Black Hole singularity resolution in infinite derivative gravity theories

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past works with Alexei Starobinsky and works in progress with O.Melichev, A.Naskar, L.Rachwal, A.Tokareva and my students

Breakdown of the problem

UV complete gravity – already a challenge for more than a century

- **Many attempts, no complete satisfaction yet**

Infinite derivatives

- **General considerations and, for example, Asymptotic Safety suggest infinite derivative Lagrangians**

Strings

- **Strings and especially string field theory strongly suggest non-local interactions in the form of infinite-derivative form factors**

Aref'eva, Barvinsky, Biswas, Dragovich, Koivisto, Krasnikov, Kuz'min, Mazumdar, Modesto, Percacci, Platania, Saueressig, Sen, Siegel, Shapiro, Tomboulis, Weinberg, Witten, Zwiebach, ...

Some old references

- **Classic one:**

  M. Ostrogradski, Mem. Ac. St. Petersburg, VI 4, 385–517 (1850)

- **Mathematical:**

- H.T. Davis, Ann. of Math. 2, no. 4, 686–714 (1931)

- H.T. Davis, The Theory of Linear Operators from the Standpoint of Differential Equations of Infinite Order (Indiana, the Principia Press, 1936)

- R.D. Carmichael, Bull. Amer. Math. Soc. 42, 193–218 (1936)

- L. Carleson, Math. Scand. 1, 31–38 (1953)

- **Physical:**

- A. Pais and G.E. Uhlenbeck, Phys. Rev. 79, 145–165 (1950)

"Convergence" (renormalizability), "definite norm" (unitarity) and causality – cannot be achieved simultaneously. Fine, but what if violation of microcausality is hidden under the uncertainty scale? de Rham, Tokareva, Tolley, ...

Action to study [1602.08475, 1606.01250, 1711.08864]

$$S=\int d^4x\sqrt{-g}\Bigg(\frac{M_P^2R}{2}-\Lambda$$
$$+\frac{\lambda}{2}\Big(R\mathcal{F}_1(\Box)R+R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu}+W_{\mu\nu\lambda\sigma}\mathcal{F}_4(\Box)W^{\mu\nu\lambda\sigma}\Big)\Bigg)$$

Here $\mathcal{F}_X(\Box)=\sum_{n\geq 0}f_{X_n}\Box^n$ with all f_{X_n} constants

We assume that $\Box$ enters form-factors in a combination $\Box/\mathcal{M}_s^2$ where the mass parameter is the non-locality scale. We put $\mathcal{M}_s=1$ for a while.

This is the most general action (still redundant, $\mathcal{F}_2$ can be zero in $D=4$ or a constant in $D>4$) to study linear perturbations around MSS.

We name it Analytic Infinite Derivative (AID) gravity.

Covariant spin-2 propagator on MSS:

$$S_2 = \frac{1}{2}\int d^4x\sqrt{-\bar{g}}\; h^{\perp}_{\nu\mu}\left(\bar{\Box}-\frac{\bar{R}}{6}\right)\left[\mathcal{P}(\bar{\Box})\right] h^{\perp\,\mu\nu}$$

$$\mathcal{P}(\bar{\Box}) = 1 + \frac{2}{M_P^2}\lambda f_{1_0}\bar{R} + \frac{2}{M_P^2}\lambda\mathcal{F}_4\left(\bar{\Box}+\frac{\bar{R}}{3}\right)\left(\bar{\Box}-\frac{\bar{R}}{3}\right)$$

$$\to e^{2\omega(\bar{\Box})}$$

We require $\mathcal{P}(\bar{\Box}) = e^{2\omega(\bar{\Box})}$ and $\omega(\bar{\Box})$ must be an entire function to avoid new poles.

The Stelle's case (and any finite degree polynomial $\mathcal{F}_4(\bar{\Box})$) results in ghost poles.

Recap on infinite derivative gravity theories

- **Graviton propagator in general is modified to**

$$\Pi = e^{2\omega(k^2)}\Pi_{GR} \sim \frac{e^{2\omega(k^2)}}{k^2}$$

  and $\omega(k^2)$ must be an entire function.

- **We thus must have an infinite number of derivatives**
- **Wick rotation is a problem but it got a resolution thanks to Pius, Sen, and also [arxiv:2103.01945]**
- **Theory is renormalizable and unitary.**
- **Full propagator yet to be computed.**
- **Many interesting solutions can be accommodated.**
- **In particular, Starobinsky inflation can be explicitly embedded.**

Action again

$$S = \int d^4x\sqrt{-g}\Bigg(\frac{M_P^2 R}{2}$$

$$+\frac{\lambda}{2}\left(R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + W_{\mu\nu\lambda\sigma}\mathcal{F}_4(\Box)W^{\mu\nu\lambda\sigma}\right)\Bigg)$$

If $\mathcal{F}_4 \neq 0$ than a Schwarzschild BH is not a solution.

Even if $\mathcal{F}_4 = 0$ we claim it is not!

WHY?

Equations and BH-s

Typical terms in EOM-s (trace equation):

$$M_P^2 R - 6\lambda\Box\mathcal{F}_1(\Box)R$$
$$-2\lambda\sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \Box^l R \Box^{n-l} R + \{8 \text{ terms}\} = -T$$

and T is the trace of the stress tensor.

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2 d\Omega^2$$

Schwarzschild metric

$$A(r) = 1 - \frac{2GM}{r}$$

Without the Weyl (or Riemann) tensor in equations Schwarzschild BH is naively a solution.

Then why not?

Schwarzschild BH: to be or not to be?

We cannot substitute the Schwarzschild metric like in GR as we need to give a meaning for instance for $R(r=0)$

Regularization

$$A(r) = 1 - \frac{2GM}{r} \to A(r) = 1 - \frac{2GM}{r}\tilde{A}(r,\alpha), \;\; \tilde{A} = e^{-\alpha/r^p}$$

such that $\tilde{A}(\infty) = 1, \tilde{A}(r)/r \to 0$ at zero and $\tilde{A}(r,0) = 1$.

We plug a regularized function in EOM-s and compute T – the stress tensor trace.

And then we compute $\int d^3x \sqrt{-g} T = E$ which is related to the energy of the object. In static case it is related to its mass.

What is a BH mass?

What we compute is $E = \int d^3x\sqrt{-g}(T_i^i + T_0^0)$.

Tolman mass is defined as $M_T = \int d^3x\sqrt{-g}(T_i^i - T_0^0)$.

ADM mass is a coefficient of $1/r$ term in a series expansion of g_{rr} metric component at infinity divided by $2G$, or equivalently $M_{ADM} = -\int d^3x\sqrt{-g}T_0^0$.

Thus E is nothing but $M_T - 2M_{ADM}$ and should correspond to $-M_{ADM}$.

It is naturally expected to be a finite quantity.

To simplify computations we actually compute

$$\lim_{\Delta t \to \infty} \frac{1}{2\Delta t}\int_{-\Delta t}^{\Delta t} dt d^3x\sqrt{-g}T$$

Schwarzschild BH in higher-derivative theories

Computing E we yield

$$-E = M - 4\pi\lambda \int_0^\infty r^2 dr \Big(R\Box\mathcal{F}_1'(\Box)R + R_{\mu\nu}\Box\mathcal{F}_2'(\Box)R^{\mu\nu} + W_{\mu\nu\lambda\sigma}\Box\mathcal{F}_4'(\Box)W^{\mu\nu\lambda\sigma}\Big)$$

If $\mathcal{F}(\Box) = \mathrm{const}$ (Stelle gravity) then it does not contribute.
Schematically

$$-E = M - 4\pi\lambda(E_0 + E_1 + E_2 + \dots)$$

Here E_n corresponds to $\Box^n$ and for $p = 1$

$$E_0 \sim 1/\alpha^3, \quad E_1 \sim 1/\alpha^6 + 1/\alpha^5, \quad \dots$$

E_0 comes from $\mathcal{F}(\Box) \sim \log(\Box)$

Convergence analysis

We can deduce for the quantity E

$$
\begin{aligned}
-E = M & \\
& - 4\pi\lambda M^2 (2\alpha)^{-\frac{3}{p}} \left(\sum_{n=0}^{\infty} (-1)^n \hat{f}_n \beta_n(p) (2\alpha)^{-\frac{2n}{p}} + \{2 \text{ terms}\} \right) \\
& + O(M^3)
\end{aligned}
$$

Here $\hat{f}_n = n f_n$ and $\hat{f}_0$ comes from a log.
Recall that $\mathcal{F}(\Box) = \sum_{n\geq 0} f_n \Box^n$.
The above series *can* converge if it is alternating with rapidly falling coefficients. Example

$$
\sum_{k\geq 0} \frac{(-1)^k}{k!\alpha^k} = e^{-1/\alpha} \xrightarrow{\alpha\to 0} 0
$$

Convergence analysis continued

By direct computations we can see that β_n grow rapidly. The series for E will converge for any p if

$$\lim_{n\to\infty} \frac{|\hat{f}_n|}{e^{qn\log n}} = 0, \text{ for any } q > 0$$

For an entire function its maximal grows rate for large z is given by $e^{sz^{\rho}}$. ρ is the order and s is the type. Computing β_n we find an acceptable order of $\mathcal{F}(\Box)$ is $\rho < 3/2$

However, from the perspective of QFT for renormalizbility and unitarity we need that $\mathcal{F}(\Box)$ grows at most polynomially along the positive real axis and this implies that the order of $\mathcal{F}(\Box)$ is infinite.

BH results briefly and what about micro-BH?

- Regularization approach is motivated by a collapse consideration. You must be able to form a BH starting with a regular matter distribution.
- Regularization of a Schwarzschild BH can be removed only in 2 and 4 derivative gravity. Any higher (finite) derivative gravity cannot have this solution.
- Infinite derivative case results in infinitely many terms like $1/\alpha^n$ and in principle a summation over n may have a good $\alpha \to 0$ limit.
  BUT this is NOT compatible with a viable propagator for a UV complete unitary gravity.
- We thus must accept that a UV complete gravity not only resolves the BH singularity but also limits the micro-BH mass from below to $\mathcal{M}_s$ which obeys $M_{inf} \ll \mathcal{M}_s < M_P$

Conclusions

- **A class of analytic infinite derivative (AID) theories has been considered targeting the goal of constructing a UV complete and unitary gravity. These models have clear connection with SFT.**
- **This gravity model features many nice properties, like native embedding of the Starobinsky inflation, finite Newtonian potential at the origin, presence of a non-singular bounce, etc.**
- **We argue that these theories disregard singular BH solutions on the example of Schwarzschild BH.**

Future directions

- Other BH solutions (charged, extremal, rotating) should be analyzed.
- BH regularity as a given feature implies that QNM may be modified.
- QNM will not test the interior of a BH as such, but higher derivatives in the action will result in new QNM shapes which is a very interesting way to support the idea that a UV complete gravity resolves BH singularities. [arxiv:2412.02678]
- Mass inflation problem should be addressed

Thank you for listening!

Let it be a Non-local scalar field [arxiv:2103.01945]

Consider Analytic Infinite Derivative (AID) scalar field action:

$$L = \frac{1}{2}\phi(\Box - m^2) f^{-1}(\Box)\phi - V(\phi)$$

We demand the form-factor to be an exponent of an entire function $\sigma(z)$

$$f(z) = \exp(2\sigma(z))$$

This is required to have no extra poles in the perturbative vacuum.
We also normalize it as $f(0) = f(m^2) = 1$ to preserve the local answers in the IR limit.

Non-local scalar field, continued

Several arguments to consider the above action:

- **It naturally appears in SFT and in p-adic strings**
- **It was proven to be unavoidable in order to build unitary and renormalizable diffeomorphism invariant gravity**
- **This construction can make any arbitrary potential renormalizable**
- **Surely, some other benefits**

Namely, we can adjust the fall rate of the propagator for large momenta by choosing the form-factor. Power-counting convergence requires the fall faster than $\sim 1/p^2$.

New excitations – Half of them are ghosts!

Linearization around a background solution ϕ_0:

$$L = \frac{1}{2}\psi\left[(\Box - m^2)f^{-1}(\Box) - V''(\phi_0)\right]\psi$$

Let's assume $V''(\phi_0) = v \approx \mathrm{const} \neq 0$.

- **In general there is an infinite number of new excitations with perhaps complex conjugate masses squared**
- **The kinetic operator is again an entire function and obeys the Weierstrass decomposition**

$$(\Box - m^2)f^{-1}(\Box) - v^2 \sim \prod_i (\Box - \mu_i^2)e^{\sigma_v(\Box)}$$

- **Each μ_i corresponds to a mass of a distinct excitation.**