Effective Field Theories, Gravity and Cosmology

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Quadratic gravity with propagating torsion: from EFT towards renormalizability and asymptotic freedom

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$$S_{\rm HE} = \int d^4x\sqrt{-g}\; m_P^2 \left[-\Lambda + \frac{1}{2}R\right]$$

[t'Hooft, Vetlman '74]
[Goroff, Sagnotti '86]
[van de Ven '92]

✗ Nonrenormalisable

$$S_{\rm 4DG} = \int d^4x\sqrt{-g}\Big[m_p^2\left(-\Lambda + \frac{1}{2}R\right) - \alpha R^2 - \beta R_{\mu\nu}R^{\mu\nu} - \gamma R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\Big]$$

✓ Renormalisable [Stelle, '77]

✓ Asymptotically free [Fradkin, Tseytlin, '81] [Avramidi, Barvinsky, '85]

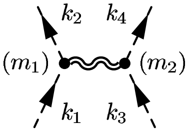
✗ Appears to contain a massive spin-2 ghost and a spin-0 tachyon

The ghost naturally has a high mass and does not appear at low energies. Furthermore, the related issue of unitarity may be resolved by changing the quantization prescription, under which they are quantized as purely virtual particles.

Tachyons, on the other hand, are generally considered unhealthy.

Both theories can be considered as **truncations** of some *infinite curvature expansion*.

Gravitational scattering potential.



$$\sim \frac{G m_1 m_2}{\mathrm{q}^2}$$

$$V(r) = \int \frac{d^3 q}{(2\pi)^3} \frac{G m_1 m_2}{\mathrm{q}^2} e^{i\mathrm{qr}} = -\frac{G m_1 m_2}{r}$$

When loop corrections are taken into account, [Donoghue '94]

$$V(r) = -\frac{G m_1 m_2}{r}\left[1 + \frac{41}{10\pi}\frac{G\hbar}{r^2 c^3} + \ldots\right]$$

Not affected by UV divergences.

Various perspectives on quantization of gravity.

UV E

String theory ?
Loop quantum gravity ?
Dynamical triangulation ?
Causal sets ?
Discrete gravity ?

Asymptotic safety ?
Infinite derivative gravity ? Hořava models ?

m_P

EFT

independent of UV-completion
computable corrections $E^2/\Lambda_{eft}^2 \ll 1$
perturbative unitarity breaks down at $E \sim \Lambda_{eft} \sim M_{Pl}$

IR GR

Perturbative renormalisability $\neq$ UV-completeness $\neq$ theory is actually valid for energies above the Planck scale

Independent connection. Palatini. MAG.

The Cartan curvature tensor is the Yang–Mills strength tensor for the $GL(4)$ group

$$F_{\mu\nu}{}^{a}{}_{b} = \partial_\mu A_\nu{}^{a}{}_{b} - \partial_\nu A_\mu{}^{a}{}_{b} + A_\mu{}^{a}{}_{c} A_\nu{}^{c}{}_{b} - A_\nu{}^{a}{}_{c} A_\mu{}^{c}{}_{b} \,.$$

The torsion tensor and the nonmetricity tensor are defined as

$$Q_{\lambda ab} = -D_\lambda g_{ab} = -\partial_\lambda g_{ab} + A_\lambda{}^{c}{}_{a}\, g_{cb} + A_\lambda{}^{c}{}_{b}\, g_{ac} \,,$$
$$T_\mu{}^{a}{}_{\nu} = \partial_\mu \theta^a{}_\nu - \partial_\nu \theta^a{}_\mu + A_\mu{}^{a}{}_{b}\, \theta^b{}_\nu - A_\nu{}^{a}{}_{b}\, \theta^b{}_\mu \,.$$

Palatini action:

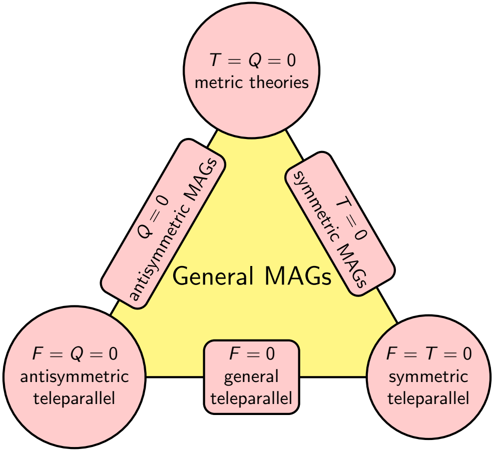
$$S = \frac{1}{2} m_P^2 \int d^4x \sqrt{-g}\, F(A, g) \,.$$

For pure gravity, it produces Einstein equations of motion.

If additional terms are present, the independent connection gains its own dynamics.

- Geometrical beauty
- Very similar to Yang–Mills theory of particle physics $\Longrightarrow$ possible unification with other interactions

Basic classification of MAGs.



The mystical triangle.

MAG incorporates several theories equivalent to GR, as well as its extensions.

For any metric theory of gravity, there exists a teleparallel equivalent (in the absence of fermionic matter). The opposite is not true.

Constructing Lagrangian. Dimension-2.

From the EFT point of view, one needs to include all invariants that are allowed by symmetries:

$$
\begin{aligned}
S &= \frac{1}{2}\int d^4x\sqrt{-g}\Big[m_P^2\left(F(A,g)-2\Lambda\right)-M_T^2T^2-M_Q^2Q^2+\dots\Big]\\
&= \frac{1}{2}\int d^4x\sqrt{-g}\Big[m_P^2\left(R(g)-2\Lambda\right)-m_T^2T^2-m_Q^2Q^2+\dots\Big]
\end{aligned}
$$

At energies lower than the torsion and nonmetricity mass scale, only $dim \leq 2$ terms dominate, making torsion and nonmetricity negligible.

One can also see this as a particular realization of the Higgs phenomenon.

Constructing Lagrangian. Dimension-4. Quadratic MAG.

Quadratic metric-affine gravity can be expressed in a schematic way:

$$\mathcal{L} = -F + Q^2 + T^2 + QT + F^2 = R + (R + \nabla\Phi + \Phi^2)^2 + \Phi^2.$$

This is a popular 28-parameter Quadratic MAG action.

Considering all the terms that are allowed by symmetries, we have rather

$$\begin{aligned}\mathcal{L} = {} & F + TT + TQ + QQ \\ & + FF + FDT + FDQ + (DT)^2 + DTDQ + (DQ)^2 \\ & + FTT + FTQ + FQQ + TTDT + TTDQ + TQDT + TQDQ \\ & + QQDT + QQDQ + TTTT + TTTQ + TTQQ + TQQQ + QQQQ\ .\end{aligned}$$

Particle content of MAG.

	ts	hs	ha	ta
TTT	3^-, 1_1^-	2_1^-, 1_2^-	2_2^-, 1_3^-	0^-
$TTL+TLT+LTT$	2_1^+, 0_1^+	-	-	1_3^+
$\frac{3}{2}LTT$	-	2_2^+, 0_2^+	1_2^+,	-
$TTL+TLT-\frac{1}{2}LTT$	-	1_1^+	2_3^+, 0_3^+	-
$TLL+LTL+LLT$	1_4^-	1_5^-	1_6^-	-
LLL	0_4^+	-	-	-

$SO(3)$ spin content of projection operators for a rank-2 and rank-3 tensors in $d=4$ (a/s=(anti)symmetric,ts/ta=totally (anti)symmetric; hs/ha=hook (anti)symmetric).

In addition to the metric degrees of freedom, MAG may propagate:
3^-, $2^+\times 3$, $2^-\times 2$, $1^+\times 3$, $1^-\times 6$, $0^+\times 4$, 0^-.

What can we say about the RG flow of such theories?

What is the space of UV-complete RG trajectories?

The heat kernel technique. Universal functional traces.

A formal way of dealing with functional traces:

$$\mathrm{Tr}\log\Delta = -\int_0^\infty \frac{ds}{s}\mathrm{Tr}\, e^{-s\Delta}\,.$$

Mostly, we are interested in the early-time asymptotic expansion, which is related to UV effects.

All hinges upon the structure of the kinetic operator, *i.e.* the Hessian:

For minimal operators $\Delta = -\Box + E$, there exists an explicit formula.

For nonminimal operators (such as $\nabla_\mu\nabla_\nu$, $\nabla_\rho\Box$, etc.), one has to expand the logarithm.

[Barvinsky, Vilkovisky '85]

Universal functional traces are related to Q-functionals (integrals over s obtained using the Laplace transform):

$$\mathrm{Tr}\left[\nabla_{\mu_1}\dots\nabla_{\mu_N} f(\Delta)\right] = \frac{1}{(4\pi)^{d/2}}\sum_{n\geq 0} Q_{-n+\frac{d}{2}+\lfloor N/2\rfloor}[f]\;\cdot\int d^dx\,\sqrt{g}\,K^{(n)}_{\mu_1\dots\mu_N}(x)\,,$$

where K are curvature invariants related to the off-diagonal heat kernel coefficients in the coincidence limit.

Running in MAG.

Consider the following action (in metric gauge):

$$\mathcal{S} = \frac{1}{2}\int d^4x\sqrt{g}\left(c_1 F_{\mu\nu\rho\lambda}F^{\mu\nu\rho\lambda} + a_1 T_{\mu\nu\rho}T^{\mu\nu\rho}\right).$$

First computation of the beta function (for theory with propagating torsion and nonmetricity):

$$\beta(c_1) = \frac{1}{(4\pi)^2}\frac{22}{3}$$

[Donoghue '17]

$$\mu\frac{\Gamma}{d\mu} = \frac{1}{(4\pi)^2}\int d^4x\sqrt{g}\Big[\frac{71}{15}L_1^{FF} + \frac{97}{60}L_3^{FF} - \frac{71}{15}L_4^{FF} + \frac{107}{30}L_7^{FF} - \frac{7}{15}L_8^{FF} - \frac{25}{12}L_{16}^{FF} - \frac{253}{30}L_1^{FT} + 4L_8^{FT}$$
$$-\frac{299}{30}L_9^{FT} - \frac{39}{5}L_{13}^{FT} + \frac{19}{30}L_1^{TT} - \frac{1}{5}L_2^{TT} - \frac{7}{30}L_3^{TT} + \frac{32}{15}L_5^{TT} + \dots\Big]$$

[OM, Percacci '23]

As expected, even the first curvature squared term contributes to all the other terms allowed by the symmetries.

This theory in *nonrenormalizable* due to the appearance of F^4 invariants.

If we want a theory that has a more acceptable UV behavior, we need to look at MAG theories, which are direct generalizations of Quadratic Gravity.

Running in Four-Derivative Gravity

$$S_{\mathrm{4DG}} = \frac{1}{2}\int d^4x\,\sqrt{g}\left[m_P^2\,(2\Lambda - R) + \frac{1}{\lambda}C^2 - \frac{1}{3\xi}R^2 + \frac{1}{\rho}E_{\mathrm{GB}}\right].$$

$$\beta_\lambda^{\mathrm{4DG}} = -\frac{1}{(4\pi)^2}\frac{133}{10}\lambda^2\,,$$

$$\beta_\xi^{\mathrm{4DG}} = \frac{1}{(4\pi)^2}\frac{5}{6}\left(2\lambda^2 + 6\lambda\xi + \xi^2\right)\,,$$

[Avramidi, Barvinsky, '85]

independently of the gauge choice.

λ is asymptotically free,

ξ is asymptotically free iff $\xi < 0 \implies$ spin-0 tachyon.

Quadratic gravity with propagating torsion

$$
\begin{aligned}
S = -\frac{1}{2}\int d^4x\sqrt{g}\Big[& m_p^2\,(2\Lambda - R) + 2\alpha R^2 + 2\beta R_{\mu\nu}R^{\mu\nu} + 2\gamma R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \\
& - b_1\, T_{\mu\nu\rho}\Box T^{\mu\nu\rho} - b_2\, T_{\mu\nu\rho}\Box T^{\mu\rho\nu} - b_3\, T_\mu \Box T^\mu - b_4\, T^{\mu\rho\lambda}\nabla_\mu\nabla_\nu T^\nu{}_{\rho\lambda} \\
& - b_5\, T^{\mu\rho\lambda}\nabla_\mu\nabla_\nu T^\nu{}_{\lambda\rho} - b_6\, T^{\rho\mu\lambda}\nabla_\mu\nabla^\nu T_{\lambda\nu\rho} - b_7\, T^{\rho\mu\lambda}\nabla_\mu\nabla^\nu T_{\nu\rho\lambda} \\
& - b_8\, T^{\rho\mu\lambda}\nabla_\mu\nabla^\lambda T^\rho - b_9\, T^\mu\nabla_\mu\nabla_\rho T^\rho + \tau R\nabla_\mu T^\mu + \sigma R^{\mu\nu}\nabla_\rho T^\rho{}_{\mu\nu} \\
& - m_1\, T_{\mu\nu\rho}T^{\mu\nu\rho} - m_2\, T_{\mu\nu\rho}T^{\mu\rho\nu} - m_3\, T_\mu T^\mu + \ldots\Big]
\end{aligned}
$$

Here $T_\mu = T_\mu{}^\alpha{}_\alpha$ is the torsion vector, $R_{\mu\nu\rho\sigma}(g)$ is the Riemann curvature of the Levi-Civita connection.

Very schematically, the kinetic operator and its inverse have the following forms:

$$
S^{(2)} = \begin{pmatrix} \Box^2 & \Box\nabla \\ \Box\nabla & \nabla\nabla \end{pmatrix} + \mathcal{O}\left(\bar{R}, \bar{T}\right)\,, \qquad G = \frac{1}{\Box^4}\begin{pmatrix} \nabla^4 & \Box\nabla^3 \\ \Box\nabla^3 & \Box^2\nabla^2 \end{pmatrix} + \mathcal{O}\left(\bar{R}, \bar{T}\right)\,.
$$

Here we visualize this matrix acting on the sets of fields $\varphi = (h_{\mu\nu}, \delta T_\rho{}^\sigma{}_\lambda)^T$.

Quadratic gravity with propagating torsion

The torsion tensor can be decomposed into irreps of the Lorentz group:

$$T_\mu = T_\mu{}^\alpha{}_\alpha \,, \qquad a_\mu = \epsilon_{\mu\nu\rho\lambda} T^{\nu\rho\lambda} \,, \qquad \kappa_{\alpha\beta\gamma} = T_{\alpha\beta\gamma} - T_{[\alpha\beta\gamma]} - \frac{1}{6} T_{[\alpha} g_{\beta|\gamma]} \,,$$

where T_μ is a vector, a_μ is an axial vector, and κ is the pure tensorial part of torsion that is traceless and hook-antisymmetric:

$$\kappa_\mu{}^\alpha{}_\alpha \equiv 0, \quad \epsilon^{\mu\nu\rho\lambda} \kappa_{\nu\rho\lambda} \equiv 0 \,.$$

After the decomposition is employed, the action becomes

$$\begin{aligned} S = -\frac{1}{2} \int d^4x \sqrt{g} \Big[& m_p^2 (2\Lambda - R) + \frac{1}{\lambda} C^2 - \frac{1}{3\xi} R^2 + \frac{1}{\rho} E_{\mathrm{GB}} \\ & + r_1 R \, \nabla_\mu T^\mu + r_2 \, C_{\mu\nu\rho\sigma} \nabla^\mu \kappa^{\nu\rho\sigma} - d_1 \, T_\mu \Box T^\mu - d_2 \, T_\mu \nabla_\mu \nabla^\nu T^\nu \\ & - d_3 \, a_\mu \Box a^\mu - d_4 \, a_\mu \nabla_\mu \nabla^\nu a^\nu - d_5 \, \kappa_{\mu\nu\rho} \Box \kappa^{\mu\nu\rho} - d_6 \, \kappa_{\mu\rho\lambda} \nabla^\mu \nabla_\nu \kappa^{\nu\rho\lambda} \\ & - d_7 \, \kappa_{\mu\rho\lambda} \nabla^\mu \nabla_\nu \kappa^{\nu\lambda\rho} - d_8 \, T_\rho \nabla_\mu \nabla_\nu \kappa^{\rho\mu\nu} - d_9 \, \eta_{\mu\nu\rho\lambda} T^\mu \nabla^\lambda \nabla_\sigma \kappa^{\sigma\nu\rho} \\ & + m_T \, T_\mu T^\mu + m_a^2 \, a_\mu a^\mu + m_\kappa^2 \, \kappa_{\mu\nu\rho} \kappa^{\mu\nu\rho} + \ldots \Big] \,. \end{aligned}$$

Here $C^2 = C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma}$, $\eta_{\mu\nu\rho\lambda} = \sqrt{g}\epsilon_{\mu\nu\rho\lambda}$, $E_{\mathrm{GB}} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$.

RG flow in gravity with propagating torsion components.

$$S = -\frac{1}{2}\int d^4x\sqrt{g}\left[m_p^2\left(2\Lambda - R\right) + \frac{1}{\lambda}C^2 - \frac{1}{3\xi}R^2 + \frac{1}{\rho}E_{\rm GB}\right] + S_{\rm torsion}\,.$$

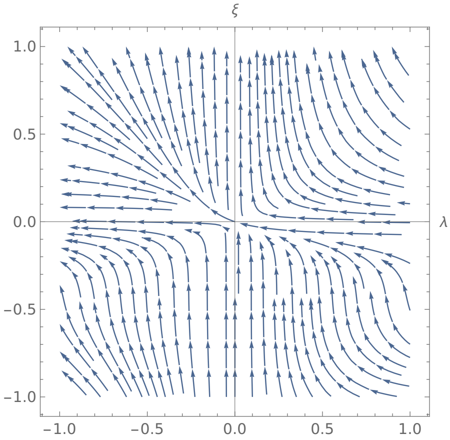


Figure: RG flow in 4DG with **torsion vector**, $d_1 = 1,\ d_2 = r_1 = -0.8$.

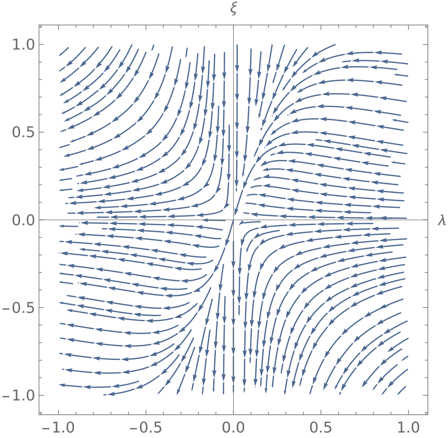


Figure: RG flow in gravity with propagating **hook-antisymmetric traceless torsion**, $d_6 = d_7 = 1/3,\ r_2 = 1/2$.

Takeaways:

1. Both metric theories and MAGs are consistent and predictive below the Planck scale, provided the ghosts are absent or have high mass. Quantum effects manifest themselves as loop corrections to the Newton coupling.

2. When treated as an EFT, MAG explains why we see the Levi-Civita connection at low energies, but lacks further predictive power at higher energies due to a large freedom in incorporating new terms.

3. The behavior of beta functions in gravity with propagating vectorial or axial torsion is qualitatively similar to the standard metric case. However, there exists a specific nonminimal kinetic term for the pure tensorial (hook-antisymmetric traceless) component of torsion that renders the gravitational couplings asymptotically free in the absence of tachyons.

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Counting terms of dimension 4.

Antisymmetric MAG (R&T):

R^2	$(\nabla T)^2$	$R\,\nabla T$	$R\,T^2$	$T^2\nabla T$	T^4	Total
3	9	2	14	31	33	92

[Christensen '80]

Symmetric MAG (R&Q):

R^2	$(\nabla Q)^2$	$R\,\nabla Q$	$R\,Q^2$	$Q^2\nabla Q$	Q^4	Total
3	16	4	22	59	69	173

[Baldazzi, OM, Percacci '21]

General MAG (R&T&Q):

R^2	$(\nabla\Phi)^2$	$R\,\nabla\Phi$	$R\,\Phi^2$	$\Phi^2\nabla\Phi$	Φ^4	Total
3	38	6	56	315	504	922

$$\Phi_\mu{}^a{}_b = A_\mu{}^a{}_b - \Gamma_\mu{}^a{}_b$$

In the general case, we have 59 contributions of dimension up to 4 to the flat-space 2-point function.

Spectrum depends on the choice of the couplings.

Almost any modified gravity theory can be fitted inside this MAG Lagrangian.

Backup

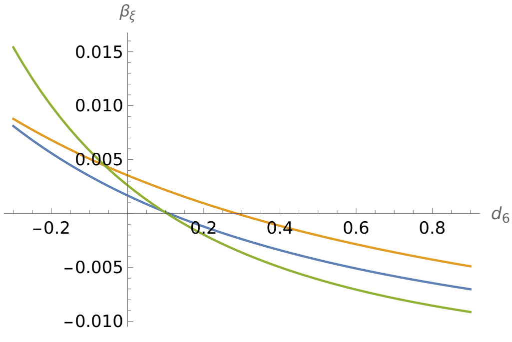

Figure: The values of the beta function of ξ coupling at $\xi = 0.1$, $\lambda = r_2 = 0$ as a function of d_6. The three lines correspond to $d_7 = 0$ (blue), $d_7 = 1/2$ (orange), $d_7 = -1/2$ (green). The change of sign observed when passing to higher values of d_6 is related to the qualitative change of the flow.

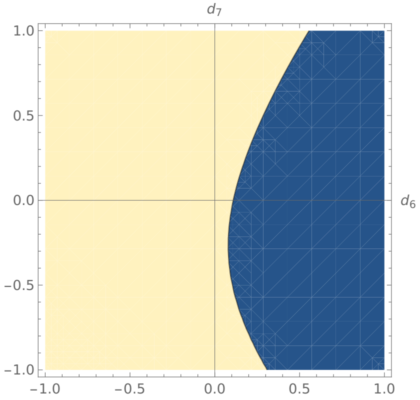

Figure: Sign of β_ξ at the positive values of ξ, with $\lambda = 0$, depending on the values of the couplings d_6 and d_7. In the blue region, $\beta_\xi < 0$ and observe asymptotic freedom. In the yellow region, $\beta_\xi > 0$. The value of r_2 is irrelevant.

GL(4) invariance in gravity

Given a coordinate system x^μ, and arbitrary bases $\{e_a\}$ in the tangent spaces and $\{e^a\}$ in the cotangent spaces:

$$e_a = \theta_a{}^\mu \partial_\mu \ , \qquad e^a = \theta^a{}_\mu dx^\mu \ .$$

Then $\theta^a{}_\mu$ and the metric g_{ab} are nonlinear objects

- metric $g_{ab} \in GL(4)/O(1,3)$,
- soldering / frame field $\theta^a{}_\mu \in GL(4)$, $(\det\theta \neq 0)$.

$$g_{\mu\nu} = \theta^a{}_\mu \, \theta^b{}_\nu \, g_{ab} \ ,$$
$$\Gamma_\lambda{}^\mu{}_\nu = \theta_a{}^\mu \Gamma_\lambda{}^a{}_b \theta^b{}_\nu + \theta_a{}^\mu \partial_\lambda \theta^a{}_\nu \ .$$

Possible gauge choices for $GL(4)$:

- $\theta^a_\mu = \delta^a_\mu$: coordinate frames - breaks $GL(4)$ completely
  $\rightarrow$ metric formulation
- $g_{ab} = \eta_{ab}$: orthonormal frames - breaks $GL(4)$ to $O(1,3)$ (Local Lorentz group)
  $\rightarrow$ vierbein formulation