Positivity Bounds at one-loop level

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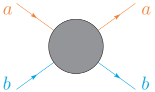
[2408.10318] Yunxiao Ye, Bin He, JG
current work with Xiao Cao, Yuhang Wu, Yunxiao Ye

Introduction

- **Can all EFTs be UV completed?**
- Dispersion relations of forward elastic amplitudes suggest that certain operator coefficients can only be positive.
  - Assuming the UV physics is consistent with the fundamental principles of QFT (analyticity, locality, unitarity, Lorentz invariance).
  - [hep-th/0602178] Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi
    ... many papers ...
- **These positivity bounds only exist for certain Dimension-8 (or higher) operators** (without additional model assumptions)**!**

$$\frac{d^2}{ds^2}\mathcal{A}(ab \to ab)_{t\to 0} \geq 0\,.$$



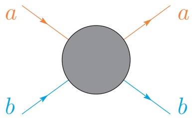
- Two important implications:
  - Assuming UV physics is described by QFT $\Rightarrow$ we can rule out a large region in the EFT parameter space.
  - **Or** we can test the fundamental principles of QFT if we measure these dim-8 coefficients well enough.

Dispersion relations

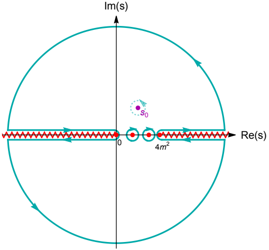
- Consider a forward ($t \to 0$) elastic amplitude ($s+t+u=4m^2$)

$$\tilde{\mathcal{A}}_{ab}(s) = \sum_n c_n (s-s_0)^n\,,$$

$$c_n = \frac{1}{2\pi i}\oint_{s=s_0} ds \frac{\tilde{\mathcal{A}}_{ab}(s)}{(s-s_0)^{n+1}}\,,$$



- Applying the fundamental principles of QFT
  - Analyticity (Cauchy's theorem applies)
  - Locality (poles from tree-level factorization, branch cuts from loops, Froissart Bound)
  - Unitarity (Optical theorem, $\mathrm{Im}\mathcal{A} \sim \sigma_{\mathrm{tot}}$)
  - Lorentz invariance (Crossing symmetry)



- Dispersion relation tells us that

$$c_n = \int_{4m^2}^{\infty} \frac{ds}{\pi} s\sqrt{1-\frac{4m^2}{s}} \left(\frac{\sigma_{\mathrm{tot}}^{ab}}{(s-s_0)^{n+1}} + (-1)^n \frac{\sigma_{\mathrm{tot}}^{a\bar{b}}}{(s-4m^2+s_0)^{n+1}} \right) + c_n^{\infty}\,,$$

Sum rules and positivity bounds

- Assuming s_0 is real and $0 < s_0 < 4m^2$,

$$c_n = \int_{4m^2}^{\infty} \frac{ds}{\pi} s\sqrt{1-\frac{4m^2}{s}} \left(\frac{\sigma_{\rm tot}^{ab}}{(s-s_0)^{n+1}} + (-1)^n \frac{\sigma_{\rm tot}^{a\bar{b}}}{(s-4m^2+s_0)^{n+1}} \right) + c_n^{\infty} \,,$$

  - Froissart bound: $\mathcal{A} < {\rm const} \cdot s \log^2 s \quad \Rightarrow \quad c_n^{\infty} = 0$ for $n > 1$.
  - For even n, the two terms with cross sections are both positive, so $c_n > 0$.

- At tree level, consider the limit $m^2,\ s_0 \ll \Lambda^2$ (massless EFT).

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4} \mathcal{O}_j^{(8)} + \cdots \,.$$

$$\mathcal{A}(s)|_{t=0} = c_0 + c_1\, s + c_2\, s^2 + \ldots$$

- $c_{n=1}$ $\Leftrightarrow$ dimension-6 (no positivity bounds, boundary can be nonzero),
  $c_{n=2}$ $\Leftrightarrow$ dimension-8 (or d6^2) (has positivity bounds!),
  ...

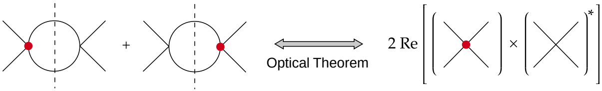
- Many generalizations and modified versions...
  - Improved positivity bounds [1710.09611] de Rham *et al.* (see also [1710.02539] Bellazzini *et al.*), Arc variable [2011.00037] Bellazzini *et al.*, [2012.15849] Arkani-Hamed, Huang, Huang, ... (See also talks by Qing, Yongjun and Shi-Lin.)

Does positivity still hold at loop level? [2408.10318] Yunxiao Ye, Bin He, JG

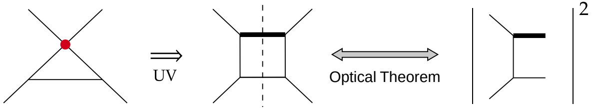
- The kinematic dependence becomes more complicated at loop level due to the log terms.
- What's the impact of RG running on positivity bounds (and vice versa)?
- Some interesting observations have been found in previous studies.
  - IR effects can be important. [2011.00037, 2112.12561] Bellazzini *et al.*
  - The naive tree-level positivity bound could appear to be violated. [2110.01624] Chala, Santiago
  - Some of the RG mixing of dim-8 coefficients are subject to positivity bounds. [2301.09995] Chala, [2309.16611] Chala, Li
- **Our main message:** To each fixed loop order in the UV model, "something" is positive, but that something can have a lot of contributions and each contribution is not necessarily positive!

Two different scenarios

- An “asymmetric” one-loop diagram of the forward elastic amplitude corresponds to the interference term of a cross section and can take either sign!



- A “symmetric” one-loop diagram corresponds to a squared contribution to the cross section and could be subject to a positivity bound (which can have several contributions in the EFT).



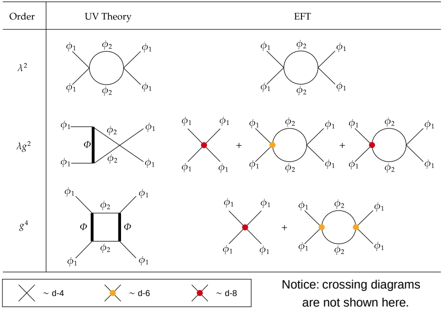
A (counter) example

- Consider a model with 2 real light scalars $\phi_{1,2}$ and a real heavy scalar Φ,

$$\mathcal{L} = \frac{1}{2}\left(\partial^\mu \Phi \partial_\mu \Phi - M^2\Phi^2\right) + \frac{1}{2}\partial^\mu \phi_1 \partial_\mu \phi_1 + \frac{1}{2}\partial^\mu \phi_2 \partial_\mu \phi_2 - gM\Phi\phi_1\phi_2 - \frac{1}{4}\lambda \phi_1^2 \phi_2^2 \,,$$

- At low energy we can integrate out Φ and match it to the EFT with $\phi_{1,2}$.

- A positivity bound can be obtained from the forward elastic amplitude $\phi_1\phi_1 \to \phi_1\phi_1$ (or $1 \leftrightarrow 2$).
- For $\lambda \gg g^2$, the new physics contributions are proportional to λg^2, which can obviously take either sign.
- **What's wrong?**



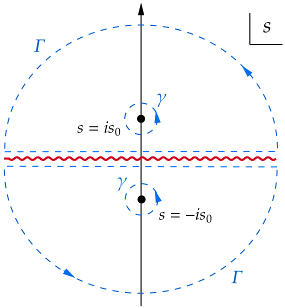
Notice: crossing diagrams are not shown here.

Back to the dispersion relation

- Two different scales:
  - s_0 labels the position of the pole in the comples s plane. (often denoted as μ^2, which we will avoid doing...)
  - μ is the renormalization scale (in $\overline{MS}$).
- Due to the cross symmetry of $\phi_1\phi_1 \to \phi_1\phi_1$, we can consider the massless case (which makes life much easier) and use a modified version of the dispersion relation.

[2011.11652] Herrero-Valea, Santos-Garcia, Tokareva

$$\Sigma \equiv \left(\oint_{s=is_0} + \oint_{s=-is_0} \right) \frac{ds}{2\pi i} \frac{s^3 \tilde{A}(s)}{(s^2+s_0^2)^3}$$
$$= \frac{2}{\pi} \int_0^\infty ds \, \frac{s^4 \sigma(s)}{(s^2+s_0^2)^3} \geq 0 \, .$$



- This holds for any real s_0, but $s_0 \ll \Lambda^2$ is required for the EFT validity.
  - Dim-10 contributions are further suppressed by a factor of s_0/Λ^2.
  - With loops, we don't want to let $s_0 \to 0$ in which case Σ may diverge.

The 2-scalar EFT

	$\mathcal{A}_1 \equiv \mathcal{A}(\phi_1\phi_1\phi_1\phi_1)$	$\mathcal{A}_2 \equiv \mathcal{A}(\phi_2\phi_2\phi_2\phi_2)$	$\mathcal{A}_{12} \equiv \mathcal{A}(\phi_1\phi_2\phi_1\phi_2)$
$D4$	$\mathcal{A}_1^{[4]} = c_1^{[4]}$	$\mathcal{A}_2^{[4]} = c_2^{[4]}$	$\mathcal{A}_{12}^{[4]} = c_{12}^{[4]}$
$D6$	$\mathcal{A}_1^{[6]} = 0$	$\mathcal{A}_2^{[6]} = 0$	$\mathcal{A}_{12}^{[6]} = c_{12}^{[6]}\, t$
$D8$	$\mathcal{A}_1^{[8]} = c_1^{[8]}(s^2+t^2+u^2)$	$\mathcal{A}_2^{[8]} = c_2^{[8]}(s^2+t^2+u^2)$	$\mathcal{A}_{12}^{[8]} = c_{12,su}^{[8]}(s^2+u^2) + c_{12,t}^{[8]} t^2$

$$
\begin{aligned}
\mathcal{L} = & \frac{1}{2}\partial_\mu\phi_1\partial^\mu\phi_1 + \frac{1}{2}\partial_\mu\phi_2\partial^\mu\phi_2 + \frac{c_1^{[4]}}{4!}\phi_1^4 + \frac{c_2^{[4]}}{4!}\phi_2^4 + \frac{c_{12}^{[4]}}{4}\phi_1^2\phi_2^2 + c_{12}^{[6]}\left(\partial_\mu\phi_1\right)\left(\partial^\mu\phi_2\right)\phi_1\phi_2 \\
& + \frac{c_1^{[8]}}{2}\left(\partial_\mu\phi_1\right)\left(\partial^\mu\phi_1\right)\left(\partial_\nu\phi_1\right)\left(\partial^\nu\phi_1\right) + \frac{c_2^{[8]}}{2}\left(\partial_\mu\phi_2\right)\left(\partial^\mu\phi_2\right)\left(\partial_\nu\phi_2\right)\left(\partial^\nu\phi_2\right) \\
& + 2c_{12,su}^{[8]}\left(\partial_\mu\phi_1\right)\left(\partial^\mu\phi_2\right)\left(\partial_\nu\phi_1\right)\left(\partial^\nu\phi_2\right) + c_{12,t}^{[8]}\left(\partial_\mu\phi_1\right)\left(\partial^\mu\phi_1\right)\left(\partial_\nu\phi_2\right)\left(\partial^\nu\phi_2\right),
\end{aligned}
$$

- The Wilson coefficients in the massless scalar EFT can be conveniently parameterized in terms of tree level on-shell amplitudes.
- (Note: We've absorbed Λ in the definition of couplings.)

$\mathcal{A}(\phi_1\phi_1\phi_1\phi_1)$ at the 1 loop level

$$\mathcal{A}_1^{[4]} = c_1^{[4]} + \frac{1}{32\pi^2}\left(\left(c_1^{[4]}\right)^2 + \left(c_{12}^{[4]}\right)^2\right)\left(-\log\frac{-s}{\mu^2} - \log\frac{-t}{\mu^2} - \log\frac{-u}{\mu^2} + 6\right),$$

$$\mathcal{A}_1^{[6]} = \frac{1}{16\pi^2}\left(c_{12}^{[4]}c_{12}^{[6]}\right)\left(-s\log\frac{-s}{\mu^2} - t\log\frac{-t}{\mu^2} - u\log\frac{-u}{\mu^2}\right).$$

$$\mathcal{A}_1^{[8],\text{tree}} = c_1^{[8]}\left(s^2 + t^2 + u^2\right),$$

$$\begin{aligned}\mathcal{A}_1^{[8],\text{1-loop}} = \frac{1}{16\pi^2}s^2\Bigg[&-\log\frac{-s}{\mu^2}\left(\frac{1}{2}\left(c_{12}^{[6]}\right)^2 + \frac{2}{3}c_{12}^{[4]}c_{12,su}^{[8]} + c_{12}^{[4]}c_{12,t}^{[8]} + \frac{5}{3}c_1^{[4]}c_1^{[8]}\right)\\ &+\left(c_{12}^{[6]}\right)^2 + \frac{13}{9}c_{12}^{[4]}c_{12,su}^{[8]} + 2\,c_{12}^{[4]}c_{12,t}^{[8]} + \frac{31}{9}c_1^{[4]}c_1^{[8]}\Bigg]\\ &+\quad(s\longleftrightarrow t)\quad+\quad(s\longleftrightarrow u).\end{aligned}$$

- $\overline{MS}$ scheme. μ is the renormalization scale at which the couplings are defined.
- Amplitudes are independent of μ (which gives RG equations).

Positivity bound at the 1-loop level

The Σ (≥ 0) for $\phi_1\phi_1 \to \phi_1\phi_1$ in the EFT up to 1-loop and dim-8 is

$$\Sigma = 2c_1^{[8]} + \frac{1}{64\pi^2}\frac{1}{s_0^2}\left(\left(c_1^{[4]}\right)^2 + \left(c_{12}^{[4]}\right)^2\right) + \frac{1}{16\pi^2}\frac{1}{s_0}\frac{3\pi}{8}c_{12}^{[4]}c_{12}^{[6]} + \left(\frac{3}{4} + \log\frac{s_0}{\mu^2}\right)\beta_1^{[8]}$$
$$+ \frac{1}{16\pi^2}\left(2\left(c_{12}^{[6]}\right)^2 + \frac{26}{9}c_{12}^{[4]}c_{12,su}^{[8]} + 4\,c_{12}^{[4]}c_{12,t}^{[8]} + \frac{62}{9}c_1^{[4]}c_1^{[8]}\right),$$

where

$$\beta_1^{[8]} \equiv \mu\frac{dc_1^{[8]}}{d\mu} = -\frac{1}{16\pi^2}\left(\frac{4}{3}c_{12}^{[4]}c_{12,su}^{[8]} + 2\,c_{12}^{[4]}c_{12,t}^{[8]} + \frac{10}{3}c_1^{[4]}c_1^{[8]} + \left(c_{12}^{[6]}\right)^2\right).$$

- Σ is independent of μ just like the amplitude. Requiring $\mu\frac{d}{d\mu}\Sigma = 0$ gives us the β function!
  - By changing μ we don't really change the positivity bound! We just change the definition of $c_1^{[8]}$.
- Σ does depend on s_0. $\Sigma \geq 0$ is valid as long as $0 < s_0 \ll \Lambda^2$.

Positivity bound at the 1-loop level

$$\Sigma = 2c_1^{[8]} + \frac{1}{64\pi^2}\frac{1}{s_0^2}\left(\left(c_1^{[4]}\right)^2 + \left(c_{12}^{[4]}\right)^2\right) + \frac{1}{16\pi^2}\frac{1}{s_0}\frac{3\pi}{8}c_{12}^{[4]}c_{12}^{[6]} + \left(\frac{3}{4} + \log\frac{s_0}{\mu^2}\right)\beta_1^{[8]}$$
$$+ \frac{1}{16\pi^2}\left(2\left(c_{12}^{[6]}\right)^2 + \frac{26}{9}c_{12}^{[4]}c_{12,su}^{[8]} + 4\,c_{12}^{[4]}c_{12,t}^{[8]} + \frac{62}{9}c_1^{[4]}c_1^{[8]}\right).$$

- If $c_1^{[8]}$ is generated at one loop, while the other coefficients are generated at the tree level, then $c_1^{[8]}$ and the 1-loop contribution of tree-level coefficients are at the same loop order.
  - The tree level bound $c_1^{[8]} \geq 0$ does not necessarily hold at loop level!
- Dim-4 and dim-6 contributions are important at the one-loop level (because of the log s terms)!
- If ϕ_1 is a NGB, the positivity bound $c_1^{[8]} \geq 0$ is robust.
- In general $\beta_1^{[8]}$ can take either sign.

Back to the $\Phi\phi_1\phi_2$ model

$$\mathcal{L} = \frac{1}{2}\left(\partial^\mu\Phi\partial_\mu\Phi - M^2\Phi^2\right) + \frac{1}{2}\partial^\mu\phi_1\partial_\mu\phi_1 + \frac{1}{2}\partial^\mu\phi_2\partial_\mu\phi_2 - gM\Phi\phi_1\phi_2 - \frac{1}{4}\lambda\phi_1^2\phi_2^2\,,$$

- $\frac{c_1^{[8]}}{2}\left(\partial_\mu\phi_1\right)\left(\partial^\mu\phi_1\right)\left(\partial_\nu\phi_1\right)\left(\partial^\nu\phi_1\right)$ is only generated at one-loop level.
- $c_1^{[8]}$ needs to be matched to the one-loop level, which is (at matching scale M):

$$c_1^{[8]}(M) = \frac{1}{16\pi^2}\frac{g^2}{M^4}\frac{1}{45}\left(55\lambda - 166g^2\right)\,,$$

- Other coefficients are matched to the tree level

$$c_{12}^{[4]}(M) = 2g^2 - \lambda\,, \qquad c_{12}^{[6]}(M) = -\frac{g^2}{M^2}\,, \qquad c_{12,su}^{[8]}(M) = \frac{g^2}{M^4}\,,$$

- Plug them in Σ, run $c_1^{[8]}$ down to μ ...

Back to the $\Phi\phi_1\phi_2$ model

- This gives (the dependence on μ cancels as intended)

$$\Sigma = \frac{\lambda^2}{64\pi^2}\frac{1}{s_0^2} + \frac{\lambda g^2}{16\pi^2}\left(-\frac{1}{s_0^2} + \frac{3\pi}{8}\frac{1}{M^2 s_0} + \frac{5}{9}\frac{1}{M^4} + \frac{4}{3}\frac{1}{M^4}\log\frac{s_0}{M^2}\right)$$
$$+ \frac{g^4}{16\pi^2}\left(\frac{1}{s_0^2} - \frac{3\pi}{4}\frac{1}{M^2 s_0} - \frac{47}{20}\frac{1}{M^4} - \frac{11}{3}\frac{1}{M^4}\log\frac{s_0}{M^2}\right),$$

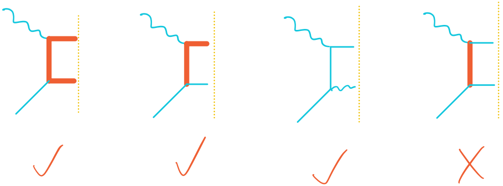
- It can be verified by calculating $\frac{2}{\pi}\int_0^\infty ds\, \frac{s^4[\sigma(\phi_1\phi_1\to\phi_2\phi_2)+\sigma(\phi_1\phi_1\to\Phi\Phi)]}{(s^2+s_0^2)^3}$ at tree level and expand to $\mathcal{O}(M^{-4})$.
- The beta function is now

$$\beta_1^{[8]} = \frac{1}{16\pi^2}\left(\frac{4}{3}\frac{\lambda g^2}{M^4} - \frac{11}{3}\frac{g^4}{M^4}\right).$$

- Consider the limit $\lambda \gg g^2$, obviously $\beta_1^{[8]}$ can take either sign in this case!
- In the opposite limit $\lambda \to 0$, it seems that the positivity bound is violated when $s_0 \approx M$, but when $s_0 \approx M$ the EFT expansion breaks down and we cannot truncate Σ at $\mathcal{O}(M^{-4})$...

Scalar QED EFT current work with Xiao Cao, Yuhang Wu, Yunxiao Ye

- The situation is quite different if external photons (gauge bosons) are involved!
  - Gauge invariance imposes strong constraints on the form of the UV completion.
- For simplicity, we focus on scalar QED EFT here, where the only light d.o.f. are a complex (charge 1) scalar and a photon.
- We assume heavy particles have spin ≤ 1.
  - No tree-level BSM contribution to $\phi\gamma \to \phi\gamma$ and $\gamma\gamma \to \gamma\gamma$.
  - At one loop, SM and BSM contributions are separately positive. Only SM contributes to $\sigma(SM \to SM)$. (By SM I mean scalar QED...)



Scalar QED EFT

$$
\begin{aligned}
\mathcal{L}_{[\mathcal{O}]\leq 4} =& -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\phi)^\dagger(D_\mu\phi) - \frac{1}{4}\lambda_1(\phi^\dagger\phi)^2\,, \\
\mathcal{L}_6\cdot\Lambda^2 =& c_{\phi^6}\frac{1}{36}(\phi^\dagger\phi)^3 + c_{D^2\phi^4}\phi^\dagger\phi(D_\mu\phi)^\dagger D^\mu\phi + c_{F^2\phi^2}\frac{1}{4}\phi^\dagger\phi F^{\mu\nu}F_{\mu\nu}\,, \\
\mathcal{L}_8\cdot\Lambda^4 =& c^{(1)}_{F^4}\frac{1}{16}F_{\mu\nu}F^{\mu\rho}F^{\nu\sigma}F_{\rho\sigma} + c^{(2)}_{F^4}\frac{1}{16}(F_{\mu\nu}F^{\mu\nu})(F_{\rho\sigma}F^{\rho\sigma}) \\
&+ c_{\phi^8}\frac{1}{576}(\phi^\dagger\phi)^4 + c_{D^2\phi^6}\frac{1}{4}(\phi^\dagger\phi)^2(D_\mu\phi)^\dagger D^\mu\phi \\
&+ c^{(1)}_{D^4\phi^4}\frac{1}{4}((D_\mu\phi)^\dagger)^2(D_\nu\phi)^2 + c^{(2)}_{D^4\phi^4}\phi^\dagger\phi(D_\mu D_\nu\phi)^\dagger(D_\mu D_\nu\phi) \\
&+ c_{F^2\phi^4}\frac{1}{16}(\phi^\dagger\phi)^2F_{\mu\nu}F^{\mu\nu} \\
&+ c^{(1)}_{F^2D^2\phi^2}\frac{1}{4}(D_\mu\phi)^\dagger(D_\nu\phi)F^{\mu\rho}F^\nu_\rho + c^{(2)}_{F^2D^2\phi^2}\frac{1}{4}(D^\mu\phi)^\dagger(D_\mu\phi)F_{\nu\rho}F^{\nu\rho}\,.
\end{aligned}
$$

- Tree level positivity bounds:

$$
c^{(1)}_{D^4\phi^4} + 2c^{(2)}_{D^4\phi^4} \geq 0\,, \quad c^{(1)}_{F^2D^2\phi^2} < 0\,, \quad c^{(1)}_{F^4} > 0\,, \quad c^{(1)}_{F^4} + 2c^{(2)}_{F^4} > 0\,.
$$

$\phi\gamma \to \phi\gamma$

- Consider γ in the helicity basis, now ϕ is a complex scalar. However, the charge conjugation symmetry in (scalar) QED ensures the amplitude still has a $s \leftrightarrow u$ symmetry.
- Repeating what we did in the scalar case, we find

$$
\begin{aligned}
\Sigma = & \frac{e^4}{8\pi^2}\frac{1}{s_0^2} - \frac{1}{\Lambda^4}\frac{1}{8}c^{(1)}_{F^2D^2\phi^2} \\
& + \frac{1}{\Lambda^4}\frac{1}{192\pi^2}\Bigg[3e^2 c^{(1)}_{F^2D^2\phi^2}\left(\frac{1}{\epsilon^2} - \frac{1}{\epsilon}\log\left(\frac{-t}{\mu^2}\right) + \frac{1}{2}\log^2\left(\frac{-t}{\mu^2}\right) - \frac{1}{2}\zeta_2\right) \\
& \quad + e^2(c^{(1)}_{D^4\phi^4} + 2c^{(2)}_{D^4\phi^4} + 9c^{(1)}_{F^2D^2\phi^2} + 9c^{(1)}_{F^4} + 12c^{(2)}_{F^4})\left(-\log\left(\frac{-t}{\mu^2}\right) + 2\right) \\
& \quad + (3e^2 c^{(1)}_{F^2D^2\phi^2} + c^2_{F^2\phi^2})\left(-\frac{3}{2} - 2\log\left(\frac{s_0}{\mu^2}\right) + 4\right)\Bigg] \\
& + \frac{1}{1152\pi^2\Lambda^4}\left[7e^2 c^{(1)}_{D^4\phi^4} + 14e^2 c^{(2)}_{D^4\phi^4} - 36e^2 c^{(1)}_{F^4} + 2c^2_{F^2\phi^2} + 6e^2 c^{(1)}_{F^2D^2\phi^2} - 48e^2 c^{(2)}_{F^2D^2\phi^2}\right].
\end{aligned}
$$

- No t-channel simple pole in this case.
- IR divergence may exist for fixed order amplitudes (but here we only need to worry about the leading order contribution of $c^{(1)}_{F^2D^2\phi^2}$) ....
- $\log t$ can be regulated by giving ϕ a small mass m.

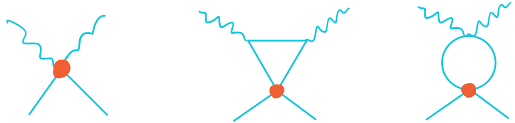
$\phi\gamma \to \phi\gamma$

- We can subtract the SM contribution by requiring at least one final state heavy particle in $\sigma_{\rm tot}(\phi\gamma)$.

$$\Sigma' = \Sigma - \Sigma_{\rm SM} \geq 0\,.$$

- Only $c^{(1)}_{D^4\phi^4}$ and $c^{(2)}_{D^4\phi^4}$ can be generated at the tree level (under our assumptions).
- Up to one loop in the UV, we have (omitting $\mathcal{O}(m^2)$ corrections)

$$\Sigma' = -\frac{1}{\Lambda^4}\frac{1}{8}c^{(1)}_{F^2D^2\phi^2} + \frac{1}{\Lambda^4}\frac{19e^2}{1152\pi^2}\left(c^{(1)}_{D^4\phi^4} + 2c^{(2)}_{D^4\phi^4}\right) - \frac{1}{\Lambda^4}\frac{e^2}{192\pi^2}\left(c^{(1)}_{D^4\phi^4} + 2c^{(2)}_{D^4\phi^4}\right)\log\frac{m^2}{\mu^2}\,.$$



(generated at one loop)

$\phi\gamma \to \phi\gamma$

- Up to one loop in the UV, we have

$$\Sigma' \approx -\frac{1}{\Lambda^4}\frac{1}{8}c^{(1)}_{F^2D^2\phi^2} + \frac{1}{\Lambda^4}\frac{19e^2}{1152\pi^2}(c^{(1)}_{D^4\phi^4} + 2c^{(2)}_{D^4\phi^4}) - \frac{1}{\Lambda^4}\frac{e^2}{192\pi^2}(c^{(1)}_{D^4\phi^4} + 2c^{(2)}_{D^4\phi^4})\log\frac{m^2}{\mu^2}\,.$$

- Here Σ' has no s_0 dependence (because only t enters these loops) ...
- The leading order β function for $c^{(1)}_{F^2D^2\phi^2}$ is (which can also be obtained by requiring $\mu\frac{d}{d\mu}\Sigma' = 0$)

$$\beta(c^{(1)}_{F^2D^2\phi^2}) = \frac{e^2}{12\pi^2}\left(c^{(1)}_{D^4\phi^4} + 2c^{(2)}_{D^4\phi^4}\right)\,.$$

- $c^{(1)}_{F^2D^2\phi^2}$ does not necessarily obey the tree-level bound ($c^{(1)}_{F^2D^2\phi^2} < 0$).
  - Taking the limit $m \to 0$, Σ' is dominated by the log term, which seems to imply a bound on β (which drives $c^{(1)}_{F^2D^2\phi^2}$ in the same direction as the tree-level bound in IR).
  - Is this always the case if we have symmetric diagrams?

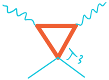
UV models

- A heavy scalar Φ with charge 2

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D^\mu\phi)^\dagger(D_\mu\phi) + (D^\mu\Phi)^\dagger(D_\mu\Phi) - M^2\Phi^\dagger\Phi$$
$$-\frac{1}{4}\lambda_1(\phi^\dagger\phi)^2 - \frac{1}{4}\lambda_2(\Phi^\dagger\Phi)^2 - \lambda_3\Phi^\dagger\Phi\phi^\dagger\phi - (\frac{1}{2}gM\Phi^\dagger\phi\phi + h.c.)\,.$$

- Tree level matching: $c^{(1)}_{D^4\phi^4} = 4g^2\,,\ c^{(2)}_{D^4\phi^4} = 0.$
- One loop matching (and running):

$$c^{(1)}_{F^2D^2\phi^2} = \frac{1}{16\pi^2}\frac{2}{3}g^2e^2\left(9 + 4\log\left(\frac{\mu^2}{M^2}\right)\right)\,,$$



- Tree-level positivity bound "violated" for $\mu \gtrsim 0.3M$.
- **Note:** λ_3 does not contribute to $c^{(1)}_{F^2D^2\phi^2}$! (Otherwise we can get a violation of positivity bound.)
- Σ' is given by (with g, e defined at the matching scale M)

$$\Sigma' = \frac{e^2g^2}{576M^4\pi^2}\left(11 - 12\log\left(\frac{m^2}{M^2}\right)\right) > 0.$$

$\gamma\gamma \to \gamma\gamma$

- Orthogonal linear polarization ($\gamma_y\gamma_x \to \gamma_y\gamma_x$)

$$\Sigma' = \frac{1}{\Lambda^4}\left[\frac{c^{(1)}_{F^4}}{4} + \frac{1}{384\pi^2} e^2 c^{(1)}_{F^2D^2\phi^2}\left(\frac{3}{2} + 2\log\left(\frac{s_0}{\mu^2}\right) + 2\log\left(\frac{-t}{\mu^2}\right) - 8\right)\right.$$
$$\left. - \frac{7}{576\pi^2} e^2 c^{(1)}_{F^2D^2\phi^2}\right].$$

- Same linear polarization ($\gamma_y\gamma_y \to \gamma_y\gamma_y$)

$$\Sigma' = \frac{1}{\Lambda^4}\left[\frac{1}{2}(c^{(1)}_{F^4} + 2c^{(2)}_{F^4}) + \frac{1}{64\pi^2} c^2_{F^2\phi^2}\left(-\frac{3}{2} - 2\log\left(\frac{s_0}{\mu^2}\right) + 4\right)\right.$$
$$- \frac{1}{384\pi^2} e^2 c^{(1)}_{F^2D^2\phi^2}\left(-\frac{3}{2} - 2\log\left(\frac{s_0}{\mu^2}\right) - 2\log\left(\frac{-t}{\mu^2}\right) + 8\right)$$
$$\left. - \frac{1}{576\pi^2} e^2 (25c^{(1)}_{F^2D^2\phi^2} + 72c^{(2)}_{F^2D^2\phi^2})\right].$$

- If all operators above are generated at one-loop level, the contributions from $c^{(1)}_{F^4}$, $c^{(2)}_{F^4}$ and $c^2_{F^2\phi^2}$ are two-loop effects from the UV model.
- If $c^{(1)}_{D^4\phi^4}$ or $c^{(2)}_{D^4\phi^4}$ are generated at tree level, their contribution to the above equations are also at the two-loop level from the UV model.

Conclusion

- The interpretation of positivity bounds are more subtle at the loop level.
- **Something** is positive, but that **something** can have several contributions!
  - It is important to include all contributions to each fixed loop order in the UV model.
  - Dim-4 and dim-6 contributions can be important.
  - Contributions that correspond to the interference term of the cross section are not necessarily positive!
- In scalar theories, we've found examples where the 1-loop generated dim-8 coefficient and the corresponding β function are not subject to the tree-level positivity bounds.
- For $\phi\gamma \to \phi\gamma$ and $\gamma\gamma \to \gamma\gamma$ scatterings in scalar QED (EFT), the one-loop diagrams are "symmetrical" and the β-functions always tend to make the coefficient more "positive" at IR.
  - Accidental? (Does it hold beyond 1-loop level?)
- Other more practical one loop cases?
  - $2f \to 2f$, $f\gamma \to f\gamma$, ...?

backup slides