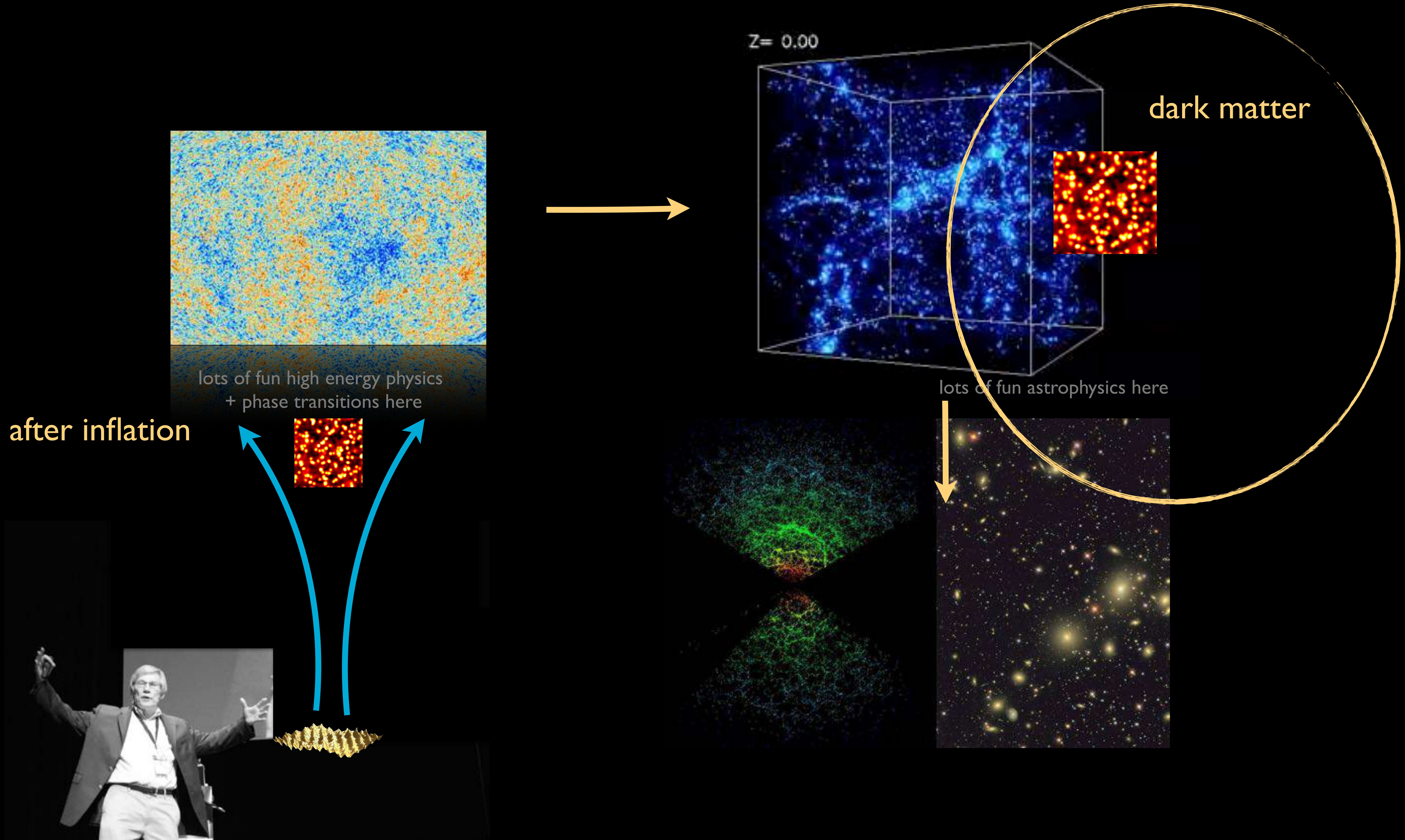
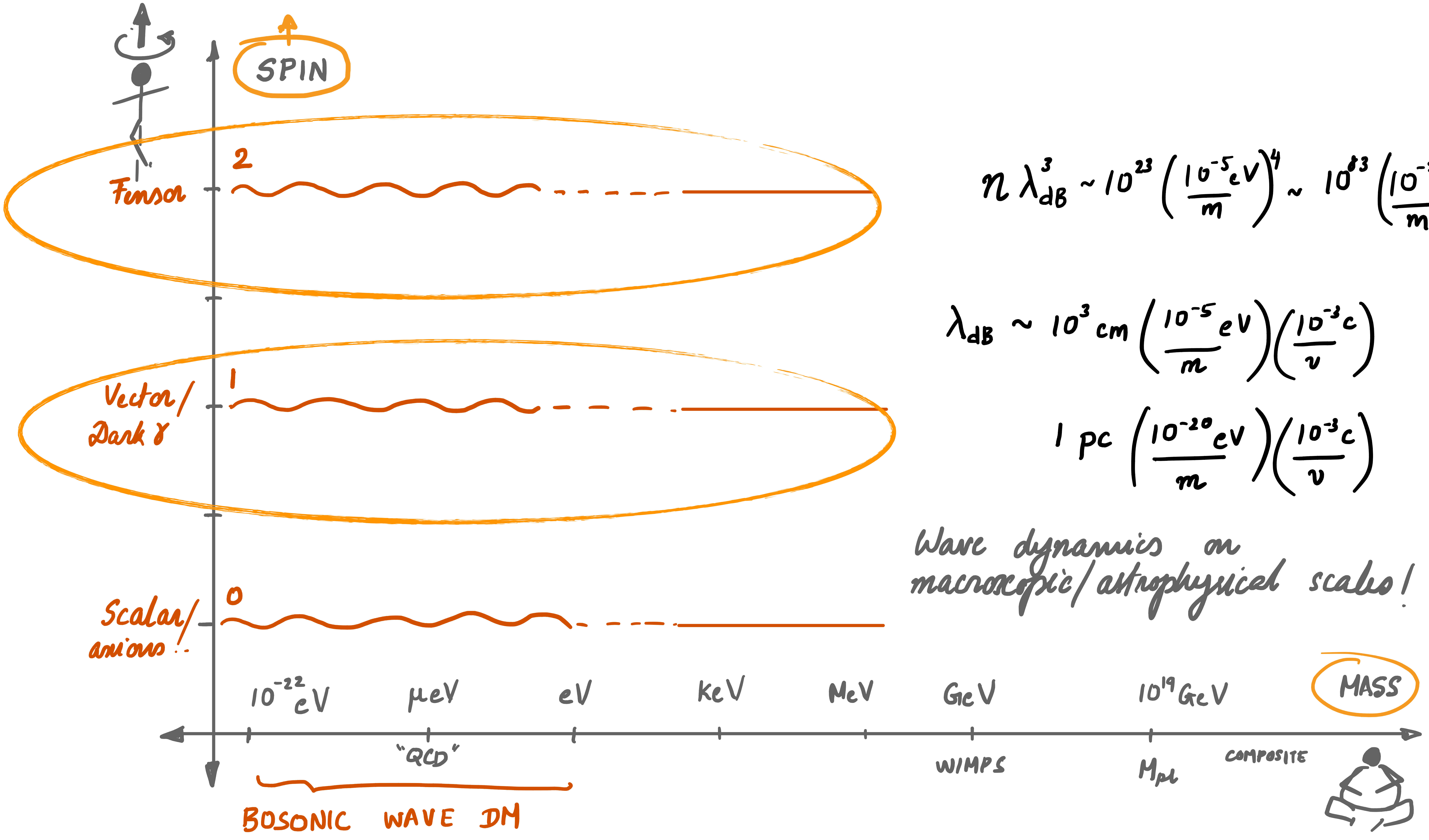
A Spin on Solitons
(in wave dark matter)

Mustafa A. Amin
RICE


Z= 0.00
dark matter
lots of fun high energy physics
+ phase transitions here
after inflation
lots of fun astrophysics here



$$n\lambda_{dB}^3 \sim 10^{23}\left(\frac{10^{-5}\,eV}{m}\right)^4 \sim 10^{83}\left(\frac{10^{-20}\,eV}{m}\right)^4$$

$$\lambda_{dB} \sim 10^{3}\,cm\left(\frac{10^{-5}\,eV}{m}\right)\left(\frac{10^{-3}c}{v}\right)$$

$$1\,pc\left(\frac{10^{-20}\,eV}{m}\right)\left(\frac{10^{-3}c}{v}\right)$$

Wave dynamics on macroscopic/astrophysical scales!

some relevant literature from our group at Rice U + coll.

Spin of wave dark matter from astrophysics?

with Jain	2109.04892
Jain, Zhang	2111.08700
Jain, Karur, Mocz	2203.11935
Jain	2211.08433
Long, Schiappacasse	2301.11470
Jain, Thomas, Waniswecharungruang	2304.01985
Helfer, Wang	2309.04345
Jain, Pu	2211.08433

Jain


Wisha


Thomas


Zhang


Schiappacasse


Long


Karur


Mocz

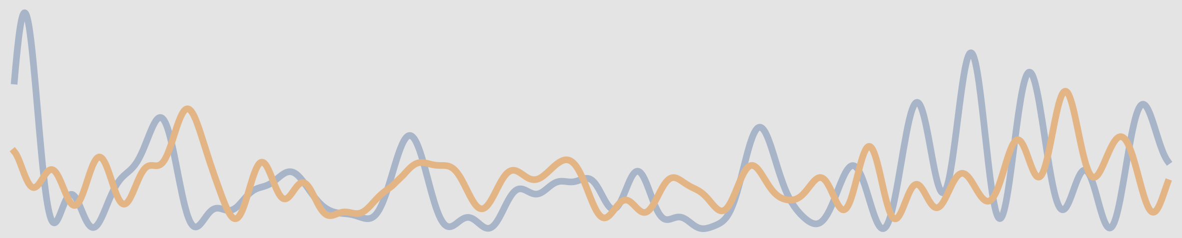

Helfer

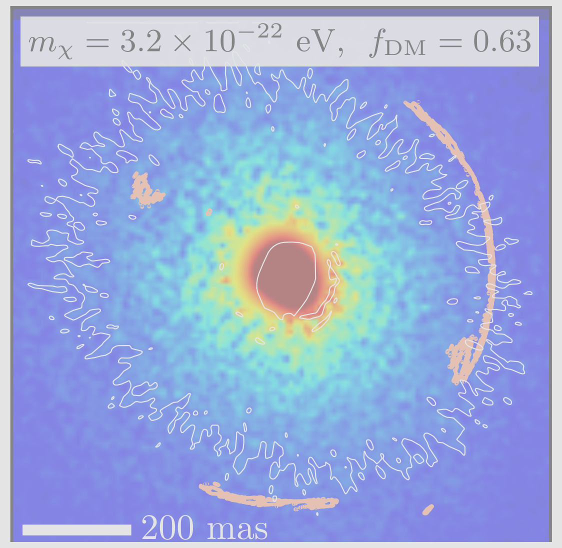

Wang

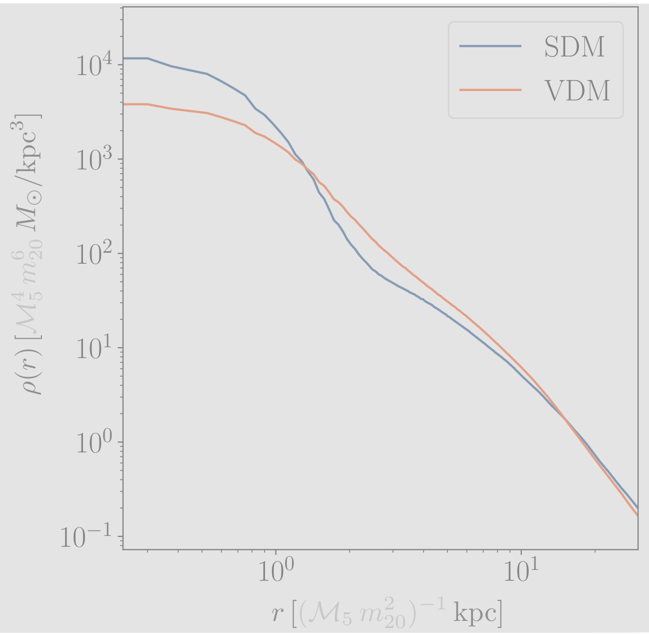

spin and dark matter sub-structure

Phenomenology

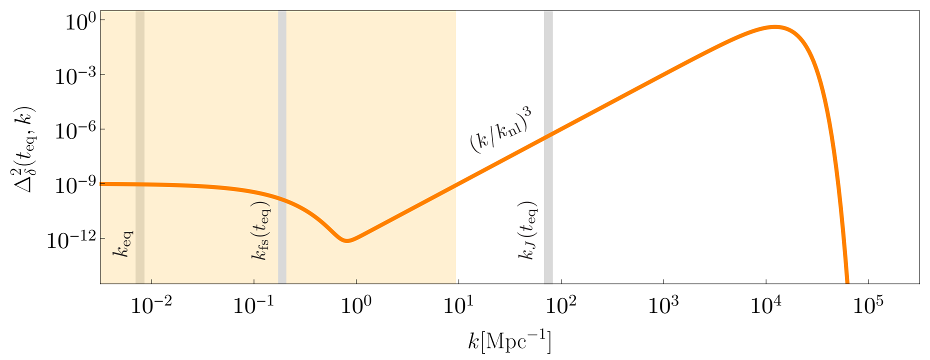
- reduced interference



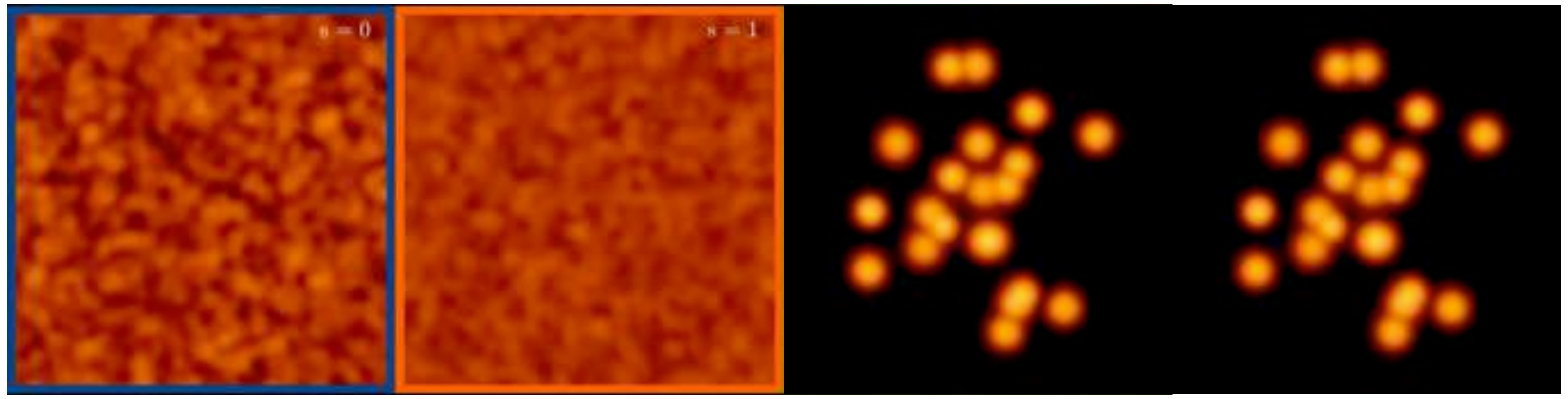




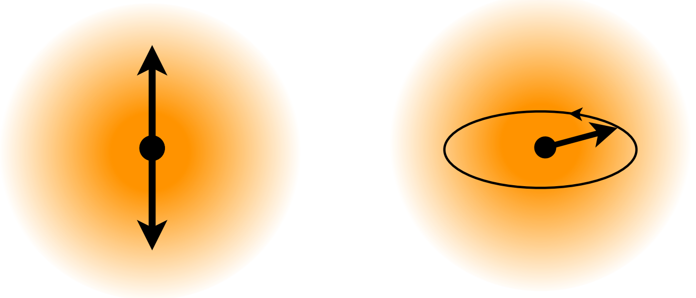
- growth of structure, nucleation time-scales

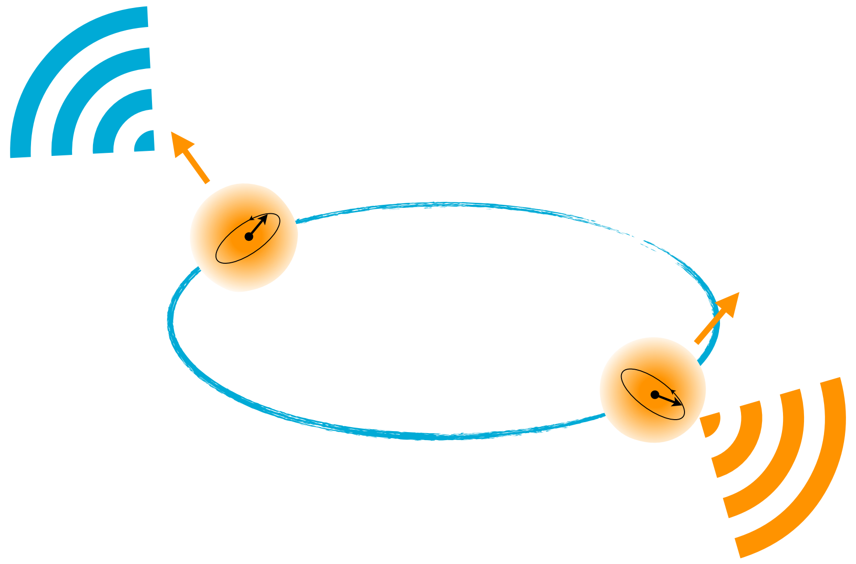


$\Longrightarrow$



- polarized solitons, with macroscopic spin





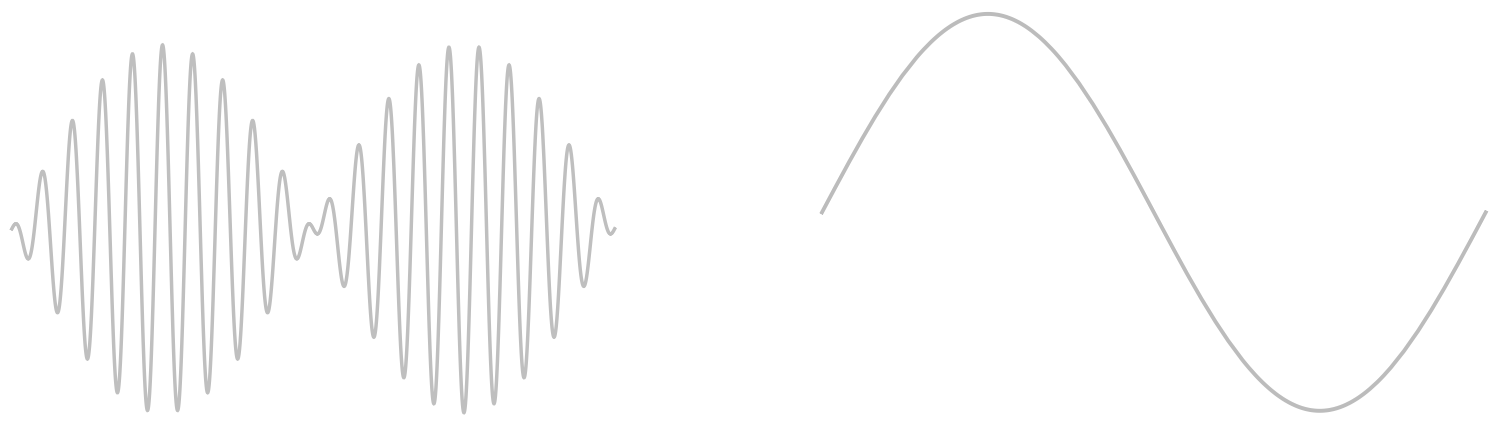
a model

non-relativistic limit = multicomponent Schrödinger-Poisson

$$S = \int d^4x\sqrt{-g}\left[-\frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}\frac{m^2c^2}{\hbar^2}X_\mu X^\mu + \frac{c^2}{8\pi G}R + \ldots\right]$$

+ non-grav, interactions

$$X_{\mu\nu} = \nabla_\mu X_\nu - \nabla_\nu X_\mu$$



split in "fast" and "slow" parts

non-relativistic limit

$$\boldsymbol{X}(t,\boldsymbol{x}) \equiv \frac{\hbar}{2mc}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imc^2t/\hbar}\right]$$

$$\mathcal{S}_{nr} = \int \mathrm{d}t\mathrm{d}^3x\left[\frac{i\hbar}{2}\boldsymbol{\Psi}^\dagger\dot{\boldsymbol{\Psi}} + \text{c.c.} - \frac{\hbar^2}{2m}\nabla\boldsymbol{\Psi}^\dagger\cdot\nabla\boldsymbol{\Psi} + \frac{1}{8\pi G}\Phi\nabla^2\Phi - m\,\Phi\,\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}\right]$$

Adshead & Lozanov (2021)

Spin - 2s+1   Jain & MA (2021)   [2109.04892]

non-relativistic limit = multicomponent Schrödinger-Poisson

$[\boldsymbol{\Psi}]_i = \psi_i$ with $i = 1, 2, 3$ vector case

$$i\hbar \frac{\partial}{\partial t}\boldsymbol{\Psi} = -\frac{\hbar^2}{2m}\nabla^2\boldsymbol{\Psi} + m\,\Phi\,\boldsymbol{\Psi}\,, \qquad \nabla^2\Phi = 4\pi G m\,\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}$$

$[\boldsymbol{\Psi}]_i = \psi_i$ with $i = 1$ scalar case

at this level this is just $2s+1$ equal-mass scalar fields
but not when non-gravitational interactions are included!

Jain & MA (2021) [2109.04892]

conserved quantities

$[\mathbf{\Psi}]_i = \psi_i$ with $i = 1, 2, 3$

$$N = \int \mathrm{d}^3x \mathbf{\Psi}^\dagger \mathbf{\Psi}, \quad \text{and} \quad M = mN, \qquad \text{(particle number and rest mass)}$$

$$E = \int \mathrm{d}^3x \left[\frac{\hbar^2}{2m} \nabla \mathbf{\Psi}^\dagger \cdot \nabla \mathbf{\Psi} - \frac{Gm^2}{2} \mathbf{\Psi}^\dagger \mathbf{\Psi} \int \frac{\mathrm{d}^3y}{4\pi|\boldsymbol{x} - \boldsymbol{y}|} \mathbf{\Psi}^\dagger(\boldsymbol{y}) \mathbf{\Psi}(\boldsymbol{y}) \right], \qquad \text{(energy)}$$

$$\boldsymbol{S} = \hbar \int \mathrm{d}^3x \, i \mathbf{\Psi} \times \mathbf{\Psi}^\dagger, \qquad \text{(spin angular momentum)}$$

$$\boldsymbol{L} = \hbar \int \mathrm{d}^3x \, \Re \left(i \, \mathbf{\Psi}^\dagger \nabla \mathbf{\Psi} \times \boldsymbol{x} \right). \qquad \text{(orbital angular momentum)}$$

spin angular momentum

$$[\boldsymbol{\Psi}]_i = \psi_i \text{ with } i = 1, 2, 3$$

$$\boldsymbol{S} = \hbar \int \mathrm{d}^3x \, i\boldsymbol{\Psi} \times \boldsymbol{\Psi}^\dagger$$

$$\boldsymbol{s} = i\hbar\boldsymbol{\Psi}^\dagger \times \boldsymbol{\Psi}$$

$$0 \leq |\boldsymbol{S}| \leq \hbar N$$

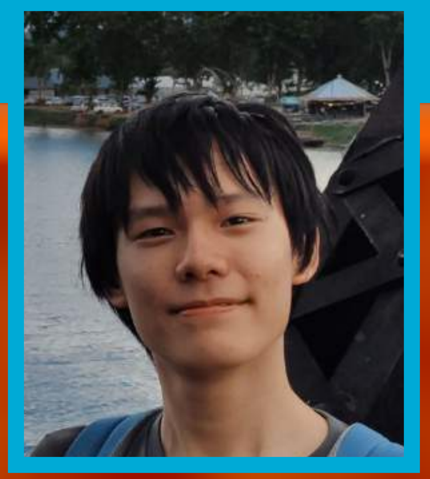
$$0 \leq |\boldsymbol{s}| \leq \hbar\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}$$

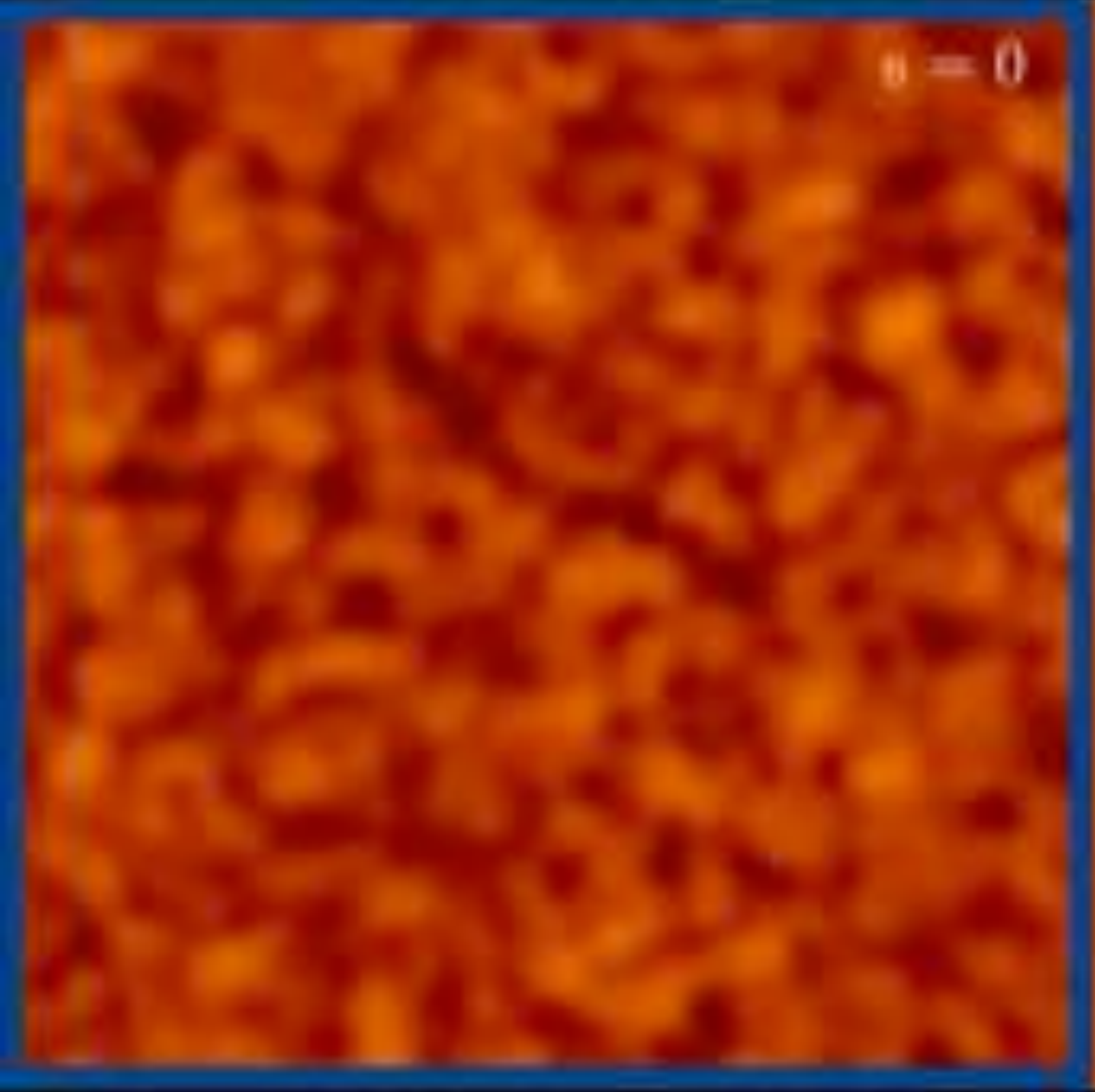
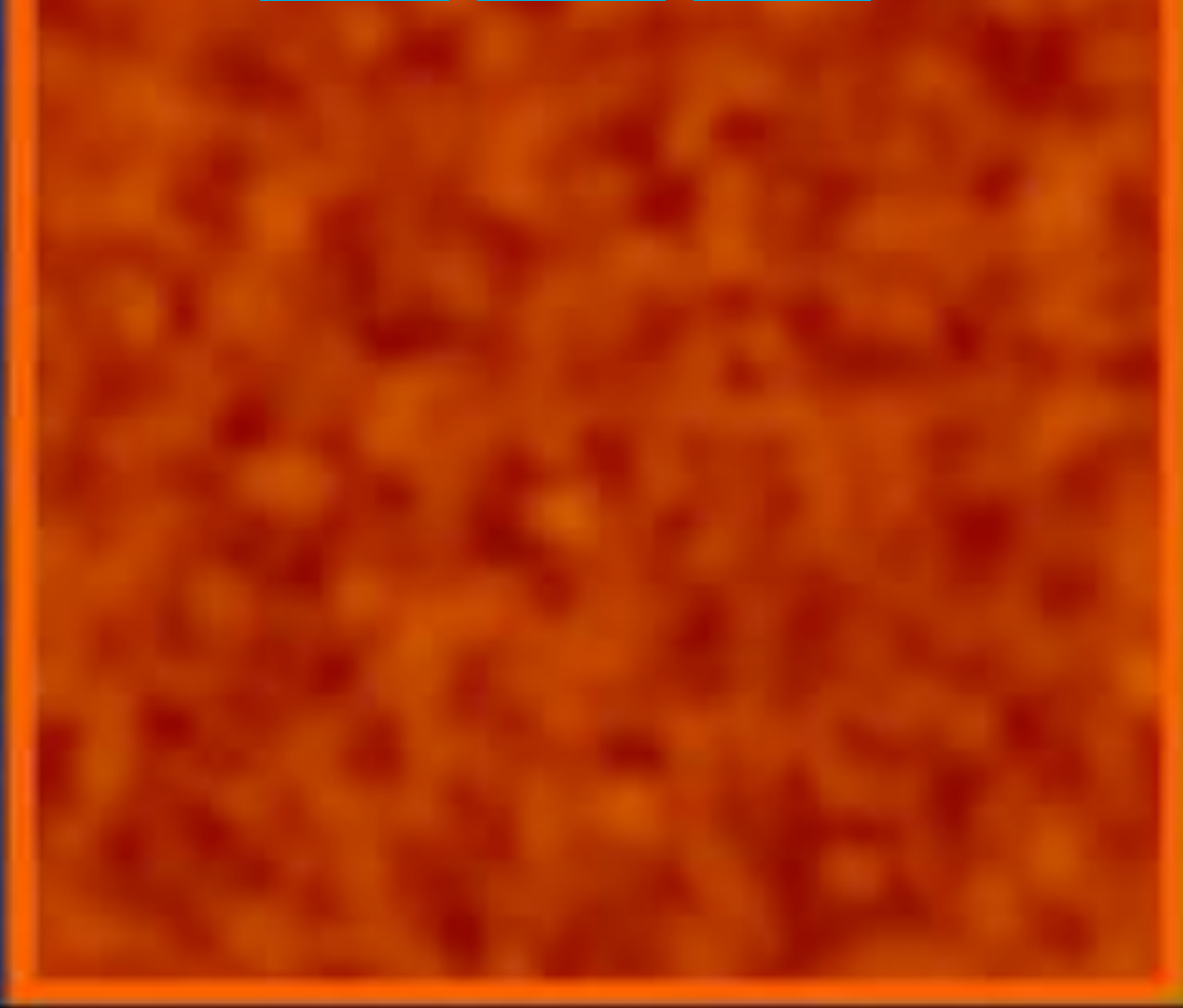
Jain & MA (2021) [2109.04892]

condensation in the kinetic regime

Jain, MA, Wanischarunarung, Thomas (2023)

[2304.01985]

condensation in the kinetic regime

Jain, MA, Wanischarunarung, Thomas (2023)

[2304.01985]

condensation in the kinetic regime

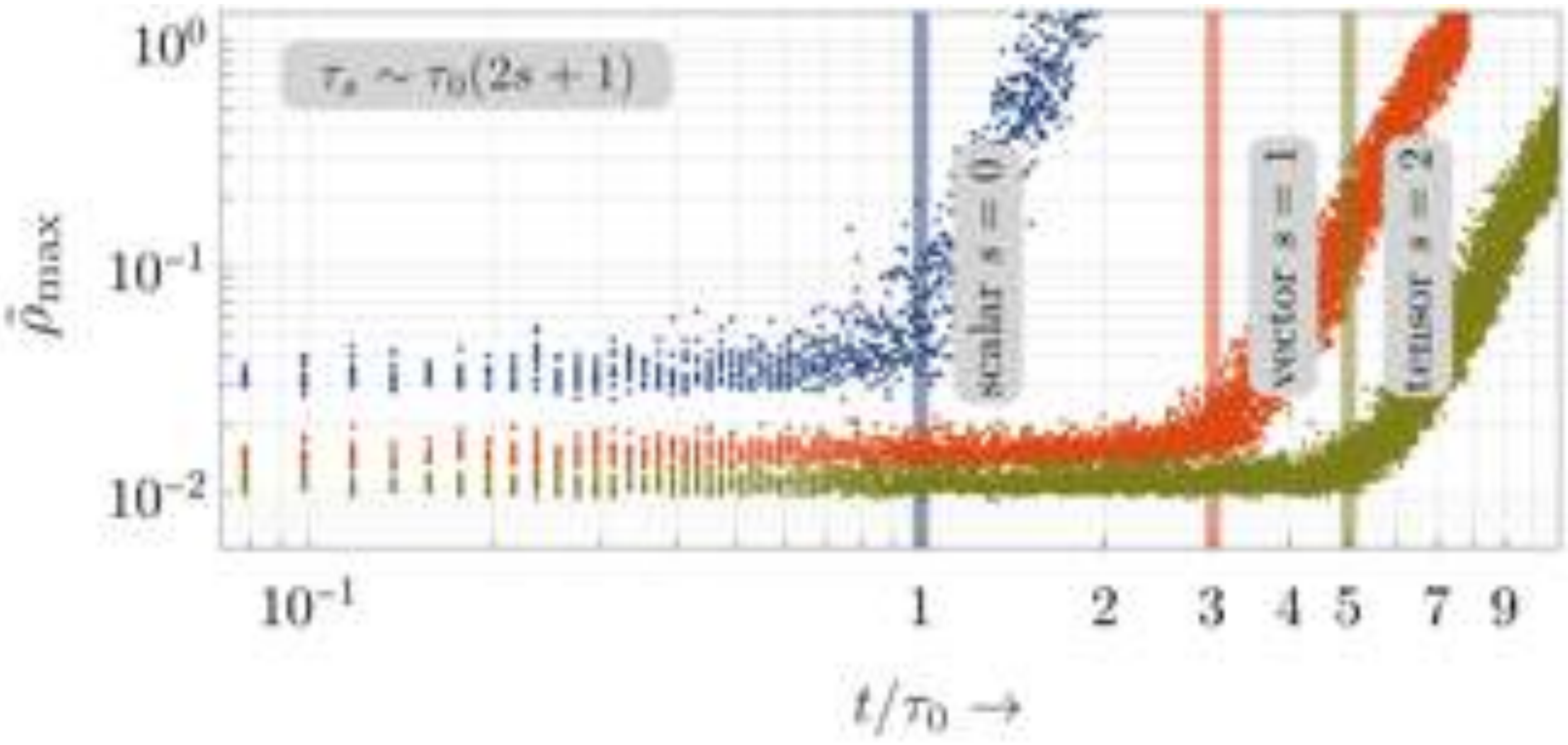
- nucleation time scale

$$\tau_s \sim (2s+1)\tau_{s=0}$$

$$\tau_{s=0} = [n\sigma_{gr} v \mathcal{N}]^{-1}$$

$$\sigma_{gr} \sim (Gm/v^2)^2, \qquad \mathcal{N} \sim n\lambda_{dB}^3$$

[2304.01985]



with Jain, Thomas, Wanichwecharungruang (2023)

see Levkov et. al (2018) for scalar case

kinetic relaxation — multi-component case

Jain, MA, Thomas, Wanichwecharungruang (2023)
[2304.01985]

$$i\frac{\partial}{\partial t}\psi_a = -\frac{1}{2m_a}\nabla^2\psi_a + m_a\Phi\,\psi_a$$

$$\nabla^2\Phi = 4\pi G\sum_a m_a\,\psi_a^*\psi_a .$$

$\Downarrow$

$$\frac{\partial f^a_{\boldsymbol{v}_a}}{\partial t} = \sum_b m_b^3\frac{\Lambda}{2\pi}\frac{(4\pi m_a m_b G)^2}{m_a}\nabla_{v_a^i}\left[\frac{\mathcal{D}^{ab}_{ij}}{2m_a}\nabla_{v_a^j}f^a_{\boldsymbol{v}_a} + \frac{\mathcal{F}^{ab}_i}{m_b}f^a_{\boldsymbol{v}_a}\right]$$

where $\mathcal{D}^{ab}_{ij} = \int\frac{\mathrm{d}\tilde{\boldsymbol{v}}_b}{(2\pi)^3}f^b_{\tilde{\boldsymbol{v}}_b}\frac{\delta_{ij}-\hat{u}_i\hat{u}_j}{u}f^b_{\tilde{\boldsymbol{v}}_b}$ and $\mathcal{F}^{ab}_i = f^a_{\boldsymbol{v}_a}\int\frac{\mathrm{d}\tilde{\boldsymbol{v}}_b}{(2\pi)^3}\frac{\hat{u}_i}{u^2}f^b_{\tilde{\boldsymbol{v}}_b}$

$\Downarrow$

$$\Gamma_a = \sum_b\frac{\Lambda\,(4\pi G)^2\bar{\rho}_b}{\sigma_a^3\sigma_b^3}\left[2\frac{\bar{\rho}_a}{m_a^3} - \beta_{ab}\frac{\bar{\rho}_b\sigma_a}{m_b^3\sigma_b}\right]$$

$\Leftarrow$

$$\Gamma_{(s)} = \Gamma_0/(2s+1)$$

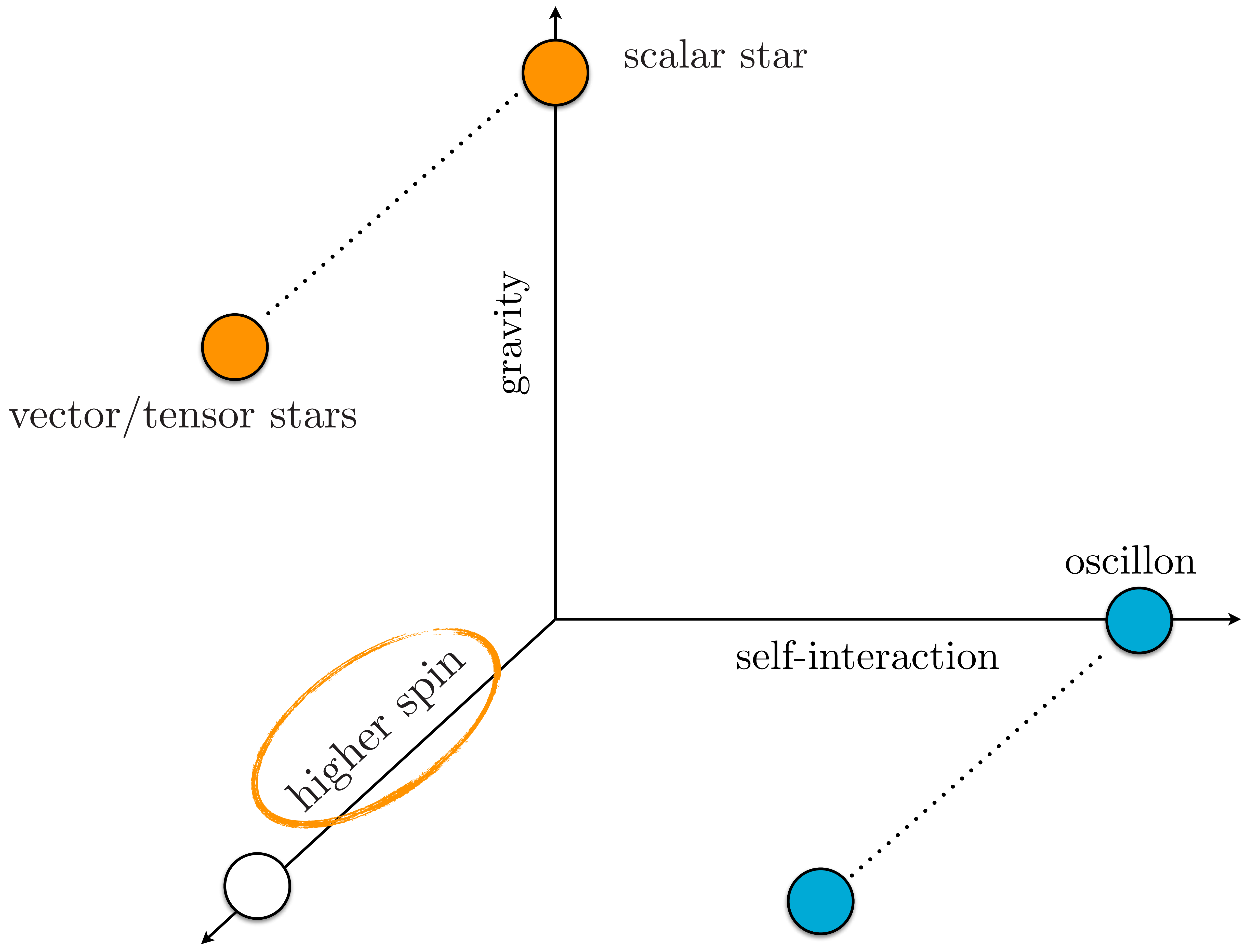
$\hbar = 1$

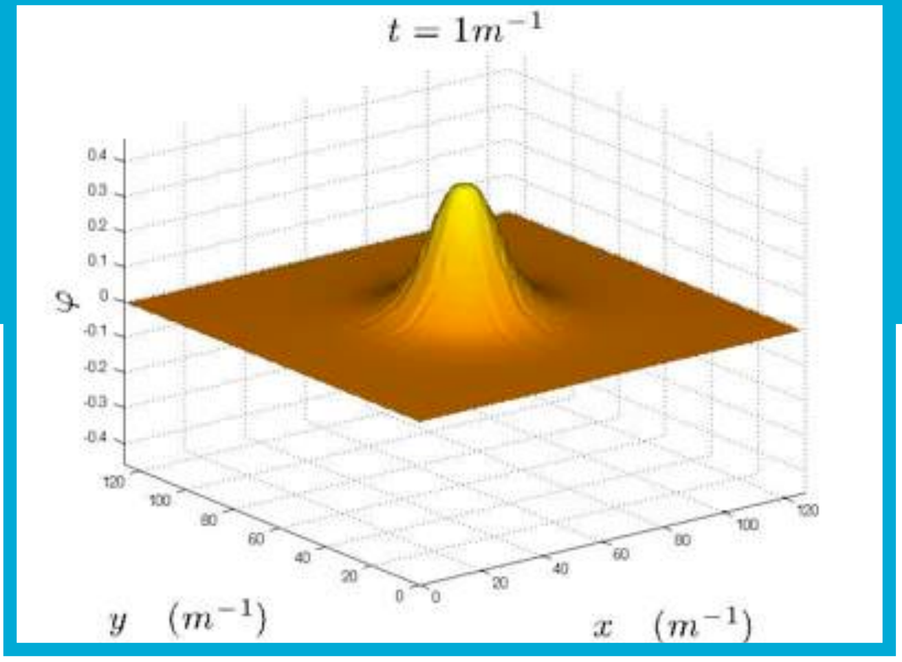
what are these “blobs” ?

Jain, MA, Wanischarunarung, Thomas (2023)
[2304.01985]

solitons in $s > 0$ fields

spatially localized, coherently oscillating, long-lived





spatially localized

coherently oscillating (components)

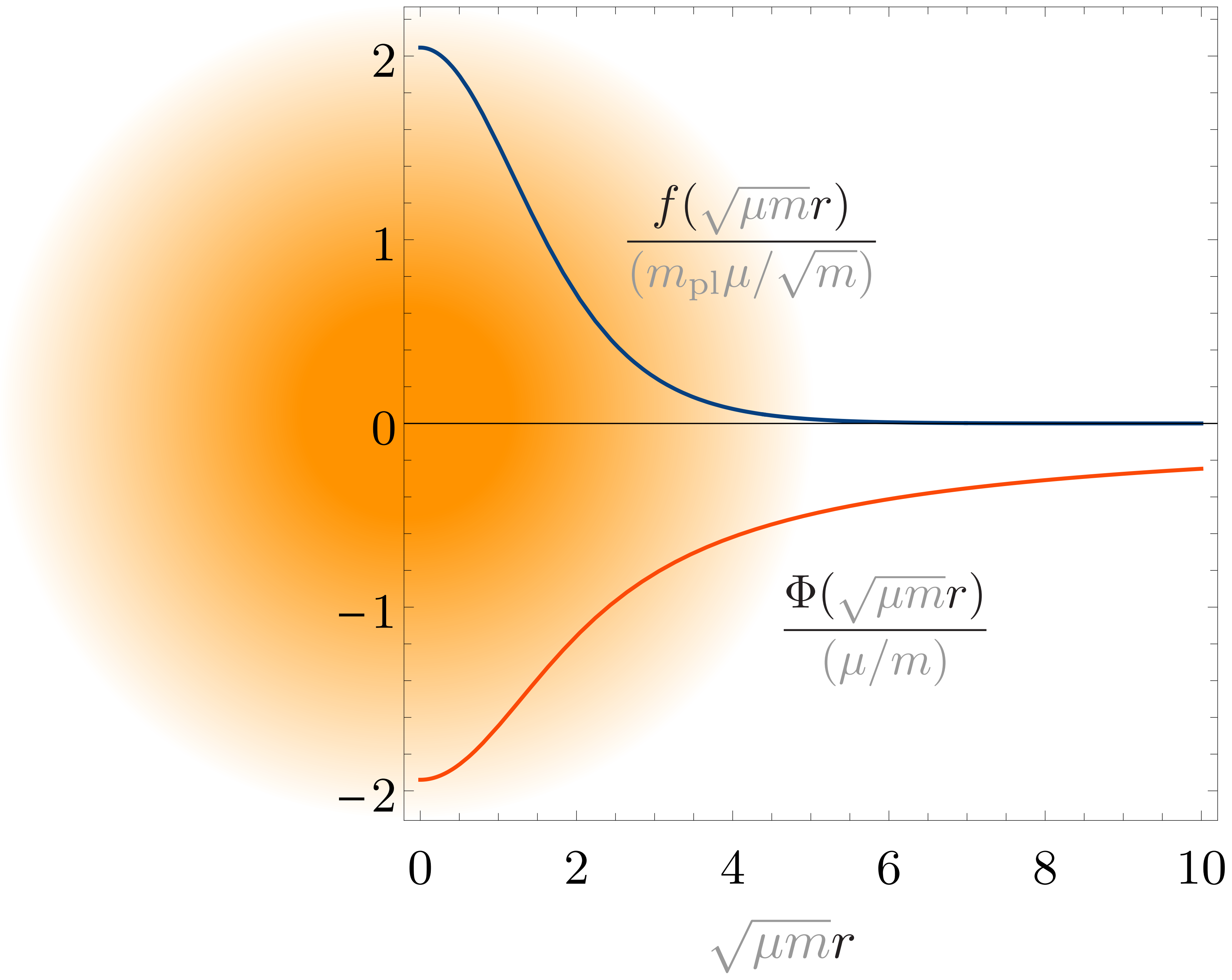
exceptionally long-lived

vector solitons ?

$$i\hbar\frac{\partial}{\partial t}\boldsymbol{\Psi} = -\frac{1}{2m}\nabla^2\boldsymbol{\Psi} + m\,\Phi\,\boldsymbol{\Psi}\,, \qquad \nabla^2\Phi = 4\pi G m\,\boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}$$

$$\boldsymbol{\Psi}(t,r) = f(r)e^{i\mu t}\boldsymbol{\epsilon} \qquad \text{with} \quad \boldsymbol{\epsilon}^\dagger\boldsymbol{\epsilon} = 1$$

$$-\mu f = -\frac{1}{2m}\nabla^2 f + m\Phi f \qquad \nabla^2\Phi = 4\pi G m f^2$$

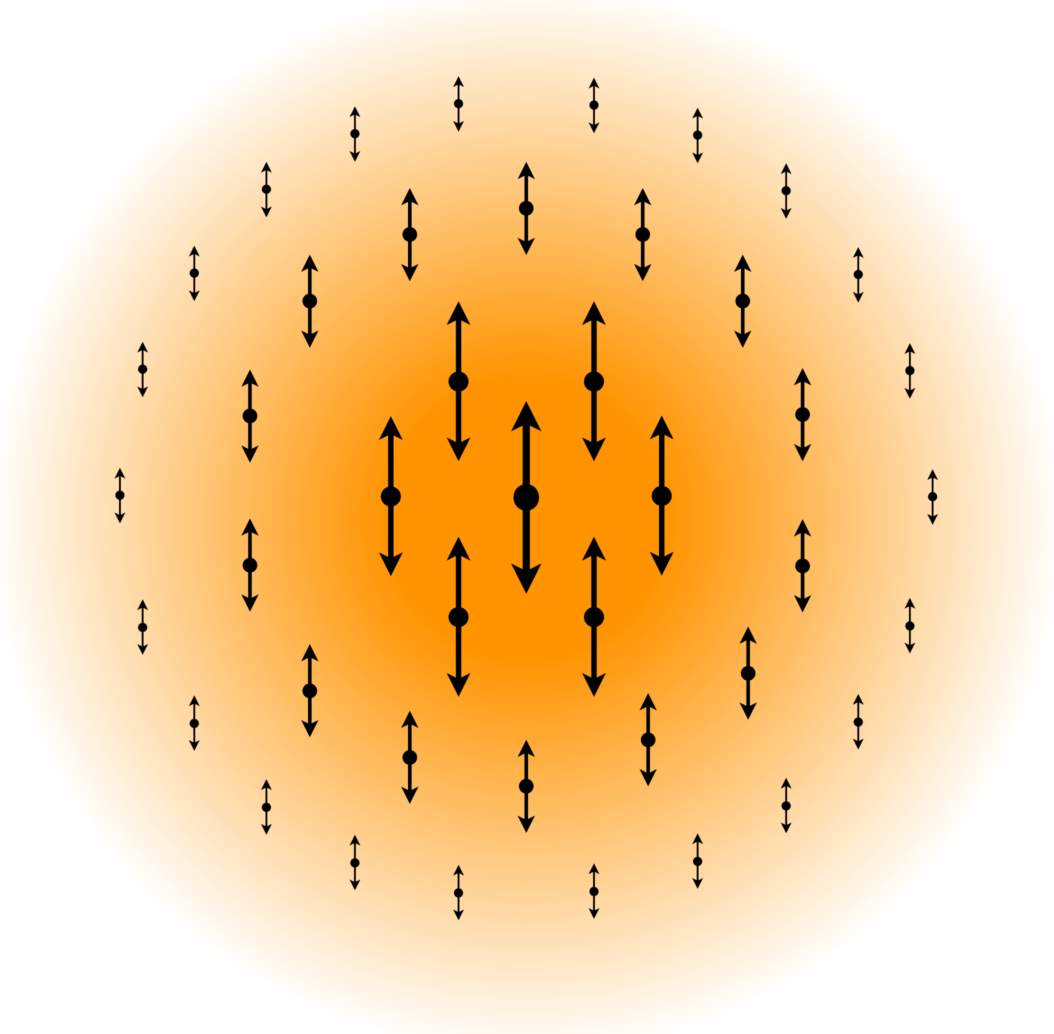


$\hbar = c = 1$

"polarized" vector solitons

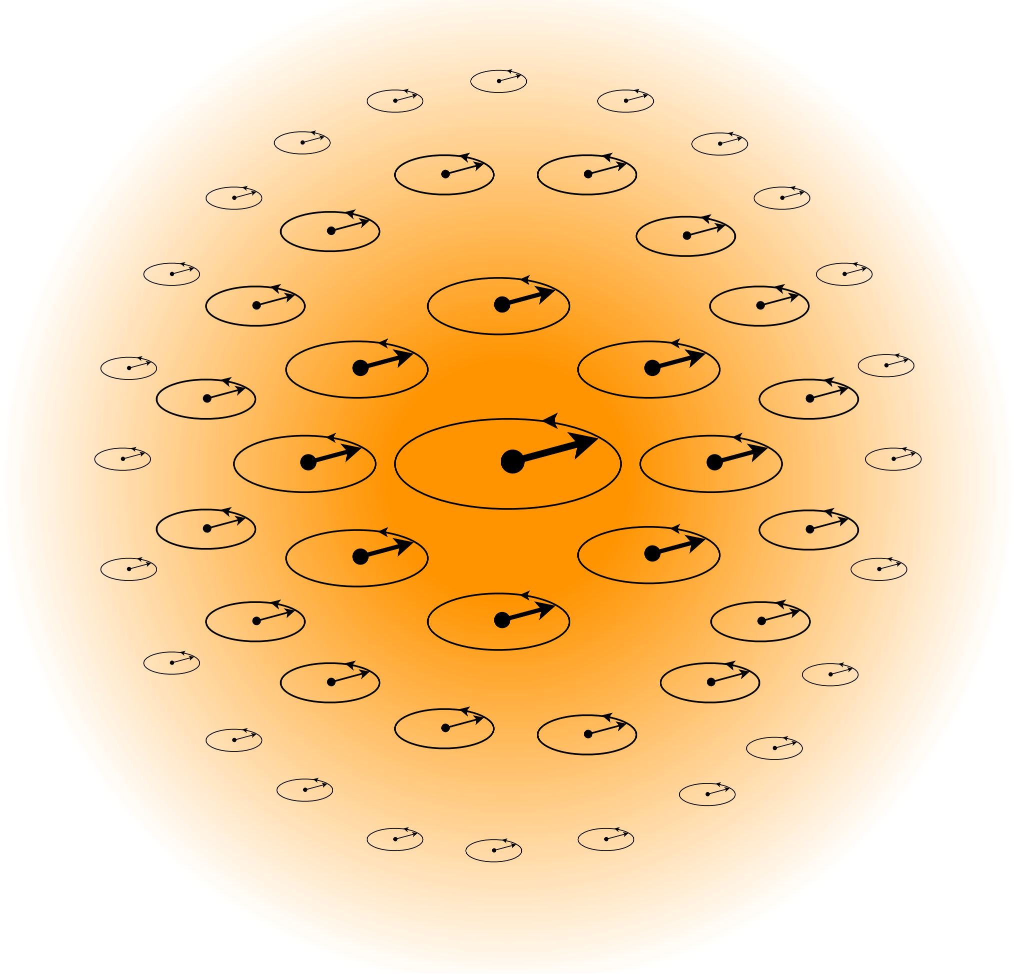
$$\boldsymbol{\Psi}(t,r) = f(r)e^{i\mu t}\boldsymbol{\epsilon} \qquad \text{with} \quad \boldsymbol{\epsilon}^\dagger\boldsymbol{\epsilon} = 1$$

$$\boldsymbol{X}(t,\boldsymbol{x}) = \sqrt{\frac{2}{m}}\Re\left[\boldsymbol{\Psi}(t,\boldsymbol{x})e^{-imt}\right]$$



$\boldsymbol{\epsilon}_0 = \hat{\boldsymbol{z}}$

$f(r)\cos[(m-\mu)t]\hat{\boldsymbol{z}}$



$\boldsymbol{\epsilon}_\pm = \hat{\boldsymbol{x}} \pm i\boldsymbol{y}/\sqrt{2}$

$f(r)[\hat{\boldsymbol{x}}\cos[(m-\mu)t] + \hat{\boldsymbol{y}}\sin[(m-\mu)t]]$

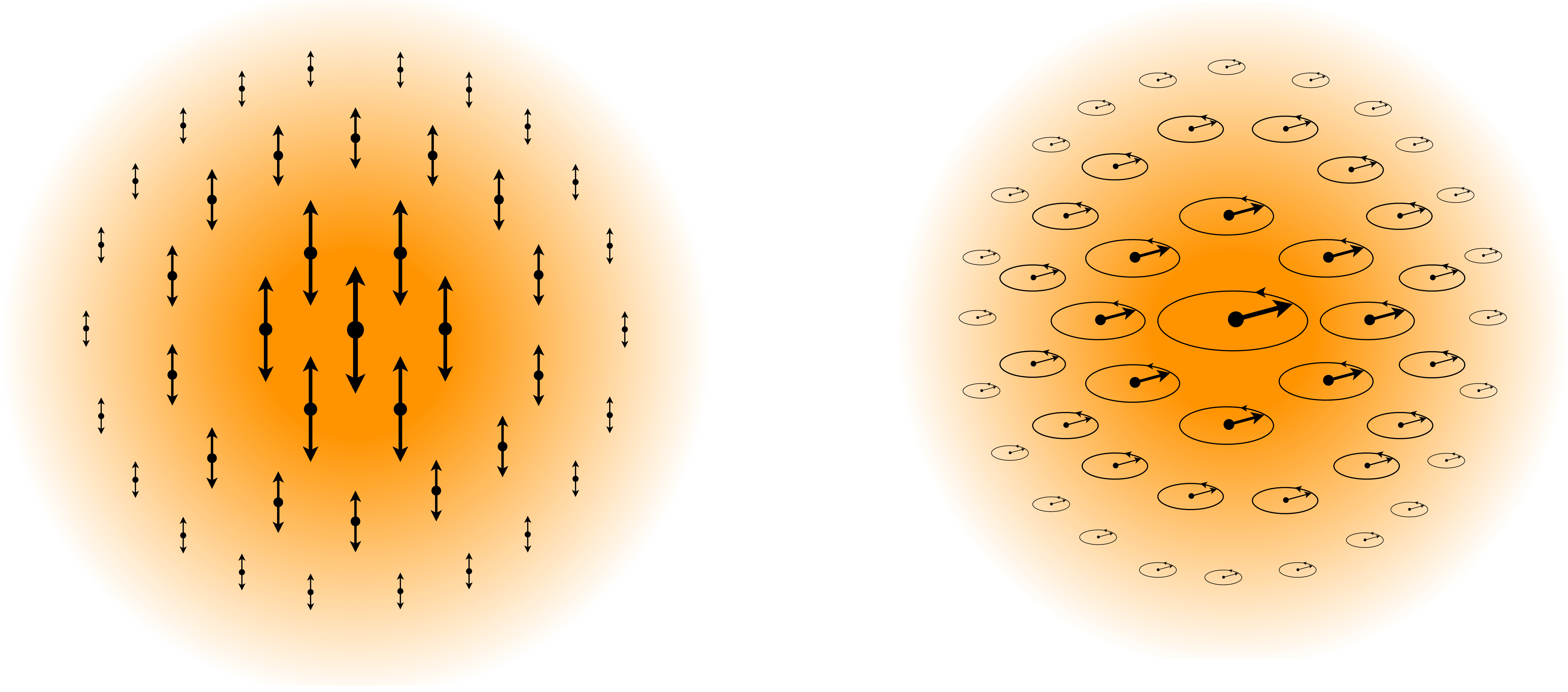
$\hbar = c = 1$

Jain & MA (2021) [2109.04892]

"polarized" vector solitons (with macroscopic spin)

macroscopic intrinsic spin!

$$\boldsymbol{S}_{\rm tot} = i(\boldsymbol{\epsilon} \times \boldsymbol{\epsilon}^\dagger)\frac{M_{\rm sol}}{m}\hbar$$



$\boldsymbol{\epsilon}_0 = \hat{\boldsymbol{z}}$

$\boldsymbol{\epsilon}_\pm = \hat{\boldsymbol{x}} \pm i\boldsymbol{y}/\sqrt{2}$

$$\boldsymbol{S}_{\rm tot} = 0$$

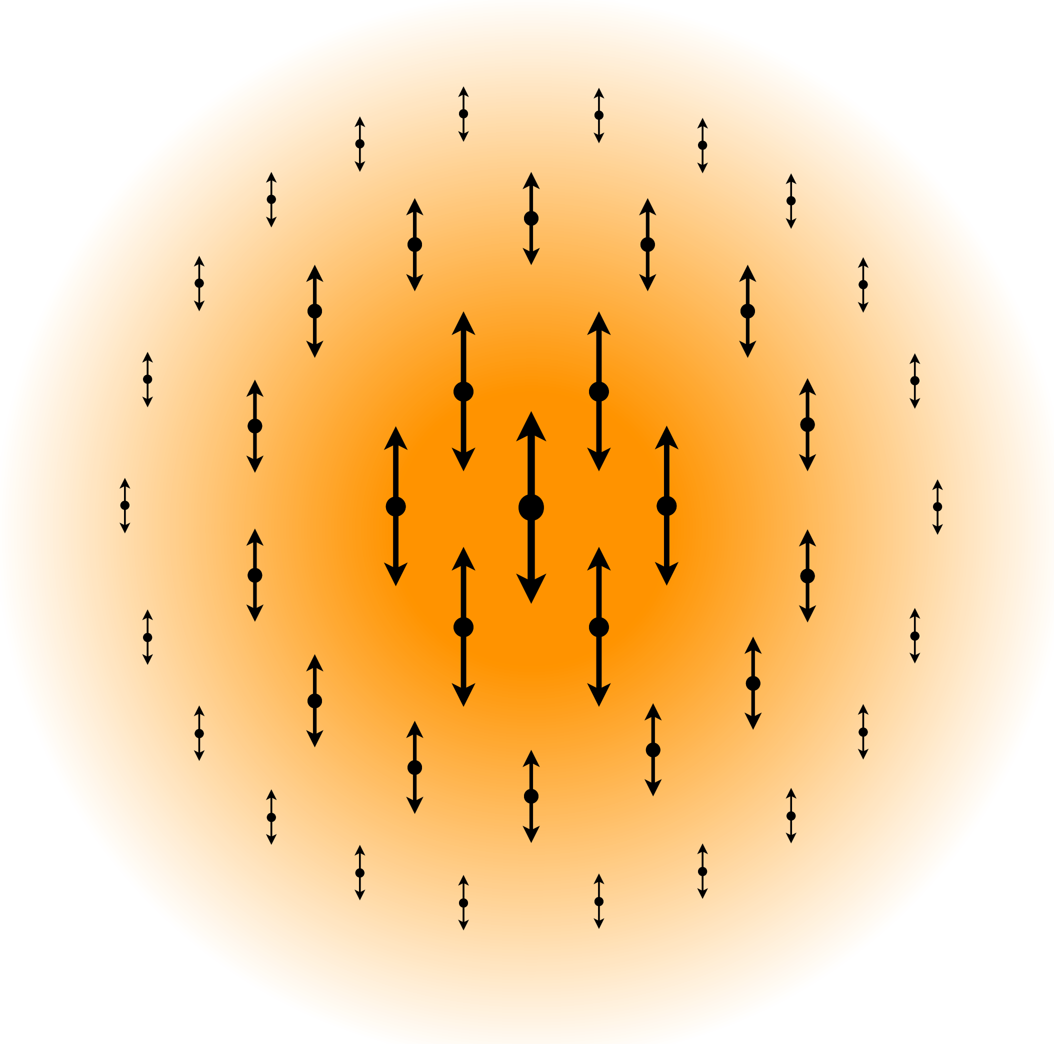
$$\boldsymbol{S}_{\rm tot} = \hbar\frac{M_{\rm sol}}{m}\hat{z}$$

Jain & MA (2021) [2109.04892]

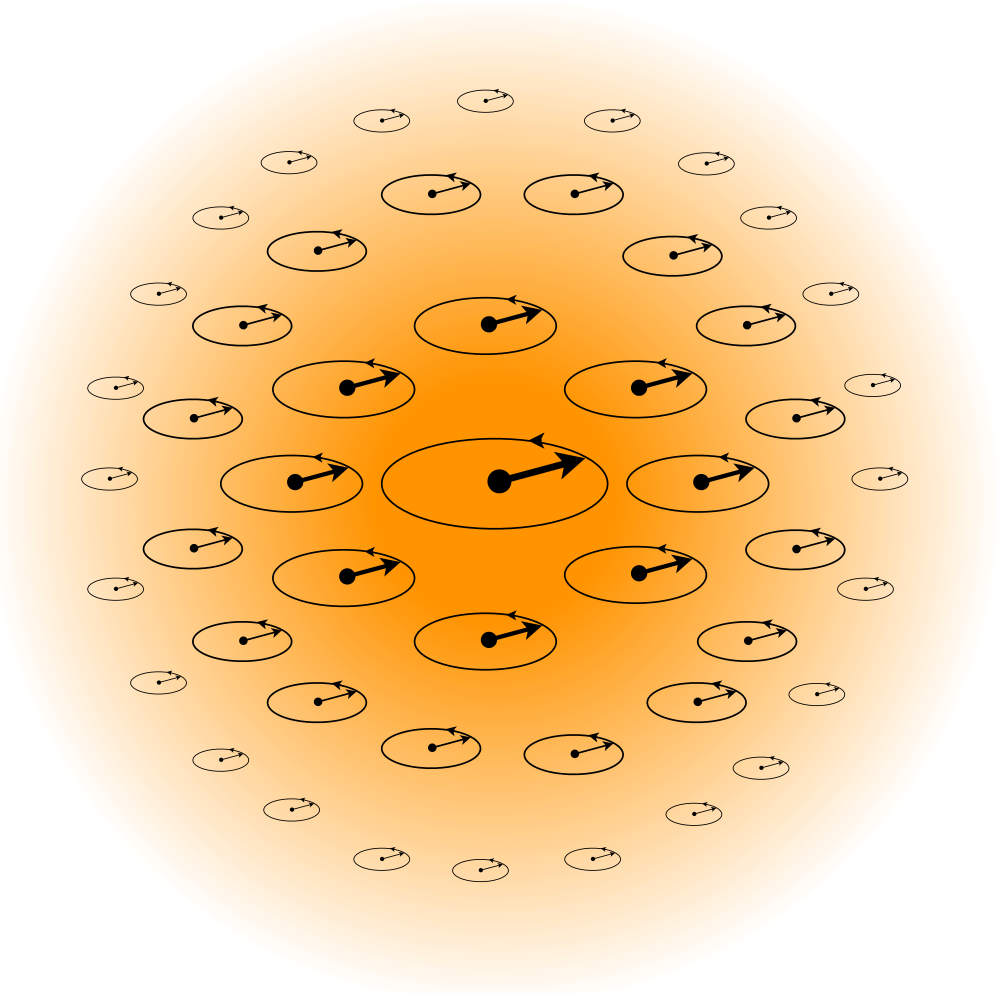
“polarized” vector solitons (with macroscopic spin)

$$\boldsymbol{S}_{\rm tot} = i(\boldsymbol{\epsilon} \times \boldsymbol{\epsilon}^{\dagger})\frac{M_{\rm sol}}{m}\hbar$$

- all lowest energy for fixed M
- bases for partially-polarized solitons

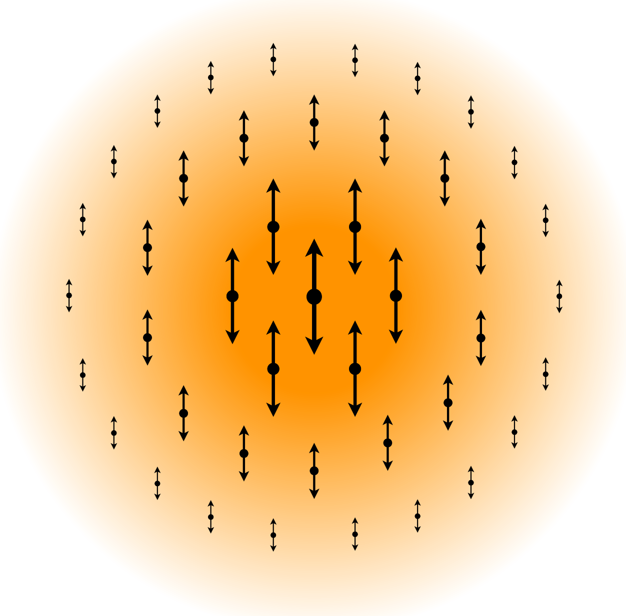
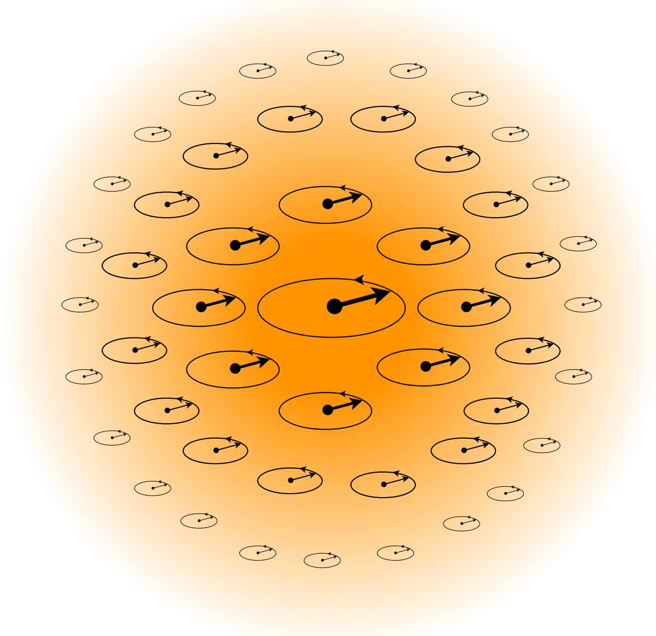
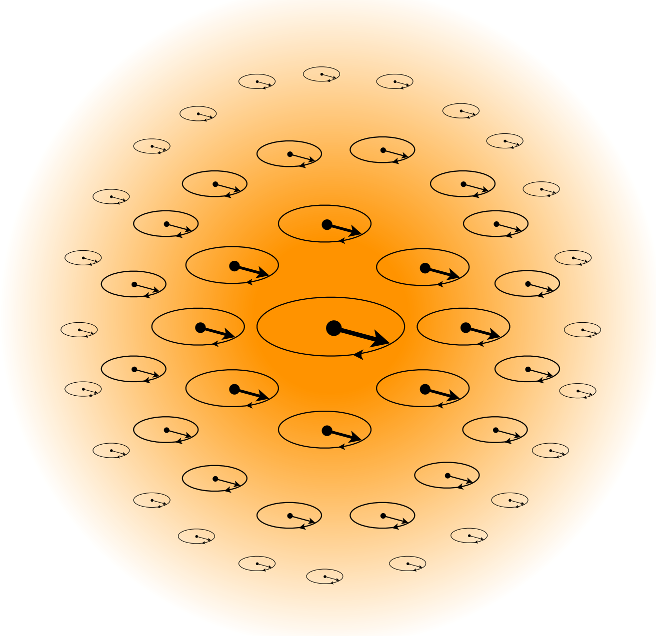


$$\boldsymbol{S}_{\rm tot} = 0$$



$$\boldsymbol{S}_{\rm tot} = \hbar\frac{M_{\rm sol}}{m}\hat{z}$$

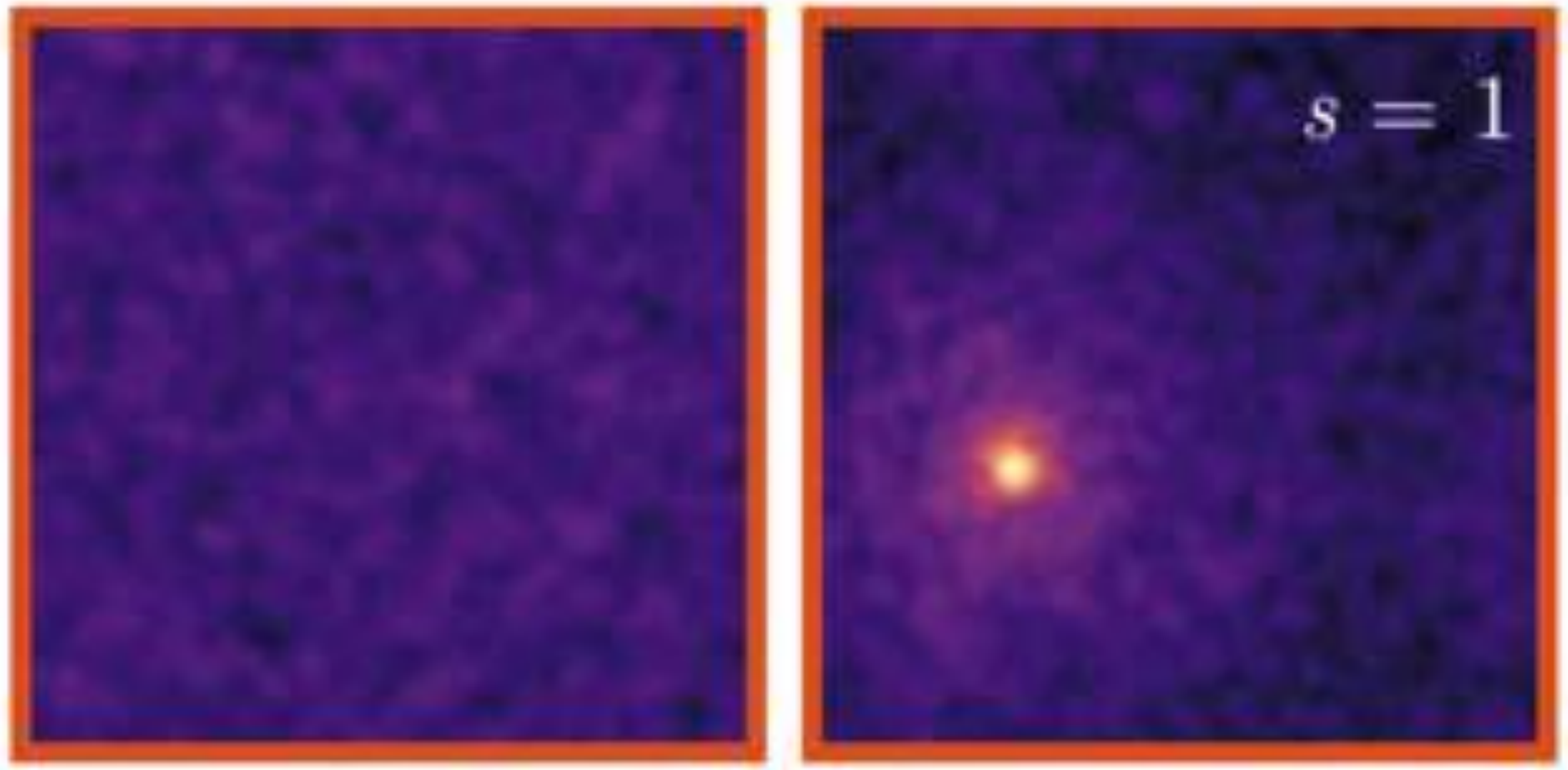
$$0 \leq |\boldsymbol{S}_{\rm tot}| \leq \frac{M_{\rm sol}}{m}\hbar$$

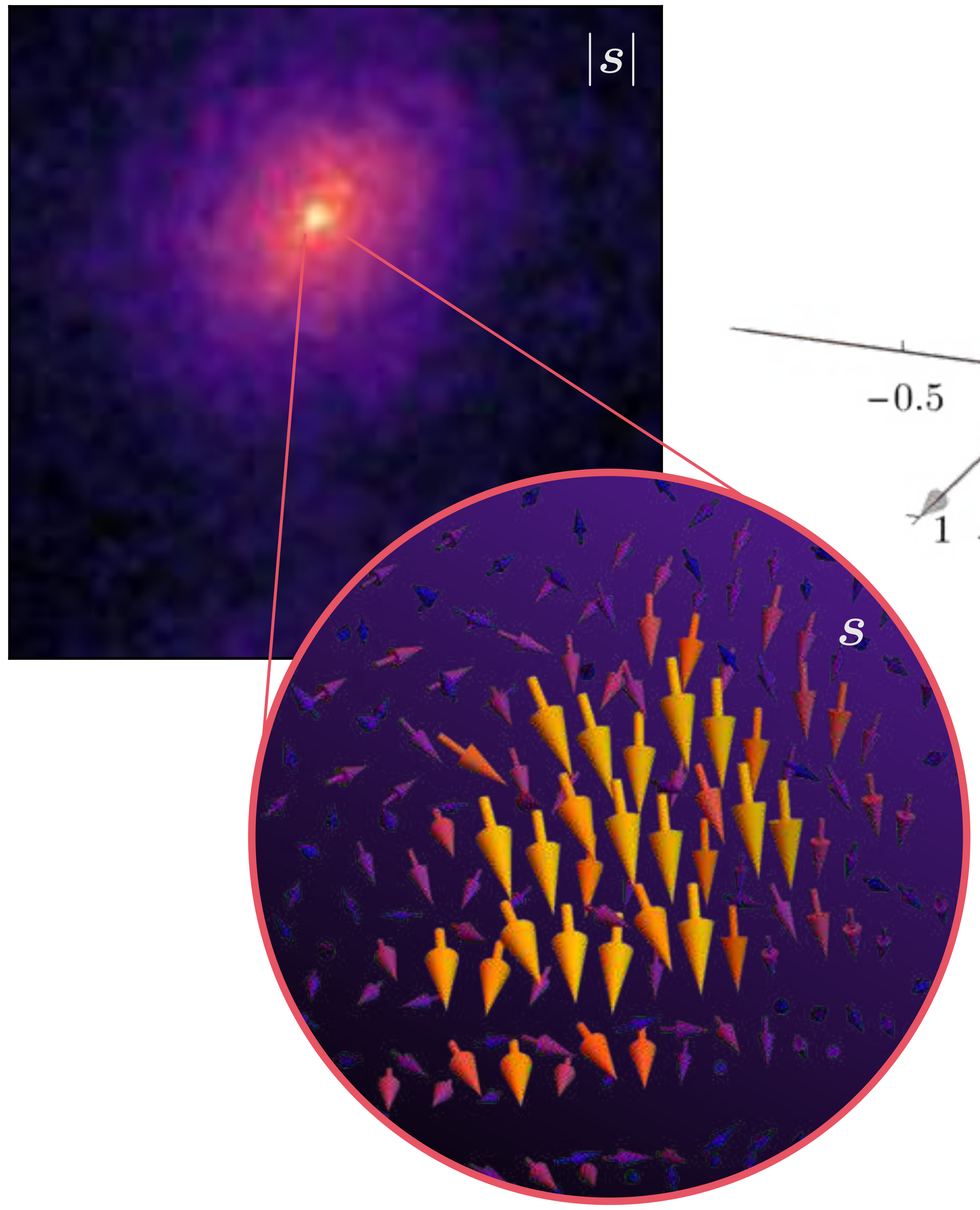
c_0  $+c_1$  $+c_{-1}$ 

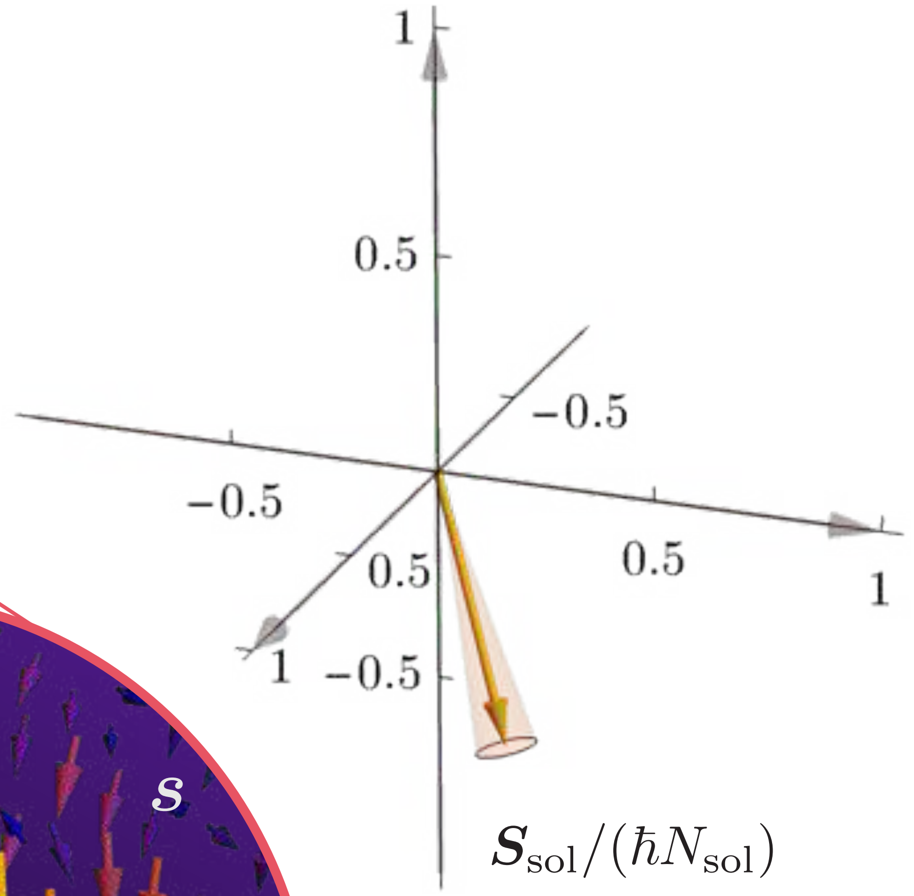
Also see: Adshead & Lozanov (2021), Jain & MA (2021)

born to spin

spin density

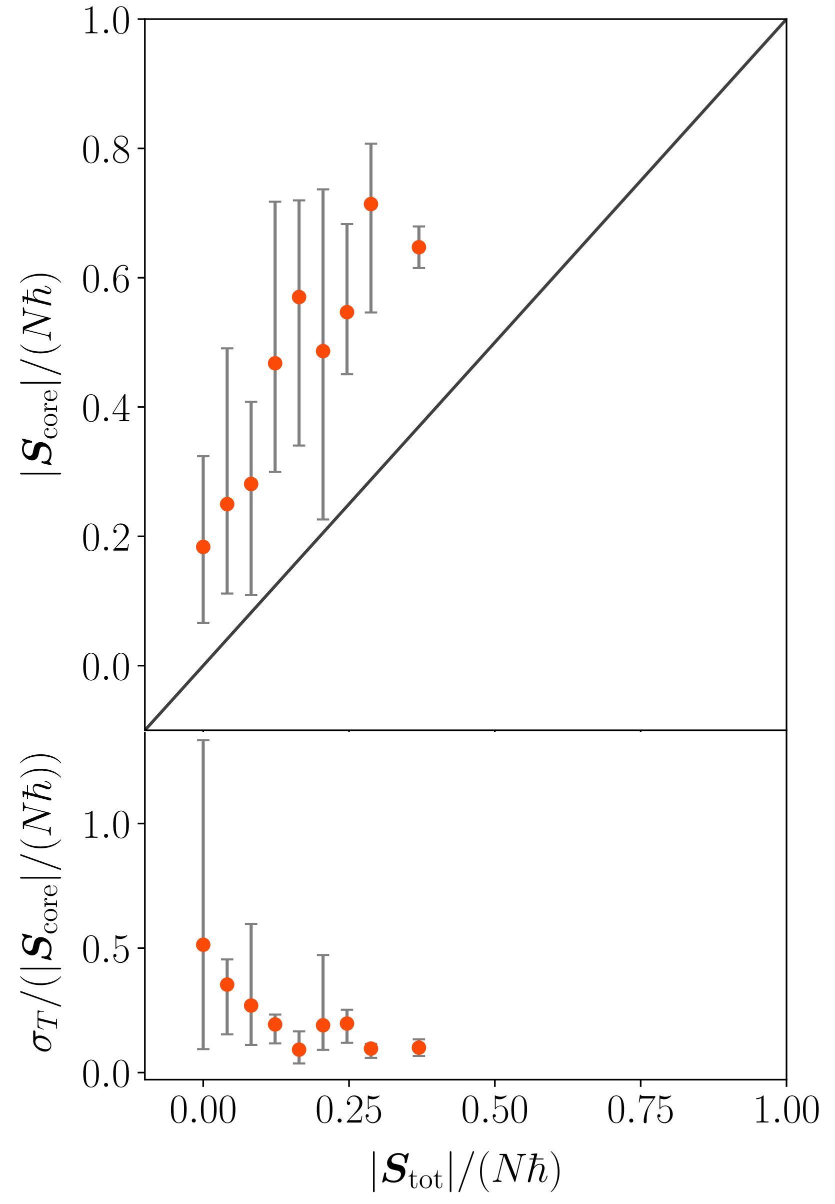






$$S_{core} \sim \hbar \frac{M_{core}}{m}$$

even when initial total spin is negligible



MA, Jain, Karur & Mocz(2022)

Jain, MA, Thomas, Wanichwecharungruang (2023)

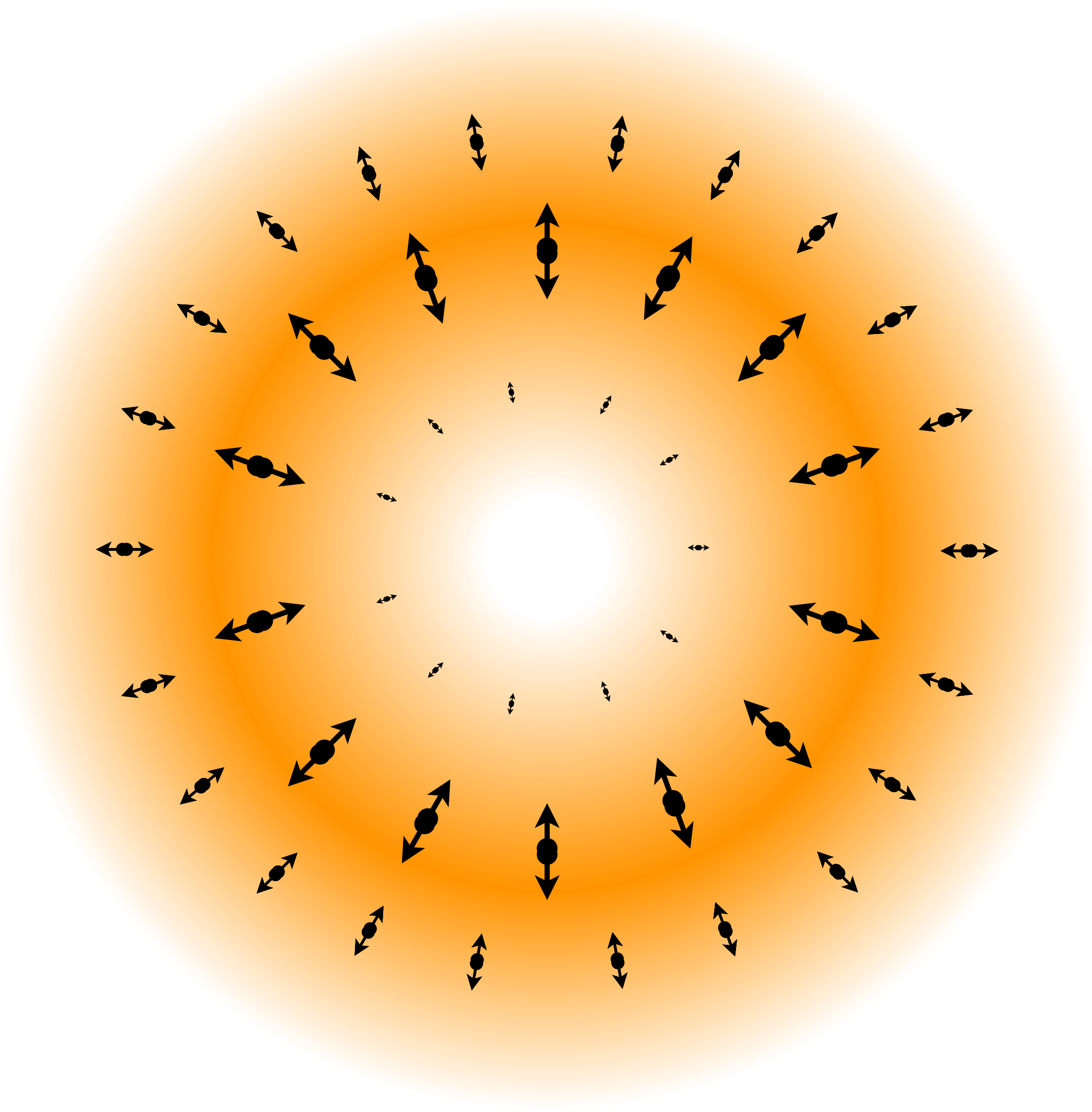
a different higher energy soliton: the “hedgehogs”

earlier literature

$$W_j(\mathbf{x},t) = f(r)\frac{x^j}{r}\cos\omega t\,,$$

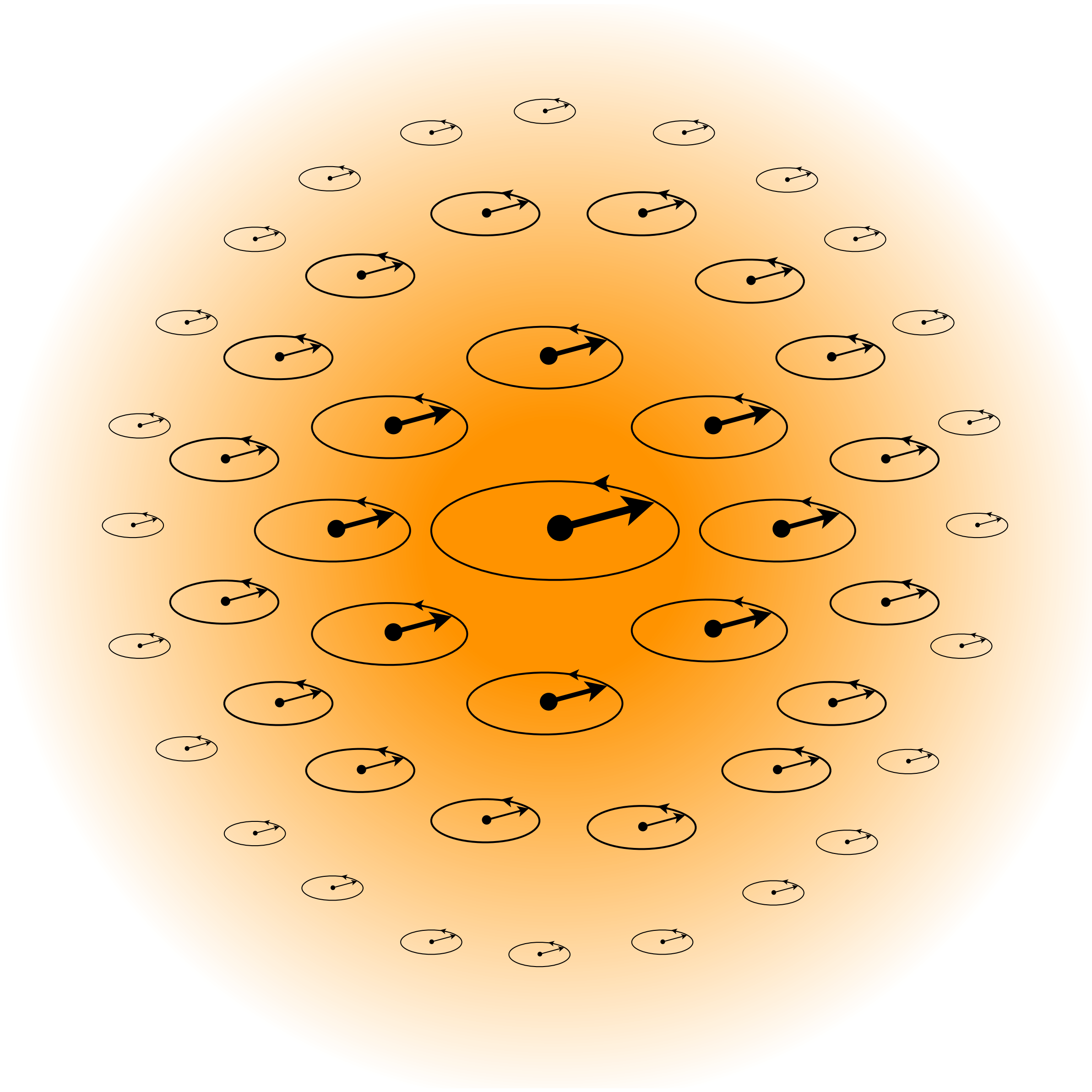
hedgehogs
not ground states

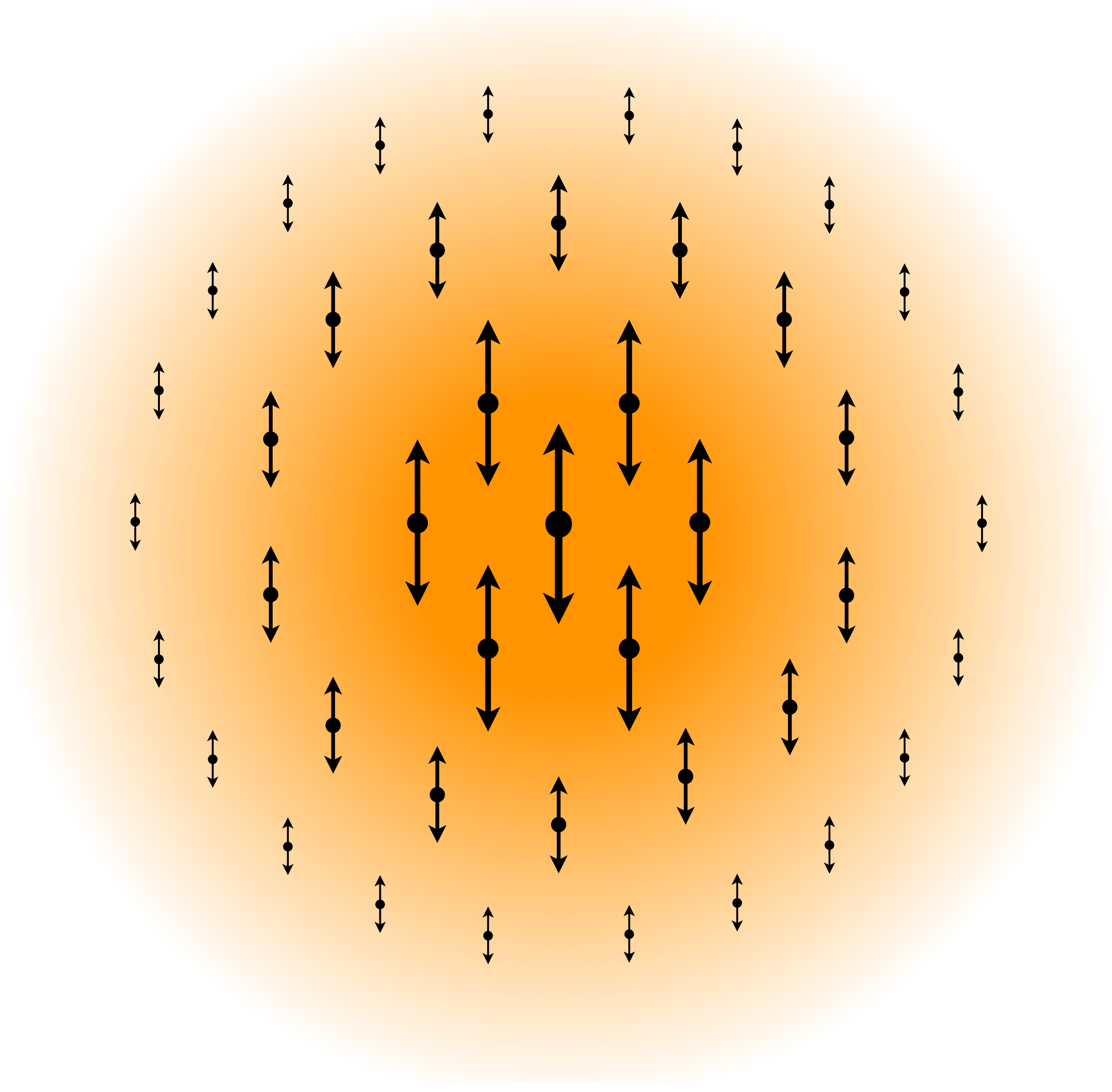
at least when non-relativistic
Lozanov & Adshead (2021)

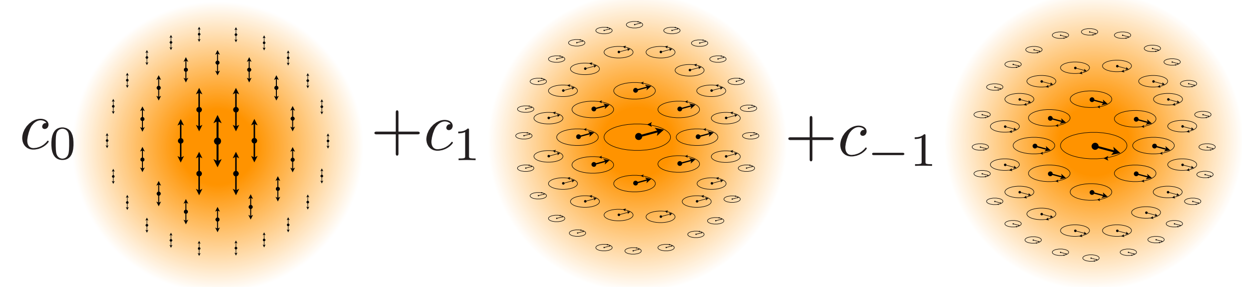


$E^s_{\rm hh} > E$

$E^{s=1}_{\rm hh} \approx 0.33E \; < 0$







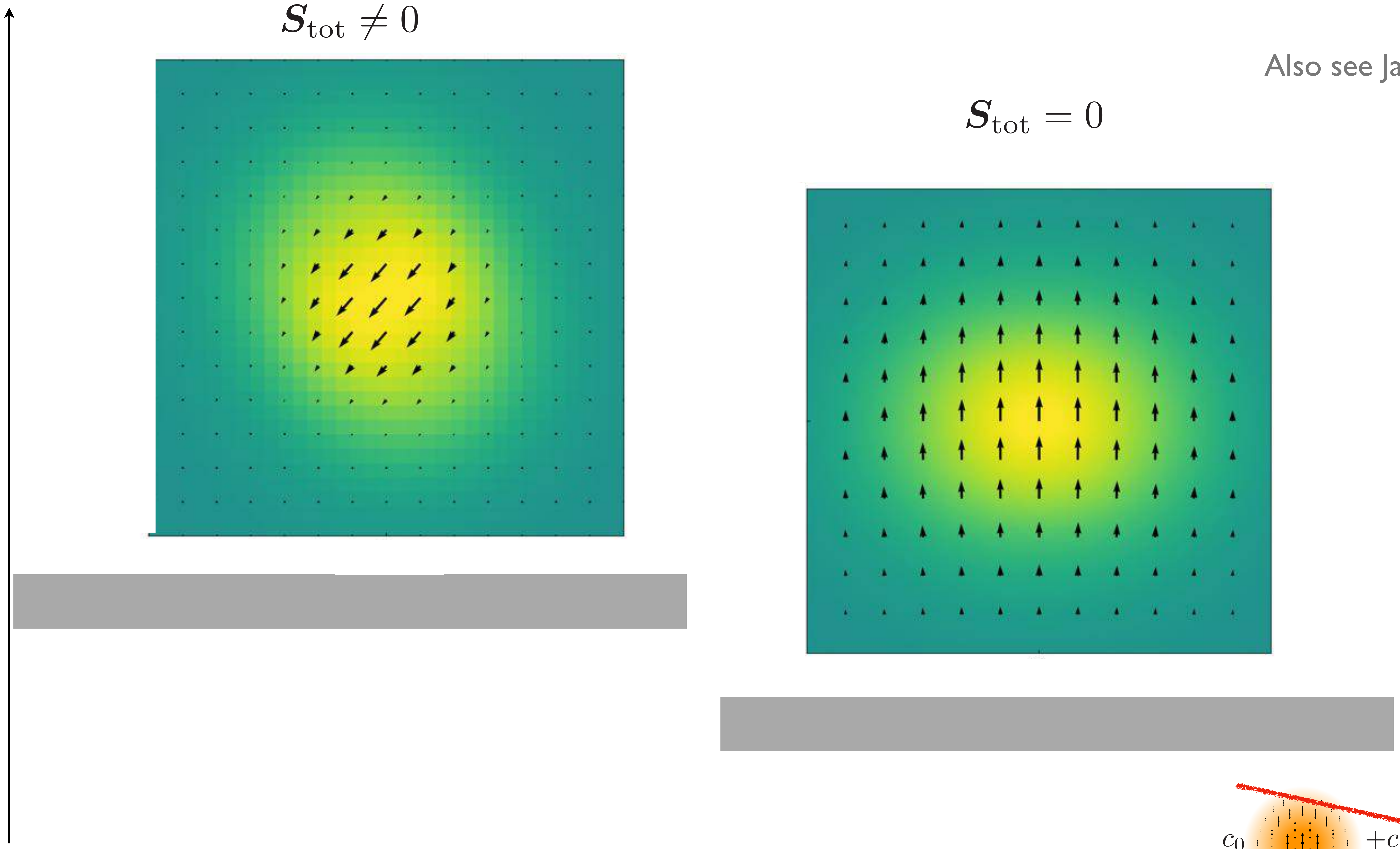
attractive non-gravitational self-interactions

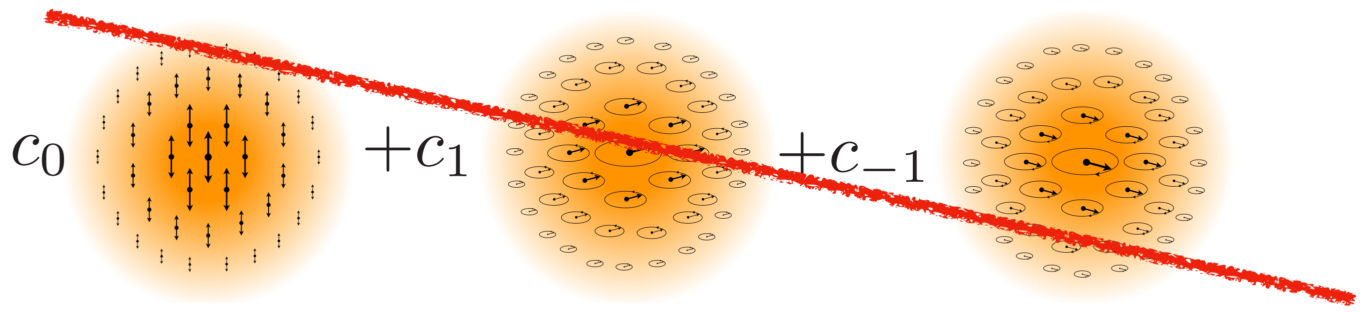
Zhang, Jain & MA (2022)

2111.08700

Also see Jain (2021), Zhang & Ling (2023)

energy





i-SPin: An integrator for multicomponent Schrodinger-Poisson systems with self-interactions

arXiv: 2211.08433
Mudit Jain & Mustafa Amin

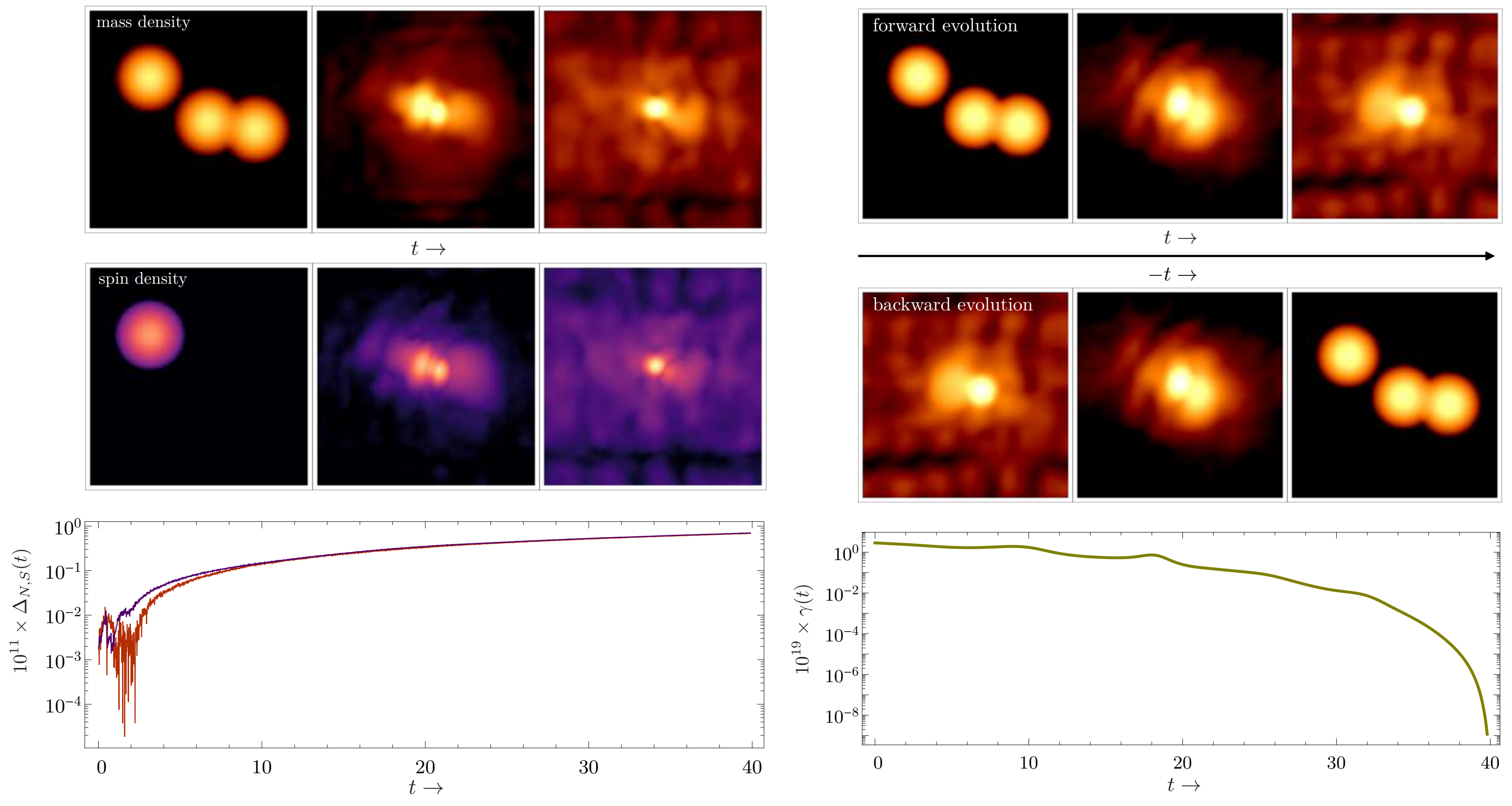
i-SPin: An algorithm (and publicly available code) to numerically evolve multicomponent Schrodinger-Poisson (SP) systems, including attractive/repulsive self-interactions + gravity

problem: If SP system represents the non-relativistic limit of a massive vector field, non-gravitational self-interactions (in particular, *spin-spin* type interactions) introduce new challenges related to mass and spin conservation which are not present in purely gravitational systems.

solution: Above challenges addressed with a novel analytical solution for the non-trivial 'kick' step in the algorithm (sec 4.3.2)

features: (i) second order accurate evolution (ii) spin and mass conserved to machine precision (iii) reversible

generalizations: n-component fields with SO(n) symmetry, an expanding universe relevant for cosmology, and the inclusion of external potentials relevant for laboratory settings



$$i\hbar\,\partial_t \boldsymbol{\Psi} = -\frac{\hbar^2}{2m}\nabla^2\boldsymbol{\Psi} + m\,\Phi\,\boldsymbol{\Psi} - \frac{\lambda(\hbar c)^3}{4(mc^2)^2}\left[(\boldsymbol{\Psi}\cdot\boldsymbol{\Psi})\,\boldsymbol{\Psi}^\dagger + 2\,(\boldsymbol{\Psi}^\dagger\cdot\boldsymbol{\Psi})\,\boldsymbol{\Psi}\right]$$

$$V_{\rm nrel}(\rho,\boldsymbol{\mathcal{S}}) = -\frac{\lambda(\hbar c)^3}{8(mc^2)^2}\left[3\rho^2 - \frac{(\boldsymbol{\mathcal{S}}\cdot\boldsymbol{\mathcal{S}})}{\hbar^2}\right]$$

number density $\rho = \boldsymbol{\Psi}^\dagger\boldsymbol{\Psi}$

spin density $\boldsymbol{\mathcal{S}} = i\hbar\boldsymbol{\Psi}\times\boldsymbol{\Psi}^\dagger$

i-SPin 2: An integrator for general spin-s Gross-Pitaevskii systems

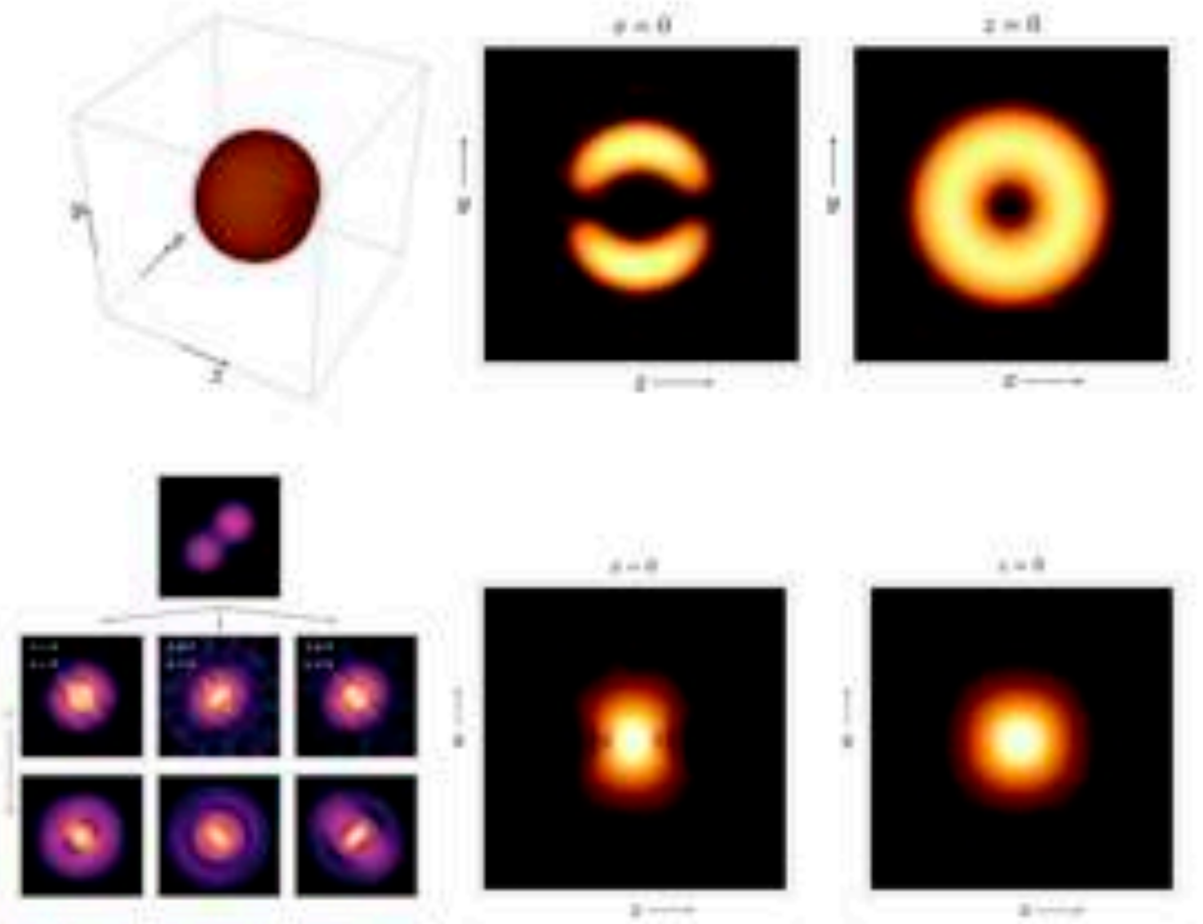
arXiv: 2305.01675
Mudit Jain, Mustafa Amin & H. Pu

i-Spin 2: An algorithm for evolving general spin-s Gross-Pitaevskii / non-linear Schrodinger systems carrying a variety of interactions, where the 2s+1 components of the 'spinor' field represent the different spin-multiplicity states.

Allowed interactions: Nonrelativistic interactions up to quartic order in the Schrodinger field (both short and long-range, and spin-dependent and spin-independent interactions), including explicit spin-orbit couplings. The algorithm allows for spatially varying external and/or self-generated vector potentials that couple to the spin density of the field.

Applications: (a) Laboratory systems such as spinor Bose-Einstein condensates (BECs). (b) Cosmological/astrophysical systems such as self-interacting bosonic dark matter.

Numerical features: Our symplectic algorithm is second-order accurate in time, and is extensible to the known higher-order accurate methods.



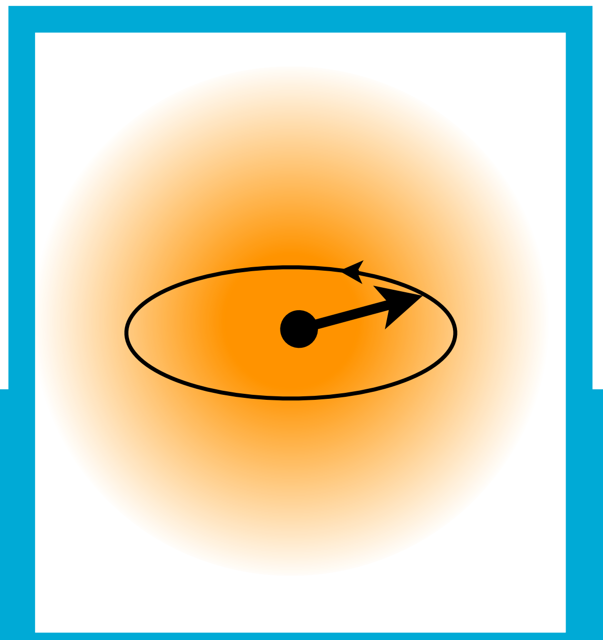
$$\mathcal{S}_{\rm nr} = \int {\rm d}t\,{\rm d}^3x\Big[\frac{i}{2}\psi_n^\dagger\dot\psi_n + {\rm c.c.} - \frac{1}{2\mu}\nabla\psi_n^\dagger\cdot\nabla\psi_n - \mu\rho V(\boldsymbol{x}) - \gamma\boldsymbol{\mathcal{S}}\cdot\tilde{\boldsymbol{B}}(\boldsymbol{x},t) - V_{\rm nrel}(\rho,\boldsymbol{\mathcal{S}}) - \frac{\xi}{2}\frac{1}{(2s+1)}|\psi_n\hat{A}_{nn'}\psi_{n'}|^2 + i\,g_{ij}\,\psi_n^\dagger\,[\hat{S}_i]_{nn'}\,\nabla_j\,\psi_n\Big],$$

with $\tilde{\boldsymbol{B}}(\boldsymbol{x},t) = f(t)\boldsymbol{B}(\boldsymbol{x})$, and

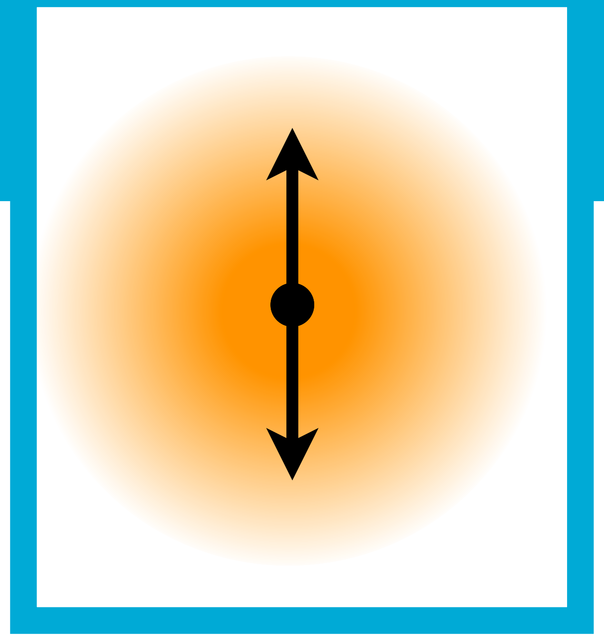
$$V_{\rm nrel}(\rho,\boldsymbol{\mathcal{S}}) = -\frac{1}{2\mu^2}\left[\lambda\rho^2 + \alpha\,(\boldsymbol{\mathcal{S}}\cdot\boldsymbol{\mathcal{S}})\right].$$

number density $\rho = \psi_n^\dagger\psi_n$

spin density $\boldsymbol{\mathcal{S}} = \psi_n^*\,\hat{\boldsymbol{S}}_{nn'}\,\psi_{n'}$

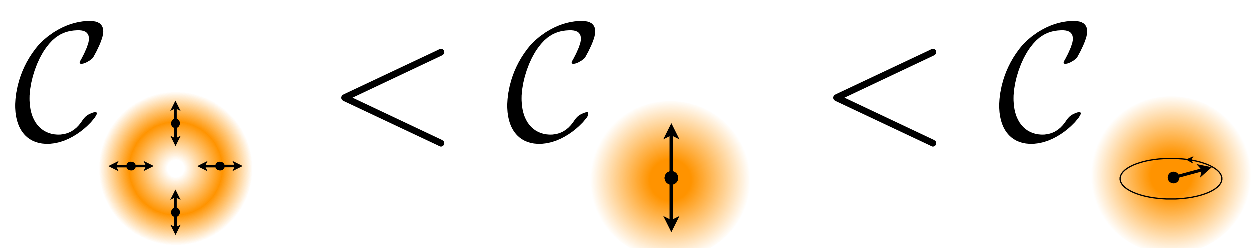


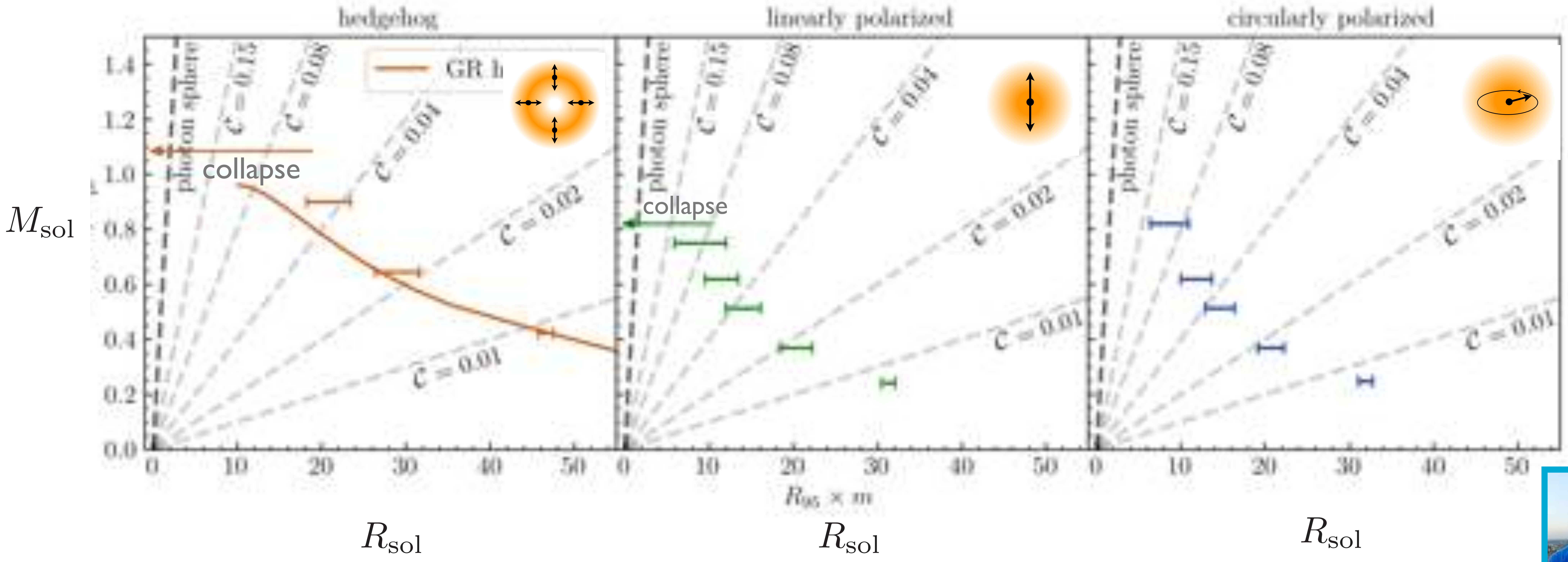
probing intrinsic spin of solitons



compactness of polarized solitons

more resistance to collapse to BH for circularly polarized stars





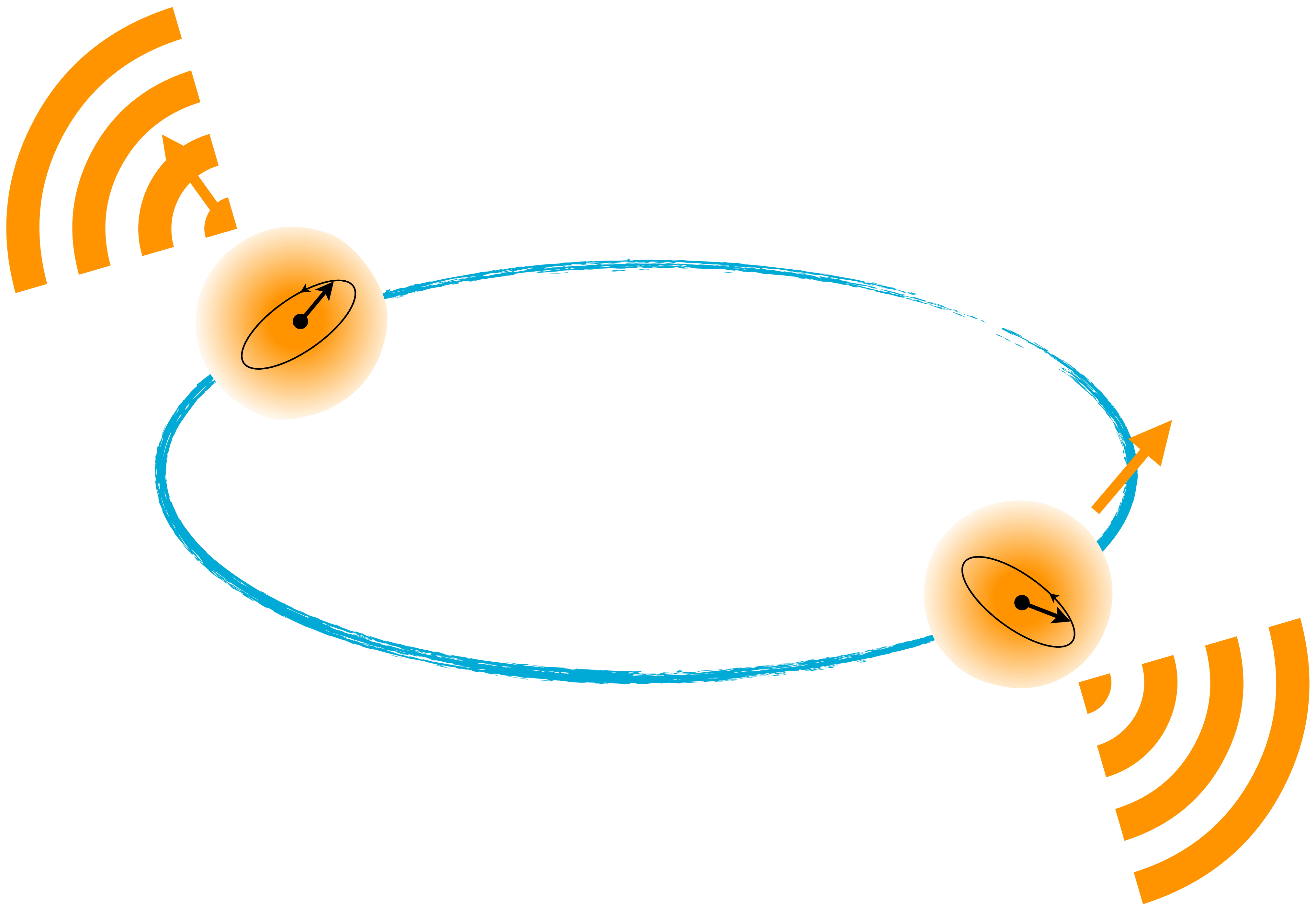
2309.04345

$\mathcal{C} = GM/Rc^2$

with Thomas Helfer & Zipeng Wang (2023)

gravitational waves and spin

$$V = -\frac{GM_1M_2}{r}\left[1 + \mathcal{O}(v^2/c^2) - \frac{2}{rc}[\hat{\boldsymbol{r}} \times (\boldsymbol{v}_1 - \boldsymbol{v_2})] \cdot \sum_{a=1}^{2} \frac{\boldsymbol{S}_a}{M_a}\right.$$
$$\left. + \frac{1}{r^2c^2}\left\{\frac{\boldsymbol{S}_1}{M_1} \cdot \frac{\boldsymbol{S}_2}{M_2} - 3\left(\frac{\boldsymbol{S}_1}{M_1} \cdot \hat{\boldsymbol{r}}\right)\left(\frac{\boldsymbol{S}_2}{M_2} \cdot \hat{\boldsymbol{r}}\right) + \sum_{a=1}^{2} \frac{C_{ES^2}^{(a)}}{2M_1M_2}\left[S_a^2 - 3(\boldsymbol{S}_a \cdot \hat{\boldsymbol{r}})^2\right]\right\} + \ldots\right]$$



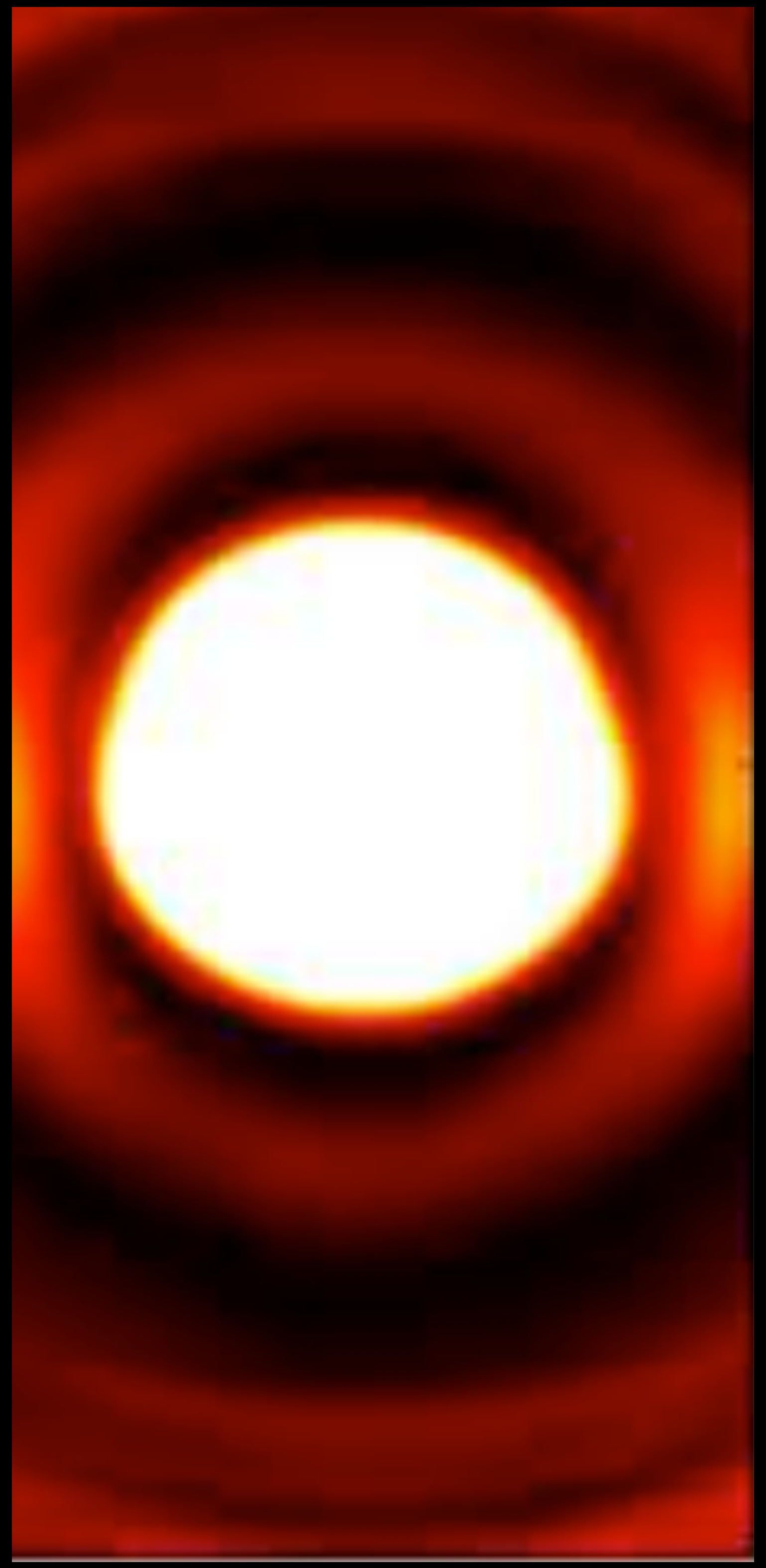
Photons from Dark Photon Solitons via Parametric Resonance

$F_{\mu\nu}G^{\mu\nu}$

$$\mathcal{L}_{\text{int}} \sim g^2 XXFF$$

MA & Mou (2019)

2009.11337



2301.11470

with Schiappacasse & Long (2022)

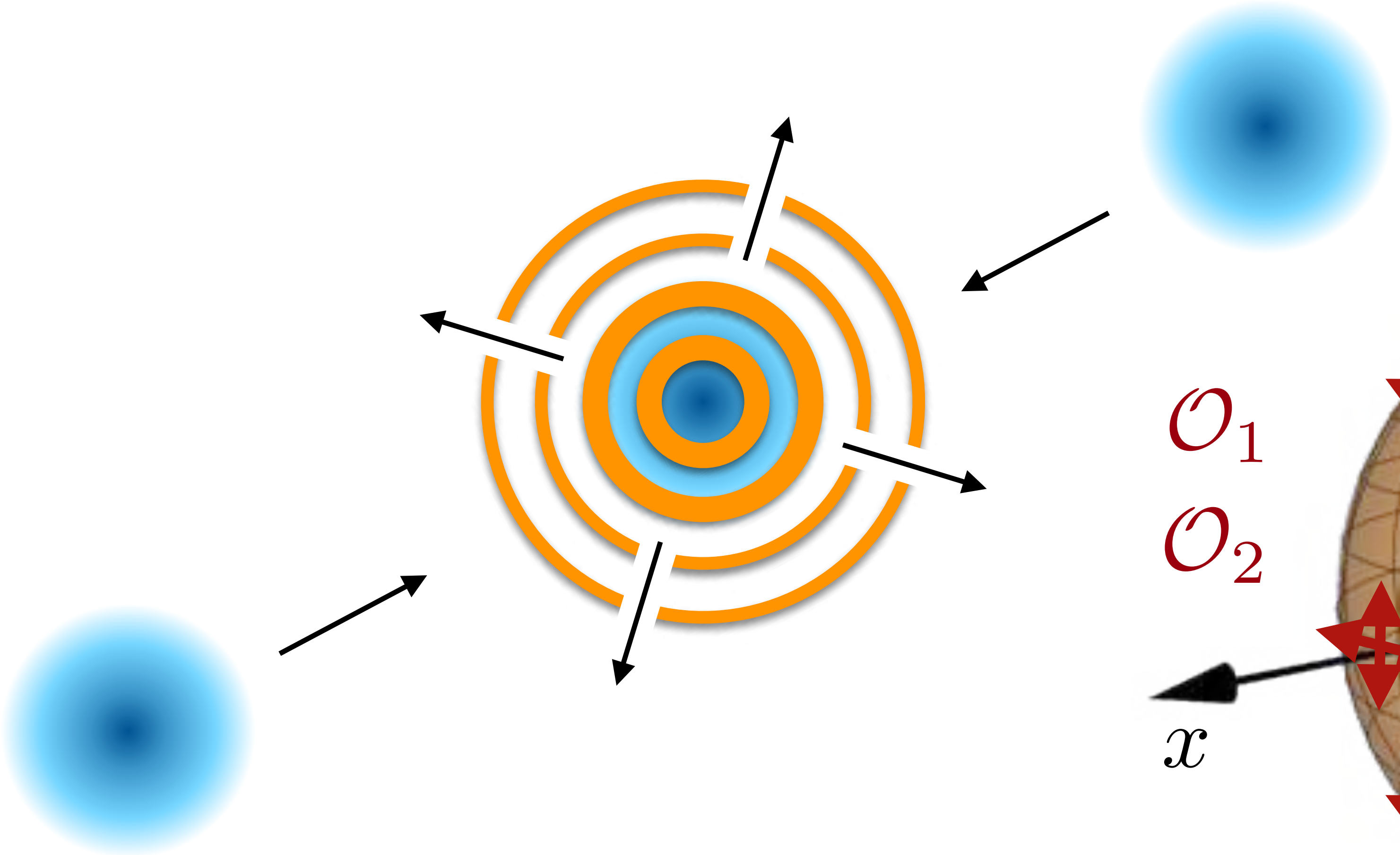
spin of soliton & polarization of photons

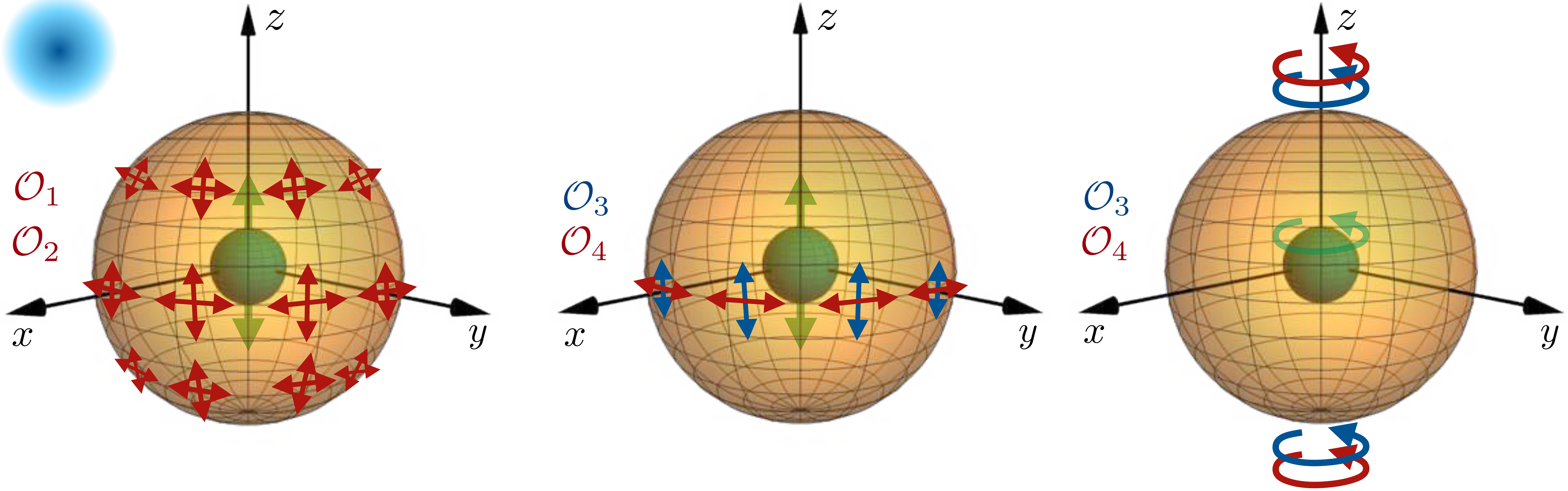
$$\mathcal{O}_1 = -\tfrac{1}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}(X\cdot X)$$
$$\mathcal{O}_2 = -\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu}(X\cdot X)$$
$$\mathcal{O}_3 = F_{\mu\rho}F^{\nu\rho}X^{\mu}X_{\nu}$$
$$\mathcal{O}_4 = \tilde{F}_{\mu\rho}\tilde{F}^{\nu\rho}X^{\mu}X_{\nu}$$
$$\mathcal{O}_5 = F_{\mu\rho}F^{\nu\rho}\partial^{\mu}X_{\nu}$$

explosive photon production (under certain conditions)

$$\mu R \gtrsim 1, \qquad \mu \sim g^2 X^2 m$$

R = soliton radius, μ = Floquet exponent



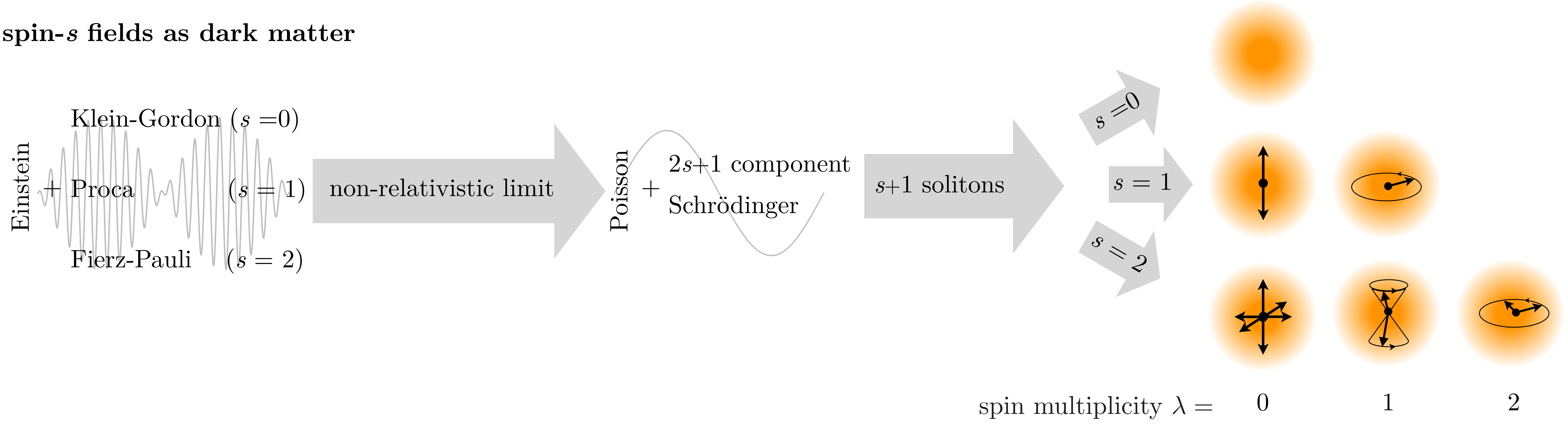


with Schiappacasse & Long (2022)

generalization to arbitrary spin

extremally polarized solitons

spin-s fields as dark matter



macroscopic spin $\boldsymbol{S}_{\rm tot}/\hbar = \lambda N \hat{z}$

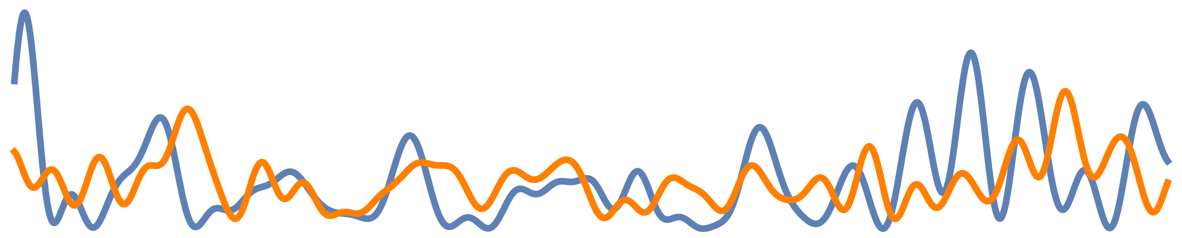
N = # of particles in soliton

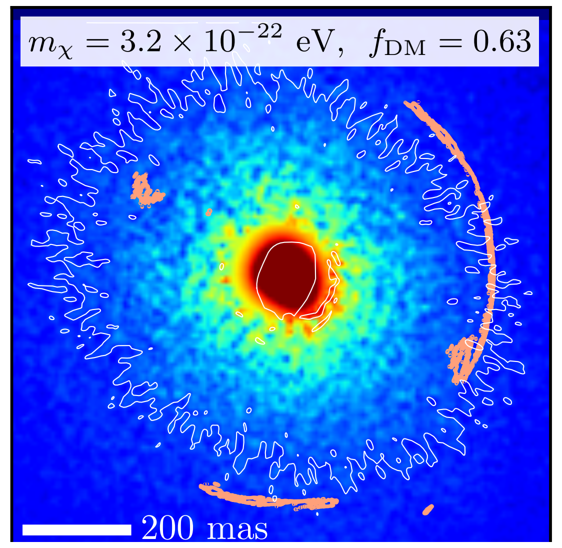
Jain & MA (2021)

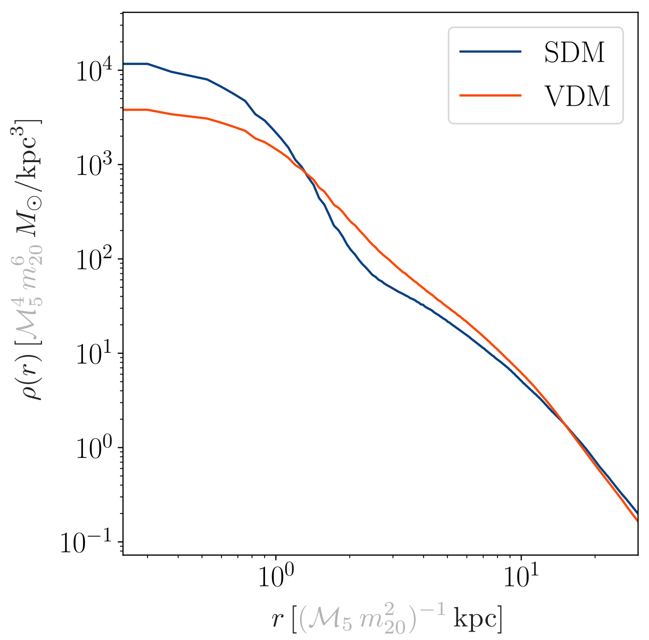
summary

Phenomenology

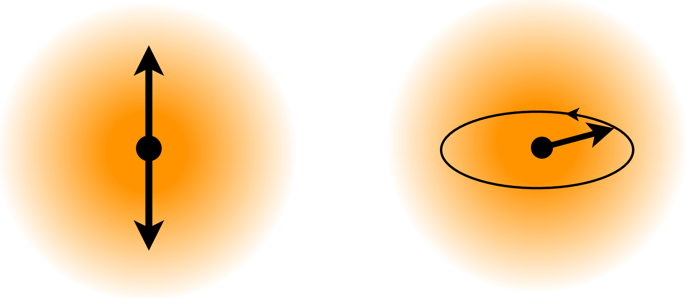
- reduced interference

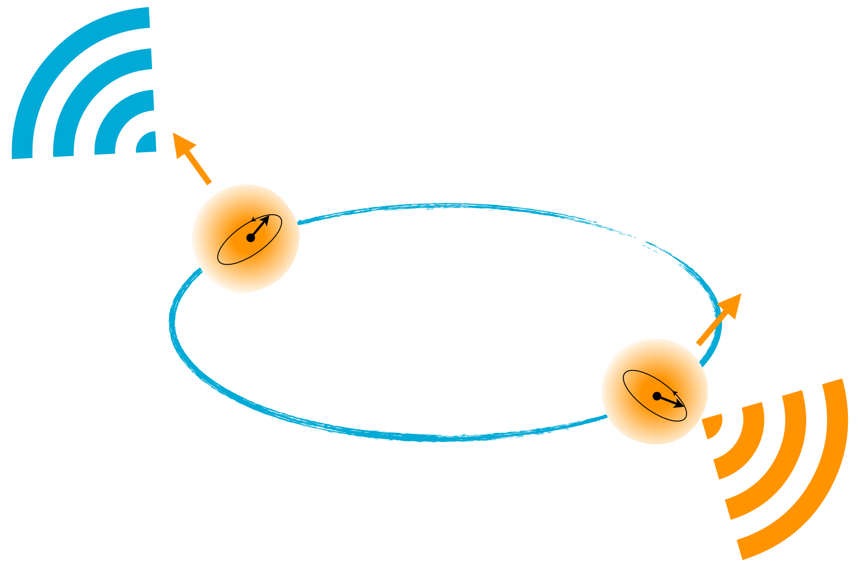






- polarized solitons, with macroscopic spin





- growth of structure, nucleation time-scales