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Muon g-2 and Tau physics

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Current status on muon g-2

[2505.21476, White Paper 25]

Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E989, E821)		Eq. (9.5)	116 592 071.5(14.5)	Refs. [5–8, 10–13]
HVP LO (lattice)	Sec. 3.6.1	Eq. (3.37)	7132(61)	Refs. [14–30]
HVP LO (e^+e^-, τ)	Sec. 2	Table 5	Estimates not provided at this point	
HVP NLO (e^+e^-)	Sec. 2.9	Eq. (2.47)	−99.6(1.3)	Refs. [31, 32]
HVP NNLO (e^+e^-)	Sec. 2.9	Eq. (2.48)	12.4(1)	Ref. [33]
HLbL (phenomenology)	Sec. 5.10	Eq. (5.69)	103.3(8.8)	Refs. [34–57]
HLbL NLO (phenomenology)	Sec. 5.10	Eq. (5.70)	2.6(6)	Ref. [58]
HLbL (lattice)	Sec. 6.2.8	Eq. (6.34)	122.5(9.0)	Refs. [59–63]
HLbL (phenomenology + lattice)	Sec. 9	Eq. (9.2)	112.6(9.6)	Refs. [34–57, 59–63]
QED	Sec. 7.5	Eq. (7.27)	116 584 718.8(2)	Refs. [64–70]
EW	Sec. 8	Eq. (8.12)	154.4(4)	Refs. [51, 71–73]
HVP LO (lattice) + HVP N(N)LO (e^+e^-)	Sec. 9	Eq. (9.1)	7045(61)	Refs. [14–33]
HLbL (phenomenology + lattice + NLO)	Sec. 9	Eq. (9.3)	115.5(9.9)	Refs. [34–63]
Total SM Value	Sec. 9	Eq. (9.4)	116 592 033(62)	Refs. [14–73]
Difference: $\Delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM}$	Sec. 9	Eq. (9.6)	38(63)	

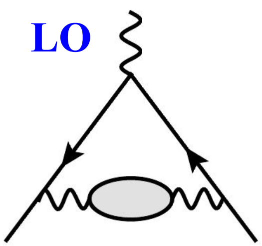
Hadron

Error budget of $a_\mu^{\rm SM}$: 61(HVP-Lat), 10(HLbL), 0.4(EW), 0.2(QED)

HVP:

(dominated by e^+e^--> hadrons below 1 GeV)

LO

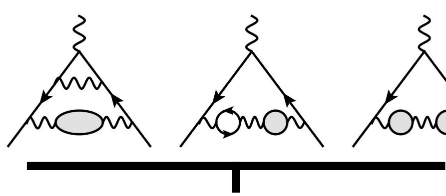


WP20: ~7000(~40)

$\pi\pi$: (>70%) (>60%)

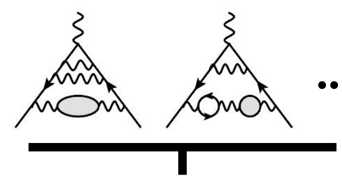
$\pi\pi\pi$, KK, ...

NLO



−99.6(1.3)

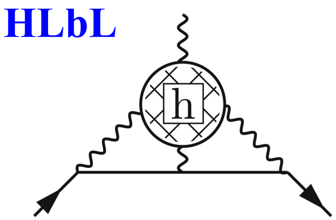
NNLO



...

12.4(1)

HLbL



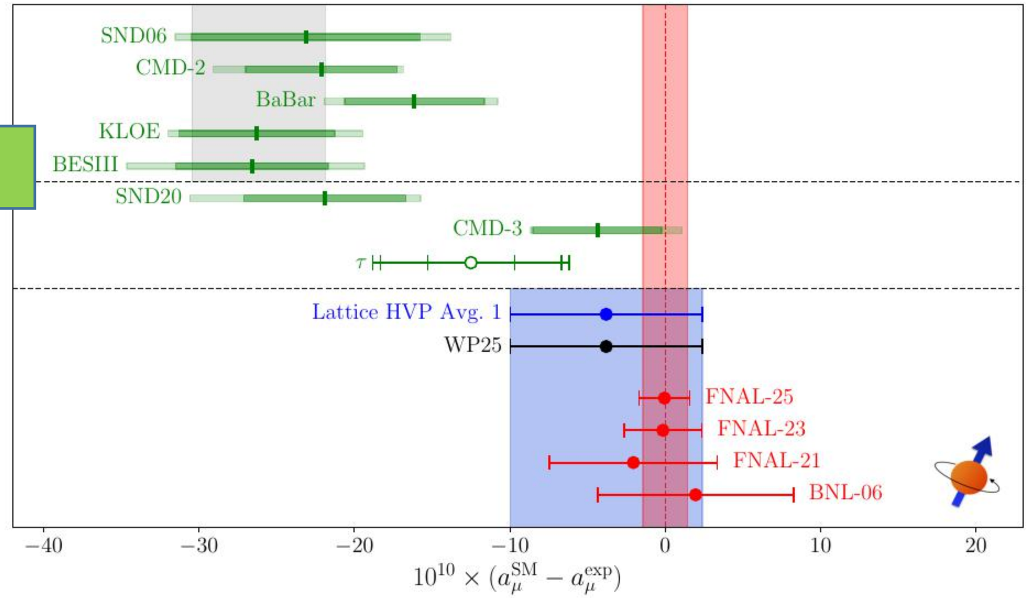
115.5(9.9)

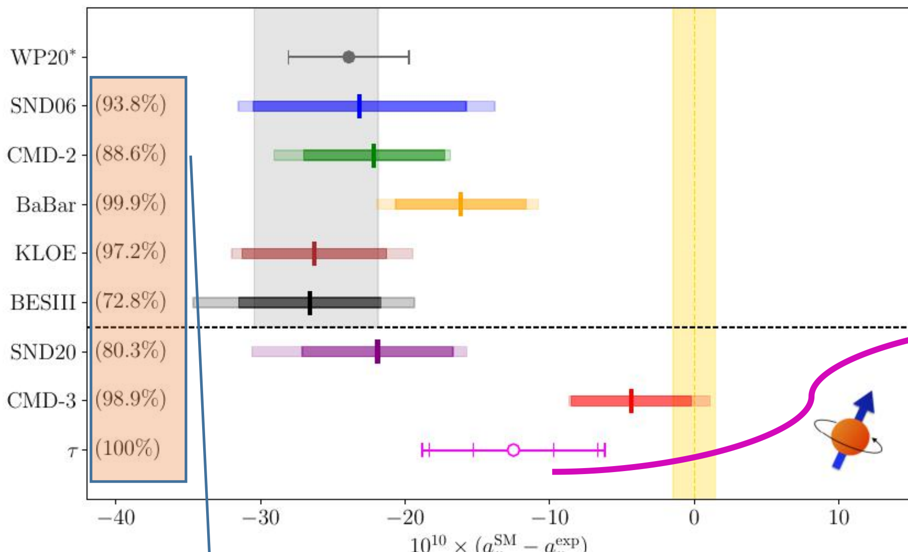
Greatly improved since WP20

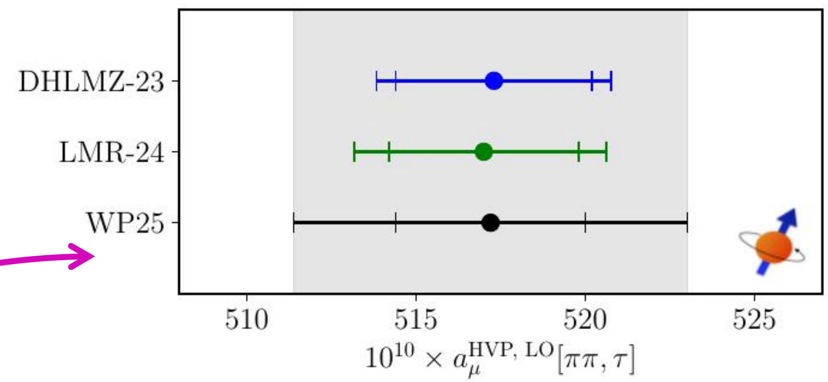
Current status on muon g-2

[2505.21476, White Paper 25]

- $e^+e^- \to \pi^+\pi^-$ from each indicated Exp
- HVP-LO beyond $\pi\pi$ from WP20
- Others, such as HLbL, EW, QED, from WP25

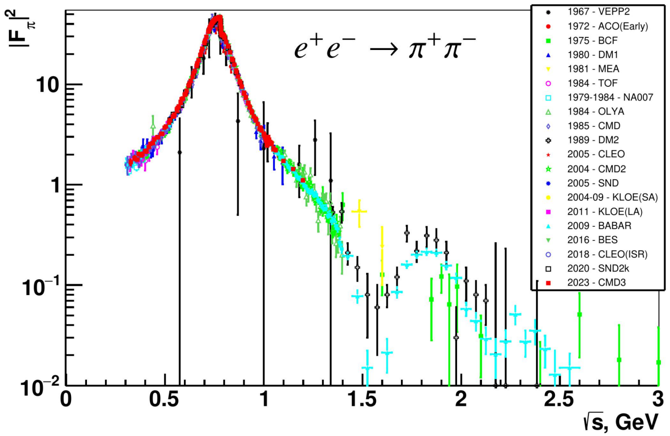




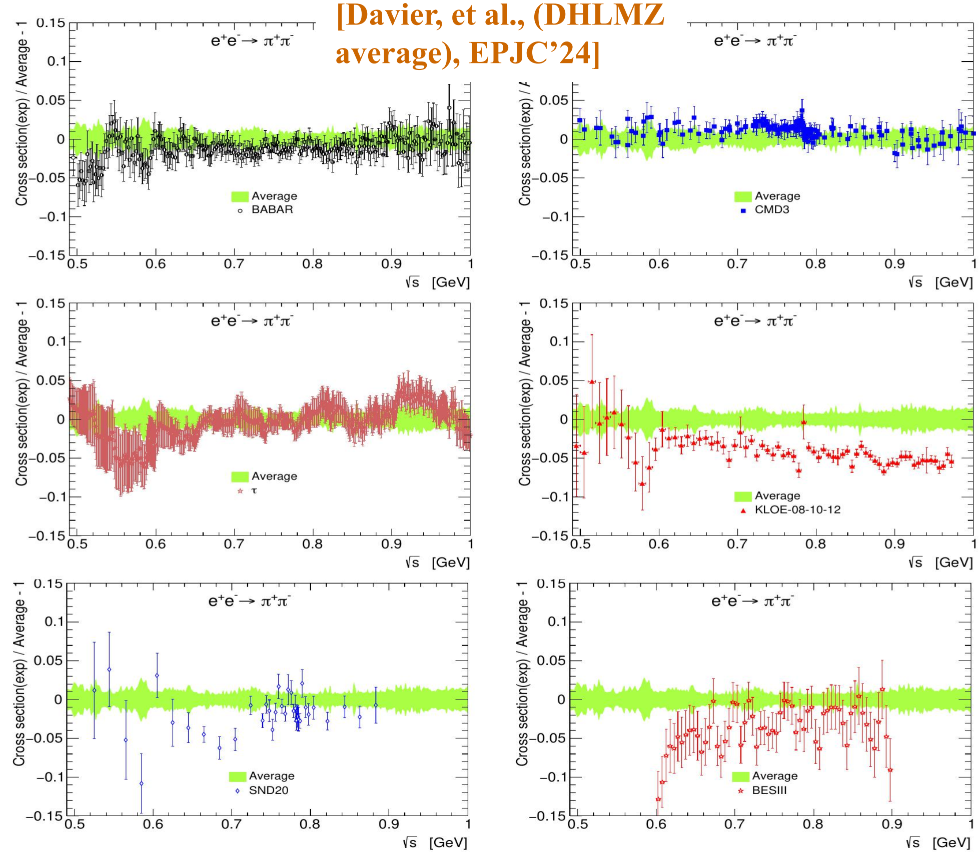


$\pi\pi$ contribution from each Exp to HVP integral

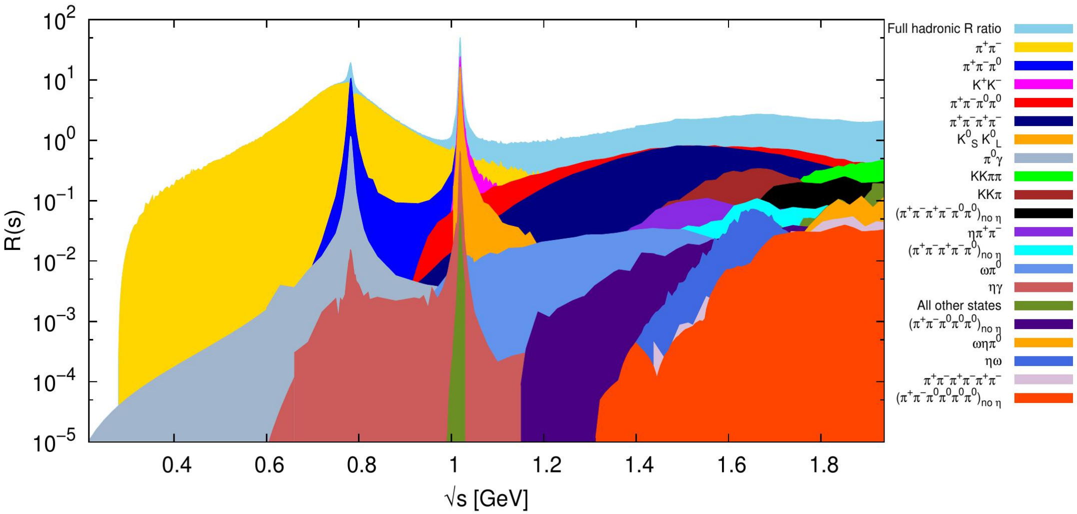
Two common methods to combine various data for $e^+e^- \to$ hadrons

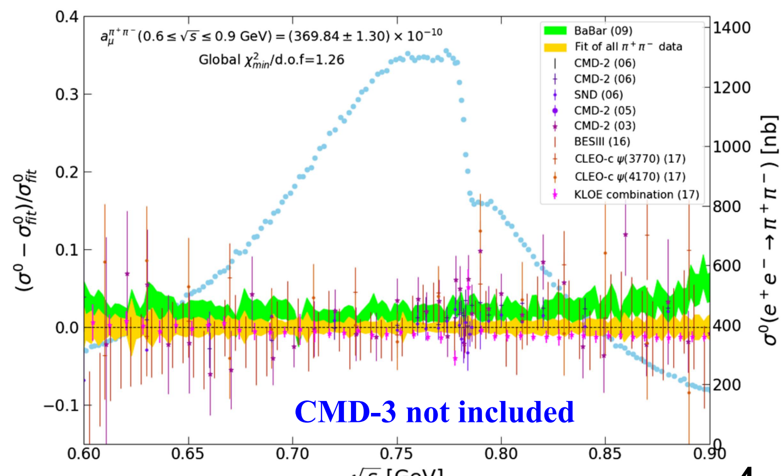


[Davier, et al., (DHLMZ average), EPJC'24]



[Keshavarzi, et al., (KNTW average), PRD'20]





CMD-3 not included

Alternative way to address HVP from $\pi\pi$

$$a_\mu^{\text{HVP,LO}} = \frac{1}{4\pi^3}\int_{4M_2^2}^{t_{max}} dt K(t)\sigma^0_{e^+e^-\to\text{hadrons}}(t)$$

Known kernel function (enhanced contribution from energy below 1GeV)

$$\sigma^0_{e^+e^-\to\pi^+\pi^-} = \frac{\pi\alpha^2}{3t}\beta_{\pi^+\pi^-}\left|F_{\pi\pi}^{(0)}(t)\right|^2$$

$$\frac{d\Gamma(\tau_{2\pi})}{dt} = \frac{G_F^2\left|V_{ud}\right|^2 m_\tau^3 S_{\text{EW}}}{384\pi^3}\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+\frac{2t}{m_\tau^2}\right)\beta_{\pi^-\pi^0}\left|F_{\pi\pi}^{(-)}(t)\right|^2$$

$\tau^- \to \pi^-\pi^0\nu_\tau$: $\langle\pi^-\pi^0|\bar{d}\gamma_\mu u|0\rangle \sim F_{\pi\pi}^{(-)}(t)$ [I=1, I_3=-1]

$e^+e^- \to \pi^+\pi^-$: $\langle\pi^+\pi^-|\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d|0\rangle \sim F_{\pi\pi}^{(0)}(t)$ [I=1, I_3=0]

Isospin limit

Conserved vector current (CVC)

$$F_{\pi\pi}^{(0)}(t) = F_{\pi\pi}^{(-)}(t)$$

$$\sigma^0_{e^+e^-\to\pi^+\pi^-} = \frac{K_\sigma(t)}{K_\Gamma(t)}\frac{\beta_{\pi^+\pi^-}}{S_{\text{EW}}\beta_{\pi^-\pi^0}}\frac{d\Gamma(\tau_{2\pi})}{dt}$$

$$K_\sigma(t) = \frac{\pi\alpha^2}{3t}\,,\; K_\Gamma(t) = \frac{G_F^2\left|V_{ud}\right|^2 m_\tau^3}{384\pi^3}\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+2\frac{t}{m_\tau^2}\right)$$

- **Isospin breaking (IB) effects become CRUCIAL at the sub-percent level.**
- **Full control of all the IB terms is yet to be reached.**

Results on the estimation of a_μ based on the tau data in WP25 are based on:

[Davier, et al., (DHLMZ), EPJC’24] [Lopez Castro, et al., (LMR) PRD’25]

Isospin breaking corrections to a_μ

$$\sigma^0_{e^+e^-\to\pi^+\pi^-} = \left[\frac{K_\sigma(t)}{K_\Gamma(t)}\frac{d\Gamma(\tau_{2\pi[\gamma]})}{dt}\right] \times \left(\frac{R_{\rm IB}(t)}{S_{\rm EW}}\right)$$

Exp tau data (photon inclusive)

$$R_{\rm IB}(t) = \frac{{\rm FSR}(t)}{G_{\rm EM}(t)}\frac{\beta^3_{\pi^+\pi^-}}{\beta^3_{\pi^-\pi^0}}\frac{\left|F^{(0)}_{\pi\pi}(t)\right|^2}{\left|F^{(-)}_{\pi\pi}(t)\right|^2}$$

$$\Delta a_\mu^{\rm HVP,LO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{t_{max}} dt K(t)\left[\frac{K_\sigma(t)}{K_\Gamma(t)}\frac{d\Gamma(\tau_{2\pi[\gamma]})}{dt}\right] \times \left(\frac{R_{\rm IB}(t)}{S_{\rm EW}} - 1\right)$$

- **Final-state radiation (FSR) corrections to $\pi^+\pi^-$**
- **$\beta_{\pi\pi}=2q_{\rm CM}(t)/\sqrt{t}$: kinematical factor caused by the π^{+-} - π^0 mass difference [important near thresh.]**
- **Raito of form factors: $F^{(0)}_{\pi\pi}(t)/F^{(0)}_{\pi\pi}(t)$ [carrying the largest uncertainty]**

  $F^{(0)}_{\pi\pi}(t)$ [$e^+e^- \to \pi^+\pi^-$]: **$M_{\rho 0}$, $\Gamma_{\rho 0}$, ρ^0-ω mixing**

  $F^{(-)}_{\pi\pi}(t)$ [$\tau^- \to \nu_\tau\pi^0\pi^-$]: **$M_{\rho -}$, $\Gamma_{\rho -}$**

Not only depend on $\Delta M_\rho = M_{\rho -} - M_{\rho 0}$, $\Delta\Gamma_\rho = \Gamma_{\rho -} - \Gamma_{\rho 0}$, ρ^0-ω mixing , but also on the FF parameterization.

[2505.21476, WP25] $\Delta a_\mu^{\rm HVP,\,LO}[\pi\pi,\tau]$ (in units of 10^{-10})

		(DHLMZ)	(LMR)
$\frac{F^V_\pi}{f_+}$ (w/o ρ–ω)	ΔM_ρ	$0.20(^{+27}_{-19})(9)$	$1.95^{+1.56}_{-1.55}$
	$\Delta\Gamma_\rho(\Delta M_\pi)$	$4.09(0)(7)$	3.37
	$\Delta\Gamma_\rho(\pi\pi\gamma)$	$-5.91(59)(48)$	$-6.66(73)$
	$\Delta\Gamma_\rho(g_{\rho\pi\pi})$	–	–
	Total	$-1.62(65)(63)$	$(-1.34)^{+1.72}_{-1.71}$

	(DHLMZ)	(LMR)
ρ–ω mixing	4.0(4)	2.87(8)

- **$G_{\rm EM}(t)$: long-distance radiative corrections to $\tau^- \to \nu_\tau\pi^0\pi^-$**

➢ $G_{EM}(t)$: long-distance EM corrections to $\tau^- \to \nu_\tau \pi^0 \pi^-$

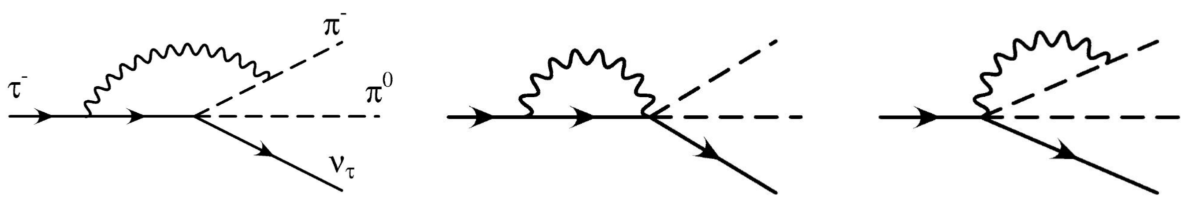
$$\frac{d\Gamma(\tau_{2\pi[\gamma]})}{dt} = \frac{G_F^2 \left|V_{ud}\right|^2 m_\tau^3 S_{\rm EW}}{384\pi^3} \left(1 - \frac{4m_\pi^2}{t}\right) \left(1 - \frac{t}{m_\tau^2}\right)^2 \left(1 + \frac{2t}{m_\tau^2}\right) \left|F_{\pi\pi}^{(-)}(t)\right|^2 G_{\rm EM}(t)$$

$d\Gamma_{\tau \to \pi\pi\nu} / dt$

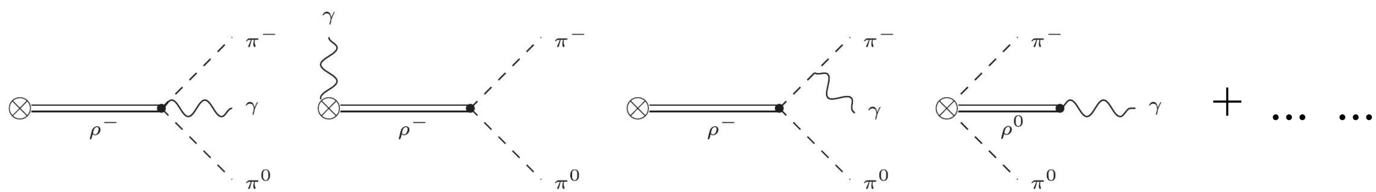
$G_{EM}(t) \sim$ virtual photon + real photon

virtual photon (loops)

[Cirigliano, et al.,PLB'01]



real photon (radiative decay, hadronic modeling needed)



➢ **G_{EM} is infrared finite: cancellation between photon loop and bremsstrahlung of the real photon.**

➢ **Experimental measurement of $\tau \to \pi\pi\gamma\nu_\tau$ is absent: theoretical estimation needed.**

. **[Cirigliano et al, JHEP'02]: Minimal Resonance Chiral Theory interactions**

. **[Flores-Baez et al., PRD'06]: VMD with anomalous vector interactions**

$$a_\mu^\tau[2\pi] = (517.3 \pm 1.9 \pm 2.2 \pm 1.9) \times 10^{-10}$$ [Davier et al., EPJC'24]

. **[Miranda, Roig., PRD'20]: extended RChT with many free parameters**

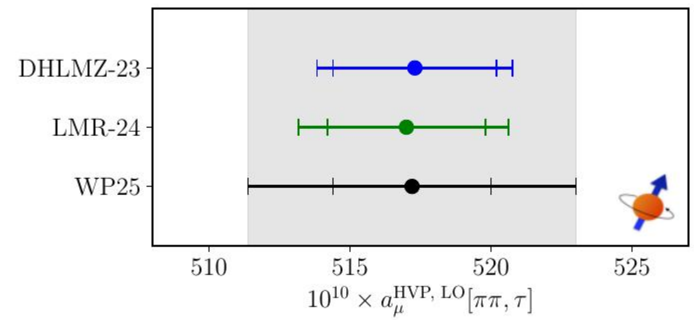
$$a_\mu^\tau[2\pi] = (519.6 \pm 2.8[\exp]^{+1.9}_{-2.1}[\mathrm{IB}]) \times 10^{-10}$$ **[O(p⁴)]**

Muon g-2 based on the $\tau \to \nu_\tau \pi\pi$ data

$\tau \to \nu_\tau \pi\pi$ data (Belle, ALEPH, CLEO, OPAL)

+

Isospin breaking factors ([Davier, et al., (DHLMZ), EPJC'24][Lopez Castro, et al., (LMR) PRD'25])



$$a_\mu^{\text{HVP, LO}}[\pi\pi, \tau] = 517.2(2.8)_{\text{exp}}(5.1)_{\text{th}} \times 10^{-10}$$

Adding other contributions to HVP-LO ($\pi\pi\pi$, KK, $\pi\gamma$ ...) from WP20

$$a_\mu^{\text{HVP, LO}}[(\pi\pi, \tau) + \text{WP20}] = 704.5(6.2) \times 10^{-10}$$

Caveat in WP25: "The above offset from WP20 is not updated in this work, we instead focus on the major tensions in the 2π channel. ... ... As described in Secs. 2.2.6 and 2.6.2, tensions between the Belle-II 3π data and previous measurements are now visible, other tensions are present in the K+ K− channel and in the comparison of the BESIII inclusive R-ratio measurement with pQCD. ... ... "

- **To futher take HLbL, HVP-N(N)LO, EW, QED from WP25, one would obtain**

$$a_\mu^{SM}\ [(\pi\pi,\tau)+\text{WP25}] = 116\,591\,946\ (63) \times 10^{-11}$$

$$a_\mu^{Exp} = 116\,592\,071.5\ (14.5) \times 10^{-11}$$

$$\Delta a_\mu\ [(\pi\pi,\tau)] = a_\mu^{Exp} - a_\mu^{SM} = 126\ (65) \times 10^{-11} \quad (1.9\sigma)$$

to compare with: $\Delta a_\mu\ [(\pi\pi,\text{lattice})] = a_\mu^{Exp} - a_\mu^{SM} = 38\ (63) \times 10^{-11}$ (reference value in WP25)

Other interesting topics on tau lepton

分支比概览

- **Br($\tau \to e\nu_\tau\overline{\nu_e}$) : 17.8%**

  Br($\tau \to \mu\nu_\tau\overline{\nu_\mu}$) : 17.4%

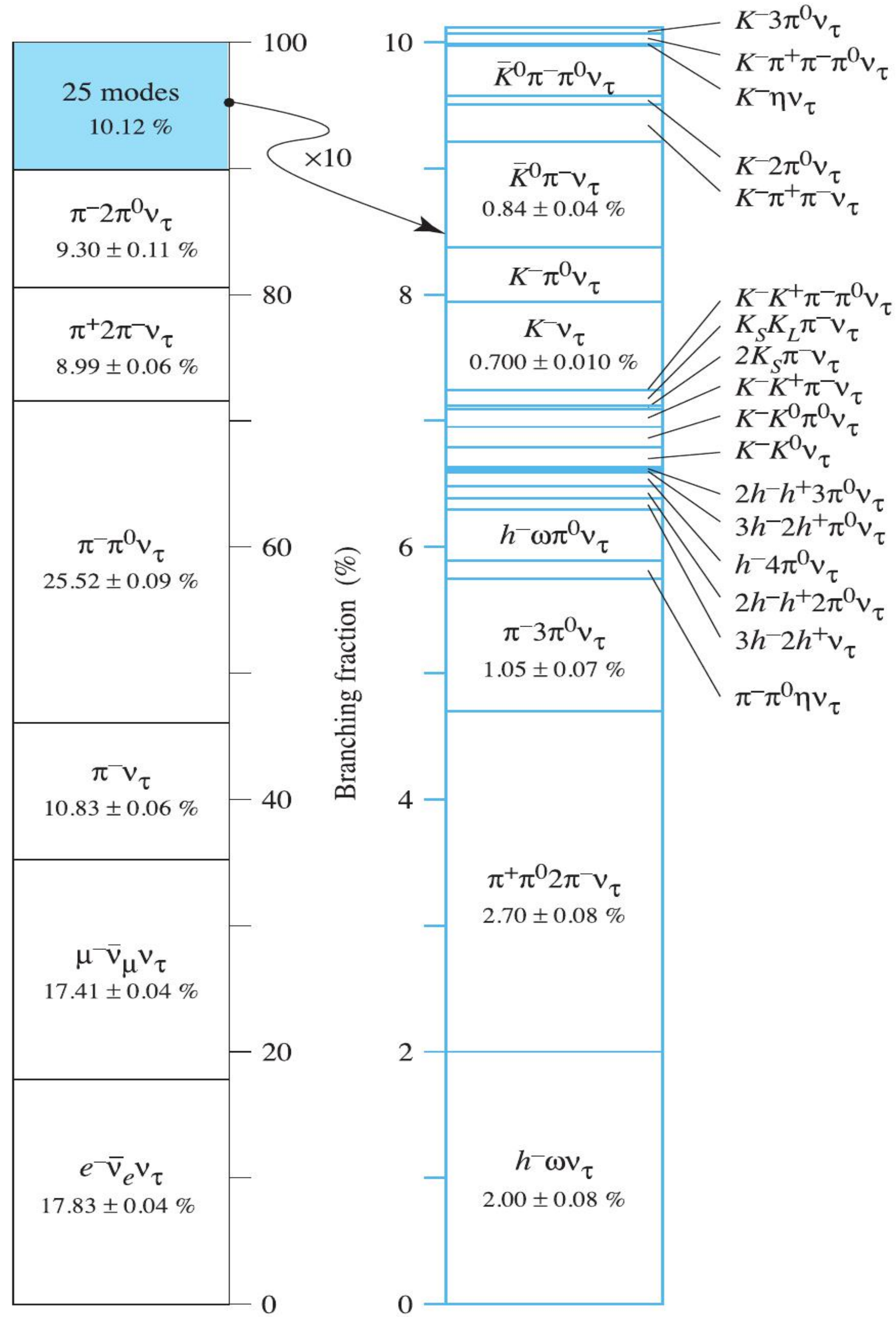
- **Br($\tau \to \nu$+Cabbibo allowed hadrons) ~ 62%**
- **Br($\tau \to \nu$+Cabbibo suppressed hadrons) : ~3%**

- □ **Br($\tau \to \nu\pi\pi$) ~ 25%, 单举衰变中分支比最大**
- □ **tau的衰变末态只有轻味强子，不涉及重味粒子 ($m_\tau < m_D$)**
- □ **在重子数守恒的假设下，tau不能衰变至含有重子的末态($m_\tau < 2m_N$)**

名词澄清：

- **单举(exclusive): 只包含某一个具体物理过程**
- **遍举(inclusive): 包含所有可能的单举过程或者包含某一类单举过程**

例如， **Cabbibo允许的遍举过程是指末态不含奇数个K介子的所有单举过程**



强衰变过程

tau的强衰变可以给我们提供什么信息？

- 遍举衰变：（某类）所有的强子末态

$$\tau^- \rightarrow \nu_\tau\,(\bar{u}d, \bar{u}s)$$

➡ 可以用来研究标准模型的基本参数：α_S, V_{us}, ...

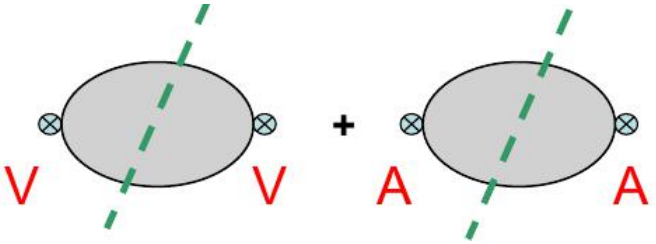
- 单举衰变：衰变至特定的强子末态

$$\tau^- \rightarrow \nu_\tau(P,\ PP,\ PPP,\ ...)$$

➡ 可以用来强作用形状因子，强子共振态，手征对称性，...

利用tau的谱函数确定$\alpha_s(m_\tau)$

$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau \,\text{mesons}\right)}{\Gamma\left(\tau^- \to e^- \overline{\nu}_e \nu_\tau\right)} \propto$$



两点关联函数

$$V_{ij}^{\mu} = \bar{\psi}_j \gamma^{\mu} \psi_i \qquad A_{ij}^{\mu} = \bar{\psi}_j \gamma^{\mu} \gamma_5 \psi_i$$

$$\Pi_{ij,J}^{\mu\nu}(q) \equiv i \int d^4x \; e^{iqx} \langle 0 | T[J_{ij}^{\mu}(x) J_{ij}^{\nu}(0)^{\dagger}] | 0 \rangle$$

$$= \left(-g^{\mu\nu} q^2 + q^{\mu} q^{\nu}\right) \Pi_{ij,J}^{(1)}(q^2) + q^{\mu} q^{\nu} \; \Pi_{ij,J}^{(0)}(q^2)$$

于是有：

$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{M_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

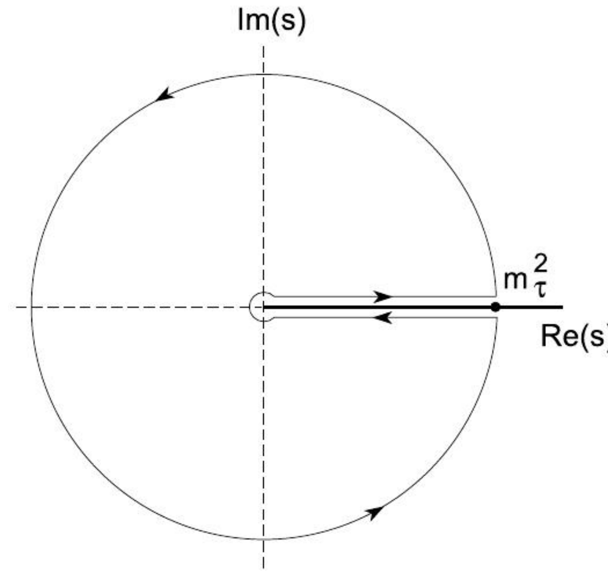
谱函数（两点关联函数的虚部）

说明：

- 谱函数ImΠ(s)实验可测：tau的遍举衰变过程
- 谱函数ImΠ(s) 在 s~(0,m_τ^2) 区间内的理论计算完全涉及非微扰QCD，很难有可靠的计算
- 理论出路？

利用函数Π(s)的解析性质

- 柯西定理
- Π(s)在除去正实轴以外的其他地方解析
- f(s)为任一解析函数



$$\frac{1}{\pi}\int_0^{s_0} ds\, f(s)\, \mathrm{Im}\Pi(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, f(s)\, \Pi(s)$$

$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2 \left[\left(1+2\frac{s}{M_\tau^2}\right)\mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

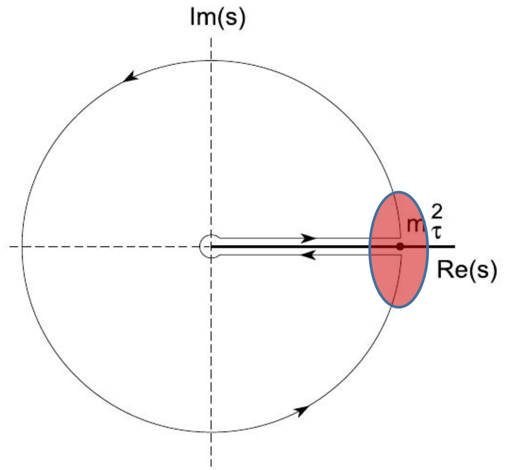
$$= 6\pi i \oint_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2 \left[\left(1+2\frac{s}{M_\tau^2}\right)\Pi^{(0+1)}(s) - \frac{2s}{M_\tau^2}\Pi^{(0)}(s)\right]$$

✓ 在$|s|=m_\tau^2$的圆周上，利用算符乘积展开(operator product expansion, OPE)，可对Π(s)进行可靠的理论计算。

算符乘积展开(OPE)

$$\Pi^{(J)}(s) = \sum_{D=0,2,4,\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim \mathcal{O}=D} C_D^{(J)}(s,\mu) \langle \mathcal{O}_D(\mu) \rangle$$

- **D=0, QCD微扰部分（以α_s为参数进行展开）**
- **D>0, QCD非微扰部分（以各种凝聚量为展开）**
- **可能的Quark-hadron duality violation (DV) 效应**



$$R_\tau = N_C \, S_{\rm EW} \left(1 + \delta_{\rm P} + \delta_{\rm NP}\right)$$

$S_{\rm EW} = 1.0201\,(3)$; $\delta_{\rm NP} = -0.0064 \pm 0.0013$

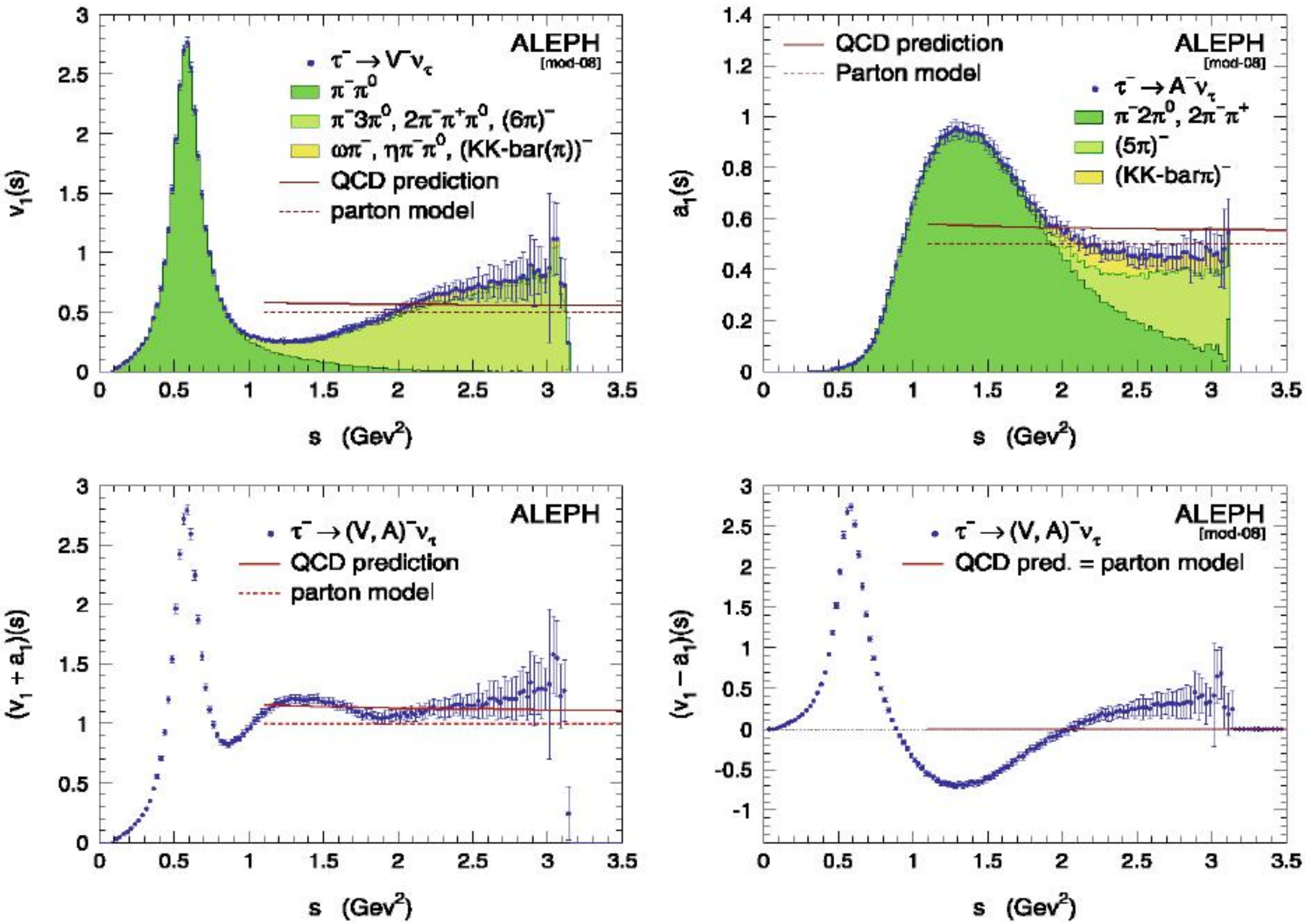
Marciano-Sirlin, Braaten-Li, Erler — **Fitted from data (Davier et al)**

$\delta_{\rm P} = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \ldots \approx 20\%$; $a_\tau \equiv \alpha_s(m_\tau)/\pi$

Baikov-Chetyrkin-Kühn

- **tau遍举衰变中的微扰修正非常重要，其对α_s依赖敏感，因此可以有效地确定α_s数值。**

$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2 \left[\left(1+2\frac{s}{M_\tau^2}\right)\mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right] = N_C\, S_{\mathrm{EW}}\left(1+\delta_{\mathrm{P}}+\delta_{\mathrm{NP}}\right)$$



- **矢量谱函数(V)：**

$I^G J^P = 1^+ 1^-$ 末态为偶数个π

- **轴矢谱函数(A)：**

$I^G J^P = 1^- 1^+$ 末态为奇数个π

- **末态含K介子时：**

需要通过理论模型分别抽取V、A谱函数

➢ **大部分数据仍来自于LEP！**

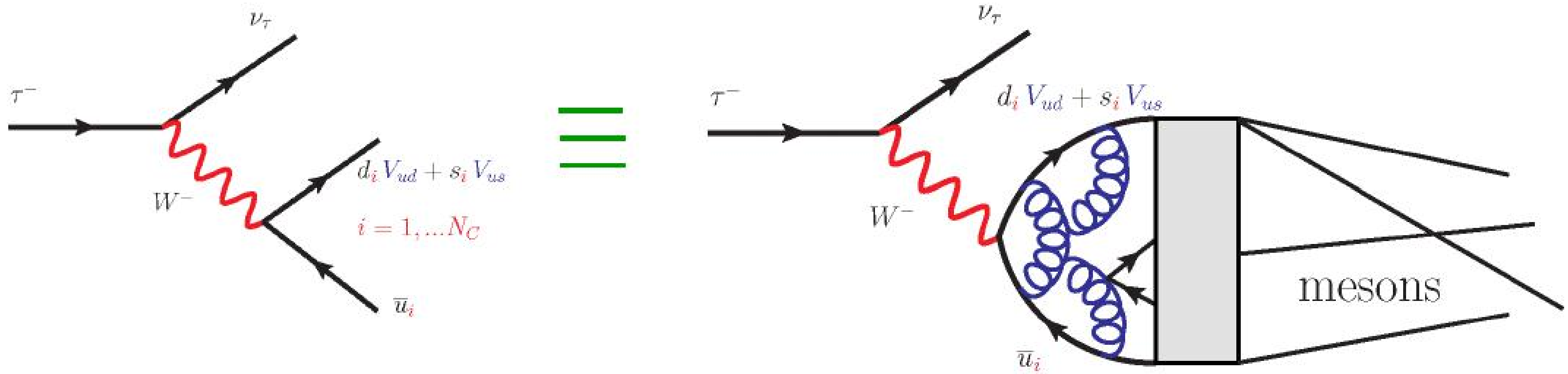
迫切需要新的测量！

$$(V-A)\big|_\chi \quad \propto \quad \text{non-perturbative} \qquad (m_u = m_d = m_s = 0)$$

$$(V+A) \quad \propto \quad \left(\text{perturbative} + \frac{1}{M_\tau^6}\,\text{non-perturbative}\right)$$

$\uparrow$ $\alpha_S(M_\tau)$

tau的单举强衰变过程



$$\mathcal{M}\left(\tau \to \nu_\tau \mathrm{H}\right) = \frac{G_F}{\sqrt{2}} V_{\mathrm{CKM}}\, \overline{u}_{\nu_\tau} \gamma^\mu \left(1-\gamma_5\right) u_\tau \left\langle H\right| \left(V_\mu - A_\mu\right) e^{i\,\mathrm{L_{QCD}}} \left|\Omega_{\mathrm{H}}\right\rangle$$

form factors

$$\left\langle H\right| \left(V_\mu - A_\mu\right) e^{i\,\mathrm{L_{QCD}}} \left|\Omega_{\mathrm{H}}\right\rangle = \sum \left(\text{Lorentz structure}\right)^i{}_\mu\, F_i\left(Q^2, s, \ldots\right)$$

$$d\Gamma\left(\tau \to \nu_\tau H\right) = \frac{G_F^2}{4\, M_\tau} \left|V_{\mathrm{CKM}}\right|^2 L_{\mu\nu}\, H^{\mu\nu}\, d\mathrm{PS} \quad \begin{cases} L_{\mu\nu} H^{\mu\nu} = \sum_X L_X W_X \\ W_X \equiv \textbf{structure functions} \end{cases}$$

$\tau \rightarrow PP' + \nu_\tau$

[Hao, Duan, ZHG, 2507.00383]

$$\langle P_1 P_2 | \bar{D}\gamma^\mu u | 0 \rangle = \left[(p_2 - p_1)^\mu - \frac{\Delta_{P_2 P_1}}{s} q^\mu \right] F_+^{P_1 P_2}(s) + \frac{\Delta_{Du}}{s} q^\mu \widehat{F}_0^{P_1 P_2}(s)$$

$$\Delta_{P_2 P_1} = m_{P_2}^2 - m_{P_1}^2 \,, \qquad \Delta_{Du} = B_0(m_D - m_u) \,, \qquad q_\mu = (p_1 + p_2)_\mu, \qquad s = q^2 .$$

$$\frac{d\Gamma_{\tau \to P_1 P_2 \nu_\tau}}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{48\pi^3 s} S_{\rm EW} \, |V_{uD}|^2 \left(1 - \frac{s}{M_\tau^2}\right) \left\{ \left(1 + \frac{2s}{M_\tau^2}\right) q_{P_1 P_2}^3(s) \left| F_+^{P_1 P_2}(s) \right|^2 + \frac{3\Delta_{Du}^2}{4s} q_{P_1 P_2}(s) \left| \widehat{F}_0^{P_1 P_2}(s) \right|^2 \right\}$$

$$A_{FB}(s) = \frac{\int_0^1 d\cos\alpha \frac{d^2\Gamma_{\tau \to P_1 P_2 \nu_\tau}}{d\sqrt{s}\, d\cos\alpha} - \int_{-1}^0 d\cos\alpha \frac{d^2\Gamma_{\tau \to P_1 P_2 \nu_\tau}}{d\sqrt{s}\, d\cos\alpha}}{\int_0^1 d\cos\alpha \frac{d^2\Gamma_{\tau \to P_1 P_2 \nu_\tau}}{d\sqrt{s}\, d\cos\alpha} + \int_{-1}^0 d\cos\alpha \frac{d^2\Gamma_{\tau \to P_1 P_2 \nu_\tau}}{d\sqrt{s}\, d\cos\alpha}} \propto \Re \left[F_+^{P_1 P_2}(s) \, \widehat{F}_0^{P_1 P_2 *}(s) \right]$$

α: angle between the momenta of P_1 and τ in the P_1P_2 rest frame

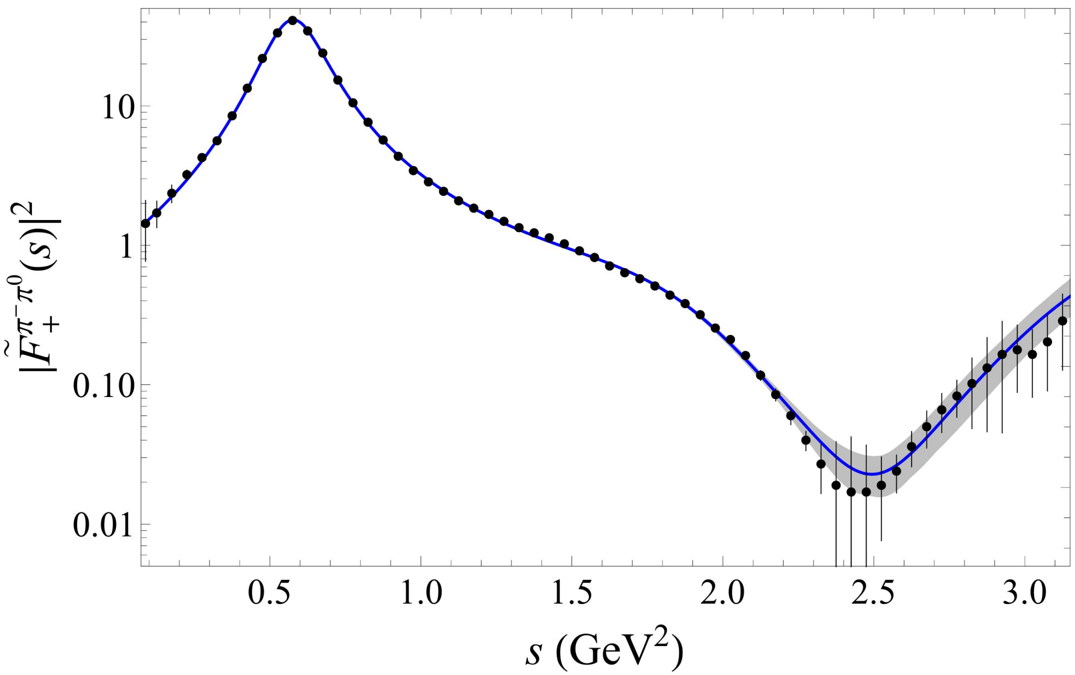
$\tau \rightarrow \pi^- \pi^0 \nu_\tau$

$\Delta_{PP'} \rightarrow 0$, 所以标量形状因子$F_S$项可以忽略，只有矢量形状因子$F_+$贡献!

$\tau \rightarrow K\pi\nu_\tau$

$\Delta_{PP'} \neq 0$, 标量形状因子F_0项以及矢量形状因子F_+项都有贡献!

$\tau \rightarrow \pi^- \pi^0 \nu_\tau$



ρ(770), ρ(1450), ρ(1700)

- **Crucial inputs to address muon g-2**
- **Most precise spectra is from Belle;**

  but most precise BR is from ALEPH: 25.47(13)%
- **Coherent precise measurements of both spectra and BR from STCF would be invaluable!**

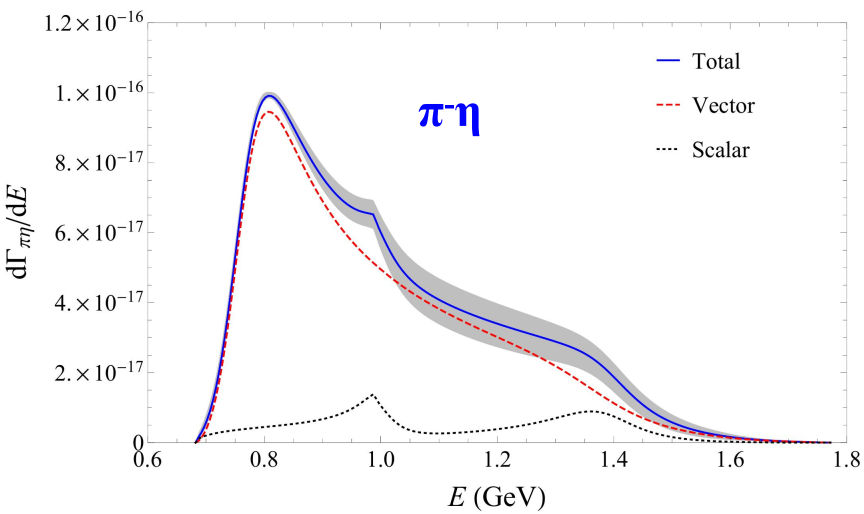
$\tau \to \pi^-\eta^{(')}\nu_\tau$ (Cabibbo allowed)：second-class currents

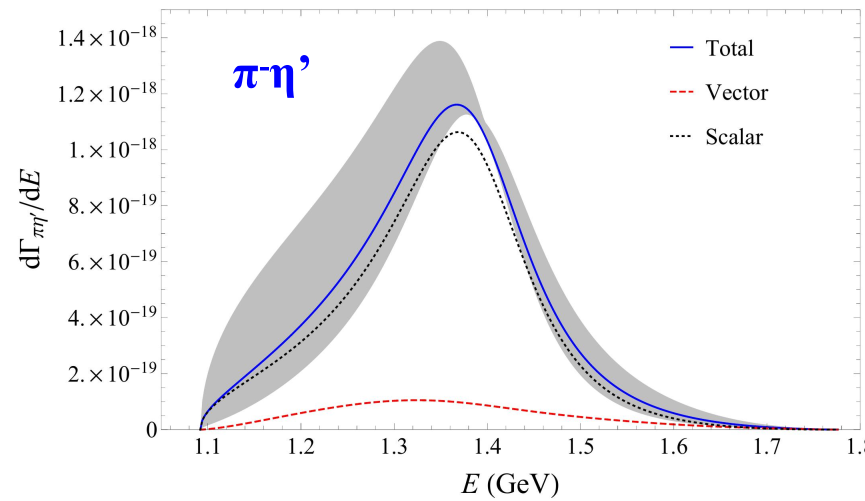
若$\pi^-\eta$ 的J=0，则其P=+1 （V-A型的流不允许）； 若J=1，则P=-1 （矢量流）

但是对于SM来讲，1-矢量流对应的G宇称为正，而$\pi^-\eta$的G宇称为负，表明这是一个破坏G宇称的过程（second-class current），因此可能是寻找新物理的一个有效途径。

$$\langle \pi^- P|\bar{d}\gamma^\mu u|0\rangle = \left[(p_P - p_\pi)^\mu - \frac{\Delta_{P\pi}}{s}q^\mu\right] F_+^{\pi^- P}(s) + \frac{\Delta_{du}^{\rm Phy}}{s} q^\mu F_0^{\pi^- P}(s)$$

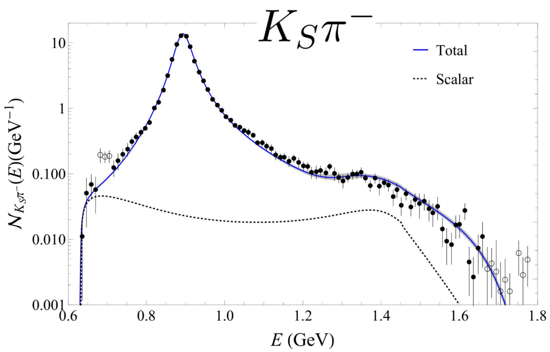
[Hao, Duan, ZHG, 2507.00383]

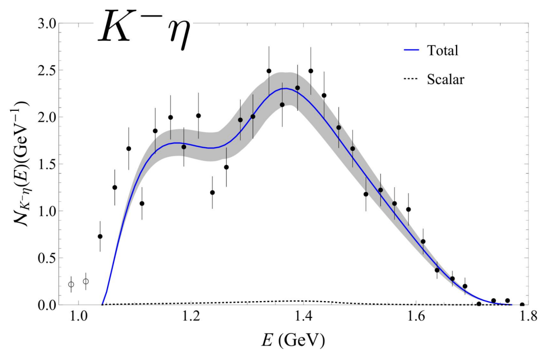


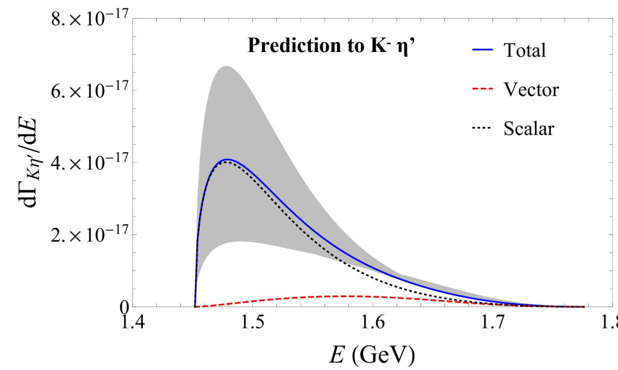


Channel	Total	Vector	Scalar	Exp Limits
$\tau^- \to \pi^-\eta\nu_\tau$ $(\times 10^5)$	$1.63^{+0.14}_{-0.14}$	$1.43^{+0.18}_{-0.21}$	$0.20^{+0.07}_{-0.04}$	< 9.9 (BaBar) [69] < 7.3 (Belle) [70]
$\tau^- \to \pi^-\eta'\nu_\tau$ $(\times 10^7)$	$1.17^{+0.36}_{-0.07}$	$0.14^{+0.09}_{-0.08}$	$1.03^{+0.44}_{-0.16}$	< 40 (BaBar) [71]

τ → (KP)⁻ ν_τ (Cabibbo suppressed)







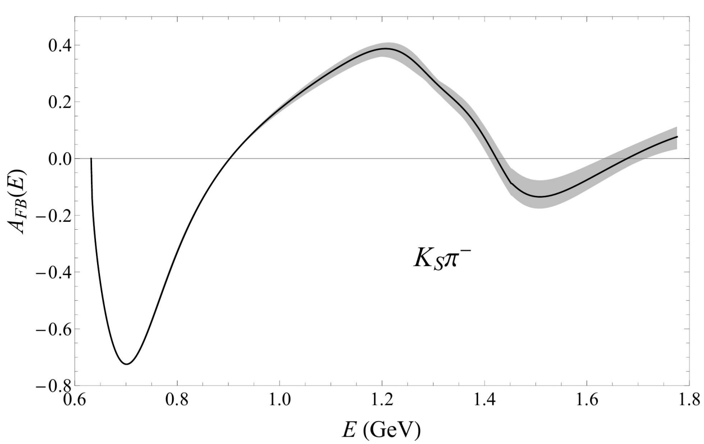
F_+^{KP}: K*, K*(1410), ...; F_0^{KP}: κ, $K^*_0(1430)$, ...

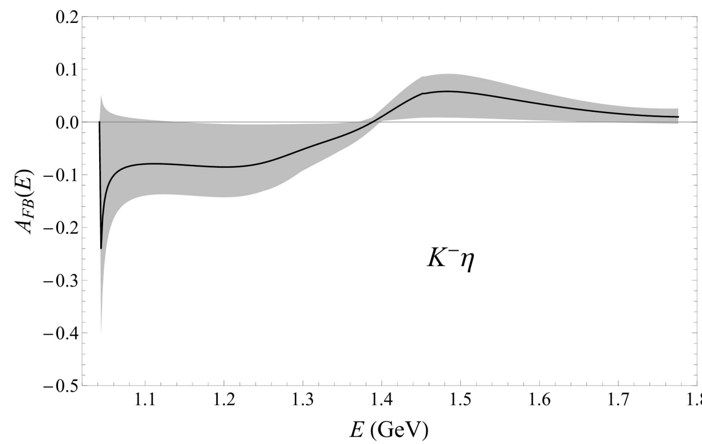
BR(K⁻ η')^{Theo} = (2.0±1.0)×10⁻⁶

BR(K⁻ η')$^{Exp,\ BaBar}$ < 2.4×10⁻⁶

Prediction to Forward-Backward asymmetries

[Hao, Duan, ZHG, 2507.00383]

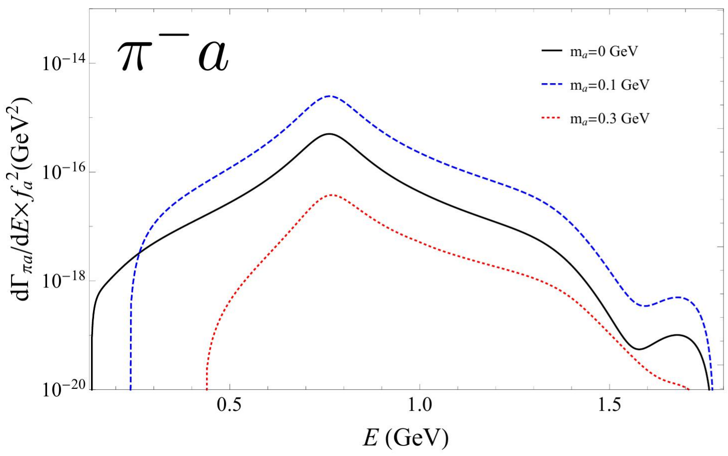


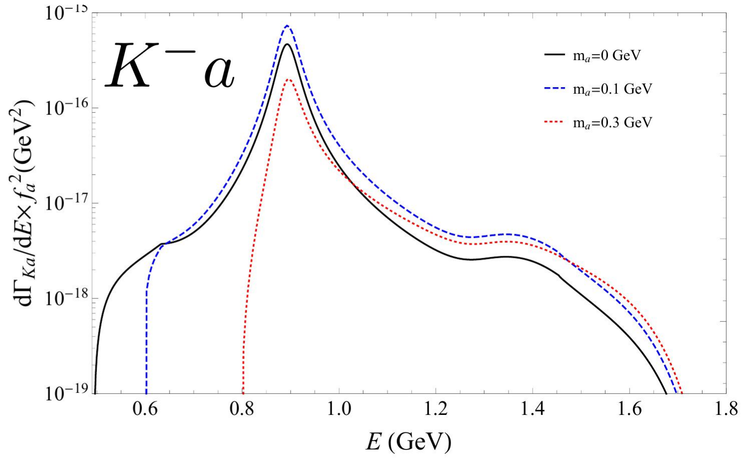


- **τ → $\pi^0 K^- \nu_\tau$: related to $\pi^- K_S$ with isospin-breaking corrections**
- **τ → π K ν_τ: crucial for precise measurement of the $R_{\tau,S}$**

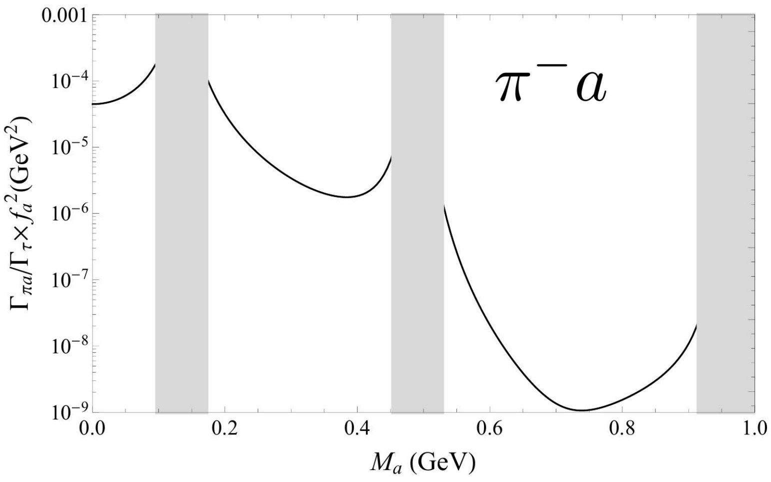
Predictions to axion-meson production in tau decays

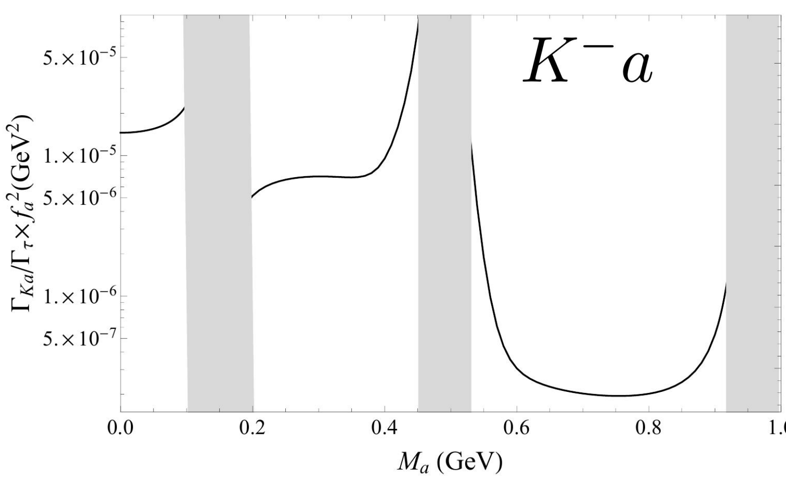
specta





branching ratios





CPV study in $\tau \to \pi^- K_S \nu_\tau$

Intensive discussions on tau -> Ks pi nu

$$A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \bar{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}$$

$$\approx (0.36 \pm 0.01)\%$$

SM prediction

$$(-0.36 \pm 0.23_{stat} \pm 0.11_{syst})\%$$

BaBar

[Bigi et al., PLB'05] [Grossman et al., JHEP'12] [Lees et al., PRD'12]

[Cirigliano et al., PRL'18] [Rendo et al., PRD'19] [Chen et al., PRD'19 JHEP'20]

- **An important subject at STCF: around 3×10^{-4} sensitivity could be reached**

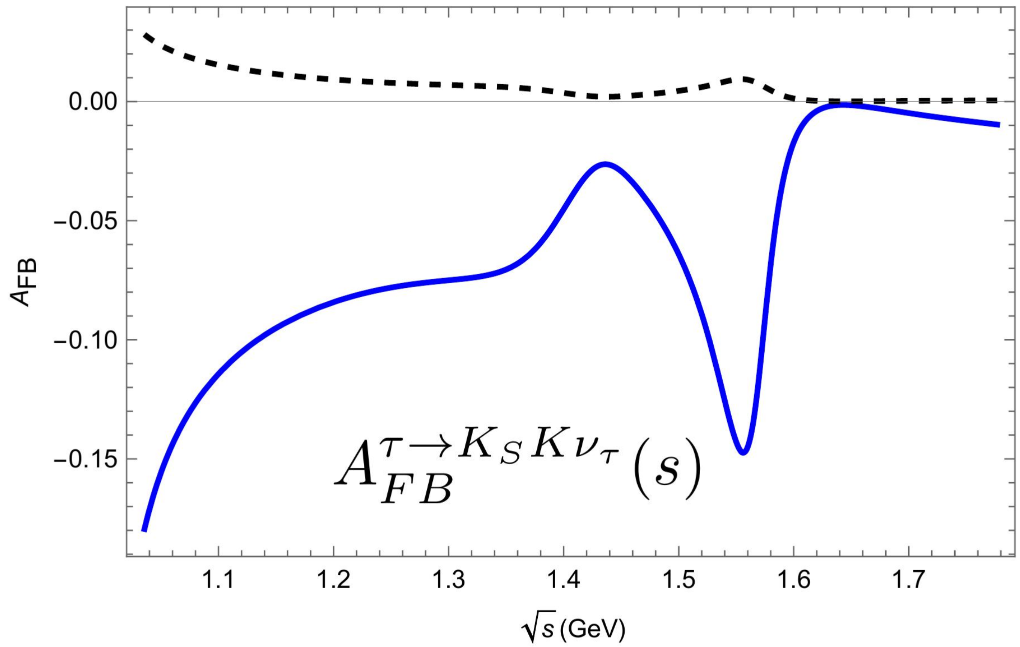
New proposal to search CPV in other $\tau \to PP' \nu_\tau$ channels

[Lopez Aguilar, et al., JHEP'25]

$$A_{CP}^{\text{rate}}|_{KK} = \frac{\Gamma(\tau^+ \to K^+ K_S \bar{\nu}_\tau) - \Gamma(\tau^- \to K^- K_S \nu_\tau)}{\Gamma(\tau^+ \to K^+ K_S \bar{\nu}_\tau) + \Gamma(\tau^- \to K^- K_S \nu_\tau)}$$

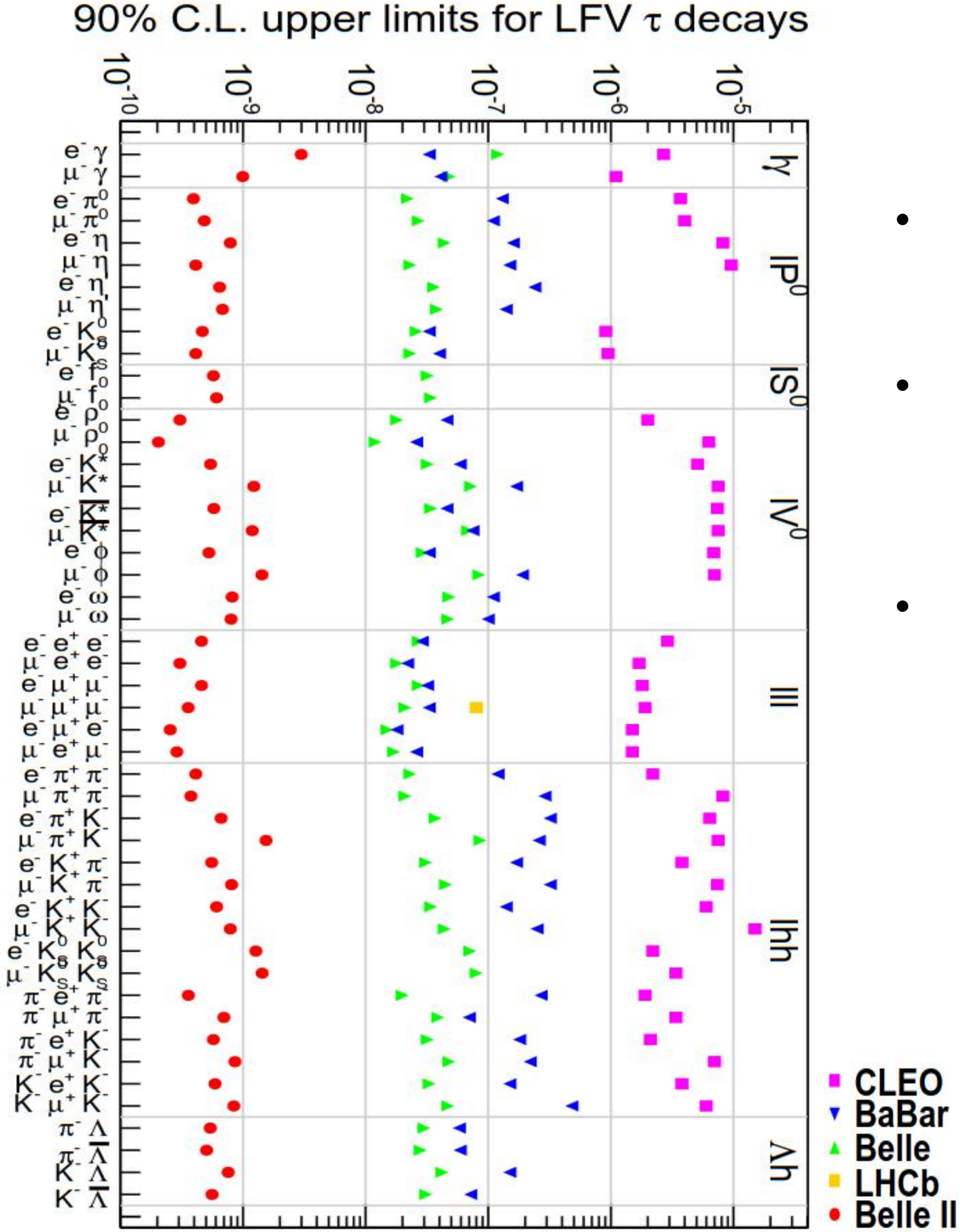
$$-3.83 \times 10^{-3} \leq A_{CP}^{rate}|_{KK} \leq -3.37 \times 10^{-3}$$

Forward-Backward asymmetry



- **Tensor current plays the decisive role in CPV of $\tau^{+/-} \to K^{+/-} K_S \nu_\tau$**

Charged lepton flavor violation in tau decays



- Not only statistic but also systematic uncertainties are important in $\tau \rightarrow l\,\gamma$
- Clean backgroud makes $\tau \rightarrow l\,l'\,l''$ one of the best channels to search for LFV signals.
- $\tau \rightarrow l + hadrons$ provides a different laboratory to probe different LFV origins, comparing with the pure leptonic processes.

结语　　Tau与g-2 包含丰富有趣的物理：

- 不仅有诗和远方 --诱人的新物理现象--：

  轻子味道破坏，轻子数破坏，新的CP破坏，… …

- 也充满了烟火气息 --亟需提升精度/澄清的SM允许的过程--：

- $e^+e^- \to \pi^+\pi^-$ ，$\tau^- \to \nu_\tau \pi^0 \pi^-$
- $\tau^- \to \nu_\tau \pi^- (K^-) K_S$ 中的CP破坏
- $\tau^- \to \nu_\tau P^- \gamma$ ，$\tau^- \to \nu_\tau \pi^0 \pi^- \gamma$　(尚未有实验测量，可有效降低很多理论误差)
- tau谱函数的精确测量 $\to \alpha_S(m_\tau)$确定
- 第二类流主导过程的发现：$\tau^- \to \nu_\tau \pi^- \eta/\eta'$
- τ衰变中的前后不对称性的测量
- 跃迁形状因子：$A \to V\gamma$, $A \to V\gamma$, $A\gamma^*\gamma^*/ \gamma^*\gamma^{(*)} \to P\ P'$ / $P\gamma^{(*)}\gamma^{(*)}$
  (为提升HLbL的精度作好准备)
- … …

谢谢大家！