# Effective field theory of inflation with strongly non-geodesic motion

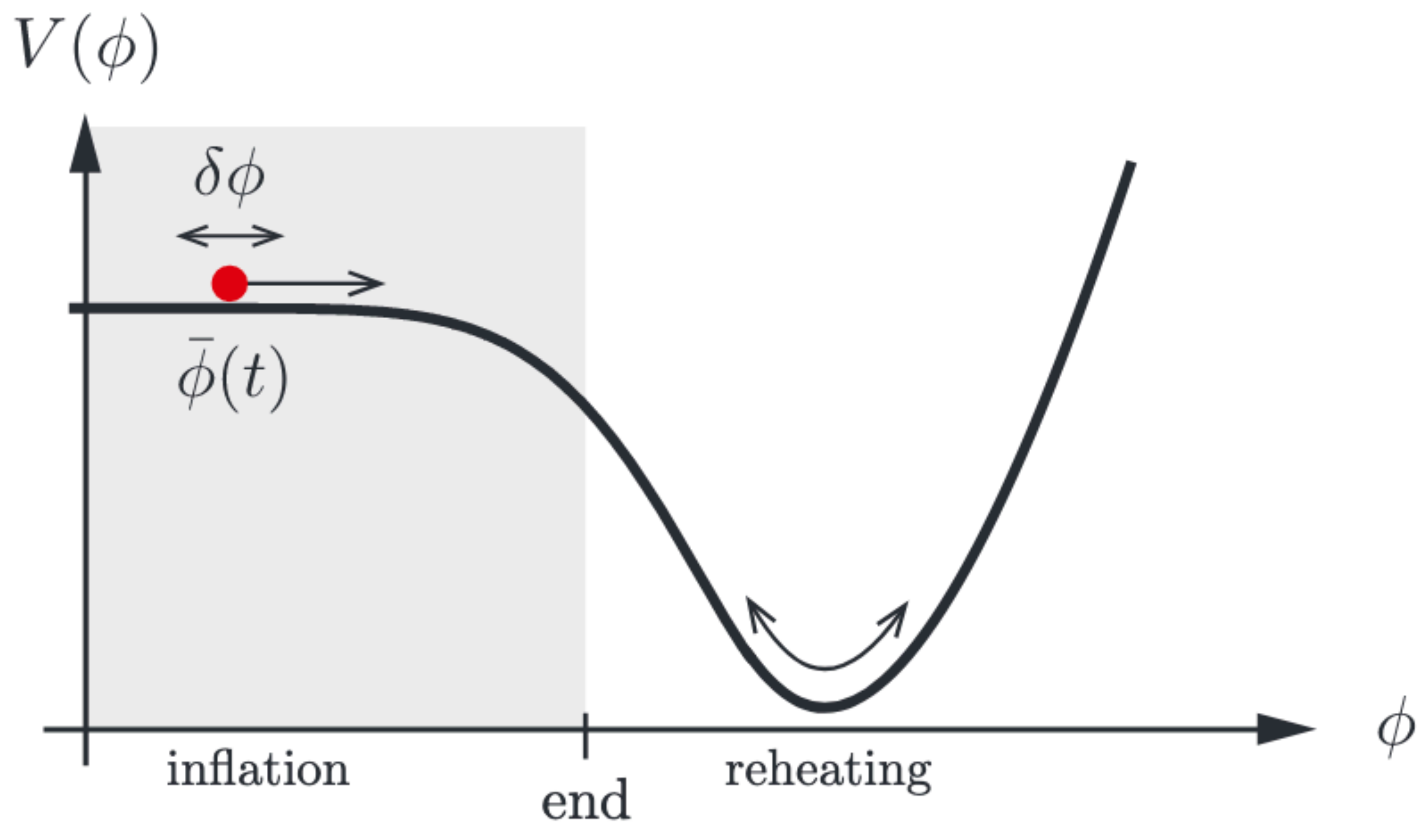
Based on **1805.12563, 1902.03221, 2503.18516**
with J. Fumagalli, Y. Lu, L. Pinol, J. Ronayne, S. Renaux-Petel

**Sebastian Garcia-Saenz**
**SUSTech**

# Plan of the talk

- Multi-field inflation and strongly non-geodesic motion
- Effective field theory with imaginary speed of sound
- Hyper-non-Gaussianities
- Loops and perturbative control

# Multi-field inflation




Credit: D. Baumann

- 'Vanilla' single-field inflation

$$S[g,\phi] = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}}{2}R - \frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - V(\phi)\right]$$

Pros: — simple

— consistent with data

Cons: — top-down realization? (UV sensitivity, large-field excursions, …)

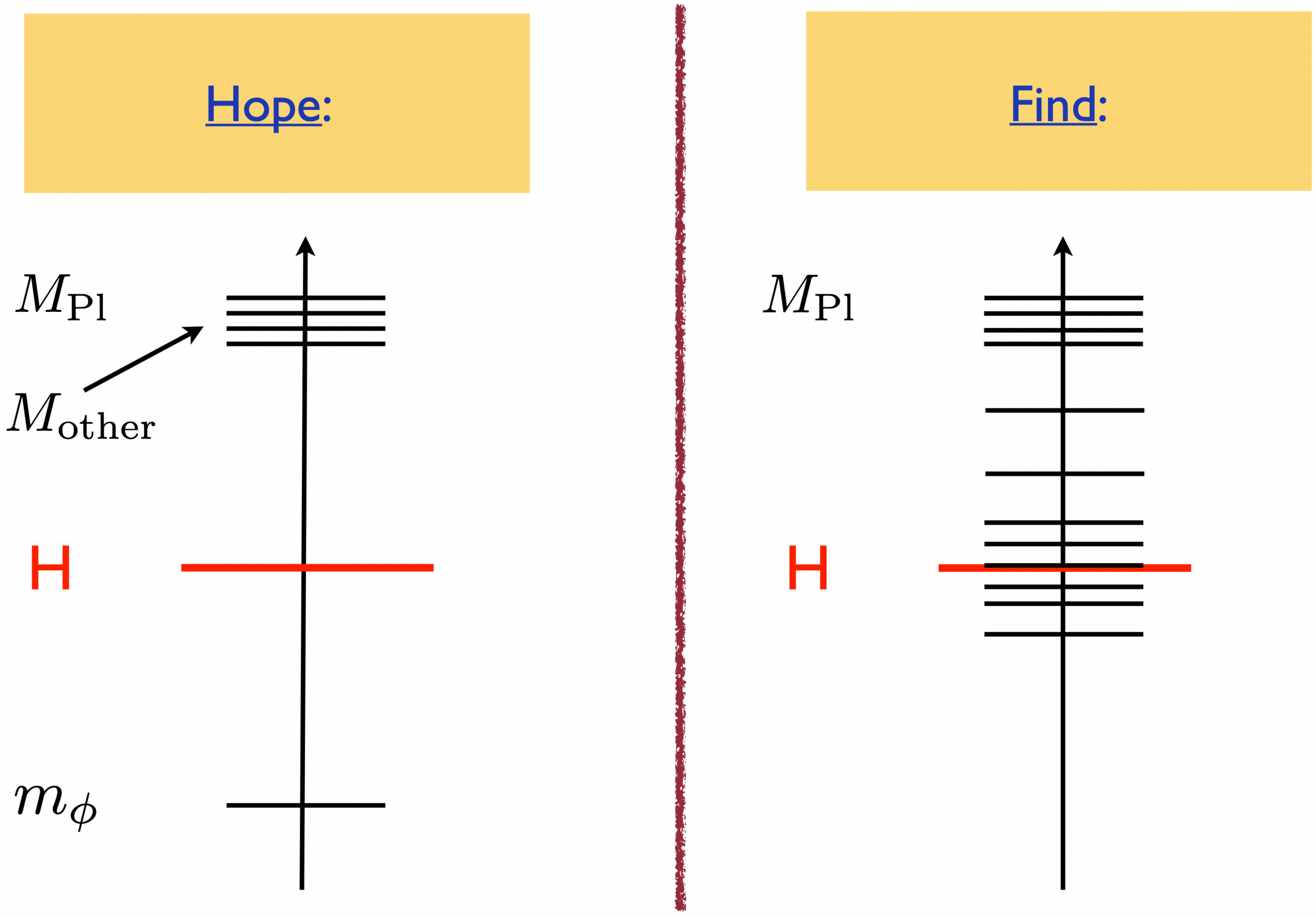
— `boring' phenomenology (non-Gaussianities, gravitational waves, …)

# Multi-field inflation

- Multi-field inflation

$$S[g,\phi^I] = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}}{2}R - \frac{1}{2}\delta_{IJ}\nabla^\mu\phi^I\nabla_\mu\phi^J - V(\phi)\right]$$

- Natural expectation from UV realizations
- Interesting observables
  - enhanced fluctuations
  - non-Gaussianities
  - destabilization




Credit: S. Renaux-Petel

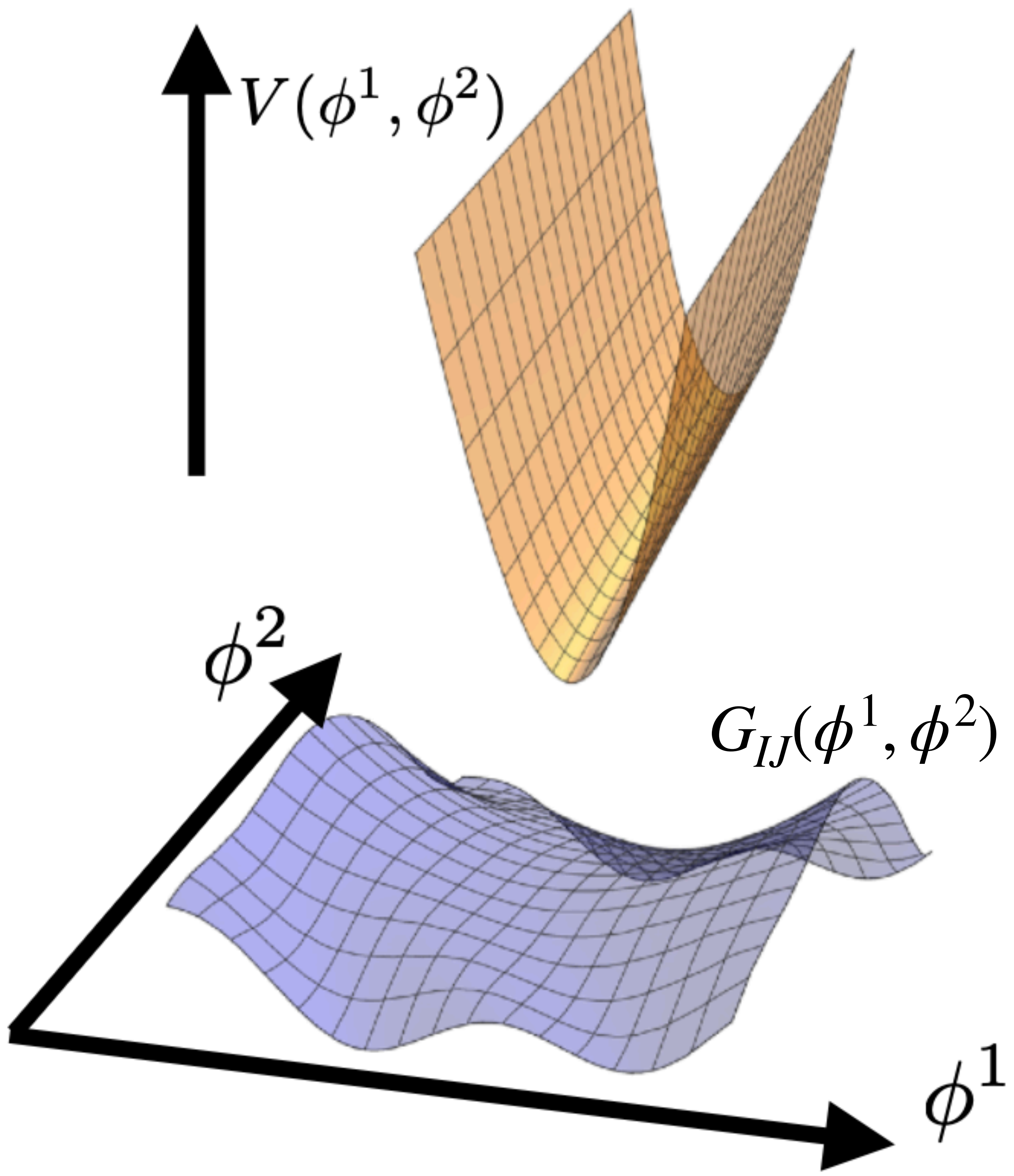
# Multi-field inflation

- Multi-field inflation with non-canonical kinetic terms

$$S[g,\phi^I] = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}}{2}R - \frac{1}{2}G_{IJ}(\phi)\nabla^\mu\phi^I\nabla_\mu\phi^J - V(\phi)\right]$$

- $G_{IJ}(\phi)$ $\rightarrow$ field space metric
- ${R^I}_{JKL} \neq 0$ $\rightarrow$ in general, curved field space
- Includes a large class of top-down constructions

  Baumann, McAllister (2014)




Credit: S. Renaux-Petel

# Strongly non-geodesic motion

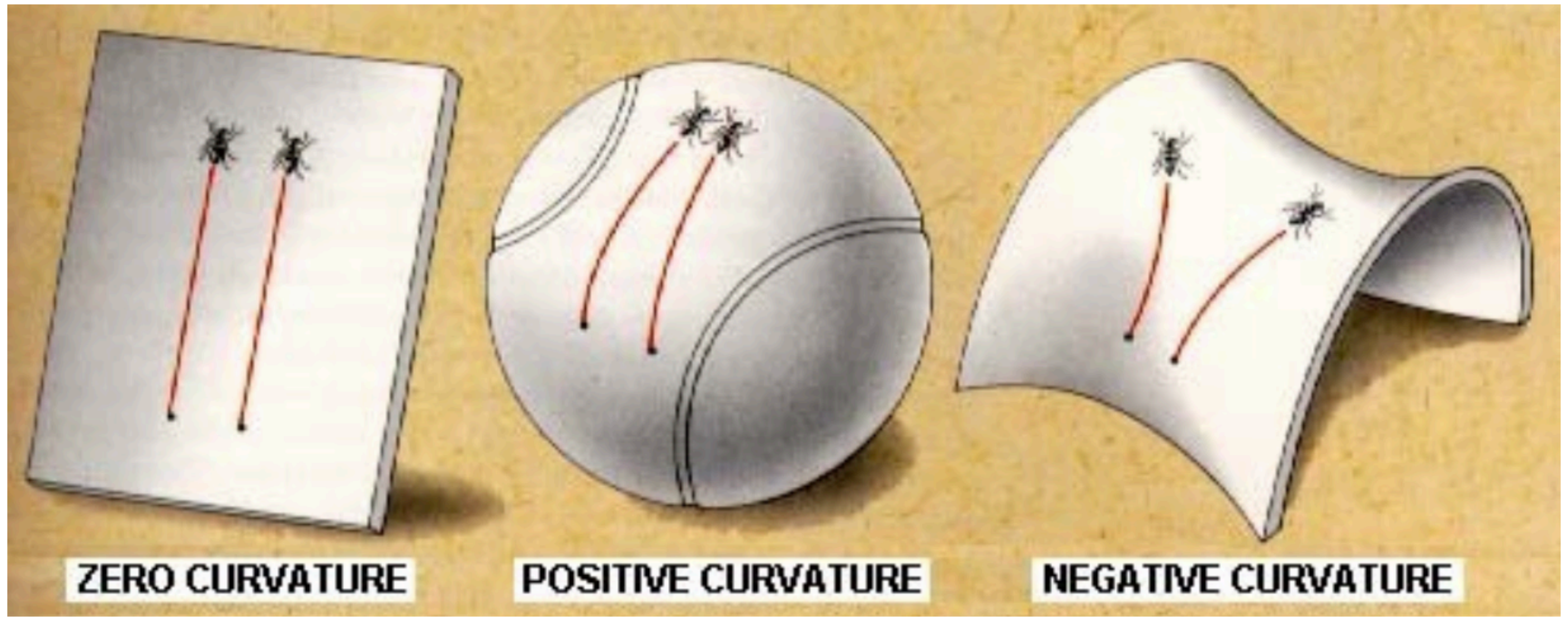
- My approach: bottom-up, but top-down motivated

- Why interesting?

  — new mechanism to inflate: inflation on steep potentials

  — new EFT of fluctuations: imaginary speed of sound

- Geometric intuition: negative curvature tends to destabilize fluctuations



# Strongly non-geodesic motion

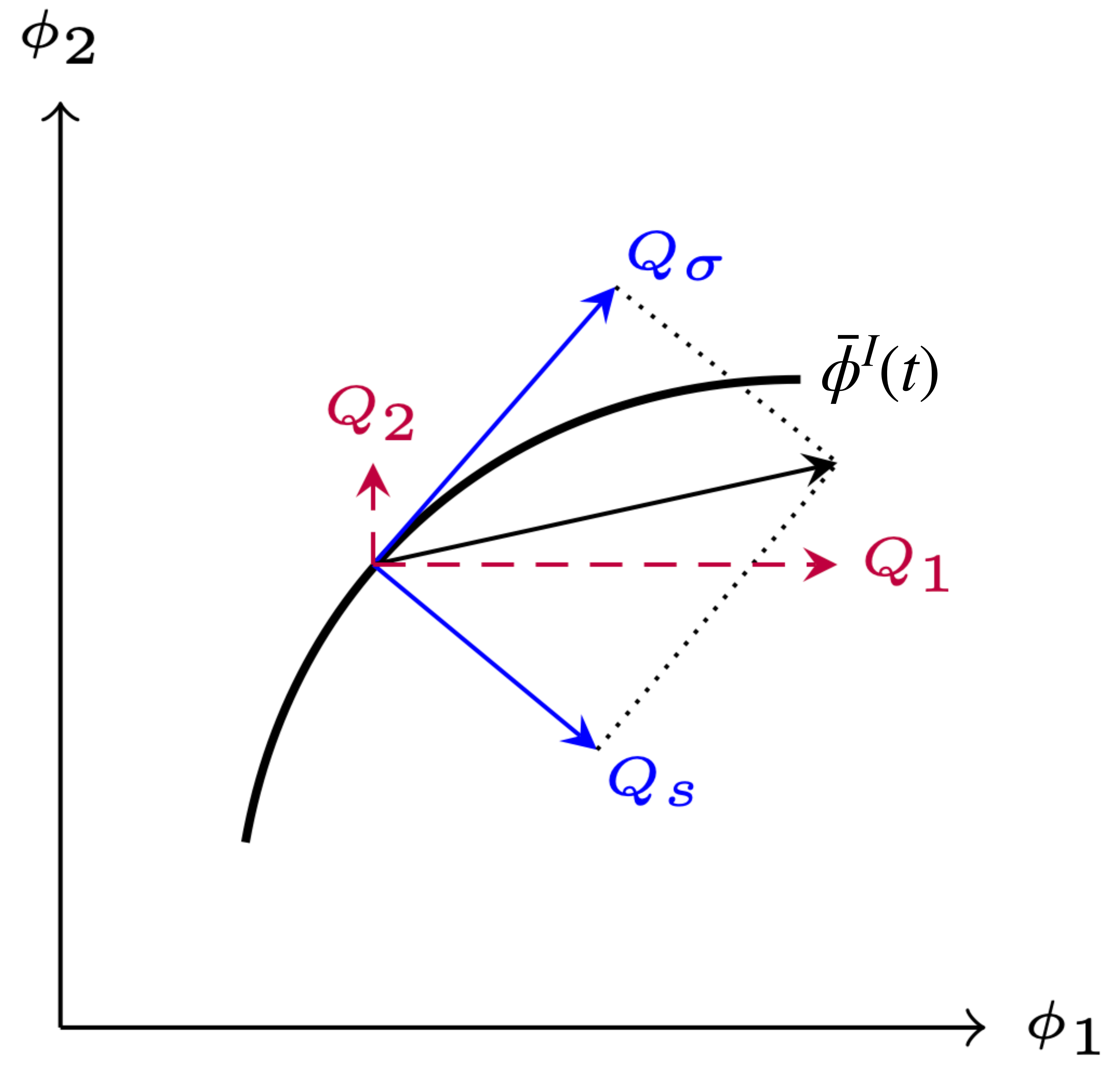
- Scalar fields: $$\phi^I(t,\mathbf{x}) = \bar{\phi}^I(t) + Q^I(t,\mathbf{x})$$

background (inflaton) fluctuation

- Adiabatic-entropic basis (two fields):

$$Q^I = Q_\sigma\, e^I_\sigma + Q_s\, e^I_s$$

adiabatic mode entropic mode



# Strongly non-geodesic motion

$$Q^I = Q_\sigma e^I_\sigma + Q_s e^I_s$$

adiabatic mode

entropic mode

$$e^I_\sigma = \frac{\dot{\bar{\phi}}^I}{\dot\sigma} \qquad Q_\sigma = \sqrt{2\epsilon} M_{\rm Pl} \zeta$$

curvature perturbation

$H$ : Hubble parameter

$\epsilon$ : slow-roll parameter

$$\mathscr{D}_t e^I_\sigma = H \eta_\perp e^I_s$$

covariant directional derivative

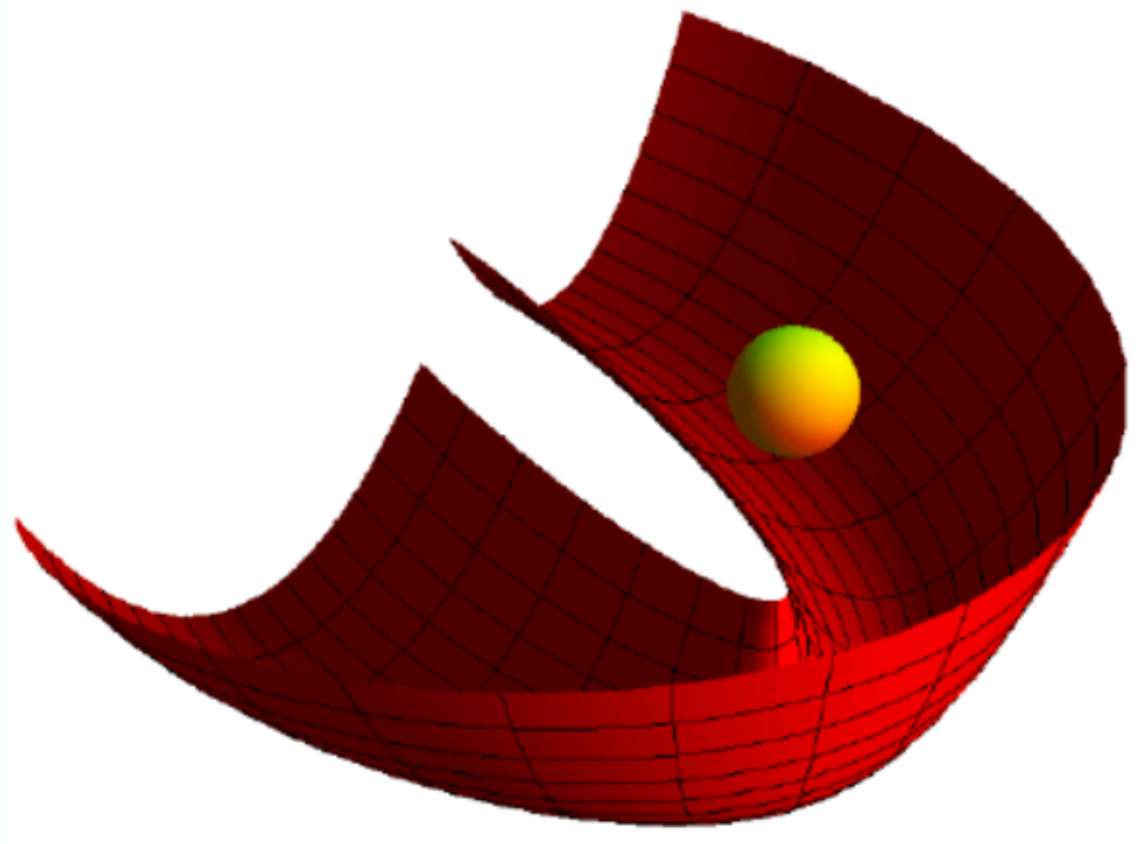
$$\boxed{\eta_\perp \equiv -\frac{V_{,s}}{H\dot\sigma}}$$

$$\dot\sigma \equiv \sqrt{G_{IJ}\dot{\bar{\phi}}^I \dot{\bar{\phi}}^J}$$

$$V_{,s} \equiv e^I_s \frac{\partial V}{\partial \bar{\phi}^I}$$

**Bending parameter**

dimensionless measure of geodesic deviation



# Strongly non-geodesic motion

$$\boxed{\eta_\perp \equiv -\frac{V_{,s}}{H\dot{\sigma}}}$$

- Quadratic perturbation Lagrangian

$$\mathscr{L}^{(2)} = M_{\rm Pl}^2\epsilon\left[\dot{\zeta}^2 - \frac{(\partial\zeta)^2}{a^2}\right] + 2\dot{\sigma}\,\eta_\perp\,\dot{\zeta}Q_s + \frac{1}{2}\left[\dot{Q}_s - \frac{(\partial Q_s)^2}{a^2} - m_s^2 Q_s^2\right]$$

coupling strength from bending

- Entropic mass:

$$\boxed{m_s^2 \equiv V_{;ss} - H^2\eta_\perp^2 + \epsilon H^2 M_{\rm Pl}^2 R_{\rm fs}}$$

Hessian of potential

field space curvature

# Strongly non-geodesic motion

$$\boxed{\mathscr{L}^{(2)} \supset 2\dot{\sigma}\,\eta_\perp\,\dot{\zeta}Q_s}$$

$$\boxed{m_s^2 \equiv V_{;ss} - H^2\eta_\perp^2 + \epsilon H^2 M_{\rm Pl}^2 R_{\rm fs}}$$

Several interesting scenarios:

$|\eta_\perp| \gg 1\,, m_s^2 \gg H^2$ : reduced speed of sound

$|\eta_\perp| \gg 1\,, |m_s^2| \ll H^2$ : modified dispersion relation — Baumann, Green (2011)

$|\eta_\perp| \gg 1\,, |m_s^2| \gg H^2\,, m_s^2 < 0$ : imaginary speed of sound — SGS, Renaux-Petel (2018)

$|\eta_\perp| \lesssim 1\,, |m_s^2| \gg H^2\,, m_s^2 < 0$ : background destabilization

Renaux-Petel, Turzynski (2016)
SGS, Renaux-Petel, Ronayne (2018)

$$\boxed{|\eta_\perp| \gg 1}$$

**Strongly non-geodesic motion**

# Imaginary speed of sound

- Integrate out entropic fluctuation

$$\mathscr{L}^{(2)} = M_{\rm Pl}^2 \epsilon \left[ \dot{\zeta}^2 - \frac{(\partial\zeta)^2}{a^2} \right] + 2\dot{\sigma}\,\eta_\perp\,\dot{\zeta} Q_s + \frac{1}{2}\left[ \dot{Q}_s - \frac{(\partial Q_s)^2}{a^2} - m_s^2 Q_s^2 \right]$$

$$\downarrow$$

$$\mathscr{L}_{\rm eff}^{(2)} = M_{\rm Pl}^2 \epsilon \left[ \frac{\dot{\zeta}^2}{c_s^2} - \frac{(\partial\zeta)^2}{a^2} \right]$$

**Speed of sound:**

$$\boxed{\frac{1}{c_s^2} = 1 + \frac{4H^2\eta_\perp^2}{m_s^2}}$$

# Imaginary speed of sound

$$\boxed{\frac{1}{c_s^2} = 1 + \frac{4H^2\eta_\perp^2}{m_s^2}}$$

- EFT is valid if $|m_s^2| \gg H^2$

- Crucially, it is possible to have $m_s^2 < 0$ and $c_s^2 < 0$ $\longrightarrow$ **Imaginary speed of sound**

- Curious (and worrisome) EFT

$$\mathscr{L}_{\rm eff}^{(2)} = M_{\rm Pl}^2\epsilon\left[\frac{\dot\zeta^2}{c_s^2} - \frac{(\partial\zeta)^2}{a^2}\right]$$

$\zeta$ is a ghost

gradient instability $\zeta \sim e^{k|c_s|t}$

- This is expected:

tachyonic instability in UV $\longrightarrow$ gradient instability in IR

$\zeta \sim e^{|m_s|t}$ $\qquad$ $\zeta \sim e^{\Lambda t}$

# Imaginary speed of sound

- Catastrophic?

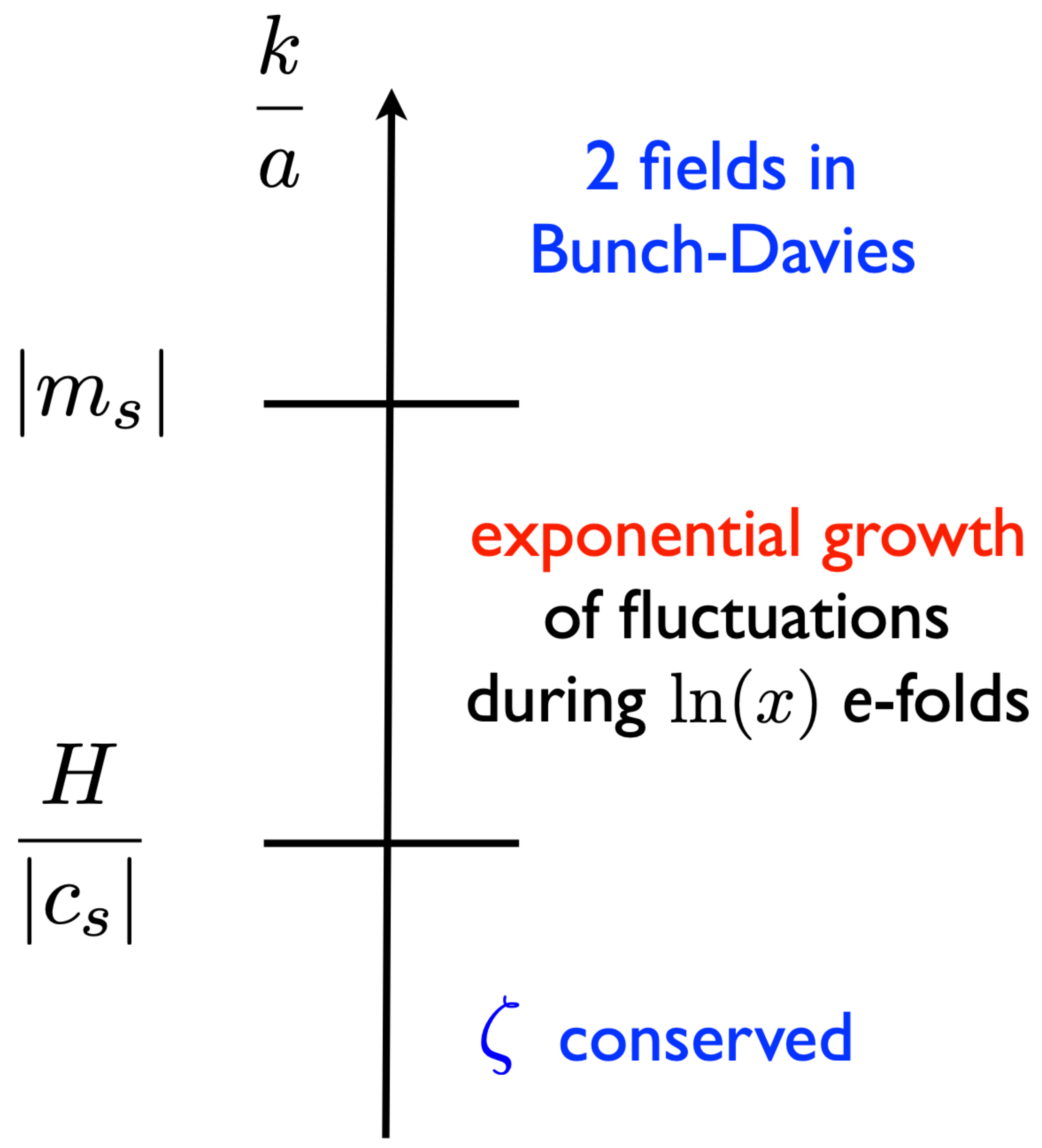
  No: by definition, EFT not applicable at arbitrarily high energies

- Useless?

  Maybe: observables expected to depend sensitively on cutoff scale

- Strategy:

  Treat cutoff as phenomenological parameter and search for robust observables



# Hyper-non-Gaussianities

comoving momentum

- Introduce dimensionless cutoff $x \gg 1$ : EFT is valid for $k|c_s|/a < xH$

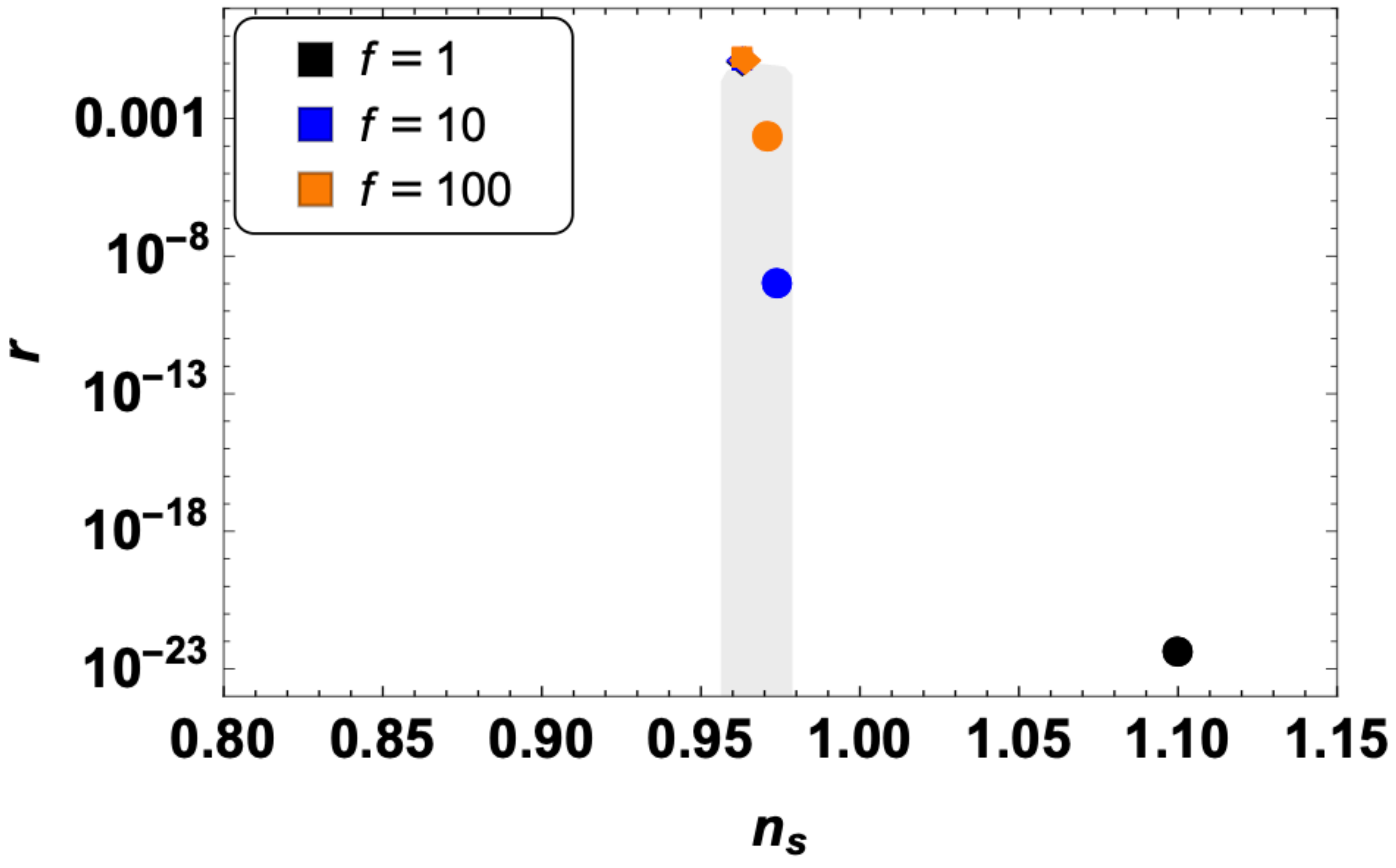
- Quantifies duration of transient instability: $\Delta N \sim \log(x)$

- Exponential enhancement of power spectrum: $$\frac{P_\zeta}{P_{\text{single field}}} \sim e^{2x}$$

- Predicts exponentially suppressed tensor-to-scalar ratio

- Ruled-out models become consistent with data (e.g. natural inflation)

  SGS, Renaux-Petel, Ronayne (2018)



# Hyper-non-Gaussianities

- Interestingly, higher-point correlation functions are "only" enhanced by powers of $x$ :

$$\boxed{\frac{\langle \zeta^n \rangle}{\langle \zeta^2 \rangle^{n-1}} \sim \left[ \left( \frac{1}{|c_s|^2} + 1 \right) x^3 \right]^{n-2}}$$

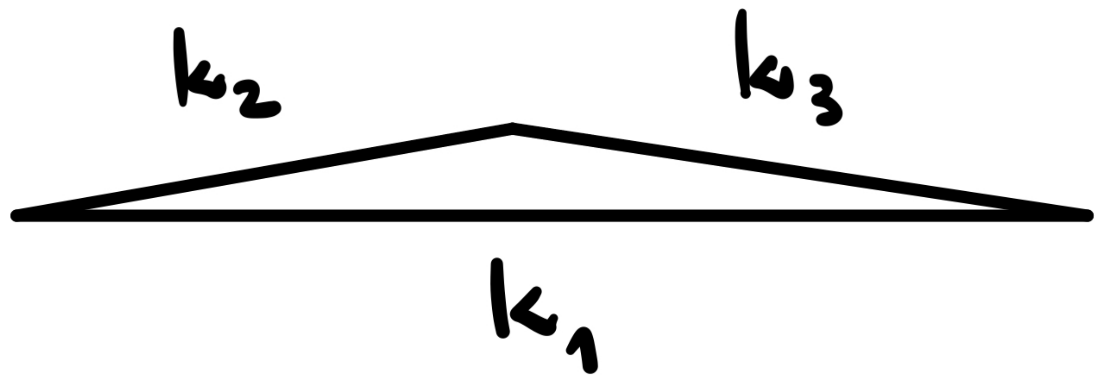
**"Hyper non-Gaussianities"**

Fumagalli, SGS, Pinol, Renaux-Petel, Ronayne (2019)

Bjorkmo, Marsh (2019)

Ferreira (2020)

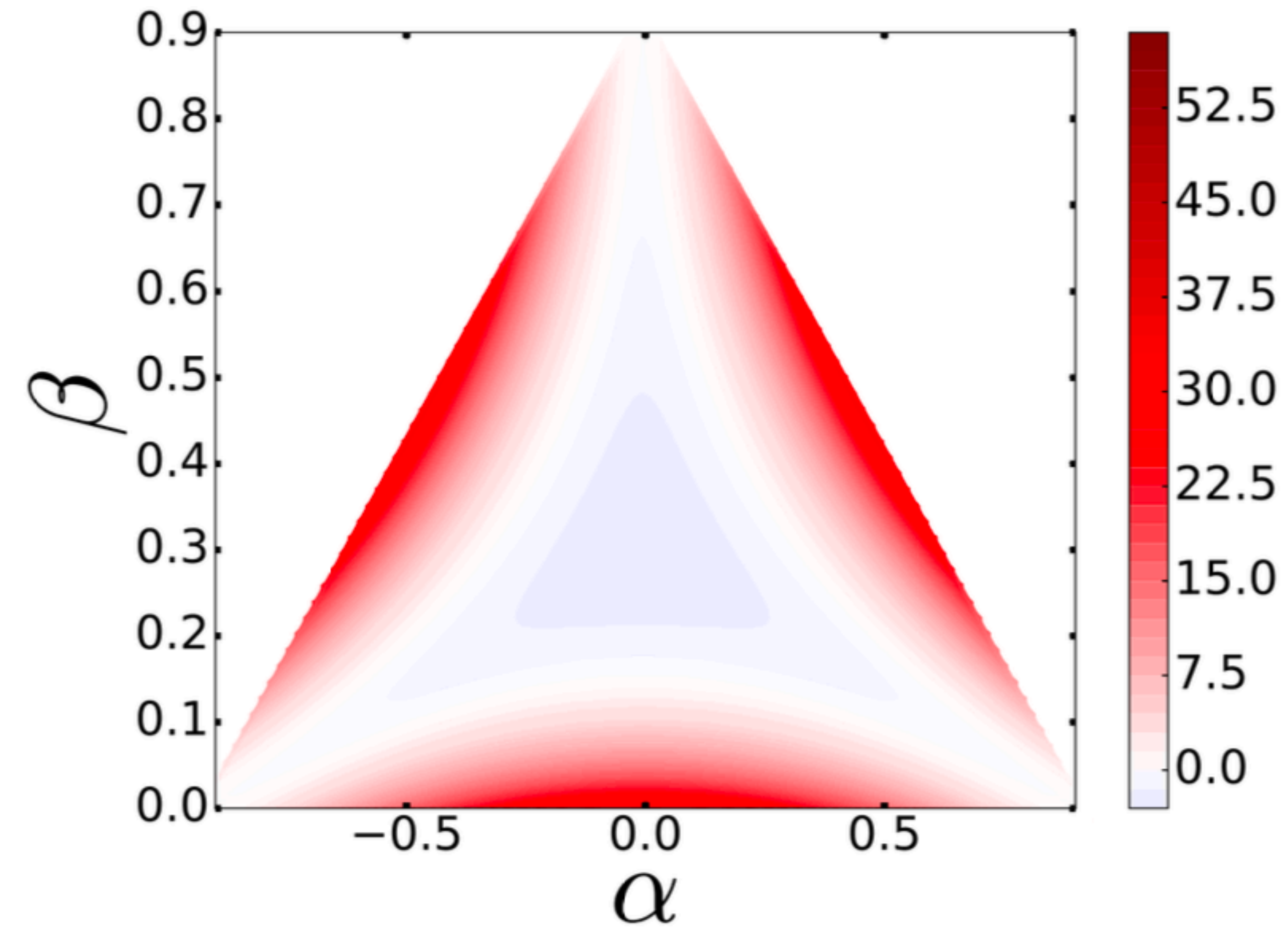
- Enhancement only occurs near **flattened** configurations





etc.

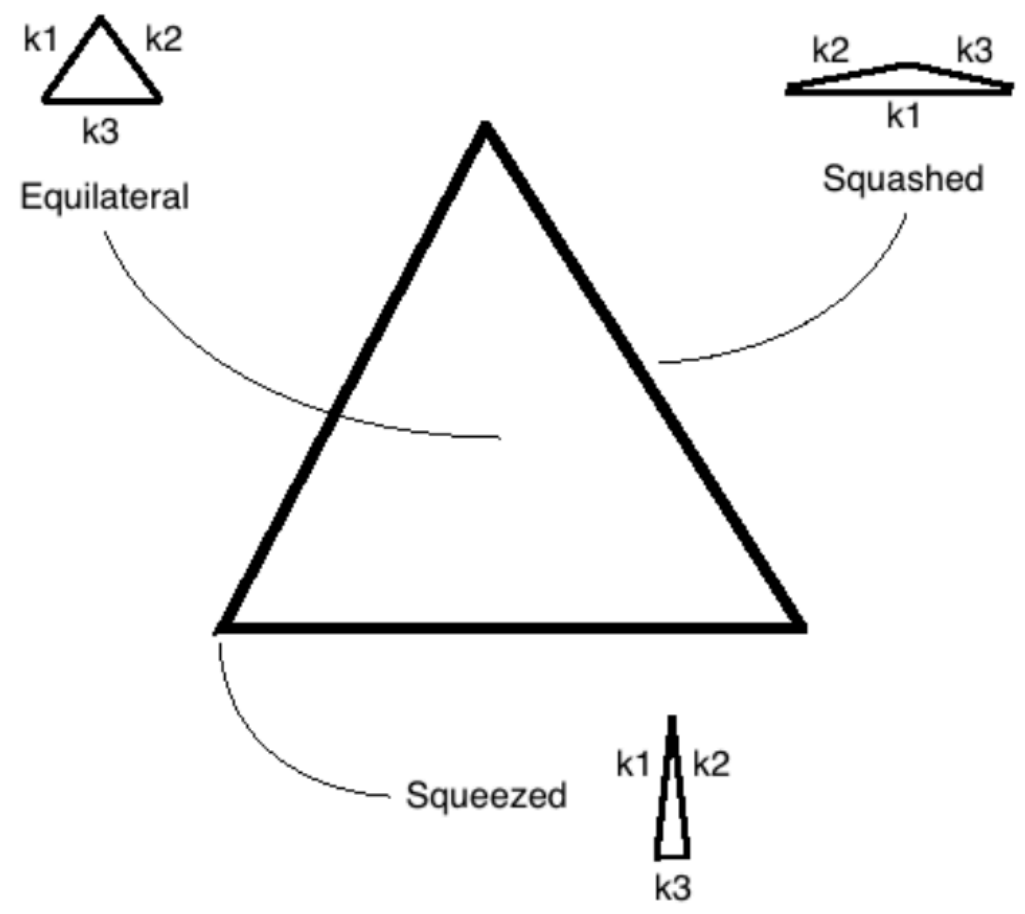
# Hyper-non-Gaussianities




Bispectrum function

$f_{\rm NL}(k_1, k_2, k_3)$

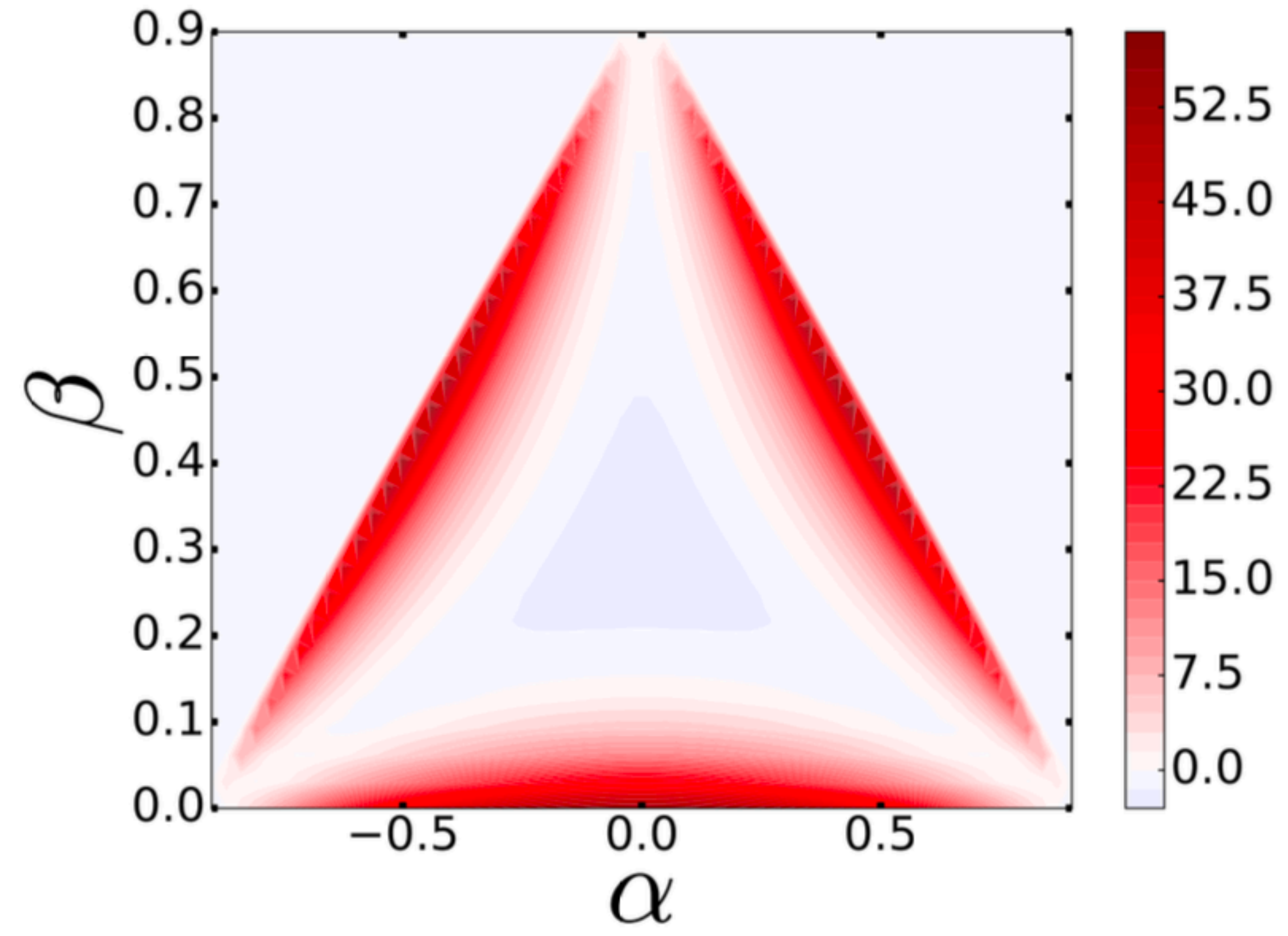
Calculated **analytically** in the EFT with $x = 10$




$$k_1 = \frac{3k_*}{4}(1+\alpha+\beta)$$

$$k_2 = \frac{3k_*}{4}(1-\alpha+\beta)$$

$$k_3 = \frac{3k_*}{2}(1-\beta)$$




Calculated **numerically** (no approximation) in a two-field model

# One-loop corrections

- Hyper-non-Gaussianities $\longrightarrow$ risk of strong coupling?

- OK at **tree-level**, on CMB scales

$$\langle\zeta^n\rangle/\langle\zeta^2\rangle^{n-1} \times \mathcal{P}_\zeta^{(n-2)/2} \sim \left(f_{NL}^{\text{flat}}\, A_s^{1/2}\right)^{n-2} \lesssim 1$$

- Beyond CMB scales, this provides **model-independent constraints** on theories of strongly non-geodesic motion

Fumagalli, SGS, Pinol, Renaux-Petel, Ronayne (2019)

- What about beyond tree-level?

# One-loop corrections

- **Goal:** calculate one-loop corrections to **scalar** and **tensor** power spectra
- **Set-up:** EFT of fluctuations at **leading non-trivial** order in slow-roll
- **Method:** in-in formalism
- 3 cubic couplings: $(\dot{\zeta})^3 \quad \dot{\zeta}(\partial_i\zeta)^2 \quad \gamma_{ij}\partial_i\zeta\partial_j\zeta$
- 5 quartic couplings: $(\dot{\zeta})^4 \quad \dot{\zeta}^2(\partial_i\zeta)^2 \quad (\partial_i\zeta)^4 \quad \gamma_{ij}^2\dot{\zeta}^2 \quad \gamma_{ij}^2(\partial_i\zeta)^2$

# One-loop corrections

Definition of $x$ from EFT validity regime:

$$k|c_s|/a < xH$$

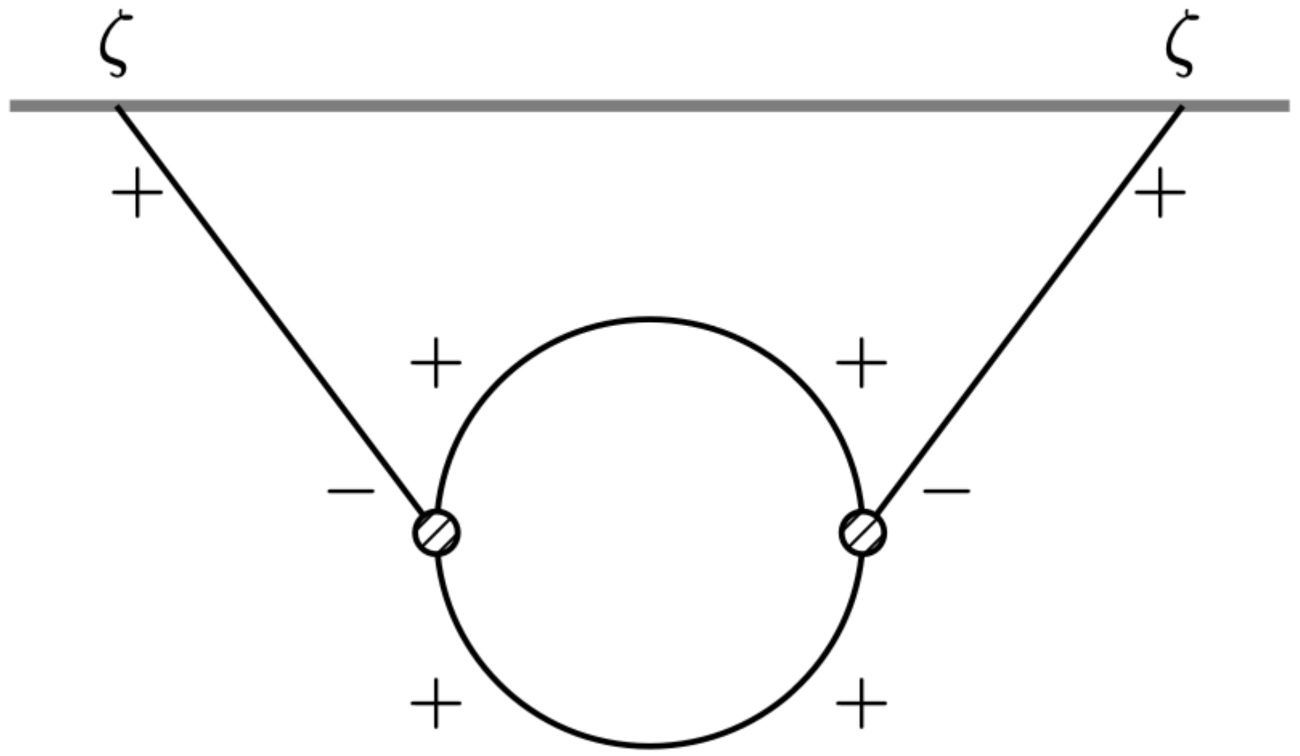
- **Approximation:** work at leading order in large $x$

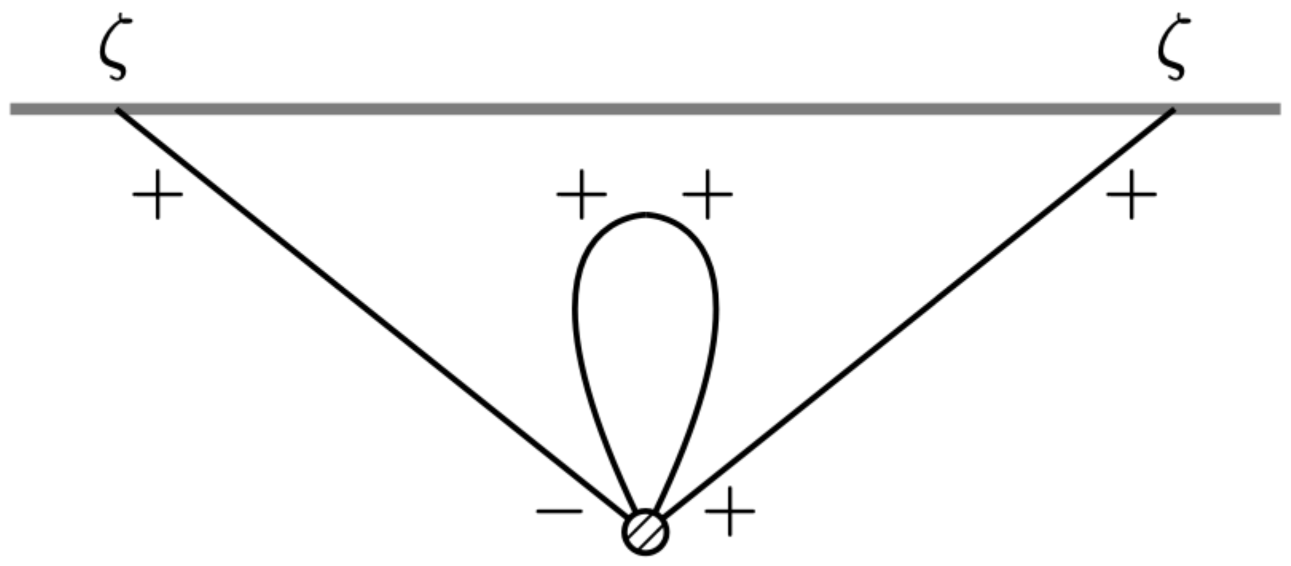
- **Technical difficulty:** mode function is a combination of growing **(+)** and decaying **(-)** modes

$$\zeta = \zeta_+ + \zeta_- \sim e^{k|c_s|t} + e^{i\psi} e^{-k|c_s|t}$$

Phase $\psi \neq 0$ from quantization condition

- Due to reality, one cannot completely neglect decaying mode (similar to usual EFT with excited states)



# One-loop corrections

Definition of $x$ from EFT validity regime:

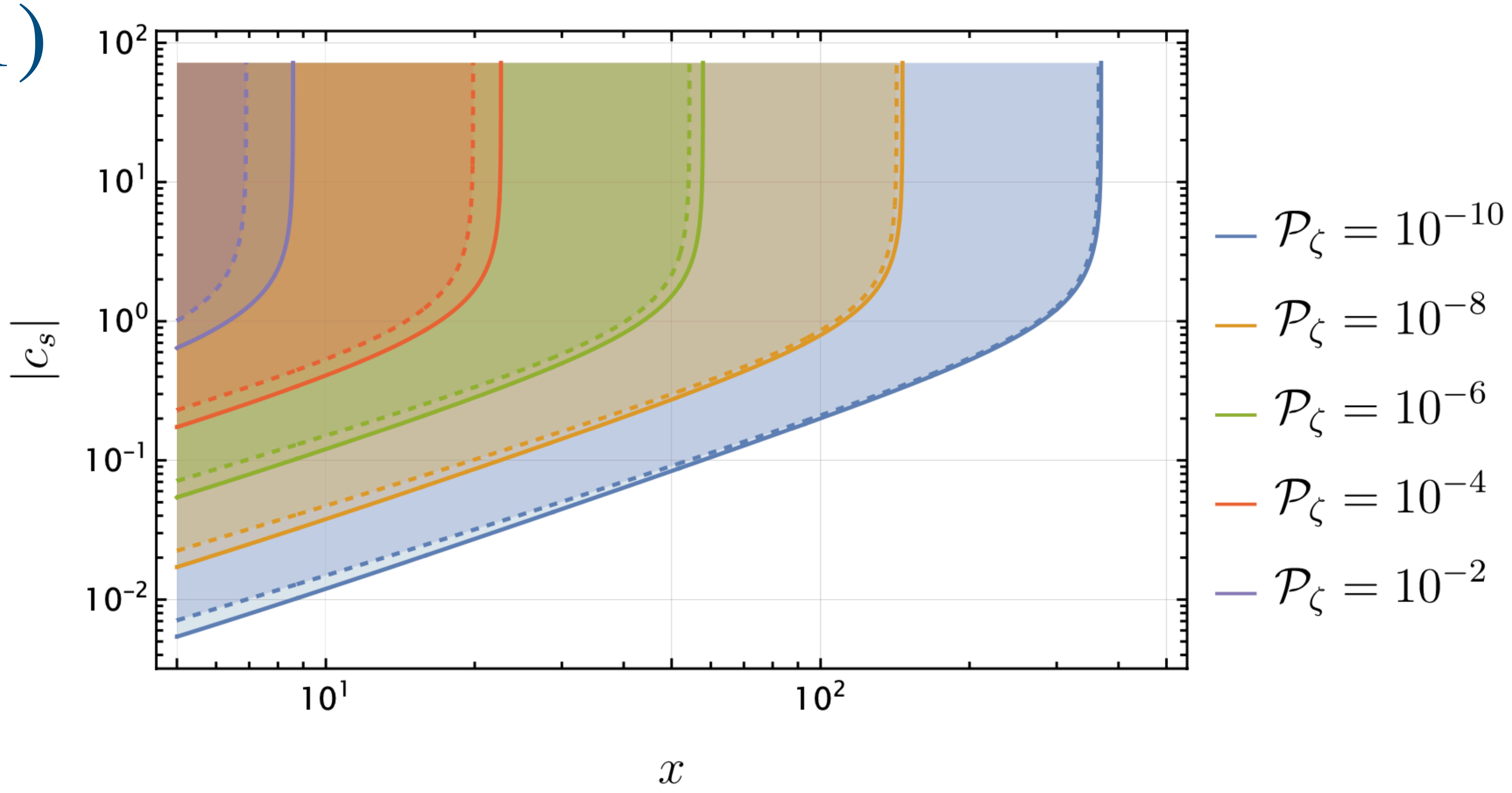
$$k|c_s|/a < xH$$

- **One-loop scalar spectrum:** enhanced by powers of $x$

$$\boxed{\frac{\mathcal{P}_\zeta^{(1-\mathrm{loop})}}{\mathcal{P}_\zeta^{(\mathrm{tree})}} \sim \mathcal{P}_\zeta \left(\frac{1}{|c_s|^2}+1\right)^2 x^5}$$

- Models with $x = \mathcal{O}(10)\,, |c_s| = \mathcal{O}(1)$ are **consistent**, even if $\mathcal{P}_\zeta$ is large (e.g. hyper-inflation)

- **Excludes** models with very large $\mathcal{P}_\zeta$ or small $|c_s|$



# One-loop corrections

Definition of $x$ from EFT validity regime:

$$k|c_s|/a < xH$$

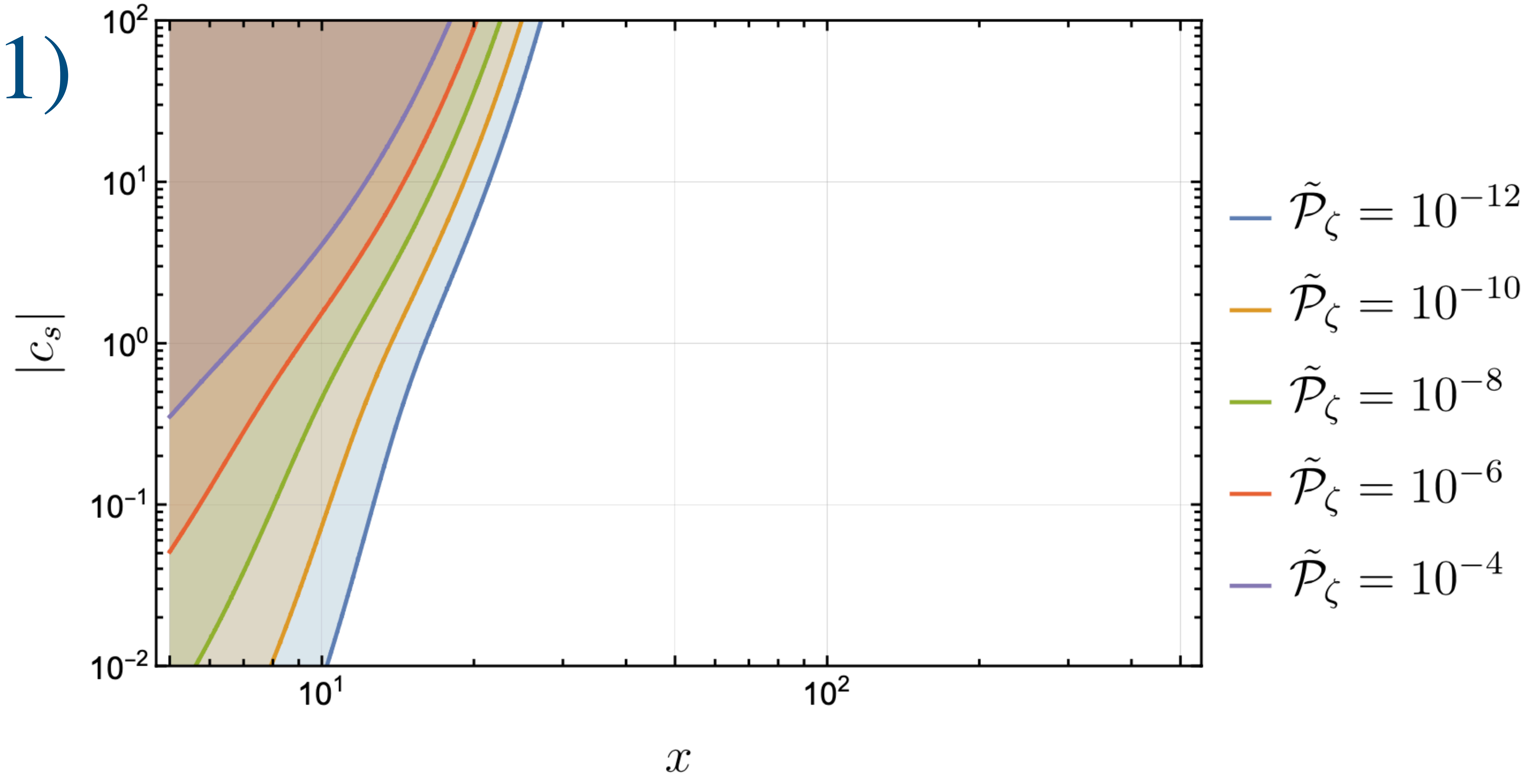
- **One-loop tensor spectrum:** enhanced by exponential of $x$

$$\frac{\mathcal{P}_\gamma^{(1-\mathrm{loop})}}{\mathcal{P}_\gamma^{(\mathrm{tree})}} \sim \frac{\epsilon}{|c_s|}\mathcal{P}_\zeta e^{2x}$$

$$\tilde{\mathcal{P}}_\zeta \sim \epsilon \mathcal{P}_\zeta$$

- Models with $x = \mathcal{O}(10)\,, |c_s| = \mathcal{O}(1)$ are again **consistent**, even if $\mathcal{P}_\zeta$ is large (e.g. hyper-inflation)

- **Excludes** models with very large $x$ (i.e. very large bending)



# Summary

- Models of multi-field inflation with strongly non-geodesic motion may exhibit transient instabilities
- Many interesting predictions (enhanced fluctuations, large non-Gaussianities, novel attractors) but worrisome due to perturbative control
- EFT of inflation with imaginary speed of sound is a powerful theoretical and computational tool
- One-loop corrections dangerously large, but still under control in many models of interest

# Summary

- Models of multi-field inflation with strongly non-geodesic motion may exhibit transient instabilities
- Many interesting predictions (enhanced fluctuations, large non-Gaussianities, novel attractors) but worrisome due to perturbative control
- EFT of inflation with imaginary speed of sound is a powerful theoretical and computational tool
- One-loop corrections dangerously large, but still under control in many models of interest

**Thank you for your attention**