Quasi-normal modes in non-perturbative quantum gravity

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Analytic infinite derivative gravity

[arXiv:2211.02070, arXiv:2209.02515]

Modified action capturing the propagator in 4D Minkowski spacetime

$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R + \frac{\lambda}{2} \left(R\mathcal{F}(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\alpha\nu\beta}\mathcal{F}_4(\Box)R^{\mu\alpha\nu\beta} \right) + \dots \right)$$

Spin-2 graviton propagator

$$\Pi_{spin-2} \sim \frac{1}{\Box\left(1 + \frac{\lambda}{M_P^2}\Box(\mathcal{F}_2(\Box) + 2\mathcal{F}_4(\Box))\right)}$$

Box is the d'Alembertian operator coming with a new scale: scale of non-locality

We should expect only 1 massless graviton in Minkowski spacetime

$$1 + \frac{\lambda}{M_P^2}\Box(\mathcal{F}_2(\Box) + 2\mathcal{F}_4(\Box)) = e^{\omega(\Box)}$$

$\omega(\Box)$ is an entire function

Analytic infinite derivative!

In a curved spacetime, new poles will appear in the propagator.

Background induced states (BIS-s)

Single Box modification can already generate BIS-s

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + \frac{\lambda}{2}(R\Box R + R_{\mu\nu}\Box R^{\mu\nu})\right]$$

the trace of the metric tensor perturbations h

$$\delta R_{\mu\nu}\Box\delta R^{\mu\nu} \sim= h(3\Box^3 + D^\mu R^\sigma_{\mu\alpha\nu}D^\alpha D_\sigma\partial^\nu + D^\mu D^\alpha R^\sigma_{\mu\alpha\nu}D_\sigma\partial^\nu)h$$

Degrees of freedom with curvature-dependent complex masses appear!

$$3\Box^3 + D^\mu R^\sigma_{\mu\alpha\nu}D^\alpha D_\sigma\partial^\nu + D^\mu D^\alpha R^\sigma_{\mu\alpha\nu}D_\sigma\partial^\nu = 0$$

Inflation aspects of BIS-s were widly discussed. But not much is done about black holes (BH).

We will focus on QNM-s, the excitation signatures of BH-s, generated by BIS-s

Quasi-normal modes

A review [arXiv:1102.4014]

Scalar perturbations in a nearly Schwarzschild BH background

$$(\nabla^\nu \nabla_\nu - \mu^2)\Phi(t,r,\theta,\phi) = 0$$

$$dr_* = \frac{dr}{1 - \frac{2GM}{r}}$$

In tortoise coordinate, it becomes a wave equation

$$\left[\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial r_*^2} + V(r)\right]\Psi(t,r) = 0$$ with potential $$V(r) = \left(1 - \frac{2GM}{r}\right)\left(\frac{l(l+1)}{r^2} + \frac{2GM}{r^3} + \mu^2\right)$$

Boundary conditions
$$\Psi(t,r_*) \sim \begin{cases} e^{-i\omega t - i\omega r_*}, & r_* \to -\infty \\ e^{-i\omega t + i\sqrt{\omega^2-\mu^2} r_*}, & r_* \to +\infty \end{cases}$$

pure ingoing wave at the horizon
pure outgoing wave at spatial infinity

$$\frac{\partial^2}{\partial r_*^2}\Psi(r_*) + (\omega^2 - V(r))\Psi(r_*) = 0$$

$$\mu = \mu(r)$$

QNM problem can be seen as finding quantum bound states in a flipped potential.

⇓

A discrete set of complex eigenvalues ω

BIS-s introduce complex, generically radius dependent masses

Restrictions from causality

Green's function formalism

BIS-s introduce a complex variable mass $\mu = \mu(r)$

$$\Psi(r_*, t) = \int [G(r_*, r'_*, t)\Psi_t(r'_*, 0) + G_t(r_*, r'_*, t)\Psi(r'_*, 0)]dr'_*$$

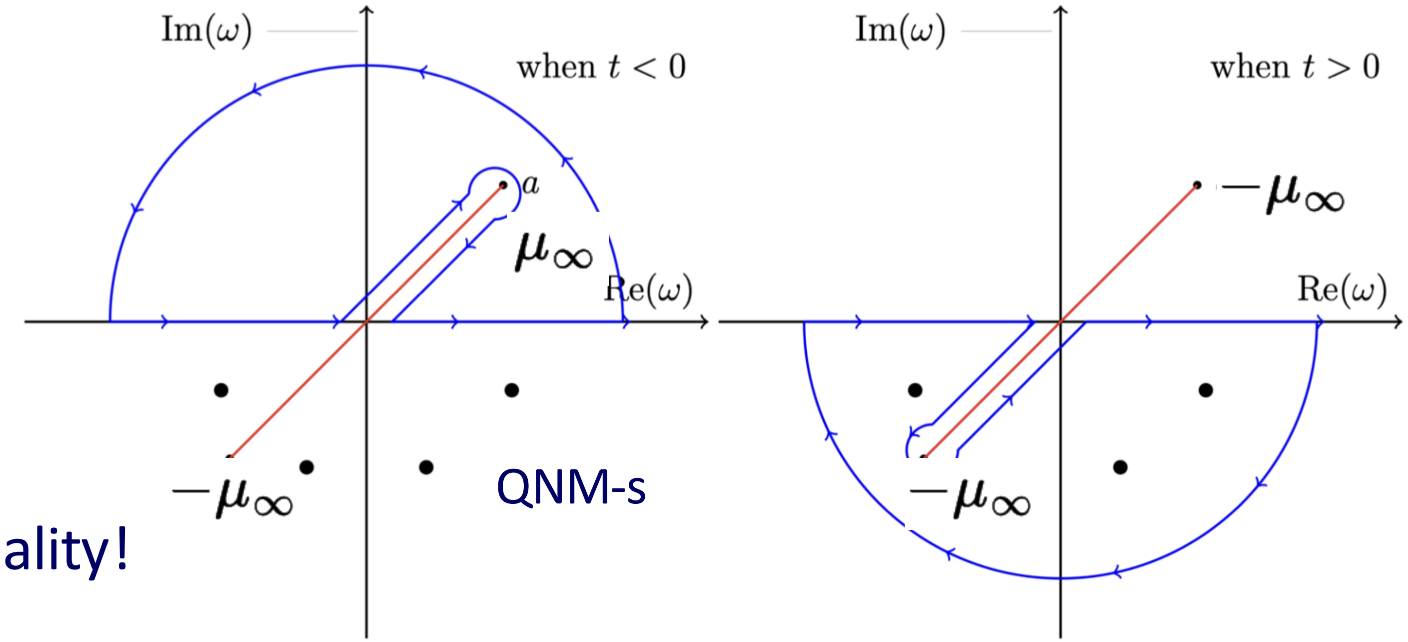
$$\tilde{G}(r_*, r'_*, \omega) = \frac{1}{\tilde{\Psi}_1 \partial_{r_*}\tilde{\Psi}_2 - \tilde{\Psi}_2 \partial_{r_*}\tilde{\Psi}_1} \begin{cases} \tilde{\Psi}_1(r'_*, \omega)\tilde{\Psi}_2(r_*, \omega), & r_* > r'_* \\ \tilde{\Psi}_1(r_*, \omega)\tilde{\Psi}_2(r'_*, \omega), & r_* < r'_* \end{cases}$$

$\tilde{\Psi}_2 \to e^{\sqrt{\omega^2 - \mu_\infty^2}\, r_*}$ r->∞

$$G(r_*, r'_*, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{G}(r_*, r'_*, \omega)e^{-i\omega t}d\omega$$

Non-zero imaginary part

at spatial infinity gives



So, μ_∞ must be real to preserve causality!

Possible unstable modes

“The mathematical theory of black holes” by Chandrasekhar

$$\Psi(t, r_*) \sim \begin{cases} e^{-i\omega t - i\omega r_*}, & r_* \to -\infty \\ e^{-i\omega t + i\sqrt{\omega^2 - \mu^2} r_*}, & r_* \to +\infty \end{cases}$$

$$\mathrm{Im}(\omega) > 0$$

From the boundary condition, the solution will grow if ω has positive imaginary part

Assume: $\mathrm{Im}(\omega) > 0$

$$\frac{\partial^2\Psi}{\partial t^2} - \frac{\partial^2\Psi}{\partial r_*^2} + V(r)\Psi = 0 \qquad (1)$$

multipy by $\frac{\partial\Psi^*}{\partial t}$ and integrate over r_*

$$\int_{-\infty}^{+\infty}\left(\frac{\partial\Psi^*}{\partial t}\frac{\partial^2\Psi}{\partial t^2} + \frac{\partial\Psi}{\partial r_*}\frac{\partial^2\Psi^*}{\partial t\partial r_*} + V(r)\Psi\frac{\partial\Psi^*}{\partial t}\right)dr_* = 0 \qquad (2)$$

(2)+(2)*

$$\frac{\partial}{\partial t}\int_{-\infty}^{+\infty}\left(\left|\frac{\partial\Psi}{\partial t}\right|^2 + \left|\frac{\partial\Psi}{\partial r_*}\right|^2 + V(r)|\Psi|^2\right)dr_* = 0 \qquad (3)$$

A constant contradicts with a growing $\Psi(t, r_*)$

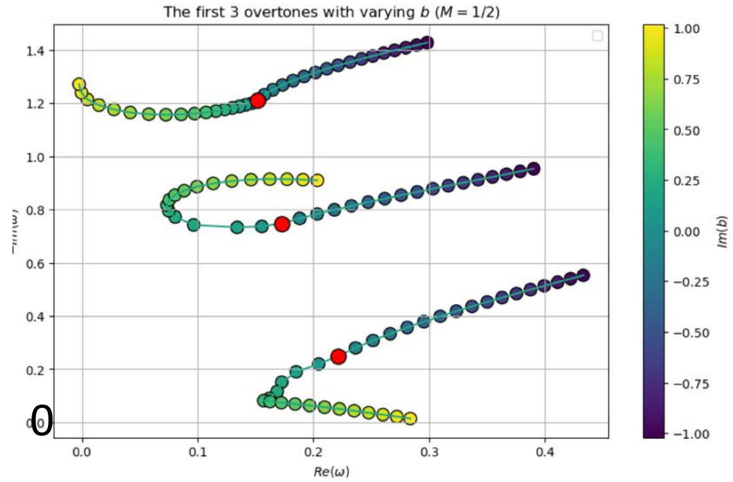
However:
In the presence of complex mass, the potential will be complex.

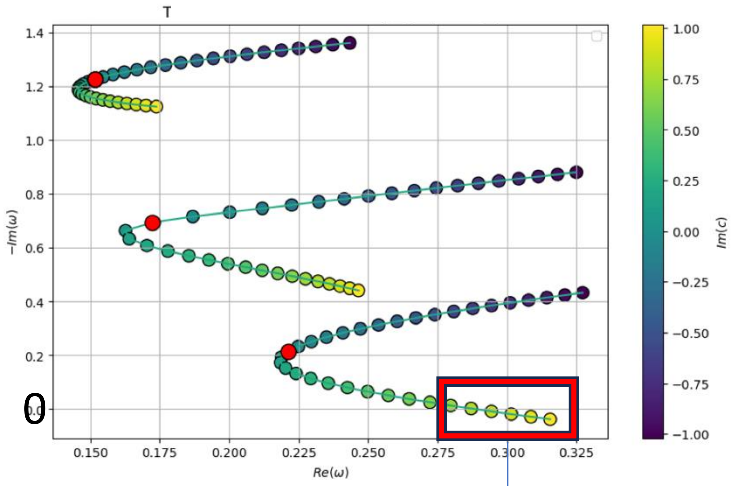
Going from (2) to (3) is no longer true

Unstable modes are possible in the presence of BIS-s

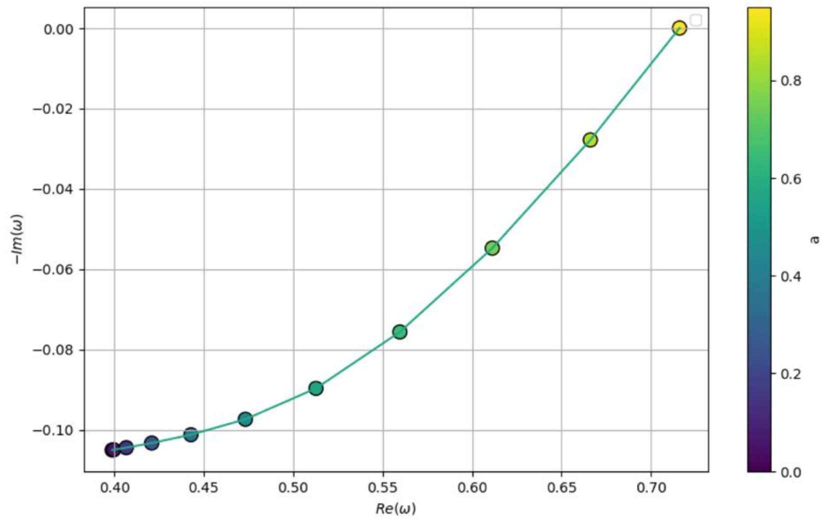
Numerical results

We consider $$\mu(r)^2 = a^2 + \frac{b}{2GMr} + \frac{c}{r^2} + \frac{2GMd}{r^3}$$

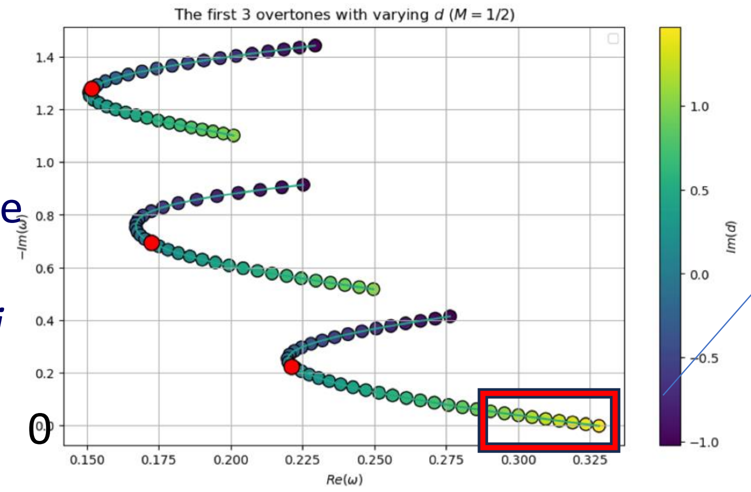




We use Leaver's method of continued fraction



The color changes from dark to light, corresponding to the value of a, b, c changing from -*i* to *i*



Unstable modes!

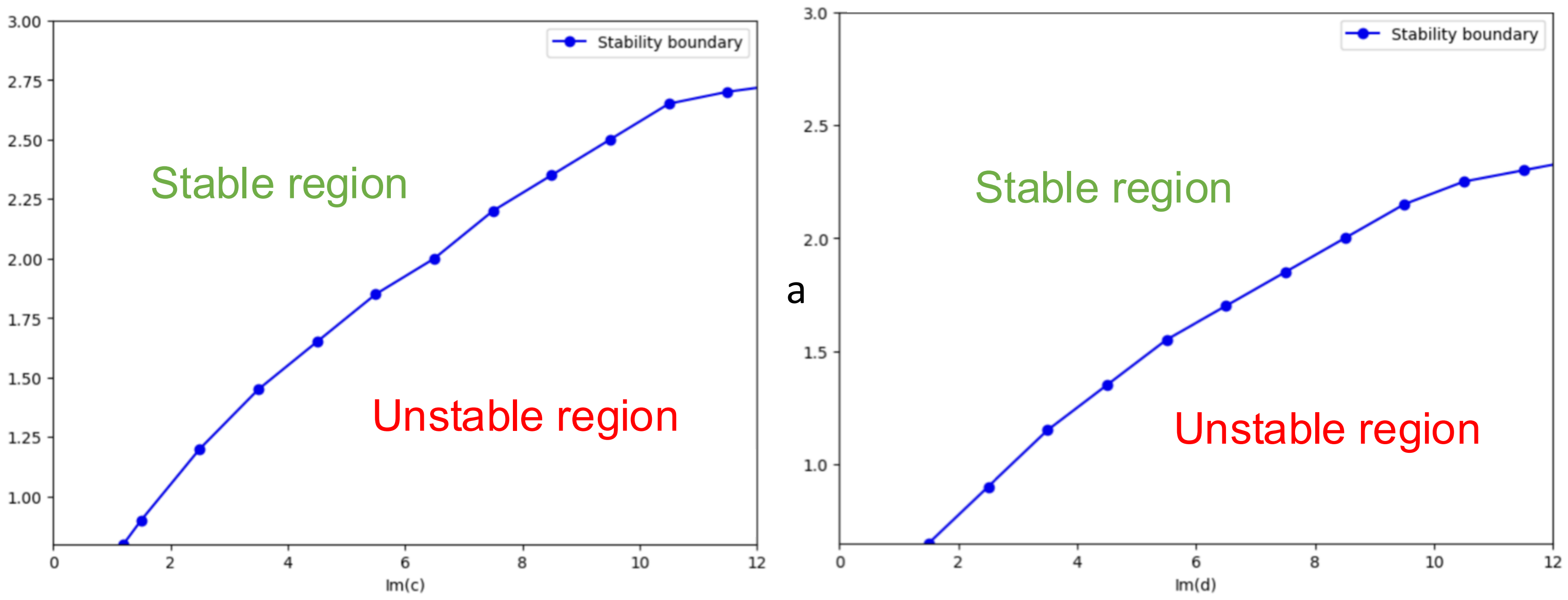
c=2*i*, *a*=0, which falls in the unstable region, moves towards the stable region as we increase *a*

Big real mass could restore stability

First three overtones with l = 0, a=0. b, c, d vary as pure imaginary values

Restrictions form stability

$$\mu(r)^2 = a^2 + \frac{b}{2GMr} + \frac{c}{r^2} + \frac{2GMd}{r^3}$$



Plot of the relation between critical values of a and c, d

For a fixed *M*, the BIS-s mass can’t exceed a critical value. This constrains parameters and form-factors of this gravity model.

Summary:

- Background induced modes will appear in a non-trivial background in the analytic infinite derivative gravity theory. Masses of such excitations are complex and depend on the curvature.
- From causality constraints, the complex, radius dependent mass must be real at spatial infinity in an asymptotically Schwarzschild black hole background.
- Unstable modes are possible in the presence of BIS-s with complex mass. We can formulate constraints to the modified gravity theory by requiring stability of QNM-s.

Outlook:

- Similar computations can be done for other BH configurations like Kerr BH and a more general radius dependence of the mass .
- More detailed constraints to the modified gravity theory can be found from stability of QNM-s.

Thank you for your attention!