A Stationary Mössbauer Scheme for Gravitational Wave detection

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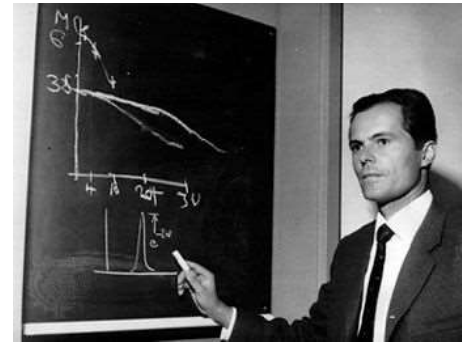
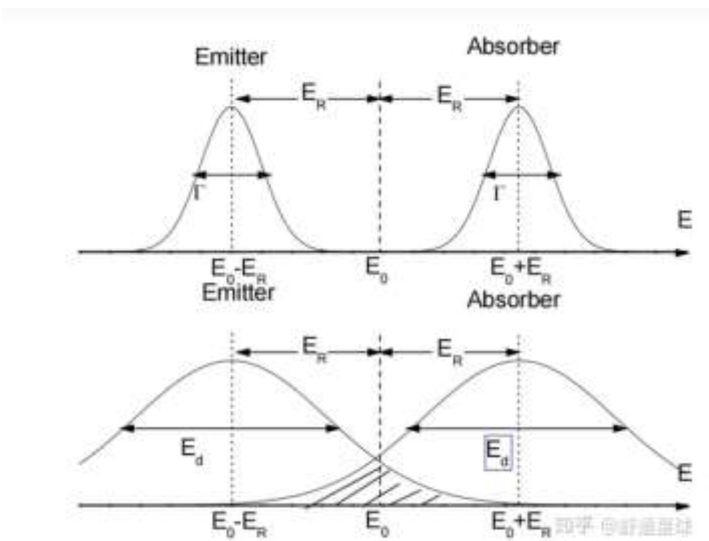
A conceptual experiment presented in 2310.06607. In collaboration with 张华桥、徐伟

Mössbauer effect

1961



- Recoil-less emission and resonant absorption of nuclear transition photons.
- W/O phonon excitations, the recoil is extremely small (against the entire lattice + other shifts)
- Resonance is sensitive to a tiny photon energy variation.



Frequency peaks with recoil-less (natural) width Γ are separated by suppressed shifts (collective recoils, Doppler shifts, energy level differences) E_R. The separation $2*E_R$ is compensated for by a manually introduced shift to achieve resonance.

E_γ Sensitivity:

^{57}Fe ~ 10^{-13} (mostly used)
^{65}Zn ~ 10^{-15}
^{109}Ag ~ 10^{-22}
+ many others.

An early role in relativity test

- Mossbauer effect can sense the gravitational redshift.

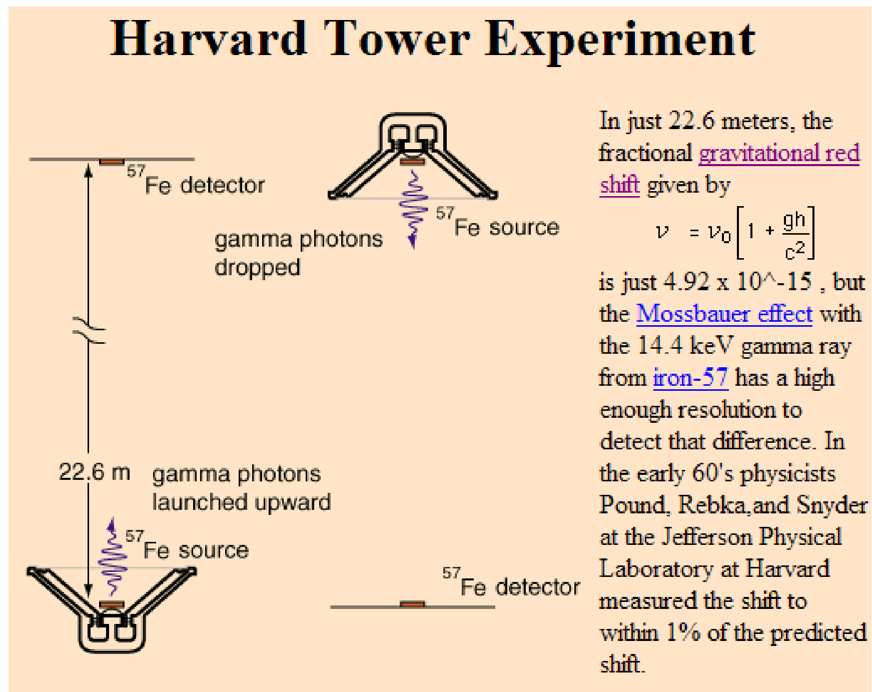
Jefferson laboratory at Harvard University. The experiment occurred in the left "tower". The attic was later extended in 2004.

Pound, Rebka & Snyder (1960-1965)

Observation of a height-induced $2ghc^{-2} \sim 4.905 \times 10^{-15}$ frequency shift

Also see: frequency shift due to acceleration

H.J. Hay, J. P. Schiffer, T. E. Cranshaw, and P. A. Egelstaff, (Atomic Energy Research Establishment,1960)



Mössbauer for GW?

- Many have thought about it, no doubt…
- Photon frequency varies when it propagates in an un-even space-time background.
- Mössbauer gave way to clock-based experi-ments in later tests of gravity.
- **Issues with line-shifts**.

See [K. Hentschel, *Annals of Science* 53, 269–295 (1996)]

Published: 11 July 1970

Redshift Fluctuations arising from Gravitational Waves

WILLIAM J. KAUFMANN

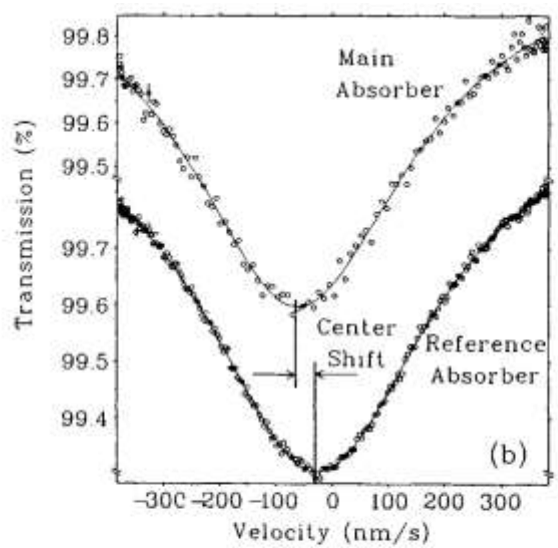
Nature **227**, 157–158 (1970) | Cite this article

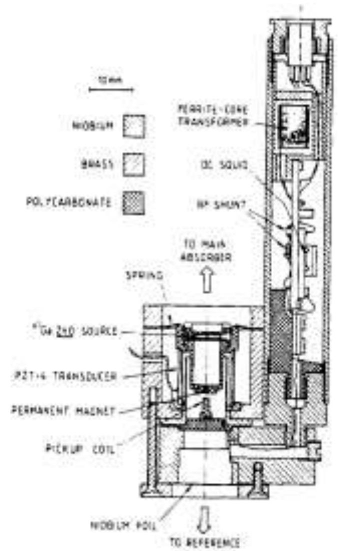
It should be noted that the gravitational waves which Weber[4,5] claims to have observed at 1,660 Hz are too weak to be detected by the method suggested in this paper. A gravitational radiation flux of 10^4 ergs cm^{-2} s^{-1} determined by Weber corresponds to the $h_{\mu\nu}$ being several orders of magnitude below the present limits of detectability of the Mössbauer effect ($h_{\mu\nu} \sim 10^{-15}$). Nevertheless, we might expect that refined techniques using the Mössbauer effect will one day become important tools in the detection of gravitational radiation.

A cryogenic ^{65}Zn measure-ment of the local *g*-value (Potzel et.al. 1992)

Differential measurement of the resonance with a sinusoidal oscillator

* resonance is achieved
* g-value is off, possibilly due to various line-shifts.



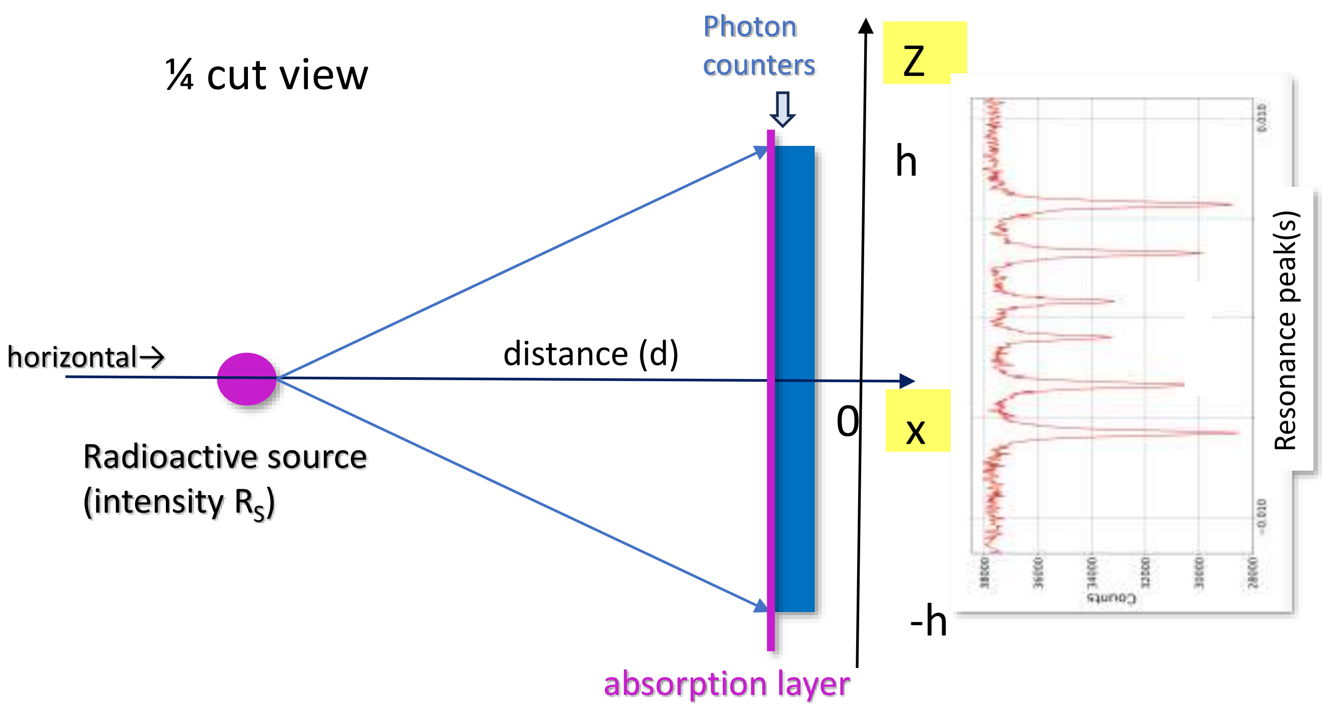


Can improve with a stationary scheme

"such solid-state effects might be difficult … … there might exist two exceptions. The first are null redshift experiments, in particular measurements with stationary source and absorber".

A stationary measurement?

- Improve on vibration-induced uncertainties. √
- Replace Doppler shift with gravitational shift. √
- GW: time-variance of resonance's ***height-shift instead of absolute height*** → avoid uncontrolled energy level uncertainties.



Energy loss E_R is compensated by a slight height difference between absorber and source: *can be calibrated in advance.*

--- the absolute height of resonance (Z_0) is affected by large systematics: 2nd Doppler, chemical composition, etc.

--- but its time-dependent shift under GW is *not* affected.

Stationary detectors' resonance

The recoil-free absorption peak spectrum. f_S:

Natural width $\Gamma \sim lifetime^{-1}$

$$\frac{\mathrm{d}N_{\mathrm{RF}}(E)}{\mathrm{d}E\ \mathrm{d}t} = \dot{N}_0 f_s \cdot \frac{\Gamma/2\pi}{[E - E_0]^2 + (\Gamma/2)^2}$$

Intrinsic transition energy

Arrival energy (with shift)

The photon count spectrum
(behind an absorption layer)

$$C(Z) = \dot{N}_0\, e^{-\mu_e t'} \cdot \Big[(1 - f_S) + \int_{-\infty}^{\infty} f_S \xi(Z_S, E_0) \cdot \mathrm{e}^{-t\xi(Z, E_0 + \Delta E_0)\Gamma/2\pi} \mathrm{d}E\Big],$$

$$\xi(Z, E_0) \equiv \frac{\Gamma/2\pi}{[E + g(Z - Z_S)E - E_0]^2 + (\Gamma/2)^2},$$

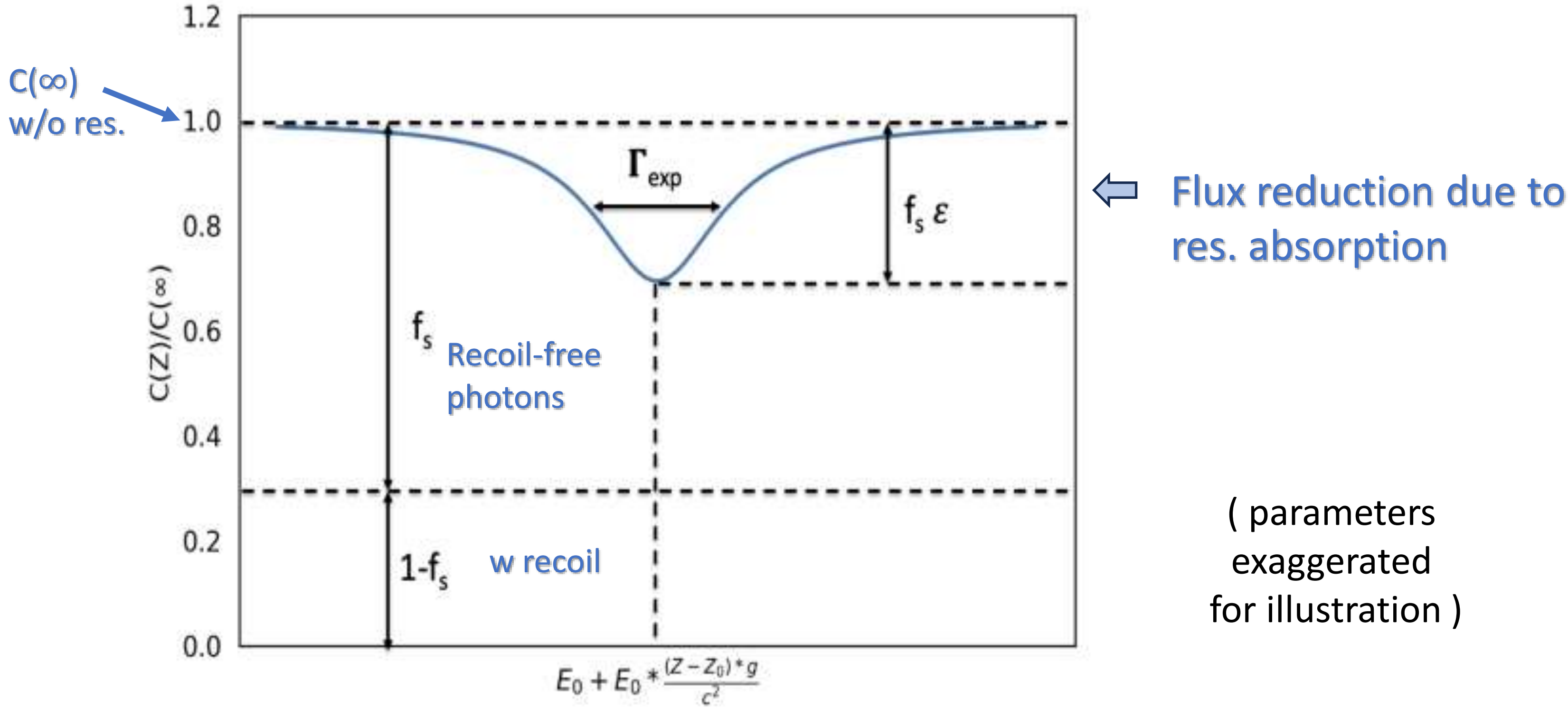
Height-induced energy shift replaces the Doppler shift

Expansion in the thin layer limit ($t \to 0$)

$$C(Z) = \dot{N}_0\, e^{-\mu_e t'} \left\{ 1 - f_S\, \epsilon \cdot \frac{\Gamma_{\mathrm{exp}}^2}{[g(Z - Z_0)E_0]^2 + \Gamma_{\mathrm{exp}}^2} \right\}$$

Effective pars: ϵ, Γ_{exp} depend nontrivially with *t*.
Benchmark: consider a high concentration with t=8 and correspondingly $\Gamma_{exp} = 4.1\Gamma$, $\epsilon = 0.8$

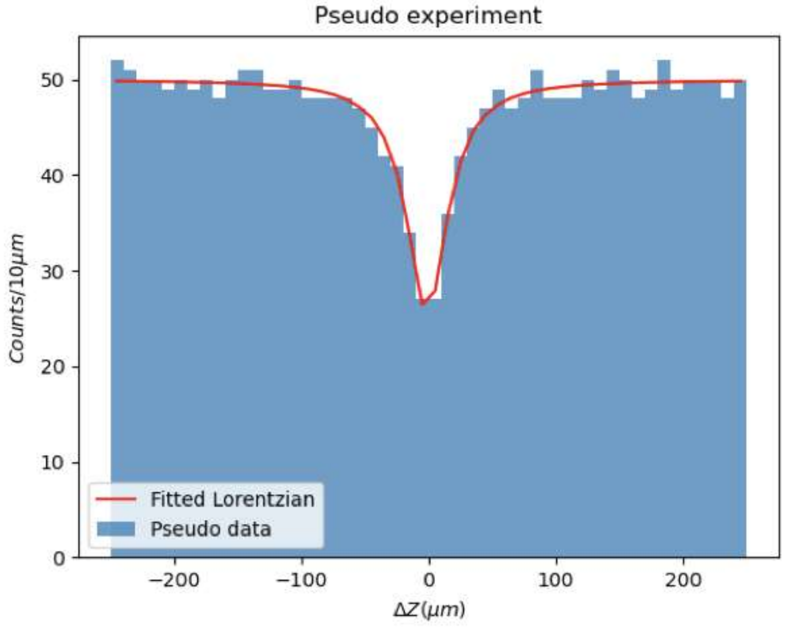
The resonance line-shape (in terms of absorber height Z)



(parameters
exaggerated
for illustration)

with extra shift/perturbation:

$$Z_0 \rightarrow Z_0(t) = Z_0 + g^{-1}\frac{\Delta f(t)}{f_\gamma}$$



Recoil free fraction	C_∞			
f_S	50	500	5000	50000
0.05*	-	-	-	1.2e-22
0.10	-	-	1.3e-22	3.8e-23
0.20	-	1.3e-22	4.5e-23	1.4e-23
0.30	-	7.9e-23	1.9e-23	7.0e-24
0.40	-	4.8e-23	1.5e-23	4.5e-24
0.50	-	3.3e-23	9.4e-24	2.9e-24
0.60	7.3e-23	2.2e-23	7.2e-24	2.1e-24
0.70	5.0e-23	1.5e-23	5.0e-24	1.5e-24
0.80	4.1e-23	1.2e-23	4.0e-24	
0.90	3.7e-23	9.5e-24	3.1e-24	

* for metallic silver

Choose height binwidth that matches Γ_{exp} :

$$\Delta Z = 0.5 \cdot g^{-1}\Gamma_{\rm exp}/E_0$$

Spatial resolution of peak relates to freq. shift:

$$\frac{\delta f}{f} = \frac{\delta Z_0}{\Delta Z} \cdot \frac{\delta f_{\rm Moss}}{f} \equiv \frac{\xi(\epsilon f_S)}{\sqrt{C_\infty}} \cdot \frac{\Gamma_{\rm exp}}{E_0},$$

* Function ξ derives from line-shape fitting and it is insensitive to ΔZ;

* Freq. resolution improves over larger statistics (C_∞)

Silver alloy/compound with higher T_{deBye} helps improve f_S

e.g. AgB_2 has a higher T_{deBye} and f_S =0.2 (@ 4K)

Spatially resolved: $\delta f/f \propto \sqrt{\Gamma_{exp}}$

The realistic Γ_{exp} may come with a `broadening' factor. While the resonance width scales linearly with Γ_{exp}, so does ΔZ and C_∞. Improved stat. in each height bin lets the overall sensitivity ∝ $\sqrt{broadening\ \#}$

Isotope of choice: ^{109}Ag

^{109}Ag Isotope Properties

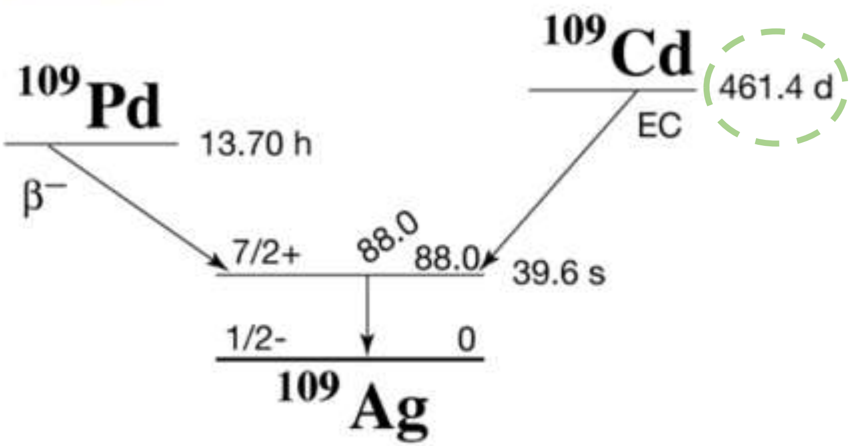
Isotopic abundance 48.161(5)%

Ground state properties:
? = -0.130563(23) nm

Excited state properties:
E =88.0341(11) keV
E_R = 4.3544(9) 10^{-2} eV
? = 4.400(6) nm
Q = 1.02(12)b
$T_{1/2}$ = 39.6(2) s
W = 7.9(2) 10^{-11}mm/s

Unit Conversion:
1mm/s = 71.0043(9) MHz
1mm/s = 2.9365(4) 10^{-7}eV

Decay Diagram



Mössbauer database (DICP, CAS)

- Long parent nuclei lifetime: 461 days allow for sufficient operation time
- Narrow 88 keV linewidth: O(10^{-22}) sensitivity
- Workable ΔZ ~ 10μm under terrestrial (1 g) gravity field for $\Gamma_{exp} = 4.1\Gamma$

R&D with high-z detectors

The quest of the ^{109}Ag resonance:

$\Gamma_{exp} \sim 30\Gamma$, (W. Wildner and U. Gonser, 1979)

Improved resonance resolution, w broadening factors down to 16 (US)

R. D. Taylor and G. R. Hoy, SPIE **875**, 126 (1988).
S.RezaieSerej, G. R. Hoy, and R. D. Taylor, Laser Phys. **5**, 240 (1995).

Russian group: improvements with Grav. Effects

V. G. Alpatov, et.al. Laser Physics 17, 1067–1072 (2007).
Yu. D. Bayukov, et.al. JETP Letters 90, 499–503 (2009).

The GW signal: frequency shift

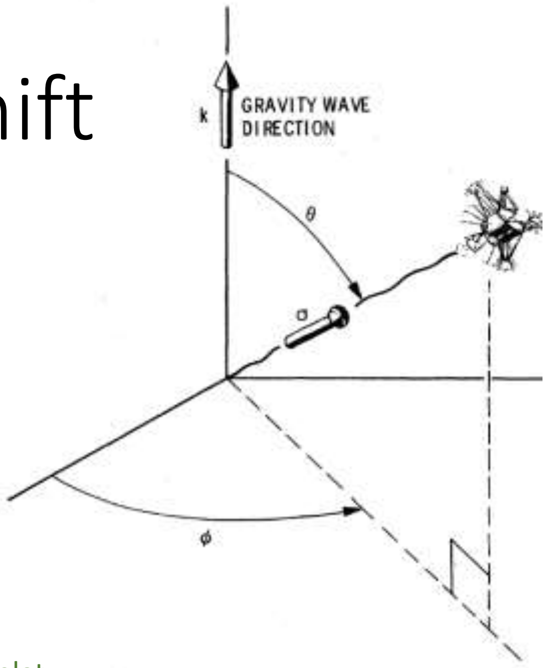


FIG. 1. Tracking geometry.

(figure from Hellings paper)

Consider a plain-wave strain perturbation

$$h(\boldsymbol{x}, t) = h_0 \, e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})}$$

$$\mathrm{ds}^2 = \mathrm{c}^2\mathrm{dt}^2 - [1+\mathrm{h}]\mathrm{dx}^2 - [1-\mathrm{h}]\mathrm{dy}^2 - \mathrm{dz}^2$$

A particle's 4-momentum response to GW strain after one-way propagation:

$$\frac{\Delta f}{f_\gamma} = \frac{\ell^\mu \ell^\nu}{1-\cos\theta}[h^{\mathrm{D}}_{\mu\nu} - h^{\mathrm{E}}_{\mu\nu}]$$

$$\ell^\mu = f_\gamma(1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

Estabrook and Wahlquist, Gen. Relat. Gravit. 6, 439–447 (1975);
Hellings, Phys. Rev. D 23, 832–843 (1981).

$$\frac{\Delta f}{f_\gamma} = 2h_0 \cos^2\frac{\theta}{2}\cos 2\phi \sin\left(\omega d \sin^2\frac{\theta}{2}\right) \cdot \sin\left(\omega t - \omega d\cos^2\frac{\theta}{2}\right),$$

Energy diff. between $E(t_E, \vec{0})$ and $D(t_E + \frac{d}{c}, \frac{\vec{d}}{c})$

When the baseline distance approaches to the GW's wavelength scale, a particle starts to see the strain difference.

$$\frac{\Delta f}{f_\gamma} = 2h_0 \cos^2\frac{\theta}{2}\cos 2\phi \sin\left(\omega d \sin^2\frac{\theta}{2}\right) \cdot \sin\left(\omega t - \omega d \cos^2\frac{\theta}{2}\right),$$

Spin-2

* Requires a perpendicular *h* component.
* Extra complication w baseline at high freq.
* Vanishes when (anti) parallel to GW direction

Maximal shift with GW frequency:

$$\left.\frac{\Delta f}{f_\gamma}\right|_{\max.} = \begin{cases} \frac{\omega d}{2} h_0, & \omega d \ll 1 \ \&\ \theta \to \frac{\pi}{2}, \\ \eta(\omega d)\cdot h_0, & \omega d > 1,\ 1^{\rm st}\ \max. \end{cases}$$

$\eta \to 2$ at high frequency, with multiple maxima.
Angular patterns becomes very complicated for $\omega d > O(10)$
Angular pattern allows for GW direction reconstruction
At low-freq, freq. shift decreases *linearly* with ωd

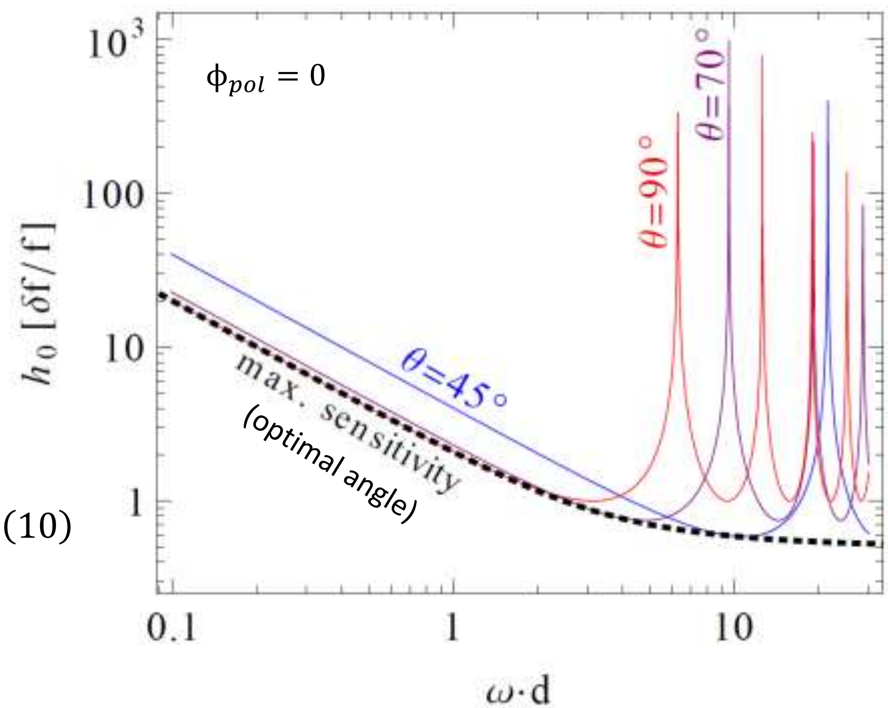
Non-trivial angular pattern with the incident GW direction:

Low GW freq: max. at 90.
High GW freq: modulated btw $0 < \theta < 2\pi$

"blind directions"

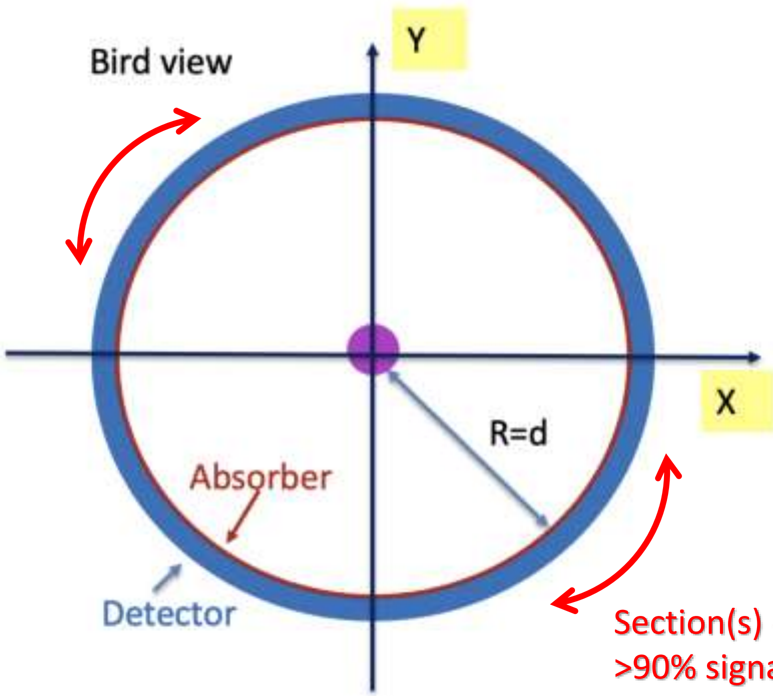
$$\omega d \sin^2\frac{\theta}{2} = n\pi, \quad n = 1, 2, 3...$$

Multiple directions can compensate for others' insensitive directions.



A circular layout

- A ring of detectors in the horizontal plane covers $\theta \in (\theta, \pi - \theta)$ when GW comes at angle θ.
- Guarantee (at least) two perpendicular directions relative to any GW incident θ angle.



Section(s) containing >90% signal strength: f_ϕ~0.29 for vertically incident GWs.

Circle stays horizontal: for height compensation of Mossbauer E_R.

Huaqiao's counting algorithm:

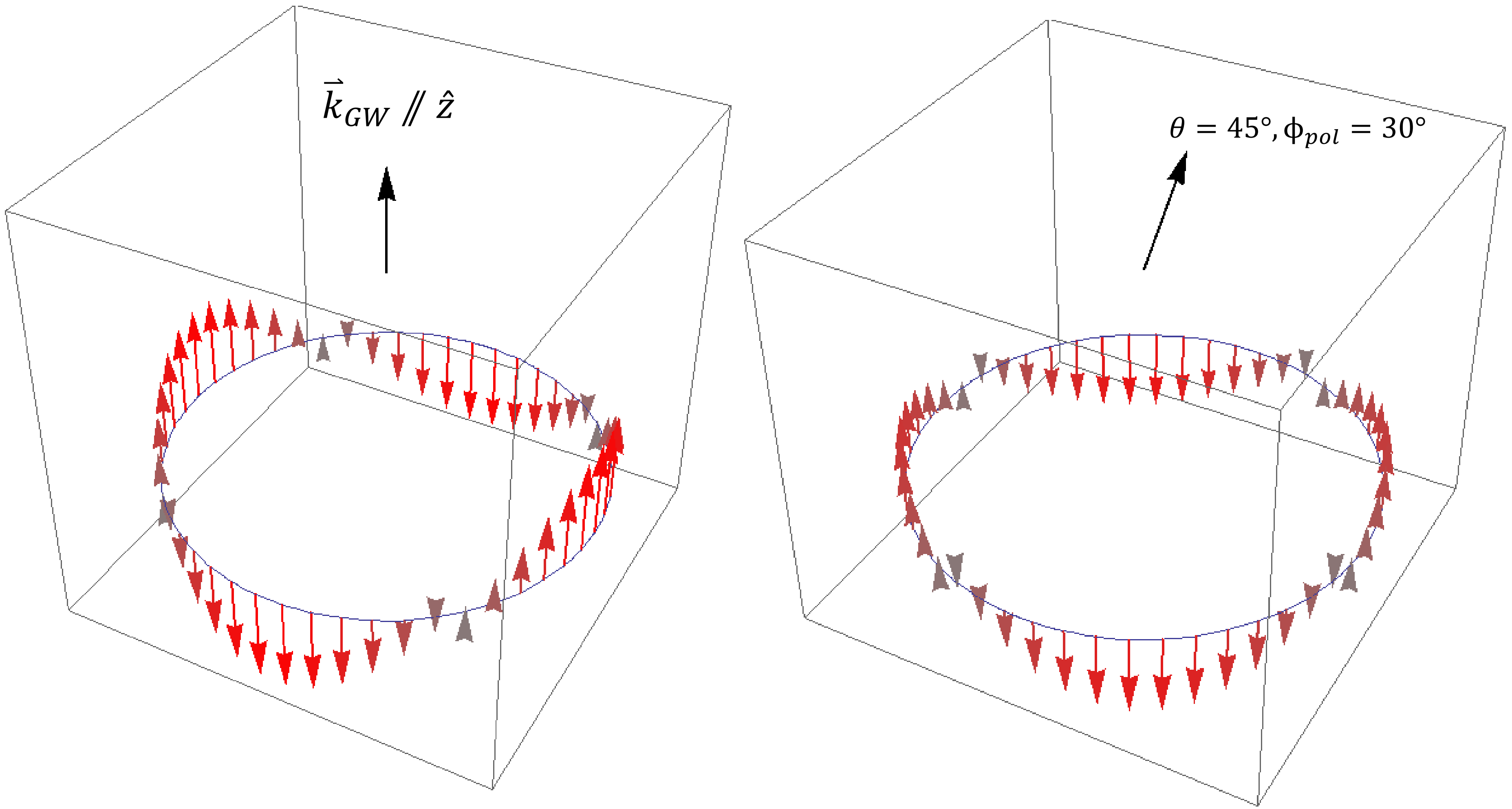
We sum up all detectors' counts within >90% signal region & identify this counting # (in each signal period) to the C_∞ in resonance peak location reconstruction.

(sacrifice angular information for statistics)

$$N_{90} = R_s \cdot \frac{2\pi f_t}{\omega} \cdot \frac{(2\pi f_\phi d) \cdot \Delta Z}{4\pi d^2}$$

f_t~0.3 for the time fraction of >90% signal in each period.

Total angular fraction: $f_\phi \Delta Z / 2d$



$\vec{k}_{GW} /\!/ \hat{z}$
$\theta = 45°, \phi_{pol} = 30°$

An estimate on the required source intensity: (R_S for an isotropic source)

$$R_s = \frac{\omega}{2\pi}\frac{C_\infty \, 2d}{\Delta Z f_\phi f_t} = \frac{2\omega d g \xi^2}{f_\phi f_t}\left(\frac{\Gamma_{\rm exp}}{E_0}\right)\left(\frac{\delta f}{f}\right)^{-2}$$
$$\approx 10^{14}\ {\rm Bq}\cdot\left(\frac{\omega/2\pi}{\rm MHz}\right)\left(\frac{d}{1\ {\rm m}}\right)\left(\frac{g}{g_\oplus}\right)$$
$$\cdot\left[\frac{\eta(\epsilon f_S)}{12.4}\right]^2\left(\frac{4\times10^{-21}}{\delta f/f}\right)^2$$

Beware: pars on the 2nd line do not scale independently.

Benchmarks:
A: table-top experiment.
B: 10-meter radius in low-*g*

	g ($g_\oplus$)	d (m)	ΔZ	ϵf_S	$h_{\rm min}$	$f_{\rm max}$	R_s (Bq.)
A	1	1	10 μm	0.04	3×10^{-15}	0.6 KHz	10^{11}
A'	1	5	10 μm	0.04	3×10^{-17}	13 KHz	10^{13}
B	10^{-4}	10	1 dm	0.4	3×10^{-23}	30 MHz	10^{14}
A^C	1	1	10 μm	0.04	1×10^{-21}	3 GHz	10^{11}

TABLE II. Sample static Mössbauer measurement configurations that corresponds to a table-top experiment with a Type-III source intensity (A) and a low-g setup with a stronger source (B). A' is scaled-up scenario by increasing the source intensity in A by two orders of magnitude. $h_{\rm min}$ and $f_{\rm max}$ denote the sensitivity to the GW strain and the maximal GW frequency that can be probed. A^C represents the sensitivity with setup A but for a periodic signal with coherence up to 10^6 periods. The source intensity is given for isotropic sources.

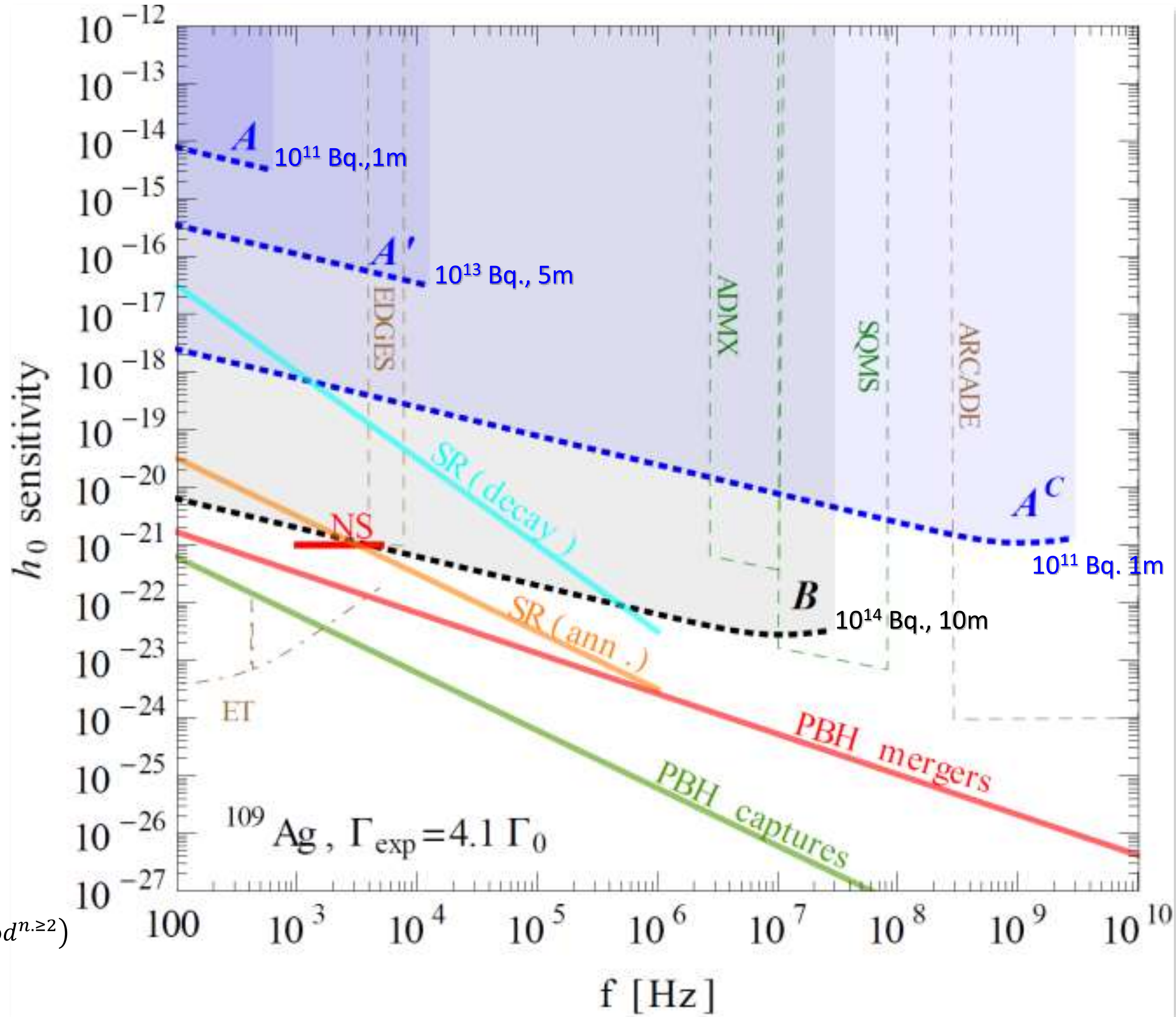
- Source intensity scales linearly with baseline length and inversely with the local gravitational acceleration.
- Need to balance between resonance shift length, detector size, and practical sources.
- Non-isotropic source / focusing would immensely enhance efficiency.
- Coherently repeated signals can boost statistics: N_{90} -> N_{90}*Q

2310.06607

A, A' & B:
single period

A^C:
10^6 periods

Max Freq. Reach:
cutoffs at:
*3σ peak with C_∞,
$C_\infty > (\sigma)^2/(f_S\epsilon)^2$
* $2\pi f d < O(10)$



Coherent GWs from NS, SR, PBHs, see
N.Aggarwal et al.,
Living Rev. Rel. 24, 4 (2021)

Inverse Gentsenshtein:
$\Delta F = F_0\, h * (form\ factor{\sim}\omega d^{n\geq 2})$
also see: 2305.00877

Take-home message

2310.06607

- A conceptual layout for a stationary Mössbauer measurement for GWs:

  *Measure the spatial peak shift instead of peak width.
  *A relatively small-scale setup (meter – 10 meter).
  *^{109}Ag gives 10^{-22} sensitivity, has long lifetime.
  *Encouraging forecast for f_{GW} > kHz, a multiband search alternative.
  *Overall Sensitivity scales as sqrt of the effective Mössbauer width.
  *Significant improvement in low-g, or any other way to enhance detectors' angular coverage, e.g. beam focusing, Laue lens, etc.