# Logistics & Introduction

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# Logistics - 1

Attending a course in English is difficult for young persons who are not too familiar with foreign languages

*I understand your difficulty and appreciate your effort*
*The world of HEP (High Energy Physics) is a world-wide collaboration and English is the standard tool of communication.*
*Attending these lectures will help you to improve your foreign language skills (… and studying abroad?)*

Course will last 16 weeks, Lectures by
- me, (Antonio) Toni Baroncelli from February 23rd to April 16th
- Prof. Haiping Peng from April 21st to June 11th

Two lectures/week
- Monday, 3 slots, 15:55-18:20
- **Wednesday 2 slots, 14:00-15:35**

| Topic | Weeks | Who | from | → | # lectures |
|---|---|---|---|---|---|
| Introduction to basic concepts | 2 | T.Baroncelli | 23/02/25 | 05/03/25 | 4 |
| Deep Inelastic Scattering | 1 | T.Baroncelli | 02/03/25 | 12/03/25 | 6 |
| Accelerators | 1 | T.Baroncelli | 09/03/25 | 19/03/25 | 8 |
| Detectors | 1 | T.Baroncelli | 16/03/25 | 26/03/25 | 10 |
| Measurements at Colliders | 3 | T.Baroncelli | 06/04/25 | 16/04/25 | 16 |
| | | | | | |
| Standard Model Theory | 2 | H.Peng | 21/04/25 | 30/04/25 | 4 |
| CPV theory and experiment (BELLE,BABAR, LHCb) | 2 | H.Peng | 05/05/25 | 14/05/25 | 8 |
| Hadron physics (BESIII, STCF) | 2 | H.Peng | 19/05/25 | 28/05/25 | 12 |
| Higher Symmetries (GUT, SUSY, Superstrings....) | 2 | H.Peng | 02/06/25 | 11/06/25 | 16 |

Slides will be made available soon after the lecture at http://cicpi.ustc.edu.cn/indico/.... Will be defined soon

Each Lecture will be preceded by a short recap of the lecture before

# Logistic - 2

*The course is not historically-organised*

*First part*

- Overall picture of how we see (today!) the microscopic world;
- How laws and structure of nature can be represented by models / mathematical formalism;
- Little formalism, just main ideas. Much more material can be found in the reference book.

*Second part*

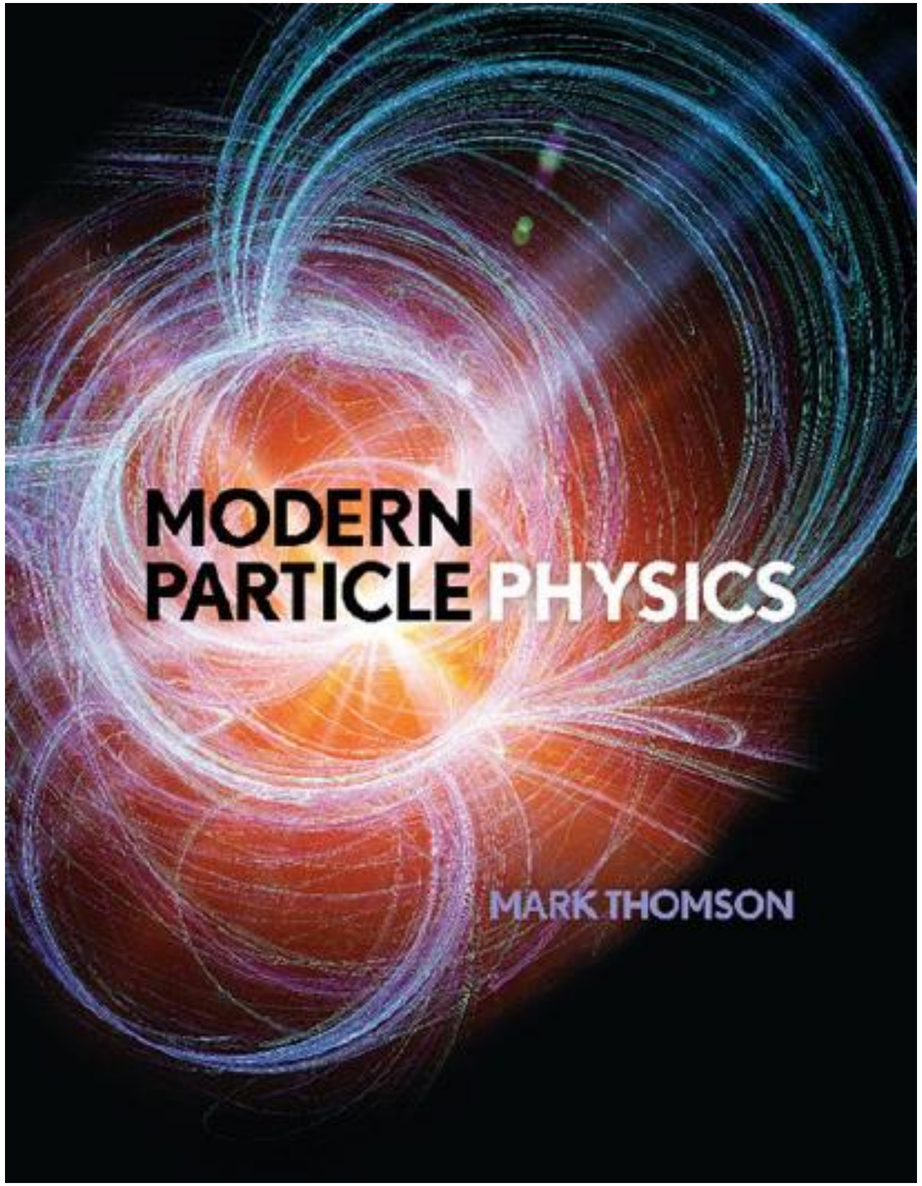
- Instruments and tools of the research in High-Energy Particle Physics (HEP)
    - Accelerators
    - Detectors and Analysis
    - Analysis of discoveries of the past 50 years

# Reference Textbook

Lectures of the first part: recent book including much more than in these lectures

- Standard Model
- Discovery of the Higgs Boson
- …

Formalism well documented




**Modern Particle Physics**

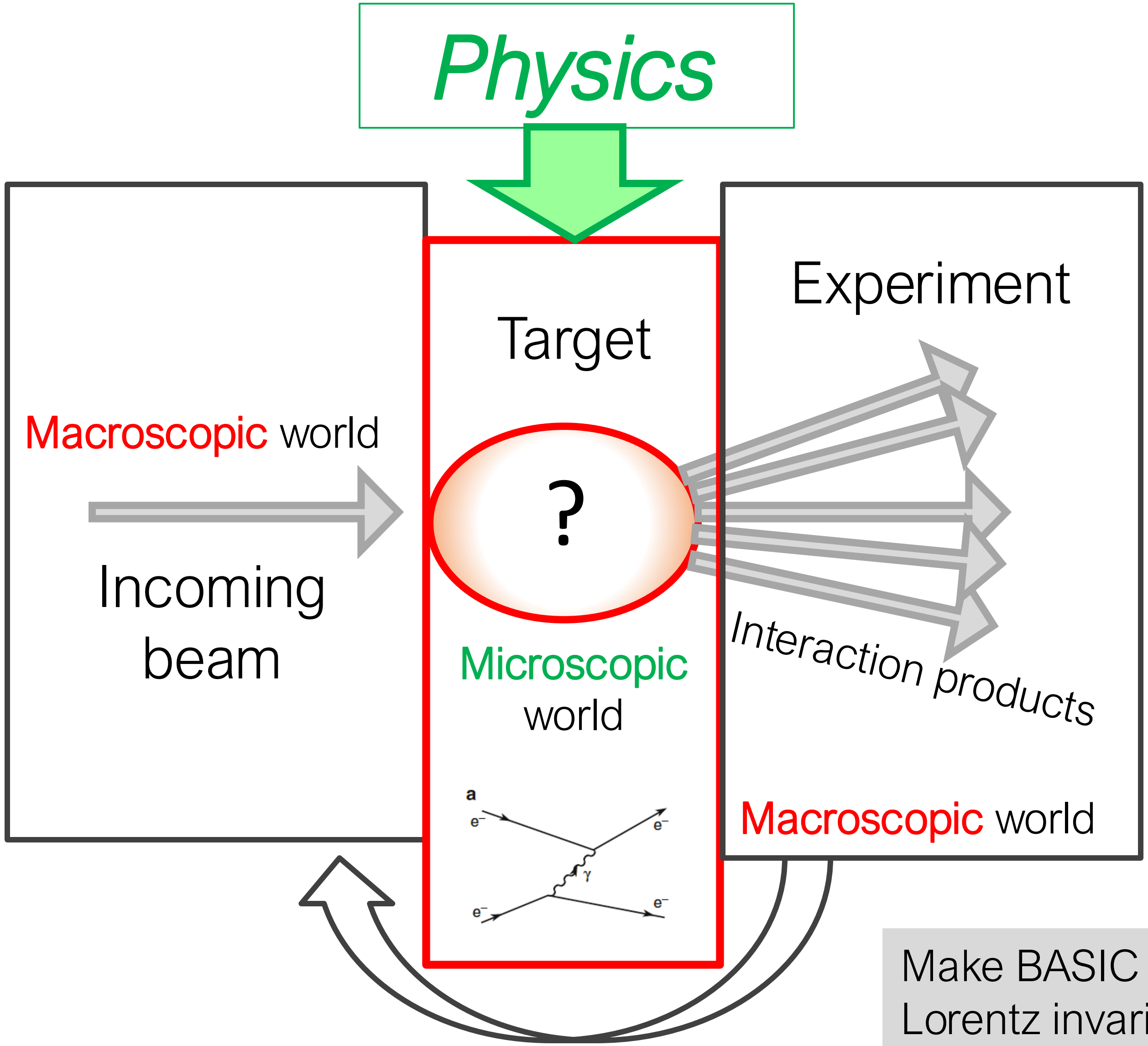
MARK THOMSON
University of Cambridge

CAMBRIDGE UNIVERSITY PRESS

Mark Thomson was recently elected as Director of CERN

# *Very Basic Ideas*

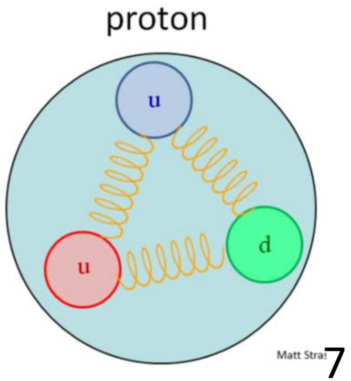
# Micro to Macro world




No way to 'see' what is in the microscopic world → can only see the effect of sending a projectile on your target

'Incoming beam' and 'Target' may be not point-like,

it may have a structure, like a proton (or a nucleus)




Make BASIC assumptions (Einstein special relativity, Lorentz invariance) and use models. Are they OK?

# Prologue: Many Order of Magnitude

(Reduced) *Planck*'s Constant ($\hbar = h/2\pi$) $h$

**The uncertainty principle**: "position x (uncertainty Δx) and momentum $p_x$ (with uncertainty $\Delta p_x$) cannot simultaneously be known to better than

$$\Delta x \Delta p_x \sim \hbar/2.$$

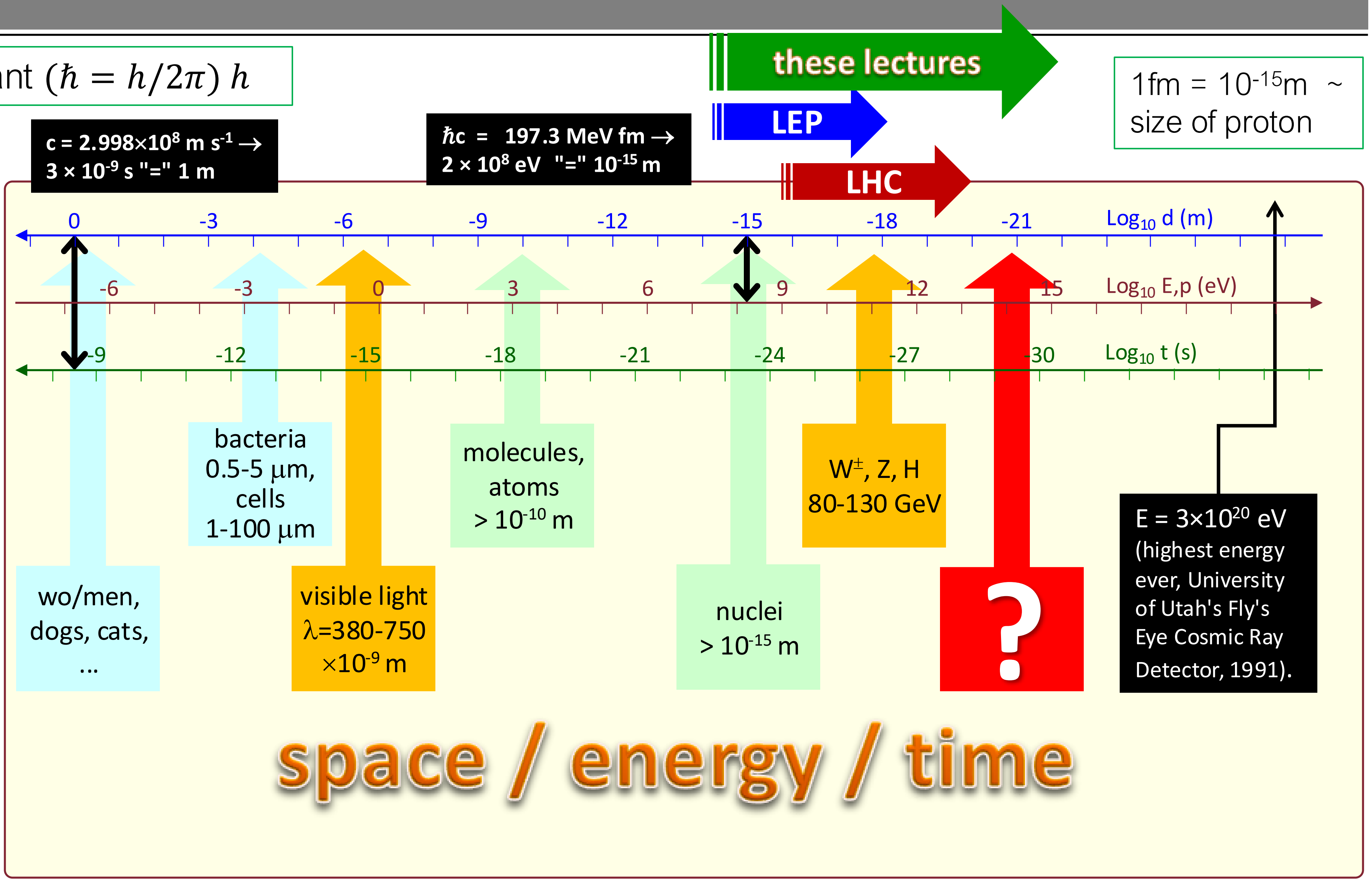
A relation for the energy is obtained by multiplying c,

$$\Delta x \Delta E \sim \frac{\hbar c}{2}$$

which gives numerically,

$$\Delta E(MeV) = \frac{1.973^{-11}(MeV\ cm)}{2\Delta x(cm)}$$

Also $\Delta x = c\Delta t \rightarrow \Delta t \Delta E \sim \frac{\hbar}{2}$

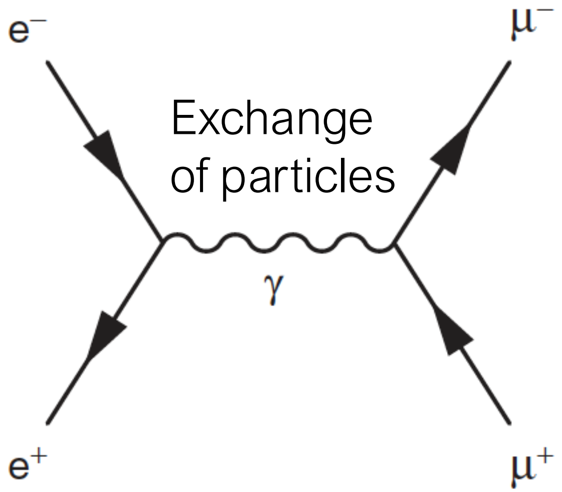


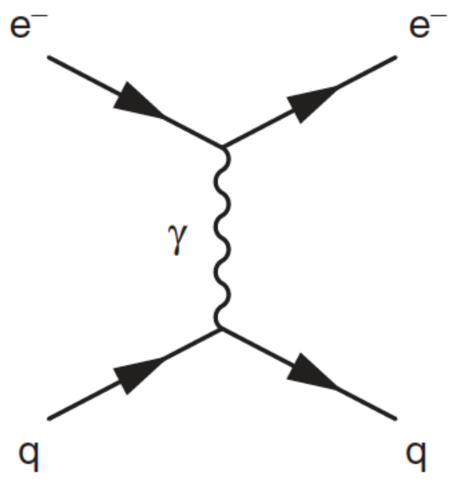
# First Part: Preview of Lectures

Content of the lectures

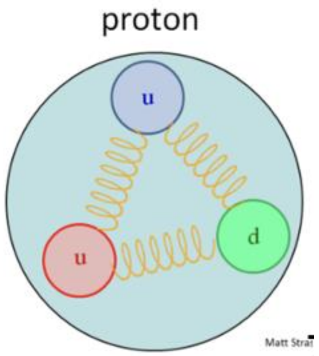
- Calculation of cross sections and decay rates
  - Fermi's golden rule
  - Phase space
- Spin ½ particles (Dirac equation)
  - Klein-Gordon equation
  - Dirac equation
  - Antiparticles
  - Spin & helicity
  - Parity of Dirac Particles
- Interaction by particle exchange
  - Perturbation theory
  - Feynman diagrams & virtual particles
  - QED
- Deep Inelastic Scattering
  - Electron-proton scattering
  - Electron-quark scattering
  - PDFs

Feynman diagrams for $e^+e^- \rightarrow \mu^+\mu^-$ and $e^-q \rightarrow e^-q$ scattering



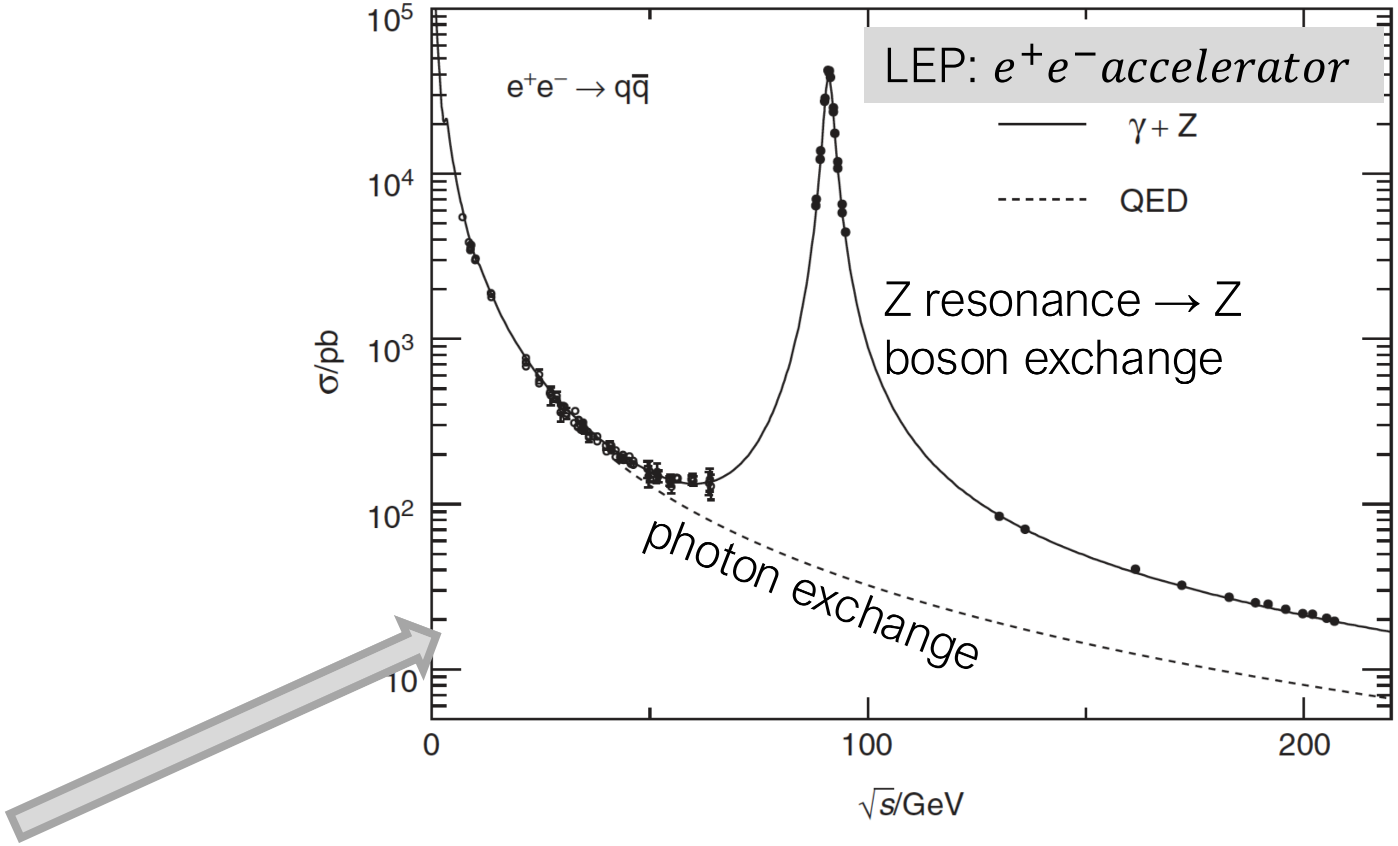

Probe proton structure




LHC: protons against protons

Content of the lectures

- Accelerators (basic ideas, future accelerators?)
- Experiments
  - Assembly of detectors
  - Analysis techniques
- Precision measurements
  - *Resonances*
  - The discovery of $charm\ and\ bottom$ quarks
  - The discovery of the $top$ quark
  - The $Z-line\ shape$ & number of neutrinos @ LEP ($e^+e^-$ collider at CERN)
  - The discovery of the $Higgs\ boson$ (pp collider at CERN)



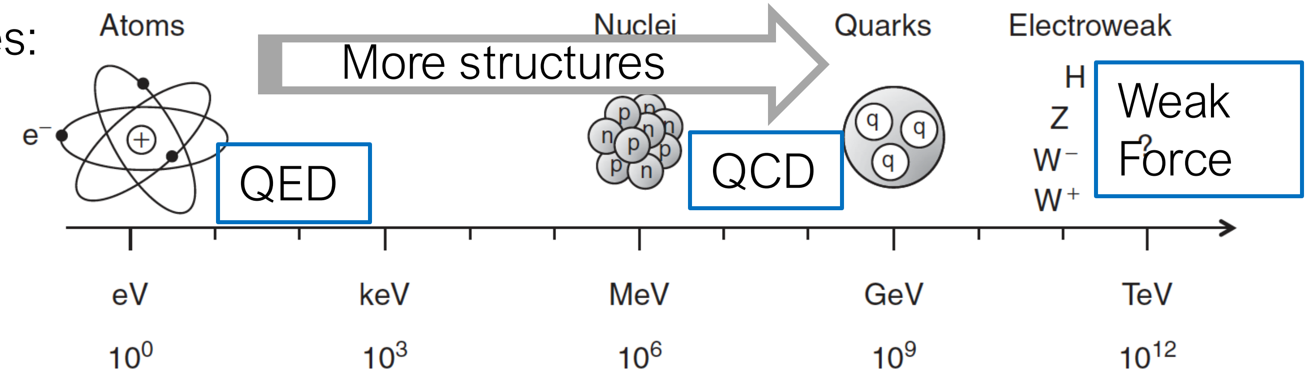

The measurements of the $e^+e^- \to q\bar{q}$ cross section from LEP close to and above Z resonance. Also shown are the lower-energy measurements from earlier experiments. The dashed line shows the contribution to the cross section from the QED process alone. Adapted from LEP and SLD Collaborations (2006).

# Fundamental Particles & Forces

Our world seems to be populated by few objects/particles:

- Atoms p,n & electrons (kept together by em forces)
- Nuclei p,n kept together by strong forces
- Radioactive decays → weak forces
- Gravity → large scale structures in the Universe
- Protons & neutrons → quarks

Elementary particles + Forces → Standard Model




The Universe at different energy scales, from atomic physics to modern particle physics at the TeV scale.

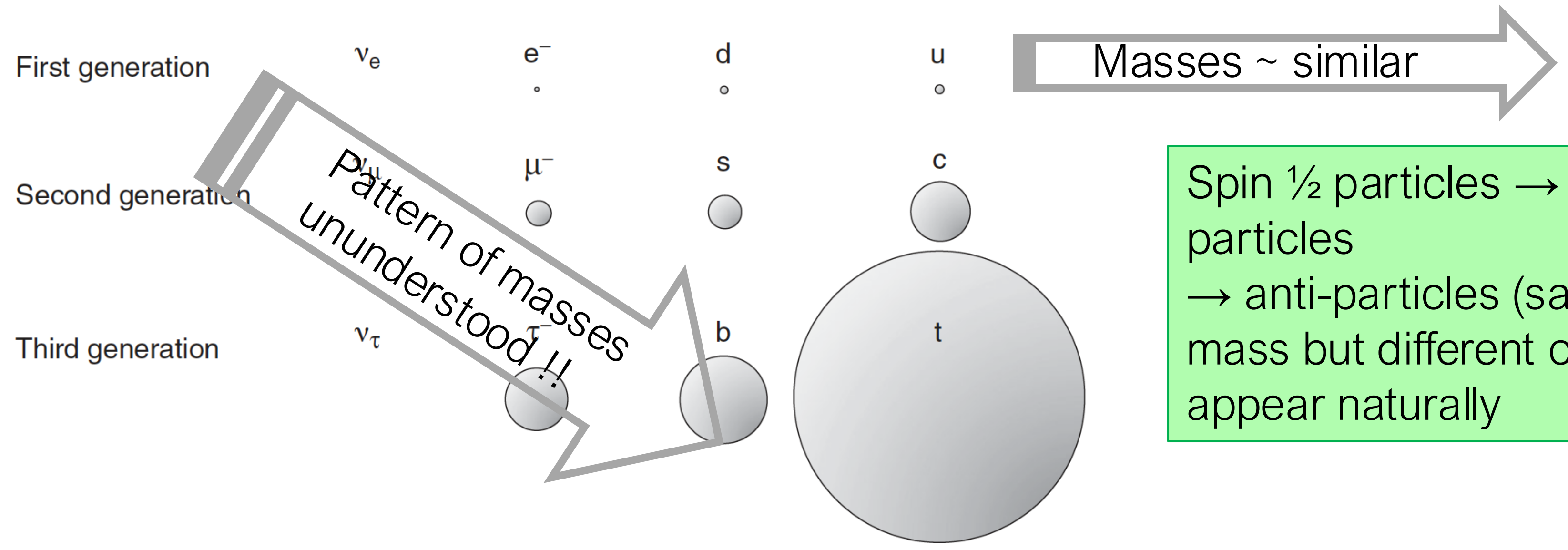
12 particles

**Table 1.1** The twelve fundamental fermions divided into quarks and leptons. The masses of the quarks are the current masses.

| | Leptons | | | | Quarks | | | |
|---|---|---|---|---|---|---|---|---|
| | Particle | | $Q$ | mass/GeV | Particle | | $Q$ | mass/GeV |
| First generation | electron | $(e^-)$ | −1 | 0.0005 | down | (d) | −1/3 | 0.003 |
| | neutrino | $(\nu_e)$ | 0 | $< 10^{-9}$ | up | (u) | +2/3 | 0.005 |
| Second generation | muon | $(\mu^-)$ | −1 | 0.106 | strange | (s) | −1/3 | 0.1 |
| | neutrino | $(\nu_\mu)$ | 0 | $< 10^{-9}$ | charm | (c) | +2/3 | 1.3 |
| Third generation | tau | $(\tau^-)$ | −1 | 1.78 | bottom | (b) | −1/3 | 4.5 |
| | neutrino | $(\nu_\tau)$ | 0 | $< 10^{-9}$ | top | (t) | +2/3 | 174 |

- **Quarks & leptons** are ~point-like, no structure inside; *spin 1/2*
- Organised into **three generations**, differing only in mass, same properties;
- Apparently, **no more generations**;
- → 4 particles x 3 generations

# Quarks & Leptons - 2




The particles in the three generations of fundamental fermions with the masses indicated by imagined spherical volumes of constant density. In reality, fundamental particles are believed to be point-like.

Spin ½ particles → Dirac particles
→ anti-particles (same mass but different charge) appear naturally

**Table 1.2** The forces experienced by different particles.

| | | | | | strong | electromagnetic | weak |
|---|---|---|---|---|---|---|---|
| Quarks | down-type | d | s | b | ✓ | ✓ | ✓ |
| | up-type | u | c | t | | | |
| Leptons | charged | $e^-$ | $\mu^-$ | $\tau^-$ | | ✓ | ✓ |
| | neutrinos | $\nu_e$ | $\nu_\mu$ | $\nu_\tau$ | | no charge | ✓ |

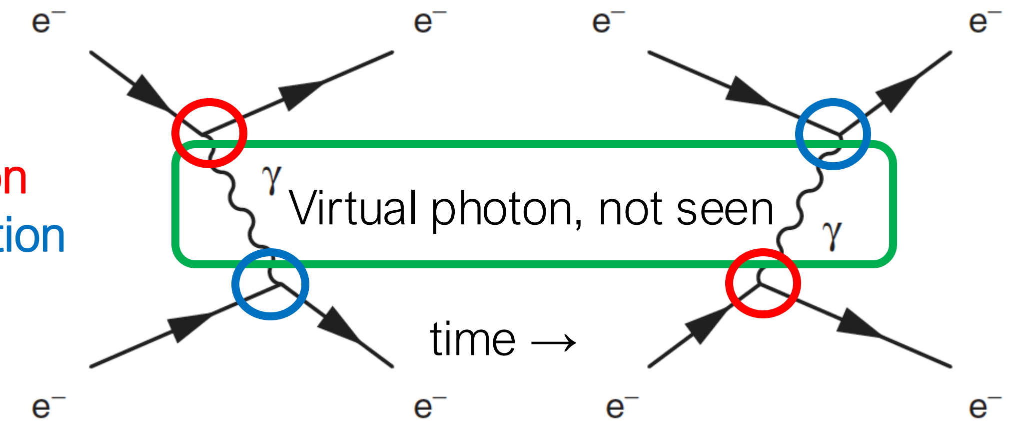
4 forces;

- Gravity (neglect, no role in particle-interactions)
- Weak force
- EM force
- Nuclear (strong) force

# Forces: potential?

In Classical Mechanics (EM) forces can be described by means of a scalar potential.
*Unsatisfactory*! Transfer of momentum without mediating body!

In QFT each force acts via virtual mediators.
No action at a distance!




Transfer of momentum: natural !

Each of the three forces (*not gravity*) is mediated by a

spin-1 force-carrying particle

Relative strength very different (we do not know why …!)

**Table 1.3** The four known forces of nature. The relative strengths are approximate indicative values for two fundamental particles at a distance of 1 fm $= 10^{-15}$ m (roughly the radius of a proton).

| Force | Strength | Boson | | Spin | Mass/GeV |
|---|---|---|---|---|---|
| Strong | 1 | Gluon | g | 1 | 0 |
| Electromagnetism | $10^{-3}$ | Photon | $\gamma$ | 1 | 0 |
| Weak | $10^{-8}$ | W boson | $W^{\pm}$ | 1 | 80.4 |
| | | Z boson | Z | 1 | 91.2 |
| Gravity | $10^{-37}$ | Graviton? | G | 2 | 0 |

# The Higgs Boson

Discovered in 2012 by ATLAS & CMS Experiments at the LHC;

- Fundamental fermions: spin ½ particles;
- Gauge bosons: spin-1 particles;

Higgs boson is a spin-0 scalar particle. As conceived in the Standard Model, the Higgs boson is the only fundamental scalar discovered to date.

$$m_{\mathrm{H}} \approx 125\,\mathrm{GeV},$$

The Higgs boson, in the SM of particles, has the role of 'giving mass' to all particles

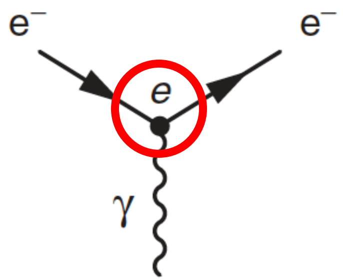
# *Forces: mediators*

Graphical representation: ***3-point vertex***, one gauge boson + incoming fermion + outgoing fermion

**Rule:**

Particle couples to a force carrying mediator ***only*** if it carries the charge of the interaction

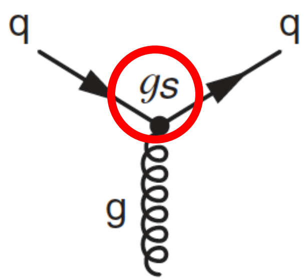
### Electromagnetism




All charged particles
Never changes flavour

$\alpha \approx 1/137$

### Strong interaction

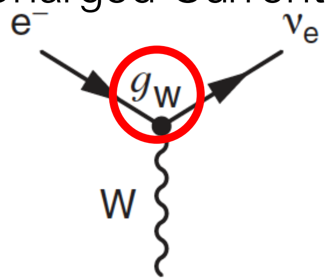



Only quarks
Never changes flavour

$\alpha_S \approx 1$

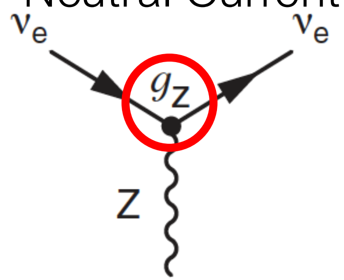
### Weak interaction

Charged Current




All fermions
Always changes flavour

Neutral Current




All fermions
Never changes flavour

$\alpha_{W/Z} \approx 1/30$ Mass of W,Z

Change of flavour

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}, \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}, \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}$$

Strength of Weak force is greatest for transitions same generation

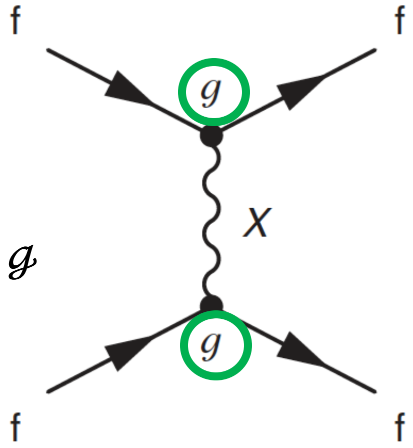
$$\begin{pmatrix}u\\ d\end{pmatrix}, \begin{pmatrix}u\\ s\end{pmatrix}, \begin{pmatrix}u\\ b\end{pmatrix}, \begin{pmatrix}c\\ d\end{pmatrix}, \begin{pmatrix}c\\ s\end{pmatrix}, \begin{pmatrix}c\\ b\end{pmatrix}, \begin{pmatrix}t\\ d\end{pmatrix}, \begin{pmatrix}t\\ s\end{pmatrix}, \begin{pmatrix}t\\ b\end{pmatrix}$$

- Interaction probability: $\mathcal{M}$ (one state to another)
- Coupling constant: $g$ (probability spin ½ fermion emits or absorbs the interaction boson)

**Table 1.2** The forces experienced by different particles.

| | | | | | strong | electromagnetic | weak |
|---|---|---|---|---|---|---|---|
| Quarks | down-type | d | s | b | ✓ | ✓ | ✓ |
| | up-type | u | c | t | | | |
| Leptons | charged | $e^-$ | $\mu^-$ | $\tau^-$ | | ✓ | ✓ |
| | neutrinos | $\nu_e$ | $\nu_\mu$ | $\nu_\tau$ | | | ✓ |

Scattering of two fermions via exchange of a vector boson $X$
Each vertex $ffX$ is described by $g$



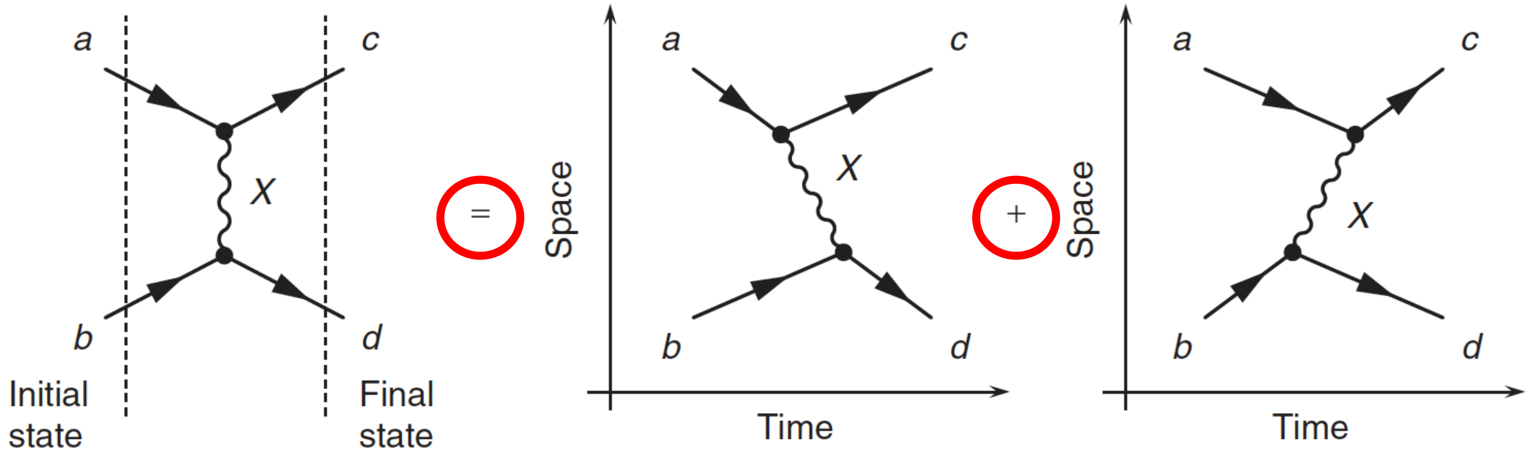

The matrix element $\mathcal{M}$ includes a factor $g$ for each vertex

$$\mathcal{M} \propto g^2$$

→ interaction probability is the square of $\mathcal{M}$

$$|\mathcal{M}|^2 \propto g^4$$

- Feynman diagrams give a graphical representation of an interaction;
- Shows possible time orderings of the interaction
- The interaction is the sum of possible time orderings




The Feynman diagram for the scattering process $a + b \rightarrow c + d$ and the two time-ordered processes that it represents.

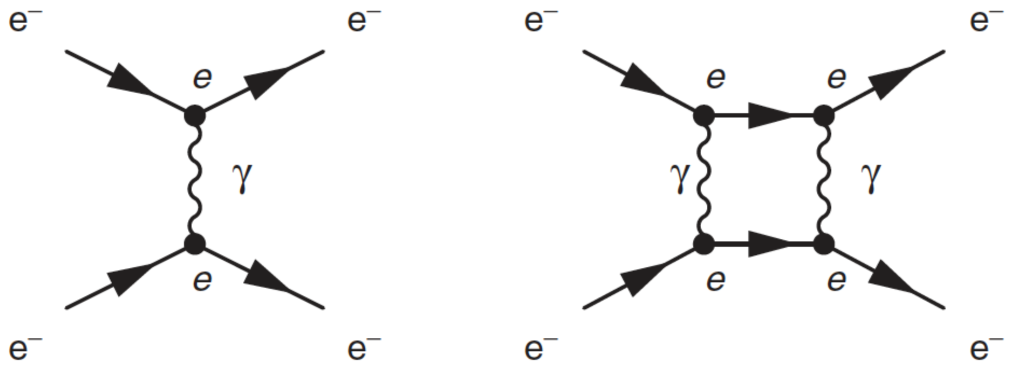
Time →

# Feynman Diagrams

One process = superposition of infinite Feynman diagrams.

Example: scattering of two electrons via the exchange of one or two photons.

- Same initial and final state;
- Use $\alpha$ (contains $e^2$) $\approx {}^1/_{137}$
- First diagram (one photon)
  $\mathcal{M} \propto \alpha^2$
- Second diagram (two photons)
  $\mathcal{M} \propto \alpha^4 \rightarrow 2nd\ diagram\ is\ \approx 10^{-4}\ times\ lower\ than\ 1st\ one$

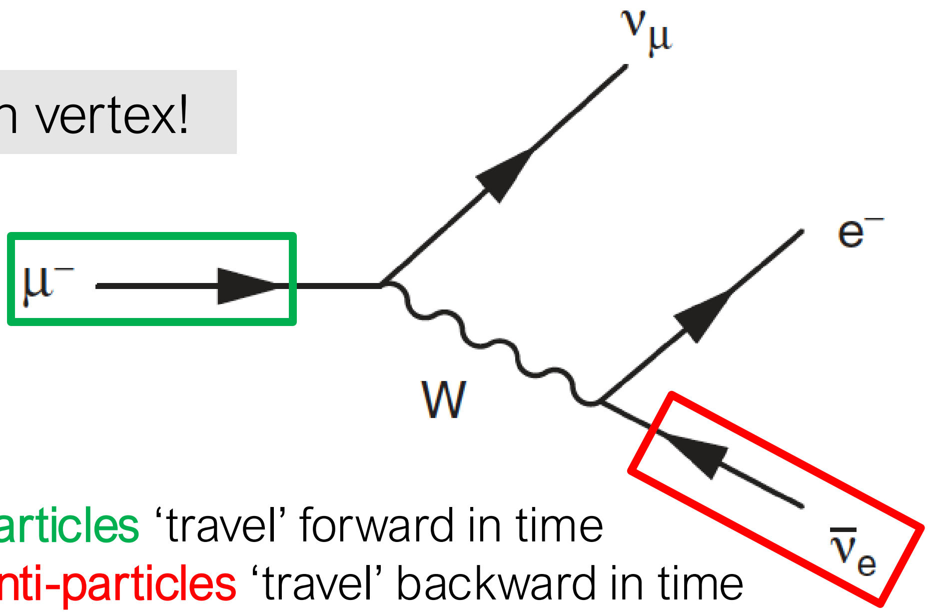



Two Feynman diagrams for $e^-e^- \rightarrow e^-e^-$ scattering.

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}.$$

$\alpha$ at each vertex!

- particles and antiparticles created/annihilated only in pairs.
- arrows on the incoming and outgoing fermion in the same sense and flow through the vertex;
- they never both point towards or away from the vertex.



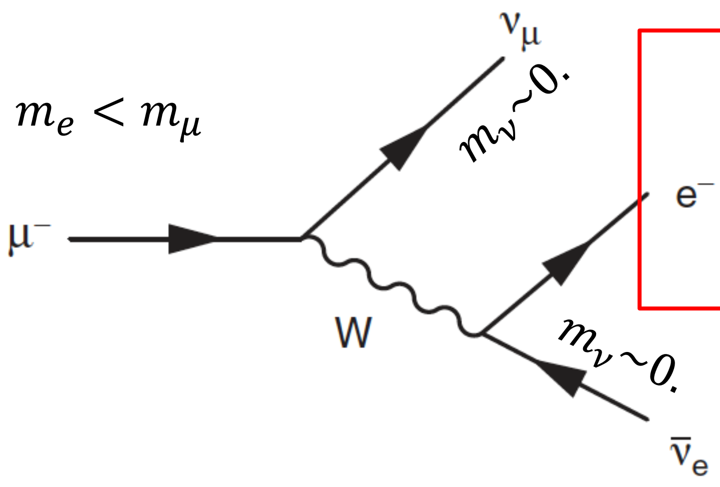

Particles 'travel' forward in time
Anti-particles 'travel' backward in time

# Unstable Particles

Most particles decay with a very short lifetime → few long-lived or stable particles detected in experiments

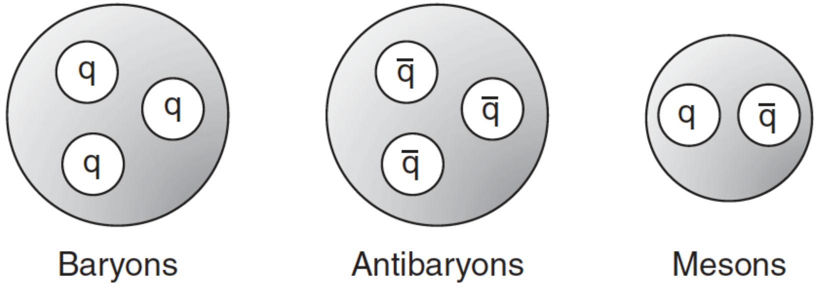
The decay of a particle can always be described in terms of a Feynman diagram

- the decay products must have a rest mass lower than the initial state:
- Weak force: all particles (and change of flavour)
- Coupling Constant increases → Lifetime decreases




Lightest charged lepton → cannot decay

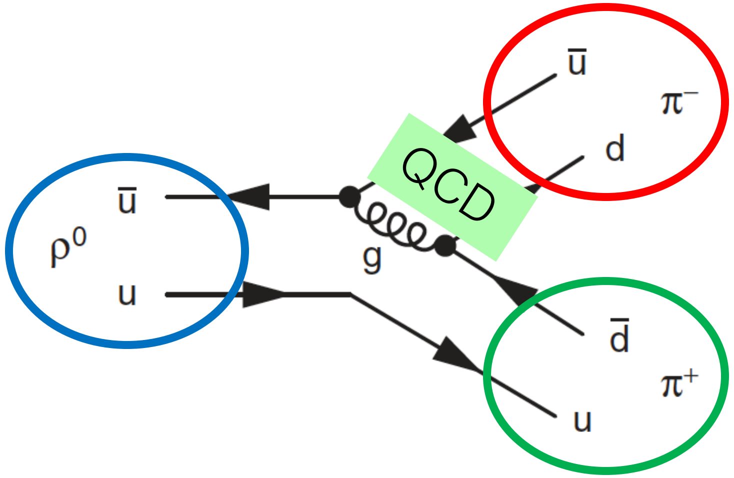
- Hadrons exist as Baryons, Antibaryons, Mesons;
- Strong force, QCD interactions:
  - quarks cannot exist as free particles
  - → only bound states
  - → decays to be interpreted as transitions between bound states

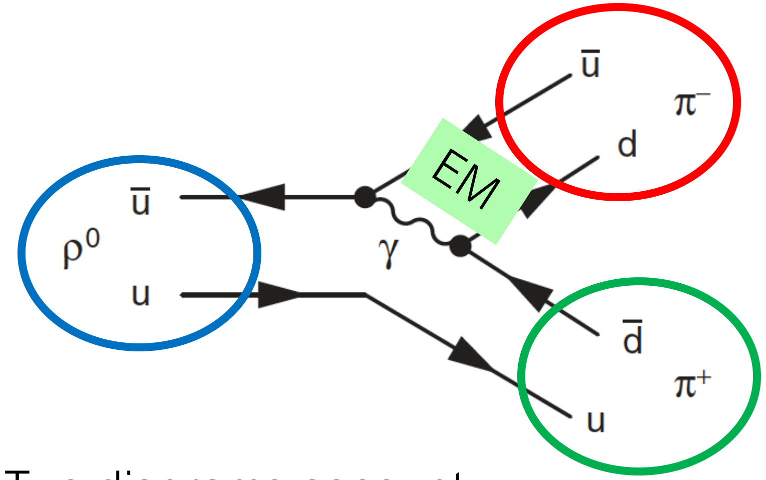


# Unstable Particles – continued

$|\mathcal{M}_g|^2 \propto \alpha_s^2 \gg |\mathcal{M}_\gamma|^2 \propto \alpha^2$
Exchange of gluon dominates

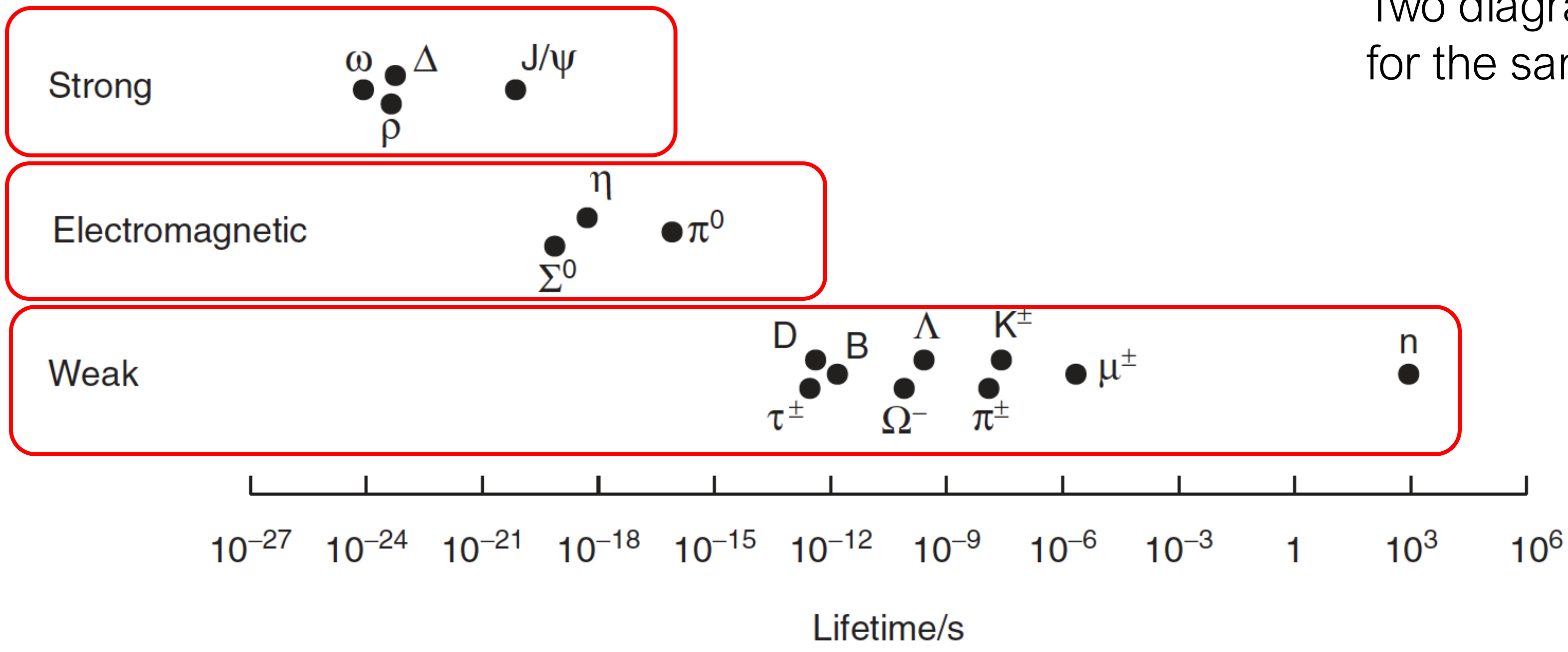
$\tau_{strong} \ll \tau_{EM} \ll \tau_{weak}$




Strong decays dominates over EM decays
EM decays dominate over weak decays

Two diagrams account for the same decay

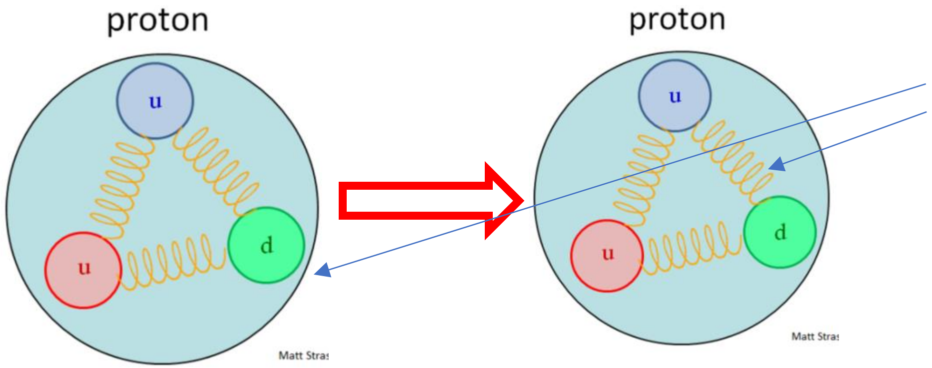



Secondary vertices *may* be detected

# Anticipation → why electron scattering?

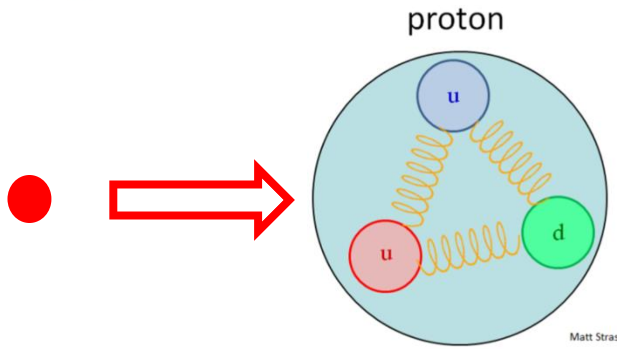
Nuclear sizes and shapes → use scattering technique → use a projectile (accelerated or from radioactivity) that hits a target

Protons are extended and complex objects




nuclear forces between the projectile and the target are complex and complex to describe

Use electrons! Point-like projectiles!



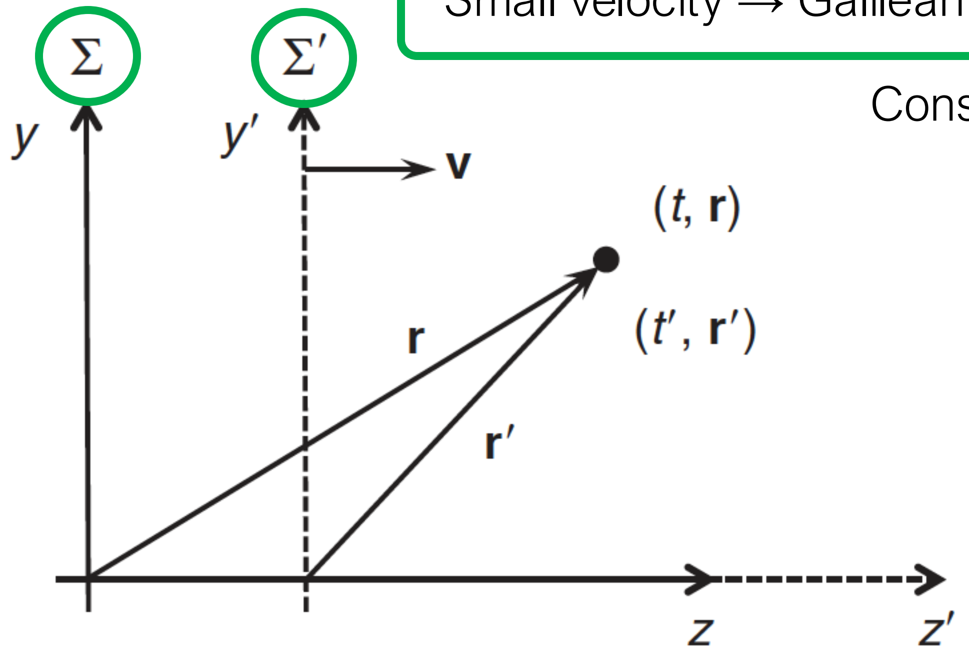

- *The interactions between an electron and a nucleus, nucleon or quark takes place via the exchange of a virtual photon — this may be very accurately calculated in quantum electrodynamics (QED).*
- *These processes are in fact manifestations of the well known electromagnetic interaction, whose coupling constant α ≈ 1/ 137 is much less than one. This last means that higher order corrections play only a tiny role*

# *Kinematics & Co*

# Reminder: Special Relativity

$\hbar = c = \varepsilon_0 = \mu_0 = 1$

Small velocity → Galilean transformation $t' = t, \quad x' = x, \quad y' = y \quad \text{and} \quad z' = z - vt.$




Consider two inertial frames: $\Sigma$ *and* $\Sigma'$, $\Sigma'$ moving with velocity v *along* $z$: $\beta = {}^{v}/_{c}$ $\gamma = (1-\beta^2)^{1/2}$

Einstein: r and r' are the same in all systems →

$$c^2t^2 - x^2 - y^2 - z^2 = c^2t'^2 - x'^2 - y'^2 - z'^2,$$

r, r': space-time interval

$$t' = \gamma(t - \beta z), \quad x' = x, \quad y' = y \quad \text{and} \quad z' = \gamma(z - \beta t).$$

When $v \ll c \;\rightarrow\; \beta = 0,\ \gamma = 1$

In matrix form:

$$\underbrace{\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} \gamma & 0 & 0 & +\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\gamma\beta & 0 & 0 & \gamma \end{pmatrix}}_{\Lambda} \underbrace{\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix}}_{X'} . \quad \begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \underbrace{\begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix}}_{\Lambda^{-1}} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix},$$

4-vector $(t, \boldsymbol{x})$, $\boldsymbol{X}' = \boldsymbol{\Lambda X}$, $\boldsymbol{X} = \boldsymbol{\Lambda}^{-1}\boldsymbol{X}'$, $\boldsymbol{\Lambda\Lambda}^{-1} = \boldsymbol{I}$.

# 4-Vectors and Lorentz Invariance

A fundamental idea in Physics is that laws of Nature do not depend on the frame where they are measured.

This translates into
- Introducing contravariant and covariant 4 vectors and
- Requiring space-time interval to be Lorentz invariant

$$x^\mu = (t, x, y, z), \qquad x_\mu = (t, -x, -y, -z).$$

$$x^\mu x_\mu = x^0 x_0 + x^1 x_1 + x^2 x_2 + x^3 x_3 = t^2 - x^2 - y^2 - z^2.$$

$$\begin{pmatrix} t' \\ -x' \\ -y' \\ -z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & +\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} t \\ -x \\ -y \\ -z \end{pmatrix}.$$

Lorentz transformation of covariant 4-vector $x'^\mu = \Lambda^\mu{}_\nu x^\nu,$

contravariant 4-vector to a covariant 4-vector $x_\mu = g_{\mu\nu} x^\nu,$

Only quantities with Lorentz transformation properties are such that $x^\mu x_\mu$ are Lorentz invariant

$$g_{\mu\nu} \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

If $a^\mu$ *and* $b^\mu$ are contravariant then the scalar product is also invariant $a^\mu b_\mu = a_\mu b^\mu = g_{\mu\nu} a^\mu b^\nu,$

# Four Momentum and Four Derivatives

Relativistic momentum and energy of a particle with mass *m* $E = \gamma m$ and $\mathbf{p} = \gamma m \boldsymbol{\beta}.$ (Use $c = 1$)

$t' = \gamma(t - \beta z), \quad x' = x, \quad y' = y \quad \text{and} \quad z' = \gamma(z - \beta t)$

Momentum and energy are conserved separately → also 4-momentum is $p^\mu = (E, p_x, p_y, p_z),$ the scalar product $p^\mu p_\mu = E^2 - \mathbf{p}^2,$

Lorentz transformation of a 4-derivative from frame Σ to Σ′

$$\frac{\partial}{\partial z'} = \left(\frac{\partial z}{\partial z'}\right)\frac{\partial}{\partial z} + \left(\frac{\partial t}{\partial z'}\right)\frac{\partial}{\partial t} \quad \text{and} \quad \frac{\partial}{\partial t'} = \left(\frac{\partial z}{\partial t'}\right)\frac{\partial}{\partial z} + \left(\frac{\partial t}{\partial t'}\right)\frac{\partial}{\partial t}.$$

$$\frac{\partial}{\partial z'} = \gamma\frac{\partial}{\partial z} + \gamma\beta\frac{\partial}{\partial t} \quad \text{and} \quad \frac{\partial}{\partial t'} = \gamma\beta\frac{\partial}{\partial z} + \gamma\frac{\partial}{\partial t}.$$

$$\left(\frac{\partial z}{\partial z'}\right) = \gamma, \quad \left(\frac{\partial t}{\partial z'}\right) = +\gamma\beta, \quad \left(\frac{\partial z}{\partial t'}\right) = +\gamma\beta \quad \text{and} \quad \left(\frac{\partial t}{\partial t'}\right) = \gamma,$$

In matrix notation

$$\begin{pmatrix} \partial/\partial t' \\ \partial/\partial x' \\ \partial/\partial y' \\ \partial/\partial z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & +\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} \partial/\partial t \\ \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix},$$

→ transforms as a covariant four-vector

# Laplacian

The corresponding contravariant four-derivative is $\partial^\mu = \left(\frac{\partial}{\partial t}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z}\right),$

The Laplacian for the four-derivative $\Box = \partial^\mu \partial_\mu = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}.$

# Notations

Quantities written as

$x, p$

are four-vectors

Quantities written **in bold** as

$\boldsymbol{x}, \boldsymbol{p}$

Are three-vectors

Four-vectors scalar product is $a \cdot b \equiv a^\mu b_\mu \equiv g_{\mu\nu} a^\mu b^\nu = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3.$

The Einstein energy-momentum relationship → $p^2 = m^2 \; since \; p^2 = p \cdot p = E^2 - \boldsymbol{p}^2$

A quantity in the Center of Mass System (cms) of a group of particles is labelled with a * , example q*

# Mendelstam Variables

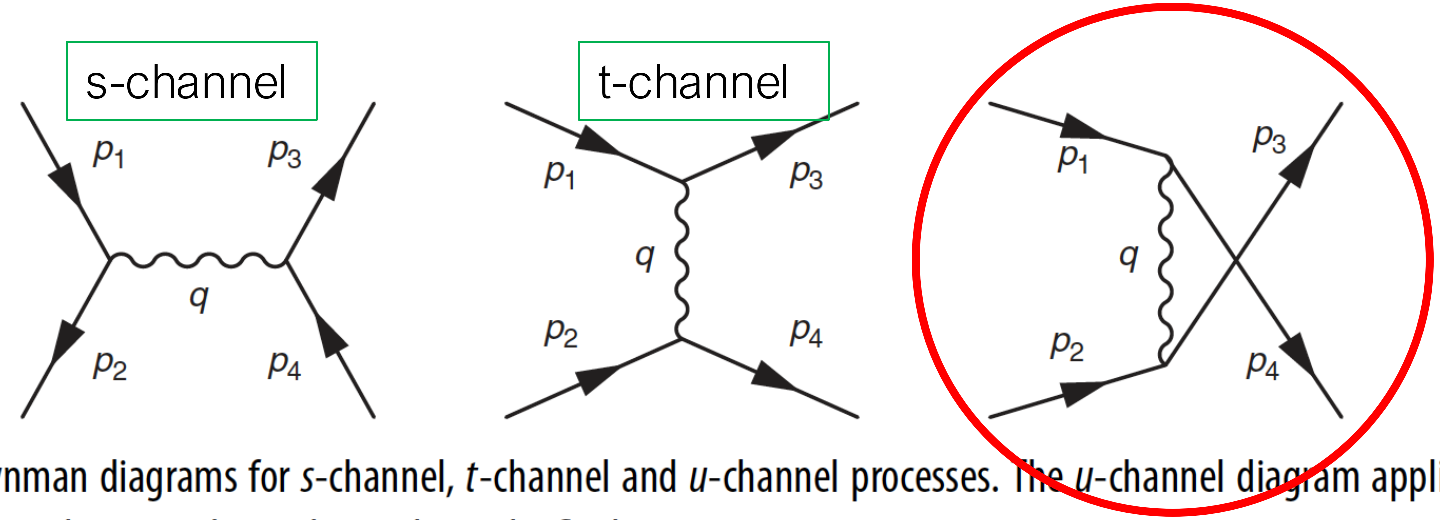
In reaction $1 + 2 \rightarrow 3 + 4$ one mediating particle is emitted/absorbed in different ways

- s-channel: particle 1 emits a mediator absorbed by particle 3
- t-channel: particle 1 emits a mediator absorbed by particle 2

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$
$$\mathrm{t} = (p_1 - p_3)^2 = (p_2 - p_4)^2$$
$$\mathrm{u} = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

$$s + u + t = m_1^2 + m_2^2 + m_3^2$$

These variables are equivalent to the four-momentum squared $q^2$ of the exchanged particle




Only relevant when there are identical particles in the final state

The Feynman diagrams for $s$-channel, $t$-channel and $u$-channel processes. The $u$-channel diagram applies only when there are identical particles in the final state.

In the cms $p_1 = (E_1^*, \boldsymbol{p}^*)$ and $p_2 = (E_2^*, -\boldsymbol{p}^*) \rightarrow s = (p_1 + p_2)^2 = (E_1^* + E_2^*)^2 - (p^* - p^*)^2 = (E_1^* + E_2^*)^2$

$\rightarrow s$ is the total available energy in the cms system

Short reminder, define notations

# Non Relativistic Quantum Mechanics

# Wave Mechanics & Schrödinger Equation

Free particles: Fourier superposition of plane waves ($\boldsymbol{k} = \boldsymbol{p}, \omega = E$)

$$\psi(\mathbf{x}, t) \propto \exp\{i(\mathbf{k} \cdot \mathbf{x} - \omega t)\}. \qquad \psi(\mathbf{x}, t) = N \exp\{i(\mathbf{p} \cdot \mathbf{x} - Et)\},$$

Classical physics: energy & momentum of a particle time-dependent real numbers;
Quantum Mechanics (Schrödinger view):

- Wave function completely defines a state;
- *time-dependent* wavefunction;
- Dynamical variables (energy & momentum): *time-independent* operators acting on the wavefunction:

$$\hat{A}\psi = a\psi$$

Identify:

$$\hat{\mathbf{p}} = -i\boldsymbol{\nabla} \quad \text{and} \quad \hat{E} = i\frac{\partial}{\partial t},$$

$$\hat{\mathbf{p}}\psi = -i\boldsymbol{\nabla}\psi = \mathbf{p}\psi,$$

$$\hat{E}\psi = i\frac{\partial \psi}{\partial t} = E\psi.$$

In classical mechanics: total energy = kinetic energy + potential energy (Hamiltonian)

$$E = H = T + V = \frac{\mathbf{p}^2}{2m} + V,$$

$$i\frac{\partial\psi(\mathbf{x},t)}{\partial t} = \hat{H}\psi(\mathbf{x},t), \qquad \hat{H}_{NR} = \frac{\hat{\mathbf{p}}^2}{2m} + \hat{V} = -\frac{1}{2m}\nabla^2 + \hat{V}.$$

Time-dependent Schrödinger Equation

$$i\frac{\partial\psi(\mathbf{x},t)}{\partial t} = -\frac{1}{2m}\frac{\partial^2\psi(\mathbf{x},t)}{\partial x^2} + \hat{V}\psi(\mathbf{x},t).$$

# Time Dependence and Conserved Quantities

Time dependence of a system $i\frac{\partial\psi(\mathbf{x},t)}{\partial t} = \hat{H}\psi(\mathbf{x},t),$ If $\psi(x,t)$ is eigenstate of $\widehat{H}$ with energy E

$$i\frac{\partial\psi_i(\mathbf{x},t)}{\partial t} = E_i\psi_i(\mathbf{x},t). \implies \psi_i(\mathbf{x},t) = \phi_i(\mathbf{x})e^{-iE_it}.$$

Consider an observable corresponding to an operator $\hat{A}\psi = a\psi$ is $a$ a conserved quantity? Expectation value

$$\langle\hat{A}\rangle = \langle\psi|\hat{A}|\psi\rangle = \int \psi^\dagger\hat{A}\psi\,\mathrm{d}^3\mathbf{x},$$

If the Hamiltonian and the operator commute, then the corresponding observable does not change with time

$$\frac{\mathrm{d}\langle\hat{A}\rangle}{\mathrm{d}t} = i\left\langle[\hat{H},\hat{A}]\right\rangle,$$

If two operators commute $[\hat{A},\hat{B}] = 0$ then they can be simultaneously measured

# Time Dependence and Conserved Quantities

The expectation value of an operator $\hat{A}$ is given by

$$\psi^\dagger = (\psi^*)^T$$

$$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle = \int \psi^\dagger \hat{A} \psi d^3\boldsymbol{x}$$

$$\frac{\mathrm{d}\langle \hat{A} \rangle}{\mathrm{d}t} = \int \left[ \frac{\partial \psi^\dagger}{\partial t} \hat{A} \psi + \psi^\dagger \hat{A} \frac{\partial \psi}{\partial t} \right] \mathrm{d}^3\mathbf{x},$$

$$\begin{aligned} \frac{\mathrm{d}\langle \hat{A} \rangle}{\mathrm{d}t} &= \int \left[ \left\{ \tfrac{1}{i} \hat{H} \psi \right\}^\dagger \hat{A} \psi + \psi^\dagger \hat{A} \left\{ \tfrac{1}{i} \hat{H} \psi \right\} \right] \mathrm{d}^3\mathbf{x} \\ &= i \int \left[ \psi^\dagger \hat{H}^\dagger \hat{A} \psi - \psi^\dagger \hat{A} \hat{H} \psi \right] \mathrm{d}^3\mathbf{x} \\ &= i \int \psi^\dagger (\hat{H}\hat{A} - \hat{A}\hat{H}) \psi \, \mathrm{d}^3\mathbf{x}. \end{aligned}$$

$$\frac{d\langle \hat{A} \rangle}{dt} = i \langle [\hat{H}, \hat{A}] \rangle$$

If $\hat{H}$ commutes with $\hat{A}$ then the derivative with respect to time is 0 and the corresponding observable does not vary with time → conserved quantity

# Schrödinger Equation & Co (Sec.2.3)

Non-relativistic Quantum Mechanics (QM), free particles = superposition of wave-packets (Fourier decomposition)

$$\psi(\boldsymbol{x}, t) \propto e^{i(\boldsymbol{k}\cdot\boldsymbol{x}-\omega t)}$$

$\psi(\boldsymbol{x}, t)$ contains all the information about a state

Use $\lambda = h/p$ or $\boldsymbol{k} = \boldsymbol{p}/\hbar$ and $E = \hbar\omega$ and put $\hbar = 1$

$$\psi(\boldsymbol{x}, t) = N \cdot e^{i(\boldsymbol{p}\cdot\boldsymbol{x}-Et)}$$

The result of an observation is the result of an operator $\hat{A}$ on the wavefunction resulting in a *real* eigenvalue $a$:

$$\hat{A}\psi = a\psi$$

In classical mechanics

$$E = H = T + V = \frac{\boldsymbol{p}^2}{2m} + V$$

We want that $\hat{\boldsymbol{p}}$ and $\hat{E}$ applied on $\psi(\boldsymbol{x}, t)$ return p and E $\rightarrow$ $\hat{\boldsymbol{p}} = -i\boldsymbol{\nabla}$ and $\hat{E} = i\frac{\partial}{\partial t}$

$$i\frac{\partial\psi(\boldsymbol{x}, t)}{\partial t} = -\frac{1}{2m}\frac{\partial^2\psi(\boldsymbol{x}, t)}{\partial x^2} + \hat{V}\psi(\boldsymbol{x}, t)$$

# Commutation Relations

In general, any state can be described as a superposition of states

$$|\varphi\rangle = \sum_i c_i |\psi_i\rangle,$$

If at time $t = 0$ the system is in the state

$$|\varphi(\boldsymbol{x}, t)\rangle = |\varphi(\boldsymbol{x})\rangle$$

then the evolution of the system is determined by the evolution of the different components.

$$|\varphi(\mathbf{x}, t)\rangle = \sum_i c_i \, |\phi_i(\mathbf{x})\rangle e^{-iE_i t}$$

If $[\hat{A}, \hat{B}] = 0$ then the observables can be determined at the same time:

$$\hat{A}|\phi\rangle = a|\phi\rangle,$$

$$\hat{A}\hat{B}\,|\phi\rangle = \hat{B}\hat{A}\,|\phi\rangle = a\hat{B}\,|\phi\rangle.$$

$$\hat{B}|\phi\rangle = b|\phi\rangle.$$

If $[\hat{A}, \hat{B}] \neq 0$ then the observables cannot be determined at the same time to better than : $\Delta A\, \Delta B \geq \frac{1}{2}\left|\langle i[\hat{A}, \hat{B}]\rangle\right|,$

Example: position and momentum: $\hat{x}\psi = x\psi$ and $\hat{p}_x\,\psi = -i\frac{\partial}{\partial x}\psi$

$$[\hat{x}, \hat{p}_x]\psi = -ix\frac{\partial}{\partial x}\psi + i\frac{\partial}{\partial x}(x\psi)$$

$$= -ix\frac{\partial \psi}{\partial x} + i\psi + ix\frac{\partial \psi}{\partial x} = +i\psi, \qquad [\hat{x}, \hat{p}_x] = +i.$$

$$\Delta x\, \Delta p_x \geq \frac{\hbar}{2}.$$

# Cross Sections and Decay Rates

# Fermi's Golden Rule

Particle Physics:

- Study of decays (→ measure decay rates, how often $a \to 1 + 2$?);
- Study of cross-sections (→ measure reaction rates, how often $a + b \to 1 + 2$?).

These processes correspond to transitions between states.

Non-relativistic quantum mechanics, $|i\rangle \to |f\rangle$: Fermi's golden rule

$$\Gamma_{fi} = 2\pi \cdot |T_{fi}|^2 \rho(E_i)$$

Non relativistic!

- If interaction potential is known or calculable → compute the cross section
- if $T_{fi}$ is not known one can measure σ and derive $T_{fi}$ from it.

The Golden Rule applies both to scattering and decay processes. In the second case the lifetime of the process will be

$$\tau = \frac{1}{W}$$

- if the lifetime is (can be) measured then $T_{fi}$ can be derived.
- If τ cannot be measured then the uncertainty principle can be used and we can take $\Delta E = \hbar/\tau$

Elaborate technicalities in next slides

# The Fermi Golden Rule - continued

According to the (second) Fermi golden rule,

- the *reaction rate* $\Gamma_{fi}$ from the initial state $|i\rangle$ to a final state $|f\rangle$ is given by

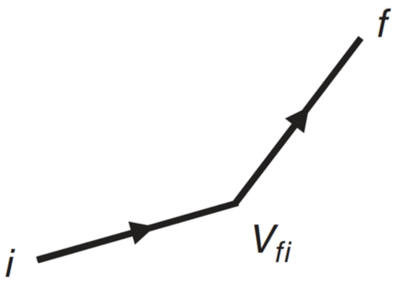
$\Gamma_{fi}$ = 'transition rate' Your Experiment

$$\Gamma_{fi} = 2\pi \cdot |T_{fi}|^2 \rho(E_i)$$

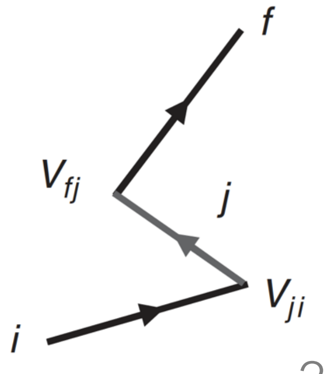
$\rho(E_i)$ = density of states Kinematics

$T_{fi}$ = Matrix element Physics

Problem: this expression is NOT Lorentz invariant




$$\mathrm{T}_{fi} = \langle f|\mathcal{H}'|i\rangle + \sum_{j\neq i} \frac{\langle f|\mathcal{H}'|j\rangle\langle j|\mathcal{H}'|f\rangle}{E_i - E_j} +' \, higher\ order\ diagrams'$$



# The Density of States

$\rho(E_i) = |\frac{dn}{dE}|_{E_i}$ $dn$ is the number of states in the interval $E \rightarrow E + dE$

in how many ways we can construct the final state (and conserve energy and momentum of the initial state $E_i$).

Alternative: counting of **all possible final states** but imposing the energy conservation by means of a $\delta$ function:

$$\rho(E_i) = |\frac{dn}{dE}|_{E_i} = \int \frac{dn}{dE} \delta(E - E_i)\, dE$$

Giving a new expression for the Fermi Golden Rule

$$\Gamma_{fi} = 2\pi \cdot |T_{fi}|^2 \rho(E_i) \rightarrow$$

$$\Gamma_{fi} = 2\pi \int |T_{fi}|^2 \delta(E_i - E_n) dn$$

Transition matrix depends on

- $|T_{fi}|$ this term contains physics;
- $\rho(E_i)$ this tern describes the kinematics of the event

Example: two body decay of particle a

$$a \rightarrow 1 + 2$$

$$\Gamma_{fi} = \langle \psi_1 \psi_2 | \widehat{H'} | \psi_a \rangle = \int_V \psi_1^* \psi_2^* \widehat{H'} \psi_a d^3\boldsymbol{x}$$

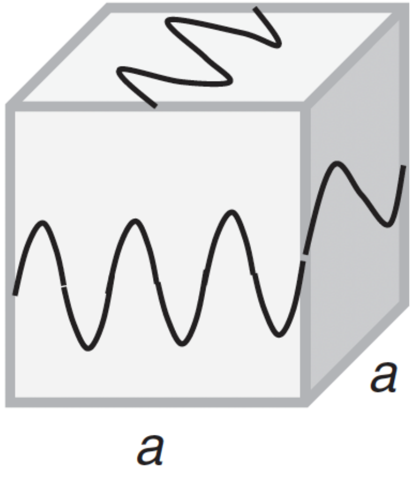
In the Born approximation & perturbation is small

$$\psi(\boldsymbol{x}, t) = A e^{i(\boldsymbol{p}\cdot\boldsymbol{x} - Et)} \quad A \rightarrow normalisation$$

A …

$$\int_0^a \int_0^a \int_0^a \psi^* \psi \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z = 1$$

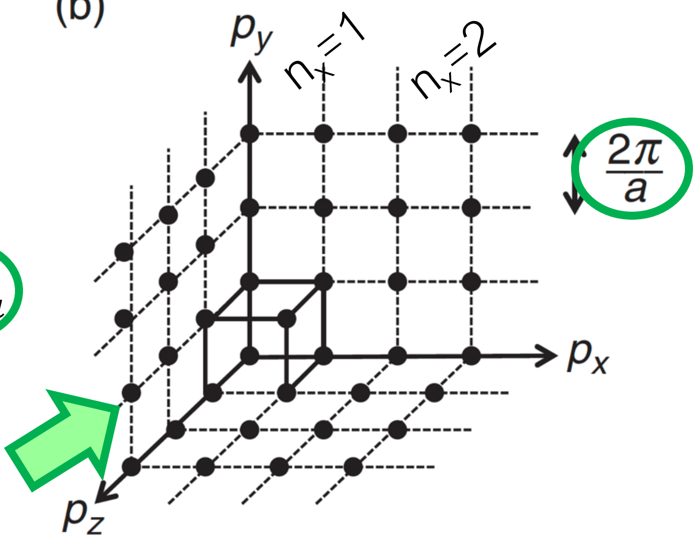
A …




1 particle in a cube of side a →

$$A^2 = {}^1/_{a^3} = {}^1/_V$$

(b)




The normalisation of one particle in a volume $a^3$ implies periodic conditions (wave function is zero at boundaries)

$$\psi(x + a, y, z) = \psi(x, y, z) \rightarrow e^{i(p_x x)} = e^{i(p_x(x+a))} \rightarrow (p_x, p_y, p_z) = (n_x, n_y, n_z) {}^{2\pi}/_a$$

Where $n_x, n_y, n_z$ are integers → momenta are quantised as shown in the figure here

# Normalisation of States - 2

Each state occupies a cubic volume

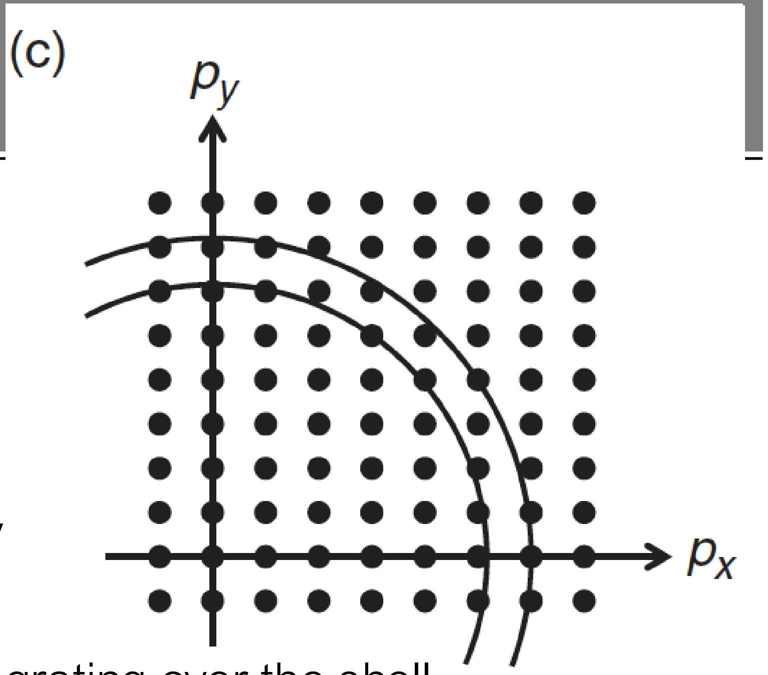
$$d^3\boldsymbol{p} = dp_x dp_y dp_z = \left(\frac{2\pi}{a}\right)^3 = \frac{(2\pi)^3}{V}$$

Density of States: how many states can I put inside this normalisation volume?

The number of available states $dn$ in the momentum interval $p \to p + dp$ is given by

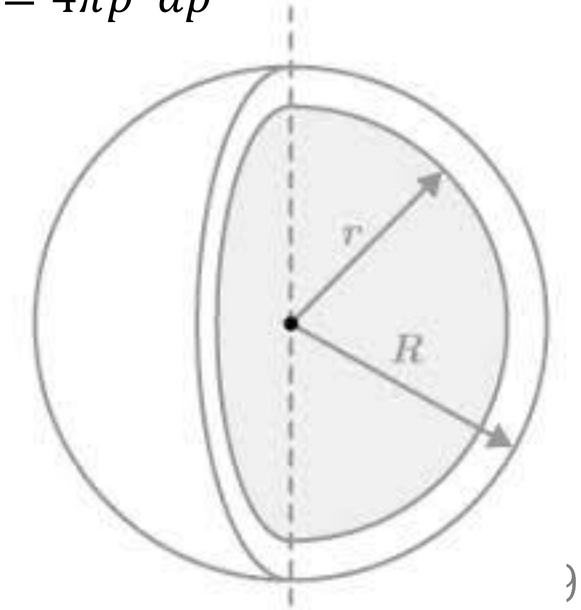
$$\frac{dn}{dp} = \frac{4\pi p^2}{(2\pi)^3} V$$

$$p = \beta E \quad \frac{dp}{dE} = \beta$$




Integrating over the shell
$d^3p = 4\pi p^2 dp$

$$\frac{volume\ of\ spherical\ shell}{(2\pi)^3/V} \to \rho(E) = \frac{dn}{dE} = \frac{dn}{dp}\frac{dp}{dE} = \frac{4\pi p^2}{(2\pi)^3} \cdot \beta$$




All above for ONE particle!

Comment: *"V"* appears in $\frac{dn}{dp}$ but will cancel with wavefunction normalisation → use V=1

# Normalisation of a System with N Particles

$$d^3\boldsymbol{p} = dp_x dp_y dp_z = \left(\frac{2\pi}{a}\right)^3 = \frac{(2\pi)^3}{V} \Rightarrow dn_i = \frac{d^3\mathbf{p}_i}{(2\pi)^3} \Rightarrow dn = \prod_{i=1}^{N-1} dn_i = \prod_{i=1}^{N-1} \frac{d^3\mathbf{p}_i}{(2\pi)^3}.$$

We have put V=1

Always non-relativistic case!

- Decay to two particles $a \to 1 + 2$ the phase space is determined by one particle, the other is constrained by $\boldsymbol{p}$ conservation → $\boldsymbol{\delta}$ function;
- When there are more than two particles, $N$ particles → $N$-1 are 'free'

$$dn = \prod_{i=1}^{N-1} \frac{d^3\mathbf{p}_i}{(2\pi)^3} \delta^3\left(\mathbf{p}_a - \sum_{i=1}^{N} \mathbf{p}_i\right) d^3\mathbf{p}_N, \rightarrow dn = (2\pi)^3 \prod_{i=1}^{N} \frac{d^3\mathbf{p}_i}{(2\pi)^3} \delta^3\left(\mathbf{p}_a - \sum_{i=1}^{N} \mathbf{p}_i\right).$$

δ Function → Momentum conservation

Nth particle

# Make Golden Rule Lorentz Invariant

We have to transform the Fermi Golden rule into a Lorentz invariant form:

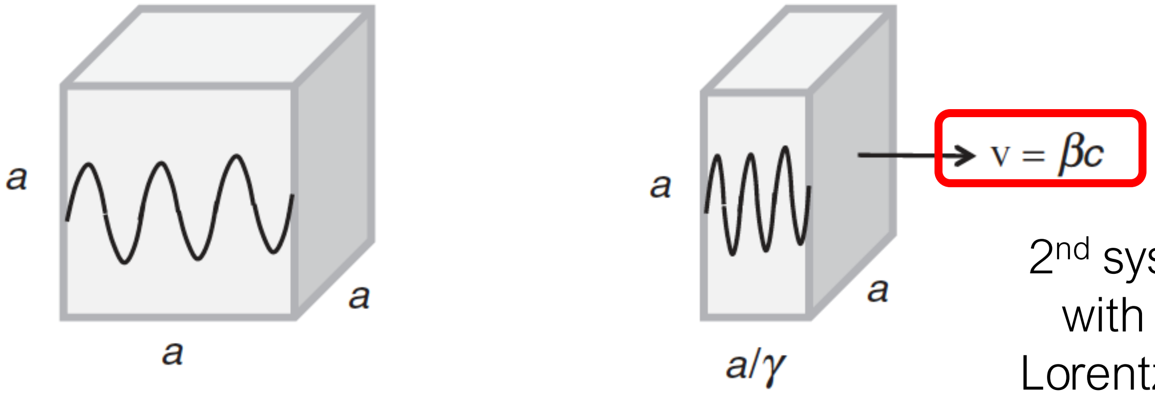
- Phase space (kinematics)
- $T_{fi}$ (Physics)

$$\Gamma_{fi} = 2\pi \cdot |T_{fi}|^2 \rho(E_i)$$

# Lorentz Invariant Normalisation

Normalise V to one particle per unit volume → V=1? Unsatisfactory…
The decay/interaction rate = f(Physics) doesn't depend on normalisation volume

Lorentz Invariance!




2[nd] system moving with velocity v → Lorentz contracted

$\psi$ normalised to 1 particle in volume V

$$\int_V \psi^* \psi \, d^3\mathbf{x} = 1.$$

$\psi'$ normalised to 2E particle in volume V

$$\int_V \psi'^* \psi' d^3\mathbf{x} = 2E,$$

$$\psi' = (2E)^{1/2}\psi.$$

$T_{fi}$ : *Fermi's golden rule matrix element*

For a process $a + b \to 1 + 2 + \cdots$

$$\mathcal{M}_{fi} = \langle \psi_1' \psi_2' \cdots | \hat{H}' | \psi_a' \psi_b' \cdots \rangle = (2E_1 \cdot 2E_2 \cdots 2E_a \cdot 2E_b \cdots)^{1/2} T_{fi},$$

Lorentz Invariance

$\mathcal{M}_{fi}$ : *Lorentz invariant matrix element*

# *Decays and Cross Sections*

# Recap Without Formalities: what we did

1. Start from Fermi's Golden Rule $\Gamma_{fi} = 2\pi \cdot |T_{fi}|^2 \rho(E_i)$
2. Rewrite it using a δ function $\Gamma_{fi} = 2\pi \int |T_{fi}|^2 \delta(E_i - E_n)\, dn$ (energy conservation)
3. Compute number of states $\rho(E) = \frac{dn}{dE} = \frac{dn}{dp}\frac{dp}{dE} = \frac{4\pi p^2}{(2\pi)^3} \cdot \beta$
4. Rewrite $\rho(E)$ by introducing a δ function

$$dn = (2\pi)^3 \prod_{i=1}^{N} \frac{d^3\mathbf{p}_i}{(2\pi)^3} \delta^3\left(\mathbf{p}_a - \sum_{i=1}^{N} \mathbf{p}_i\right).$$

$\boldsymbol{p}$ conservation

$\rho(E) = dn/dE$

5. Introduce Lorentz invariant normalization

$$\mathcal{M}_{fi} = \langle \psi_1' \psi_2' \cdots | \hat{H}' | \psi_a' \psi_b' \cdots \rangle = (2E_1 \cdot 2E_2 \cdots 2E_a \cdot 2E_b \cdots)^{1/2} T_{fi},$$

$$\Gamma_{fi} = \int_V \psi_1^* \psi_2^* \widehat{H'} \psi_a d^3\boldsymbol{x}$$

$$T_{fi}^2 = M_{fi}^2 / (2E_1 \cdot 2E_1 \ldots)$$

# 2 Body Particle Decays $a \to 1 + 2$

For a decay process $a \to 1 + 2$

$$\Gamma_{fi} = \frac{(2\pi)^4}{2E_a} \int |\mathcal{M}_{fi}|^2 \delta(E_a - E_1 - E_2)\delta^3(\mathbf{p}_a - \mathbf{p}_1 - \mathbf{p}_2)\frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1}\frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2},$$

$\mathcal{M}_{fi}$ : *Lorentz invariant matrix element*: contains physics and has to be computed for each type of process

*The phase space is the same for all types of processes, depends only on the number of particles*

Since $\Gamma_{fi}$ is Lorentz invariant → can be computed in any reference system → cms where

$$E_a = m_a, \qquad \boldsymbol{p_2} = -\boldsymbol{p_1} \qquad E_2^2 = (m_2^2 + p_1^2)$$

$$\Gamma_{fi} = \frac{1}{8\pi^2 m_a} \int |\mathcal{M}_{fi}|^2 \frac{1}{4E_1 E_2} \delta(m_a - E_1 - E_2)\, d^3\mathbf{p}_1,$$

If we use polar coordinates

$$d^3\mathbf{p}_1 = p_1^2 dp_1 \sin\theta\, d\theta\, d\phi = p_1^2\, dp_1 d\Omega, \quad p^* = \frac{1}{2m_a}\sqrt{\left[(m_a^2 - (m_1 + m_2)^2\right]\left[m_a^2 - (m_1 - m_2)^2\right]}.$$

We arrive (few steps in the book) to an expression valid for all two bodies decays

$$\Gamma_{fi} = \frac{p^*}{32\pi^2 m_a^2} \int |\mathcal{M}_{fi}|^2\, d\Omega.$$

# Cross Section Measurement

Interaction rate ⟺ Cross Section

The cross section can be defined through the relation

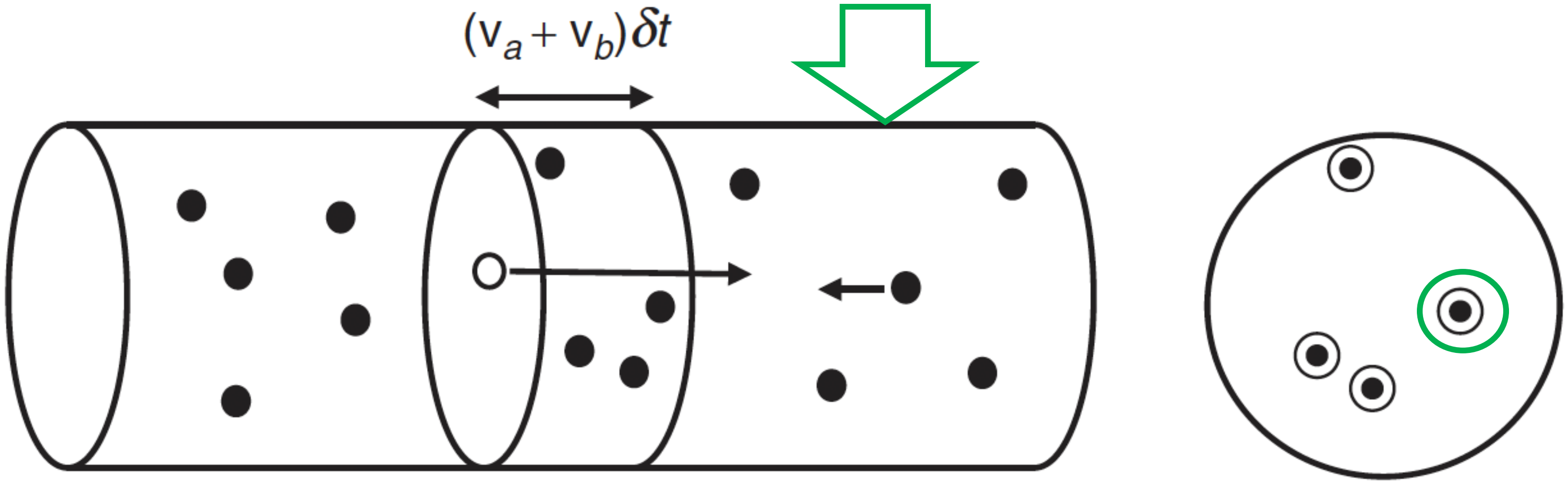
Physics: $\sigma$

Experiment: $r_b$

$$r_b = \sigma \phi_a$$

Technique: $\phi_a$

- Slightly more complicated than the calculation of decay rates: account for the flux of incoming particles hitting a target with Nb scattering centres;
- In modern experiments two beams colliding against each other;




- One may think of $\sigma$ as an "*effective area*";
- Rarely it is the case (scattering of 'big' objects);
- More correct to think of $\sigma$ as a Quantum Mechanics observable associated to the interaction probability.

Calculation of interaction rates more complex: account for flux of incoming particles. You cannot do $a + b \rightarrow 1 + 2 + \cdots$
You do beam on target or beam against beam

Structure of the matter is studied with scattering experiments. Energetic projectiles → small equivalent wave length

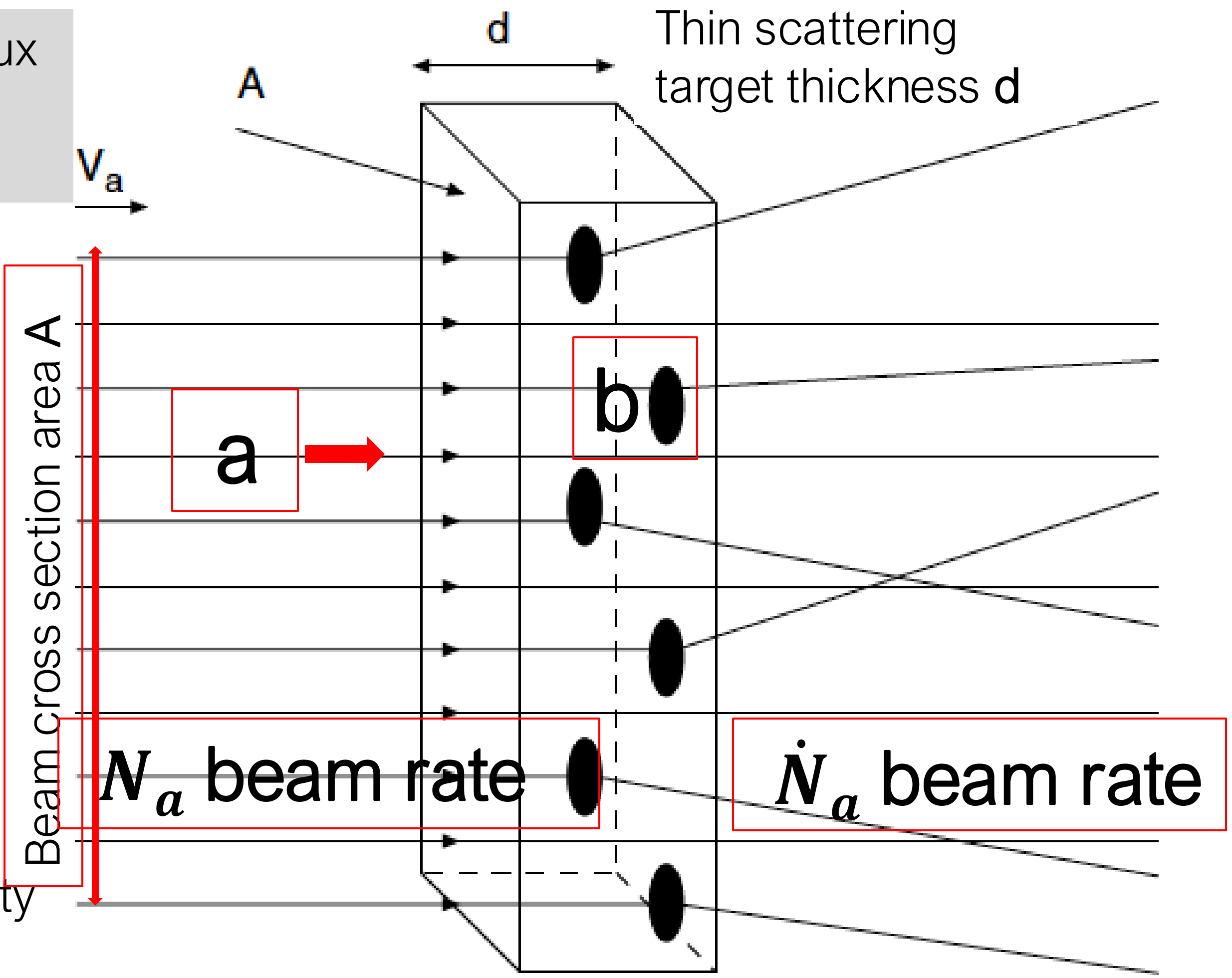
$$\lambda = \hbar / p$$

## Ideal Simplified Experiment:

Beam particles **a** bombard scattering centres b.

- reaction occurred when **a** hits **b**.
- The beam particle **a** disappears after the interaction

$$a + b \rightarrow anything$$




**Particle beam a** coming from left with density $n_a$ and velocity $v_a$ The corresponding flux is

$$\phi_a = n_a \times v_a$$

**Target** with $N_b$ scattering centres **b** and particle density $n_b$

$$N_b = A\, d\, n_b$$
= (density x Volume)$_{target}$

## Ideal Simplified Experiment:

After the interaction beam particles disappear (we do not distinguish different final topologies, we sum elastic + inelastic cross sections). Reaction rate is

$$\dot{N} = N_a - \dot{N}_a$$

Particle beam **a** coming from left with density $n_a$ and velocity $v_a$ The corresponding flux is

$$\phi_a = n_a \times v_a = \frac{N_a}{A} (area \times time)^{-1}$$

Target with $N_b$ scattering centres **b** and particle density $n_b$. Target particles within the beam area A are

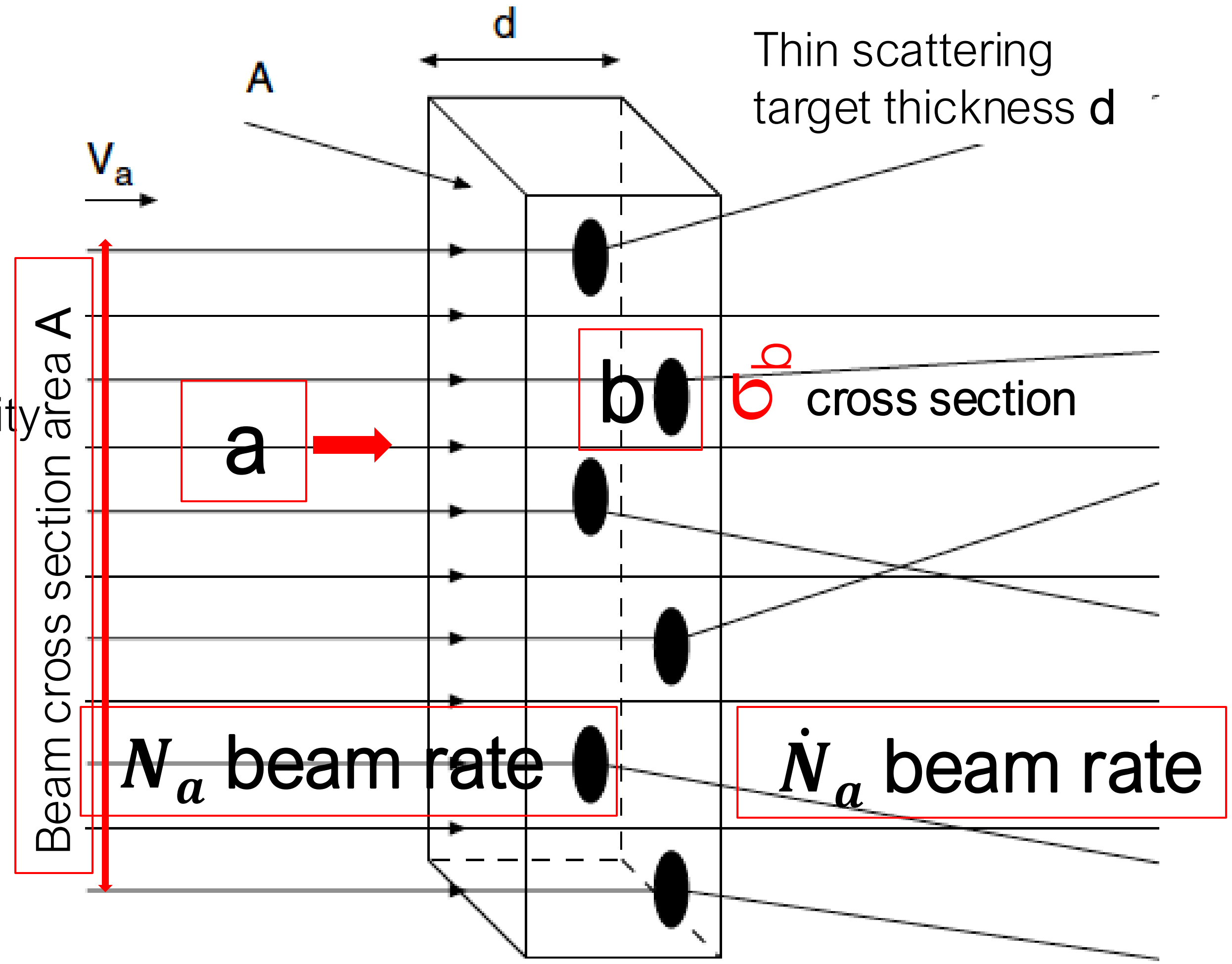
$$N_b = A \times d \times n_b$$

→ the reaction rate $\dot{N}$ is

$$\dot{N} = \phi_a \times N_b \times \sigma_b$$

$$\sigma_b = \frac{\dot{N}}{\phi_a \times N_b}$$

$$= \frac{number\ or\ reactions\ per\ unit\ time}{beam\ particles\ per\ unit\ time\ per\ unit\ area \times scattering\ centres}$$




**Limitations**: HP, scattering centres do not overlap + only one scattering

# (Geometric) Cross Sections - 3 (Povh…)

If beam is not uniform

$$\sigma_b = \frac{\dot{N}}{\phi_a \times N_b} = \frac{\textbf{number or reactions per unit time}}{(\textbf{beam particles per unit time} \times \textbf{scattering centres})\ \textbf{per unit area}}$$

In the expression

$$\sigma_b = \frac{\dot{N}\ \text{Physics!}}{\phi_a \times N_b\ \ \text{Experiment}}$$

$(\phi_a \times N_b)$ = Luminosity, $\mathcal{L}$ in this case

- Energy dependence
- Particle types..

$$\dot{N} = \mathcal{L} \times \sigma_b$$

The total cross section $\sigma_{tot}$ is as the sum of elastic and inelastic cross section

$$\sigma_{tot} = \sigma_{el} + \sigma_{inel}$$

and has dimensions of area. a common unit to define cross sections is the *barn*

$\sigma_{pp}$(10 GeV) ~ 40 mb, $\sigma_{\nu p}$(10 GeV) ~ 70 fb (ratio is → $10^{-12}$)

| Unit | Symbol | m² | cm² |
|---|---|---|---|
| megabarn | Mb | $10^{-22}$ | $10^{-18}$ |
| kilobarn | kb | $10^{-25}$ | $10^{-21}$ |
| barn | b | $10^{-28}$ | $10^{-24}$ |
| millibarn | mb | $10^{-31}$ | $10^{-27}$ |
| microbarn | µb | $10^{-34}$ | $10^{-30}$ |
| nanobarn | nb | $10^{-37}$ | $10^{-33}$ |
| picobarn | pb | $10^{-40}$ | $10^{-36}$ |
| femtobarn | fb | $10^{-43}$ | $10^{-39}$ |
| attobarn | ab | $10^{-46}$ | $10^{-42}$ |
| zeptobarn | zb | $10^{-49}$ | $10^{-45}$ |
| yoctobarn | yb | $10^{-52}$ | $10^{-48}$ |

# The Luminosity (~ Technology, not Physics)

$$\mathcal{L} = \phi_a \cdot N_b$$

*Beam on a target*

*Luminosity* : [(area x time)$^{-1}$ ]. From $\phi_a = n_a \times v_a$ and $N_b = n_b \cdot d \cdot A$ we have

$$\mathcal{L} = \varphi_a \cdot N_b = \boxed{\dot{N}_a \cdot n_b \cdot d} = \boxed{n_a \cdot v_a \cdot N_b}$$

***Luminosity*** → defined as one of two products below

1. number of incoming beam particles per unit time $N_a$, the target particle density in the scattering material $n_b$, and the target's thickness d;
2. beam particle density $n_a$, their velocity $v_a$ and the number of target particles $N_b$ exposed to the beam.

j packets with $N_a$ or $N_b$ particles, a ring of circumference U. velocity v ~ c in opposite directions and cross at an interaction point

*two beams in a storage ring*.

The luminosity is then:

$$\mathcal{L} = \frac{N_a \cdot N_b \cdot j \cdot v/U}{A}$$

A = beam cross-section at the collision point. For a Gaussian distribution of the beams ($\sigma_x$ and $\sigma_y$ respectively),

$$A = 4\pi\sigma_x\sigma_y \ .$$

… and have to be well aligned: LHC ~27Km circumference!

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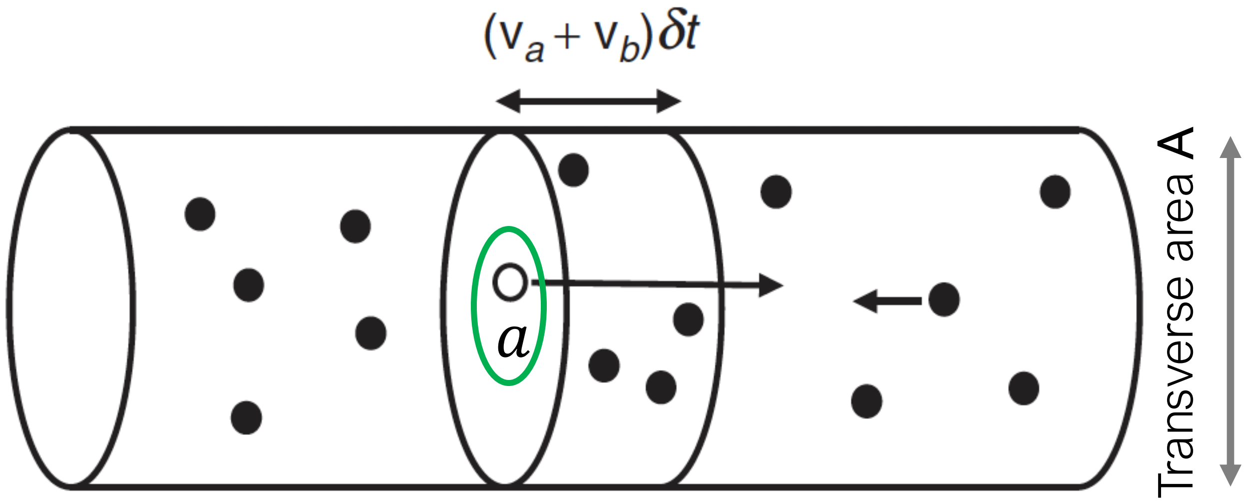
→ beams must be focused at the interaction point into the smallest possible area possible. Typical beam diameters are of the order of tenths of millimetres or less.

# A Simple Experiment




Simplified Experiment: *ONE* particle '$a$' travels in a medium with a particle density $n_b$ of type '$b$'

- $v_a, v_b$ velocities of particles of type a, b respectively;
- a, b travel opposite to each other;
- 

In time δt, traverses a volume with $\delta N = n_b(v_a + v_b)\delta t A$;
The interaction probability will be,

$$\delta P = \frac{\delta N\, \sigma}{A} = \frac{n_b(\mathrm{v}_a + \mathrm{v}_b)A\, \sigma \delta t}{A} = n_b \mathrm{v} \sigma \delta t,$$

where $\sigma$ can be considered as the 'effective area' of the particle

$$\frac{d\sigma}{d\Omega} = \frac{\text{number of particles scattered into } d\Omega \text{ per unit time per target particle}}{\text{incident flux}}.$$

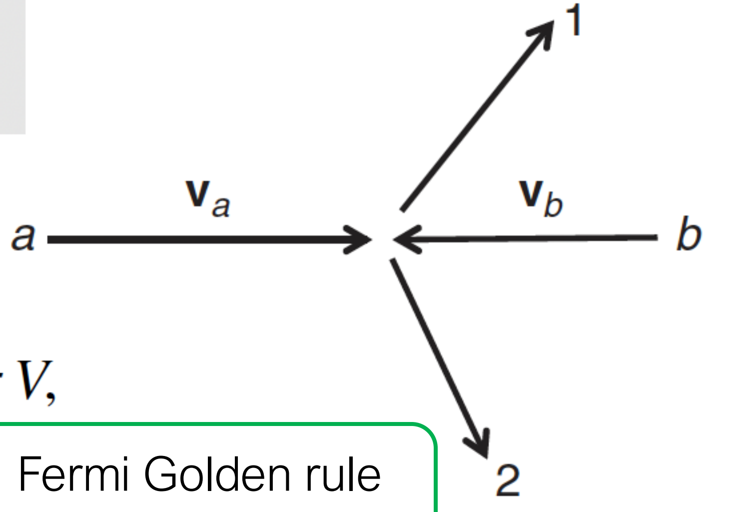
# Interaction Cross Section

$$\sigma = \frac{\text{number of interactions per unit time per target particle}}{\text{incident flux}}.$$

Consider the process $a + b \rightarrow 1 + 2$; observed in a rest frame with
- $v_a,\ v_b$ velocities;
- $n_a,\ n_b$ particle densities
- Cross section $\sigma$
- Normalized in a volume V

$$\phi_a = n_a(\mathrm{v}_a + \mathrm{v}_b).$$

$$\text{rate} = \phi_a n_b V \sigma = (\mathrm{v}_a + \mathrm{v}_b)\, n_a n_b\, \sigma\, V,$$




$$\sigma = \frac{(2\pi)^4}{(\mathrm{v}_a + \mathrm{v}_b)} \int |T_{fi}|^2 \delta(E_a + E_b - E_1 - E_2)\delta^3(\mathbf{p}_a + \mathbf{p}_b - \mathbf{p}_1 - \mathbf{p}_2)\frac{\mathrm{d}^3\mathbf{p}_1}{(2\pi)^3}\frac{\mathrm{d}^3\mathbf{p}_2}{(2\pi)^3}$$

Fermi Golden rule (non relativistic)

The cross section can be expressed in a Lorentz invariant form as

$$\sigma = \frac{(2\pi)^{-2}}{4\, E_a E_b(\mathrm{v}_a + \mathrm{v}_b)} \int |\mathcal{M}_{fi}|^2 \delta(E_a + E_b - E_1 - E_2)\delta^3(\mathbf{p}_a + \mathbf{p}_b - \mathbf{p}_1 - \mathbf{p}_2)\frac{\mathrm{d}^3\mathbf{p}_1}{2E_1}\frac{\mathrm{d}^3\mathbf{p}_2}{2E_2}.$$

$$F = 4E_a E_b(\mathrm{v}_a + \mathrm{v}_b)$$

Lorentz invariant flux

The most convenient way is to express the cross section in the center of mass system
- $\boldsymbol{p_a} = -\boldsymbol{p_b}. \boldsymbol{p_1} = -\boldsymbol{p_2} = \boldsymbol{p_f^*}$
- $\sqrt{s} = (E_a^* + E_b^*)$

In cms:

$$F = \sqrt{(p_a p_b)^2 - (m_a m_b)^2}$$

$$\sigma = \frac{1}{(2\pi)^2}\frac{1}{4\mathrm{p}_i^*\sqrt{s}} \int |\mathcal{M}_{fi}|^2 \delta\left(\sqrt{s} - E_1 - E_2\right)\delta^3(\mathbf{p}_1 + \mathbf{p}_2)\frac{\mathrm{d}^3\mathbf{p}_1}{2E_1}\frac{\mathrm{d}^3\mathbf{p}_2}{2E_2}.$$

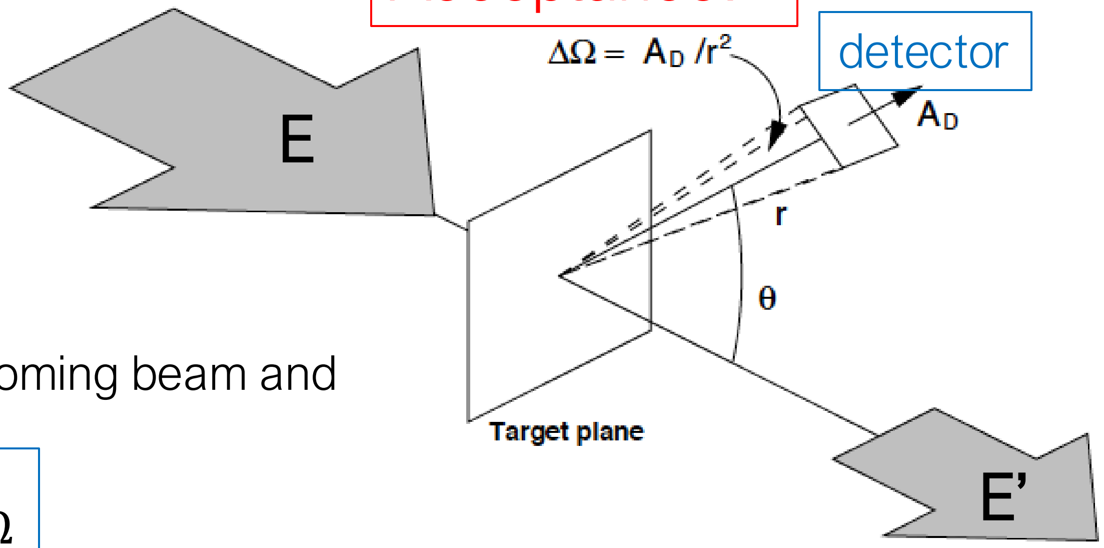
It may be shown to give

$$\sigma = \frac{1}{64\pi^2 s}\frac{\mathrm{p}_f^*}{\mathrm{p}_i^*}\int |\mathcal{M}_{fi}|^2 \mathrm{d}\Omega^*.$$

# Differential and Doubly-Differential Cross Sections

Real life: In all experiments only a fraction of all reactions are measured or accessible because of limited acceptance of the experimental set-up.

Acceptance!




Detector of area $A_D$ at a distance r and at an angle θ, it covers a solid angle equal to $\Delta\Omega = A_D/r^2$.

The reaction rate (assumed to depend on the energy of the incoming beam and on the angle θ) will be:

$$N(E,\theta,\Delta\Omega) = \mathcal{L}\frac{d\sigma(E,\vartheta)}{d\Omega}\Delta\Omega$$

If the energy & direction of the products is measured then the doubly differential cross section is also measured $d^2\sigma(E,E',\theta)/d\Omega dE'$ . The total cross section, in this case, will be the integral over the solid angle and over the scattering energies

$$\sigma_{tot}(E) = \int_{E_{min}}^{E_{max}}\int_{\theta_{min}}^{\theta_{max}} \frac{d^2\sigma(E,E',\theta)}{d\Omega dE} d\Omega dE'$$

Only a part of $\sigma_{tot}(E)$ measured due to acceptance

# Differential Cross Sections

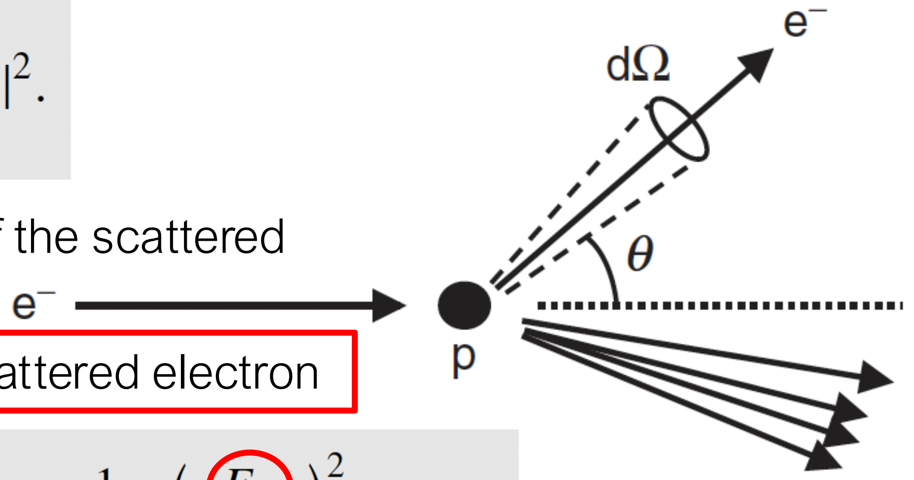
It may be important to measure the distribution of kinematic quantities, like angle and/or energy → derive information on the nature of the interaction

You measure the reaction rate in a solid angle element: but not only directions: single (E) or double differential cross-section (E & direction)

$$\frac{d\sigma}{dE} \quad \text{or} \quad \frac{d^2\sigma}{dE\,d\Omega}.$$

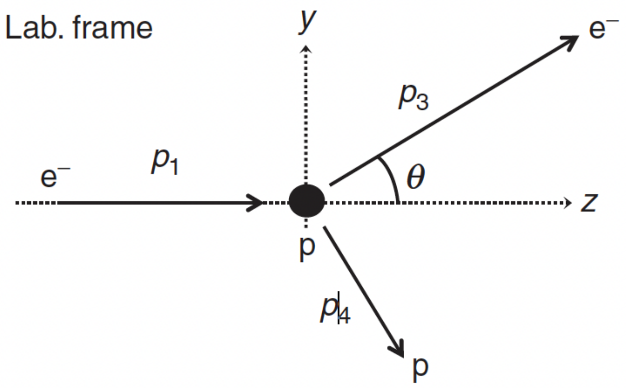
$$\sigma = \frac{1}{64\pi^2 s}\frac{p_f^*}{p_i^*}\int |\mathcal{M}_{fi}|^2 d\Omega^*.$$

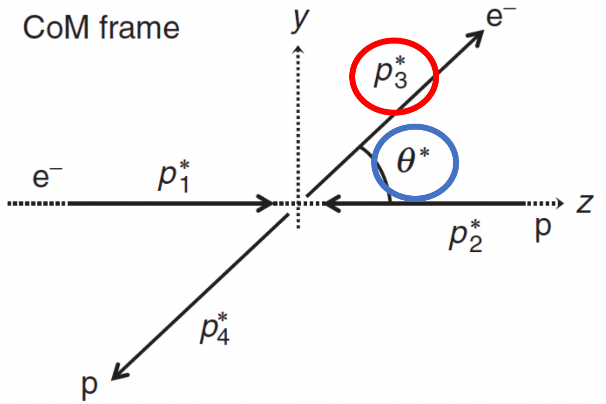
$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s}\frac{p_f^*}{p_i^*}|\mathcal{M}_{fi}|^2.$$




Example: Electron on a proton → Measure the direction (energy?) of the scattered electron (in the laboratory frame)

Measure angle or energy of the scattered electron




$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\left(\frac{E_3}{m_p E_1}\right)^2 |\mathcal{M}_{fi}|^2.$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\left(\frac{1}{m_p + E_1 - E_1\cos\theta}\right)^2 |\mathcal{M}_{fi}|^2.$$