Supersymmetry

Beyond the Standard Model

Chiung Hwang


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- The Higgs boson (1964, 2012) was the last piece of the standard model, describing three fundamental forces in nature.

- Gravitational waves (1916, 2015) are ripples of spacetime, proving that spacetime is really dynamical and is the source of gravity.

These two events complete two major pillars of 20c physics:

Quantum Field Theory and General Relativity.

What’s next?

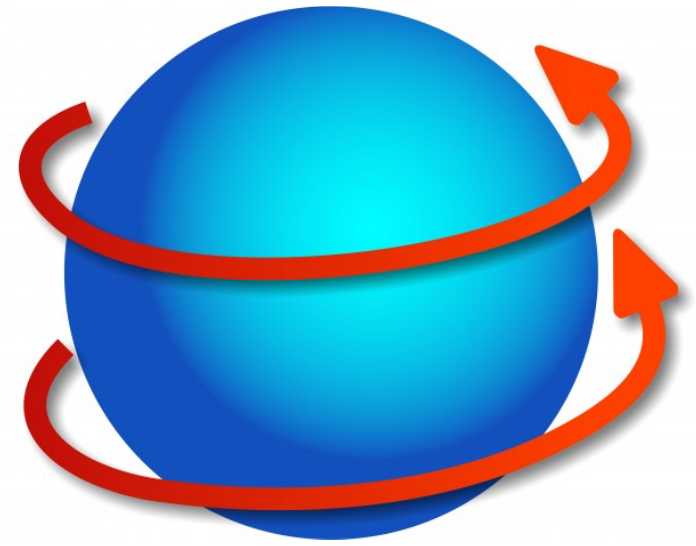
What’s next?

Supersymmetry

What Is Supersymmetry?

- A symmetry between boson (integer spin) and fermion (half-integer spin).

$S = 1$



$S = \frac{1}{2}$

- Since spin is a representation of the spacetime symmetry, supersymmetry inevitably mixes with that of spacetime.

Supersymmetry $\longleftarrow$ $QQ^\dagger + Q^\dagger Q = 2H$ $\longrightarrow$ Time translation

- For 4-dimensional Minkowski spacetime,

$$\left\{ Q_a , Q^\dagger_{\dot{a}} \right\} = -\, 2\, \sigma^\mu_{a\dot{a}}\, P_\mu$$

- How does it act on fields?
- E.g., for a scalar field A and a fermion field ψ_a,

$$\left[A , Q_a \right] = -\, i \sqrt{2}\, \psi_a \,, \quad \left\{ \psi_a , Q^\dagger_{\dot{a}} \right\} = -\sqrt{2}\, \sigma^\mu_{a\dot{a}}\, \partial_\mu A$$

- Leads to a theory with symmetry between bosonic spectrum and fermionic spectrum.

- Why interesting?

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Euclidean symmetry

- Translations, rotations
- Euclidean space+time
- Newtonian mechanics

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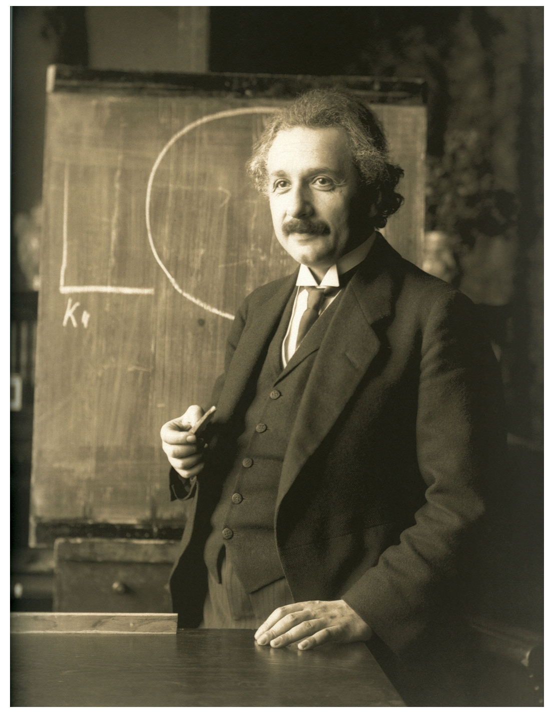
Poincare symmetry

- Translations, rotations + boosts
- Minkowskian spacetime
- (Special) Relativity



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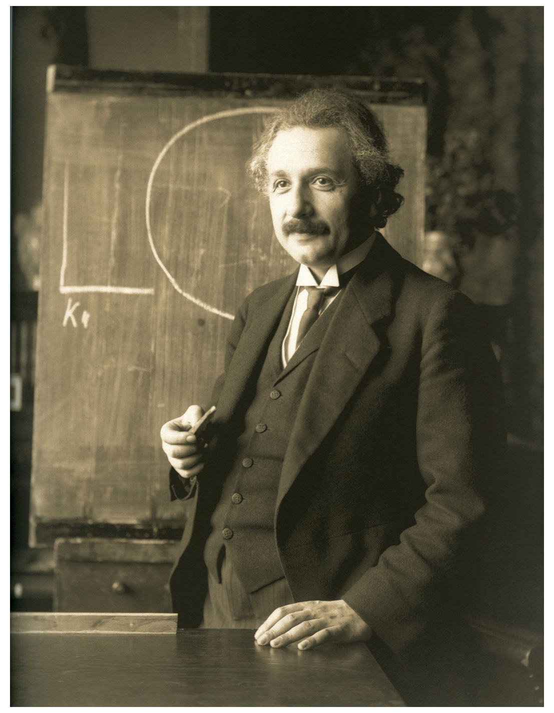

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Super-Poincare symmetry

- Translations, rotations, boosts + supersymmetry
- Superspace
- Supersymmetric theories

- Why interesting?
- Haag–Łopuszański–Sohnius theorem (1975)

Euclidean symmetry

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- Translations, rotations + boosts
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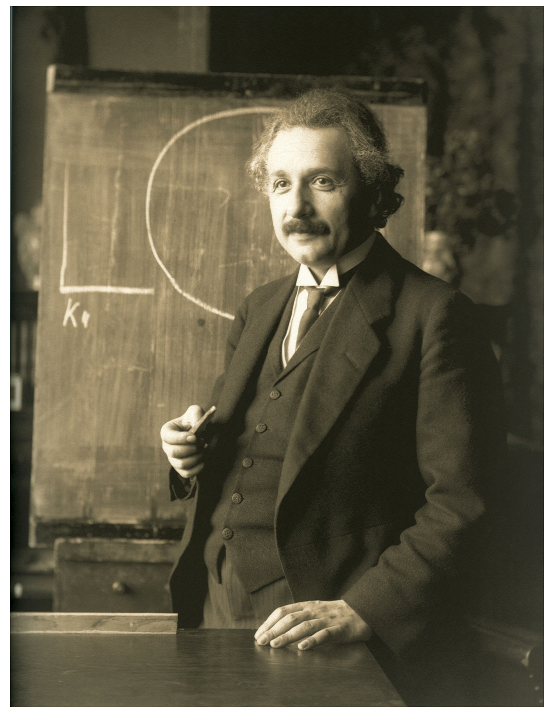


Super-Poincare symmetry

- Translations, rotations, boosts + supersymmetry
- Superspace
- Supersymmetric theories

Supersymmetry is the only possible extension of the Poincare symmetry!

Early History of SUSY

- 1971 Ramond, Neveu-Schwarz develop **string theory** with fermions and bosons.
- 1971 Gervais-Sakita show that this theory obeys **supersymmetry algebra in two dimensions**.
- 1971 Golfand-Likhtman extend the Poincare algebra into a superalgebra and discover **supersymmetry in four spacetime dimensions**.
- 1974 Wess-Zumino rediscover **supersymmetry in four spacetime dimensions**.
- 1975 Motivated by WZ, **Haag–Łopuszański–Sohnius** prove that the super-Poincare symmetry is the unique generalization of the Poincare symmetry.

Why do we need it?

Outline

- Part I: Why Supersymmetry?
- Part II: Application: From Higgs Mass to Black Hole Entropy

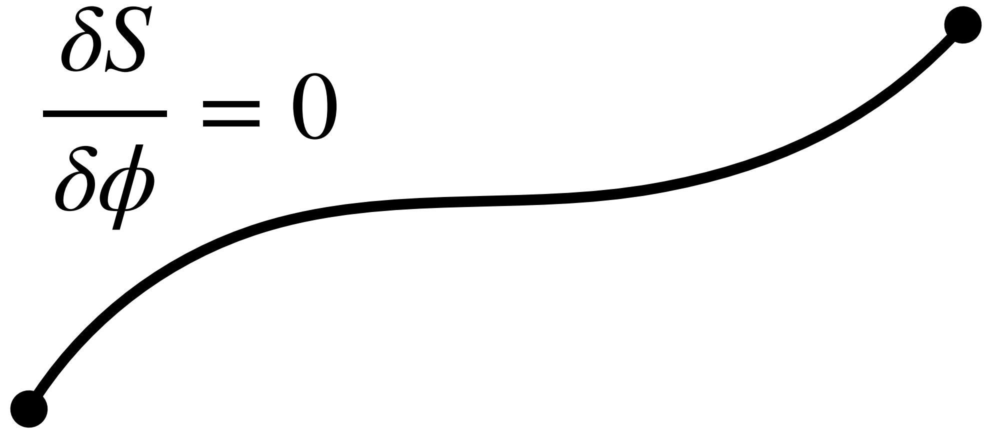
Part I

Why Supersymmetry?

Preliminary: Classical vs Quantum

Classical vs Quantum

$$S(\phi) = \int dt\, L(\phi) = \int d^4x\, \mathscr{L}(\phi)$$



$$\frac{\delta S}{\delta \phi} = 0$$

$$G(p) = \frac{1}{p^2 - m_0^2}$$

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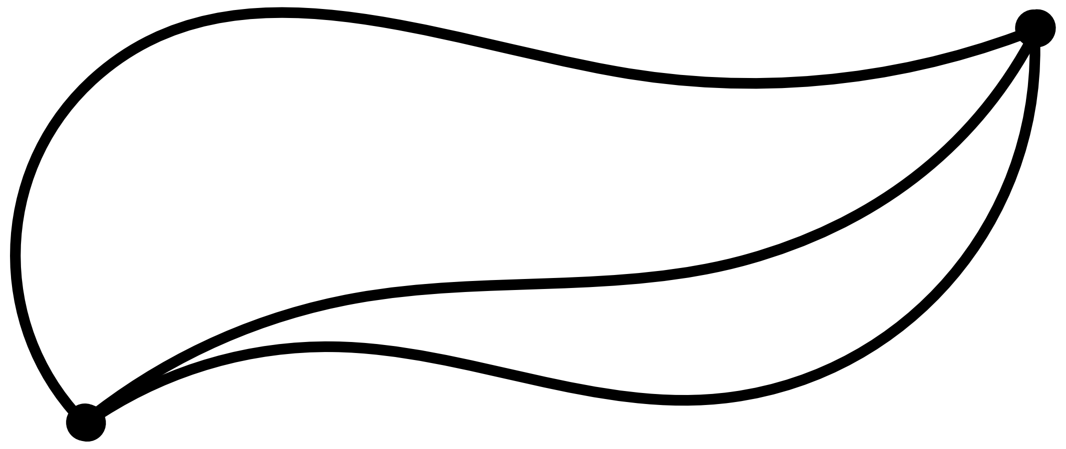
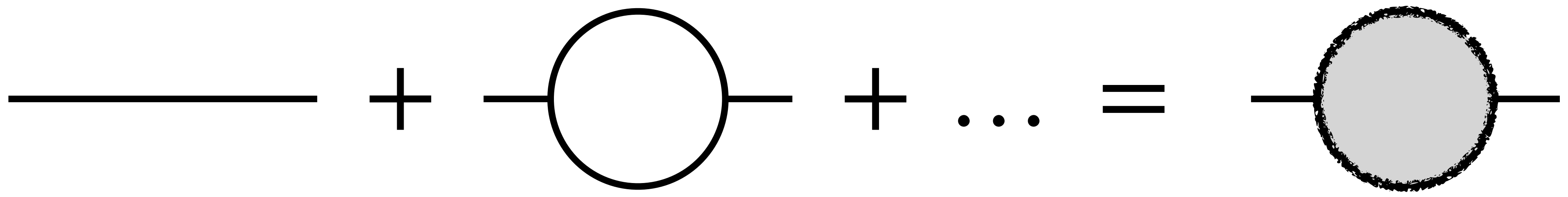


$$\int d\phi\, e^{iS(\phi)}$$

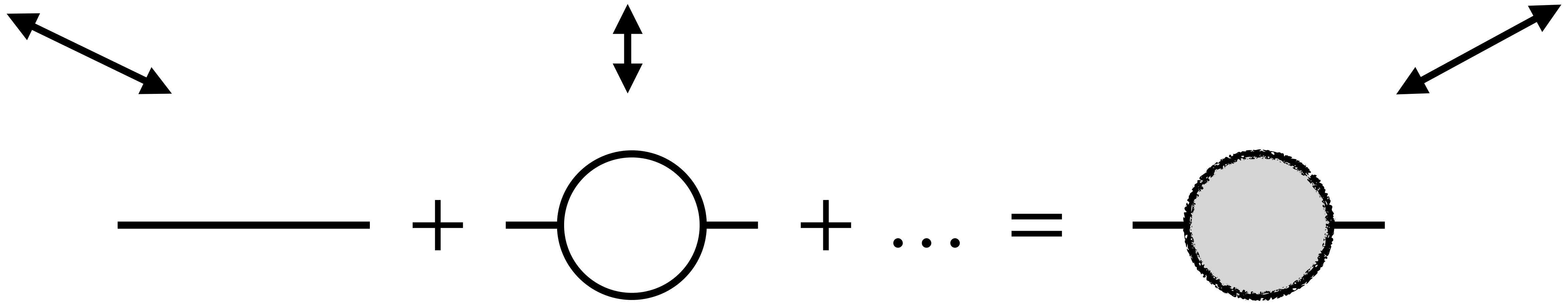
$$G(p) = \frac{1}{p^2 - m_0^2} + \int d^4 l\, \frac{g^2}{\left(p^2 - m_0^2\right)^2 \left(l^2 - m_0^2\right) \left((p+l)^2 - m_0^2\right)} + \dots$$

$$= \frac{1}{p^2 - m_{phy}^2}$$



- Propagator of the ϕ^3 theory → $\mathscr{L} = \frac{1}{2}(\partial\phi_0)^2 - \frac{1}{2}m_0^2\phi_0^2 - \frac{1}{3\,!}g_0\phi_0^3$

$$G(p) \quad = \quad \frac{1}{p^2 - m_0^2} + \int d^4l \, \frac{g^2}{\left(p^2 - m_0^2\right)^2 \left(l^2 - m_0^2\right)\left((p+l)^2 - m_0^2\right)} + \dots \quad = \quad \frac{1}{p^2 - m_{phy}^2}$$



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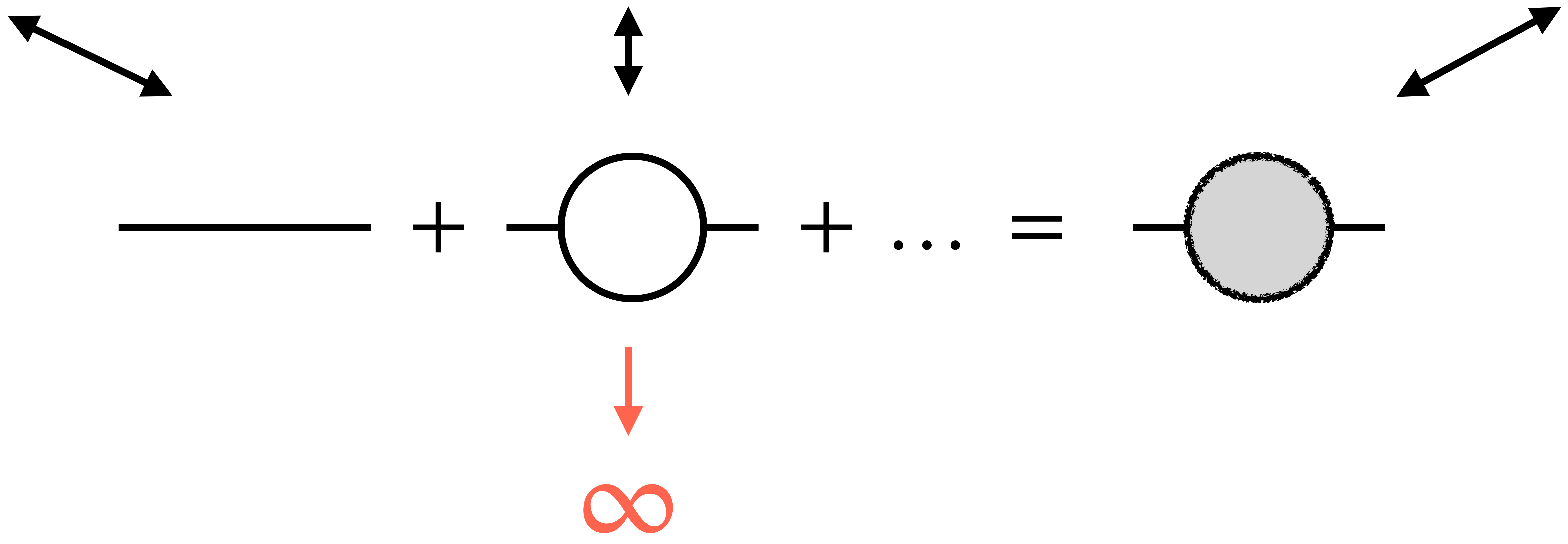

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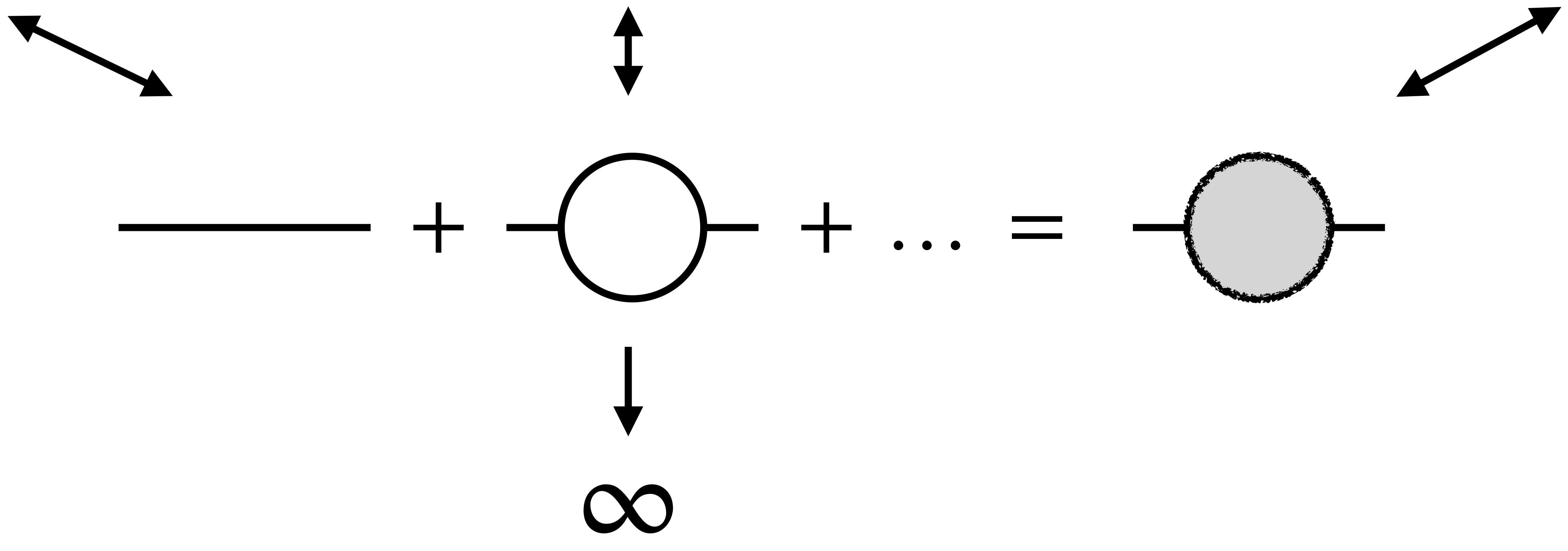

$$⇨ \qquad \underset{\substack{\downarrow \\ \textbf{Finite}}}{m_{phy}^2} = \underset{\substack{\downarrow \\ \infty}}{m_0^2} + \underset{\substack{\downarrow \\ \infty}}{\ldots}$$

$$\begin{array}{ccc} m_{phy}^2 & = m_0^2 & + \dots \\ \downarrow & \downarrow & \downarrow \\ \textbf{Finite} & \infty & \infty \end{array}$$

↔

Taylor series

$$\frac{1}{x} = \frac{1}{0} - \frac{x}{0^2} + \dots$$

$$\underset{\text{Finite}}{\underset{\downarrow}{m_{phy}^2}} = \underset{\infty}{\underset{\downarrow}{m_0^2}} + \underset{\infty}{\underset{\downarrow}{\ldots}}$$

$$\Longleftrightarrow$$

Taylor series

$$\cancel{\frac{1}{x} = \frac{1}{0} - \frac{x}{0^2} + \ldots}$$

$$\frac{1}{a + (x - a)} = \frac{1}{a} - \frac{x - a}{a^2} + \ldots$$

- **Renormalization**

$$\mathscr{L} = \frac{1}{2}(\partial\phi_0)^2 - \frac{1}{2}m_0^2\phi_0^2 - \frac{1}{3!}g_0\phi_0^3$$

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$$\Big\downarrow \qquad \phi_0 = \alpha\phi + \beta\,, \quad m_0 = \dots$$

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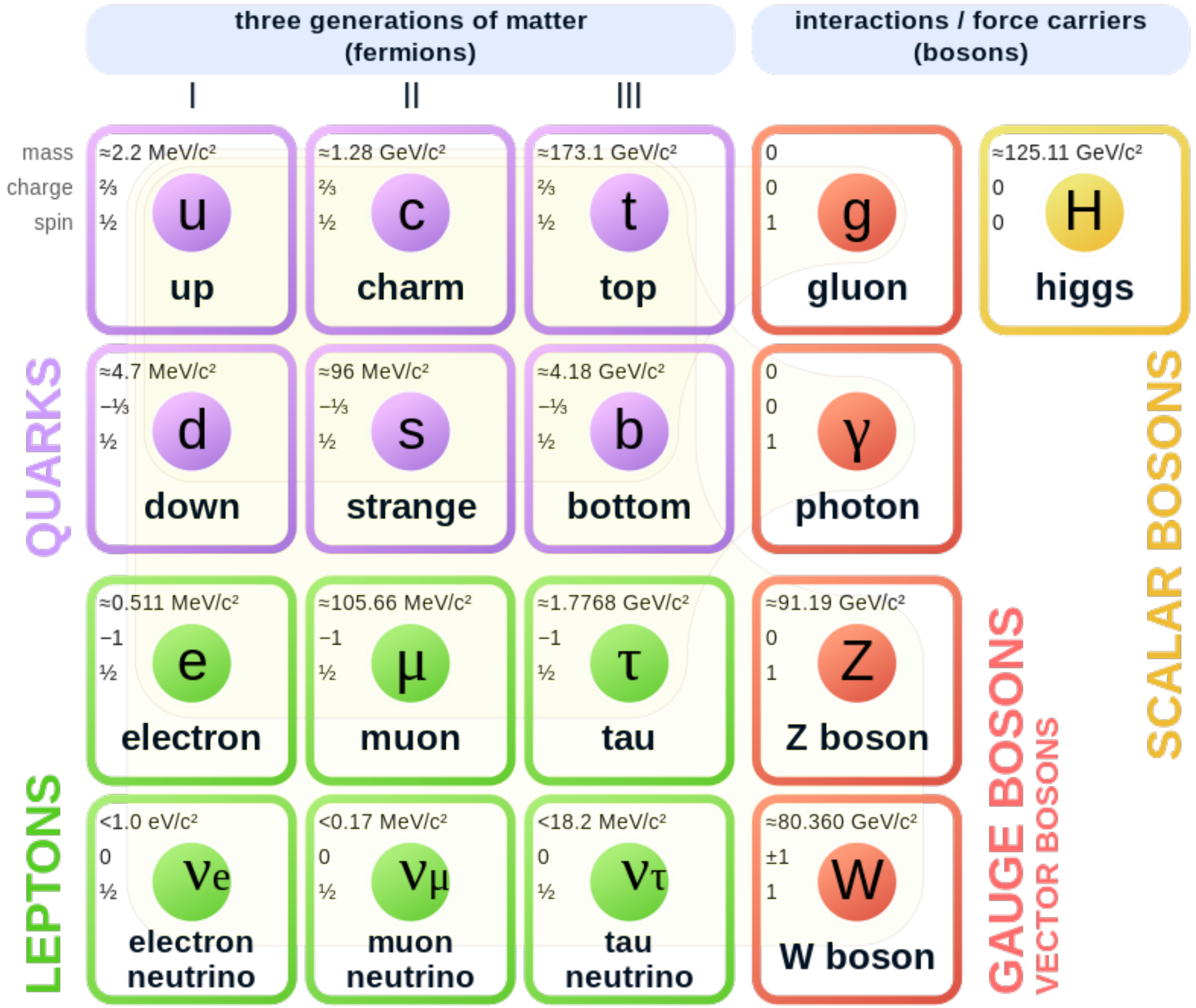
Counter-terms, tuned to cancel the infinities from the first line

Called *renormalizable* if a finite number of counter-terms are required

Standard Model of Elementary Particles



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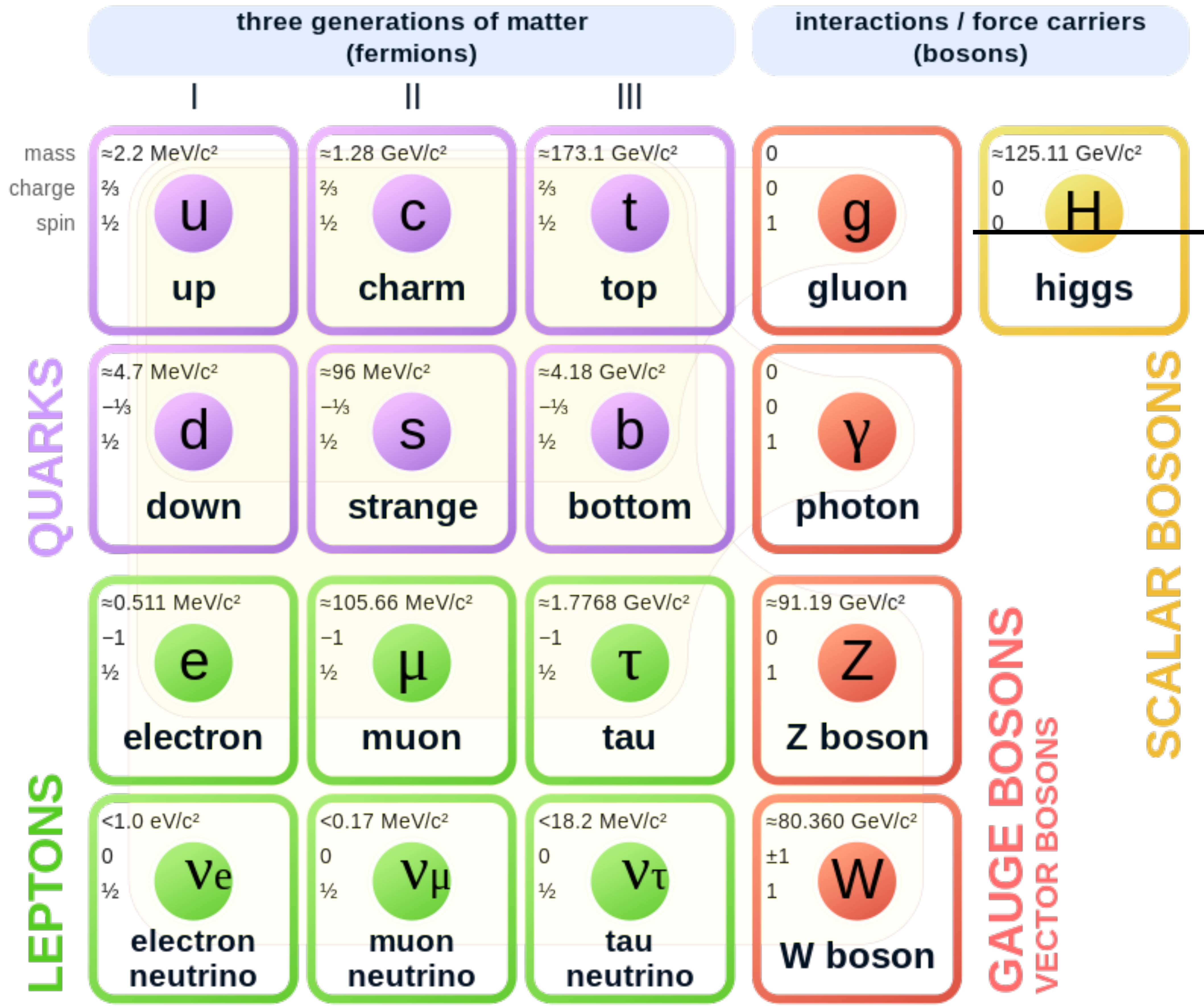
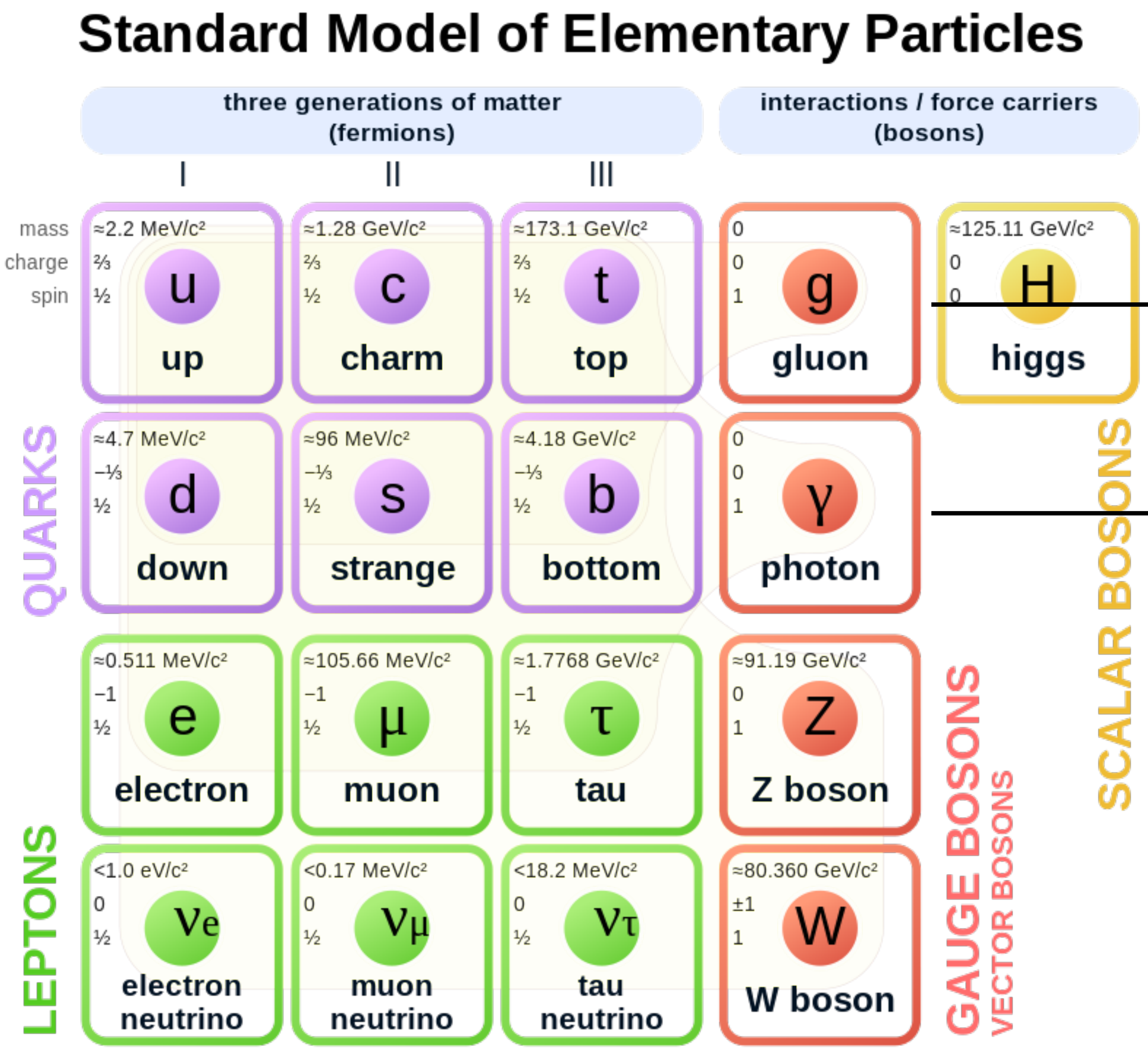


	three generations of matter (fermions)			interactions / force carriers (bosons)		
	I	II	III			
mass charge spin	≈2.2 MeV/c^2 ⅔ ½ u up	≈1.28 GeV/c^2 ⅔ ½ c charm	≈173.1 GeV/c^2 ⅔ ½ t top	0 0 1 g gluon	≈125.11 GeV/c^2 0 0 H higgs	SCALAR BOSONS
QUARKS	≈4.7 MeV/c^2 −⅓ ½ d down	≈96 MeV/c^2 −⅓ ½ s strange	≈4.18 GeV/c^2 −⅓ ½ b bottom	0 0 1 γ photon		
	≈0.511 MeV/c^2 −1 ½ e electron	≈105.66 MeV/c^2 −1 ½ μ muon	≈1.7768 GeV/c^2 −1 ½ τ tau	≈91.19 GeV/c^2 0 1 Z Z boson	GAUGE BOSONS VECTOR BOSONS	
LEPTONS	<1.0 eV/c^2 0 ½ ν_e electron neutrino	<0.17 MeV/c^2 0 ½ ν_μ muon neutrino	<18.2 MeV/c^2 0 ½ ν_τ tau neutrino	≈80.360 GeV/c^2 ±1 1 W W boson		

Strong force



Strong force

Electromagnetic force

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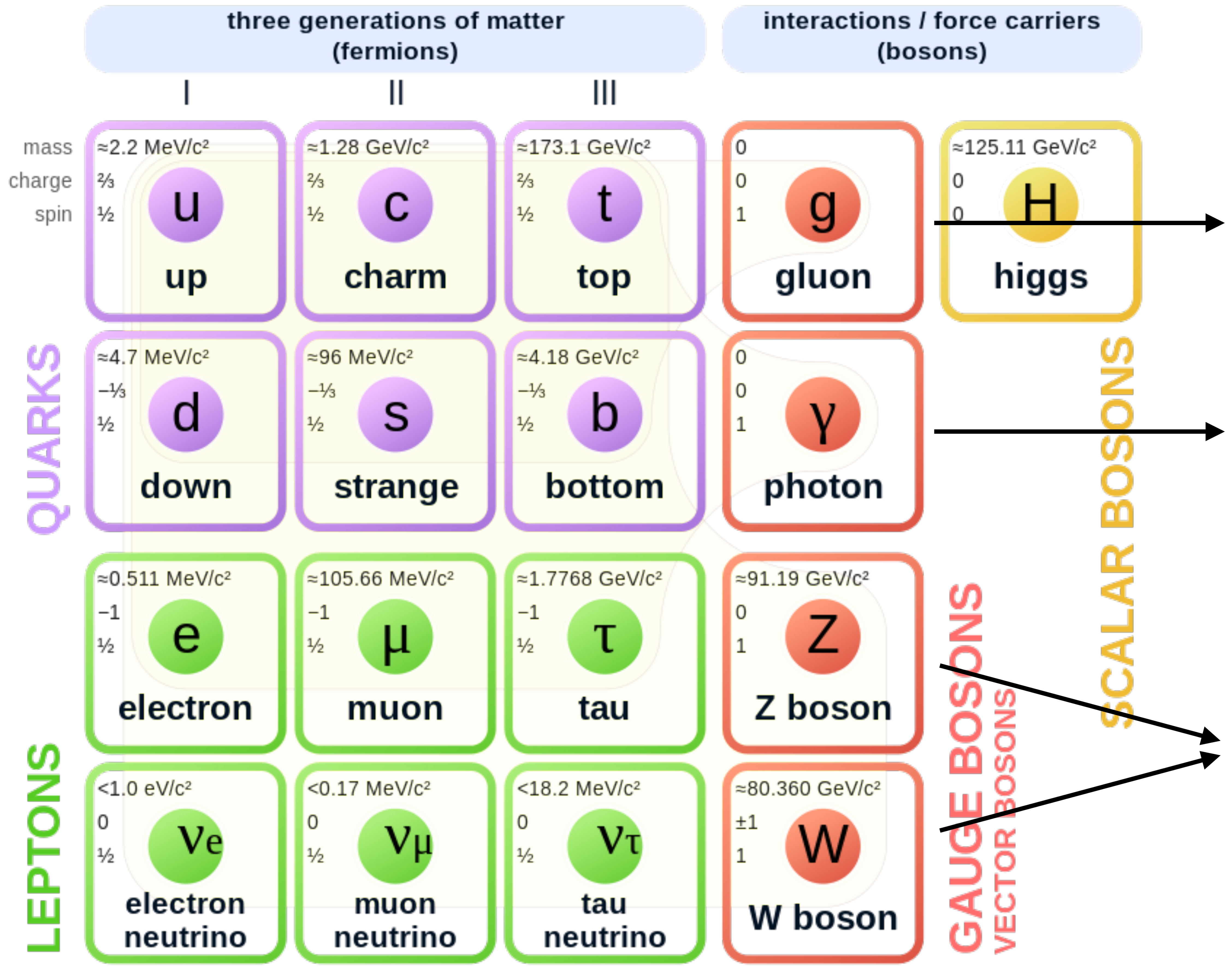

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GAUGE BOSONS
VECTOR BOSONS

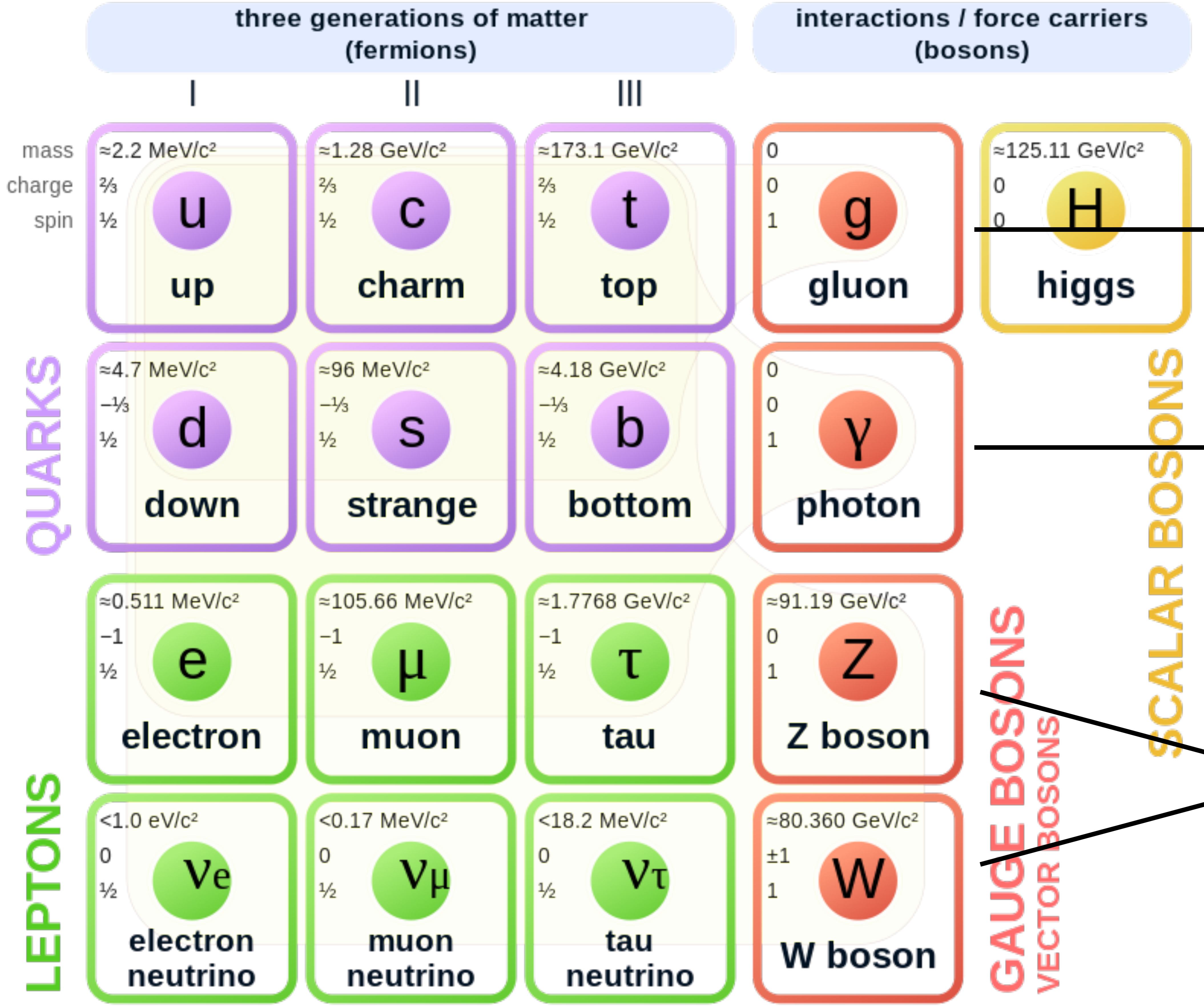
SCALAR BOSONS

Strong force

Electromagnetic force

Weak force

Standard Model of Elementary Particles



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Strong force

Electromagnetic force

Weak force

QFT is the most successful framework describing fundamental forces in nature.

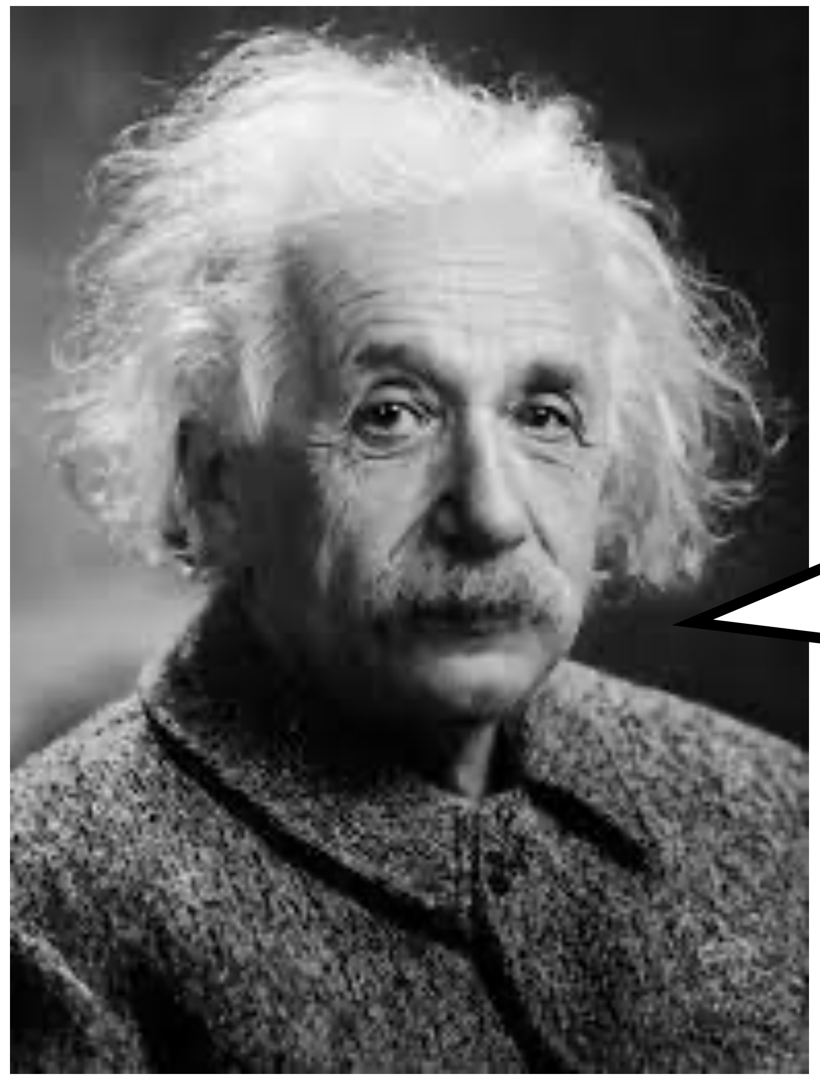
Even though the standard model and the QFT framework are so successful…

- *What about gravity?*
- *Other than gravity, is the standard model enough?*

Q1. What about gravity?

Gravity = Curved Spacetime

- Gravity is explained by Einstein's general relativity (1915), which teaches us the gravitational force originates from dynamical and curved spacetime.



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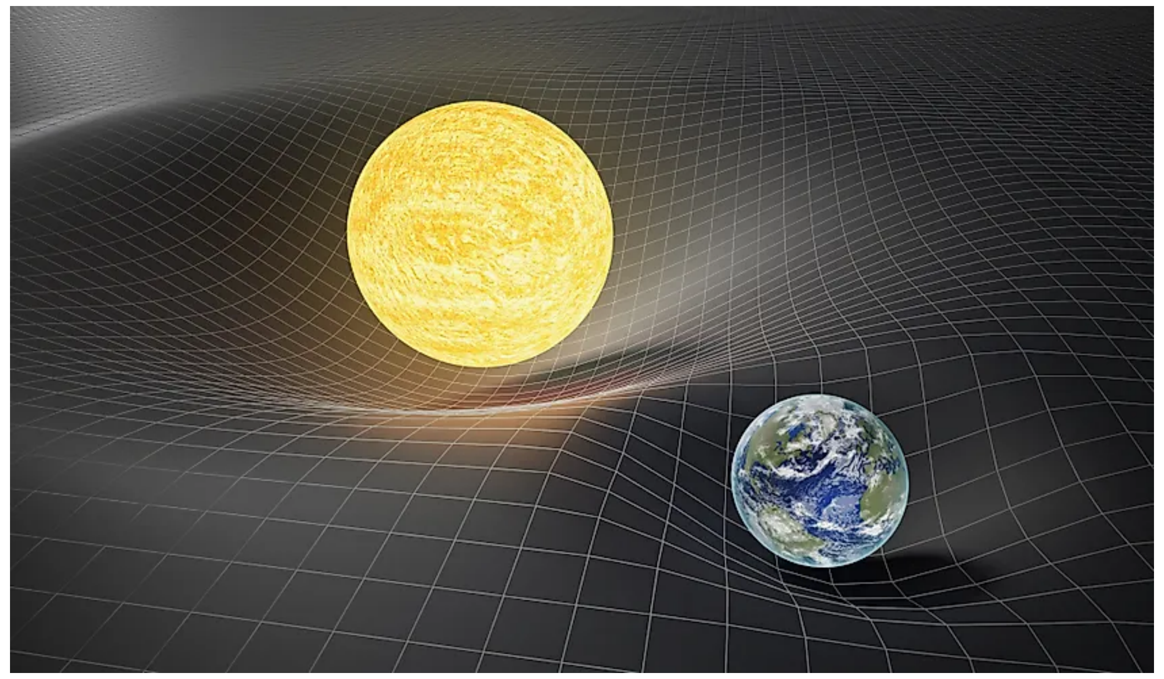
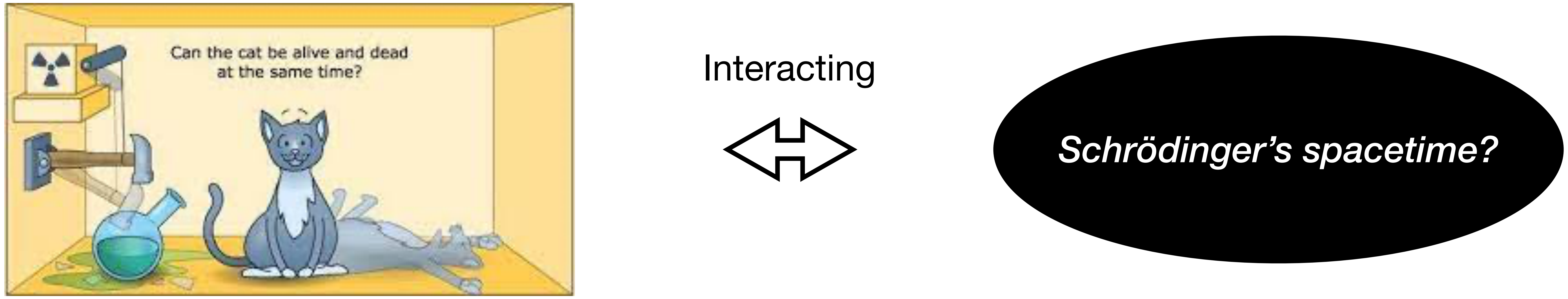


$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Gravity as QFT

- General relativity is a classical theory.
- On the other hand, spacetime interacts with matters, which are quantum —so is the spacetime?
- What is quantum spacetime?



Interacting

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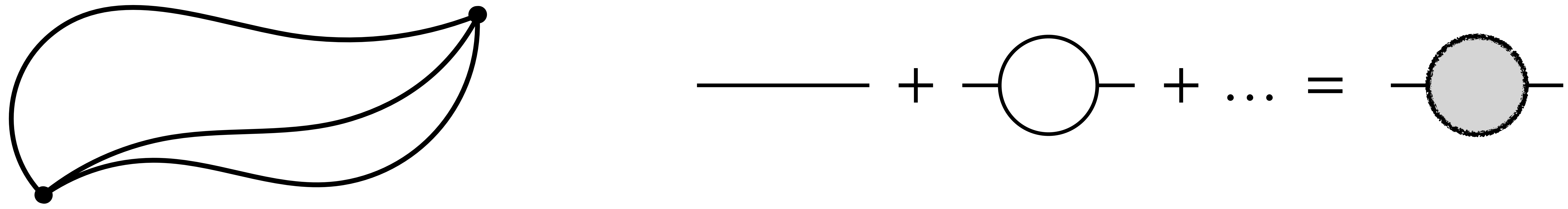
Schrödinger’s spacetime?

Gravity Is Non-renormalizable



- However, GR is non-renormalizable—there are infinitely many counterterms to be determined.

- No predictability in the high energy regime.

- Big bang? Black holes?

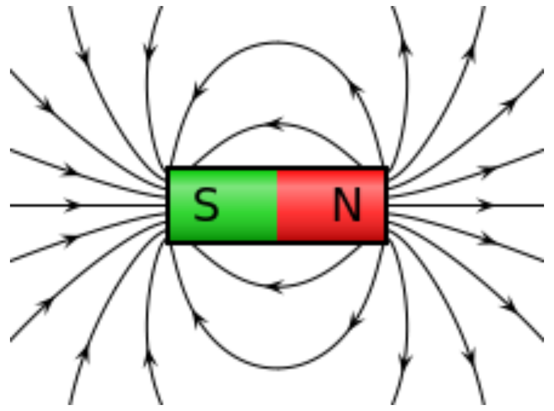
What do we learn from history?

Gravitational force = curved spacetime

History of forces: unification

Gravitational force = curved spacetime

Electric Force



Magnetic Force

Electric Force

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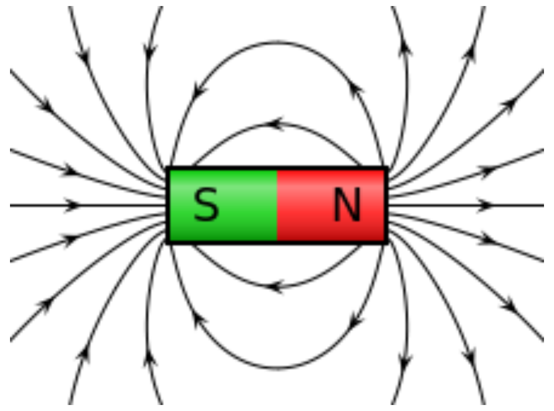
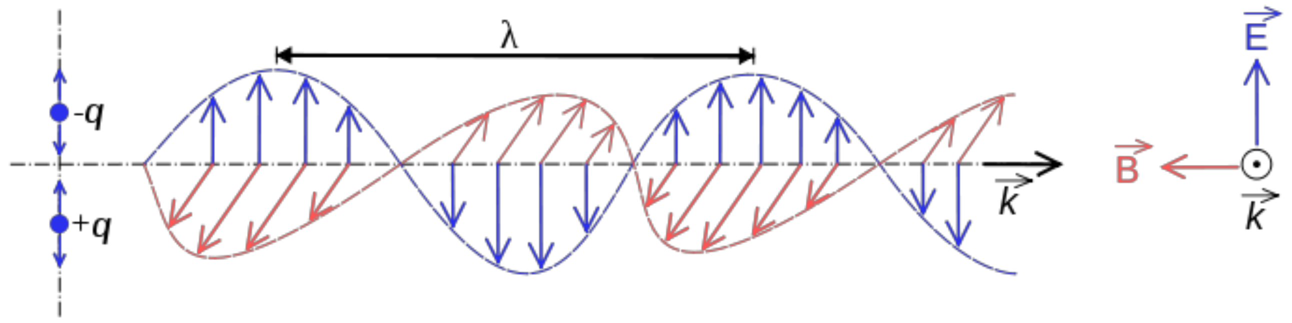


Magnetic Force

Electromagnetic Force



Electric Force

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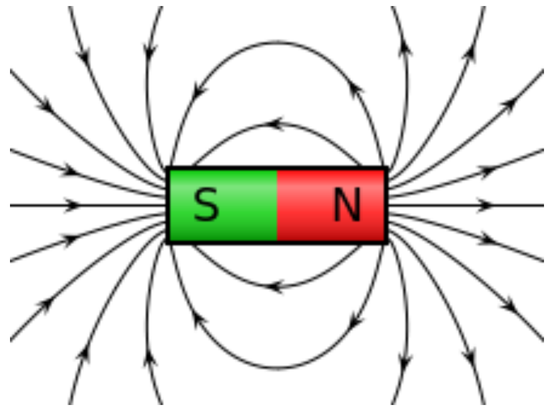

Magnetic Force

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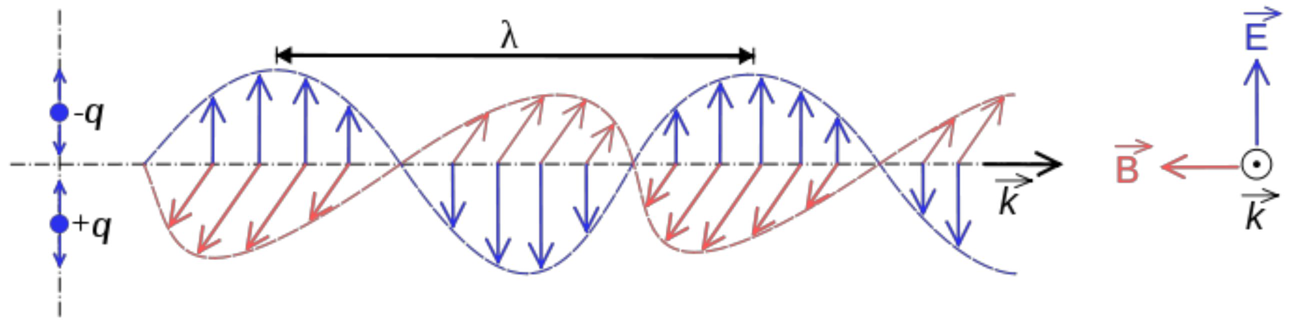
Electromagnetic Force



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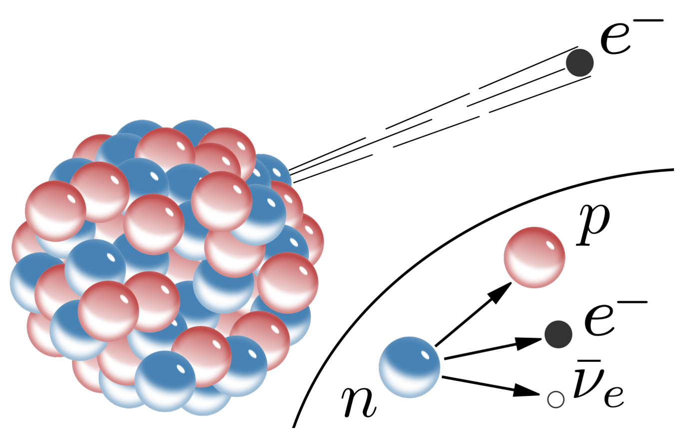

Weak Force

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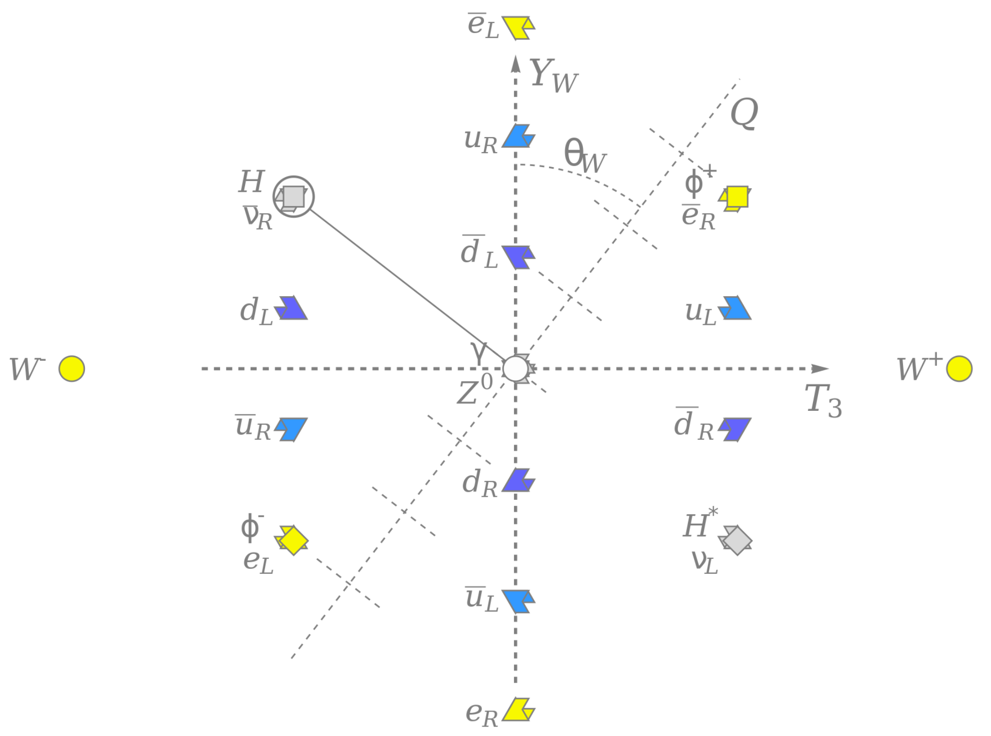
Electroweak Force



Electric Force

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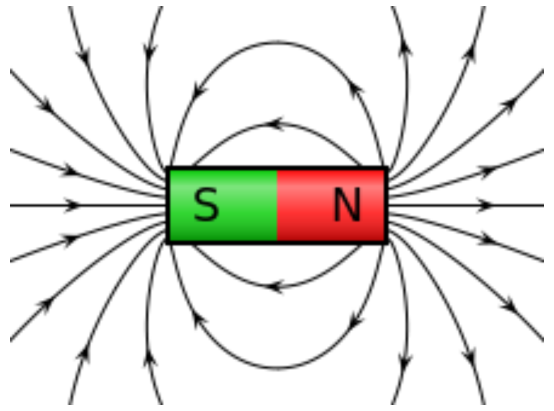
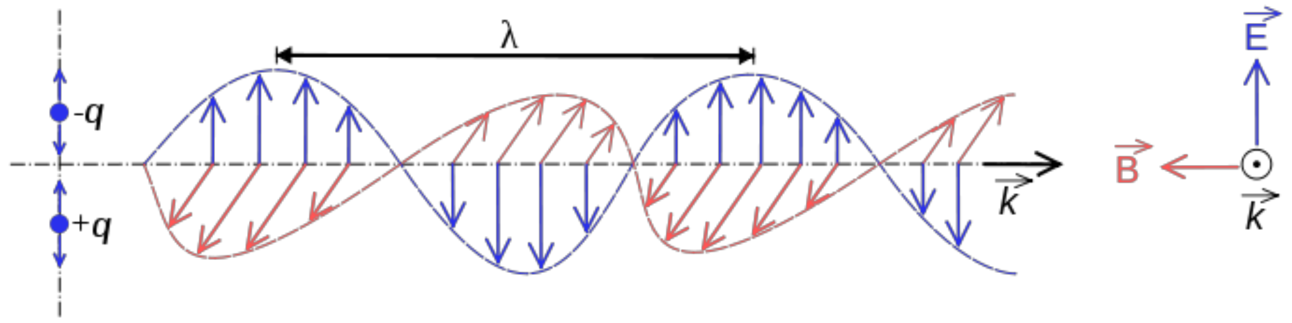


Magnetic Force

Electromagnetic Force



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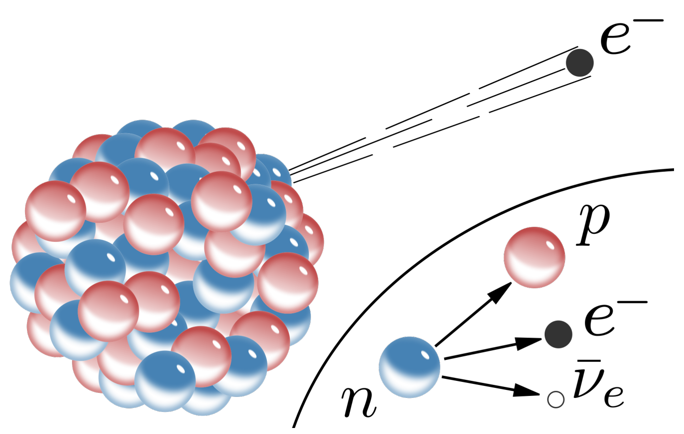
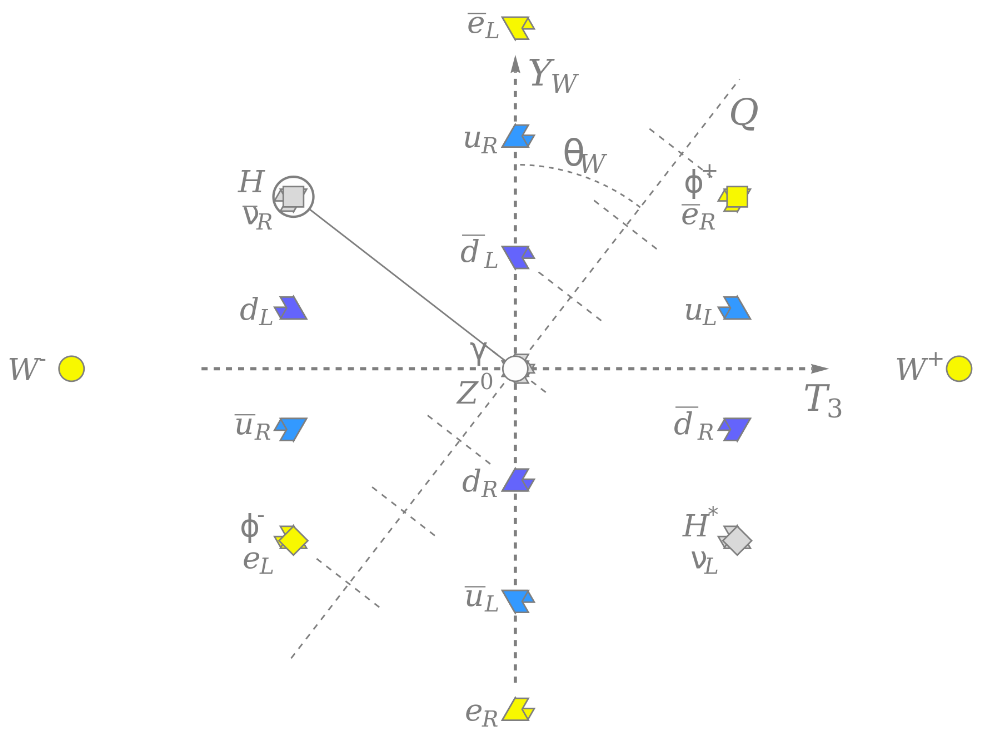


Weak Force

Electroweak Force



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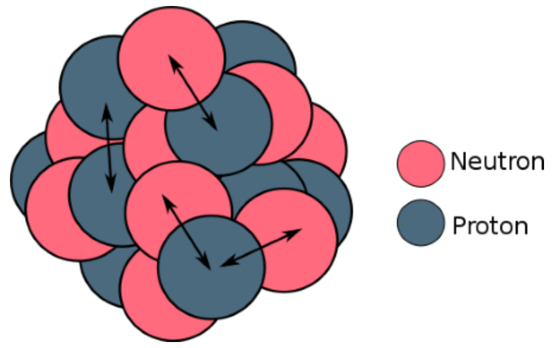
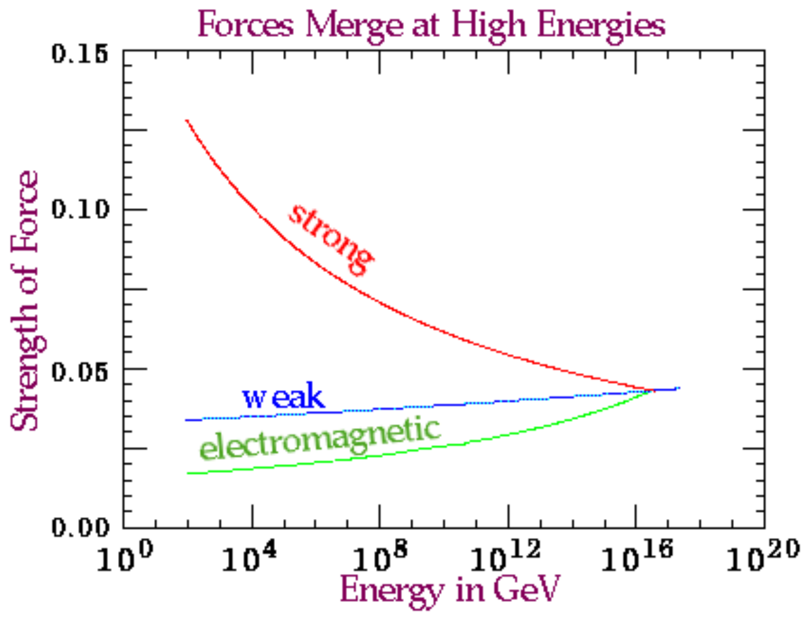
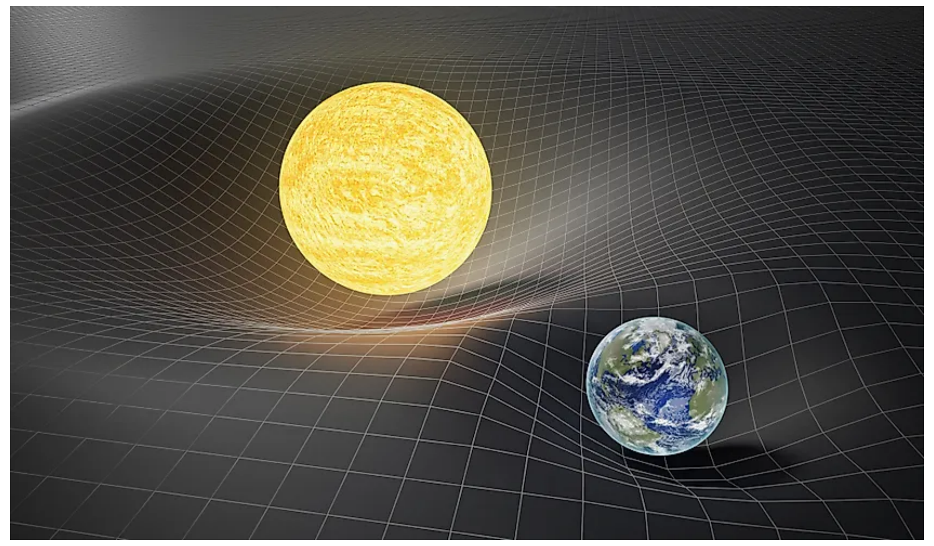


Strong Force



GUT?



Gravity = curved spacetime

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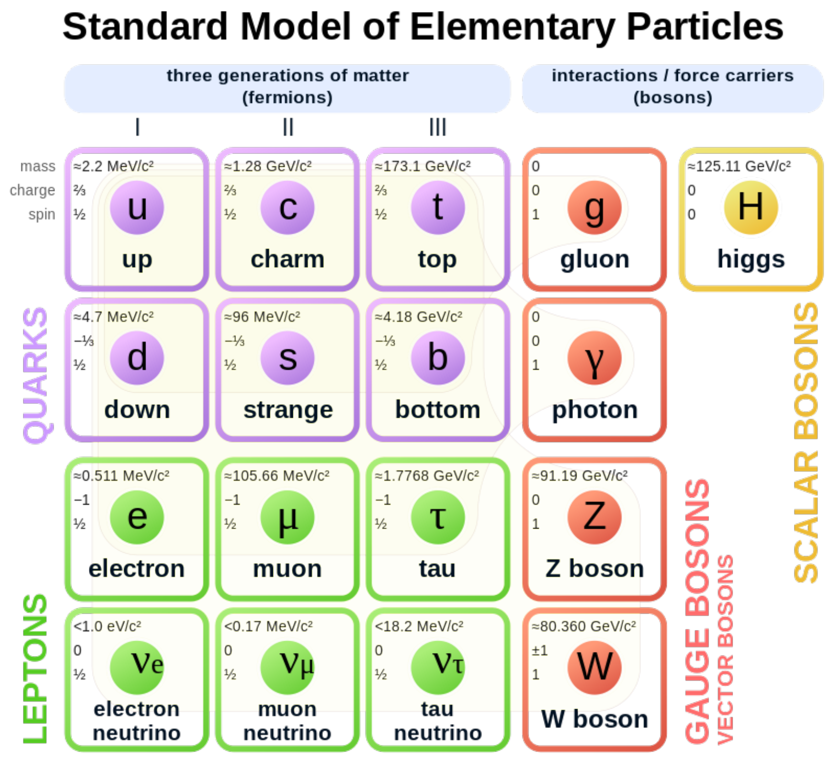

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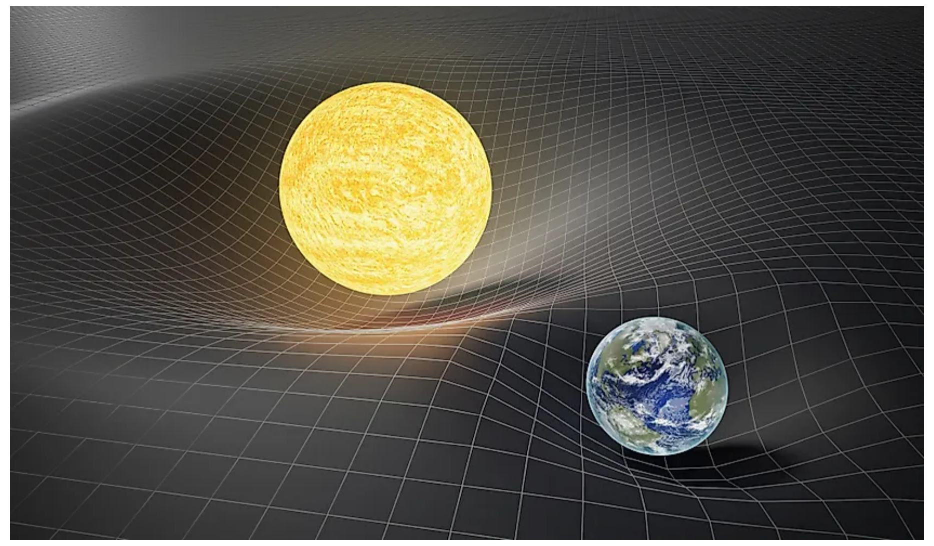
The other forces?



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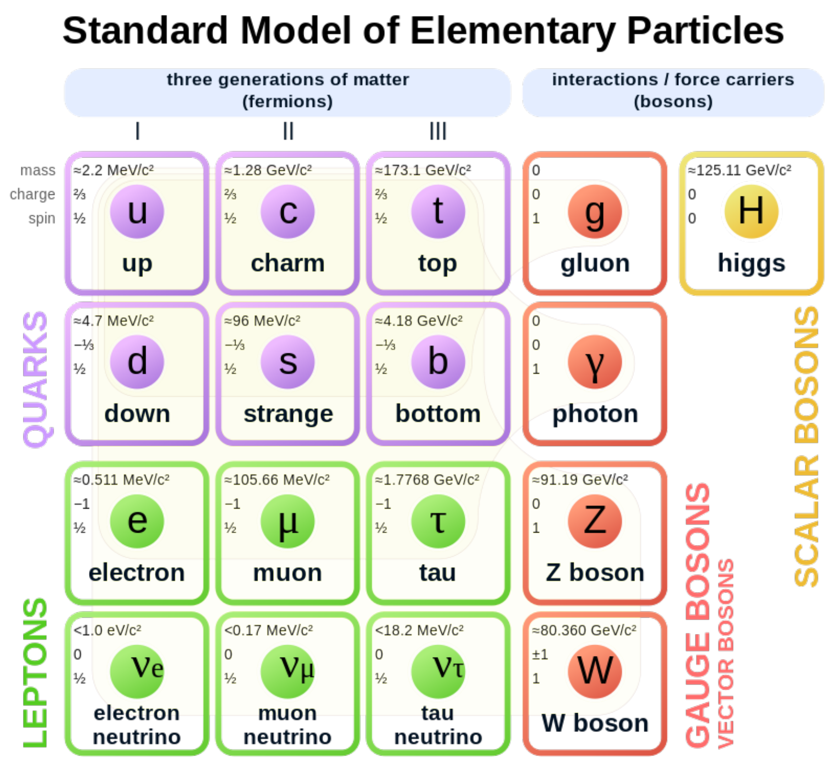
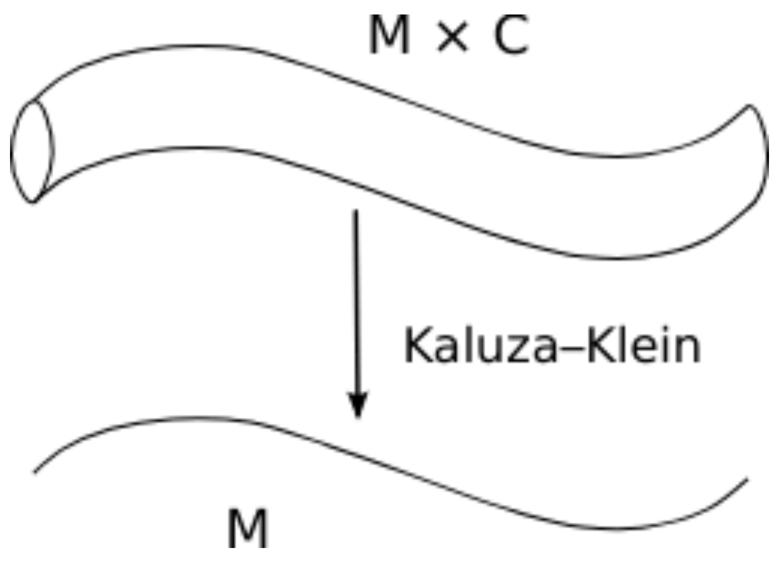


Standard Model of Elementary Particles

	three generations of matter (fermions)			interactions / force carriers (bosons)	
	I	II	III		
mass charge spin	≈2.2 MeV/c^2 ⅔ ½ u up	≈1.28 GeV/c^2 ⅔ ½ c charm	≈173.1 GeV/c^2 ⅔ ½ t top	0 0 1 g gluon	≈125.11 GeV/c^2 0 0 H higgs
QUARKS	≈4.7 MeV/c^2 −⅓ ½ d down	≈96 MeV/c^2 −⅓ ½ s strange	≈4.18 GeV/c^2 −⅓ ½ b bottom	0 0 1 γ photon	SCALAR BOSONS
	≈0.511 MeV/c^2 −1 ½ e electron	≈105.66 MeV/c^2 −1 ½ μ muon	≈1.7768 GeV/c^2 −1 ½ τ tau	≈91.19 GeV/c^2 0 1 Z Z boson	
LEPTONS	<1.0 eV/c^2 0 ½ ν_e electron neutrino	<0.17 MeV/c^2 0 ½ ν_μ muon neutrino	<18.2 MeV/c^2 0 ½ ν_τ tau neutrino	≈80.360 GeV/c^2 ±1 1 W W boson	GAUGE BOSONS VECTOR BOSONS

Gravity = curved spacetime

The other forces?



Kaluza-Klein theory—the other forces originate from gravity in the **extra dimensions**

Gravitational force = curved spacetime

History of
the spacetime symmetry

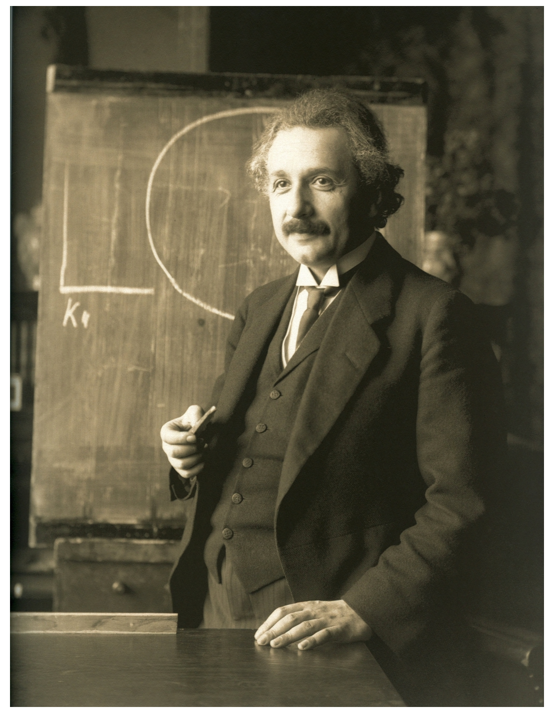
Gravitational force = curved spacetime

Euclidean symmetry

- Translations, rotations
- Euclidean space+time
- Newtonian mechanics

Poincare symmetry

- Translations, rotations + boosts
- Minkowskian spacetime
- (Special) Relativity



Super-Poincare symmetry

- Translations, rotations, boosts + supersymmetry
- Superspace
- Supersymmetric theories

Kaluza-Klein Theory + Supersymmetry

Kaluza-Klein Theory + Supersymmetry

⇩

11D Supergravity

- Large enough dimension to include the standard model
- Small enough dimension to exclude higher spin (> 2) particles
- However, still *non-renormalizable*
- Solution?

Kaluza-Klein Theory + Supersymmetry

⇩

11D Supergravity

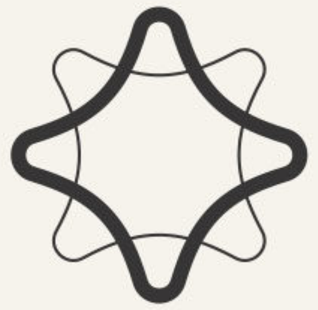
- Large enough dimension to include the standard model
- Small enough dimension to exclude higher spin (> 2) particles
- However, still *non-renormalizable*
- Solution?

⇩ UV completion

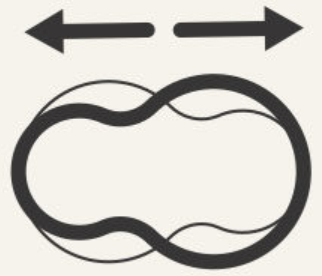
M-Theory

Type I
string theory

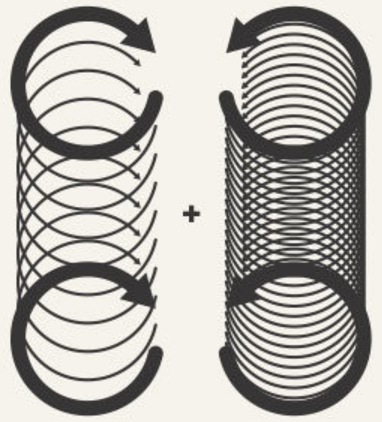
Type IIA
string theory



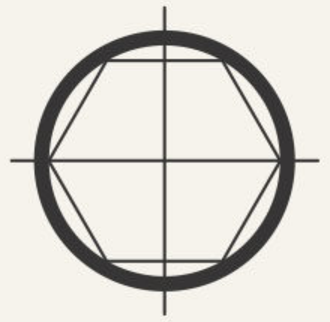
Type IIB
string theory

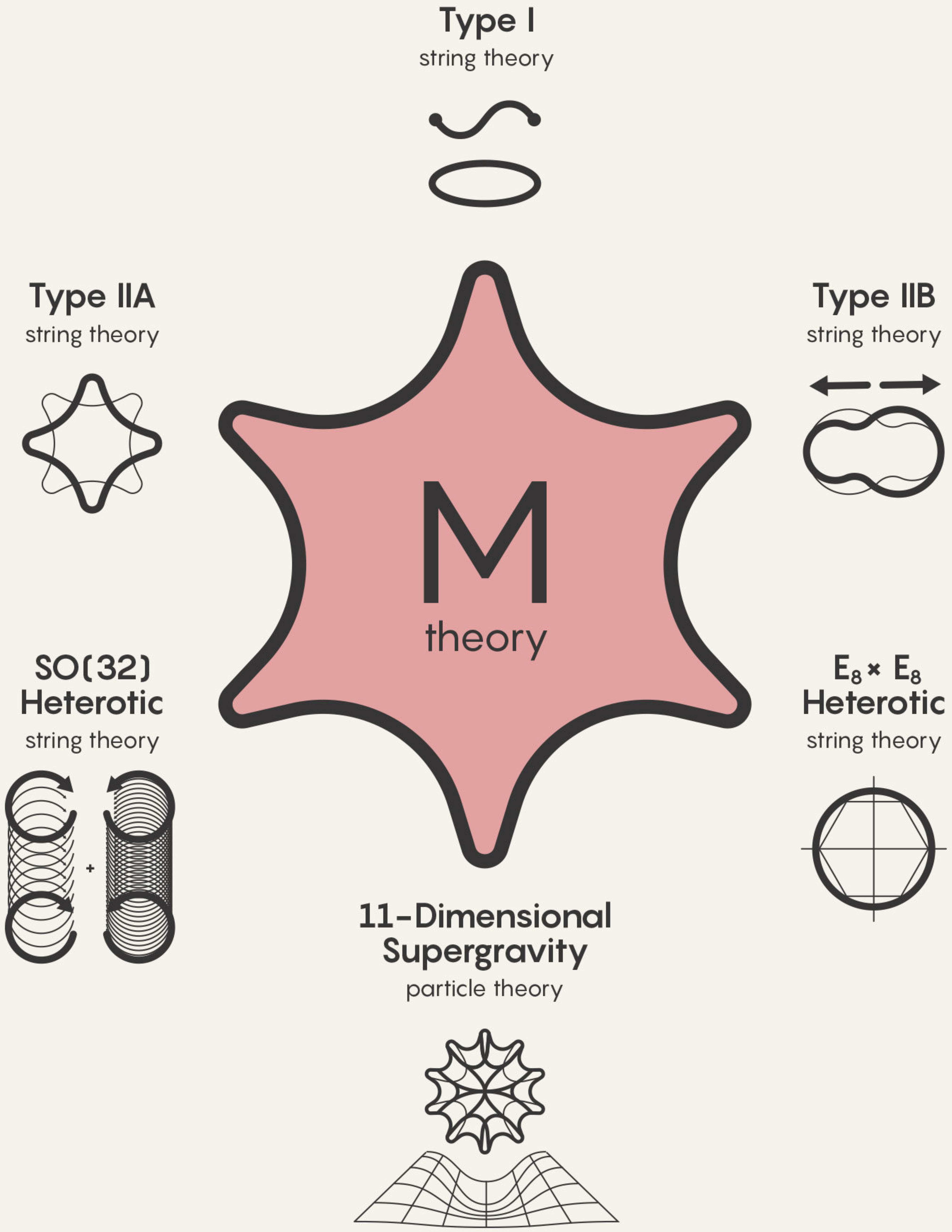


SO(32) Heterotic
string theory



$E_8 \times E_8$ Heterotic
string theory





Type I
string theory

Type IIA
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Type IIB
string theory

M
theory

SO(32)
Heterotic
string theory

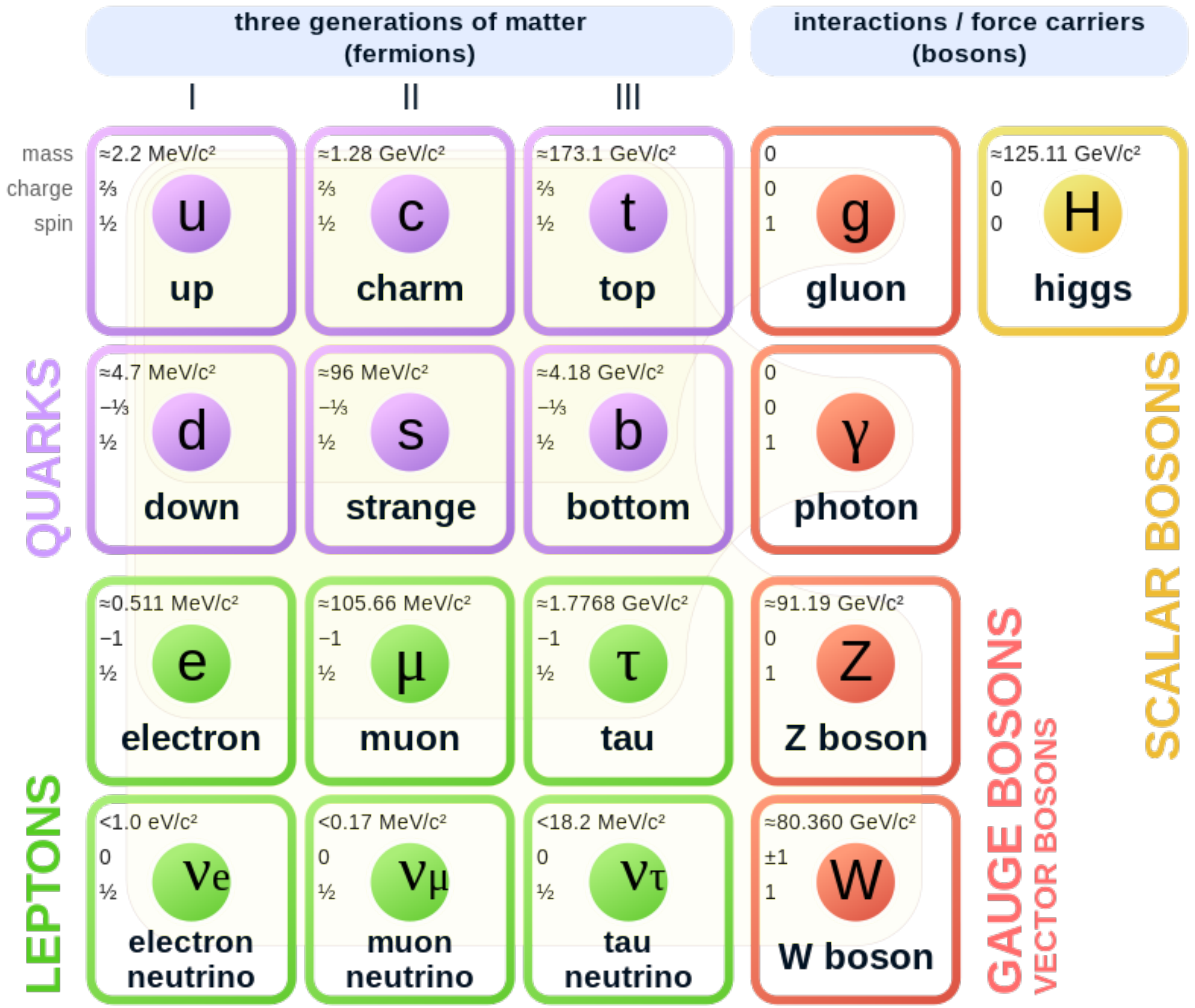
$E_8 \times E_8$
Heterotic
string theory

11-Dimensional
Supergravity
particle theory

- Supersymmetry and Kaluza-Klein’s idea lead to M/string theory, which provides a consistent framework for quantum gravity.
- Among other things, M/string theory provides the most concrete understanding of black hole entropy and its microstates. (Part II)

Q2. Other than gravity, is the standard model enough?

Standard Model of Elementary Particles



three generations of matter
(fermions)

interactions / force carriers
(bosons)

	I	II	III		
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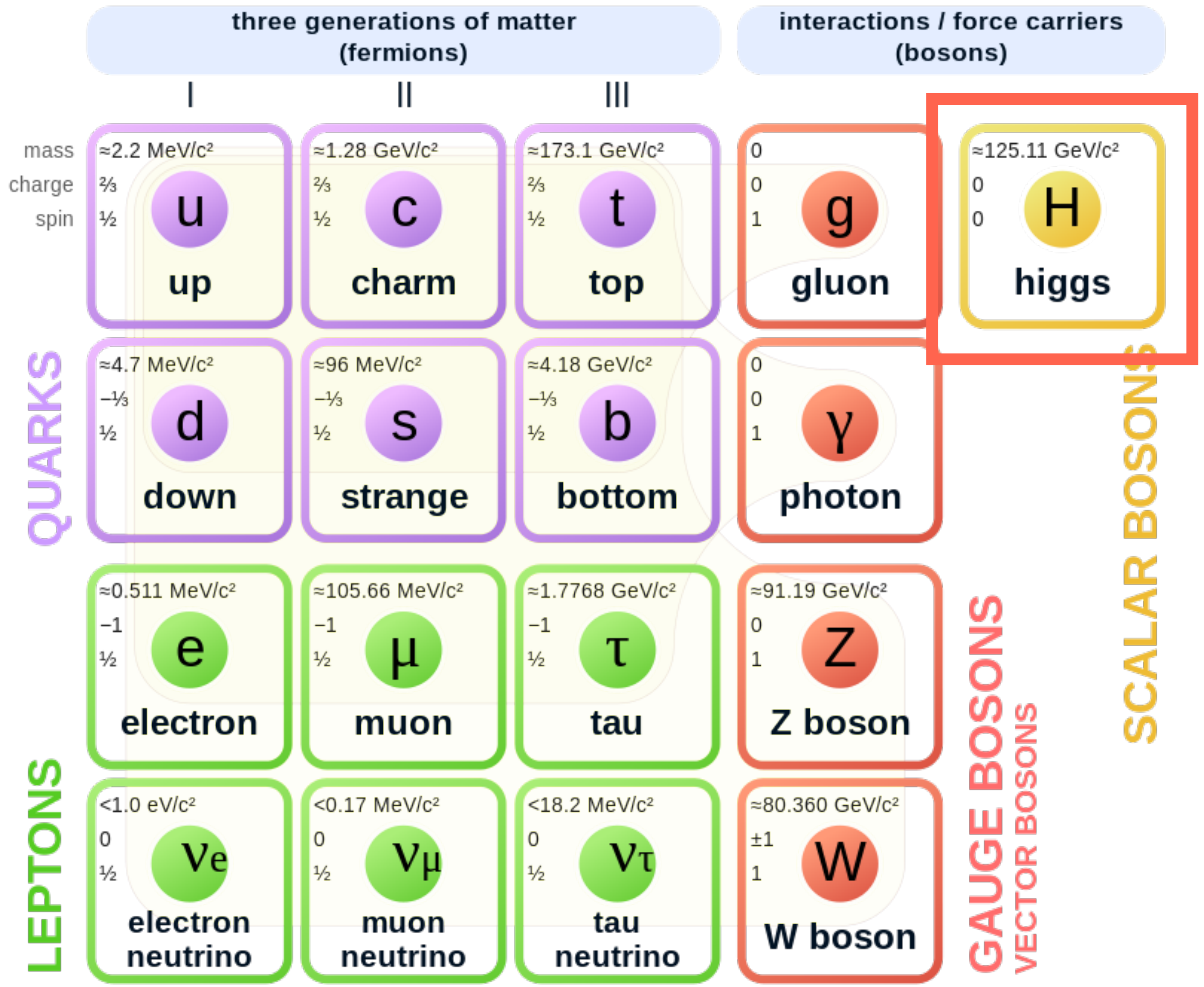
QUARKS	≈4.7 MeV/c^2 −⅓ ½ d **down**	≈96 MeV/c^2 −⅓ ½ s **strange**	≈4.18 GeV/c^2 −⅓ ½ b **bottom**	0 0 1 γ **photon**	SCALAR BOSONS
LEPTONS	≈0.511 MeV/c^2 −1 ½ e **electron**	≈105.66 MeV/c^2 −1 ½ μ **muon**	≈1.7768 GeV/c^2 −1 ½ τ **tau**	≈91.19 GeV/c^2 0 1 Z **Z boson**	GAUGE BOSONS VECTOR BOSONS
	<1.0 eV/c^2 0 ½ ν_e **electron neutrino**	<0.17 MeV/c^2 0 ½ ν_μ **muon neutrino**	<18.2 MeV/c^2 0 ½ ν_τ **tau neutrino**	≈80.360 GeV/c^2 ±1 1 W **W boson**	

Standard Model of Elementary Particles



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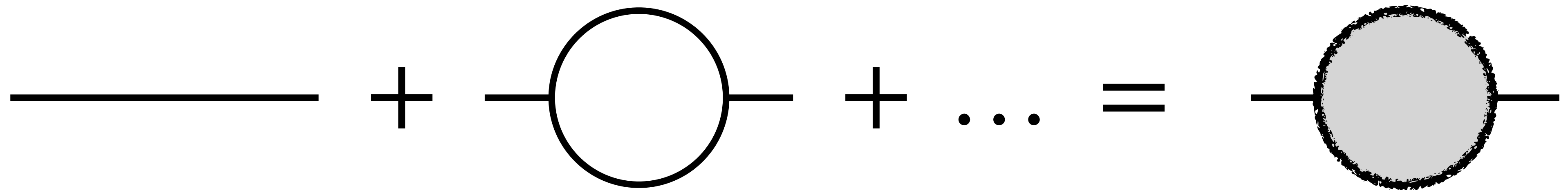
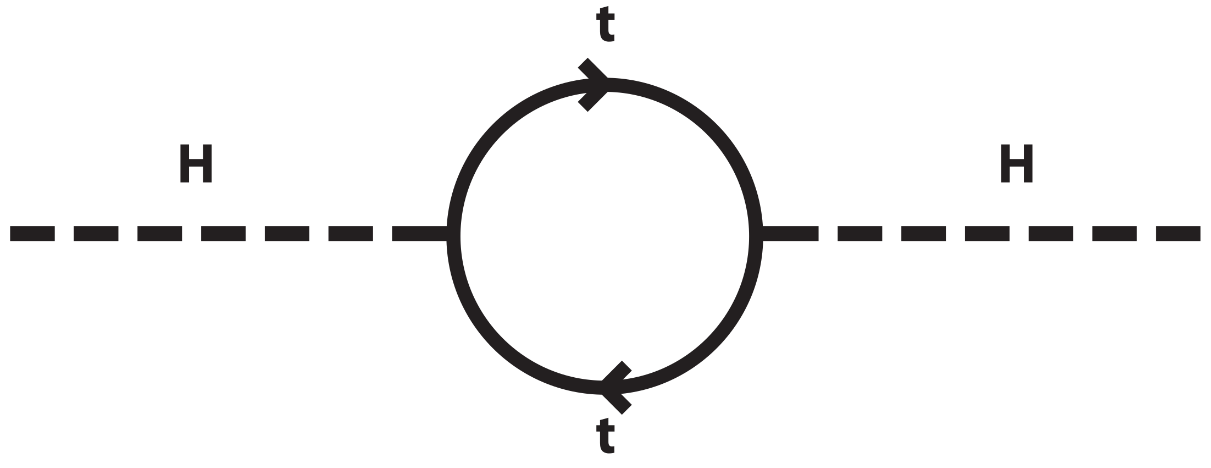
I. Hierarchy Problem

- Recall that QFT adds quantum corrections to classical (bare) quantities



- For Higgs mass,

$$\Delta m^2_{Higgs} = -\frac{|\lambda_f|^2}{8\pi^2}\left[\Lambda^2_{UV} + m_f^2 \times \ldots\right]$$



- Λ_{UV} : the energy scale at which new physics kicks in

- For example, the Planck scale $\left(\sim 10^{19}\,\mathrm{GeV} \right)$ where the quantum gravity effect becomes important.

- The GUT scale $\left(\sim 10^{16}\,\mathrm{GeV} \right)$ where all the forces except gravity are unified.

- Both are much higher than the observed Higgs mass $\left(\sim 10^{2}\,\mathrm{GeV} \right)$.

- A unnaturally precise cancelation for Higgs mass

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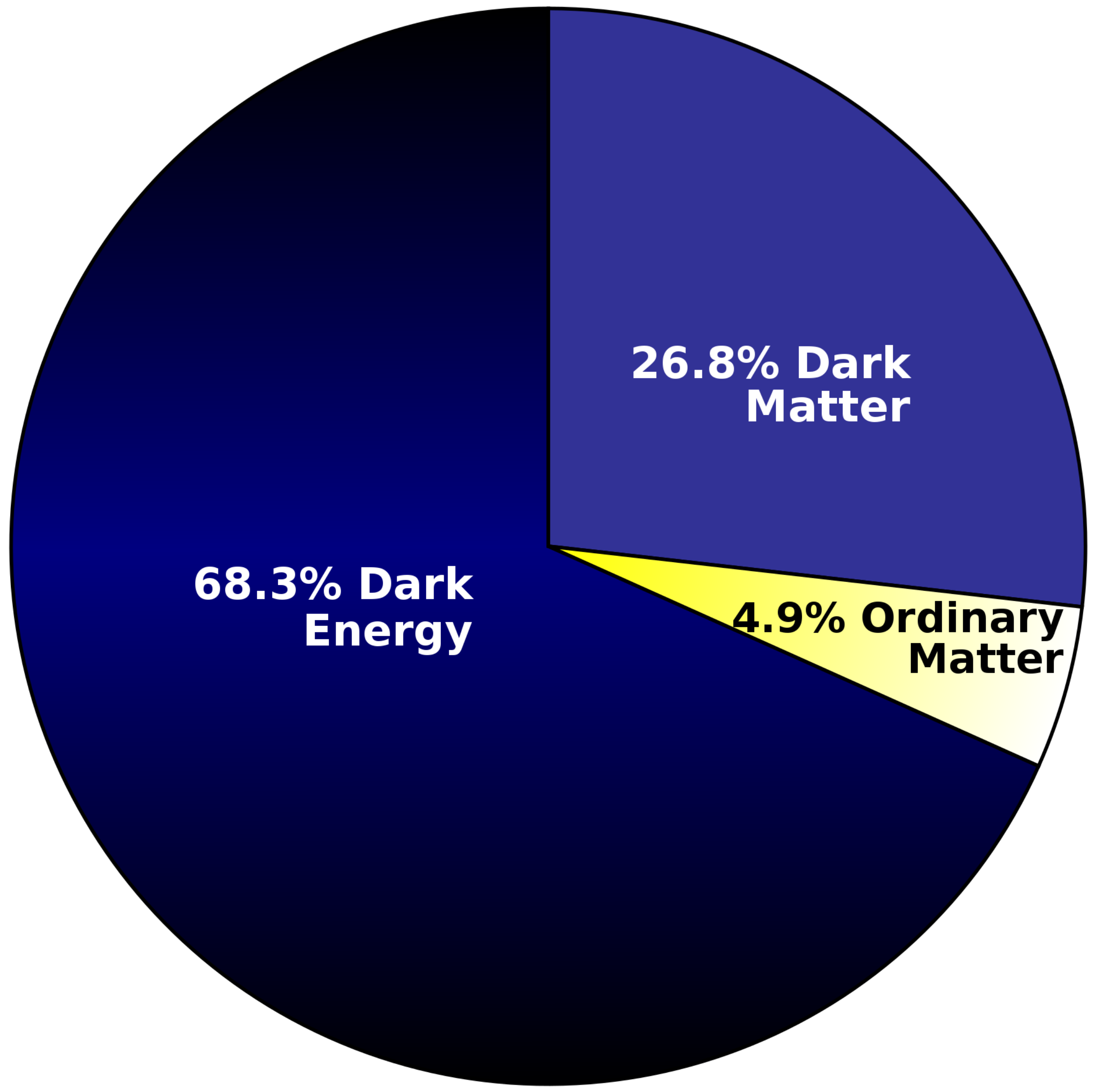
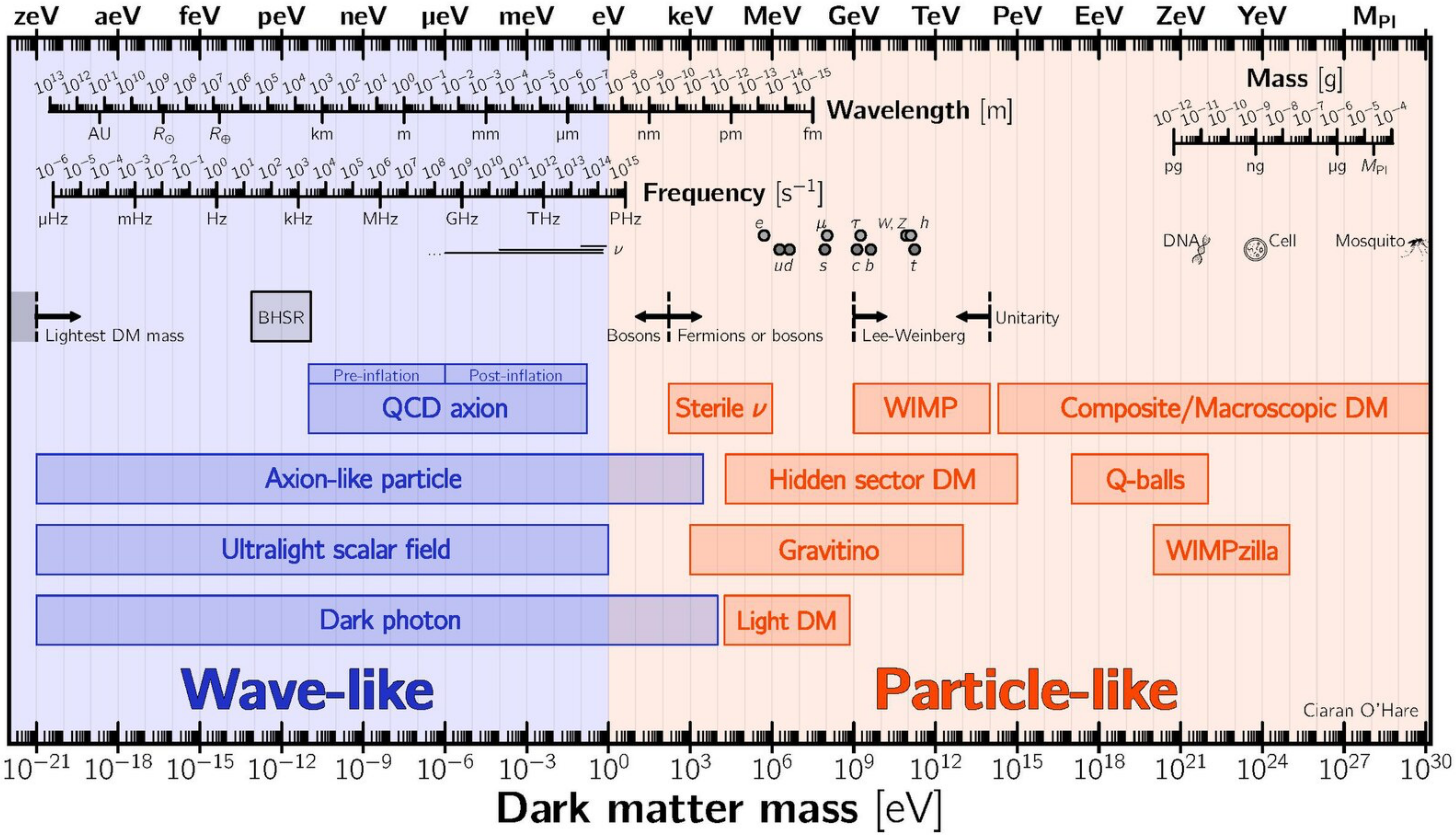
- The GUT scale $\left(\sim 10^{16}\,\mathrm{GeV} \right)$ where all the forces except gravity are unified.
- Both are much higher than the observed Higgs mass $\left(\sim 10^{2}\,\mathrm{GeV} \right)$.
- A unnaturally precise cancelation for Higgs mass

Why is Higgs mass so fine-tuned to cancel such a large quantum correction?

II. Dark Matter

- Our universe consists of ordinary matter, dark matter, and dark energy.
- Dark matter, 85% of the total amount of matter, doesn't interact with light and is only observed by gravitational effects.
- Many scenarios.

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µeV
meV
eV
keV
MeV
GeV
TeV
PeV
EeV
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Wavelength [m]
AU
$R_\odot$
$R_\oplus$
km
m
mm
µm
nm
pm
fm
Frequency [s^{-1}]
µHz
mHz
Hz
kHz
MHz
GHz
THz
PHz
ν
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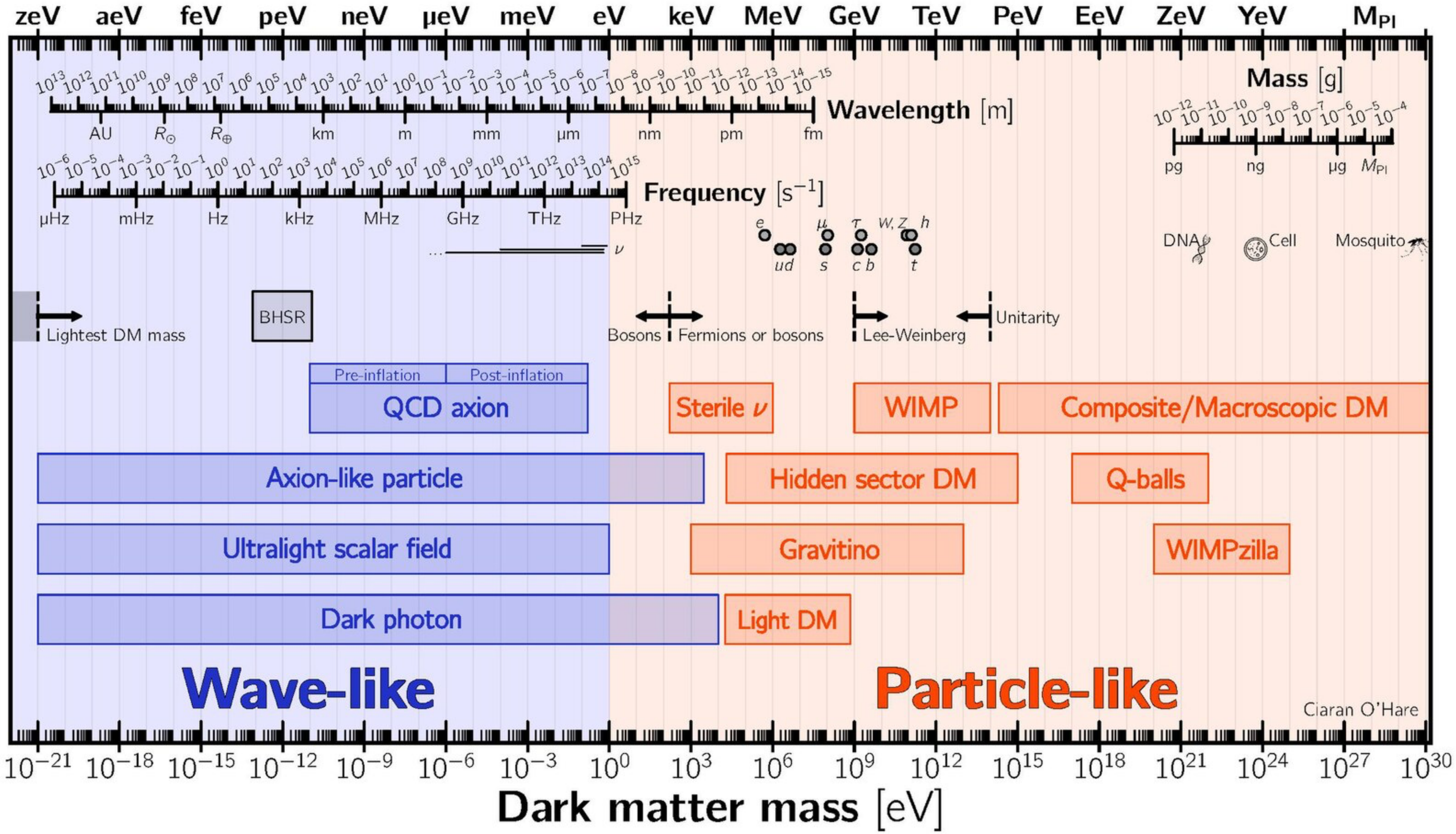
h
t
DNA
Cell
Mosquito
Lightest DM mass
BHSR
Bosons
Fermions or bosons
Lee-Weinberg
Unitarity
Pre-inflation
Post-inflation
QCD axion
Axion-like particle
Ultralight scalar field
Dark photon
Sterile ν
WIMP
Composite/Macroscopic DM
Hidden sector DM
Q-balls
Gravitino
WIMPzilla
Light DM
Wave-like
Particle-like
Ciaran O'Hare
10^{-21} 10^{-18} 10^{-15} 10^{-12} 10^{-9} 10^{-6} 10^{-3} 10^{0} 10^{3} 10^{6} 10^{9} 10^{12} 10^{15} 10^{18} 10^{21} 10^{24} 10^{27} 10^{30}
Dark matter mass [eV]



Supersymmetry requires new particles, some of which are DM candidates.

III. Grand Unified Theory

- Why do the electric charges of the electron and the proton exactly cancel each other?
- $U(1)$ symmetry alone doesn't necessarily require quantized charges.
- On the other hand, if $U(1)$ is part of a larger simple Lie group, charges must be quantized.

III. Grand Unified Theory

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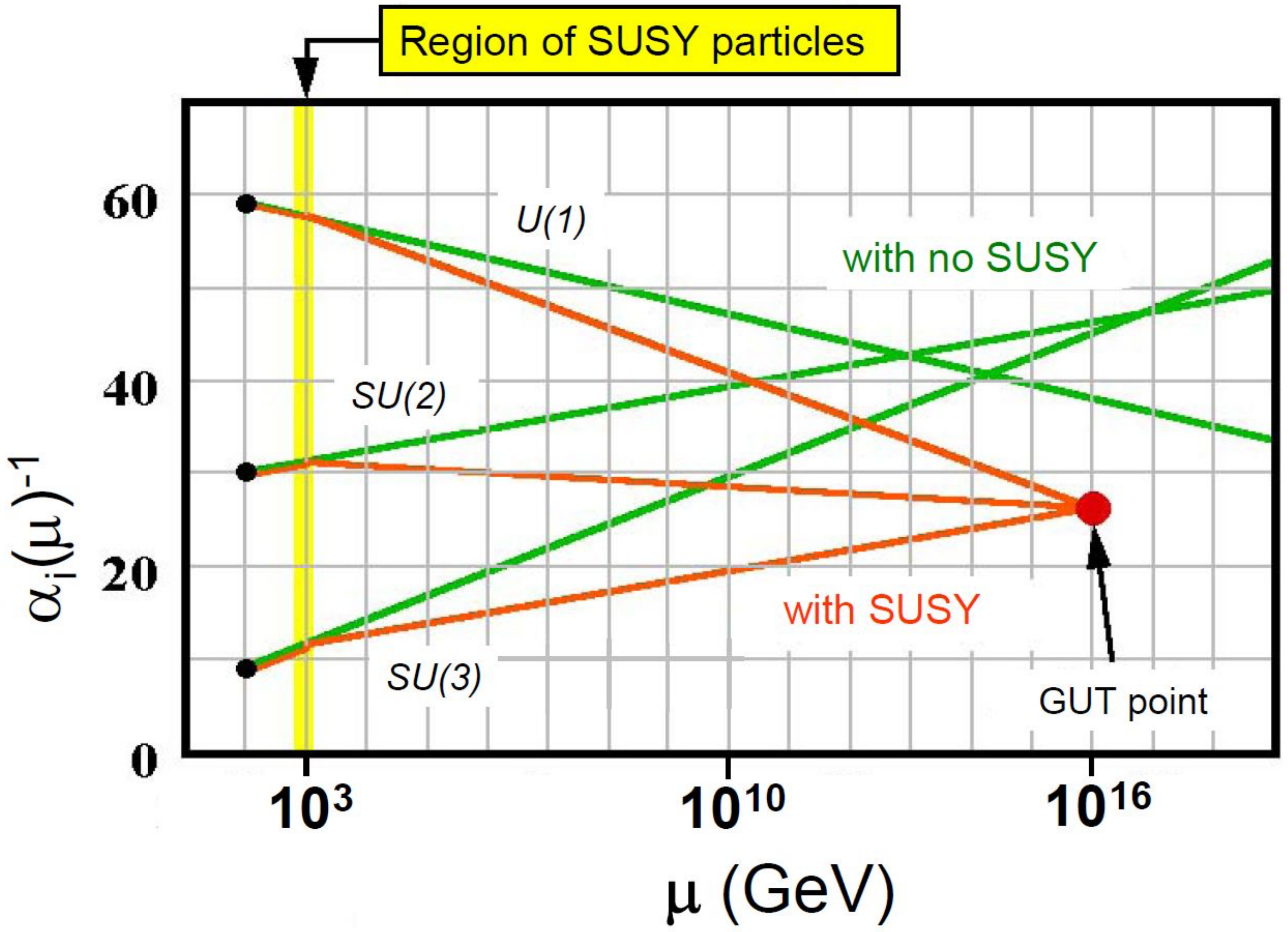
- On the other hand, if $U(1)$ is part of a larger simple Lie group, charges must be quantized.

➡ **Grand Unified Theory**

- There are several models based on $SU(5)$, $SO(10)$, E_6, … , which should be broken to the standard model gauge group $SU(3) \times SU(2) \times U(1)$.

- However, the couplings of $SU(3) \times SU(2) \times U(1)$ do not seem to agree at any energy scale.

- **Supersymmetry can cure this problem.**



Supersymmetry
provides (at least partial) solutions to
these problems.

Part II

Application: From Higgs Mass to Black Hole Entropy

- Example I: Higgs Mass and the Hierarchy Problem

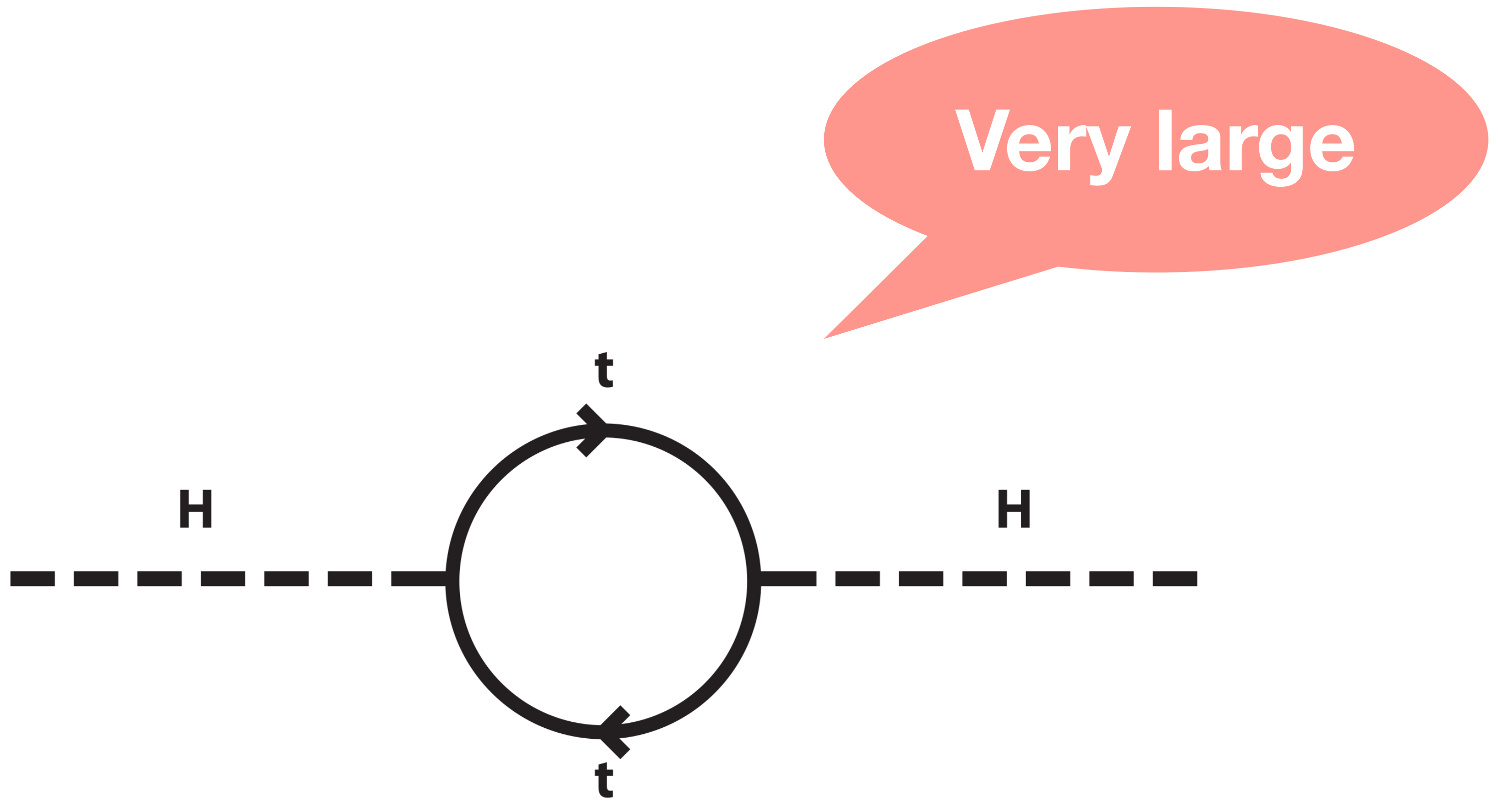
- Example II: Holographic Duality and Black Hole Entropy

Example I: Higgs Mass and the Hierarchy Problem

Higgs Mass

- The quantum correction to Higgs mass

$$\Delta m_{Higgs}^2 = -\frac{|\lambda_f|^2}{8\pi^2}\left[\Lambda_{UV}^2 + m_f^2 \times \dots\right]$$



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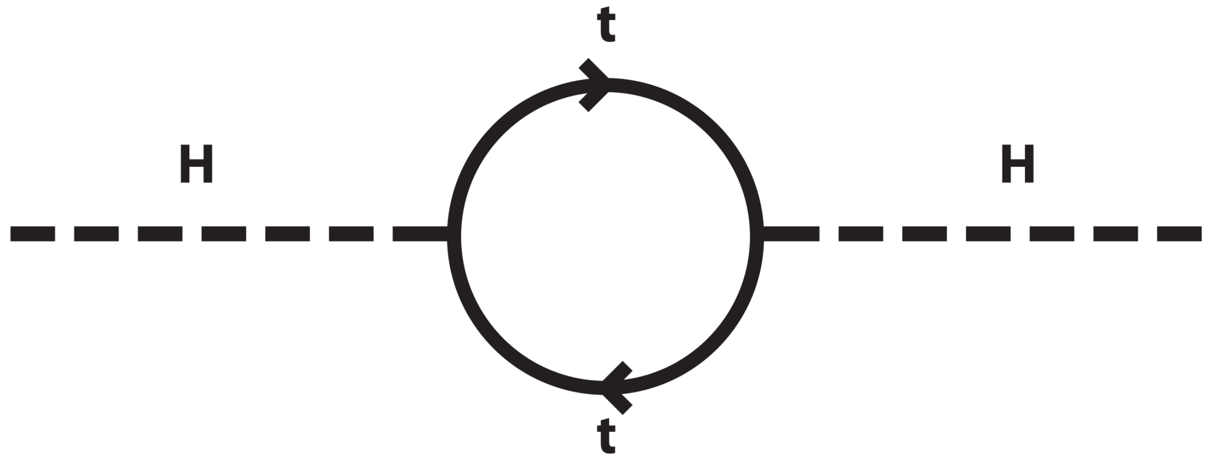
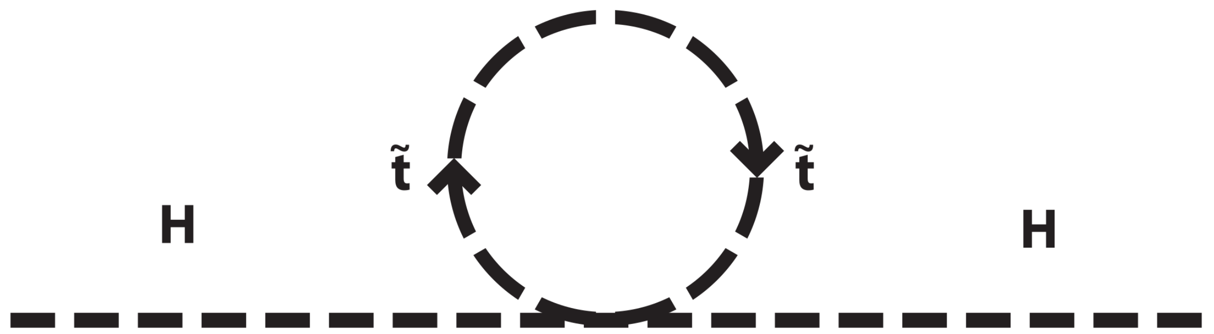


Very large

- If there are two complex scalar fields with $\lambda_S = |\lambda_f|^2$ for each of the quarks and the leptons, the Λ^2_{UV} contributions completely cancel.

$$\Delta m^2_{Higgs} \leftarrow +2 \times \frac{\lambda_S}{16\pi^2}[\Lambda^2_{UV} + m_S^2 \times \ldots]$$



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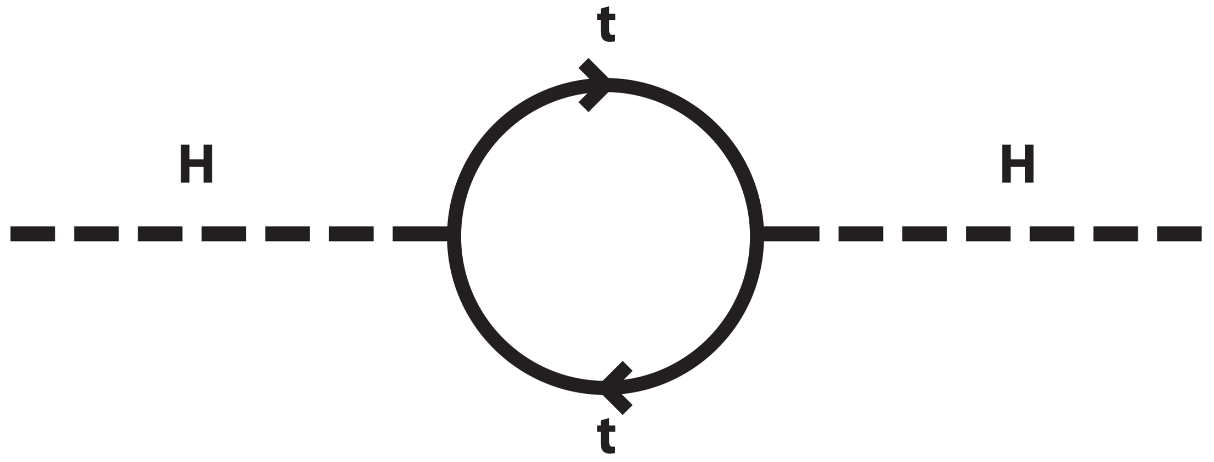

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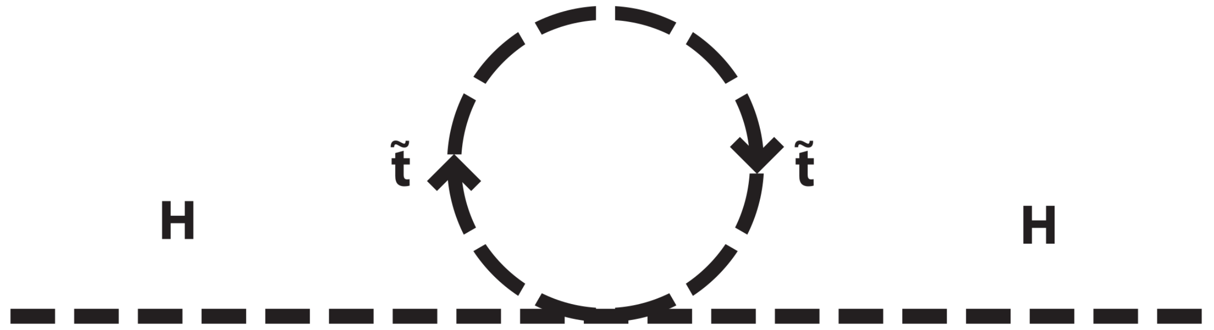


- *Such cancelations persist to higher orders if the theory is **supersymmetric**!*

Universal Properties of SUSY Theories

- The supersymmetric ground state, satisfying $Q_\alpha \,|vac\rangle = Q^\dagger_{\dot\alpha} \,|vac\rangle = 0$, must have the vanishing energy.

$$0 = \left\langle vac \middle| \left\{ Q_\alpha , Q^\dagger_{\dot\alpha} \right\} \middle| vac \right\rangle = -\,2\,\sigma^\mu_{\alpha\dot\alpha}\,\langle vac |\, P_\mu \,| vac \rangle$$

- For a supersymmetric multiplet with given four-momentum p^μ, the number of the bosonic states and that of the fermionic states must be the same unless $p^\mu = 0$.

$$0 = \mathrm{tr}\left[(-1)^F \left\{ Q_\alpha , Q^\dagger_{\dot\alpha} \right\}\right] \sim \mathrm{tr}\left[(-1)^F P^\mu\right] = p^\mu\,\mathrm{tr}\,(-1)^F = p_\mu\,(n_B - n_F)$$

The Simplest Model: A Free Chiral Supermultiplet

- Consider a complex scalar field ϕ and a Weyl fermion ψ with action

$$S = \int d^4x \left(\mathscr{L}_{\text{scalar}} + \mathscr{L}_{\text{fermion}} \right)$$

$$\mathscr{L}_{\text{scalar}} = -\,\partial^\mu \phi^* \partial_\mu \phi\,, \qquad \mathscr{L}_{\text{fermion}} = i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi$$

- S is invariant under the following transformation

$$\delta\phi = \epsilon\psi\,, \qquad \delta\phi^* = \epsilon^\dagger \psi^\dagger$$

$$\delta\psi_\alpha = -\,i(\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi\,, \qquad \delta\psi^\dagger_{\dot\alpha} = i(\epsilon\sigma^\mu)_{\dot\alpha} \partial_\mu \phi^*$$

Useful identities

$$\left[\sigma^\mu \overline{\sigma}^\nu + \sigma^\nu \overline{\sigma}^\mu\right]_\alpha{}^\beta = -\,2\eta^{\mu\nu}\delta^\beta_\alpha\,,$$

$$\left[\overline{\sigma}^\mu \sigma^\nu + \overline{\sigma}^\nu \sigma^\mu\right]^{\dot\beta}{}_{\dot\alpha} = -\,2\eta^{\mu\nu}\delta^{\dot\beta}_{\dot\alpha}$$

- Satisfy the SUSY algebra?

$$\left\{ Q_\alpha \, , Q^\dagger_{\dot\alpha} \right\} = -\, 2\, \sigma^\mu_{\alpha\dot\alpha}\, P_\mu$$

$$\left(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2}\right)\phi = i\left(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger\right)\partial_\mu\phi$$

$$\left(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2}\right)\psi_\alpha = i\left(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger\right)\partial_\mu\psi_\alpha + \underbrace{i\epsilon_{1\alpha}\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi - i\epsilon_{2\alpha}\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi}_{\text{= 0 only when EOM is satisfied}}$$

- To make it closed regardless of EOM, i.e., off-shell, introduce an auxiliary complex scalar field F with

$$\mathscr{L}_{\text{auxiliary}} = F^*F$$

- The off-shell SUSY action

$$S = \int d^4x \left(\mathscr{L}_{\text{scalar}} + \mathscr{L}_{\text{fermion}} + \mathscr{L}_{\text{auxiliary}} \right)$$

$$\mathscr{L}_{\text{scalar}} = -\partial^\mu \phi^* \partial_\mu \phi \,, \qquad \mathscr{L}_{\text{fermion}} = i\psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi \,, \qquad \mathscr{L}_{\text{auxiliary}} = F^* F$$

- The SUSY transformation

$$\delta\phi = \epsilon\psi \,, \qquad \delta\phi^* = \epsilon^\dagger \psi^\dagger$$

$$\delta\psi_\alpha = -i(\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi + \epsilon_\alpha F \,, \qquad \delta\psi^\dagger_{\dot\alpha} = i(\epsilon\sigma^\mu)_{\dot\alpha} \partial_\mu \phi^* + \epsilon^\dagger_{\dot\alpha} F^*$$

$$\delta F = -i\epsilon^\dagger \bar{\sigma}^\mu \partial_\mu \psi \,, \qquad \delta F^* = i\partial_\mu \psi^\dagger \bar{\sigma}^\mu \epsilon$$

- The SUSY algebra is closed regardless of EOM for arbitrary field X:

$$\left(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2}\right) X = i\left(-\epsilon_1\sigma^\mu\epsilon_2^\dagger + \epsilon_2\sigma^\mu\epsilon_1^\dagger\right)\partial_\mu X$$

- $\Phi = \left(\phi\,,\psi_\alpha\,,F\right)$ forms an irreducible representation of the SUSY algebra, called a **chiral multiplet**.

- It also preserves the $U(1)$ R-symmetry acting on Φ as follows:

$$e^{ir\theta}\phi\,, \qquad e^{i(r-1)\theta}\psi\,, \qquad e^{i(r-2)\theta}F$$

Interacting Chiral Multiplet

- A renormalizable Lagrangian of interacting chiral multiplets is governed by a single *holomorphic* function $W(\Phi_i)$ of chiral fields Φ_i, called the **superpotential**.

$$W(\Phi_i) \quad \longrightarrow \quad \mathscr{L}_{superpotential} = \underset{\substack{\Big\downarrow \\ \text{Scalar potential}}}{\frac{\partial W}{\partial \Phi_i}} F_i - \frac{1}{2} \underset{\substack{\Big\downarrow \\ \text{Yukawa couplings}}}{\frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j}} \psi_i \psi_j + h.c.$$

Gauge Theory: A Vector Multiplet

- A **vector multiplet** consists of a gauge field A_μ, a Weyl fermion field λ, and an auxiliary real scalar field D.
- A renomalizable Lagrangian of a vector multiplet is completely fixed by SUSY.

$$\mathscr{L}_{vector} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\lambda^{\dagger a}\overline{\sigma}^\mu\nabla_\mu\lambda^a + \frac{1}{2}D^aD^a$$

$$\delta A^a_\mu = -\frac{1}{\sqrt{2}}\left(\epsilon^\dagger\overline{\sigma}_\mu\lambda^a + \lambda^{\dagger a}\overline{\sigma}_\mu\epsilon\right)\,,$$

$$\delta\lambda^a_\alpha = \frac{i}{2\sqrt{2}}(\sigma^\mu\overline{\sigma}^\nu\epsilon)_\alpha F^a_{\mu\nu} + \frac{1}{\sqrt{2}}\epsilon_\alpha D^a\,,$$

$$\delta D^a = \frac{i}{\sqrt{2}}\left(-\epsilon^\dagger\overline{\sigma}^\mu\nabla_\mu\lambda^a + \nabla_\mu\lambda^{\dagger a}\overline{\sigma}^\mu\epsilon\right)$$

Gauged Chiral Multiplet

- The gauge invariant Lagrangian

$$\mathscr{L}_{chiral} = -\partial^\mu \phi^* \partial_\mu \phi + i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi + F^* F$$

$$\Downarrow$$

$$\mathscr{L}_{chiral} = -\nabla^\mu \phi^* \nabla_\mu \phi + i\psi^\dagger \overline{\sigma}^\mu \nabla_\mu \psi + F^* F$$

$$-\sqrt{2}g(\phi^* T^a \psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a \phi) + g(\phi^* T^a \phi)D^a \longrightarrow$$

Gaugino
Yukawa couplings

- The SUSY transformation

$$\begin{aligned} \delta\phi &= \epsilon\psi\,, \\ \delta\psi_\alpha &= -i(\sigma^\mu \epsilon^\dagger)_\alpha \partial_\mu \phi + \epsilon_\alpha F\,, \\ \delta F &= -i\epsilon^\dagger \overline{\sigma}^\mu \partial_\mu \psi \end{aligned} \quad \Rightarrow \quad \begin{aligned} \delta\phi &= \epsilon\psi\,, \\ \delta\psi_\alpha &= -i(\sigma^\mu \epsilon^\dagger)_\alpha \nabla_\mu \phi + \epsilon_\alpha F\,, \\ \delta F &= -i\epsilon^\dagger \overline{\sigma}^\mu \nabla_\mu \psi + \sqrt{2}g(T^a\phi)\epsilon^\dagger \lambda^{\dagger a} \end{aligned}$$

Supersymmetric Gauge Theory

$$\boxed{\mathscr{L}_{vector} + \mathscr{L}_{chiral} + \mathscr{L}_{superpotential}}$$

$$\mathscr{L}_{vector} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\lambda^{\dagger a}\overline{\sigma}^{\mu}\nabla_{\mu}\lambda^a + \frac{1}{2}D^aD^a$$

$$\mathscr{L}_{chiral} = -\nabla^{\mu}\phi^*\nabla_{\mu}\phi + i\psi^{\dagger}\overline{\sigma}^{\mu}\nabla_{\mu}\psi + F^*F$$
$$-\sqrt{2}g(\phi^*T^a\psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^{\dagger}T^a\phi) + g(\phi^*T^a\phi)D^a$$

➡ $$V = \left|\frac{\partial W}{\partial \Phi_i}\right|^2 + \frac{1}{2}\sum_a g_a^2(\phi^*T^a\phi)^2$$

$$\mathscr{L}_{superpotential} = \frac{\partial W}{\partial \Phi_i}F_i - \frac{1}{2}\frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j}\psi_i\psi_j + h.c.$$

Supersymmetric Gauge Theory

$$\boxed{\overbrace{\mathscr{L}_{vector} + \mathscr{L}_{chiral}}^{\text{Gauge group \& representations}} + \overbrace{\mathscr{L}_{superpotential}}^{W}}$$

$$\mathscr{L}_{vector} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + i\lambda^{\dagger a}\overline{\sigma}^\mu \nabla_\mu \lambda^a + \frac{1}{2}D^aD^a$$

$$\begin{aligned}\mathscr{L}_{chiral} = &- \nabla^\mu\phi^* \nabla_\mu\phi + i\psi^\dagger\overline{\sigma}^\mu \nabla_\mu\psi + F^*F \\ &-\sqrt{2}g(\phi^*T^a\psi)\lambda^a - \sqrt{2}g\lambda^{\dagger a}(\psi^\dagger T^a\phi) + g(\phi^*T^a\phi)D^a\end{aligned}$$

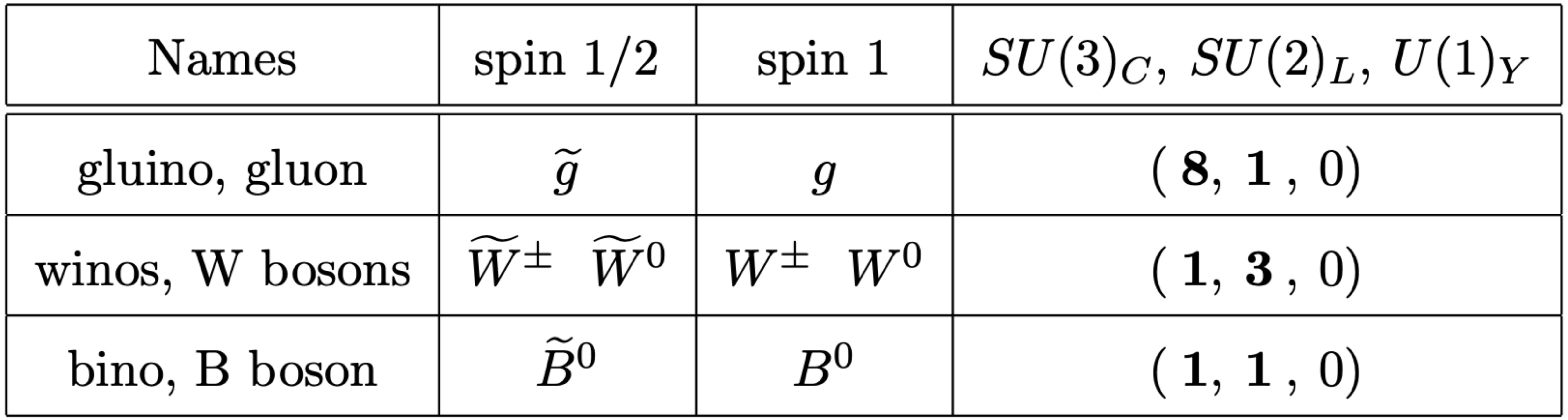
➡ $$V = \left|\frac{\partial W}{\partial \Phi_i}\right|^2 + \frac{1}{2}\sum_a g_a^2(\phi^*T^a\phi)^2$$

$$\mathscr{L}_{superpotential} = \frac{\partial W}{\partial \Phi_i}F_i - \frac{1}{2}\frac{\partial^2 W}{\partial \Phi_i \partial \Phi_j}\psi_i\psi_j + h.c.$$

Minimal Supersymmetric Standard Model

- One can construct a supersymmetric version of the standard model by introducing extra superpartners.

- Vector multiplets



Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	$(\ \mathbf{8},\ \mathbf{1}\ ,\ 0)$
winos, W bosons	$\widetilde{W}^{\pm}\quad \widetilde{W}^{0}$	$W^{\pm}\quad W^{0}$	$(\ \mathbf{1},\ \mathbf{3}\ ,\ 0)$
bino, B boson	$\widetilde{B}^{0}$	B^{0}	$(\ \mathbf{1},\ \mathbf{1}\ ,\ 0)$

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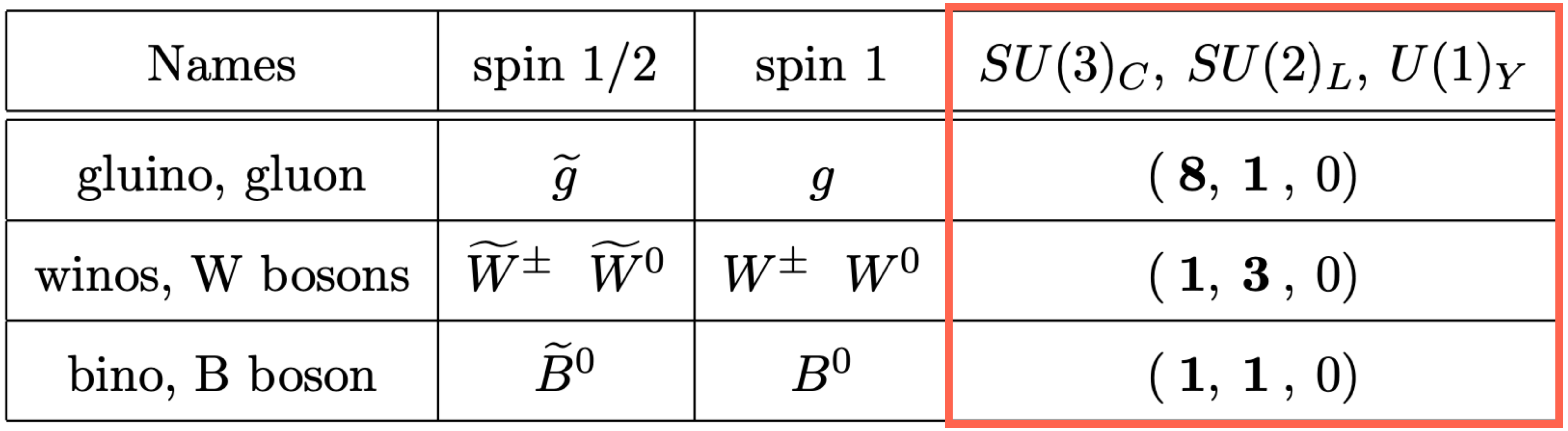

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Minimal Supersymmetric Standard Model

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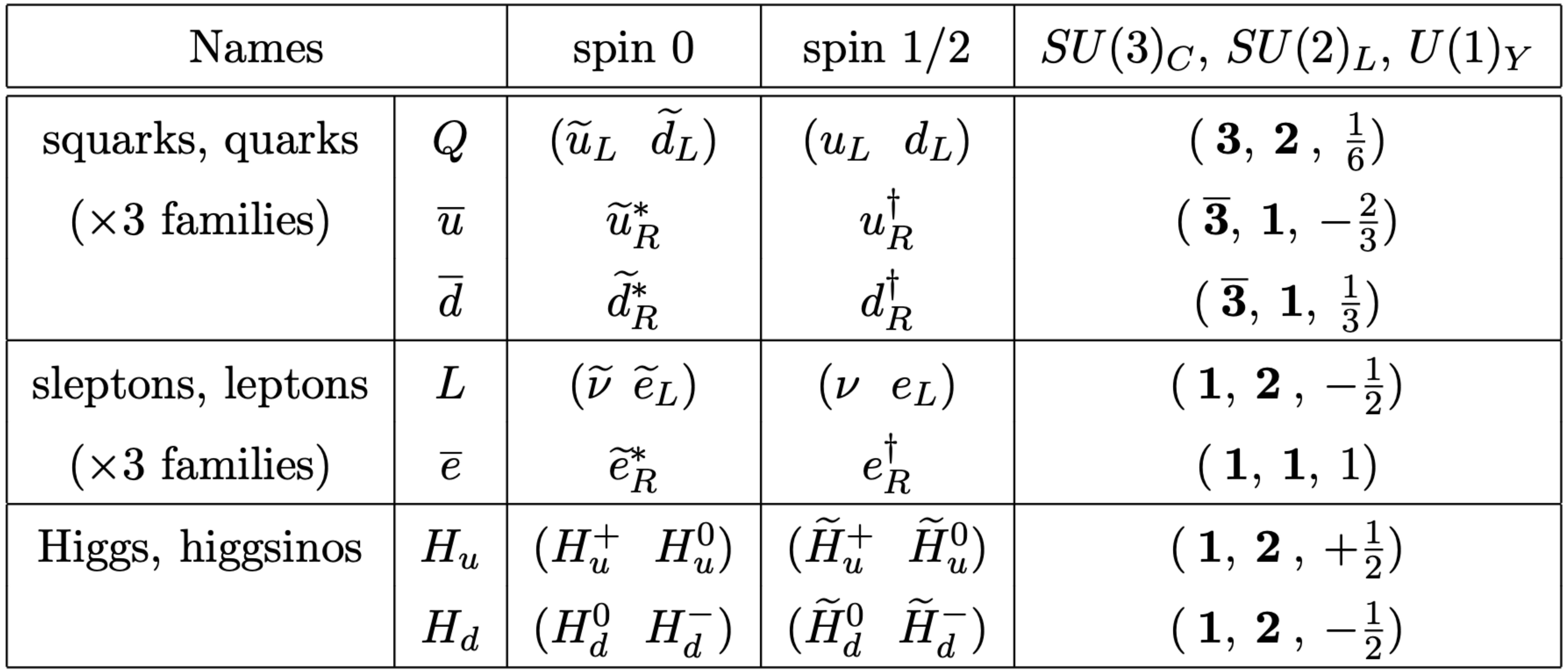

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\widetilde{u}_L\ \ \widetilde{d}_L)$	$(u_L\ \ d_L)$	$(\,\mathbf{3},\ \mathbf{2}\,,\ \frac{1}{6})$
(×3 families)	$\overline{u}$	$\widetilde{u}^*_R$	$u^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\overline{d}$	$\widetilde{d}^*_R$	$d^\dagger_R$	$(\,\overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\widetilde{\nu}\ \ \widetilde{e}_L)$	$(\nu\ \ e_L)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$
(×3 families)	$\overline{e}$	$\widetilde{e}^*_R$	$e^\dagger_R$	$(\,\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H_u^+\ \ H_u^0)$	$(\widetilde{H}_u^+\ \ \widetilde{H}_u^0)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ +\frac{1}{2})$
	H_d	$(H_d^0\ \ H_d^-)$	$(\widetilde{H}_d^0\ \ \widetilde{H}_d^-)$	$(\,\mathbf{1},\ \mathbf{2}\,,\ -\frac{1}{2})$

- Superpotential

$$W_{MSSM} = \overline{u} y_u Q H_u - \overline{d} y_d Q H_d - \overline{e} y_e L H_d + \mu H_u H_d$$

Minimal Supersymmetric Standard Model

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$$W_{MSSM} = \overline{u} y_u Q H_u - \overline{d} y_d Q H_d - \overline{e} y_e L H_d + \mu H_u H_d$$

Holomorphic

Minimal Supersymmetric Standard Model

- Chiral multiplets

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- Superpotential

Two Higgs'

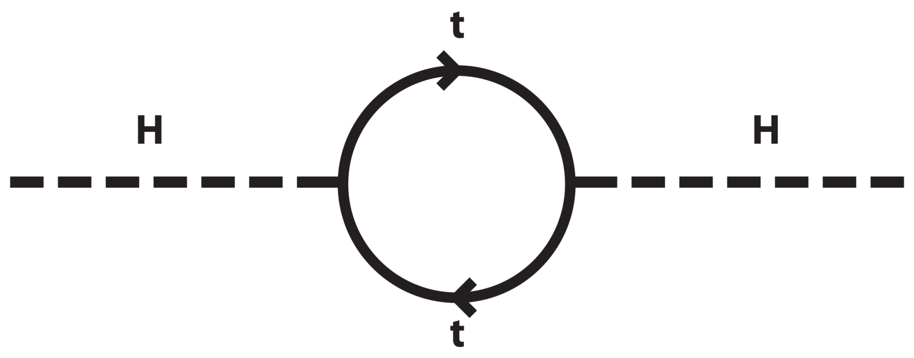
$$W_{MSSM} = \boxed{\bar{u} y_u Q H_u - \bar{d} y_d Q H_d} - \bar{e} y_e L H_d + \mu H_u H_d$$

Holomorphic

- However, we haven’t observed any of superpartners
- *Supersymmetry should be spontaneously broken.*

Soft SUSY Breaking

$$\Delta m^2_{Higgs} \leftarrow -\frac{|\lambda_f|^2}{8\pi^2}\left[\Lambda^2_{UV} + m_f^2 \times \ldots\right]$$



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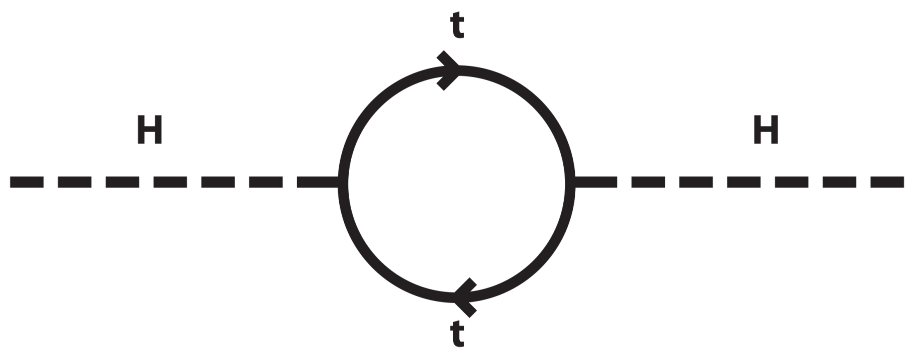
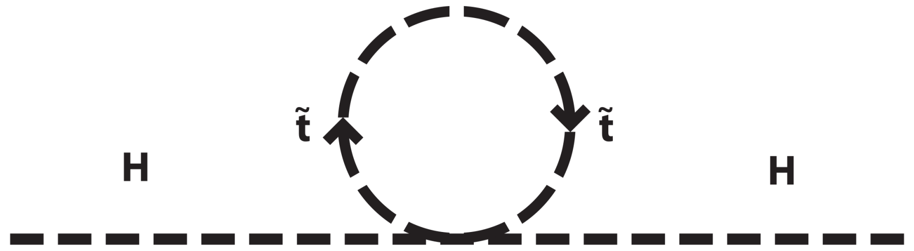


$$\Delta m^2_{Higgs} \leftarrow +2\times\frac{\lambda_S}{16\pi^2}\left[\Lambda^2_{UV} + m_S^2 \times \dots\right]$$



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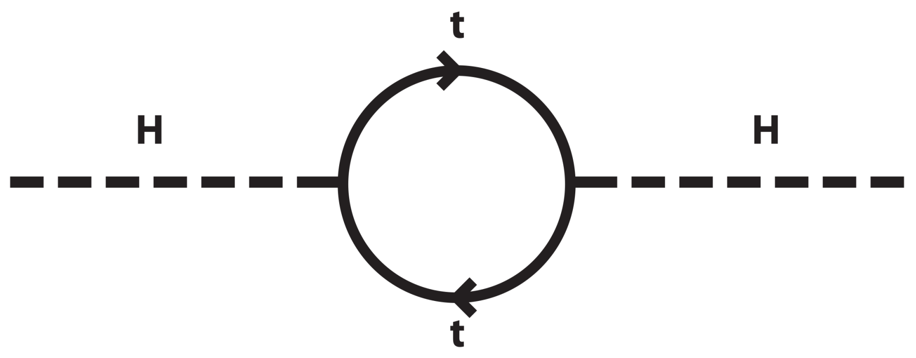

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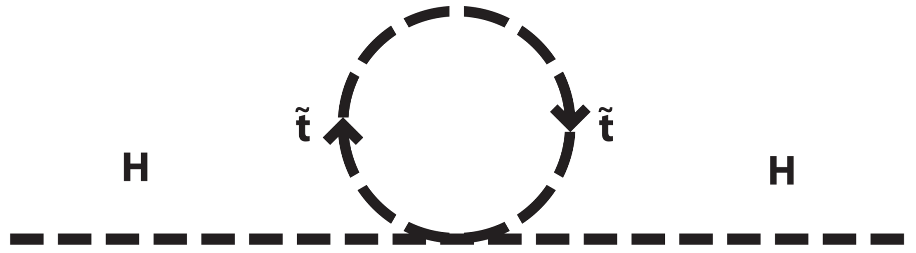

- If SUSY is broken severely such that $\lambda_S \neq |\lambda_f|^2$ anymore, the quantum correction to Higgs mass would be

$$\Delta m^2_{Higgs} = \frac{1}{8\pi^2}\left(\lambda_S - |\lambda_f|^2\right)\Lambda^2_{UV} + \ldots$$

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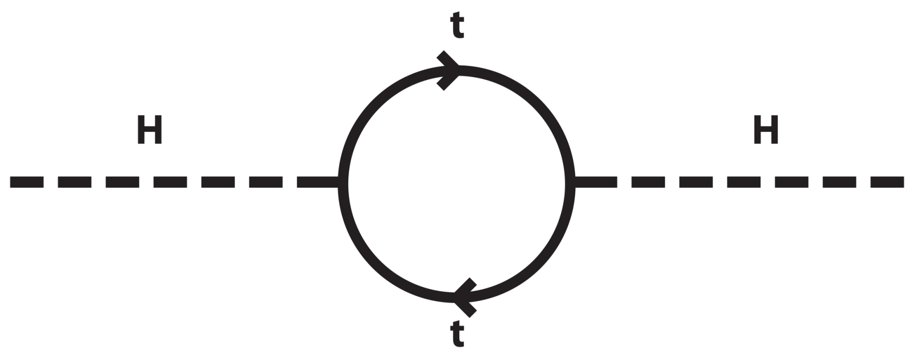

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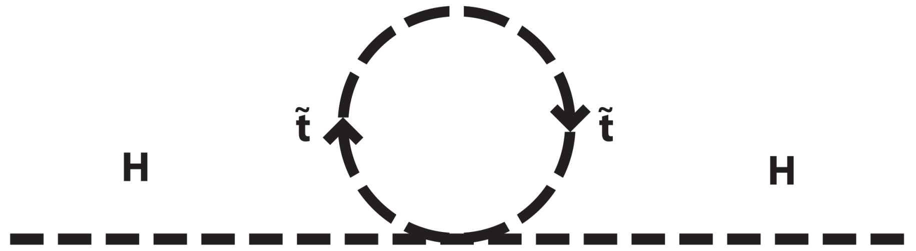

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- SUSY must be broken maintaining $\lambda_S = |\lambda_f|^2$.

- Soft supersymmetry breaking:

$$\mathscr{L} = \mathscr{L}_{SUSY} + \mathscr{L}_{soft}$$

- $\mathscr{L}_{soft}$ contains **SUSY breaking terms** maintaining $\lambda_S = |\lambda_f|^2$ with an **additional scale** m_{soft}, suppposed to originate from the spontaneous SUSY breaking of the microscopic theory.

$$m_{soft} \to 0 \quad \Rightarrow \quad \mathscr{L}_{soft} \to 0$$

$$\Delta m^2_{Higgs} = m^2_{soft} \left[\frac{\lambda}{16\pi^2} \ln \left(\Lambda_{UV}/m_{soft} \right) + \dots \right]$$

- Soft SUSY breaking terms in the MSSM

$$
\begin{aligned}
\mathscr{L}_{soft}^{MSSM} = & -\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{W}\tilde{W} + M_1\tilde{B}\tilde{B} + h.c.\right) \\
& -\left(\tilde{\bar{u}}a_u\tilde{Q}H_u - \tilde{\bar{d}}a_d\tilde{Q}H_d - \tilde{\bar{e}}a_e\tilde{L}H_d + h.c.\right) \\
& -\tilde{Q}^\dagger m_Q^2\tilde{Q} - \tilde{L}^\dagger m_L^2\tilde{L} - \tilde{\bar{u}}m_{\bar{u}}^2\tilde{\bar{u}}^\dagger - \tilde{\bar{d}}m_{\bar{d}}^2\tilde{\bar{d}}^\dagger - \tilde{\bar{e}}m_{\bar{e}}^2\tilde{\bar{e}}^\dagger \\
& -m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (bH_uH_d + h.c.)
\end{aligned}
$$

$M_1, M_2, M_3, a_u, a_d, a_e \quad \sim m_{soft}$

$m_Q^2, m_L^2, m_{\bar{u}}^2, m_{\bar{d}}^2, m_{\bar{e}}^2, m_{H_u}, m_{H_d}^2, b \quad \sim m_{soft}^2$ ⇨ **105 parameters**

Phenomenology of MSSM

$$\Delta m^2_{Higgs} = m^2_{soft}\left[\frac{\lambda}{16\pi^2}\ln\left(\Lambda_{UV}/m_{soft}\right) + \ldots\right]$$

- Assuming $\Lambda_{UV} \sim M_p$ and $\lambda \sim 1$, the SUSY breaking scale m_{soft} shouldn't be much greater than the $10^3\,\mathrm{GeV}$ scale to avoid further miraculous cancelations.
- The lightest superpartner at this scale?
- Seems no, but not conclusive.

At SUSY 2023...

Status and Future of Supersymmetry

Stephen P. Martin
Northern Illinois University
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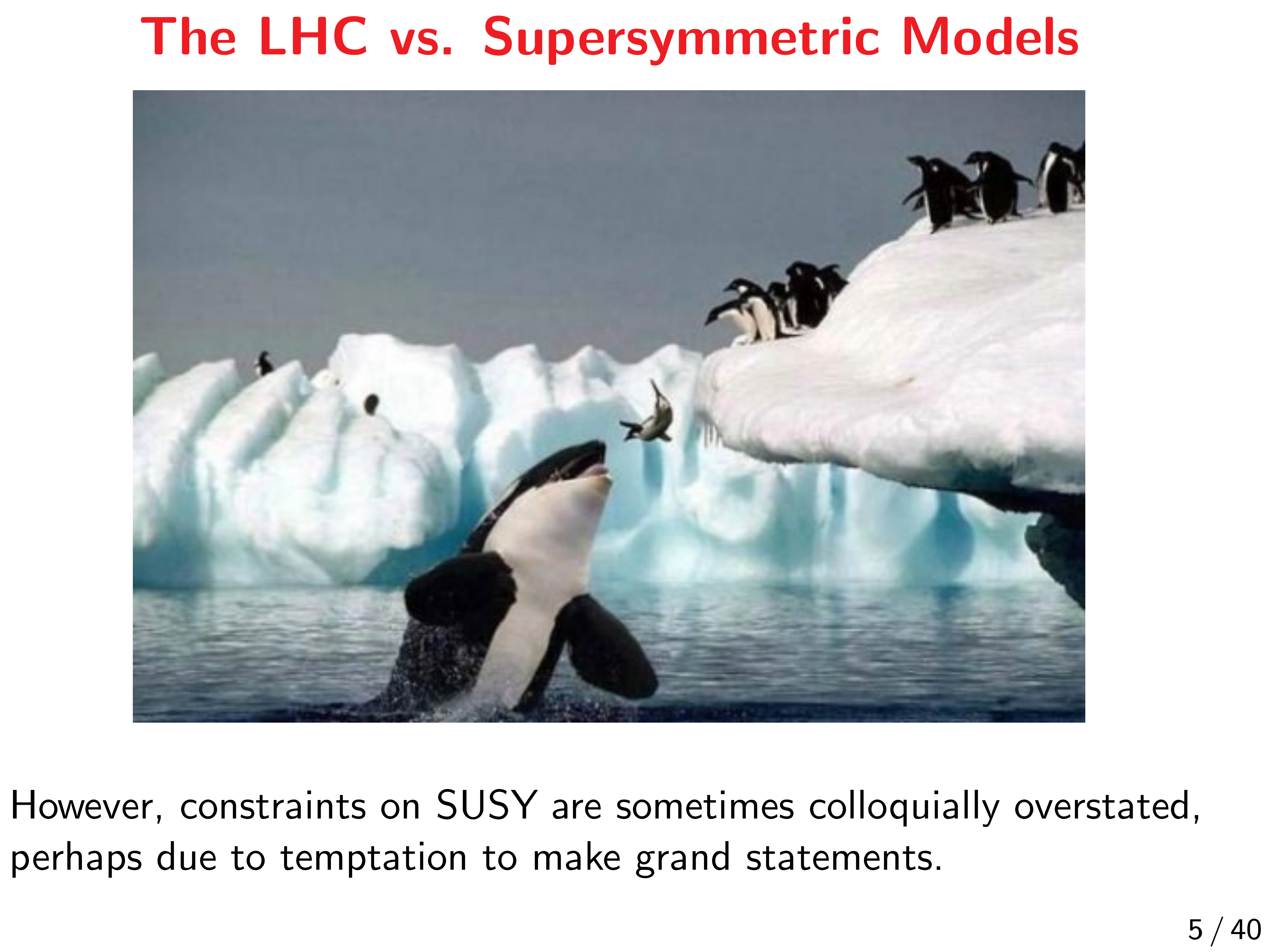
SUSY 2023
July 17-21, 2023
University of Southampton



The LHC vs. Supersymmetric Models

However, constraints on SUSY are sometimes colloquially overstated, perhaps due to temptation to make grand statements.

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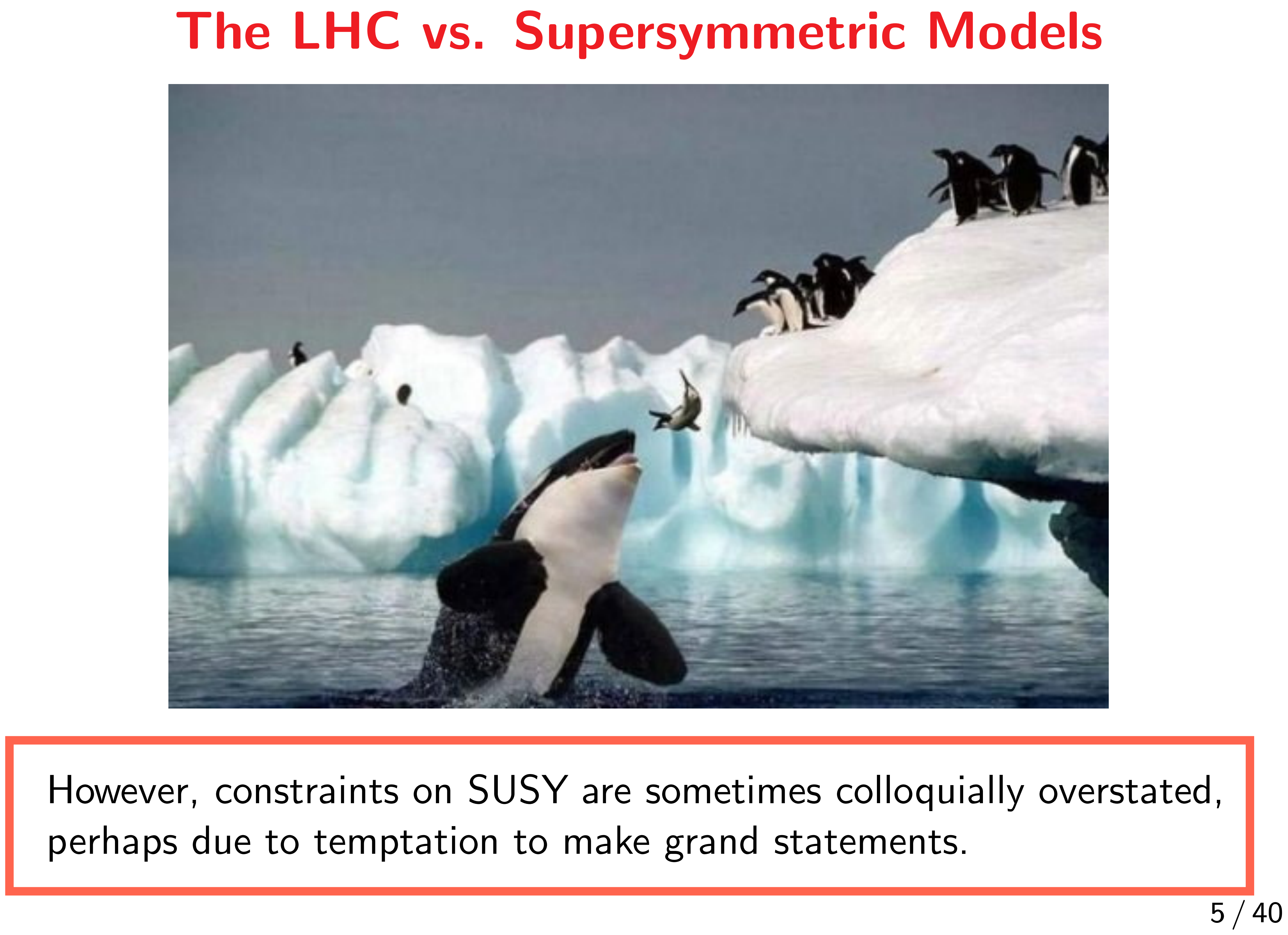

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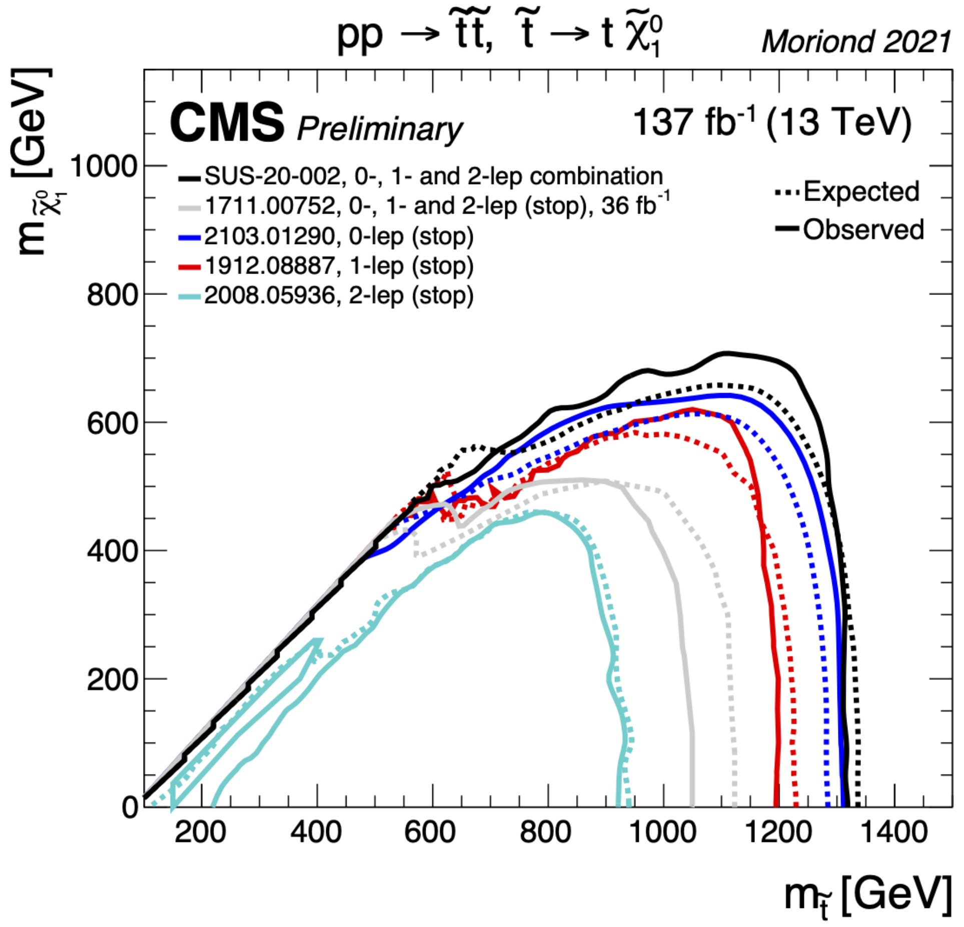
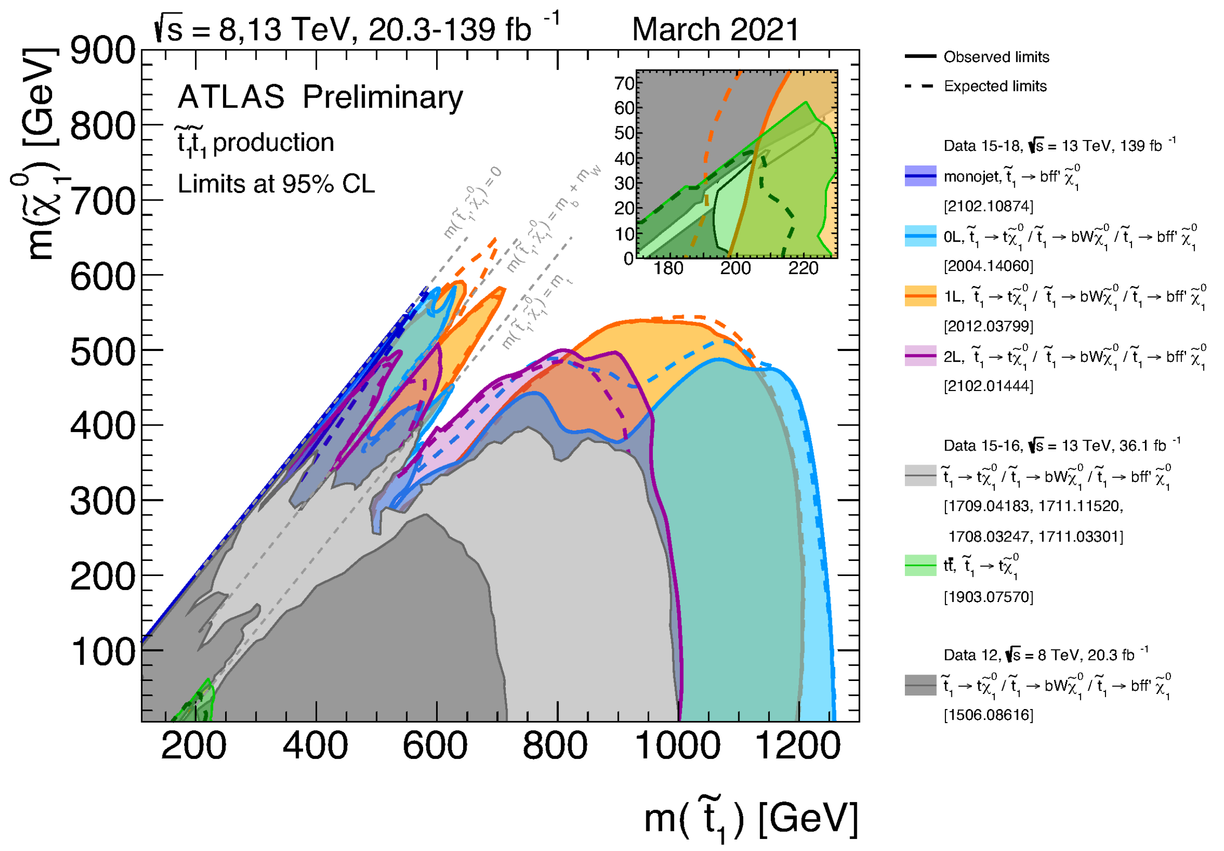
At SUSY 2023…

Limits on top squarks

See Vellidis (CMS) and Maurer (ATLAS) talks for details.



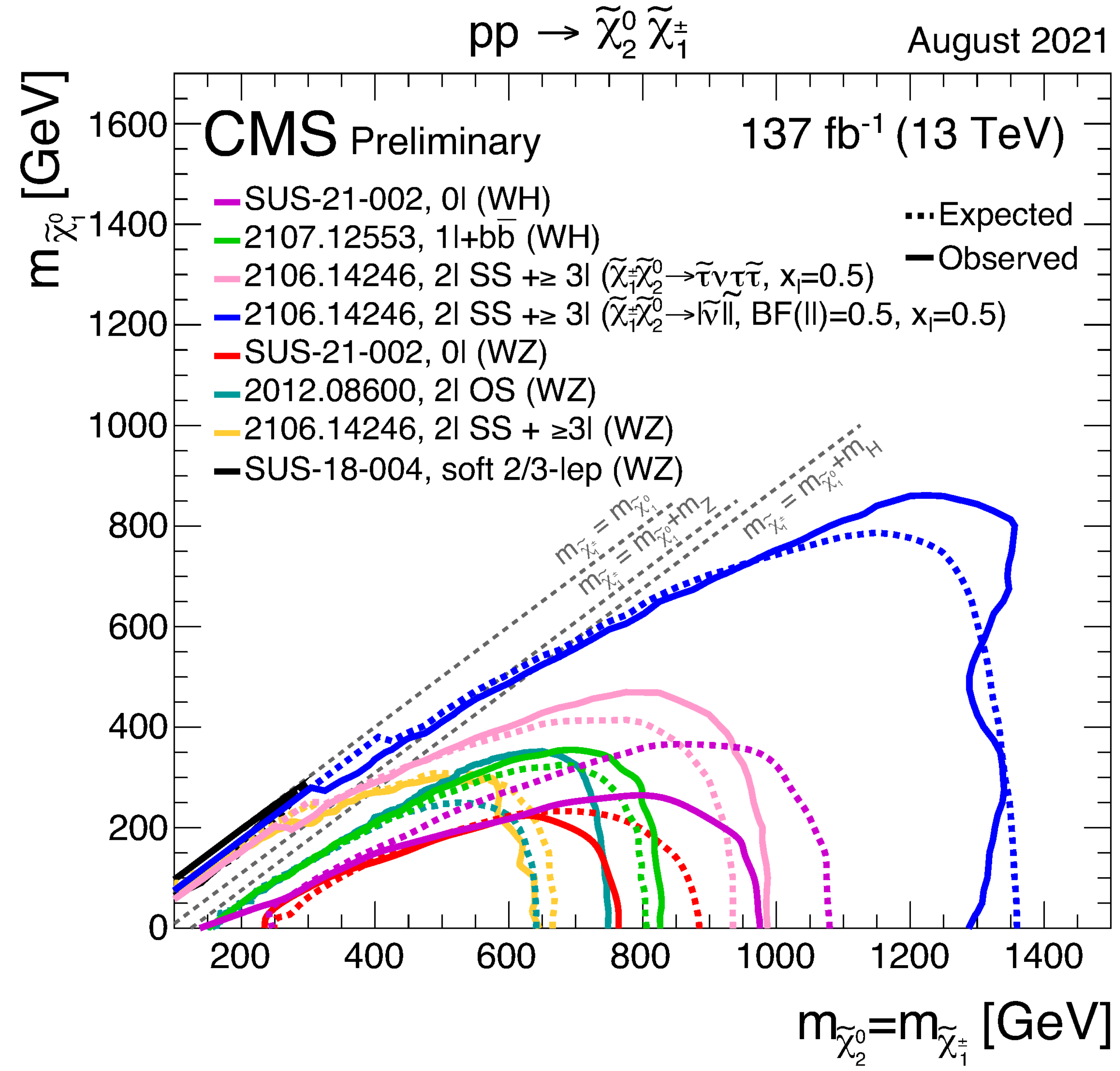


Pessimist: Exclusion of top-squark masses now up to 1300 GeV!

Optimist: No constraints at all on direct pair production of top squarks, if LSP mass exceeds 700 GeV.

More generally, "Compressed SUSY" models with small mass differences are more difficult because visible energy in each event is smaller.

Constraints on wino-like charginos and neutralinos that decay through sleptons: $pp \to \tilde{C}_1 \tilde{N}_2 \to$ leptons $+ \not{E}_T$



Pessimist: Exclusion of electroweakinos above 1300 GeV!

Optimist: Constraints on decays through staus are much weaker, with no exclusion for $m_{\tilde{C}_1} > 1000$ GeV or LSP mass > 450 GeV. Furthermore…

At SUSY 2023...

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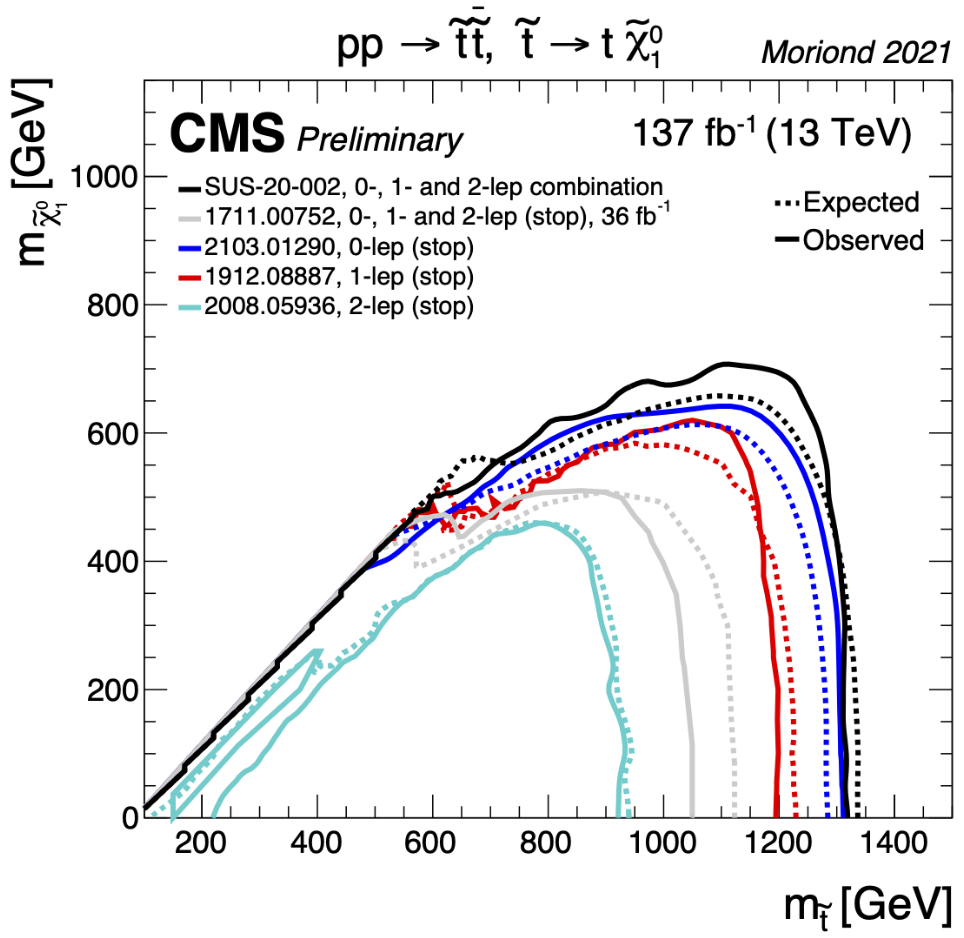

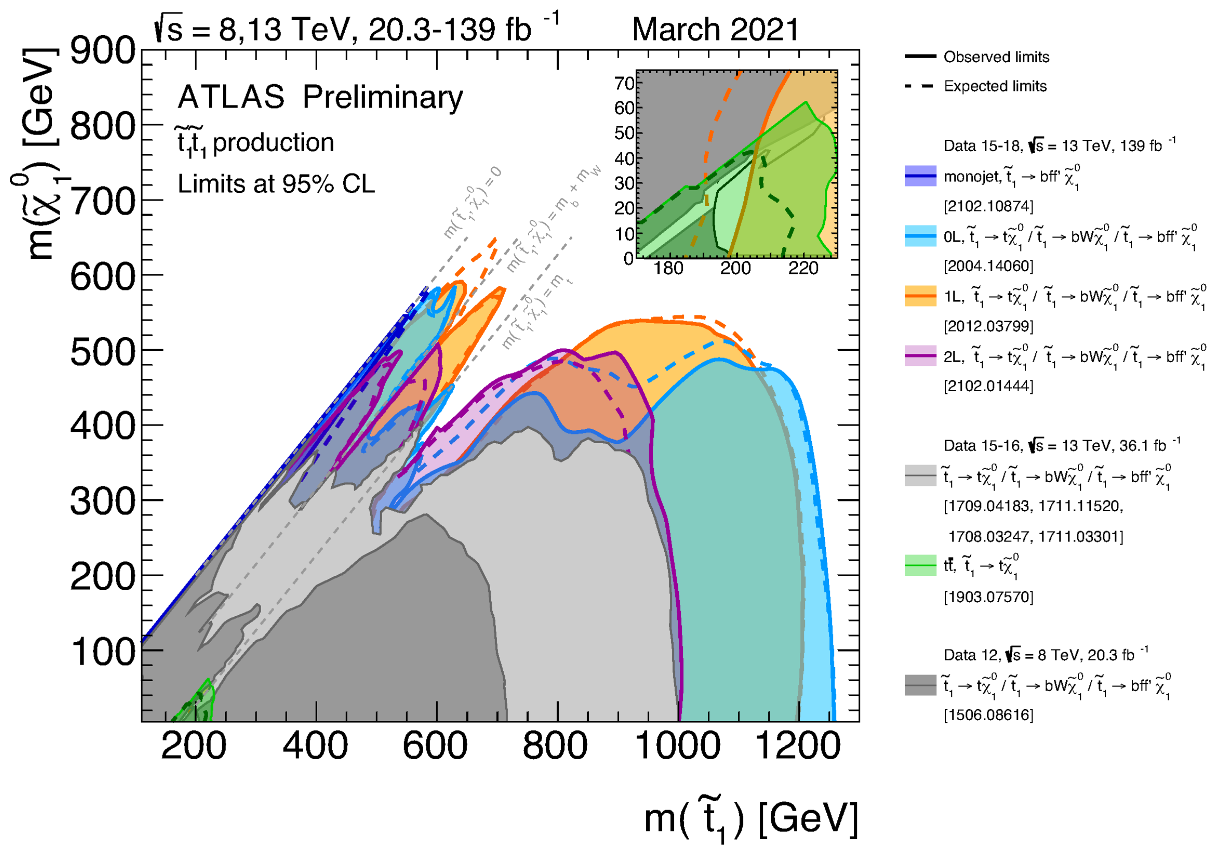

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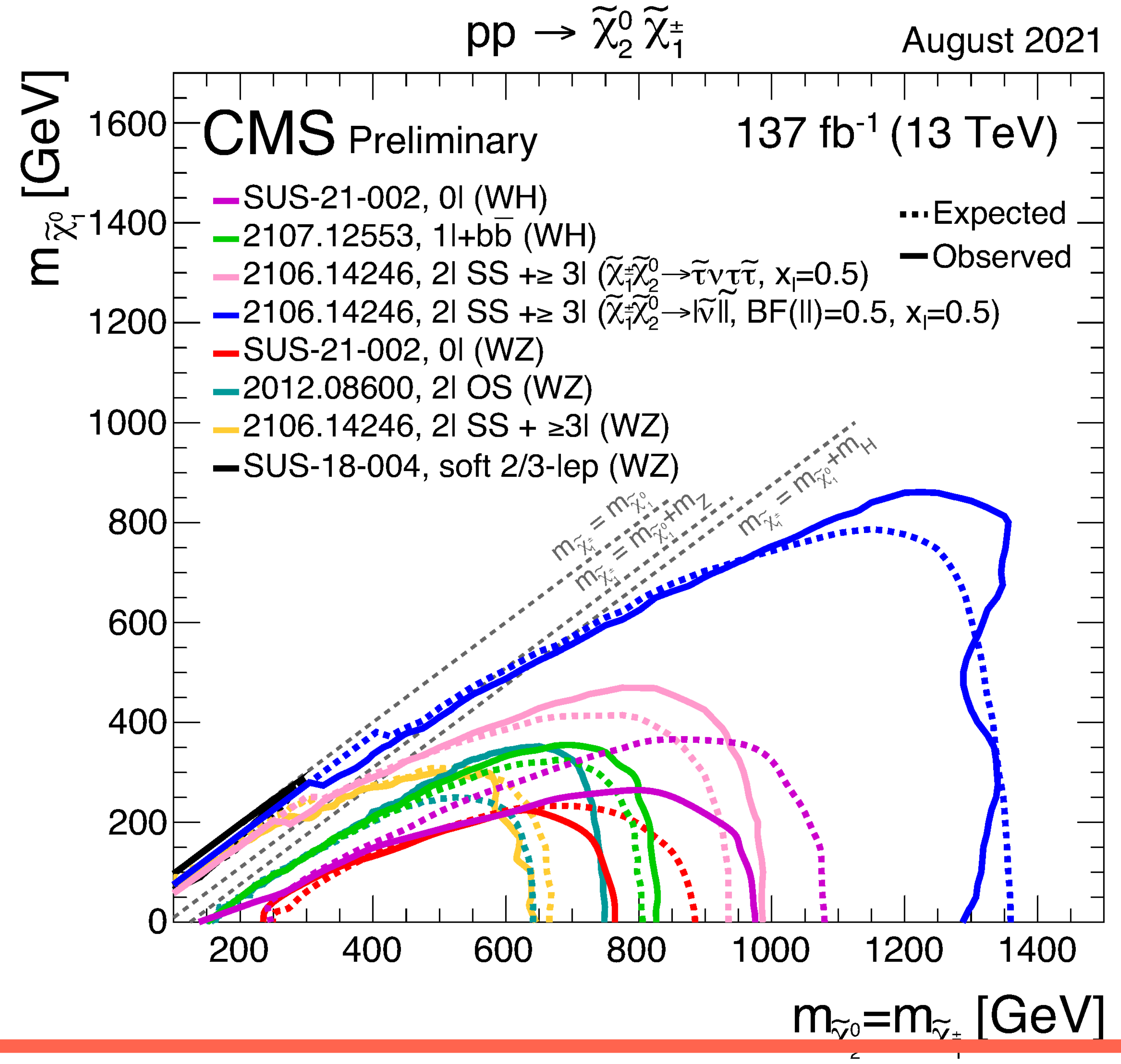

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Furthermore...

The current analysis relies on some simplifying assumptions. It is hard to make a conclusion in full generality due to the large number of parameters.

- Our nature seems not explained by the MSSM in the simplest way but doesn’t rule it out.

- More models: Next-to-Minimal Supersymmetric Standard Model, Split supersymmetry, …

Example II: Holographic Duality and Black Hole Entropy

Black Hole Entropy:
The Heart of Quantum Gravity

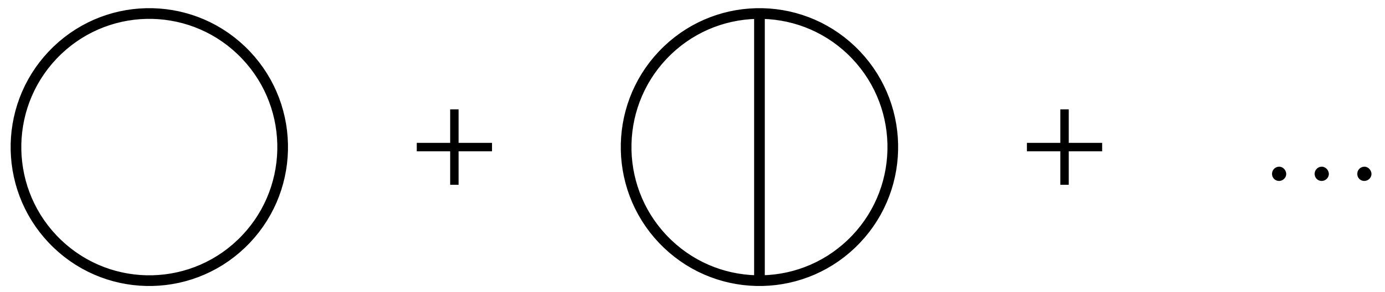
Unruh Effect

- Although we don’t know the complete quantum theory of gravity, one can consider *quantum field theory of matters* in curved (still *classical*) spacetime.
- Surprisingly, the vacuum in QFT is an observer dependent notion.
- E.g., the Unruh effect

$$|vac\rangle_{inertial} \quad = \quad |thermal\rangle_{accelerating}$$

- In QFT, even the vacuum state receives quantum corrections

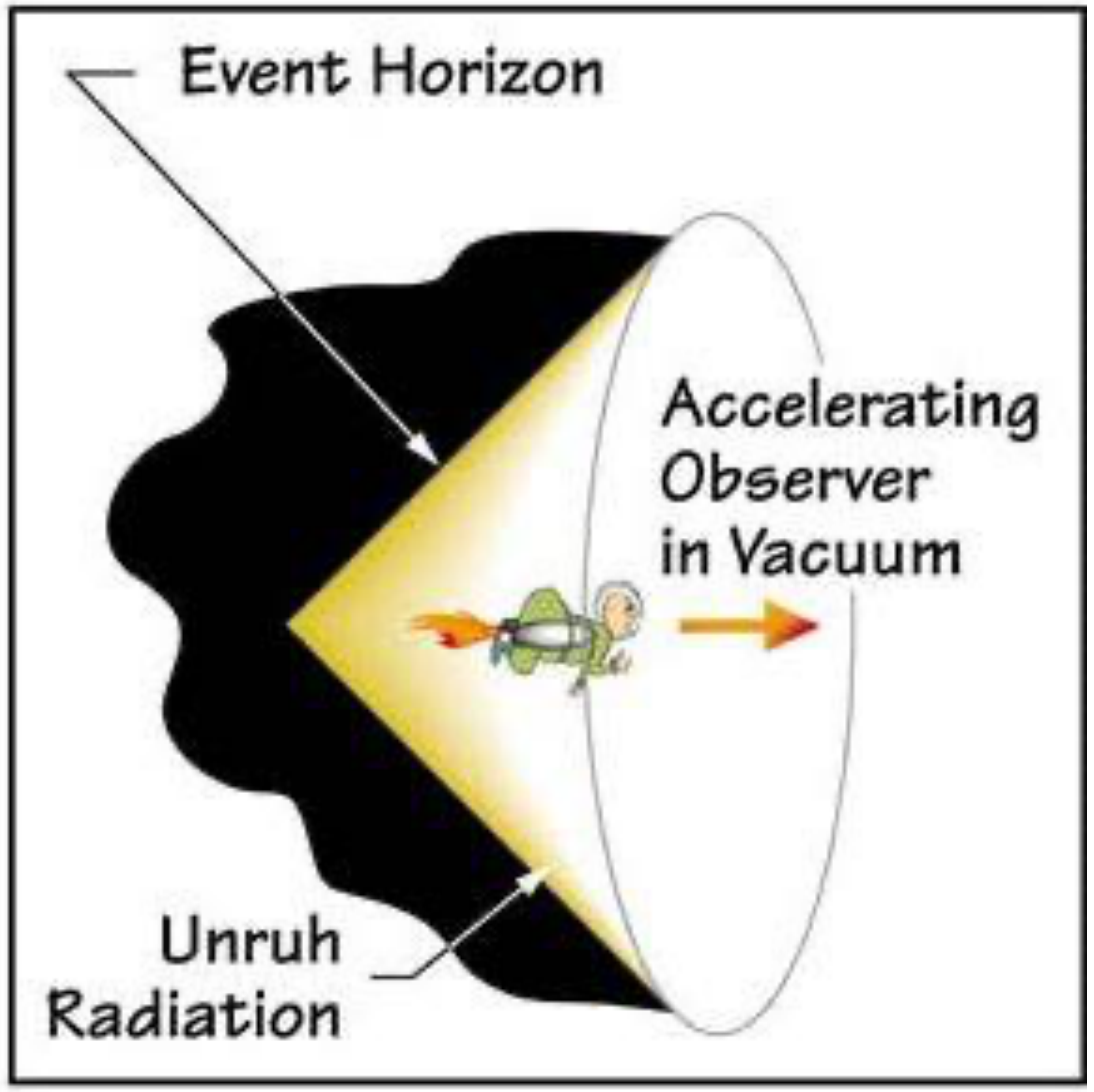
$$\langle vac \,|\, vac \rangle =$$



- A pair of a virtual particle and an anti-particle repeatedly appear and disappear.

- An accelerating observer in the Minkowski spacetime has a Rindler event horizon.

- From the observer’s perspective, if the virtual particle falls behind the horizon, its anti-particle is left alone and cannot be annihilated, resulting in thermal radiation from the horizon.

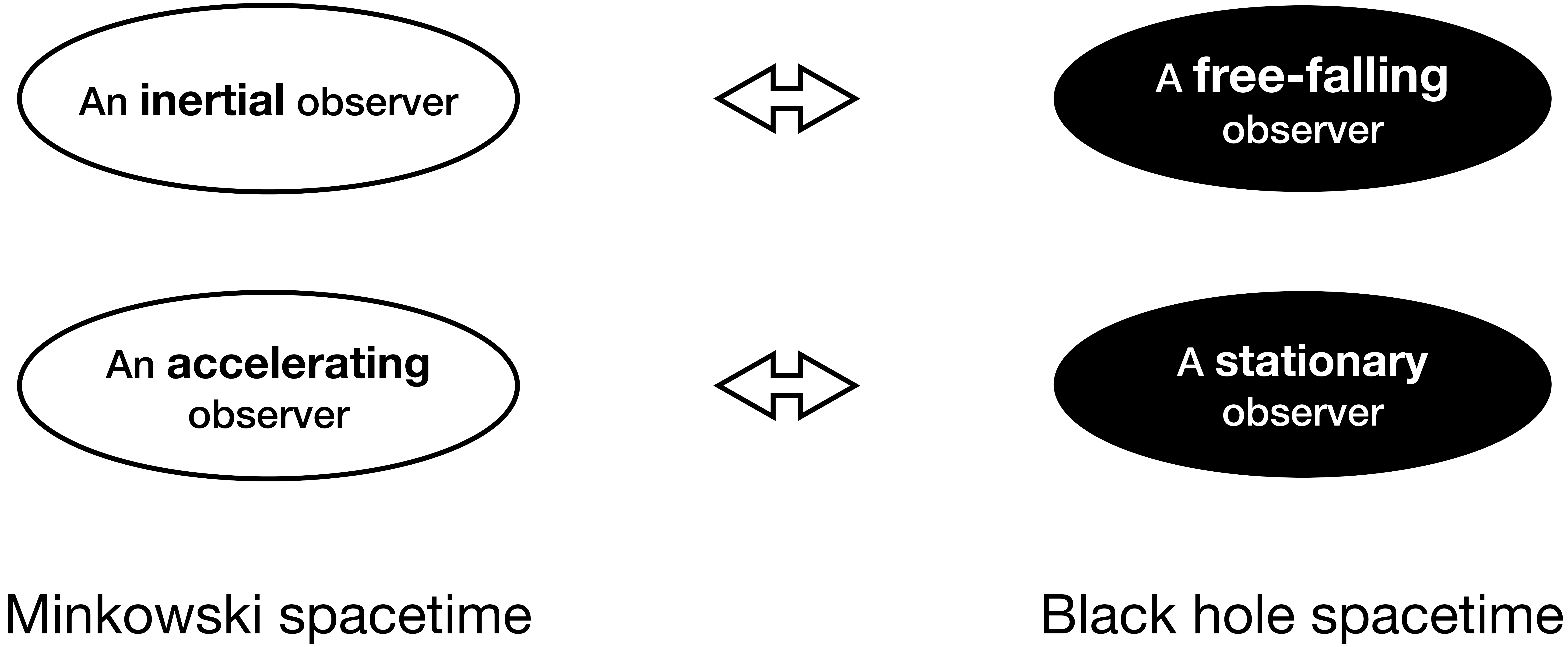


An accelerating observer in vacuum would see a similar Hawking-like radiation called Unruh radiation.

$$T_{Unruh} = \frac{\hbar a}{2\pi c k_B}$$

Hawking Radiation and Black Hole Entropy

- Exactly the same phenomenon occurs in a black hole spacetime.



An **inertial** observer

A **free-falling** observer

An **accelerating** observer

A **stationary** observer

Minkowski spacetime

Black hole spacetime

Hawking Radiation and Black Hole Entropy

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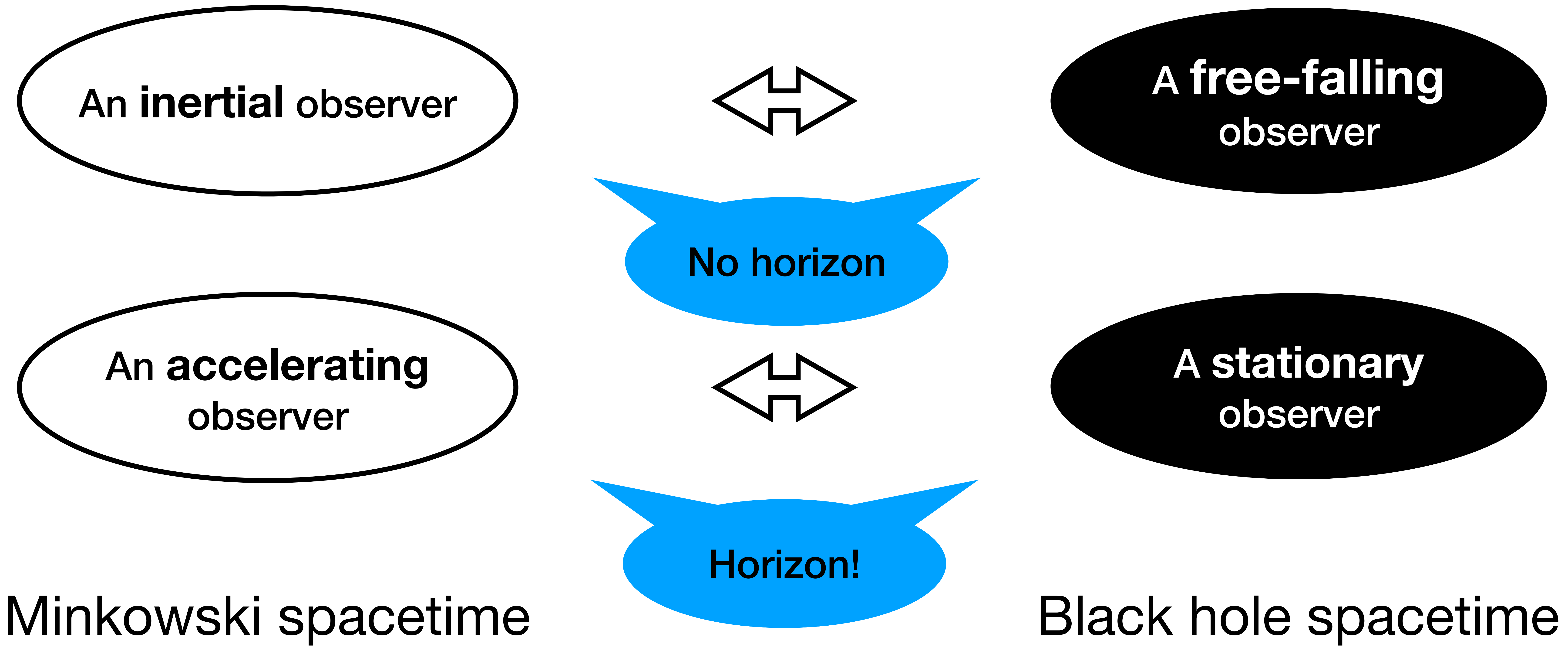


An **inertial** observer ⇔ A **free-falling** observer

No horizon

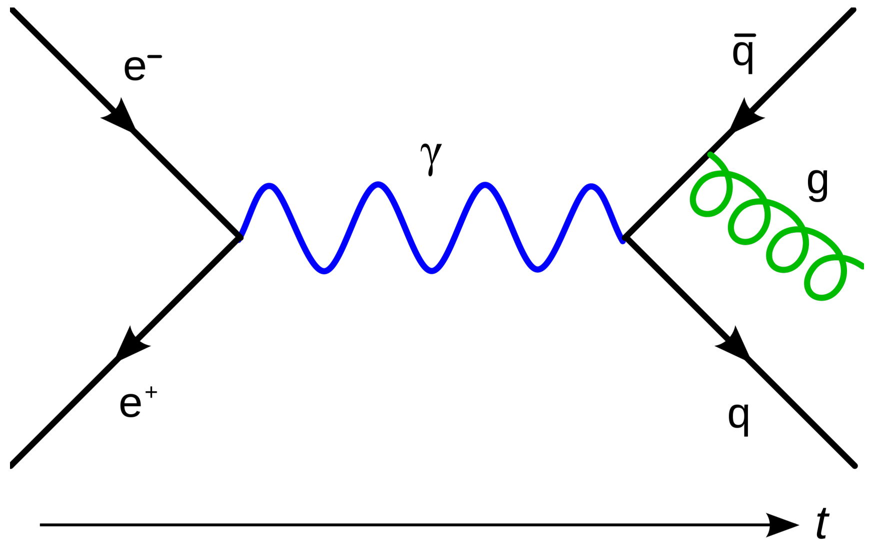
An **accelerating** observer ⇔ A **stationary** observer

Horizon!

Minkowski spacetime

Black hole spacetime

- **Quantum** field theory + the **curved** spacetime -> the Hawking radiation (1974)



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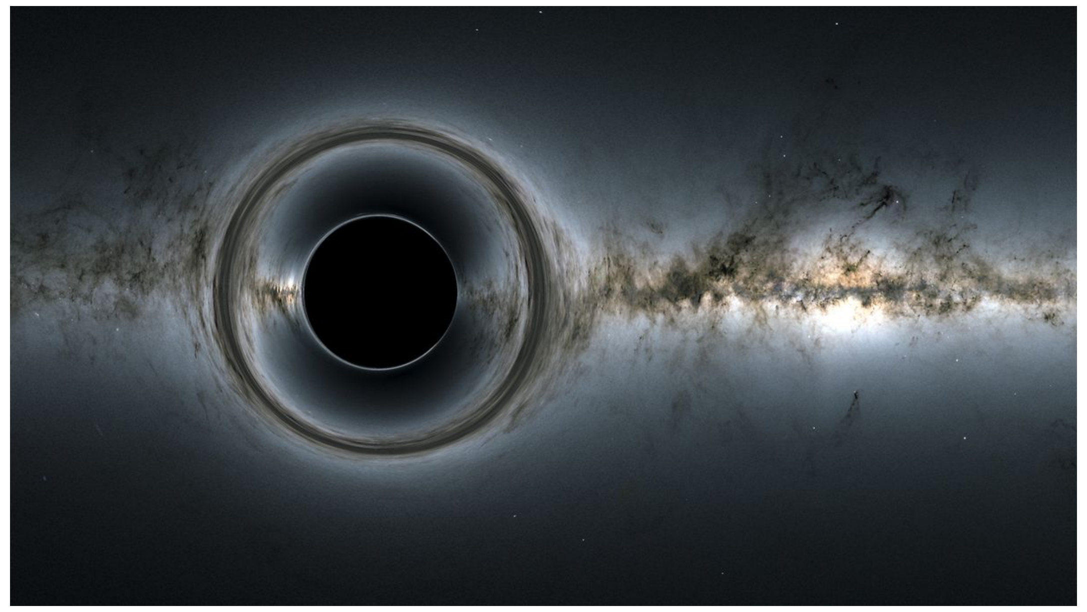

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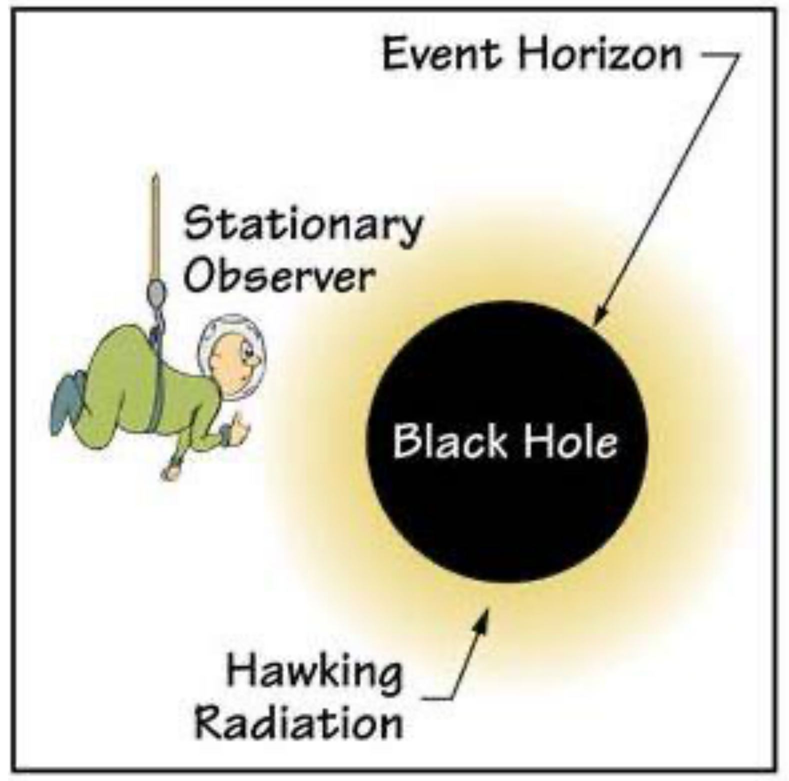

A stationary observer outside the black hole would see the thermal Hawking radiation.

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}, \qquad S_{BH} = \frac{k_B A c^3}{4 G \hbar}$$

A black hole is not black!

- A black hole is an thermal object.
- The microstates of a black hole? $e^{S/k_B} \sim e^{10^{44}}$ for 10 $M_\odot$.
- However, everything is squeezed into a single point in a (Schwarzschild) black hole.
- Where does this vast number of states come from?

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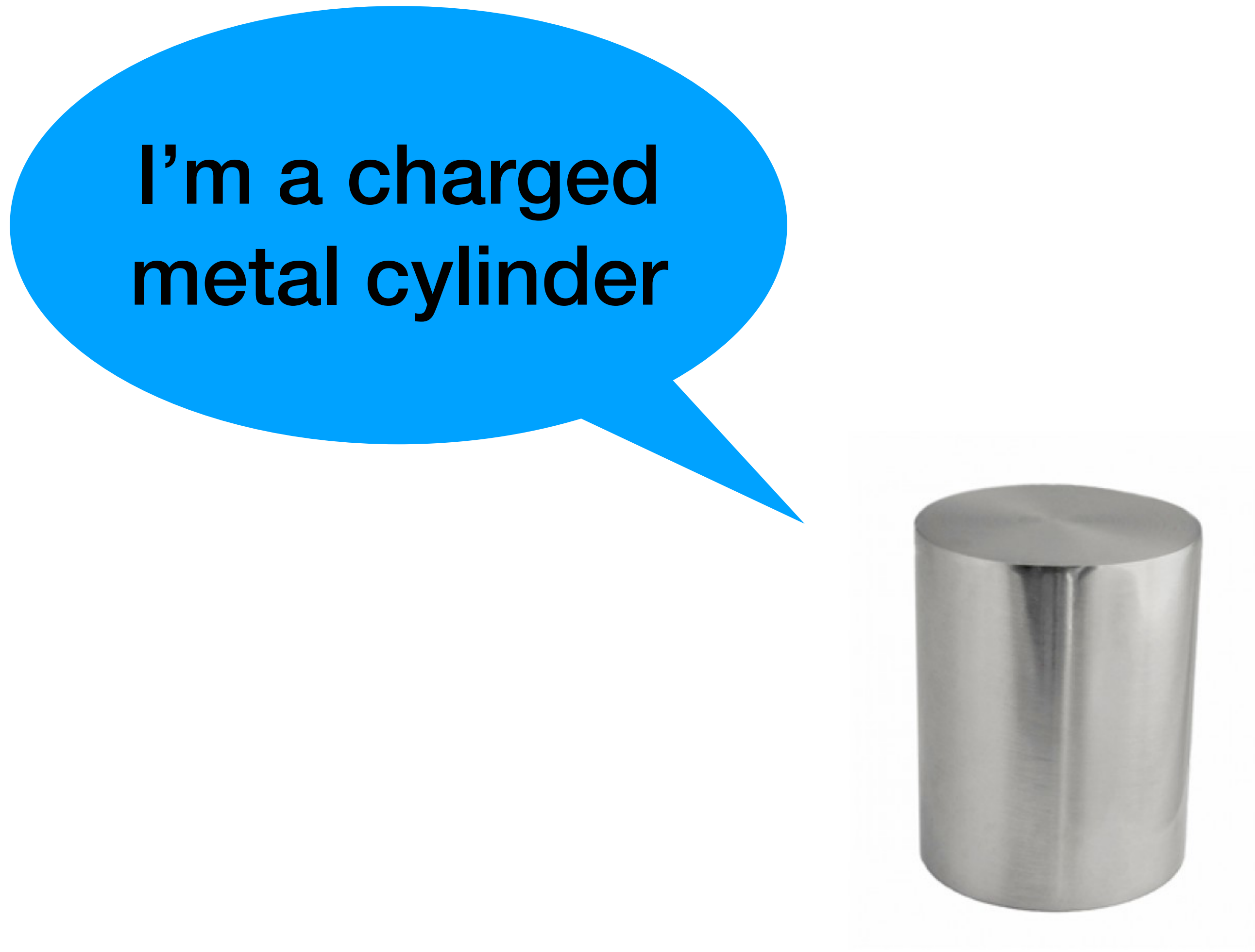
Quantum nature of the spacetime itself!

- A black hole entropy is a key property that any quantum theory of gravity must be able to explain.

- Our current understanding?


I'm a charged metal cylinder

I’m a charged metal cylinder

I’m a spherical chicken

I’m a charged metal cylinder

What is the electric field I’m producing?

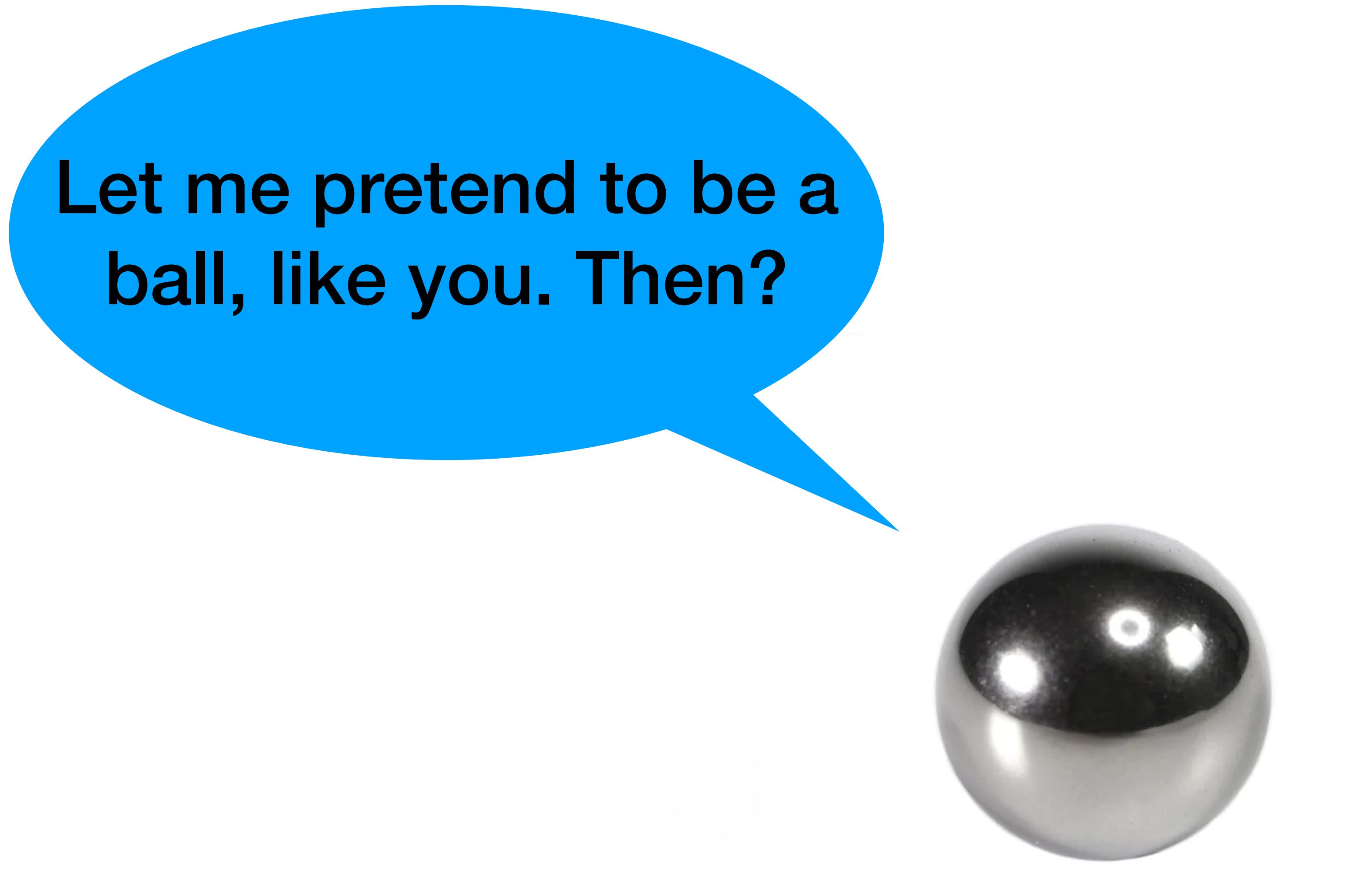
I’m a spherical chicken

I’m a charged metal cylinder

What is the electric field I’m producing?

I’m a spherical chicken

I don’t know


Let me pretend to be a ball, like you. Then?

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Easy!

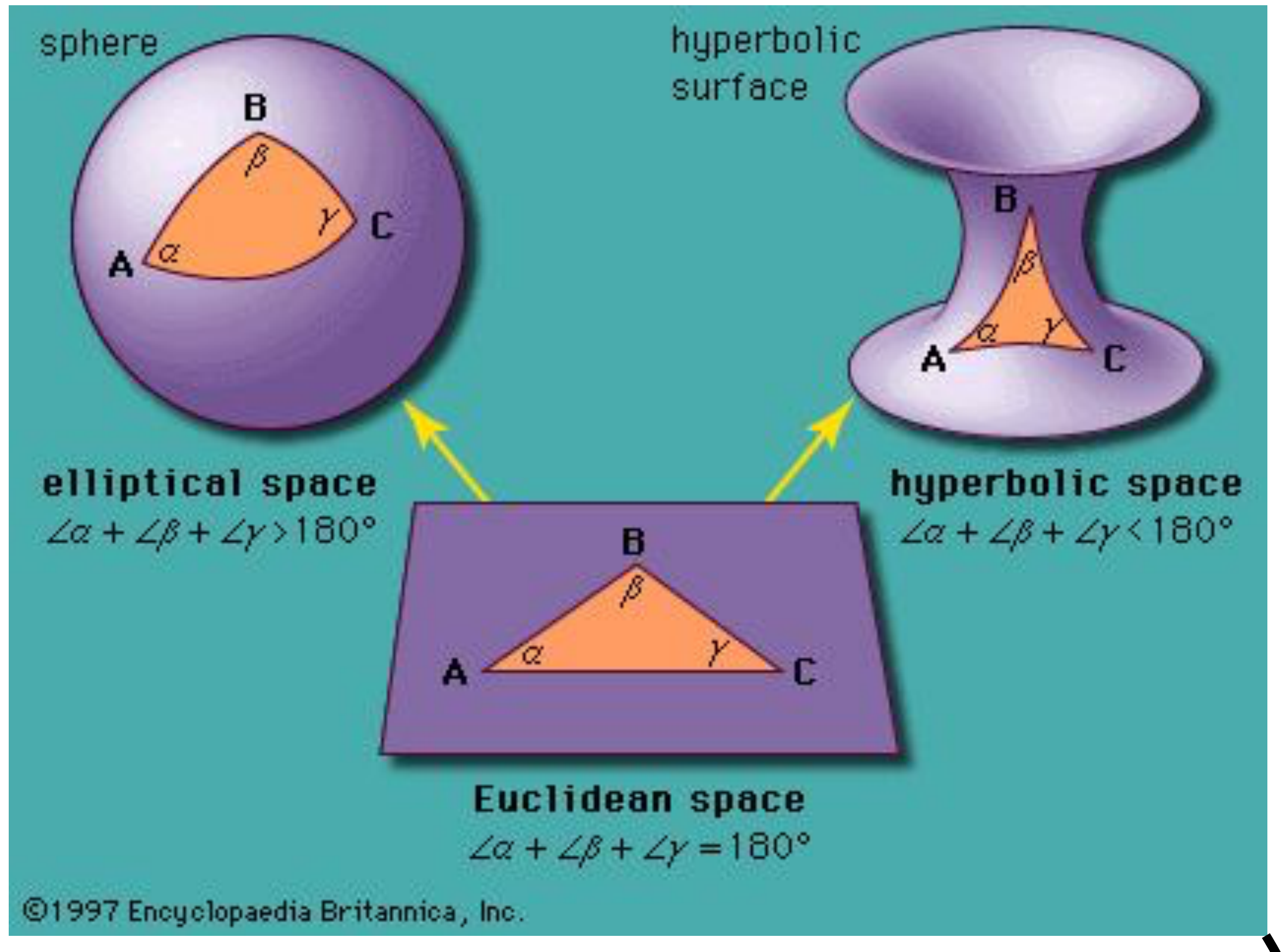
- Let’s consider the most symmetric (but exotic) black hole.

- A **supersymmetric** black hole in the 5-dimensional **Anti-de Sitter** spacetime.

- Let’s consider the most symmetric (but exotic) black hole.
- A **supersymmetric** black hole in the 5-dimensional **Anti-de Sitter** spacetime.

Three types of spacetime with the constant curvature

- De Sitter (positive curvature)
- Minkowski (vanishing curvature)
- Anti-de Sitter (negative curvature)



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- Let’s consider the most symmetric (but exotic) black hole.
- A **supersymmetric** black hole in the 5-dimensional **Anti-de Sitter** spacetime.

Spacetime version

Three types of spacetime with the constant curvature

- De Sitter (positive curvature)
- Minkowski (vanishing curvature)
- Anti-de Sitter (negative curvature)

- Let’s consider the most symmetric (but exotic) black hole.

- A **supersymmetric** black hole in the 5-dimensional **Anti-de Sitter** spacetime.

Why AdS?

- The Hawking temperature of a black hole in the flat spacetime:

$$T_H = \frac{\hbar c^3}{8\pi G M k_B}$$

- Negative specific heat.
- On the other hand, the AdS spacetime is a gravitational potential trap.
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AdS is a nice playground to study black hole thermodynamics!

Why Supersymmetry?

As mentioned, the largest spacetime symmetry. Furthermore…

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The AdS/CFT correspondence

- A ground breaking duality between d-dimensional QFT and d+1-dimensional gravity (Maldacena 1997)
- The most cited paper in High-Energy Physics (>20,000 citations, HEP database INSPIRE)

String theory picture

String length
# of D3-branes
String coupling

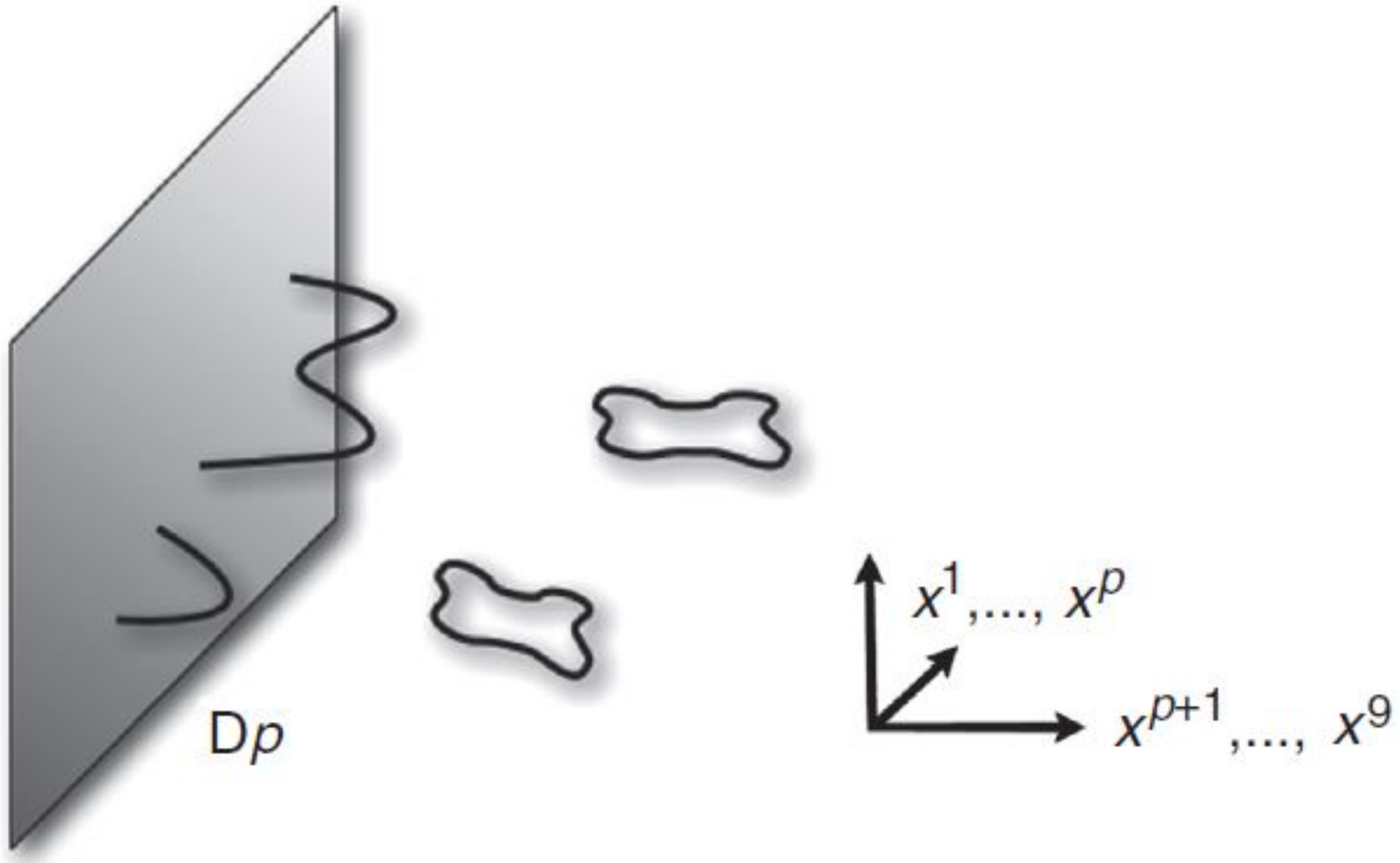

Figure 6.1 String theory in the presence of a Dp-brane, along the directions $x^0, \ldots, x^p$. The closed string sector describes the fluctuations of the theory around the vacuum, while the sector of open strings characterizes the fluctuations of the non-perturbative object.

Zwiebach, "A First Course in String Theory"

String theory picture

String length -> 0
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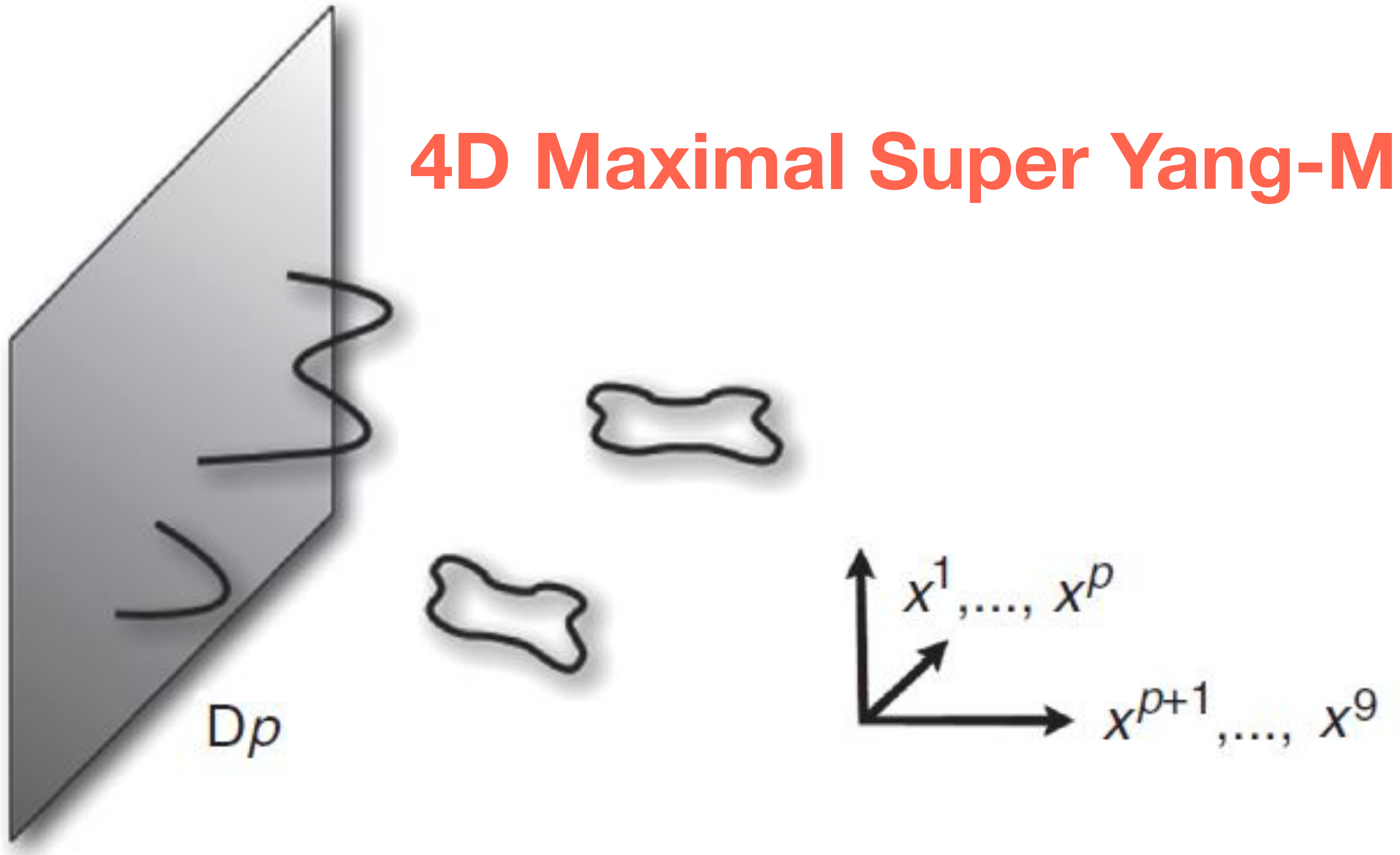

4D Maximal Super Yang-Mills (MSYM)

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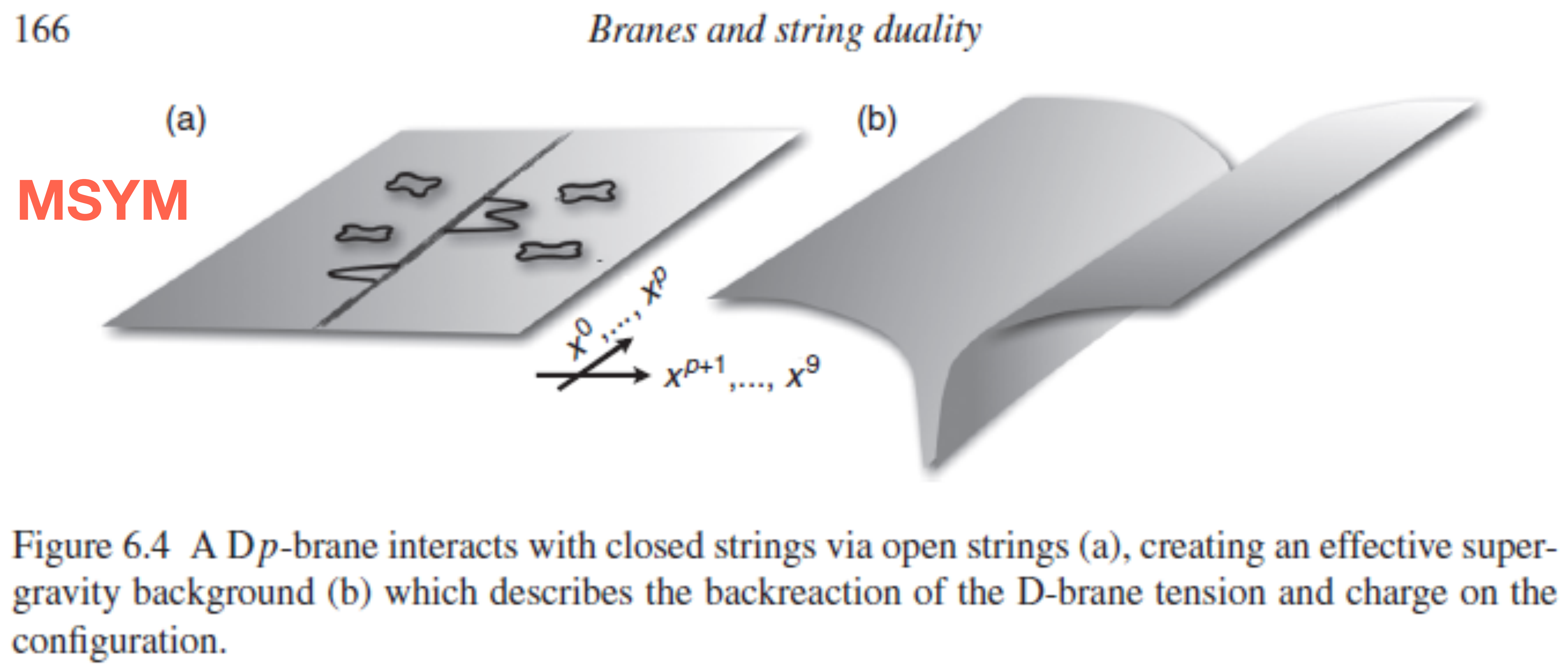

166 *Branes and string duality*

4D MSYM

Figure 6.4 A Dp-brane interacts with closed strings via open strings (a), creating an effective supergravity background (b) which describes the backreaction of the D-brane tension and charge on the configuration.

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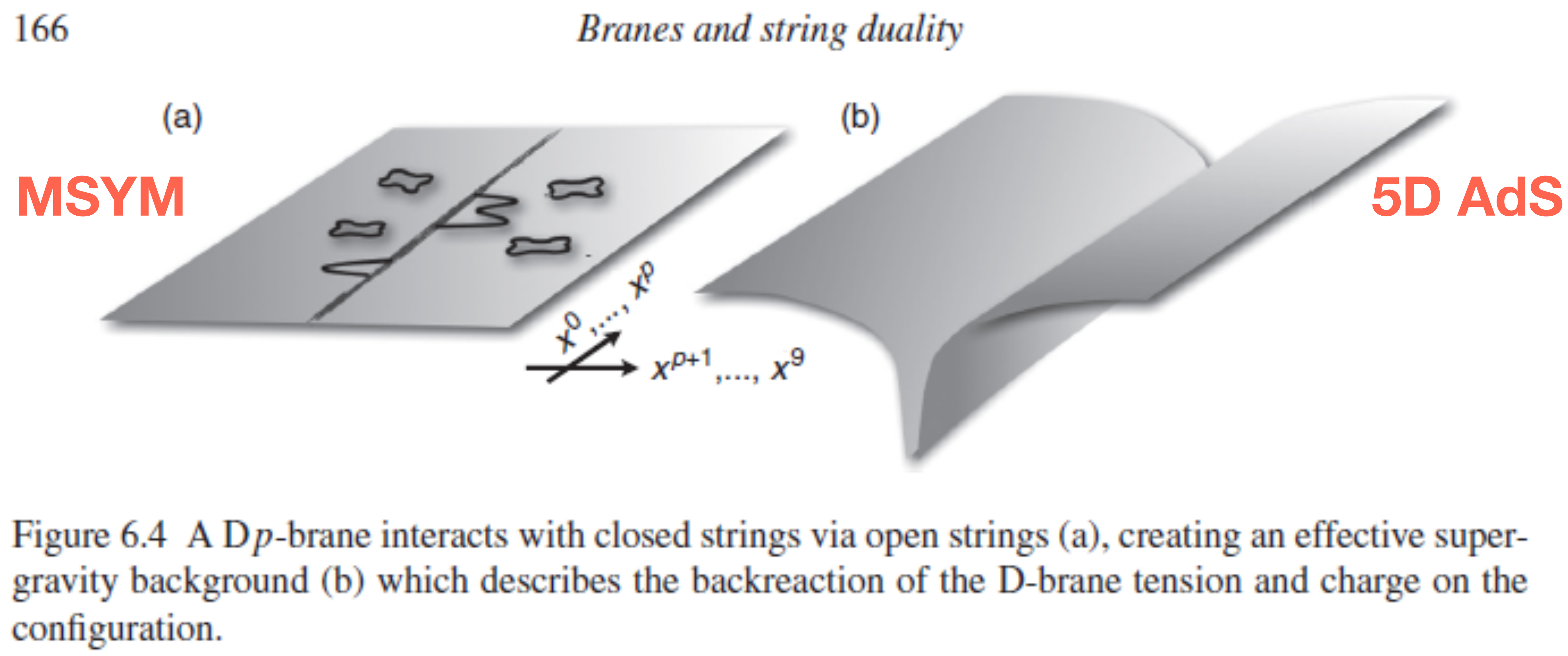

166 *Branes and string duality*

4D MSYM

5D AdS Gravity

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4D MSYM

5D AdS Gravity

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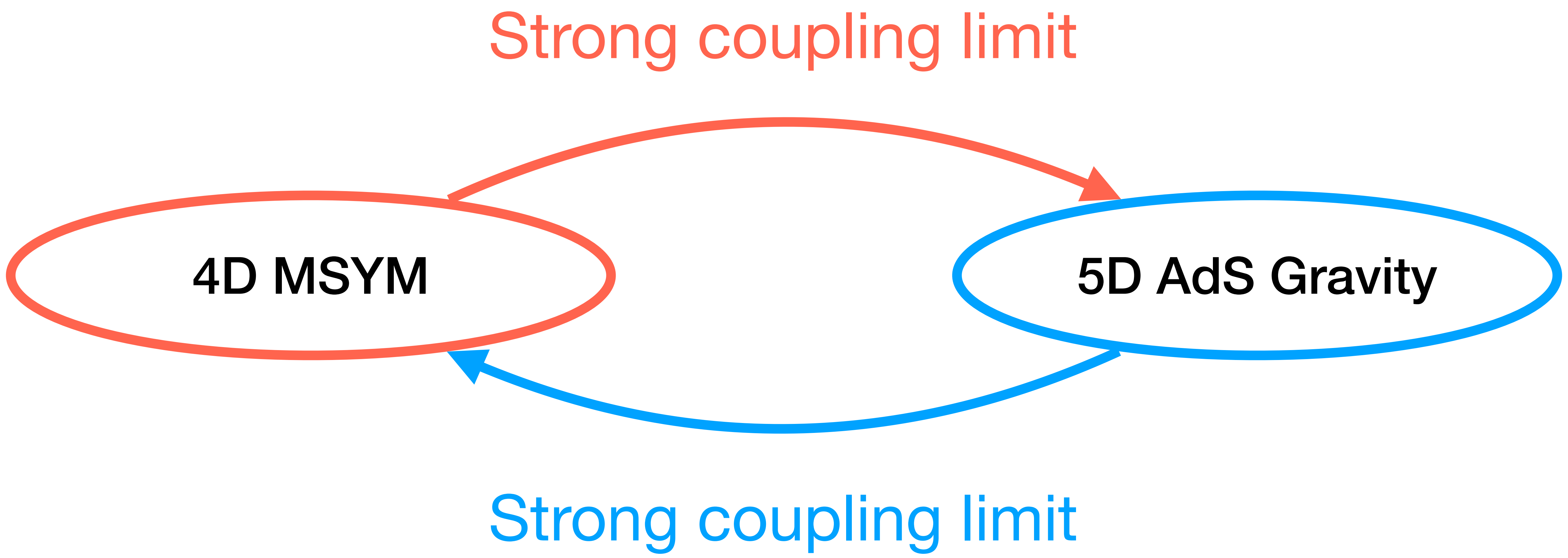


Strong coupling limit

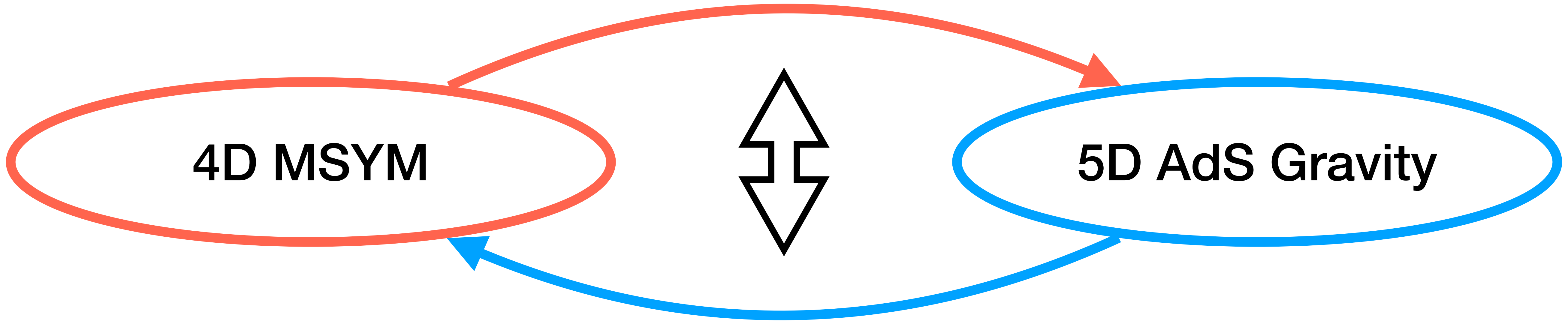
4D MSYM

5D AdS Gravity

Strong coupling limit

String coupling

0 <-------------> ∞



Strong coupling limit

4D MSYM

5D AdS Gravity

Strong coupling limit

AdS/CFT correspondence

AdS/CFT Correspondence

	4d MSYM	5d AdS Gravity
More (Less) **D3-Branes**	More (Less) Colors	Small (Large) Quantum Correction
Strong (Weak) **String Coupling**	Strong (Weak) ('t Hooft) Coupling	Small (Large) Higher Curvature Correction

- Weakly coupled 5d AdS gravity is equivalent to strongly coupled 4d MSYM, and vice-versa.

- Quantum corrections to the gravity correspond to 1/N corrections to the MSYM.

Microstates of an AdS Black Hole

- Black hole entropy is obtained from the horizon area of the (classical) black hole solution.

- A consistent quantum gravity should provide a microscopic explanation of this entropy.

- Via the AdS/CFT correspondence

$$Z_{\text{(strongly coupled) 4d MSYM}} = Z_{\text{5d AdS (Einstein) gravity}}$$

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Hard

Partition Function vs Index

$$Z_g \quad = \quad \mathrm{Tr}_g\, e^{-\beta H - \mu Q} \qquad \longrightarrow \qquad \text{Hard}$$

Partition Function vs Index

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(Again, magic of supersymmetry)

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(Again, magic of supersymmetry)

If the cancelation is not too large, the index would also capture enough microstates to reproduce the black hole entropy.

Entropy of an AdS5 Black Hole

- With some computational techniques,

$$\log \hat{Z} \quad = \quad \frac{N^2}{2} \frac{\Delta_1 \Delta_2 \Delta_3}{\omega^2}$$

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Legendre transformation $\Big\downarrow$ $S = \log \hat{Z} + \sum_i \Delta_i Q_i + \omega J$

$$S \quad = \quad 2\pi\sqrt{Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - N^2 J}$$

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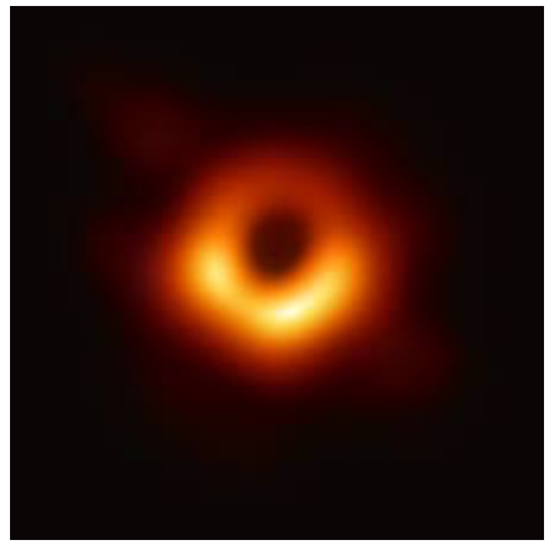
Exactly match the geometric entropy of the black hole solution!

QFT

GR

Strong QFT

A non-perturbative description of QFT

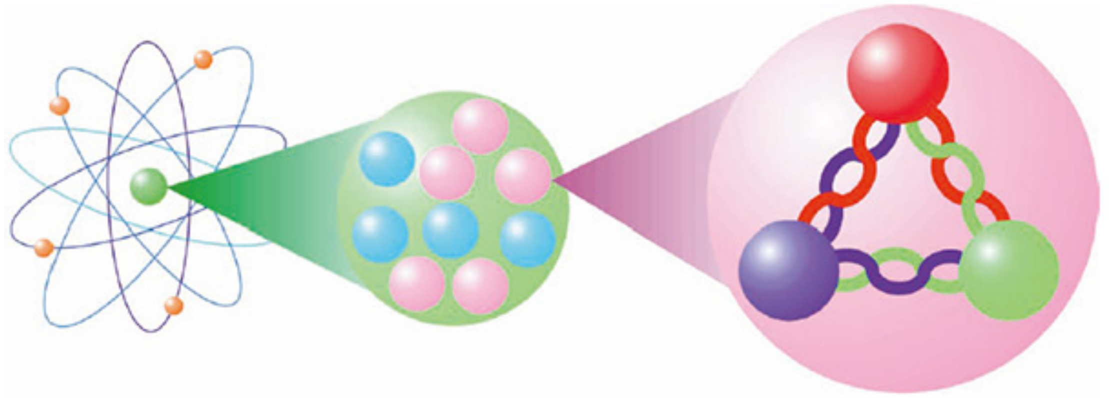


Weak gravity

AdS/CFT correspondence

⇔

Weak QFT



A non-geometric description of gravity

Strong gravity

QFT

GR

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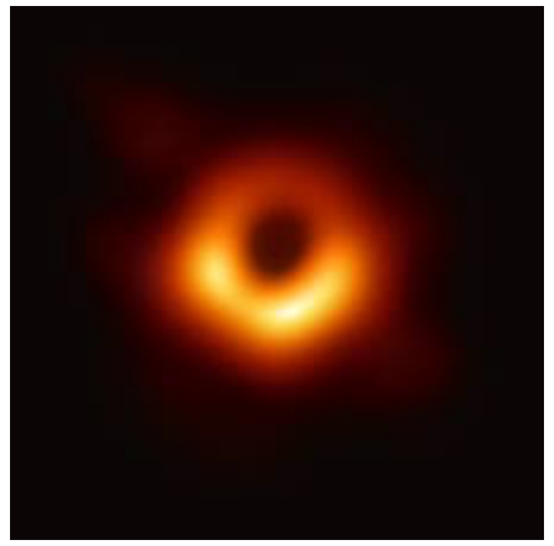


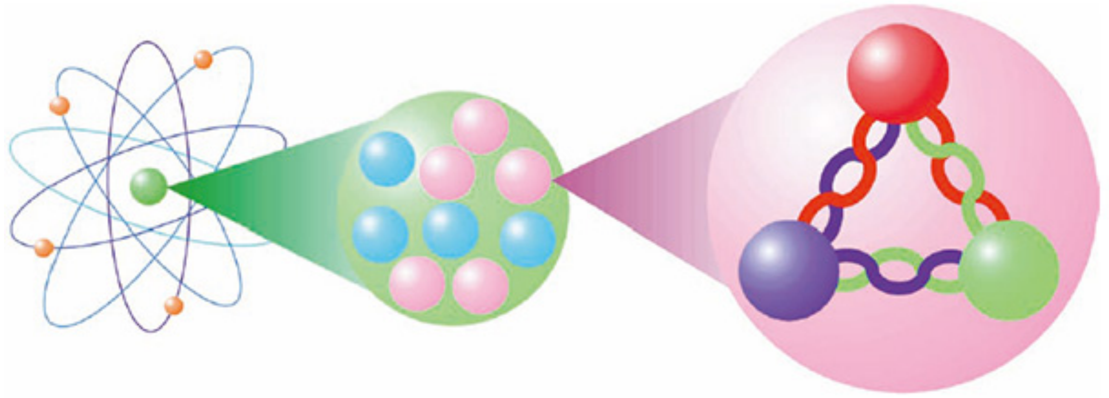
Weak gravity

? = **A black hole**

↑ ↓

A thermal ensemble of perturbative quantum states = **?**

Weak QFT



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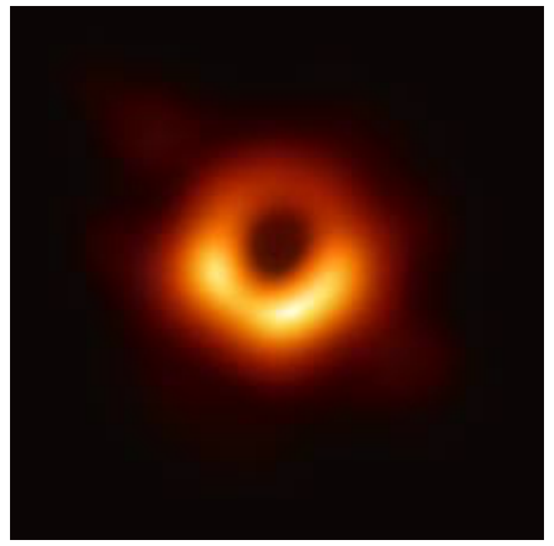

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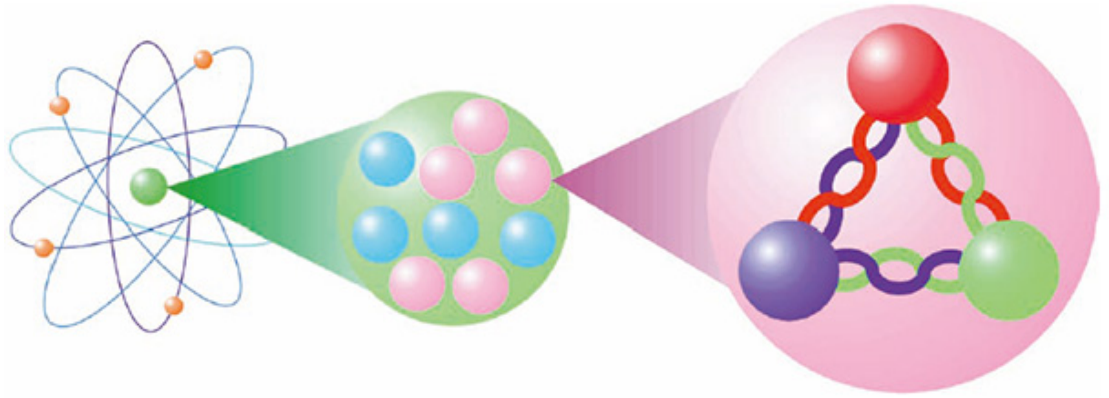

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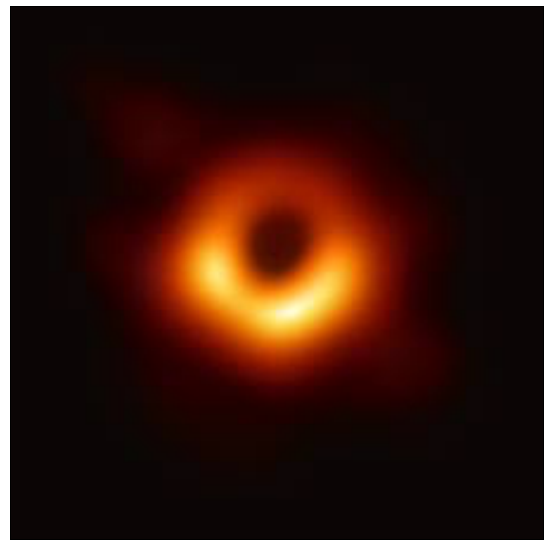

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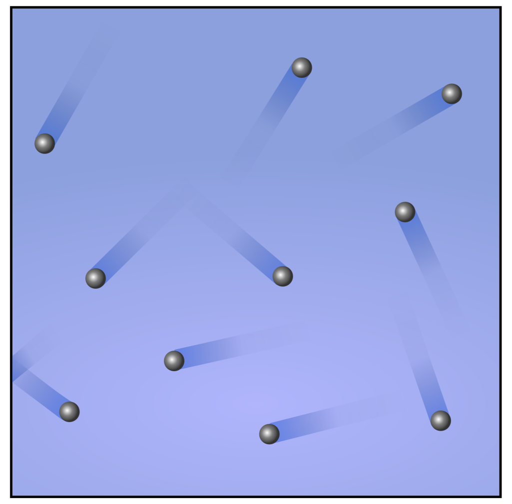

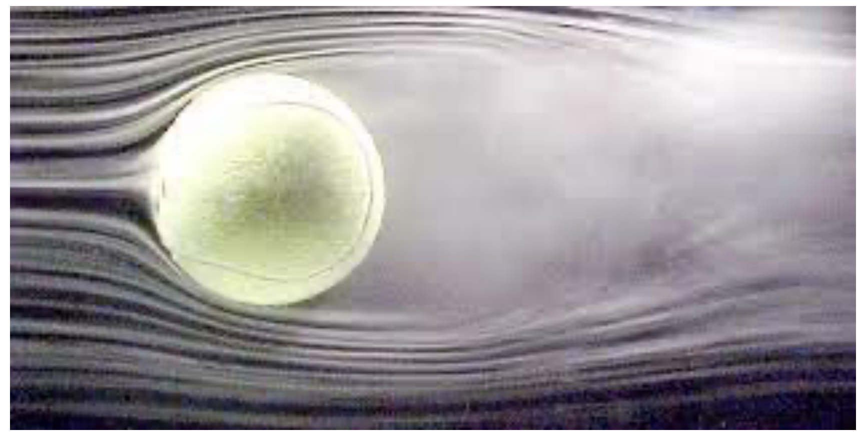

Strong gravity

- Considering a supersymmetric black hole, we derive the black hole entropy microscopically, which is a key feature of quantum gravity.

- More applications to strongly coupled theories, exhibiting confinement, duality, and other non-perturbative phenomena.

- New mathematics: mirror symmetry, the Seiberg-Witten theory, …

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Much more to explore in the SUSY world!