New factorization of Yang-Mills amplitudes from hidden zeros

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- - Reflects the principles of: Locality, Unitarity, Gauge invariance.
 - Used extensively in modern amplitude theory: Britto–Cachazo–Feng– Witten (BCFW) recursion, Collinear limits, Soft theorems, Unitarity cuts
 - Takeaway: Factorization is not just a computational trick. It is a structural manifestation of fundamental physical principles.
- Natural question: Are poles the only loci where meaningful factorization occurs?

The Role of Factorization in Quantum Field Theory



Beyond Traditional Factorization

Recent developments in scalar theories have revealed new types of splittings:

- Semi-local factorization [Cachazo et al., 2112]:
- Three-part splitting near zeros [Arkani-Hamed et al.,2312]:
- Unified by the 2-split framework [Cao et al., 2403][Cao et al., 2406]:
- Factorization under algebraic kinematic constraints
- Extended to biadjoint ϕ^3 , Nonlinear Sigma Model (NLSM), special Galileon, and even Yang-Mills/gravity

Key phenomenon underlying all these structures: The appearance of hidden zeros [Arkani-Hamed et al., 2312] amplitude contributions that vanish without any apparent symmetry or pole.

These results suggest that:

Factorization may also emerge away from poles, driven by internal algebraic constraints, not just physical propagation.

Hidden Zeros in Scattering Amplitudes

What are hidden zeros?

- Amplitude contributions that vanish under specific kinematic constraints, but not due to any symmetry or gauge invariance
- Hidden zeros were first observed in scalar theories when certain Mandelstam variables within a rectangle are set to zero: $s_{ij} = k_i \cdot k_j = \frac{1}{2} \left(k_i + k_j \right)^2 \to 0$ for specific (i, j)
- In Yang-Mills and gravity, additional polarization constraints are needed, such as setting $\epsilon \cdot k$ and $\epsilon \cdot \epsilon$ to zero to achieve cancellation [Arkani-Hamed et al., 2312].
- Not obvious in Feynman diagrams. See recent progress [Rodina, 2406] [Rodina et al, 2503] [Zhou, 2411] [Zhou et al, 2502]
- Extended to cosmological waveform [Spradlin et al., 2503] \bullet
- What is the structure of the amplitude near such a zero?

 $S_{1 m+2}, S_{1 m+3}, \ldots, S_{1 n-1},$ $\begin{array}{c} s_{2m+2}, s_{2m+3}, \dots, s_{2n-1}, \\ \vdots & \vdots & \vdots \end{array} \rightarrow 0$ $S_{m\,m+2}, S_{m\,m+3}, \ldots, S_{m\,n-1}$





New Factorization Formula

Setting certain two-point Mandelstam variables s_{ij} in a rectangle to zero, the full YM color-ordered amplitude A_n decomposes into a sum of gluings of lower-point YM amplitudes [YZ, 2406] [YZ, 2412]

$$A^{\text{YM}}\left(\mathbb{I}_{n}\right) \rightarrow \sum_{(i,j) \text{ in a rectangle}} \frac{(-1)^{n+1}}{s_{12\ldots m+1}} \sum_{\epsilon_{\hat{j}}} A^{\text{YM}}_{3}(ij-\hat{j}) \sum_{\rho \in S_{m-1}} X(s,\rho) A^{\text{YM}}_{n-1}(\rho(12\ldots i-1i+1\ldots m)m+1m+2\ldots \hat{j}\ldots m)$$



$$S_{1 m+2}, S_{1 m+3}, \dots, S_{1 n-1},$$

 $S_{2 m+2}, S_{2 m+3}, \dots, S_{2 n-1},$
 \vdots \vdots \vdots \vdots $S_{m m+2}, S_{m m+3}, \dots, S_{m n-1}$











The structure is nonlocal but recursive If We can make hidden zeros explicit by turning off polarization contractions involving the constrained particles: For each vanishing $s_{ij} \to 0$, we let $\epsilon_i \cdot \epsilon_j$, $\epsilon_i \cdot k_j$, $k_i \cdot \epsilon_j \to$ \rightarrow Under these conditions, the entire gluon amplitude vanishes.

New Factorization Formula $\sum_{(i,j) \text{ in a rectangle}} \frac{(-1)^{n+1}}{s_{12...m+1}} \sum_{\epsilon_{\hat{j}}} A_3^{\text{YM}}(ij-\hat{j}) \sum_{\rho \in S_{m-1}} X(s,\rho) A_{n-1}^{\text{YM}}(\rho(12...i-1i+1...m)m+1m+2...\hat{j}...n)$

gluon pair contribution

$$s_{1 m+2}, s_{1 m+3}, \dots, s_{1 n-1},$$

 $s_{2 m+2}, s_{2 m+3}, \dots, s_{2 n-1},$
 \vdots
 $s_{m m+2}, s_{m m+3}, \dots, s_{m n-1}$

$$0 \quad \Rightarrow A_3^{\rm YM}(ij-\hat{j}) = 0 \; .$$







Example – 5-Point Yang-Mills Amplitude

$$\overset{\text{YM}}{\longrightarrow} \text{ Under a rectangular constraint (e.g. } s_{13}, s_{14} \to 0 \text{), the 5-point of } A^{\text{YM}}(\mathbb{I}_5) \to \frac{1}{s_{12}} \sum_{\epsilon_3^2} A_3^{\text{YM}}(1,3, -\hat{3}) A_4^{\text{YM}}(2,\hat{3},4,5) + \frac{1}{s_{12}} \sum_{\epsilon_4^2} A_3^{\text{YM}}(1,3, -\hat{3}) A_4^{\text{YM}}(1,3, -\hat{3}) A_4^{\text{YM}}(2,\hat{3},4,5) + \frac{1}{s_{12}} \sum_{\epsilon_4^2} A_4^{\text{YM}}(1,3, -\hat{3}) A_4^{\text{YM}}(1,3, -\hat{3}) A_4^{\text{YM}}(2,\hat{3},4,5) + \frac{1}{s_{12}} \sum_{\epsilon_4^2} A_4^{\text{YM}}(1,3, -\hat{3}) A_4^{\text{YM}}(1,3, -$$

- Each term corresponds to a gluon pair contribution from a vanishing s_{ij} inside the rectangle
- The internal gluon \hat{j} is exchanged

Key features of the example:

• Polarization structure is respected term by term

Setting
$$\epsilon_1 \cdot \epsilon_3, \epsilon_1 \cdot k_3, k_1 \cdot \epsilon_3, \quad \epsilon_1 \cdot \epsilon_4, \epsilon_1 \cdot k_4, k_1 \cdot \epsilon_4 \to 0$$
,

oint YM amplitude decomposes into:

 $A_{3}^{\text{YM}}(1,4,-\hat{4})A_{4}^{\text{YM}}(2,3,\hat{4},5)$

This gives a structured, recursive-like decomposition of the 5-point amplitude without relying on any physical pole.

$$\Rightarrow A_3^{\mathrm{YM}}(1,3,-\hat{3}), A_3^{\mathrm{YM}}(1,4,-\hat{4}) \to 0 \qquad \Rightarrow A^{\mathrm{YM}}(\mathbb{I}_5) \to 0.$$

More Examples

• For four points, setting $s_{13} \to 0$, $A^{\text{YM}}(\mathbb{I}_4) \to -\frac{1}{s_{12}} \sum_{\epsilon_3} A_3^{\text{YM}}(1,3,-\hat{3}) A_3^{\text{YM}}(2,\hat{3},4)$

• For six points, there are two types of new factorizations: Setting $s_{13}, s_{14}, s_{15} \to 0$, $A^{\text{YM}}(\mathbb{I}_6) \to -\frac{1}{s_{12}} \Big[\sum_{\epsilon_3} A_3^{\text{YM}}(1,3, -\hat{3}) A_5^{\text{YM}}(2,\hat{3},4,5,6) + \sum_{\epsilon_{\hat{4}}} A_5^{\text{YM}}(1,3, -\hat{3}) A_5^{\text{YM}}($

Setting
$$\frac{s_{14}, s_{15}}{s_{24}, s_{25}} \to 0$$
,
 $A^{\text{YM}}(\mathbb{I}_6) \to \frac{\frac{s_{26}}{s_{12}s_{123}} \left[\sum_{\epsilon_{\hat{4}}} A_3^{\text{YM}}(1,4,-\hat{4}) A_5^{\text{YM}}(2,3,\hat{4},5,6) + \sum_{\epsilon_{\hat{5}}} A_3^{\text{YM}}(1,5,-\hat{5}) A_5^{\text{YM}}(2,3,4,\hat{5},6) \right]}{+ \frac{s_{13}}{s_{12}s_{123}} \left[\sum_{\epsilon_{\hat{4}}} A_3^{\text{YM}}(2,4,-\hat{4}) A_5^{\text{YM}}(1,3,\hat{4},5,6) + \sum_{\epsilon_{\hat{5}}} A_3^{\text{YM}}(2,5,-\hat{5}) A_5^{\text{YM}}(1,3,4,\hat{5},6) \right]$

$$\sum_{3}^{\text{YM}} (1,4,-\hat{4}) A_5^{\text{YM}}(2,3,\hat{4},5,6) + \sum_{\epsilon_{\hat{5}}} A_3^{\text{YM}}(1,5,-\hat{5}) A_5^{\text{YM}}(2,3,4,\hat{5},6) \Big]$$

CHY Formalism Recap

The Cachazo–He–Yuan (CHY) formalism provides a powerful worldsheet representation of scattering amplitudes [CHY, 1307]: $A^{\mathrm{YM}}(\mathbb{I}_n) = \int d\mu_n \operatorname{PT}(\mathbb{I}_n) \operatorname{Pf}'(\Psi)$

• Integration is over the moduli space of n punctures on the Riemann sphere

Localized on solutions of the scattering equations: $E_a :=$

Key ingredients:

 $d\mu_n = (\sigma_{1,n-1}\sigma_{n-1,n}\sigma_{n,1})^2 \prod_{a=2}^{n-2} d\sigma_i \delta(E_i)$: CHY measure, includes delta functions for the scattering equations $PT(\mathbb{I}_n) := \frac{1}{\sigma_{12}\sigma_{23}\dots\sigma_{n1}}$: Parke–Taylor factor encoding color ordering

• $Pf'(\Psi)$: reduced Pfaffian encoding polarization and kinematics

$$\Psi = \begin{pmatrix} A & C \\ -C^T & B \end{pmatrix}, \quad A_{ab} = \begin{cases} \frac{k_a \cdot k_b}{\sigma_{ab}}, & a \neq b \\ 0, & a = b \end{cases}, \quad B_{ab} = \begin{cases} \frac{\epsilon_a \cdot \epsilon_b}{\sigma_{ab}}, & a \neq b \\ 0, & a = b \end{cases}, \quad C_{ab} = \begin{cases} \frac{\epsilon_a \cdot k_b}{\sigma_{ab}}, & a \neq b \\ -\sum_{b' \neq a} \frac{\epsilon_a \cdot k_{b'}}{\sigma_{ab'}}, & a = b \end{cases}$$

$$\sum_{b \neq a} \frac{k_a \cdot k_b}{\sigma_{ab}} = 0 \quad \text{with } \sigma_{ab} := \sigma_a - \sigma_b$$

Hidden Zeros as Vanishing CHY Residues

► In the CHY formalism, tree-level amplitudes are expressed as **sums of residues**: $A^{\text{YM}}(\mathbb{I}_n) = \sum_{\substack{n \in \mathbb{C}}} \frac{PT(\mathbb{I}_n)Pf'(\Psi)}{\det'\Phi} \text{ where the typical element of } \Phi \text{ is } \frac{\Phi}{\Phi}$ solns of scattering eqs

(6) Key observation: Let's parameterize all vanishing $s_{ij} \rightarrow \tau \hat{s}_{ij}$ with $\tau \rightarrow 0$. All of the solutions are singular with one or more pairs of pinched punctures $\sigma_{ij} \sim O(\tau)$ Obviously, $\frac{1}{\sigma_{ij}} \sim \mathcal{O}(\tau^{-1}), \quad \frac{s_{ij}}{\sigma_{ij}} \sim \mathcal{O}(\tau^0), \quad \frac{s_{ij}}{\sigma_{ij}^2} \sim \mathcal{O}(\tau^{-1}).$ $PT(\mathbb{I}_n)$ is regular. \Rightarrow $det'\Phi$ is divergent. The degree of divergence depends on the number of pinched pairs of punctures in the singular solution. det' A is regular.

If further setting $\epsilon_i \cdot \epsilon_j, \epsilon_i \cdot k_j, k_i \cdot \epsilon_j \to 0$, obviously all elements in Ψ are regular and hence $Pf'(\Psi)$ are regular. \Rightarrow YM amplitudes vanish.

Easily extended to prove the hidden zeros of other theories, GR, biadjoint ϕ^3 , NLSM, sGal, etc.. A universal proof across many theories, without relying on intricate combinatorial structures from double-copy relations. [Trnka et al., 2403] [Li et al., 2403]

$$\frac{\partial E_b}{\partial \sigma_b} = \frac{s_{ab}}{\sigma_{ab}^2}$$

CHY-Based Proof of the New Factorization

pinched pairs of punctures.

Key observation: Only single-pinch solutions survive, while all other contributions cancel algebraically — proven for arbitrary multiplicity. Solution with $\sigma_{ij} \sim \mathcal{O}(\tau)$, $Pf'(\Psi_n) \rightarrow \sum A_3^{YM}(i, j, -\hat{j})Pf'(\Psi_{n-1})$

resulting in a gluon pair contribution in the new factorization formula $A^{\text{YM}}(\mathbb{I}_n) \rightarrow \sum_{(i,j) \text{ in a rectangle}} \frac{(-1)^{n+1}}{s_{12\dots m+1}} \sum_{e_{\hat{j}}} A^{\text{YM}}_{3}(ij-\hat{j}) \sum_{\rho \in S_{m-1}} X(s,\rho) A^{\text{YM}}_{n-1}(\rho(12\dots i-1i+1\dots m)m+1m+2\dots\hat{j}\dots n)$

where $X(s, \rho)$ as ratios of Mandelstam variables are determined by Bern–Carrasco–Johansson relations of (m + 2)-pt Yang-Mill amplitudes.

$$s_{12...m+1}\hat{\mathrm{PT}}(12...m+1) = \sum_{\nu=1}^{m} \sum_{\rho_{\nu} \in S_{m-1}} X(s,\rho_{\nu}) \hat{\mathrm{PT}}(\rho_{\nu},m+1) \left(\frac{s_{\nu 1}}{\sigma_{\nu 1}} + \frac{s_{\nu 2}}{\sigma_{\nu 2}} + \dots + \frac{s_{\nu,m+1}}{\sigma_{\nu,m+1}}\right)$$

- If $\epsilon_i \cdot \epsilon_j$, $\epsilon_i \cdot k_j$, $k_i \cdot \epsilon_j$ are regular, $Pf'(\Psi)$ is as divergent as det' Φ , depending on the number of
 - $\epsilon_{\hat{i}}$



Applicability to Other Theories

The core mechanism behind our factorization is:

Hidden zeros in CHY residues, arising under rectangular Mandelstam constraints

This suggests the structure may extend to many other theory with a CHY representation.

Theory	CHY Integrand	Hidden Zero Potential
Yang-Mills	$\operatorname{PT}(\mathbb{I}_n)\operatorname{Pf}'(\Psi)$	Proven
Gravity	$\mathrm{Pf}'(\Psi)\mathrm{Pf}'(\tilde{\Psi})$	Strong candidate
Born-Infeld	$\det' A \operatorname{Pf'}(\Psi)$	🔁 Likely
NLSM	$\operatorname{PT}(\mathbb{I}_n) \det'(A)$	Subleading behavior may carry structure
Special Galileon	$\left(\det'(A)\right)^2$	Subleading behavior may carry structure
String integrands	CHY-like low-energy limits	? Worth exploring

Comparison with the 2-Split Structure

The 2-split framework [Cao et al., 2403][Cao et al., 2406] organizes amplitudes by:

- Selecting a subspace of Mandelstam variables
- Imposing zero conditions on both Mandelstam and polarization products
- Producing a factorization into **two currents** under strong constraints



Feature	2-Split	Our Structure
Constraint type	$(s_{ij} = 0) + (\epsilon \cdot k, \epsilon \cdot \epsilon = 0)$	Only $(s_{ij} = 0)$
Output	Single product of two currents	Sum over gluon pair contributions
Applicable to	Scalar, YM, GR, stringy models	Proven in YM, generalizable

The combination of the two factorizations gives a split of 3 currents. For example,

Setting
$$s_{14}, s_{24} \to 0, \epsilon_2 \cdot \epsilon_4, \epsilon_2 \cdot k_4, k_2 \cdot \epsilon_4 \to 0,$$

 $\Rightarrow A^{\text{YM}}(12345) \to \frac{s_{12} + s_{23}}{s_{12}s_{123}} A^{\mu,\text{YM}}(14; -\hat{4})J$

 $\epsilon_2 \cdot \epsilon_1, \epsilon_2 \cdot \epsilon_3, \epsilon_2 \cdot \epsilon_5 \to 0, k_2 \cdot \epsilon_1, k_2 \cdot \epsilon_3, k_2 \cdot \epsilon_5 \to 0$ $J^{\text{YM}+\phi^3}(5^{\phi}23^{\phi};\tilde{4}'^{\phi})J^{\text{YM}}_{\mu}(53;\tilde{4})$



We proposed a new factorization structure for tree-level Yang-Mills amplitudes, based not on poles, but on hidden zeros under kinematic constraints.

We proved this structure using the CHY formalism, by analyzing singular solutions and showing how residues organize into a sum over lower-point gluon-pair glueings.

W This new perspective reveals:

- A recursive-like, dimension-independent organization of amplitudes
- A structural role for hidden zeros beyond symmetry or gauge redundancy
- A blueprint for extending to gravity, EFTs, loop amplitudes, and beyond

► Our work suggests that hidden zeros are not exceptions they are building blocks of new, non-pole-based factorization.

Summary of Main Results

- geometry.
- **Soft & Collinear Behavior**
- **Loop-Level** Extensions [Rodina et al, 2503]
- **Moduli Space & Geometry**
- **Recursion & Algebra**

©^{*} Next Steps

- Extend to GR, Born-Infeld, NLSM, special Galileon
- Apply in waveform generation [Spradlin et al., 2503], AdS mellin amplitudes, BCJ structure, symbolic simplification

Takeaway: Hidden-zero-based factorization may offer a new organizing principle beyond poles — with geometric, computational, and physical consequences.

Thank you!

Broader Implications and Future Directions

Q Our new factorization raises deeper questions in amplitude theory, effective field theory, and moduli space