-wavelength suppressed for long-wavelength background f 2.5 and we do not discuss the neglected \sim 2.5 $\,$

explosive photon production (under certain conditions)

2*.*5

with Schiappacasse & Long (2022) magnetism though several dimension-6 operators via the phenomenon of parametric resonance. *Top:*

 \mathcal{O}_3 and \mathcal{O}_4 , linear pol. \mathcal{O}_3 and \mathcal{O}_4 , circular pol.

² (*^B · ^X*) $\kappa =$ soliton radius, $\mu =$ $R =$ soliton radius, $\mu =$ Floquet exponent

$$
\mathcal{O}_1 = -\frac{1}{2} F_{\mu\nu} \tilde{F}^{\mu\nu} (X \cdot X_5)
$$

\n
$$
\mathcal{O}_2 = -\frac{1}{2} F_{\mu\nu} F^{\mu\nu} (X \cdot X_5)
$$

\n
$$
\mathcal{O}_3 = F_{\mu\rho} F^{\nu\rho} X^{\mu} X_{\nu} \quad 0.5
$$

\n
$$
\mathcal{O}_4 = \tilde{F}_{\mu\rho} \tilde{F}^{\nu\rho} X^{\mu} X_{\nu} \quad 0.0
$$

\n
$$
\mathcal{O}_5 = F_{\mu\rho} F^{\nu\rho} \partial^{\mu} X_{\nu} \quad 0.0
$$

\n
$$
\mathcal{O}_6 = F_{\mu\rho} F^{\nu\rho} \partial^{\mu} X_{\nu} \quad (k-m)/(g^2 \bar{X}^2 m)
$$

\n
$$
\mathcal{O}_7 = F_{\mu\rho} F^{\nu\rho} \partial^{\mu} X_{\nu} \quad (k-m)/(g^2 \bar{X}^2 m) \quad (k-m)/(g^2 \bar{X}^2 m) \quad (k-m)
$$

$$
\mu R \gtrsim 1, \qquad \mu \sim g^2
$$

 $g^2\bar{X}$

generalization to arbitrary spin

extremally polarized solitons

spin-*s* **felds as dark matter**

massive $\frac{1}{2}$ Inales $\mathsf{e}\mathsf{s}$ \mathbf{b} \mathbf{H} $|ca|$ $\overline{}$ \overline{z}

 $e \mathsf T$ $\overline{}$ - growth of structure, nucleation time-scales

Phenomenology

- reduced interference

- polarized solitons, with macroscopic spin \sim poiarized somons, with macroscopic spin \mathbf{i} \overline{a} *s s with* r Schrödinger non-relativistic limit *s*+1 solitons *s* = 1 $\frac{1}{2}$ \sim

a baryonic mass of ⁸*.*0⇥109 ^M. The total projected mass of the lens within the critical curve is set by the Einstein radius at ²*.*7⇥1010 ^M. Allowing for an uncertainty of ±0*.*2 dex in the baryonic mass, we adopt a uniform prior on 5DM between 0.5 and 0.8 (see Table 1). This prior range is consistent with dark matter fractions in massive early-type lens galaxies studied by Oldham & Auger (2018). We assume that all small-scale inhomogeneities in the lensing convergence are produced by FDM granules in the lens itself. We do