A Semi-analytic Approach Towards Curvaton

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- In standard inflationary scenario inflaton field generates the observed curvature perturbation
- An alternative method to generate curvature perturbation where inflaton is not responsible for it
- Possibility of producing large non-Gaussianities
- Can be interesting for PBH or secondary GWs

- The curvaton field is subdominant in energy during inflation
- Curvaton field is almost frozen during inflation
- Curvaton does not affect the background inflationary evolution
- Curvaton produces isocurvature perturbations
- After inflation the curvaton starts to oscillate and converts it's isocurvature perturbation to observed curvature perturbation

Curvaton dynamics

- Background curvaton: $\overline{\phi}$; curvaton perturbation: $\delta\phi$
- The system

$$S = \int d^4x \sqrt{-g} \left(\frac{M_{\rm Pl}^2}{2} R - \frac{1}{2} \nabla_\mu \chi \nabla^\mu \chi - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi - V(\phi, \chi)\right)$$
(1)

Inflaton and curvaton do not interact with each other

• The background and perturbed curvaton equation of motion:

$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + V_{,\phi} = 0 \tag{2}$$

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \left(\frac{k^2}{a^2} + V_{,\phi\phi}\right)\delta\phi_k \simeq 0 \tag{3}$$

- $\bar{\phi}$ is almost frozen during inflation
- Gaussian fluctuation: $\delta \phi \sim \frac{H_{inf}}{2\pi}$

δN formalism

- Perturbed quantities can be computed from their unperturbed solutions
- If we follow the volume expansion rate starting from a flat hypersurface to uniform density hypersurface for any field φ curvature perturbation can be estimated as,

$$\zeta \sim rac{\partial N(\phi)}{\partial \phi} \delta \phi$$

• The powerspectrum:

$$P_{\zeta} = \left(rac{\partial N}{\partial \phi}
ight)^2 \langle \delta \phi \delta \phi
angle = \left(rac{H_{inf}}{2\pi}
ight)^2 \left(rac{\partial N}{\partial \phi}
ight)^2$$

The bispectrum:

$$f_{NL} = rac{\partial^2 N}{\partial \phi^2} / \left(rac{\partial N}{\partial \phi}
ight)^2$$

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The Lagrangian:

$$\mathcal{L}=-rac{1}{2}(\partial_{\mu}\phi)^2-rac{m^2}{2}\phi^2$$

- Conisderation: $m \ll H_{inf}$
- After $H \sim m$, ϕ starts to oscillate
- Curvaton energy density redshifs as, a^{-3} as opposed to the radiation behavior a^{-4}
- If curvaton lives long enough it will dominate the energy density of the universe

Start with energy equation at the time of ϕ decay with Γ as decay width:

$$3M_p^2\Gamma^2 = \rho_{rad,0}e^{-4N} + \rho_{\phi,0}e^{-3N}$$

Differentiating w.r.t ϕ

$$\frac{\partial N}{\partial \phi} = \frac{2r}{4+3r} \frac{1}{\phi_*}$$

with, $r = \frac{\rho_{\phi,decay}}{\rho_{rad,decay}}$

• For sufficient curvaton domination, $r \to \infty \Rightarrow \frac{\partial N}{\partial \phi} \sim \frac{2}{3\phi_*}$

• For $r \to 0 \Rightarrow \frac{\partial N}{\partial \phi} = \frac{r}{2} \frac{1}{\phi_*}$ • Non-Gaussianity: $f_{NL} = \frac{5}{12}(-3 + \frac{4}{r} + \frac{8}{4 + 3r});$ For $r \to \infty \Rightarrow f_{NL} = -\frac{15}{12} \Rightarrow \text{ constant and negative}$ The system:

$$\mathcal{L} = -rac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi - \Lambda^4\left[1-\cos\left(rac{\phi}{f}
ight)
ight]$$

 $\Lambda^4 = m^2 f^2$ with f^2 is the decay constant

- Considering curvaton energy density redshifts as a^{-3} , analytical estimation results in: $P_{\zeta} = \left(\frac{H_{inf}}{2\pi}\right)^2 \left(\frac{r}{4+3r}\right)^2 \left(\frac{1+\cos\theta_*}{\sin\theta_*}\right)^2 \text{ with,}$ $\theta = \frac{\phi}{f}, \quad \text{[arXiv:2007.01741, T. Kobayashi]}$
- At $\theta \to 0$ limit the axion potential behaves like vanilla curvaton and the powerspectrum follows the same form

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Numerical analysis

- Any potential beyond quadratic $(m^2\phi^2)$ needs to be analyzed numerically. Reason: Analytical computation assumes energy of the curvaton redshifts as a^{-3} irrespective of the potential's nature
- We need to numerically solve (post-inflation):

$$\frac{dN}{dx} = \left[\alpha e^{-4N} + \frac{1}{3M_P^2} \left\{ \frac{1}{2} \left(\frac{d\phi}{dx}\right)^2 + V(\phi) \right\} \right]^{\frac{1}{2}}$$

$$\frac{d^2\phi}{dx^2} = -3\frac{dN}{dx}\frac{d\phi}{dx} - \frac{dV(\phi)}{d\phi}$$
(4)

[arXiv: 0902.2619; P. Chingangbam, Q. Huang]

• x = mt; The integration has to be done from x = 1 (H = m) to $x = \frac{m}{\Gamma}$ ($H = \Gamma$)

Comparison Between Analytical and Numerical Approach



[Eur.Phys.J.C 84 (2024)(2302.00668); A. Ghoshal, A.N.]

- We use δN formalism
- For any polynomial potential the curvaton evolution is separated into distinct region where a single term of the potential dominates
- The whole scenario can be characterised by transition field value and time from one region to another
- We compute the post-inflationary Hubble parameter by properly connecting transitions from one region to another
- Compute number of e-folds *N* from *H* (*dN* = *Hdt*); compute derivatives of *N* w.r.t curvaton initial field value to estimate the observables

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- Consider a λ and initial curvaton field value such that initially ϕ^4 term dominates
- Curvaton potential dominated by ϕ^4 ; radiation energy density dominates $\rightarrow \phi^4$ domination ends at $\phi_T = \sqrt{\frac{2}{\lambda}}m$ and curvaton transits to ϕ^2 region; radiation still dominates \rightarrow radiation domination ends and curvaton starts to dominate the energy density
- Two important times: 1. $\phi^4 \rightarrow \phi^2$ transition time $t_T = t_I \frac{\lambda}{2} \left(\frac{\phi_I}{m}\right)^2$ 2. Radiation domination ending time: $t_{MD} = t_T \left(\frac{\rho_{R,T}}{\rho_{\phi,T}}\right)^2$

- During ϕ^4 domination the equation of state of the curvaton, w = 1/3; during ϕ^2 domination w = 0
- Solve the curvaton and radiation energy evolution equations with scale factor $a(t) \propto t^{1/2}$ for radiation domination and $a(t) \propto t^{2/3}$ for matter domination
- The corresponding first derivative of *N* can be written as,

$$\frac{dN_{R,1}}{d\phi_I} = \int_{t_I}^{t_T} dt \, \frac{dH_{R,1}(\phi_I, t)}{d\phi_I} + H_{R,1}(\phi_I, t_T) \frac{dt_T}{d\phi_I}, \\
\frac{dN_{R,2}}{d\phi_I} = \int_{t_T}^{t_{MD}} dt \, \frac{dH_{R,2}(\phi_I, t)}{d\phi_I} + H_{R,2}(\phi_I, t_{MD}) \frac{dt_{MD}}{d\phi_I} - H_{R,2}(\phi_I, t_T) \frac{dt_M}{d\phi_I}, \\
\frac{dN_M}{d\phi_I} = \int_{t_{MD}}^{t_F} dt \, \frac{dH_M(\phi_I, t)}{d\phi_I} - H_M(\phi_I, t_{MD}) \frac{dt_{MD}}{d\phi_I}.$$

• For a scenario where curvaton does not dominate one can only evaluate upto $dN_{R,2}/d\phi_I$ $(\Box \rightarrow \langle \Box \rangle \land \langle \Xi \land \langle \Xi \rangle \land \langle \Xi \land \langle \Xi \rangle \land \langle \Xi \land$

Comparison with analytic vanilla curvaton



Figure: Powerspectrum and bispectrum of vanilla curvaton: comarison between our method and analytic expression in $r \to \infty$ limit

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Quadratic + Quartic Curvaton Results



Figure: Powerspectrum and bispectrum of vanilla curvaton: Quadratic + Quartic system at $r \rightarrow \infty$ limit

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An Application to Affleck-Dine Curvaton

Affleck-Dine mechanism for baryogenesis is realized with a potential like:

$$V(\Phi)=m^2|\Phi|^2+\lambda|\Phi|^4+i\lambda'(\Phi^4-\Phi^{\dagger 4})$$

A non-zero λ' is responsible for the baryon number violation. AD mechanism can also be realised with the following potential (Lloyd-Stubbs,McDonald (2021)):

$$V(\Phi)=m^2(\Phi^\dagger\Phi)+\lambda(\Phi^\dagger\Phi)^2+\epsilon m^2\left[\Phi^2+(\Phi^\dagger)^2
ight],$$

with,

$$\Phi=rac{1}{\sqrt{2}} oldsymbol{\phi}_r e^{i heta}=rac{1}{\sqrt{2}}\left(oldsymbol{\phi}_1+ioldsymbol{\phi}_2
ight),$$

- For ε ≪ 1 we identify the radial part φ_r as the curvaton sourcing observable scalar fluctuations
- But as ε ≠ 0, φ₁ and φ₂ evolves slightly differently and baryon asymmetry is generated and stored as a difference between Φ and Φ[†]

- The scenario: $\frac{\partial N}{\partial \theta} = 0, \frac{\partial N}{\partial \phi_r} \neq 0$
- We explore a scenario where curvaton decays dominate the energy density $r \gg 1$
- Benchmark: $H_I = 3.21 \times 10^{-6}, m = 10^{-7}, \phi_T = 10^{-3}, \phi_I = 4\phi_T, r_d = 10 \Rightarrow f_{NL} \sim -0.55, \epsilon \sim 2.08 \times 10^{-8}, n_B/s \sim 10^{-10}.$
- Curvaton needs to dominate the energy density in order to be consistent with observational bounds on baryon isocurvature perturbation
- AD mechanism where higher dimensional operator is responsible for the generation of baryon number may not be realised as a curvaton because of isocurvature bounds

Summary

- Standard analytical computation for generic curvaton potential does not agree with numerical estimations
- We introduce a methodology for polynomial curvaton potential, where a single term dominates the for curvaton evolution depending on field value
- The system is characterised by transition field value and time
- We explored an Affleck-Dine curvaton scenario where AD field can produce observable perturbations with sufficient baryogenesis

Thank You

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Observable curvature perturbation gets generated after inflation

• The metric can be written as,

$$ds^2 = -dt^2 + a^2(1+\zeta)g^{ij}dx_i dx_j$$

• ζ at large scales can be written as,

$$\dot{\zeta} = -rac{H}{
ho+P}\delta P_{nad}$$

 $\delta P_{nad} \rightarrow$ Non-adiabatic perturbation

Amplification process

- After inflation we have two components: radiation (rad) and curvaton (χ)
- Gauge invariant perturbations : $\zeta_i = -\psi + \frac{\delta \rho_i}{\dot{\rho}_i}$
- The total curvature perturbation: $\zeta = (1 f)\zeta_{rad} + f\zeta_{\chi}$ $f = \frac{3\rho_{\chi}}{3\rho_{\chi} + 4\rho_{rad}}$
- ζ_i 's are conserved seperatly but the total curvature perturbation evolves as,

$$\dot{\zeta} = \dot{f}(\zeta_{\chi} - \zeta_{rad}) \tag{5}$$

- $\dot{f} > 0 \Rightarrow$ Growth of curvature perturbations
- The non-adaibatic pressure: $\delta P_{nad} \propto (\zeta_{\chi} \zeta_{rad})$

- Solve the axion equation of motion during inflation even if the axion evolution is neglible
- The coupled equation in post-inflationary evolution is hard to handle; after certain *x* the code does not behave well
 - We solve the coupled equation up to some x = x₁ where the code behaves well ⇒ This is not r → ∞ limit
 - After this x_1 we assume the curvaton potential behaves like quadratic one and hence its energy density evolves as a^{-3}
 - We solve

$$\frac{dN}{dx} = \left[\alpha e^{-4N} + \rho_{\theta,x_1} e^{-3N}\right]^{1/2}$$

from x_1 to $x_{decay} = \frac{m}{\Gamma}$

During inflation the N does not depend on χ After inflation if χ startes dominating the energy density,

$$\frac{\partial N}{\partial \phi} \neq 0$$

We get the amplification of initial curvaton density fluctuations and can generate observable CMB fluctuations.

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$$N_B = 2Q_{\phi}m^2\epsilon\phi_{r,I}^2\sin(2\theta_I)J,$$

where $Q_{\phi} \rightarrow$ baryon/lepton number of the AD field

$$J = \frac{\Gamma_{\phi}}{8\epsilon^2 m^2} \text{ for } \epsilon \gg \frac{\Gamma_{\phi}}{m},$$
$$= \frac{1}{2\Gamma_{\phi}} \text{ for } \epsilon \ll \frac{\Gamma_{\phi}}{m},$$

$$n_B = N_B \left(\frac{a_I}{a_{dec}}\right)^3,$$

$$s = \frac{2\pi^2}{45} g_{\star} T_{dec}^3.$$

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