Gravitational waves from ultralight dark matter

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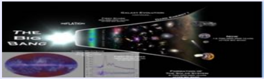
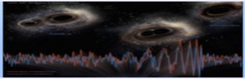
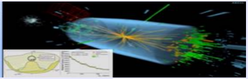
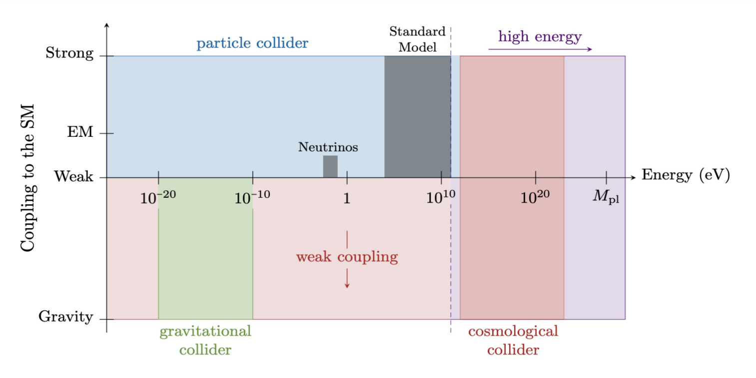
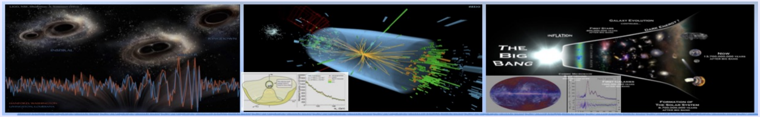


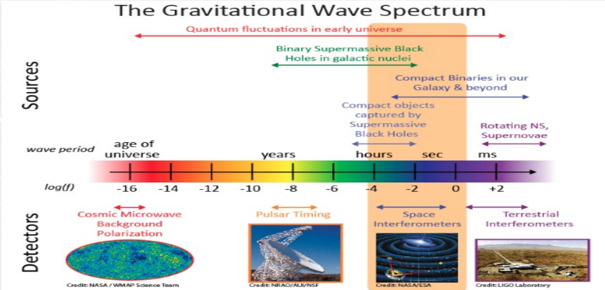
Motivation: new physics in ultra-low energy

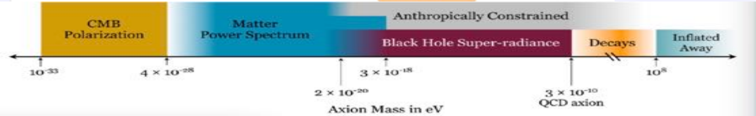


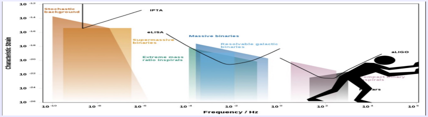
[cf. Baumann-Chia-Porto-Stout, Gravitational Collider Physics, 2019]



Spectrum of Gravitational Wave and Axion Mass







[cf. LISA/Ultra-High-Frequency Gravitational Waves Initiative]

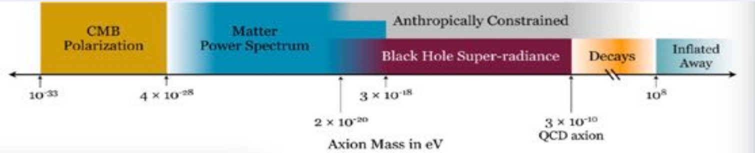
Spectrum of Ultra Light Dark Matter

QCD axion

- $\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a - \alpha_\theta \theta F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a + \bar{\psi} \left(i\gamma^\mu D_\mu - m e^{i\theta' \gamma_5} \right) \psi$
- Strong CP problem: no CP violation in measurement
- Peccei–Quinn (1977): introduce new pseudoscalar
- Wilczek- Weinberg: relax the CP-violation parameter

Axion-like: ultra light dark matter

- e.g. Fuzzy Dark Matter, Dark photon dark matter



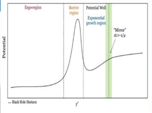
Ultralight dark matter and gravitational wave

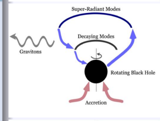
Black Hole Superradiance & GW signal

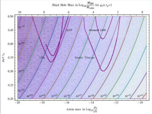
- Axion annihilation $\vartheta + \vartheta \to h$(Stochastic GW)
  Energy transition $\vartheta^{+} \to \vartheta^{-} + h$(Monochromic)
- Superradiance $\alpha \equiv \frac{R_{BH}}{\lambda_\vartheta} \simeq \left(\frac{M_{BH}}{M_\odot}\right)\left(\frac{m_\vartheta}{10^{-10}\text{eV}}\right)$
- Fast Radio Burst from Axion $\sim \vartheta F\tilde{F}$ $(\vartheta \to \gamma\gamma)$
- GW burst from Axion $\sim \vartheta R\tilde{R}$ $(\vartheta \to hh)$

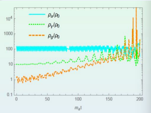
Ultra-light DM and multi band GW detection

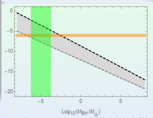
- Tabletop exp: Axion star & GW burst ($\sim$ GHz)
- LISA & LVK: Superradiance ($\sim$ mHz - kHz)
- FAST & SKA : Ultra-light dark matter ($\sim$ nHz)







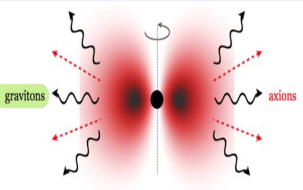


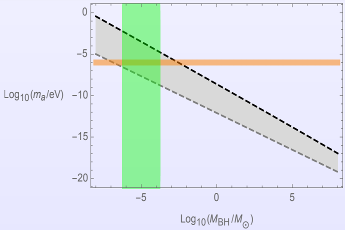


Axion-like Cloud and Black Hole Superradiance

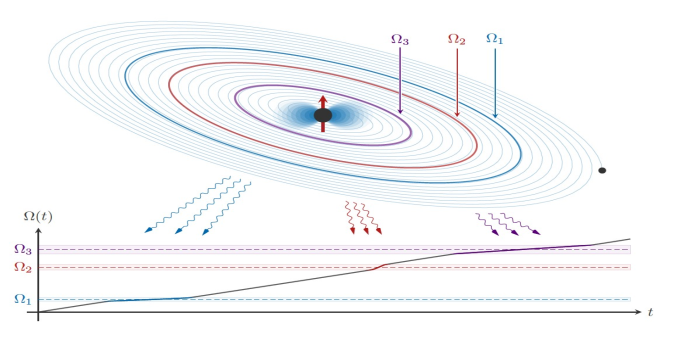
Schwarzschild $R_{BH} = G_N M_{BH}/c^2$, de Broglie wave length $\lambda_\vartheta = \hbar/(m_\vartheta v_\vartheta)$

- Characteristic $\alpha \equiv \frac{R_{BH}}{\lambda_\vartheta} \simeq \left(\frac{M_{BH}}{M_\odot}\right)\left(\frac{m_\vartheta}{10^{-10}\mathrm{eV}}\right)\left(\frac{v_\theta}{c}\right)$.
- Formation time $\tau_\vartheta <$ Universe's age $\tau_U \simeq 10^{23}\left(\frac{M_\odot}{M_{BH}}\right) R_{BH}$
- $\tau_{\vartheta\uparrow} \simeq 10^7 e^{1.84\alpha} R_{BH},\ \alpha \gg 1, \quad \tau_{\vartheta\downarrow} \simeq 24\,\alpha^{-9} R_{BH},\ \alpha \ll 1.$

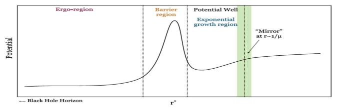


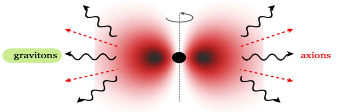


Gravitational Collider: surrounding the black hole



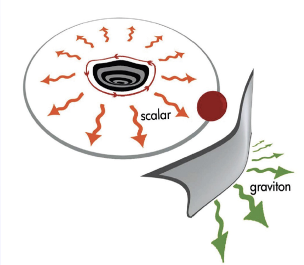
[cf. Baumann-Chia-Porto-Stout, Gravitational Collider Physics(2019)]

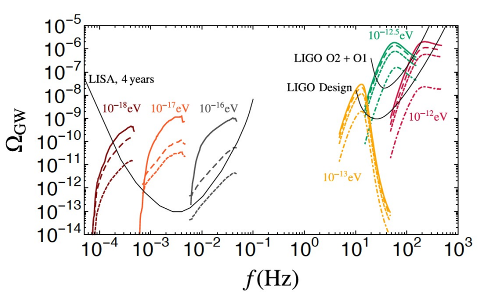




[cf. Arvanitaki-Dubovsky, String Axiverse -2011]

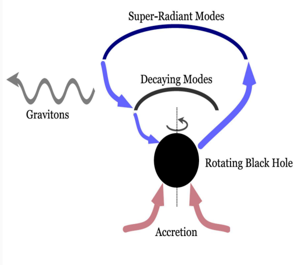
Axion annihilation and Stochastic GWs

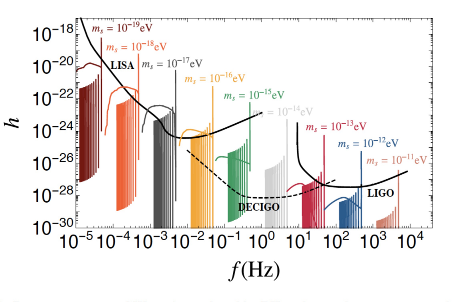




- Axion annihilation $\vartheta + \vartheta \to hh$, Strain $h \sim 10^{-21} - 10^{-32}$.
- Stochastic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

Energy level transition and Monochromic GW





- Energy transition $\vartheta^+ \to \vartheta^- + h$, Strain $h \sim 10^{-19} - 10^{-27}$
- Monochromic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

i. Axion-photon coupling ($\vartheta \to \gamma\gamma$)

Interaction

- The interaction
  $S_{EM} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}F^2 + \mathcal{L}_\vartheta + \mathcal{L}_{\vartheta F\tilde{F}}\right).$
- Axion term $\mathcal{L}_\vartheta = -\frac{1}{2}(\partial\vartheta)^2 - m_\vartheta^2 f_\vartheta^2\left(1 - \cos\frac{\vartheta}{f_\vartheta}\right).$
- Interaction $\mathcal{L}_{\vartheta F\tilde{F}} = -\frac{\alpha_\gamma}{4}\vartheta F_{\mu\nu}\tilde{F}^{\mu\nu}, \ \tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}.$

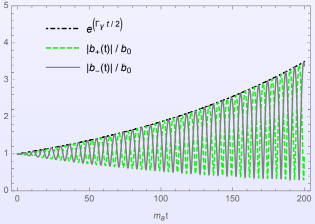
Equation of motion

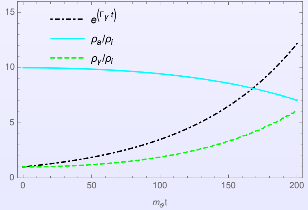
- $(\partial_\mu\partial^\mu - m_\vartheta^2)\vartheta = \frac{\alpha_\gamma}{4}F\tilde{F}, \ \partial_\mu F^{\mu\nu} = -\alpha_\gamma\tilde{F}^{\mu\nu}\partial_\mu\vartheta$
- Axion like field $\langle\vartheta\rangle = \bar{\vartheta}(t) = \vartheta_0 \sin\left(m_\vartheta t + \phi_0\right),$
- Electromagnetic field $A_\pm(t,z) \equiv [A_x(t,z) \pm iA_y(t,z)]/\sqrt{2}$

Mathieu equation and parametric resonance

$$\left[-\partial_t^2 + \partial_z^2 \mp i\alpha_\gamma \dot{\bar{\vartheta}}(t)\partial_z\right] A_\pm(t,z) = 0,\ A_\pm(t,z) = b_\pm(t)e^{ikz}.$$

- In momentum space $\ddot{b}_\pm(t) + \left[k^2 \mp \alpha_\gamma k \dot{\bar{\vartheta}}\right] b_\pm(t) = 0.$
- Amplification factor $e^{\Gamma_\gamma t_\gamma}$, $\Gamma_\gamma = \alpha_\gamma \vartheta_0 \frac{m_\vartheta}{2}$, $\quad t_\gamma \simeq \frac{1}{m_\vartheta v_\vartheta c}$.
- Time evolution of energy density: photon ρ_γ & axion ρ_ϑ.





Fast radio burst and axion cloud

$$\left[-\partial_t^2+\partial_z^2\mp i\alpha_\gamma\dot{\bar{\vartheta}}(t)\partial_z\right]A_\pm(t,z)=0,\ A_\pm(t,z)=b_\pm(t)e^{ikz}.$$

- Stimulated axion decay in superradiant clouds around primordial black holes

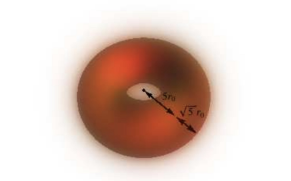
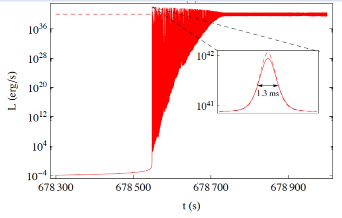


FIG. 1. Model of the superradiant 2p-axion cloud around a central Kerr black hole.



- [cf. Rosa-Kephart , PRL(2018)]

ii.Axion gravitation coupling ($\vartheta \to hh$)

Gravitational interaction

- Action $S_{GW} = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa_4}R + \mathcal{L}_\vartheta + \mathcal{L}_{\vartheta R\tilde{R}}\right)$.
- Axion like field $\mathcal{L}_\vartheta = -\frac{1}{2}(\partial\vartheta)^2 - \frac{1}{2}m_\vartheta^2\vartheta^2$.
- Chern-Simons term
  $\mathcal{L}_{\vartheta R\tilde{R}} = \frac{\alpha_g}{4}\vartheta R^{\beta}{}_{\alpha\gamma\delta}\tilde{R}^{\alpha}{}_{\beta}{}^{\gamma\delta}$, $\tilde{R}^{\alpha}{}_{\beta}{}^{\gamma\delta} \equiv \frac{1}{2}\epsilon^{\gamma\delta\mu\nu}R^{\alpha}{}_{\beta\mu\nu}$.

Equation of motion

- GWs: $\Box h_{ij} = \kappa_4\alpha_g\tilde{\epsilon}^{pk}{}_{(i}\left[\dot{\vartheta}(\partial_p\Box h_{j)k}) - \ddot{\vartheta}(\partial_p\partial_t h_{j)k})\right]$,
- Axion like field: $(\Box - m_\vartheta^2)\vartheta = -\frac{\alpha_g}{4}R\tilde{R}$,
- Polarization: $h_{ij}(t,z) = \left[h_R(t){\epsilon_0}_{ij}^R + h_L(t){\epsilon_0}_{ij}^L\right]e^{ikz} + \text{h.c.}$

Pseudoscalar or Axial current to Two-graviton

Volume 40B, number 3 PHYSICS LETTERS 10 July 1972

THE GRAVITATIONAL CORRECTION TO PCAC

R. DELBOURGO and A. SALAM
Physics Department, Imperial College, London S.W. 7 2B7, U.K.

$$\partial_\alpha A_\alpha = 2m\,P + e^2 \epsilon_{\kappa\lambda\mu\nu} F_{\kappa\lambda} F_{\mu\nu}/16\pi^2 + \epsilon_{\kappa\lambda\mu\nu} R_{\kappa\lambda\rho\sigma} R_{\mu\nu\rho\sigma}/768\pi^2 .$$

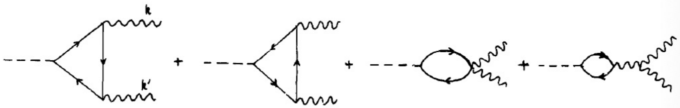


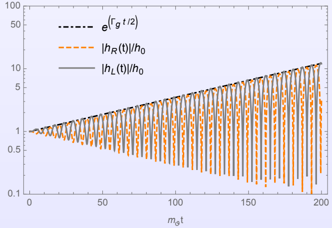
Fig. 1. One-fermion loop contributions to the two-graviton mode.

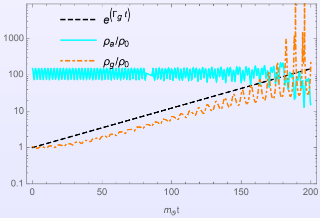
cf. Partially Conserved Axial vector Current theory (PCAC)

Parametric Resonance and GW Amplification

E.O.M. of gravitational wave ($I = L, R$)

- $\left[\ddot{h}_I(t) + k^2 h_I(t)\right]\left[1 - \varepsilon_I \kappa_4 \alpha_g k \dot{\vartheta}(t)\right] = \varepsilon_I \kappa_4 \alpha_g k \ddot{\vartheta}(t) \dot{h}_I(t)$.
- Factor $e^{\Gamma_g t_g}$, & $\frac{\Gamma_g}{m_\vartheta} \sim \left(\frac{\kappa_4 \alpha_g}{1\text{eV}^{-3}}\right)\left(\frac{m_\vartheta}{10^{-9}\text{eV}}\right)^2\left(\frac{\vartheta_0}{10^9\text{GeV}}\right)$.
- Time evolution of gravitational field ρ_g & axion field ρ_ϑ.





[cf. S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps, PRD'21]

Joint analysis & Branch Ratio ($\vartheta \to \gamma\gamma$ & $\vartheta \to hh$)

The interaction

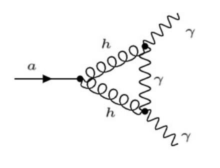
- $S_{total} = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa_4}R - \frac{1}{4}F^2 + \mathcal{L}_\vartheta + \mathcal{L}_{\vartheta F\tilde{F}} + \mathcal{L}_{\vartheta R\tilde{R}}\right)$
- EM field $\mathcal{L}_\vartheta = -\frac{1}{2}(\partial\vartheta)^2 - \frac{1}{2}m_\vartheta^2\vartheta^2$.
- $\mathcal{L}_{\vartheta F\tilde{F}} = -\frac{\alpha_\gamma}{4}\vartheta F_{\mu\nu}\tilde{F}^{\mu\nu}$, $\mathcal{L}_{\vartheta R\tilde{R}} = \frac{\alpha_g}{4}\vartheta R^{\beta}{}_{\alpha\gamma\delta}\tilde{R}^{\alpha}{}_{\beta}{}^{\gamma\delta}$.

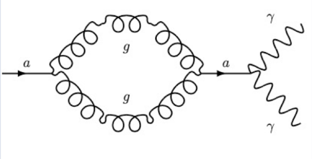
Equation of motion

- $\Box h_{ij} =$
  $\kappa_4\alpha_g\tilde{\epsilon}^{pk}{}_{(i}\left[\dot{\vartheta}(\partial_p\Box h_{j)k}) - \ddot{\vartheta}(\partial_p\partial_t h_{j)k})\right] - 2\kappa_4\left(T_{ij}^{(\gamma)} + T_{ij}^{(\vartheta)}\right)$
- axion like field $(\Box - m_\vartheta^2)\vartheta = \frac{\alpha_\gamma}{4}F\tilde{F} - \frac{\alpha_g}{4}R\tilde{R}$
- Polarization $\nabla_\mu F^{\mu\nu} = -\alpha_\gamma\partial_\mu\vartheta\tilde{F}^{\mu\nu}$

Branch Ratio of GWs and EMs

- The triangle Feynman diagram: where the axion-photon coupling is generated from Chern-Simon gravity coupling.
- $\mathcal{L}_{\vartheta F\tilde{F}} = -\frac{\alpha_\gamma}{4}\vartheta F_{\mu\nu}\tilde{F}^{\mu\nu}, \quad \mathcal{L}_{\vartheta R\tilde{R}} = \frac{\alpha_g}{4}\vartheta R^{\beta}{}_{\alpha\gamma\delta}\tilde{R}^{\alpha}{}_{\beta}{}^{\gamma\delta}.$



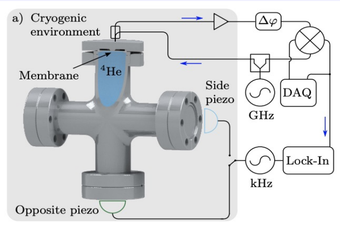


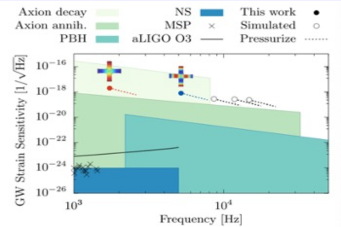
- The triangle diagram is divergent as $\alpha_\gamma \sim \alpha_g(\Lambda_{cs}/M_{pl})^4$, where Λ_{cs} is the cut-off for Chern-Simons theory.
- Two powers of M_{pl} from $h_{\mu\nu}T^{\mu\nu}$ coupling.

Parametric resonance and GW amplification

Branch Ratio and GWs

- $\frac{\mathrm{Br}(\vartheta\to gg)}{\mathrm{Br}(\vartheta\to\gamma\gamma)} \simeq \frac{\alpha_g^2}{\alpha_\gamma^2} \simeq \left(\frac{M_{pl}}{\Lambda_{cs}}\right)^8$, (Power of FRB $P_{(\gamma)} \sim 10^{42}$ergs/s).
- High frequency $h_{(g)} \sim 10^{-26}\left(\frac{1\mathrm{GHz}}{\nu}\right)\left(\frac{P_{(g)}}{P_{(\gamma)}}\right)^{1/2}\left(\frac{1\mathrm{kpc}}{L}\right)$
- Low freq. $h_{(g)} \sim 10^{-21}\left(\frac{10^{-2}\mathrm{Hz}}{\nu}\right)^{1/2}\left(\frac{M_{\mathrm{BH}}}{10^7 M_\odot}\right)^{1/2}\left(\frac{1\mathrm{kpc}}{L}\right)$



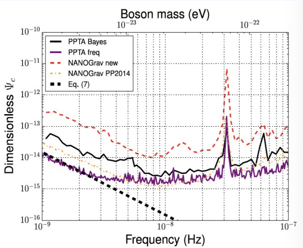


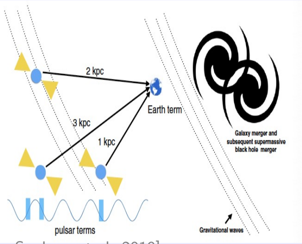
cf. PRD'21, S. Sun, Y. L. Zhang, Gravitational Wave Burst from Axion Clumps.
PRD'21, V. Vadakkumbatt et al, Prototype superfluid gravitational wave detector.

Ultra light dark matter and pulsar timing

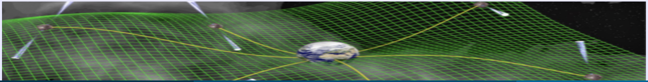
Ultra light dark matter

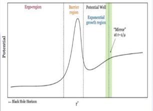
- PTA & SKA: ultra light DM($\sim$ nHz)
- $\lambda_{\rm dB} = \dfrac{2\pi\hbar}{mv} \simeq 4{\rm kpc}\left(\dfrac{10^{-23}{\rm eV}}{m}\right)\left(\dfrac{10^{-3}}{v}\right)$
- $f_c = \frac{m}{\pi} \simeq 4.8\,{\rm nHz}\left(\frac{m}{10^{-23}{\rm eV}}\right)$

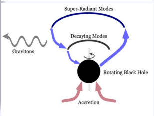


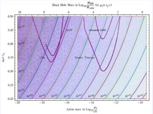


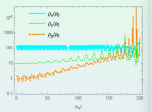
[cf. X. Xue, X. J. Zhu et al. 2018] & [cf. Burke-Spolaor, et al. 2019]

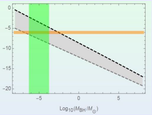




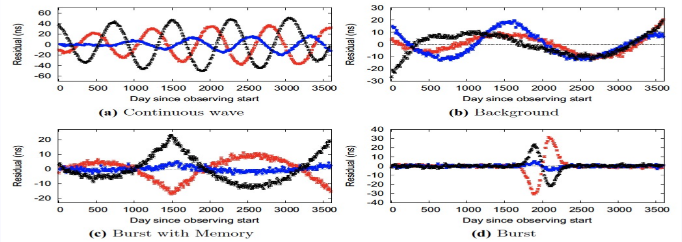


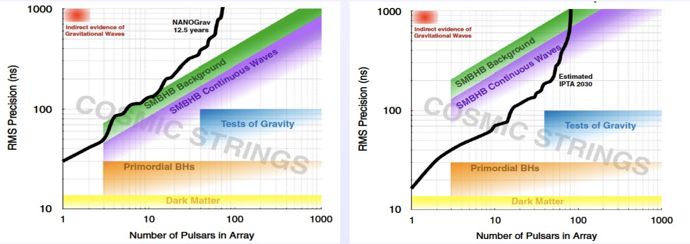






The astrophysics of nanohertz gravitational waves





[cf. Burke-Spolaor, *et al.*, "The astrophysics of nanohertz gravitational waves"]

DM oscillation induced time residual

- Metric: $ds^2 = -(1+2\Phi)\,\mathrm{d}t^2 + [(1-2\Psi)\,\delta_{ij} + h_{ij}]\,\mathrm{d}x^i\mathrm{d}x^j$.
- e.g. the scalar field $\phi(x,t) = \phi(x)\cos[mt + \theta_0(x)]$,
- Oscillating potential $\Psi \simeq \bar{\Psi}(x) + \Psi_\phi \cos[2\,(mt + \theta_0(x))]$
- Doppler effect: $z_\phi(t) \equiv \frac{\omega_0 - \omega_\phi(t)}{\omega_0} \simeq \Psi(x_\phi, t_\phi) - \Psi(x_0, t_0)$.
- Timing residual in the pulse $R_\phi(t) = \int_0^t z_\phi(t')dt'$
- Strain $h_\phi = 2\sqrt{3}\,\Psi_\phi = \frac{\sqrt{3}}{4M_{pl}^2}\frac{\rho_\phi}{m^2} \simeq 5.2 \times 10^{-17}\alpha_0 \left(\frac{f_{yr}}{f}\right)^2$,
- GW Timing residual: $R_c(f) \equiv \sqrt{\frac{S_c(f)}{T_s}} = \frac{1}{\sqrt{3}}\frac{h_c(f)}{2\pi f}\left(\frac{f_s}{f}\right)^{1/2}$

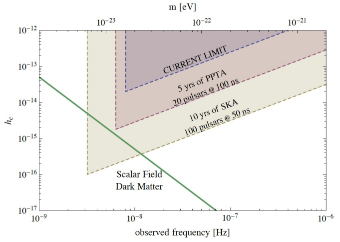
[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014)]

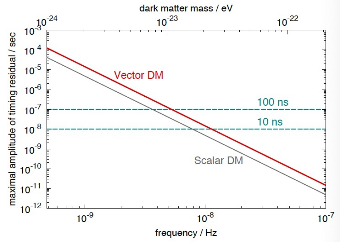
Pulsar timing residual and ultralight dark matter

DM oscillation induced time residual

- Spin-0: massive scalar field $\mathcal{L}_{(0)} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$
- Spin-1: massive vector field $\mathcal{L}_{(1)} = -\frac{1}{4}F^2 - \frac{1}{4}m^2A^2$

[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014)]



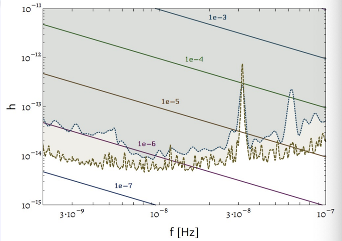


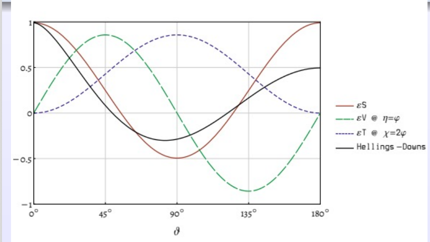
[cf. Nomura-Itoy-Soda, "Pulsar timing residual induced by ultralight vector DM" PRD(2020)]

Pulsar timing constraints on spin-2 ULDM

spin-2 ultralight fields

- Spin-2: massive tensor field(Fierz-Pauli): Bi-metric gravity,
$$\mathcal{L}_{(2)} = \frac{1}{2} M_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} M_{\rho\sigma} - \frac{1}{4} m^2 \left(M_{\mu\nu} M^{\mu\nu} - M^2 \right)$$
- The oscillating solution $M_{ij} = \mathcal{M} \cos\left[mt + \theta_2(x)\right] \varepsilon_{ij}$
- Effective metric perturbations: $\tilde{g}_{ij} = \delta_{ij} + \frac{\alpha_2}{M_{pl}} M_{ij}$
- The redshift $z(t) = \frac{\omega(t) - \omega_0}{\omega_0} = \frac{\alpha_2}{2M_{pl}} \int dt \omega_0 \partial_t M_{ij} n^i n^j$





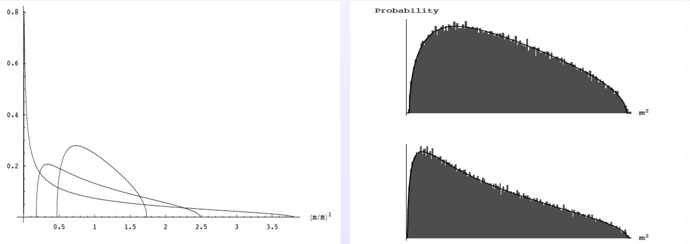
[cf. Armaleo-Nacir-Urbanb, "Pulsar timing array constraints on spin-2 ULDM", JCAP(2020)]

Multi fields: Marcenko-Pastur distribution

Mass spectrum and ultralight fields

- Marcenko-Pastur: $P_M(m^2) = \frac{\sqrt{(m^2 - m_-^2)(m_+^2 - m^2)}}{2\pi\beta m_0^2 m^2}$,
- Energy density: $\rho_\phi \equiv \int dm \tilde{\rho}(m) = \int dm \frac{1}{2} m^2 \tilde{\phi}(m)^2 P(m)$.
- Convenient choice: $\tilde{\rho}(m) \simeq \rho_\phi P(m), \quad \int dm P(m) = 1$.

[cf. Marcenko-Pastur, "Distributions of Eigenvalues for Some Sets of Random Matrices,"(1967)]

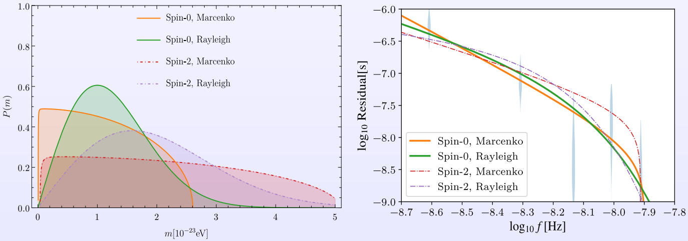


[cf. Easthera-McAllister, "Random Matrices and the Spectrum of N-flation" JCAP(2006)
Cai-Hu-Piao, "Entropy Perturbations in N-flation" PRD(2009)]

Our phenomenological fitting results

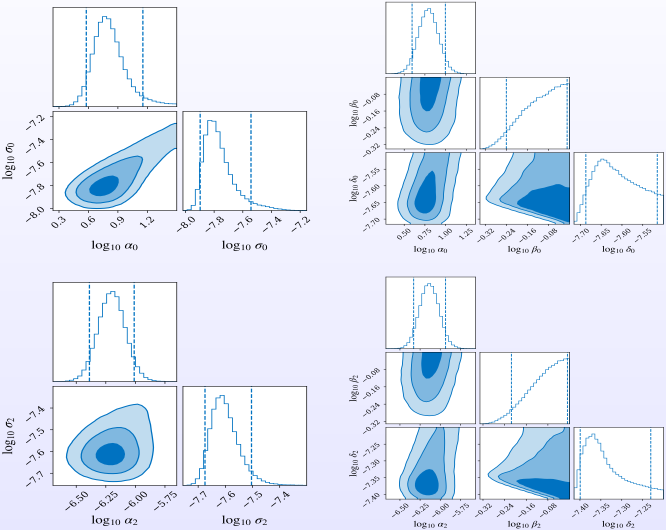
Mass spectrum and ultralight fields

- Marcenko-Pastur: $P_M(m^2) = \frac{\sqrt{(m^2 - m_-^2)(m_+^2 - m^2)}}{2\pi\beta m_0^2 m^2}$,
- Rayleigh distribution: $P_\sigma(m) = \frac{m}{\sigma^2} e^{-\frac{m^2}{2\sigma^2}}$.



[Sun-Yang-Zhang, PRD(2022) "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter"]

Corner Figures of Bayesian Fitting



Our phenomenological fitting results

The effective strain

- $h_c^{\phi}(f) = \frac{\alpha_0}{M_{pl}^2}\frac{\sqrt{3}\rho_{DM}}{4\pi f}P(\pi f)$
- $h_c^{M}(f) = \frac{\alpha_2}{M_{pl}}\frac{m\mathcal{M}P(m)}{\sqrt{5}} = \frac{\alpha_2}{M_{pl}}\frac{2\sqrt{\rho_M}}{\sqrt{5}}P(2\pi f).$

	Parameters	spin-0	spin-1	spin-2
Marcenko	α_i	$5.9^{+1.9}_{-1.3}$	$\sim 3\alpha_0$	$7.6^{+2.2}_{-1.7}\times 10^{-7}$
Marcenko	$m_-^i/(10^{-23}\text{eV})$	$2.9^{+3.6}_{-0.3}\times 10^{-3}$	$\sim \delta_0(1-\sqrt{\beta_0})$	$6.3^{+6.0}_{-1.7}\times 10^{-3}$
Marcenko	$m_+^i/(10^{-23}\text{eV})$	$2.61^{+0.21}_{-0.01}$	$\sim \delta_0(1+\sqrt{\beta_0})$	$5.08^{+0.02}_{-0.01}$
Rayleigh	α_i	$5.6^{+3.8}_{-1.0}$	$\sim 3\alpha_0$	$6.1^{+2.1}_{-1.3}\times 10^{-7}$
Rayleigh	$\sigma_i/(10^{-23}\text{eV})$	$1.0^{+0.4}_{-0.1}$	$\sim \sigma_0$	$1.6^{+0.3}_{-0.1}$

[Sun-Yang-Zhang, PRD(2022), "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter"]

Summary and Outlook

Black Hole Superradiance & Gravitational waves

- Superradiance $\alpha \equiv \frac{R_{BH}}{\lambda_\vartheta} \simeq \left(\frac{M_{BH}}{M_\odot}\right)\left(\frac{m_\vartheta}{10^{-10}\text{eV}}\right)$
- Axion annihilation $\vartheta + \vartheta \to h$ (stochastic GW)
  Energy transition $\vartheta^+ \to \vartheta^- + h$ (monochromic)
- Fast Radio Burst from Axion $\sim \vartheta F\tilde{F}$ $(\vartheta \to \gamma\gamma)$
- Fast GW burst from Axion $\sim \vartheta R\tilde{R}$ $(\vartheta \to hh)$

Ultra-light DM and multi band GW detection

- Tabletop exp: GW burst & Axion clump ($\sim$ GHz)
- LVK & LISA-Taiji: Superradiance ($\sim$ mHz - kHz)
- PTA & SKA : Ultra-light dark matter ($\sim$ nHz)

Thanks a lot for your attention!