Study of $J/\psi \to \ell^+\ell^-\pi^0/\eta^{(\prime)}$ at BESIII/STCF

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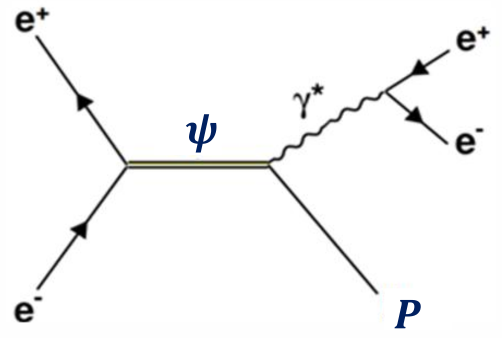
Motivation



- Electromagnetic Dalitz decays, such as $\psi \to l^+l^-P$ (P = π^0, η, η'), proceed via an offshell photon (γ^*).

✓ Four-momentum transfer square (q^2) dependent decay rate of an electromagnetic Dalitz decay

$$\frac{dB(\psi \to l^+l^-P)}{dq} = \frac{dB(\psi \to l^+l^-P)}{dq}\bigg|_{Point-like} |F(q)|^2$$

Transition form factor (TFF)

Experimentally measured differential decay rate.

Predicted by Quantum electrodynamics (QED)

✓ Within the **Vector Meson Dominance (VMD)** model, coupling of the γ^* to the ψ is governed by the TFF through intermediate vector mesons V' in the time-like region.

[Phys. Rept. 128, 301 (1985)]

$$F_{AB}(q^2) = N\sum_{V'} \frac{m_{V'}^2}{m_{V'}^2 - q^2 - i\Gamma_{V'}m_{V'}}$$

Here, V′=ρ, ω, ϕ resonances

Possible **rho-omega interference** arises because the ρ and ω mesons have nearly degenerate masses but different isospin properties (I=1 for ρ, I=0 for ω), leading to quantum mechanical interference in processes like $e^+e^-\to\pi^+\pi^-$.

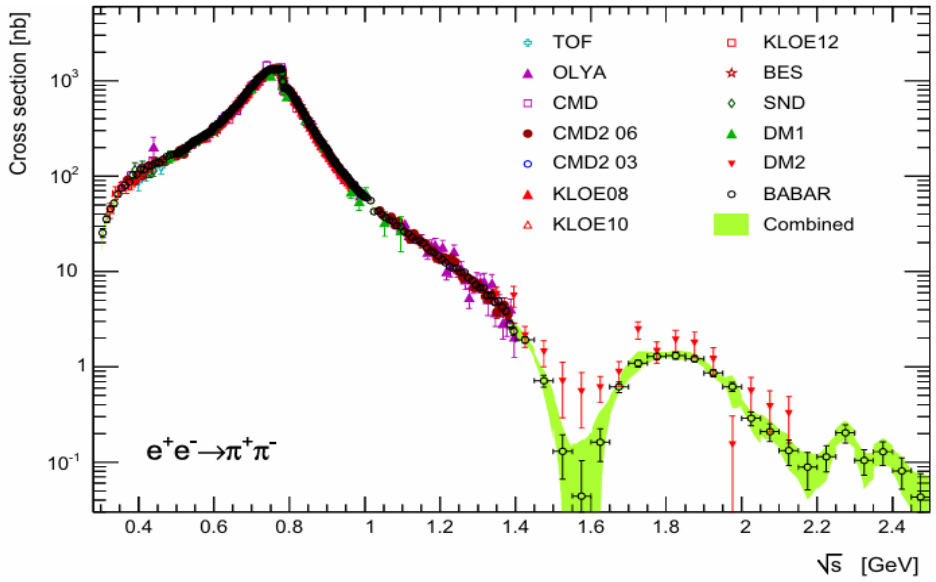
$$\sigma(e^+e^- \to \pi^+\pi^-) \propto \left| \frac{1}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho} + \frac{\epsilon}{q^2 - m_\omega^2 + im_\omega\Gamma_\omega} \right|^2$$

where ϵ represents the relative coupling strength ($\epsilon\sim1/10$ due to isospin suppression for $\omega\to\pi^+\pi^-$).

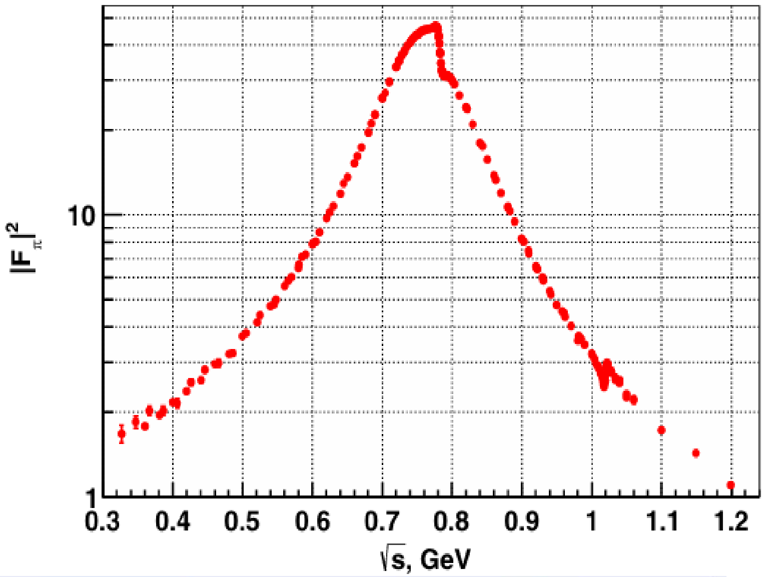
Why VMD

- The VMD provides a very accurate prediction for the production of pions in e^+e^- annihilation processes.

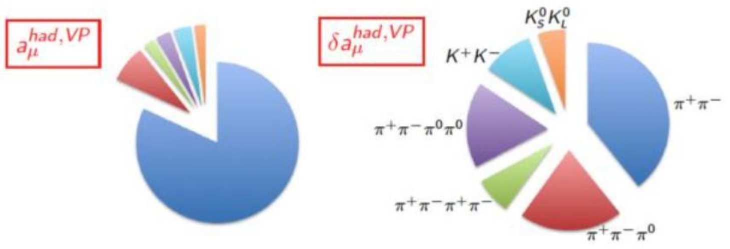
Eur. Phys. J. C **77**, 827 (2017)

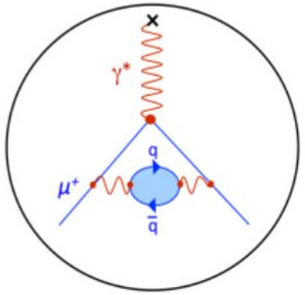


Phys. Rev. Lett. **132**, 231903 (2024)



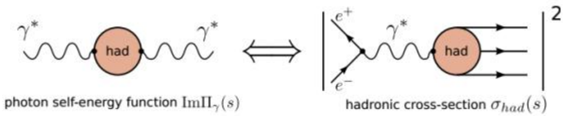
Hadronic cross-section measurements are used for the evaluation of the hadronic vacuum polarization contribution to the muon g-2





Optical theorem

$$\sigma(s)_{e^+e^-\to hadrons} = \frac{4\pi\alpha}{s} Im\,\Pi_\gamma(s)$$

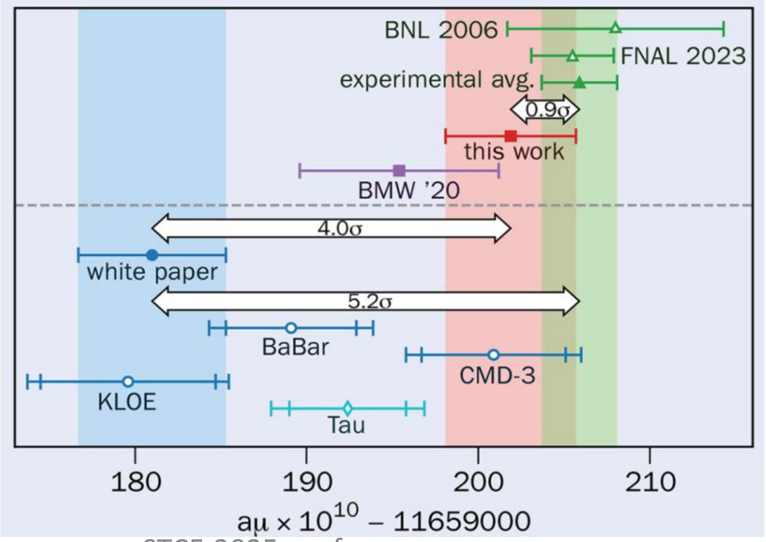


Dispersion integral

$$a_{\mu,LO}^{\rm HVP} = \frac{1}{4\pi^3}\int_{m_{\pi^0}}^{\infty} ds\, K(s)\, \sigma_{\rm had}(s)$$

Kernel function ~ 1/s

$\sigma_{had} = \sigma(e^+e^-\to\ hadrons) \sim 1/s$



- Discrepancy in Standard Model (SM) predictions of the muon *g-2* arises from inconsistencies in the measurements of the cross-section of $e^+e^-\to\pi^+\pi^-$ by various e^+e^- collider experiments.
- Observing the **ρ–ω interference pattern in $J/\psi\to V\pi^0$ (where V=ρ,ω)** would be useful for understanding this discrepancy

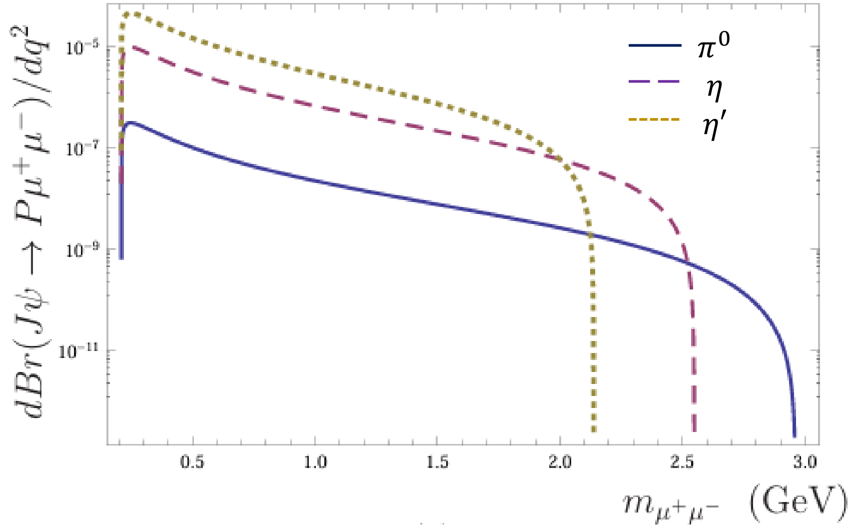
VMD prediction of $\psi \to \ell^+\ell^- P(=\pi^0, \eta, \eta')$

- Expected decay rate of EM Dalitz decays $\psi \to \ell^+\ell^- P$:

$$\frac{d\Gamma(\psi \to P l^+ l^-)}{dq^2 \Gamma(\psi \to P\gamma)} = \frac{\alpha}{3\pi} \left| \frac{f_{\psi P}(q^2)}{f_{\psi P}(0)} \right|^2 \frac{1}{q^2} \left(1 - \frac{4m_l^2}{q^2}\right)^{1/2} \left(1 + \frac{2m_l^2}{q^2}\right) \left[\left(1 + \frac{q^2}{m_\psi^2 - m_P^2}\right)^2 - \frac{4m_\psi^2 q^2}{(m_\psi^2 - m_P^2)^2} \right]^{3/2}$$
$$= |F_{\psi P}(q^2)|^2 \times [\mathrm{QED}(q^2)],$$

Phys. Rept. **B128**, 301 (1985)

Mod. Phys. Lett. A **27**, 1250223 (2012).



- The form factor may be parameterized in the simple pole approximation as

$$F_{\psi P}(q^2) = \frac{1}{1 - \frac{q^2}{\Lambda^2}}$$

Phys. Rev. D **100**, 016018 (2019)

Int. J. Mod. Phys. A **34**,1950129(2019)

Decay mode	e^+e^-		$\mu^+\mu^-$	
	VMD prediction	Experiment	VMD prediction	Experiment
$J/\psi \to \ell^+\ell^-\pi^0$	$(3.89^{+0.37}_{-0.33}) \times 10^{-7}$	$(8.06 \pm 0.31 \pm 0.38) \times 10^{-7}$	$(1.01^{+0.10}_{-0.09}) \times 10^{-7}$	
$J/\psi \to \ell^+\ell^-\eta$	$(1.21 \pm 0.04) \times 10^{-5}$	$(1.39 \pm 0.06 \pm 0.07) \times 10^{-5}$	$(0.30 \pm 0.01) \times 10^{-5}$	
$J/\psi \to \ell^+\ell^-\eta'$	$(5.66 \pm 0.04) \times 10^{-5}$	$(6.59 \pm 0.07 \pm 0.17) \times 10^{-5}$	$(1.31 \pm 0.04) \times 10^{-5}$	

Some other BESIII measurements:

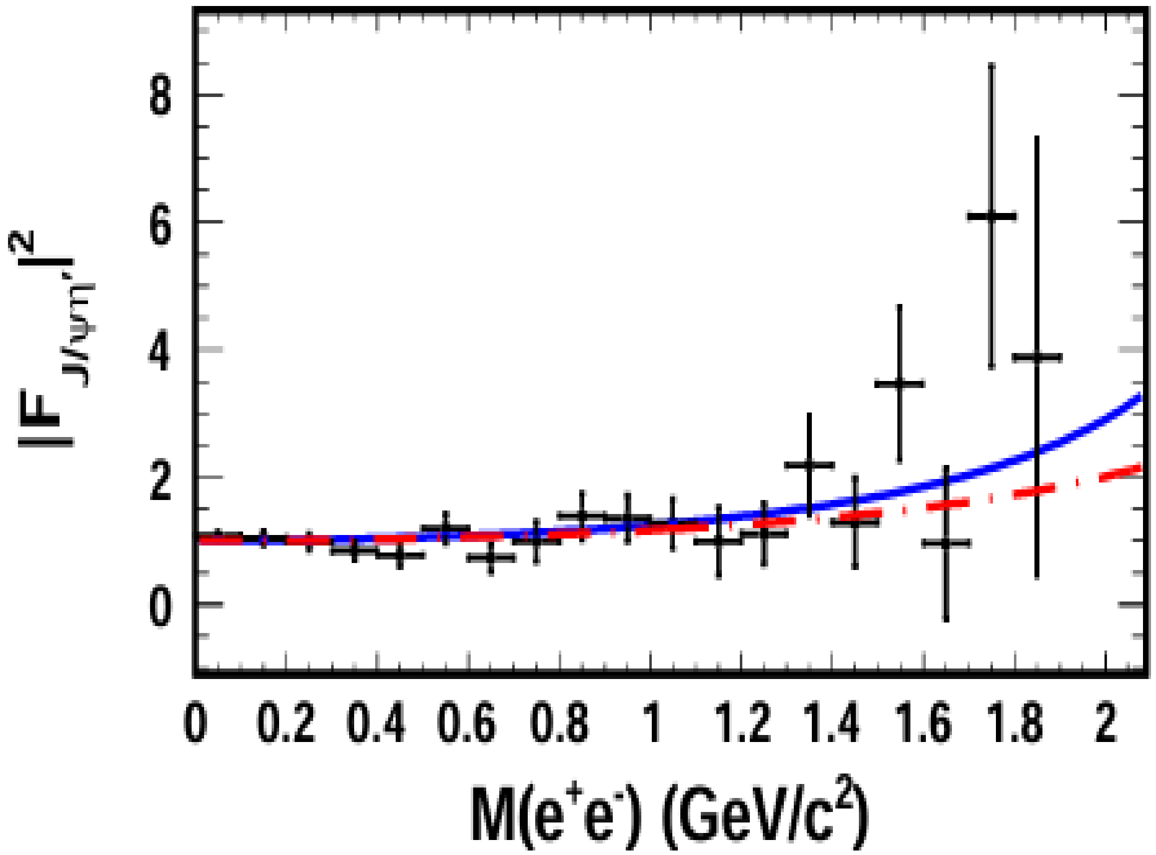
$J/\psi \to e^+e^-\eta(1405)$, $\psi(3686) \to \ell^+\ell^-\eta_c$, $h_c \to e^+e^-\eta_c$, $\eta/\eta' \to e^+e^-\gamma$, $J/\psi \to \ell^+\ell^- X(1835), X(2120), X(2370)$ etc.

- **Large discrepancy between experimental and theoretical results in $J/\psi \to \ell^+\ell^-\pi^0$ decay.**

Our objective is to update the VMD predictions of $\psi \to \ell^+\ell^-\pi^0$ including $\rho - \omega$ interference using a Data-driven method

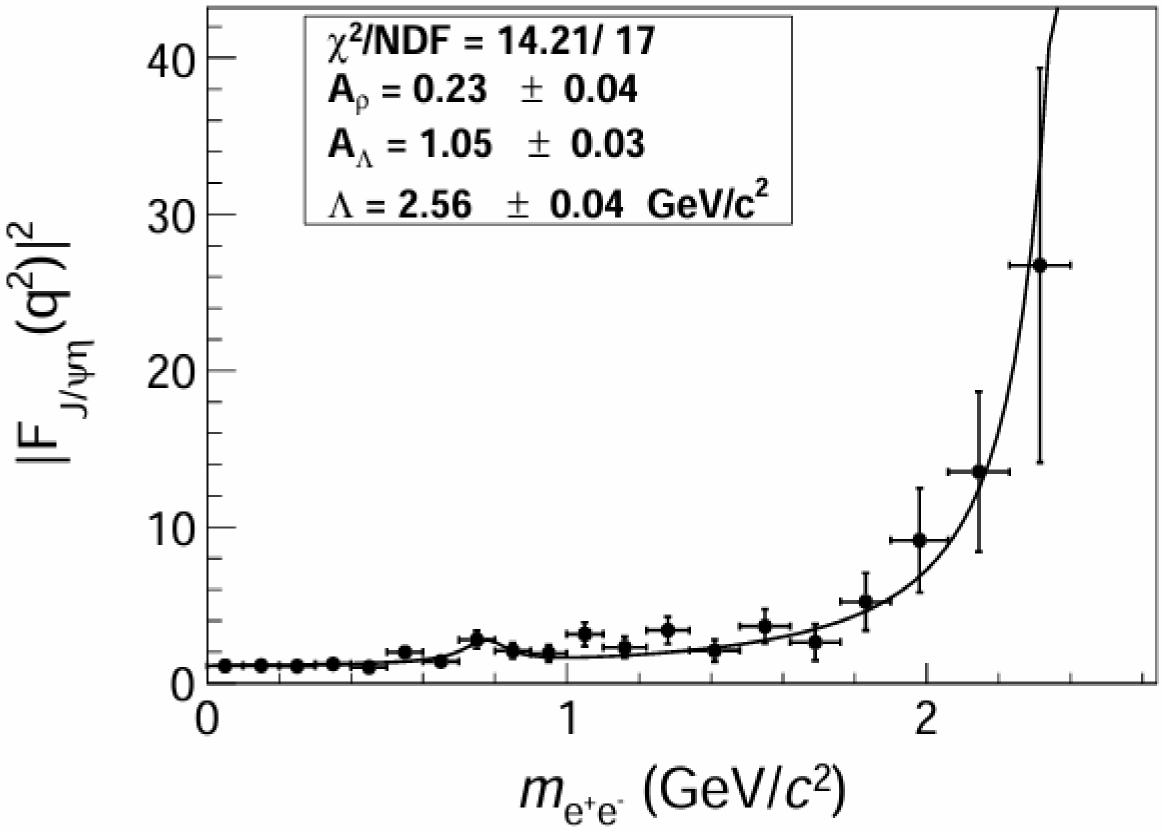
TFF measurements of $J/\psi \to e^+e^-\eta/\eta'$

For $J/\psi \to e^+e^-\eta'$ (based on 225 million J/ψ events)



Phys. Rev. D **89**, 092008 (2014)

For $J/\psi \to e^+e^-\eta$ (based on 1310.6 million J/ψ events)

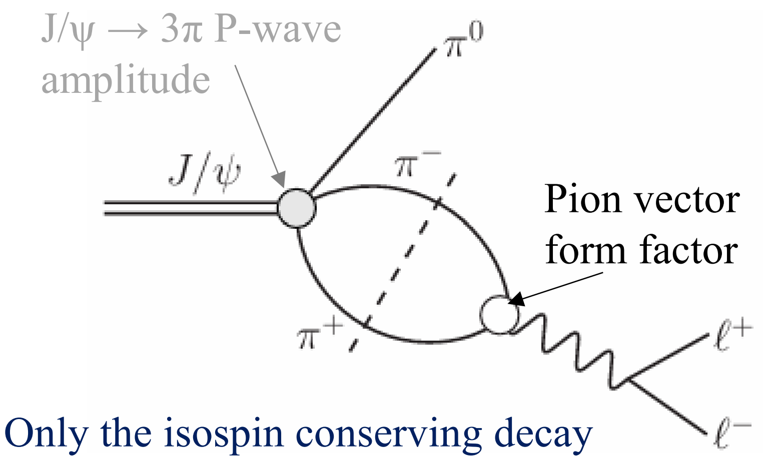


Phys. Rev. D **99**, 012006 (2019)

Erratum: Phys. Rev. D **104**, 099901 (2021)

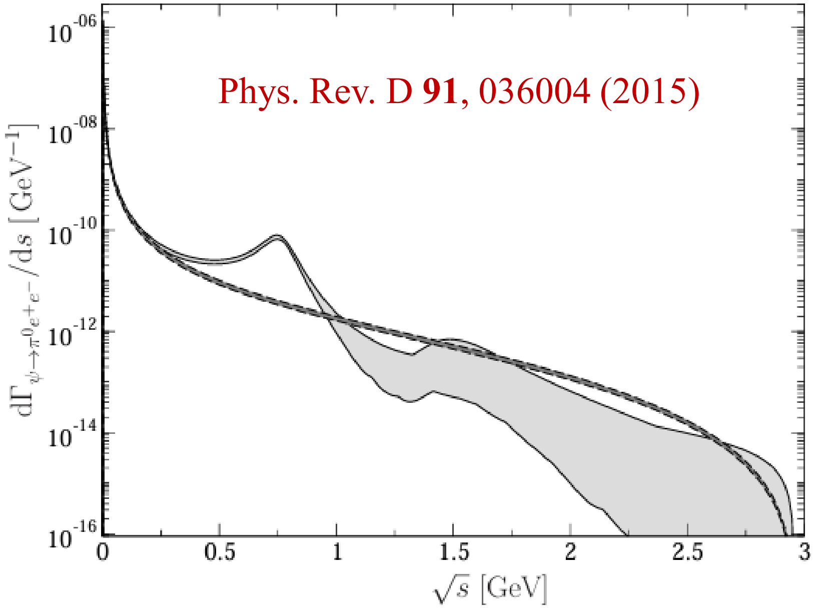
Precision of these measurements can be improved using 10 billion J/ψ events collected by the BESIII experiment

Theoretical status of $\psi \to \ell^+\ell^- \pi^0$

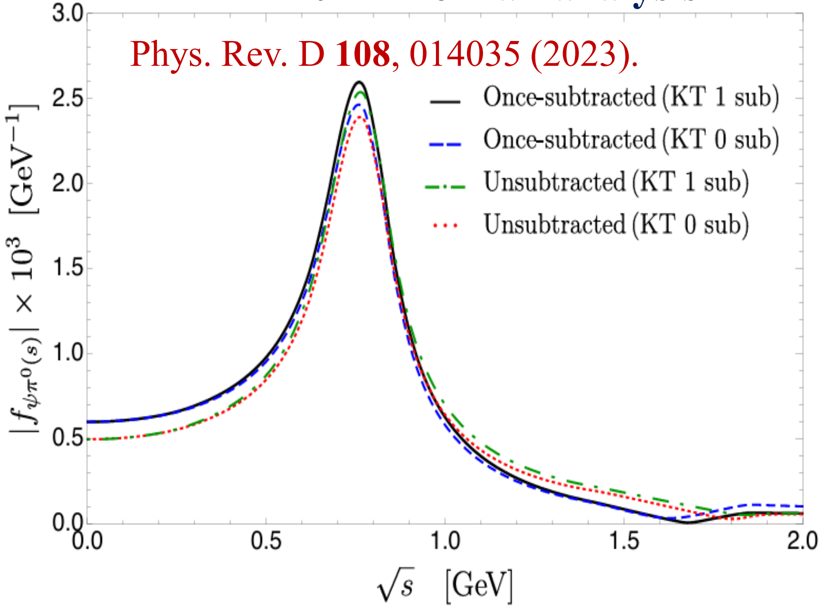


Only the isospin conserving decay $J/\psi \to \rho\pi$ was considered

Dispersive theory



Khuri–Treiman analysis



Effective Lagrangian-based analysis

Phys. Rev. D **107**, 034022 (2023) Phys. Rev. D **91**, 014010 (2015) Mod. Phys. Lett. A **27**, 1250223 (2012).

	Experiment	This Work	Ref. [20]	Ref. [54]	Ref. [55]
$\psi \to \pi^0 e^+ e^-$	0.076 ± 0.014	0.1294 ± 0.0044	0.1191 ± 0.0138	$0.0389^{+0.0037}_{-0.0033}$	...
$\psi \to \eta e^+ e^-$	1.42 ± 0.08	1.35 ± 0.02	1.16 ± 0.08	1.21 ± 0.04	1.38
$\psi \to \eta' e^+ e^-$	6.59 ± 0.18	6.08 ± 0.05	5.76 ± 0.16	5.66 ± 0.16	6.06
$\psi \to \pi^0 \mu^+ \mu^-$	...	0.0304 ± 0.0010	0.0280 ± 0.0032	$0.0101^{+0.0010}_{-0.0009}$	...
$\psi \to \eta \mu^+ \mu^-$	...	0.40 ± 0.01	0.32 ± 0.02	0.30 ± 0.01	0.46
$\psi \to \eta' \mu^+ \mu^-$	...	1.64 ± 0.02	1.46 ± 0.04	1.31 ± 0.04	1.72

Effective Lagrangian models also fall short. Large inconsistency in predictions underscores the need for a data-driven update.

Experimental status of $J/\psi \to e^+e^-\pi^0$

Previous BESIII measurement: $B(J/\psi \to e^+e^-\pi^0) = (7.56 \pm 1.32 \pm 0.50) \times 10^{-7}$ for $m_{e^+e^-} < 0.4$ GeV/c^2
(@ 225 million J/ψ events)

Phys. Rev. D **89**, 092008 (2014)

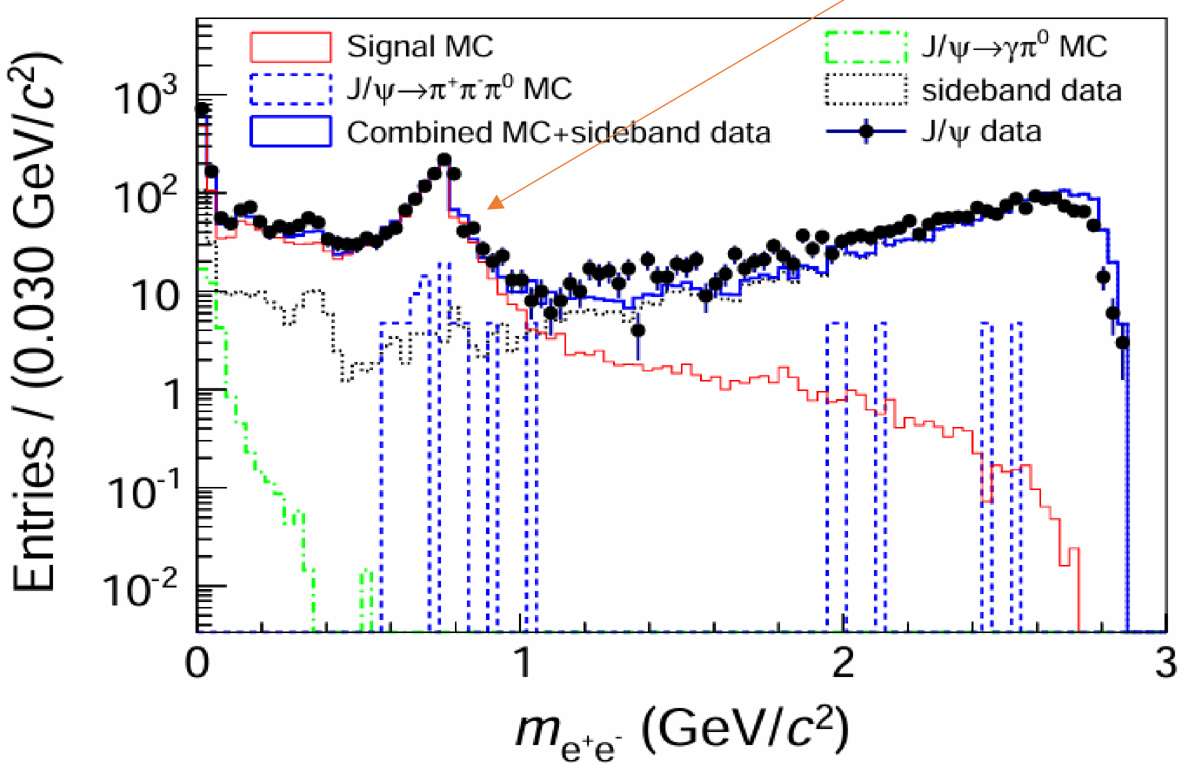
Non-resonant VMD prediction: $B(J/\psi \to e^+e^-\pi^0) = (3.89^{+0.37}_{-0.33}) \times 10^{-7}$

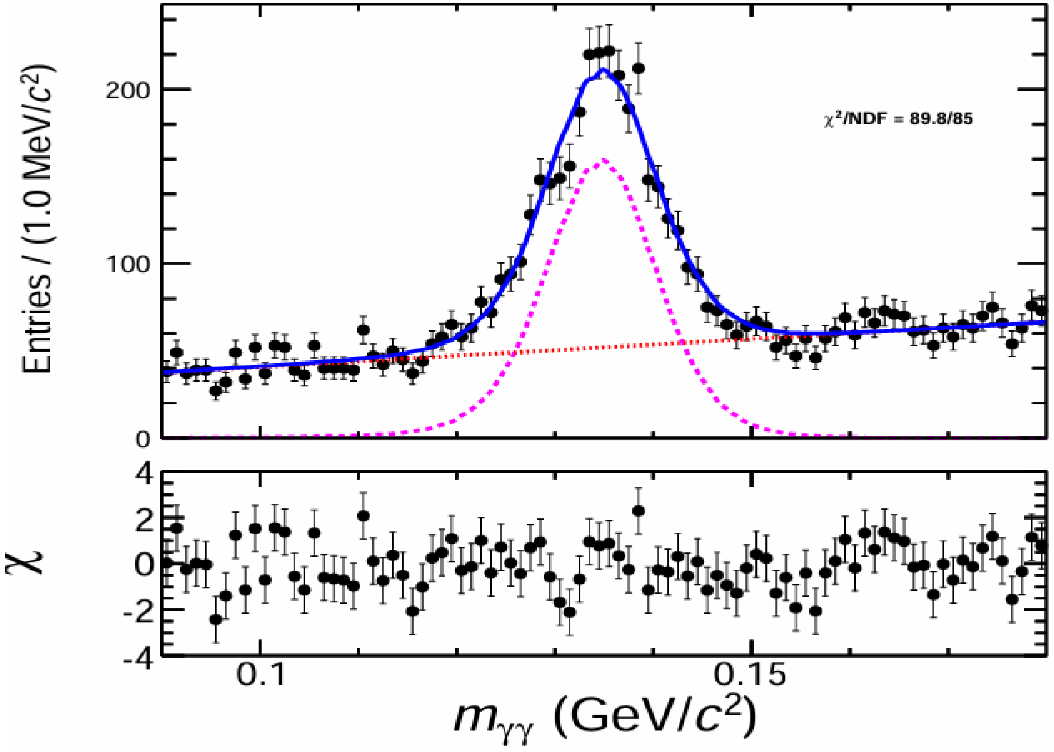
Mod. Phys. Lett. A **27**, 1250223 (2012).

New BESIII measurement (in full $m_{e^+e^-}$ spectrum)
(@ 10 billion J/ψ events)

BESIII Collaboration: arXiv:2501.04344 (2025)

A clear $\rho - \omega$ interference pattern

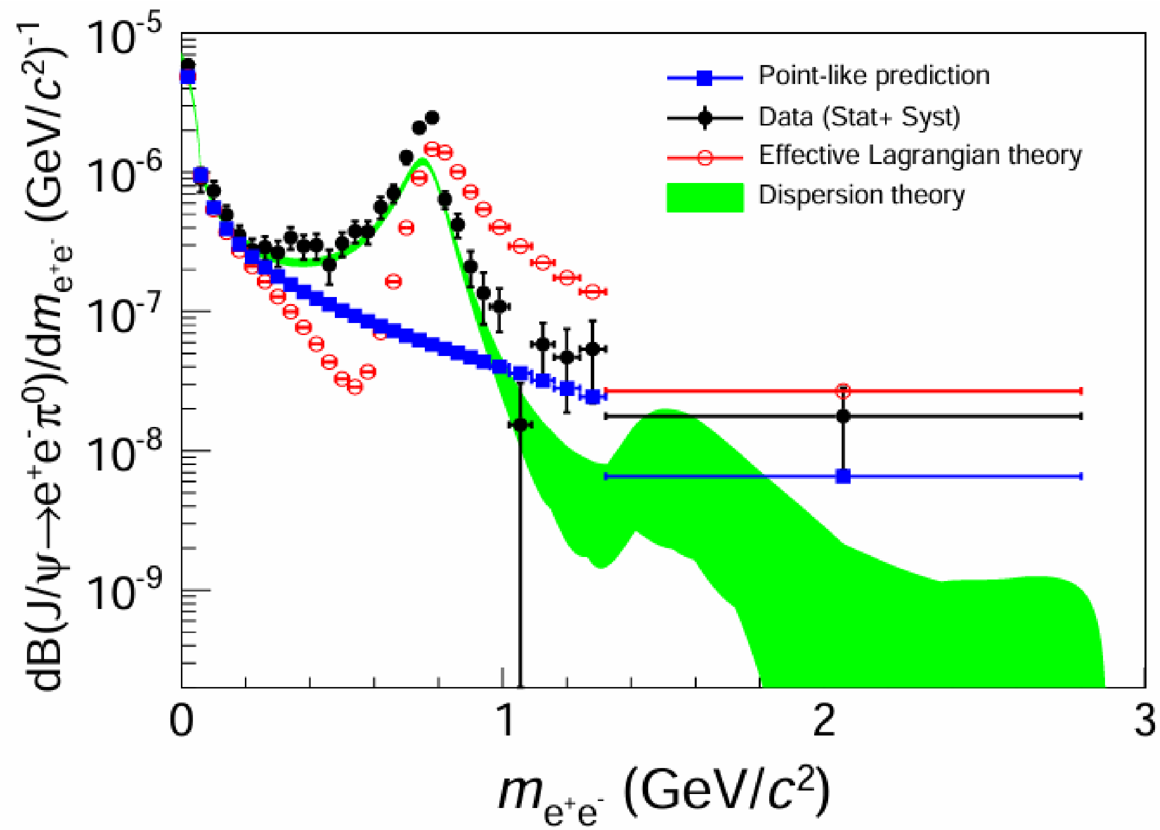


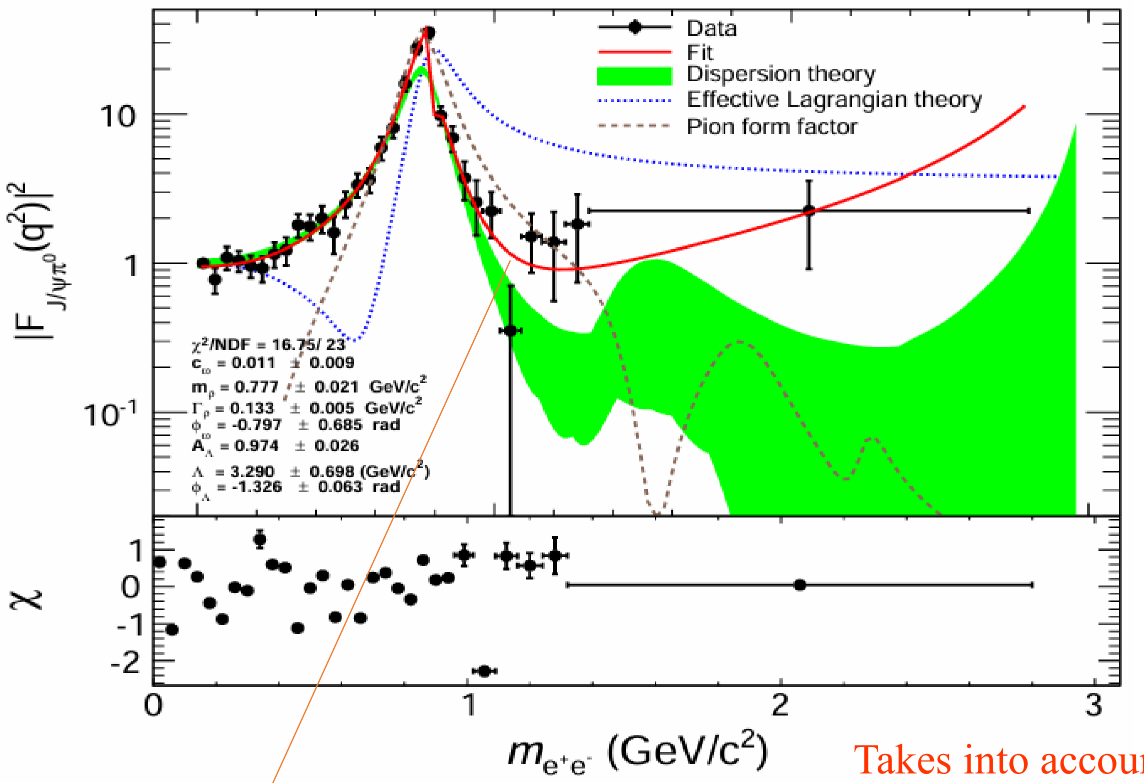


New BESIII measurement of $J/\psi \to e^+e^-\pi^0$

Results @ 10 billion J/ψ events

BESIII Collaboration: arXiv:2501.04344 (2025)





Takes into account of ρ-ω interference

New BESIII measurement (in full $m_{e^+e^-}$ spectrum)

$$B(J/\psi \to e^+e^-\pi^0) = (8.06 \pm 0.31 \pm 0.38) \times 10^{-7}$$

$$F_{J/\psi\pi^0}(q^2) = F_V^\pi(q^2) + A_\Lambda \frac{1.}{1 - q^2/\Lambda^2}$$

Large deviation from non-resonant VMD prediction: $B(J/\psi \to e^+e^-\pi^0) = (3.89^{+0.37}_{-0.33}) \times 10^{-7}$

TFF is an important input in the calculation of the hadronic light by-light contribution of the anomalous magnetic moment of the muon, $a_\mu = (g_\mu - 2)/2$ J. High Energy Phys. **09**, 074 (2015)

VMD formula for pion form factor

VMD formula for the pion form factor taken from BaBar's paper [Phys. Rev. D **86**, 032013 (2012)]

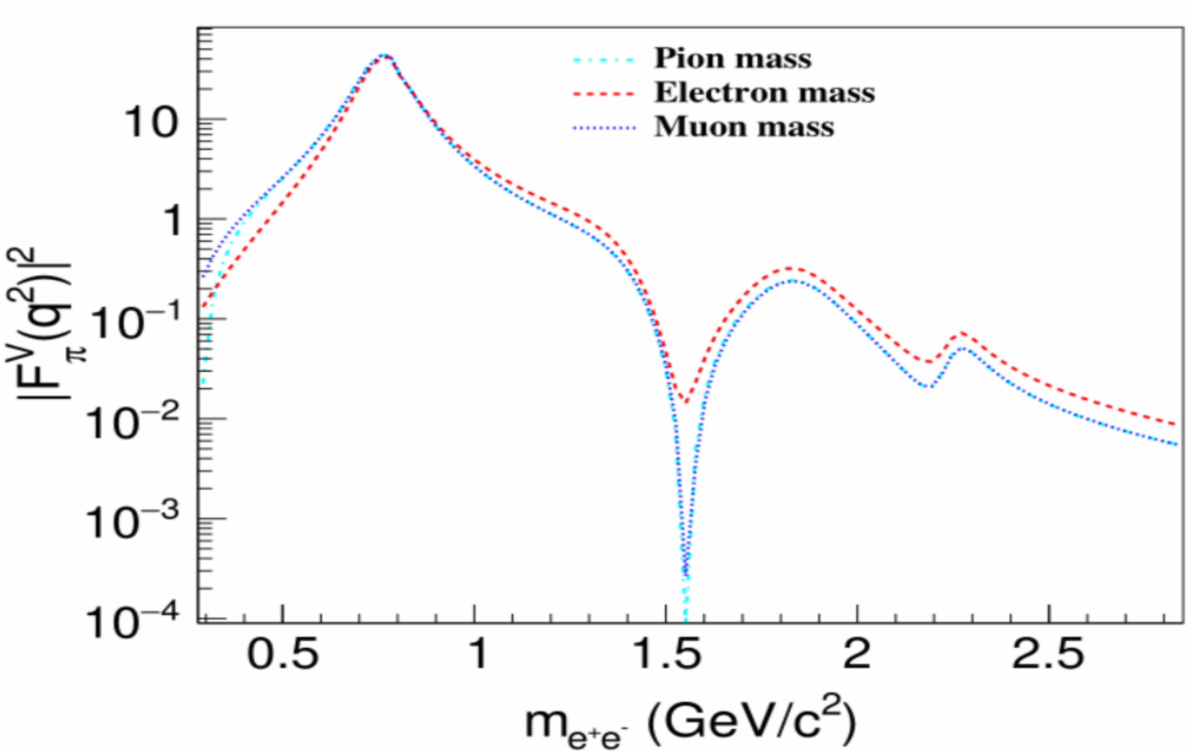
Gounaris-Sakurai (GS) model takes into account for the energy dependent ρ width

$$F_\pi^V(s) = \frac{BW_\rho^{GS}(s, m_\rho, \Gamma_\rho)\frac{1+c_\omega BW_\omega^{KS}(s,m_\omega,\Gamma_\omega)}{1+c_\omega} + c_{\rho'} BW_{\rho'}^{GS}(s, m_{\rho'}, \Gamma_{\rho'}) + c_{\rho''} BW_{\rho''}^{GS}(s, m_{\rho''}, \Gamma_{\rho'''}) + c_{\rho'''} BW_{\rho'''}^{GS}(s, m_{\rho'''}, \Gamma_{\rho'''})}{1 + c_{\rho'} + c_{\rho''} + c_{\rho'''}}$$

Higher-order ρ resonances

$\rho - \omega$ interference term

Phys. Rev. Lett. **21**, 244 (1968).

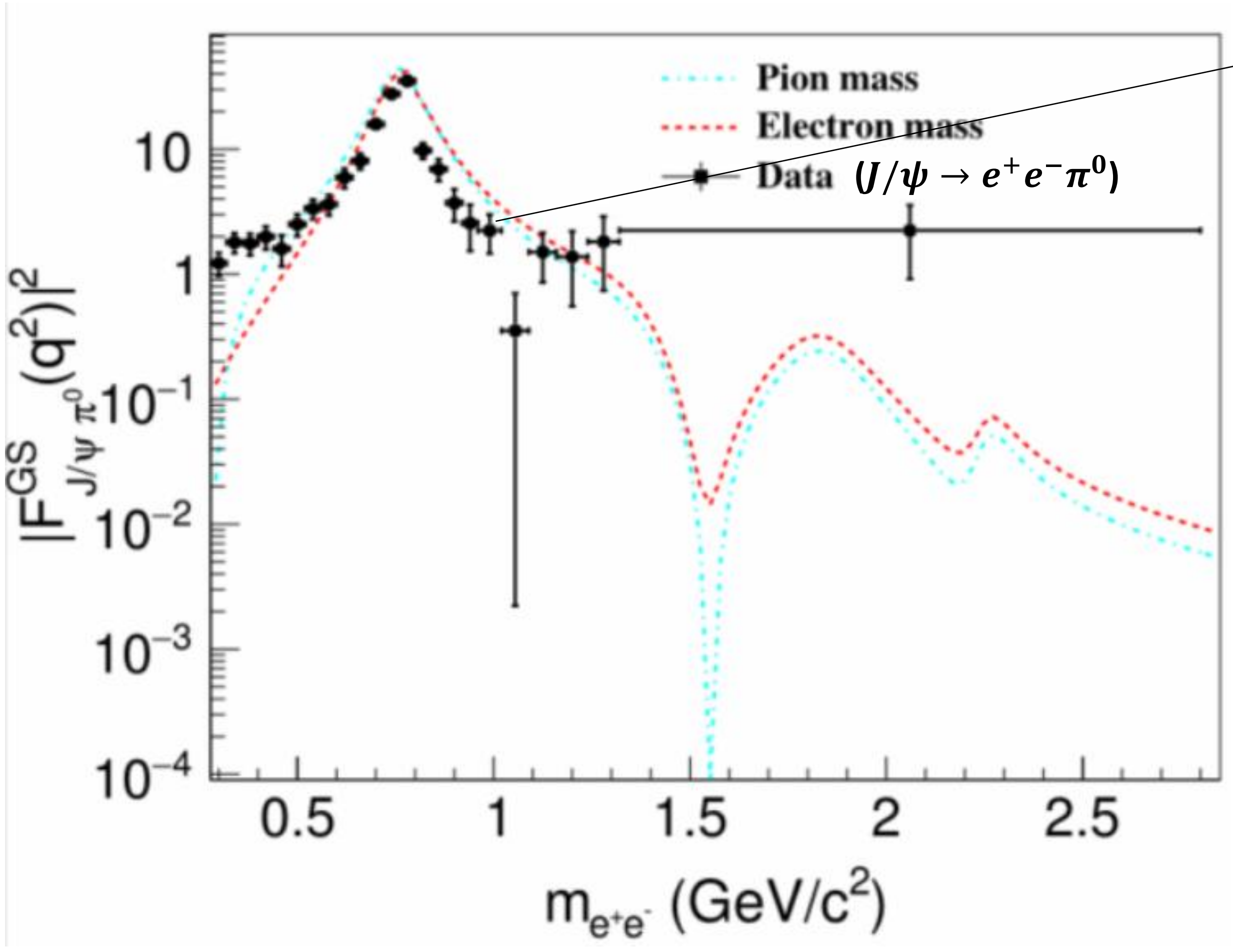


Measured parameters by BaBar

Parameter	Value ± Error
m_ρ (MeV/c^2)	775.02 ± 0.31 (± 0.16)
Γ_ρ (MeV)	149.59 ± 0.67 (± 0.02)
m_ω (MeV/c^2)	781.91 ± 0.18 (± 0.16)
Γ_ω (MeV)	8.13 ± 0.36 (± 0.27)
$\lvert c_\omega \rvert$	$(1.644 \pm 0.061) \times 10^{-3}$
ϕ_ω (rad)	−0.011 ± 0.037
$m_{\rho'}$ (MeV/c^2)	1493 ± 15
$\Gamma_{\rho'}$ (MeV)	427 ± 31
$\lvert c_{\rho'} \rvert$	0.158 ± 0.018
$\phi_{\rho'}$ (rad)	3.76 ± 0.10
$m_{\rho''}$ (MeV/c^2)	1861 ± 17
$\Gamma_{\rho''}$ (MeV)	316 ± 26
$\lvert c_{\rho''} \rvert$	0.068 ± 0.009
$\phi_{\rho''}$ (rad)	1.39 ± 0.20
$m_{\rho'''}$ (MeV/c^2)	2254 ± 22
$\Gamma_{\rho'''}$ (MeV)	109 ± 76
$\lvert c_{\rho'''} \rvert$	$0.0051^{+0.0034}_{-0.0019}$
$\phi_{\rho'''}$ (rad)	0.70 ± 0.51

[Phys. Rev. D **86**, 032013 (2012)]

TFF of $J/\psi \to e^+e^-\pi^0$ versus pion form factor

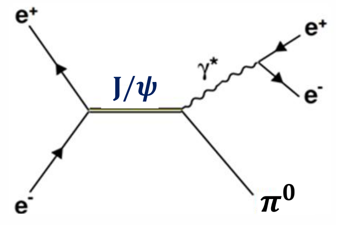


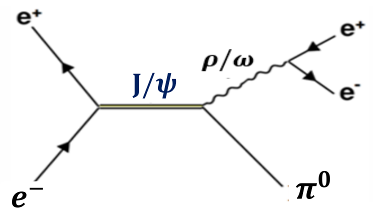
ρ width of the TFF of $J/\psi \to e^+e^-\pi^0$ data seems to be narrow than the pion form factor (measured by BaBar measurement).

BESIII data reveals both

- Non-resonant component
- Resonant contributions via intermediate vector mesons

  $J/\psi \to V(\to e^+e^-)\pi^0, \quad V = \rho, \omega$





- Interference arises from:
  - $J/\psi \to \rho\pi^0$ — isospin-conserving ($I = 1$)
  - $J/\psi \to \omega\pi^0$ — isospin-violating ($I = 0$)
- Analogous to:

$$\rho \to \pi^+\pi^- \quad (I = 1), \qquad \omega \to \pi^+\pi^- \quad (I = 0)$$

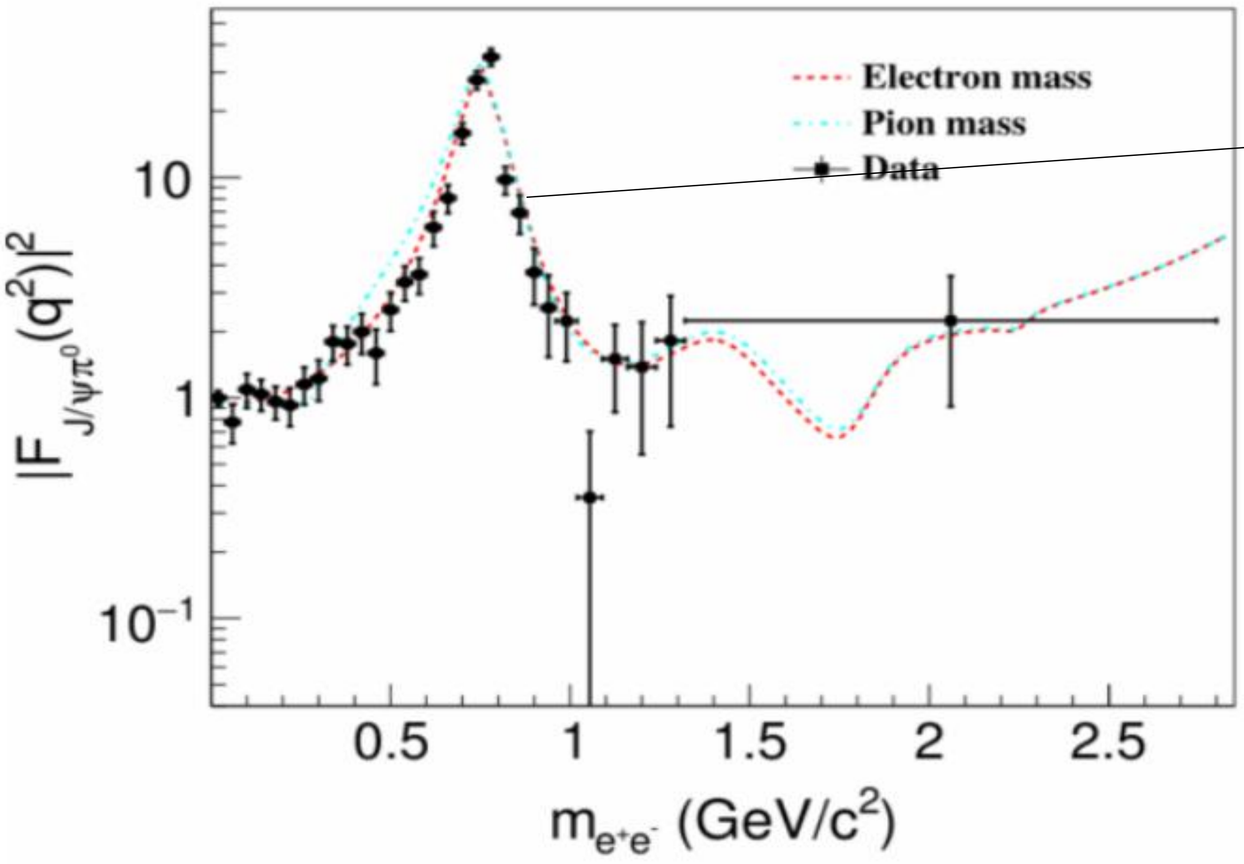
But, need to understand the reason of narrow ρ resonance

TFF of $J/\psi \to e^+e^-\pi^0$ versus pion form factor

Use this modified TFF formula to take into account of resonant and non-resonant processes of $J/\psi \to e^+e^-\pi^0$

$$F_{\psi\pi^0}(q^2) = N\sum_{V'} \frac{m_{V'}^2}{m_{V'}^2 + q^2 - i\Gamma_{V'}m_{V'}} \approx F_\pi^V(q^2) + A_\Lambda \frac{1}{1 - q^2/\Lambda^2}$$

(values of A_Λ and Λ are taken from the BESIII measurement and rest of the other parameters are taken from BaBar measurement)



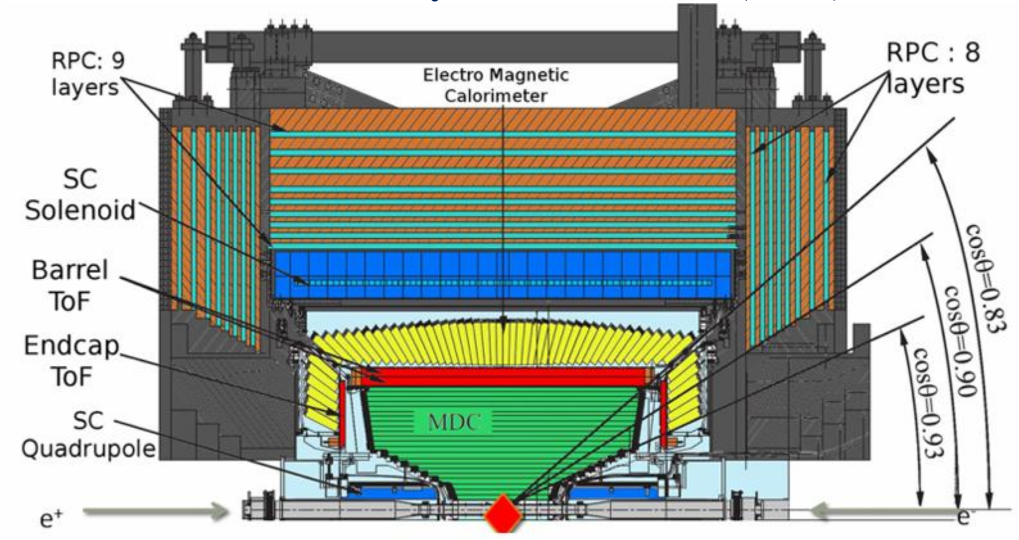
Perfect agreement between BaBar and BESIII measurements in the ρ/ω mass region.

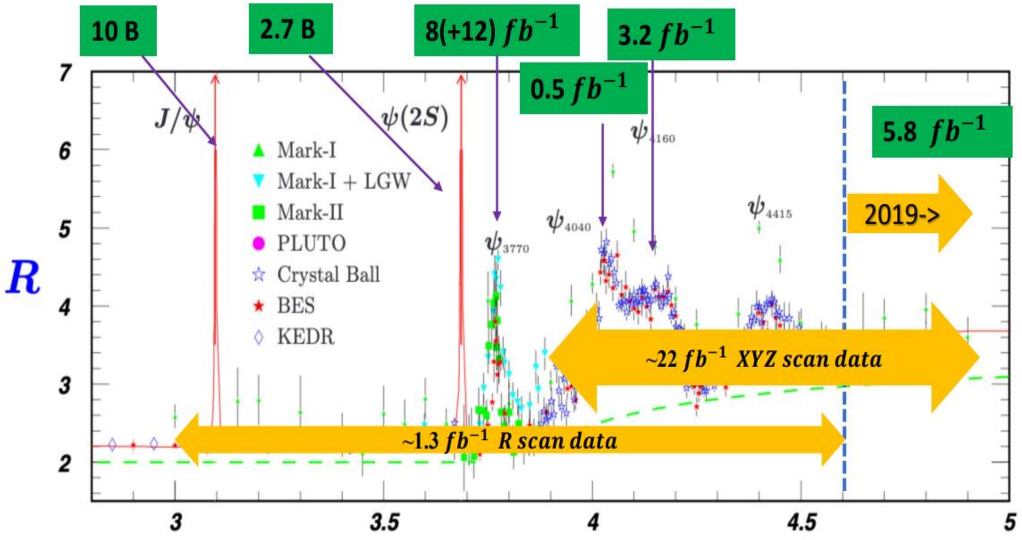
Thus, BESIII measurement about TFF of $J/\psi \to e^+e^-\pi^0$ can provide an alternative way to extract the hadronic vacuum polarization (HVP) contribution to the muon g-2.

STCF can play an important role to develop the ρ-ω interference in the leptonic-sector via EM Dalitz decays of $\psi \to e^+e^-P$ and to extract the HVP contribution of muon g-2

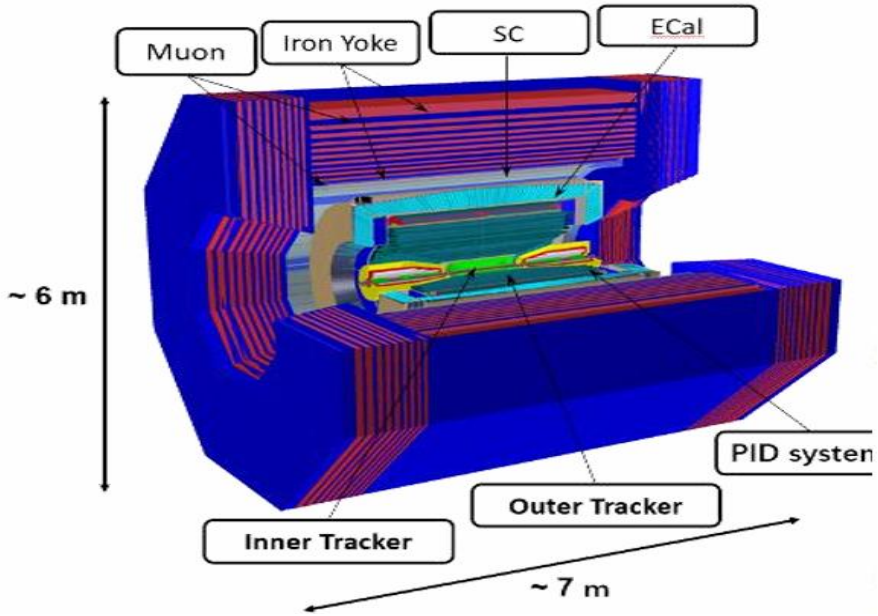
BESIII vs. Super Tau-Charm Facility (STCF) experiments

Chin. Phys. C 44, 040001 (2020)





https://stcf.ustc.edu.cn



CME (GeV)	Lumi (ab^{-1})	Samples	σ(nb)	No. of Events	Remarks
3.097	1	J/ψ	3400	3.4×10^{12}	
3.670	1	$\tau^+\tau^-$	2.4	2.4×10^{9}	
3.686	1	$\psi(3686)$	640	6.4×10^{11}	
		$\tau^+\tau^-$	2.5	2.5×10^{9}	
		$\psi(3686) \to \tau^+\tau^-$		2.0×10^{9}	
3.770	1	$D^0\bar{D}^0$	3.6	3.6×10^{9}	
		$D^+\bar{D}^-$	2.8	2.8×10^{9}	
		$D^0\bar{D}^0$		7.9×10^{8}	Single tag
		$D^+\bar{D}^-$		5.5×10^{8}	Single tag
		$\tau^+\tau^-$	2.9	2.9×10^{9}	

Multipole Transition form factor of $\psi \to \ell^+\ell^-\pi^0$

- ✓ VMD Prediction: $F_{AB}(q^2) = N\sum_{V'} \frac{m_{V'}^2}{m_{V'}^2 - q^2 - i\Gamma_{V'} m_{V'}}$ [Phys. Rept. 128, 301 (1985)]

  Here, V′=ρ, ω, ϕ resonances

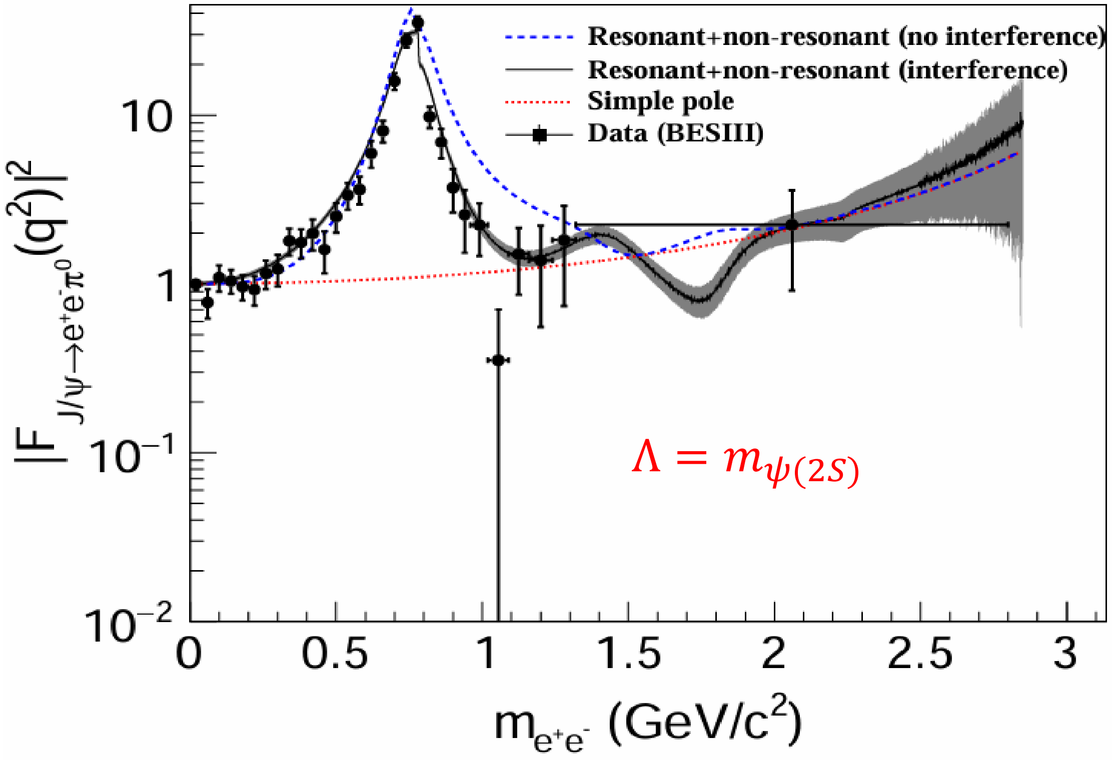
- ✓ TFF function (including ρ-ω interference): $F_{J/\psi\pi^0}(q^2) = F^{GS}_{J/\psi\pi^0}(q^2) + A_\Lambda \frac{1.}{1 - q^2/\Lambda^2}$

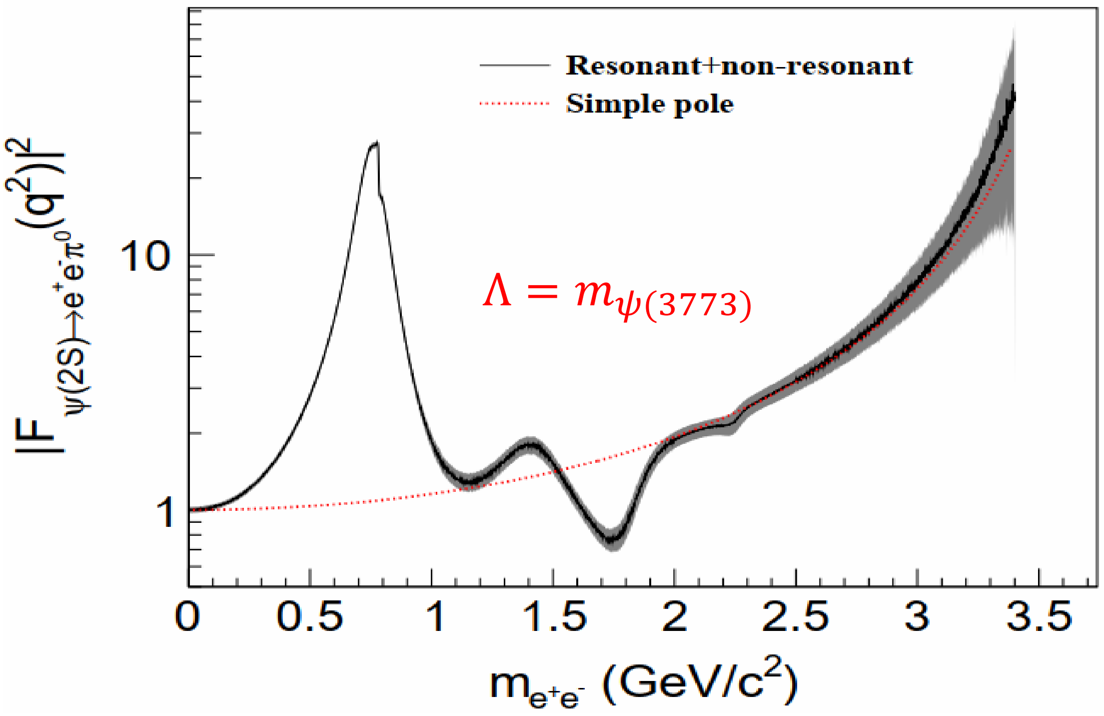
Gounaris-Sakurai function including higher-order ρ resonances)

Pole mass

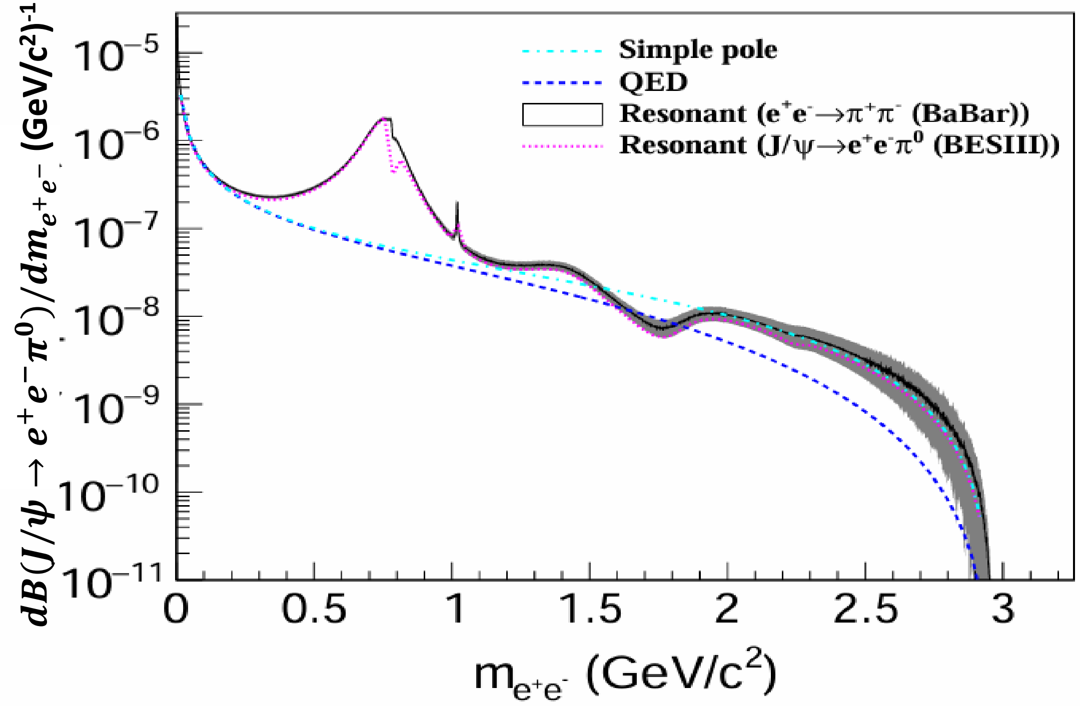
BaBar Collaboration, Phys. Rev. D **86**, 032013 (2012)

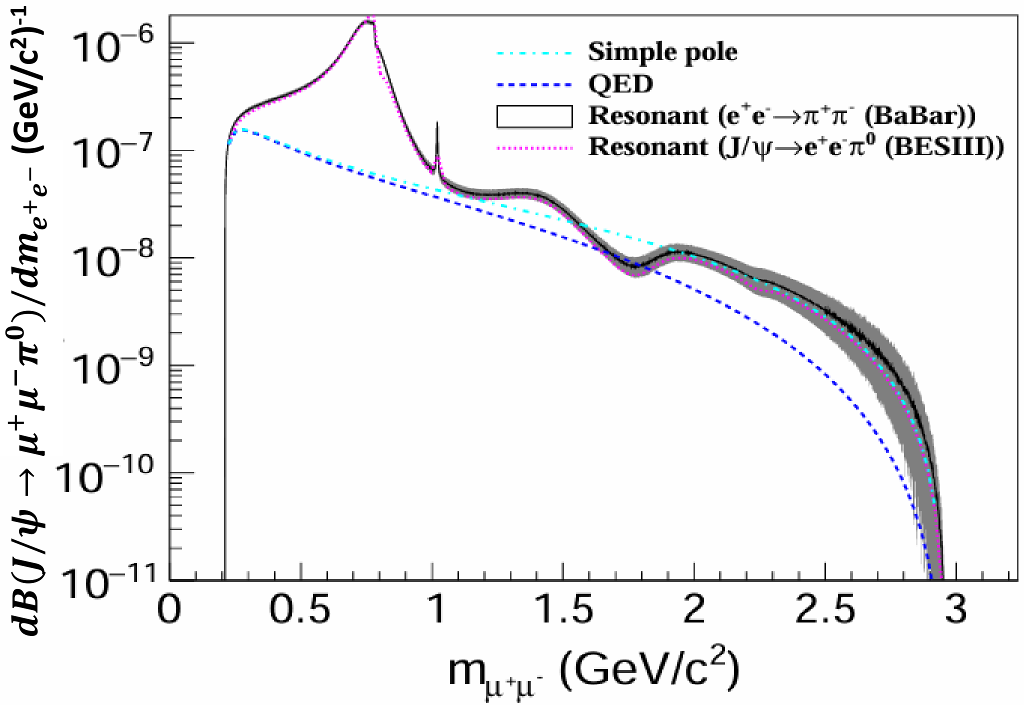
BESIII Collaboration: arXiv:2501.04344 (2025)





Expected branching fraction $J/\psi \to \ell^+\ell^-\pi^0$





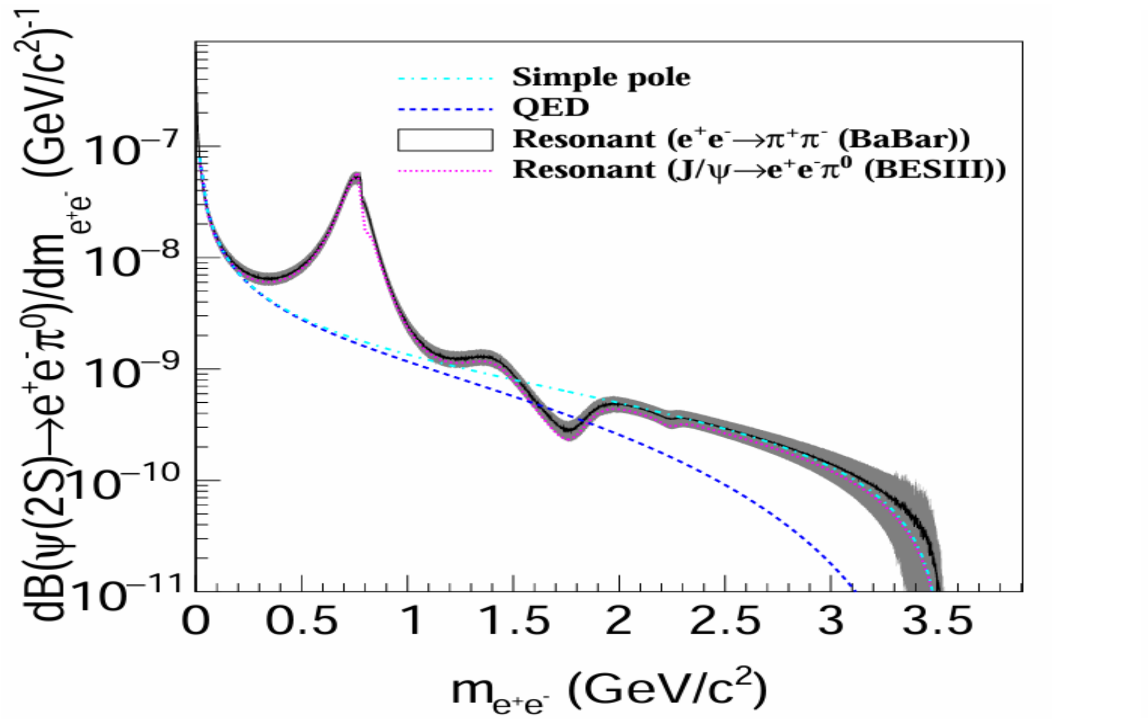
N_{sig} prediction is based on the parameters of BaBar's measurement

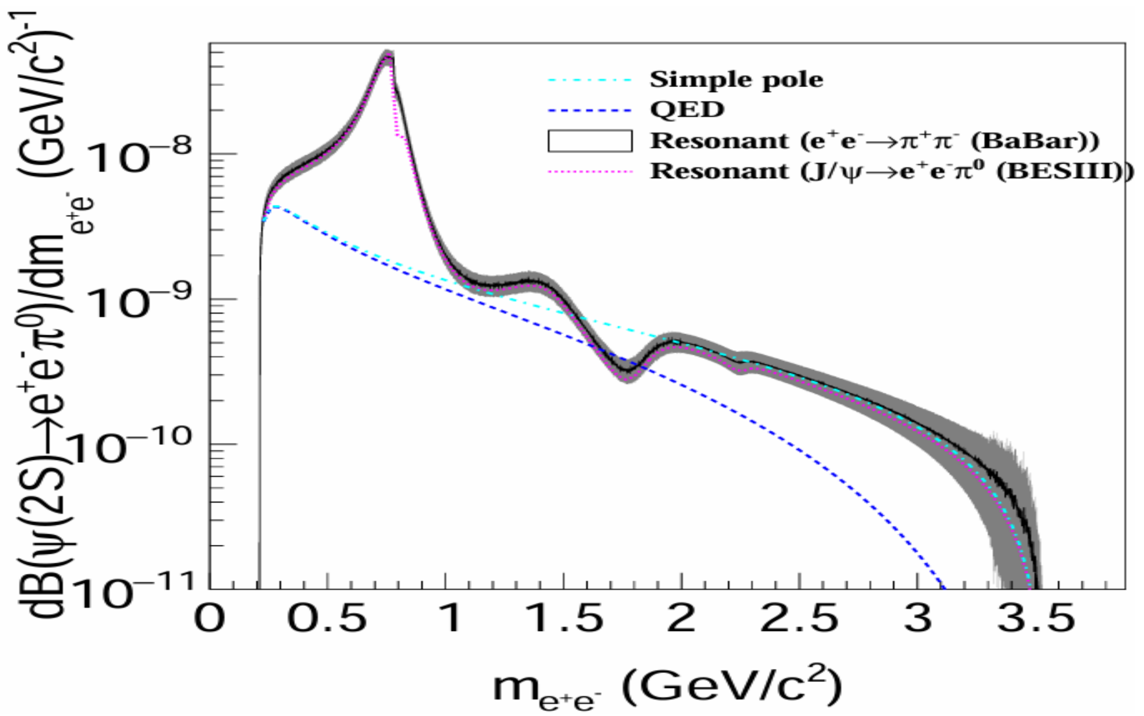
BaBar Collaboration, Phys. Rev. D **86**, 032013 (2012)

Decay mode	Branching fraction (10^{-8})					Efficiency (%)	Expected N_{sig} (events)	
	QED	Single pole	Resonant (BaBar)	Resonant BESIII	Experiment		BESIII	STCF (10^4)
$J/\psi \to e^+e^-\pi^0$	36.52±0.86	37.83±3.07	69.20±5.62	66.44±5.40	80.6±3.1±3.8	26.3	1814.5±147.5	61.16±5.00
$J/\psi \to \mu^+\mu^-\pi^0$	8.58±0.20	9.86±1.10	42.07±3.4	39.77±3.23		26.3	1102.8±89.6	37.17±3.02

BESIII Collaboration: arXiv:2501.04344 (2025)

Expected branching fraction $\psi(2S) \to \ell^+\ell^-\pi^0$





N_{sig} prediction is based on the parameters of BaBar's measurement

BaBar Collaboration, Phys. Rev. D **86**, 032013 (2012)

Decay mode	Branching fraction (10^{-9})					Efficiency (%)	Expected N_{sig} (events)	
	QED	Single pole	Resonant (BaBar)	Resonant BESIII	Experiment		BESIII	STCF
$\psi(2S) \to e^+e^-\pi^0$	11.50±2.43	12.09±2.56	20.30±3.60	19.50±3.50		26.3	14.2±2.5	3374±604
$\psi(2S) \to \mu^+\mu^-\pi^0$	2.91±0.62	3.50±0.74	12.60±2.30	12.20±2.30		26.3	8.9±1.6	2204±377

BESIII Collaboration: arXiv:2501.04344 (2025)

Large uncertainty occurs because of $B(\psi(3686) \to \gamma\pi^0) = (1.04 \pm 0.22) \times 10^{-6}$

Summary and future plan

- The study of electromagnetic Dalitz decays of charmonium mesons is important for understanding the internal structure of the involved hadrons..

- A significant discrepancy is observed between experimental measurement of $J/\psi \to \ell^+\ell^-\pi^0$ and the predction based on a simple pole approximation within the VMD model.

- This discrepancy can't be explained without including contributions from possible intermediate light unflavored vector meson resonances and the interference between ρ and ω.

- Try to update the VMD predictions for $\psi \to \ell^+\ell^-\pi^0$ by incorporating ρ-ω interference using a data-driven method.

- Numerical study indicates that $J/\psi \to \mu^+\mu^-\pi^0$ can be observed at BESIII, and $\psi(2S) \to \ell^+\ell^-\pi^0$ decays can be observed at STCF.

- STCF can play an important role in making a precise measurement of the ρ-ω interference pattern in the leptonic sector.

- Next to do:

  Try to extract the HVP contribution to the muon (g-2) from the TFF of $J/\psi \to e^+e^-\pi^0$

Thanks!