Coupled-channel analysis of the near-threshold $e^+e^- \to N\bar{N}$ cross sections

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贾召赛
曲阜师范大学
中国科学院理论物理研究所

Z.-S. Jia, Z.-H. Zhang, F.-K. Guo, and G. Li, Phys. Rev. D 111, 054014 (2025).

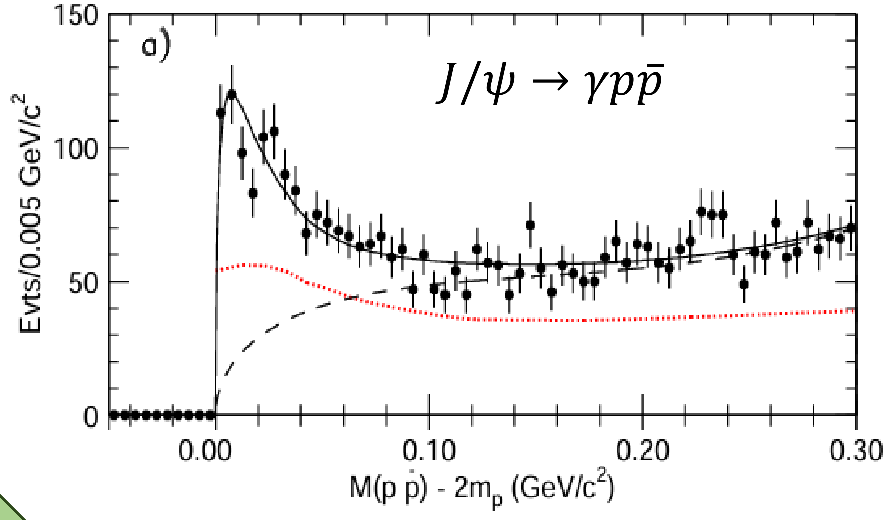
Outline

- Background
- Method: NREFT
- Results
- Summary

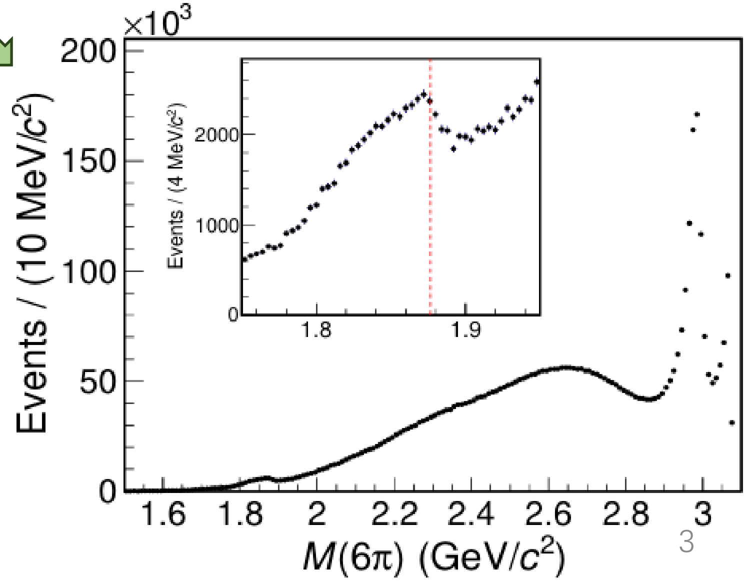
Background: Experiment

The interest in the nucleon-antinucleon bound states has been lasting for decades. They are analogs of the deuteron but with a vanishing baryon number.

- J/ψ decay
  - BES collaboration, Phys. Rev. Lett. 91, 022001 (2003).
  - $X(1835)$: $J/\psi \to \gamma\eta'\pi^+\pi^-$ Phys. Rev. Lett. 95, 262001 (2005).
  - $X(1840)$: BESIII Collaboration, Phys. Rev. D 88, 091502 (2013).
  - $X(1880)$: $J/\psi \to \gamma 3(\pi^+\pi^-)$ Phys. Rev. Lett. 132, 151901 (2024). …



- B decays
  - Belle Collaboration, Phys. Rev. Lett. 88, 181803 (2002).
  - Belle Collaboration, Phys. Rev. Lett. 89, 151802 (2002).
  - Belle Collaboration, Phys. Lett. B 659, 80 (2008). …
- $\psi(2S)$ decays
  - BESIII Collaboration, Chin. Phys. C 34, 421 (2010).
  - CLEO Collaboration, Phys. Rev. D 82, 092002 (2010).
  - BESIII Collaboration, Phys. Rev. Lett. 108, 112003 (2012). …
- $e^+e^- \to p\bar{p}$ process: $J^{PC} = 1^{--}$
  - BABAR Collaboration, Phys. Rev. D 73, 012005 (2006).
  - BABAR Collaboration, Phys. Rev. D 87, 092005 (2013).
  - CMD-3 Collaboration, Phys. Lett. B 759, 634 (2016). …



Background : Theory

- $p\bar{p}$ bound state

  G.-J. Ding and M.-L. Yan, Eur. Phys. J. A 28, 351 (2006).
  X.-H. Liu, Y.-J. Zhang, and Q. Zhao, Phys. Rev. D 80, 034032 (2009).
  P.-Y. Niu, Z.-Y. Zhang, Y.-Y. Li, Q. Wang, and Q. Zhao, Phys. Rev. D 110, 094020 (2024).
  P. G. Ortega, D. R. Entem, F. Fernandez, and J. Segovia, Phys. Lett. B 862, 139281 (2025). ...

- Final state interaction (FSI) effect

  B. S. Zou and H. C. Chiang, Phys. Rev. D 69, 034004 (2004).
  G. Y. Chen, H. R. Dong, and J. P. Ma, Phys. Lett. B 692, 136 (2010).
  G. Y. Chen and J. P. Ma, Phys. Rev. D 83, 094029 (2011).
  X.-W. Kang, J. Haidenbauer, and U.-G. Meißner, Phys. Rev. D 91, 074003 (2015). ...

- Pseudoscalar glueballs

  B. A. Li, Phys. Rev. D 74, 034019 (2006).
  G. Hao, C.-F. Qiao, and A.-L. Zhang, Phys. Lett. B 642, 53 (2006).
  N. Kochelev and D.-P. Min, Phys. Rev. D 72, 097502 (2005).
  L.-C. Gui, J.-M. Dong, Y. Chen, and Y.-B. Yang, Phys. Rev. D 100, 054511 (2019). ...

- Radial excitation states of the η' meson

  T. Huang and S.-L. Zhu, Phys. Rev. D 73, 014023 (2006).
  J.-S. Yu, Z.-F. Sun, X. Liu, and Q. Zhao, Phys. Rev. D 83, 114007 (2011).
  L.-M. Wang, Q.-S. Zhou, C.-Q. Pang, and X. Liu, Phys. Rev. D 102, 114034 (2020). ...

Background: $e^+e^- \to N\bar{N}$

Experiment:

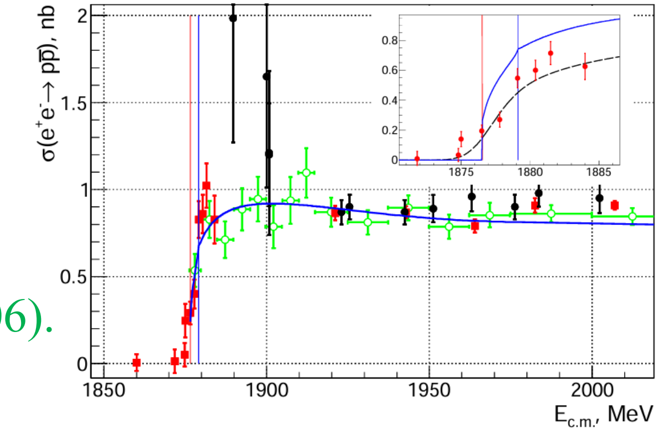
PS170 Collaboration: $p\bar{p} \to e^+e^-$ G. Bardin et al., Nucl. Phys. B411, 3 (1994).

FENICE Collaboration: $e^+e^- \to p\bar{p}$, and $e^+e^- \to n\bar{n}$ A. Antonelli et al., Phys. Lett. B 334, 431 (1994).

BABAR Collaboration, Phys. Rev. D 87, 092005 (2013). CMD-3 Collaboration, Phys. Lett. B 794, 64 (2019).

BESIII Collaboration, Phys. Lett. B 817, 136328 (2021).

SND Collaboration, Phys. At. Nucl. 87, 604 (2024).



Theory:

- FSI between the p and $\bar{p}$ J. Haidenbauer et al., Phys. Lett. B 643, 29 (2006).

  Optical potentials: Paris, Julich, ChEFT…

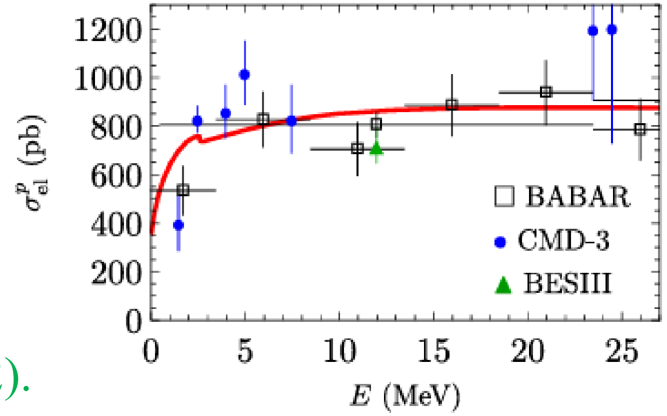
  e.g., $n\pi$ $(n \geq 2)$. J. Carbonell et al., Eur. Phys. J. A 59, 259 (2023).

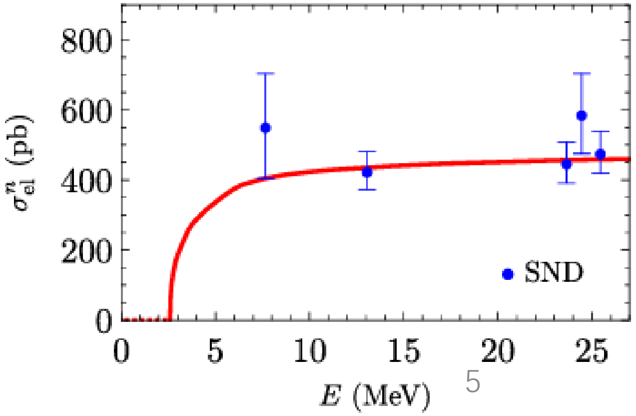
- Position-space formulism:

  Coulomb interaction between p and $\bar{p}$; proton-neutron mass difference.

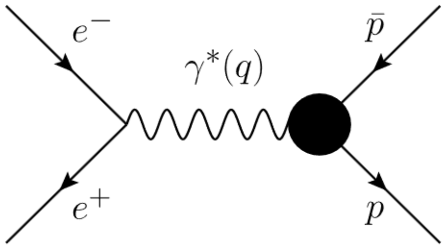
  A.I. Milstein et al., Nucl. Phys. A977, 60 (2018).

  A.I. Milstein et al., Phys. Rev. D 106, 074012 (2022).



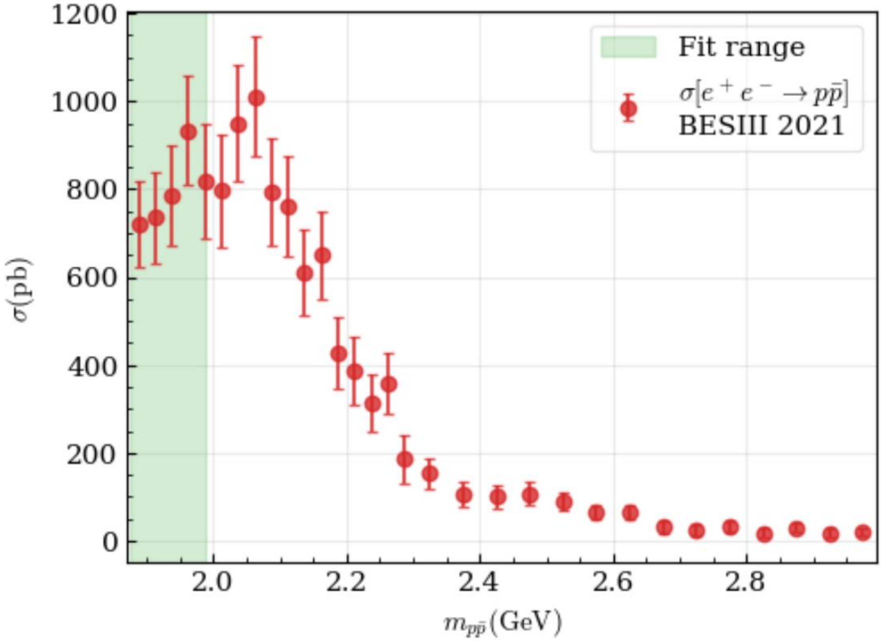


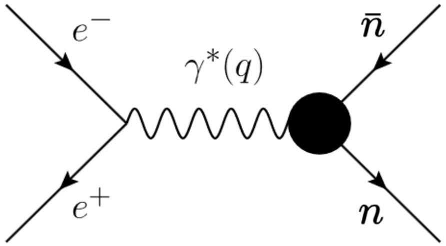
Near-threshold experimental data



BESIII Collaboration, Phys. Lett. B 817, 136328 (2021)

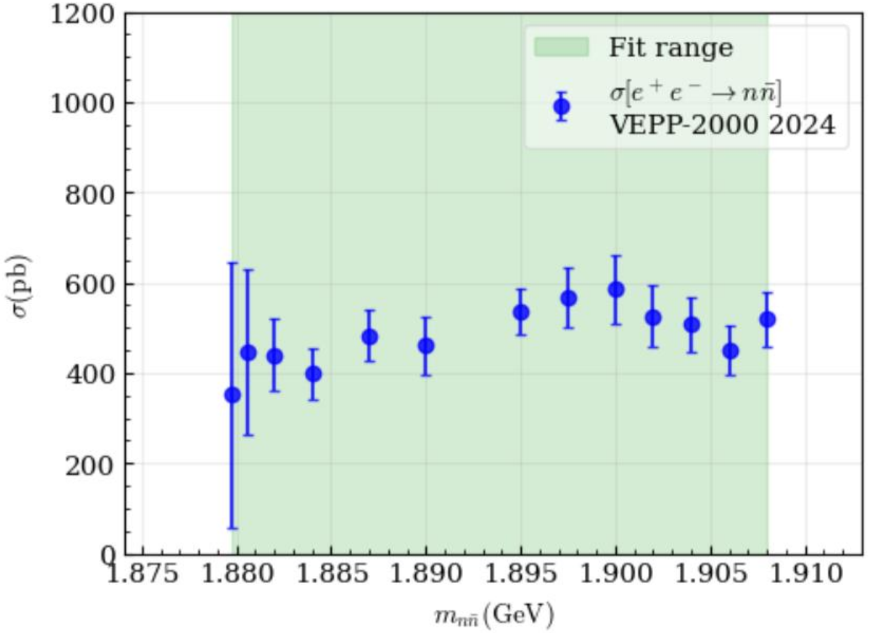
Fit range: [1.889, 1.988] GeV
Maximum k_p: 0.30 GeV





SND Collaboration, Phys. At. Nucl. 87, 604 (2024).

Fit range: [1.880, 1.908] GeV
Maximum k_n: 0.16 GeV



Method: Coupled-channel NREFT

- Two channels: CH1: $p\bar{p}$ CH2: $n\bar{n}$

Including:

- Coulomb interaction between p and $\bar{p}$
- Strong interaction between N and $\bar{\text{N}}$

- Two channel S-wave **T** matrix: two-potential formalism

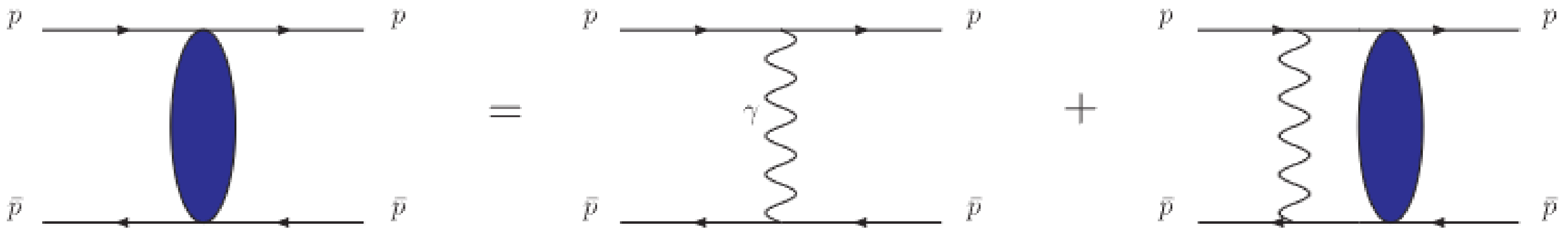
X. Kong and F. Ravndal, Nucl. Phys. A665, 137 (2000).

$$\mathbf{T}(E) = \begin{pmatrix} T_C(E) & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} W_C(E) & 0 \\ 0 & 1 \end{pmatrix} \mathbf{T}_{SC}(E) \begin{pmatrix} W_C(E) & 0 \\ 0 & 1 \end{pmatrix}$$

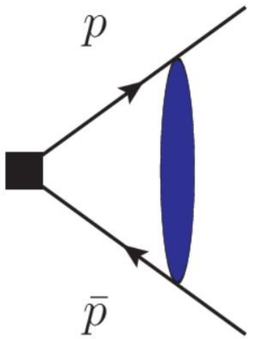
- Coulomb amplitude in CH1: $T_C(E) = \frac{\pi}{i\mu_p k_p}\left(\frac{\Gamma(1-ix)}{\Gamma(1+ix)} - 1\right)$

$$\mu_p = \frac{m_p}{2}, \quad k_p = \sqrt{2\mu_p(E - 2m_p)},$$

$$x = \alpha\mu_p/k_p, \quad \alpha = \frac{1}{137}.$$



- Photon exchange in the initial/final states: $W_C(E) = \left(\frac{2\pi x}{1-e^{-2\pi x}}\frac{\Gamma(1-ix)}{\Gamma(1+ix)}\right)^{\frac{1}{2}}$



E. Braaten, E. Johnson, and H. Zhang, J. High Energy Phys. 02 (2018) 150.
P.-P. Shi, Z.-H. Zhang, F.-K. Guo, and Z. Yang, Phys. Rev. D 105, 034024 (2022).

Method: Coupled-channel NREFT

- Coulomb-Strong scattering amplitude: $\mathbf{T}_{SC}(E) = \mathbf{V}_S(\Lambda) + \mathbf{V}_S(\Lambda)\mathbf{G}_C(E,\Lambda)\mathbf{T}_{SC}(E)$

- Strong potential with only contact term:

LECs for $I = 0, 1$

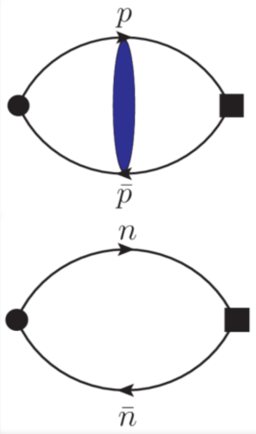
Electromagnetic and isospin-breaking correction

$$\mathbf{V}_S = \frac{1}{2}\begin{pmatrix} C_{0N} + C_{1N} + \delta_{em} & C_{0N} - C_{1N} \\ C_{0N} - C_{1N} & C_{0N} + C_{1N} \end{pmatrix} = \begin{pmatrix} a_1 + i\, a_2 & b_1 + i\, b_2 \\ b_1 + i\, b_2 & c_1 + i\, a_2 \end{pmatrix}$$

5-parameters: a_1, a_2, b_1, b_2, c_1

Unitarity constrain: $a_2 < 0, \mathrm{Im}\mathbf{T}_{ii} < 0$

- Green's function: $\mathbf{G}_C(E,\Lambda) = \mathrm{diag}(G_{C11},\ G_{C22})$



$$G_{C11}(E,\Lambda) = -\frac{\mu_p\Lambda}{\pi^2} - \frac{\alpha\mu_p^2}{\pi}\left(\ln\frac{\Lambda}{\alpha\mu_p} - \gamma_E\right) - \frac{\mu_p}{2\pi}\kappa_p(E)$$

$$G_{C22}(E,\Lambda) = -\frac{\mu_n\Lambda}{\pi^2} - i\,\frac{\mu_n}{2\pi}\,k_n$$

$\mu_n = \frac{m_n}{2},\ \gamma_E \approx 0.577,$

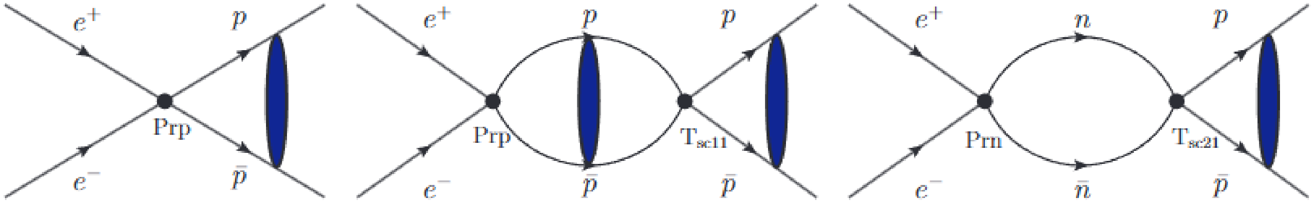
$k_n = \sqrt{2\mu_n(E - 2m_n)}\,,$

$\kappa_p(E) = 2\alpha\mu_p\left[\ln(ix) + \frac{1}{2ix} - \psi(-ix)\right]$

J. Carbonell, G. Hupin, and S. Wycech, Eur. Phys. J. A 59, 259 (2023).
X.-K. Dong, F.-K. Guo, and B.-S. Zou, Phys. Rev. Lett. 126, 152001 (2021).

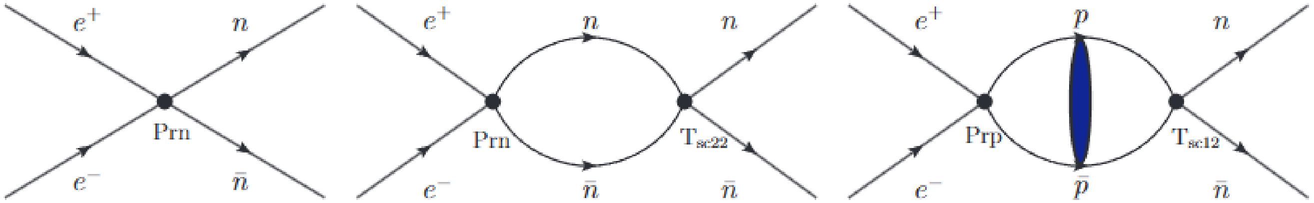
Method: Production amplitude and cross section

$e^+e^- \to p\bar{p}$



- $p\bar{p}$ production amplitude: $\mathcal{M}_{p\bar{p}} = \mathrm{Prp}(\Lambda) \times \left(1 + G_{C11}(\Lambda) \times \mathbf{T}_{sc11} + R_{n/p} \times G_{C22}(\Lambda) \times \mathbf{T}_{sc21}\right) \times W_C$

$e^+e^- \to n\bar{n}$



- $n\bar{n}$ production amplitude: $\mathcal{M}_{n\bar{n}} = \mathrm{Prp}(\Lambda) \times \left(R_{n/p} + G_{C11}(\Lambda) \times \mathbf{T}_{sc12} + R_{n/p} \times G_{C22}(\Lambda) \times \mathbf{T}_{sc22}\right)$
- Production sources: $\mathrm{Prp}(\Lambda)(p\bar{p})$, $\mathrm{Prn}(\Lambda)(n\bar{n})$, $R_{n/p} = \mathrm{Prn}/\mathrm{Prp}$
- Production cross sections: $\sigma[e^+e^- \to p\bar{p}] = \frac{|\vec{k}_p|}{16\pi E^2|\vec{k}_e|}\mathrm{Prp}^2\left(\left|\mathcal{M}_{p\bar{p}}/\mathrm{Prp}\right|^2 + \mathrm{bkg}\right)$

$$\sigma[e^+e^- \to n\bar{n}] = \frac{|\vec{k}_n|}{16\pi E^2|\vec{k}_e|}\mathrm{Prp}^2(|\mathcal{M}_{n\bar{n}}/\mathrm{Prp}|^2 + \mathrm{bkg})$$

- ☐ Fixed parameter: bkg = 0 ⟵ from $e^+e^- \to p\bar{p}\pi^0$ or $e^+e^- \to K_SK_L\pi^0$ and was subtracted
- ☐ 7 Free parameters: $a_1, a_2, b_1, b_2, c_1, \mathrm{Prp}^2, R_{n/p}$

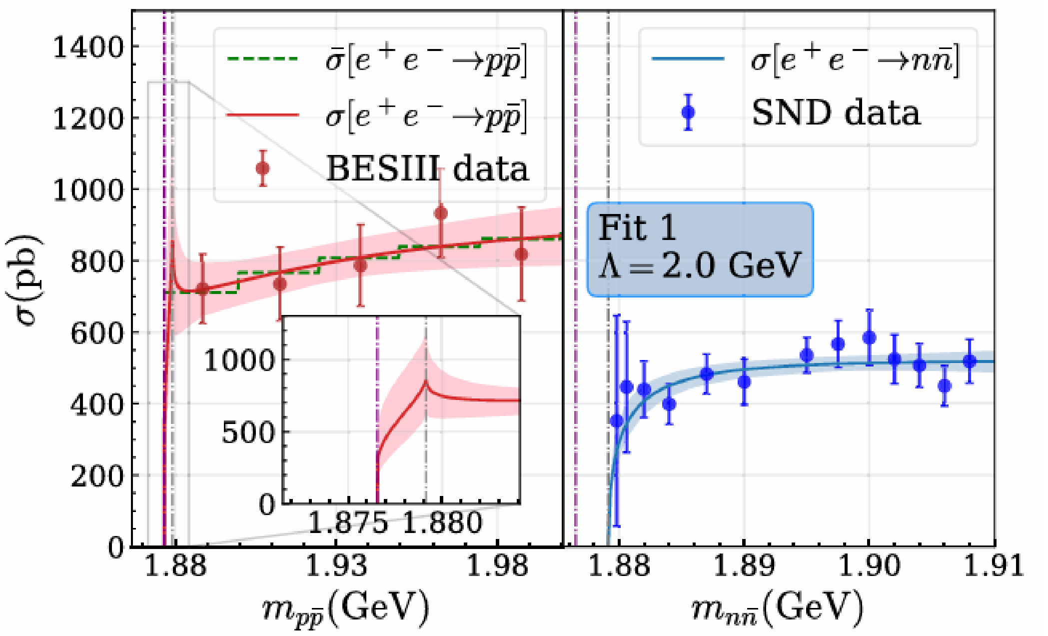
Result: Line shapes

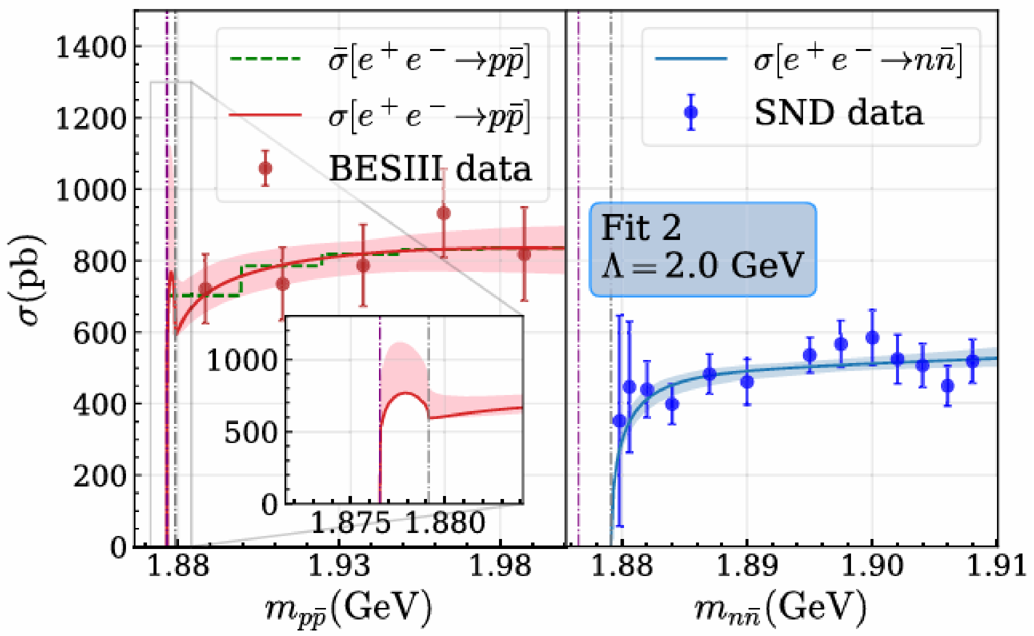
- Two kinds of near-threshold line shapes of $\sigma[e^+e^- \to p\bar{p}]$

$\Lambda \in [2.0, 2.6]$ GeV

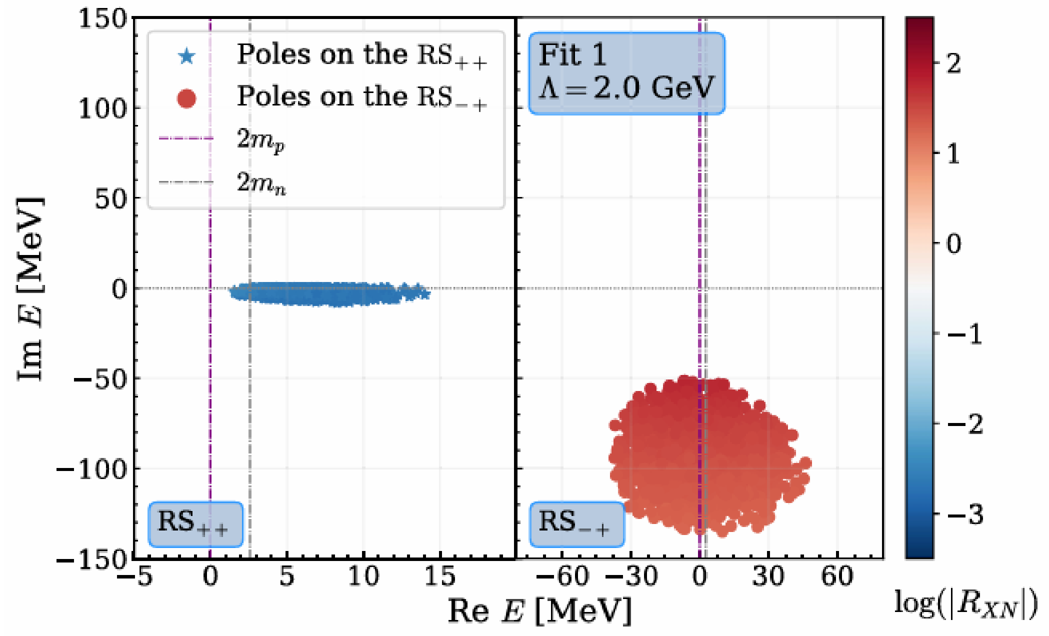
Fit 1: one sharp cusp at the $n\bar{n}$ threshold

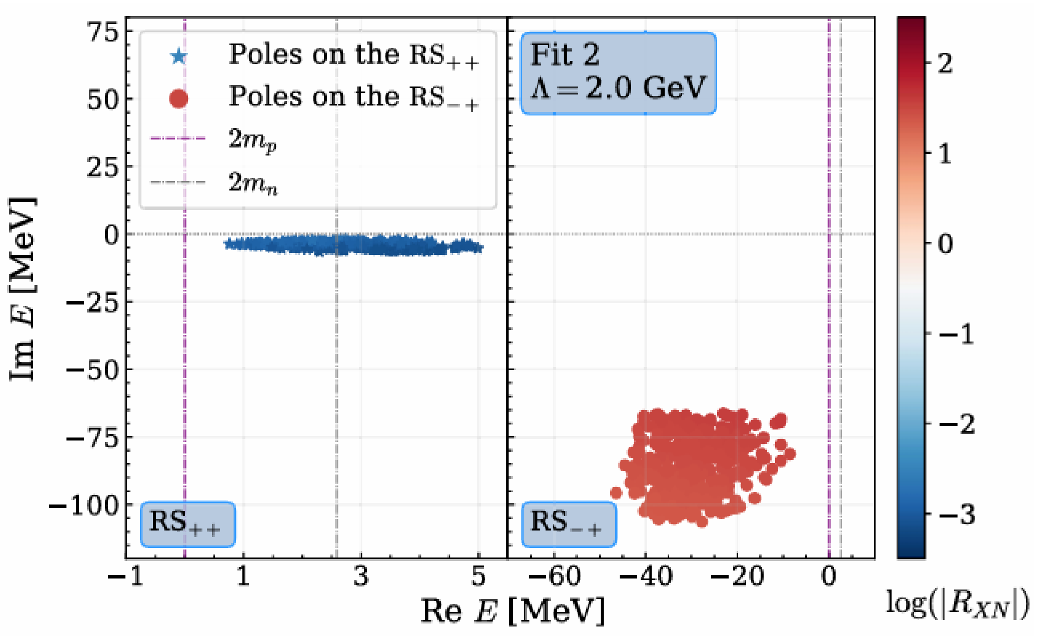
Fit 2: one bump between the two thresholds





Result: Pole positions





Two poles :
Pole-1 on the RS_{++}
Pole-2 on the RS_{-+}

$$R_{XN} = \frac{g_{I=1}}{g_{I=0}} = \frac{g_{Xp} - g_{Xn}}{g_{Xp} + g_{Xn}}$$

Fit 1:

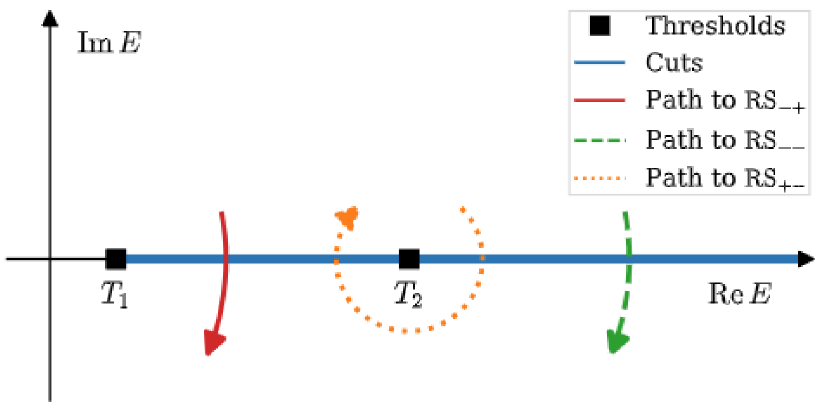
$$E_1 = 1882.2^{+9.4}_{-4.5} - i(1.2^{+6.3}_{-1.2})\ \text{MeV}$$

$$E_2 = 1882.2^{+79.4}_{-46.5} - i\left(106.4^{+83.5}_{-65.1}\right)\ \text{MeV}$$

Fit 2:

$$E_1 = 1878.2^{+4.5}_{-1.0} - i\left(5.9^{+1.5}_{-4.7}\right)\ \text{MeV}$$

$$E_2 = 1817.7^{+53.1}_{-13.1} - i(99.4^{+46.7}_{-34.2})\ \text{MeV}$$

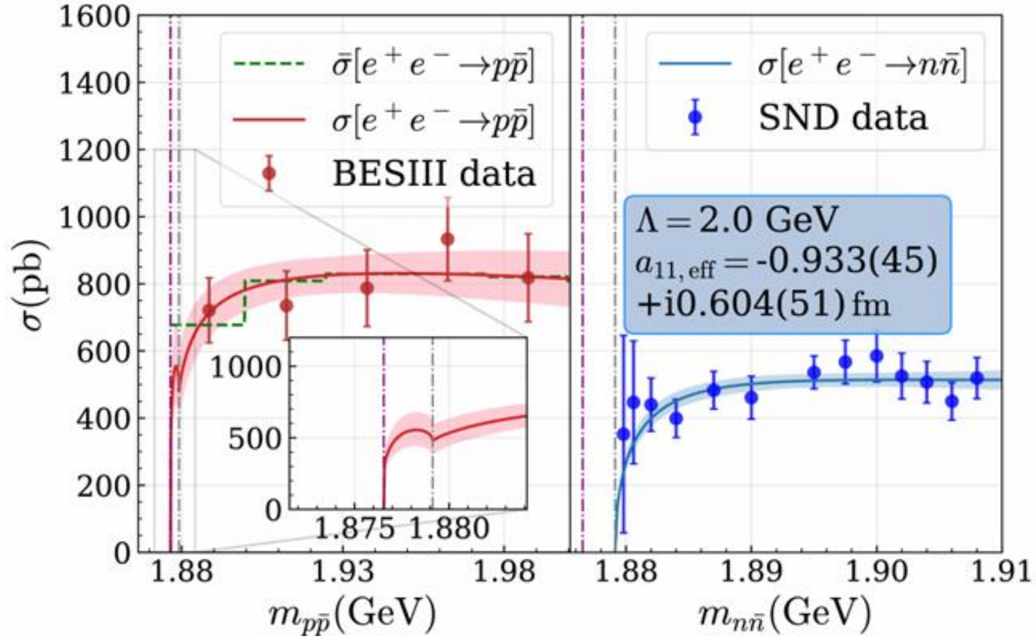


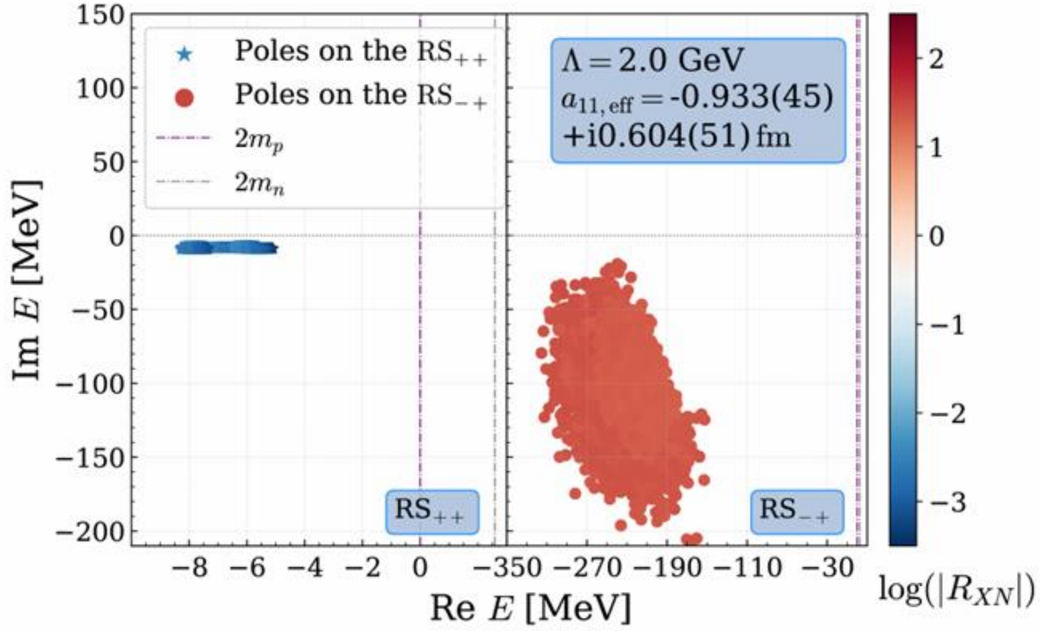
Result: with constraint from antiprotonic hydrogen

Fit	$a_{11,\text{eff}}$ [fm]	$a_{12,\text{eff}}$ [fm]	$a_{22,\text{eff}}$ [fm]
1	$0.10^{+0.63}_{-0.63} + i(1.14^{+1.76}_{-0.51})$	$-0.21^{+0.69}_{-0.68} - i(1.24^{+1.84}_{-0.55})$	$-0.24^{+0.41}_{-0.62} + i(1.04^{+0.70}_{-0.34})$
2	$-0.60^{+0.68}_{-0.23} + i(1.09^{+0.65}_{-0.13})$	$0.51^{+0.25}_{-0.72} - i(1.16^{+0.71}_{-0.14})$	$-0.61^{+0.30}_{-0.16} + i(0.83^{+0.44}_{-0.06})$

- From $\bar{p}$-H atom: $a_{p\bar{p}} = -0.933(45) + i\,0.604(51)$ fm

J. Carbonell, G. Hupin, and S. Wycech, Eur. Phys. J. A 59, 259 (2023).





$$E_1 = 1869.7^{+1.7}_{-1.7} - i(8.5^{+1.4}_{-1.6})\ \text{MeV}$$
$$E_2 = 1582.6^{+141.6}_{-251.1} - i(144.7^{+148.2}_{-124.7})\ \text{MeV}$$

$$a_{12,\text{eff}} = 0.770^{+0.093}_{-0.089} - i(0.517^{+0.092}_{-0.109})\ \text{fm}$$
$$a_{22,\text{eff}} = -0.628^{+0.062}_{-0.084} + i(0.396^{+0.108}_{-0.090})\ \text{fm}$$

Summary

For the $e^+e^- \to N\bar{N}$ process, using coupled-channel NREFT with isospin breaking from the p-n mass difference and the Coulomb interactions between p and $\bar{p}$ and fitting to the experimental data, we obtain

- Near-threshold line shape: one sharp cusp at the $n\bar{n}$ threshold for Fit 1; one bump between the two thresholds for Fit 2.
- Poles: an isoscalar near-threshold quasibound state pole on the physical RS (RS_{++}); an isovector pole on the unphysical RS_{-+}.
- Constraint from $\bar{p}$-H atom: similar to the Fit 2.
- Amplitudes for $N\bar{N}$ FSIs: $J/\psi \to \gamma p\bar{p}$, $B^\pm \to K^\pm p\bar{p}$.

Thank you for your attention!

Backup

- Scattering length: S. Sakai, F.-K. Guo, and B. Kubis, Phys. Lett. B 808, 135623 (2020).

$$a_{ij,\text{eff}} = -\frac{1}{2\pi}\left(\boldsymbol{\mu}^{\frac{1}{2}}\,\boldsymbol{T}_{SC}(E=2m_i)\,\boldsymbol{\mu}^{\frac{1}{2}}\right)_{ij}, \qquad \boldsymbol{\mu} = \begin{pmatrix}\mu_p & 0\\ 0 & \mu_n\end{pmatrix}$$

- Coulomb-Strong scattering amplitude at the threshold $E = 2m_p$:

$$\mathbf{T}_{\text{SC}}^{\text{thr}} = -2\pi\boldsymbol{\mu}^{-\frac{1}{2}}\,\boldsymbol{a}_{NN,\text{eff}}\,\boldsymbol{\mu}^{-\frac{1}{2}}, \qquad \boldsymbol{a}_{NN,\text{eff}} = \begin{pmatrix} a_{11,\text{eff}}^{\text{Re}} + a_{11,\text{eff}}^{\text{Im}} & a_{12,\text{eff}}^{\text{Re}} + a_{12,\text{eff}}^{\text{Im}} \\ a_{12,\text{eff}}^{\text{Re}} + a_{12,\text{eff}}^{\text{Im}} & \tilde{a}_{22,\text{eff}}^{\text{Re}} + \tilde{a}_{22,\text{eff}}^{\text{Im}} \end{pmatrix}$$

- Coulomb-Strong scattering amplitude:

$$\mathbf{T}_{SC}^{-1}(E) = \mathbf{V}_S^{-1}(\Lambda) - \mathbf{G}_{\text{C}}(E,\Lambda) = \frac{1}{2\pi}\boldsymbol{\mu}^{\frac{1}{2}}\begin{pmatrix} -\frac{1}{a_{11}} & \frac{1}{a_{12}} \\ \frac{1}{a_{12}} & -\frac{1}{a_{22}} \end{pmatrix}\boldsymbol{\mu}^{\frac{1}{2}} - \mathbf{G}_{\text{C}}^{\text{R}}(E),$$

$$\mathbf{T}_{\text{SC}}^{-1}(E) = \left(\mathbf{V}_{\text{S}}^{\text{R}}\right)^{-1} - \mathbf{G}_{\text{C}}^{\text{R}}(E)$$

$$= \left(\mathbf{T}_{\text{SC}}^{\text{thr}}\right)^{-1} + \mathbf{G}_{\text{C}}^{\text{R}}(E=2m_p) - \mathbf{G}_{\text{C}}^{\text{R}}(E) = \left(\mathbf{T}_{\text{SC}}^{\text{thr}}\right)^{-1} - \tilde{\mathbf{G}}_{\text{C}}^{\text{R}}$$

- Renormalized Green's function:

$$\mathbf{G}_{\text{C}}^{\text{R}}(E) = \begin{pmatrix} -\frac{\mu_p}{2\pi}\kappa_p(E) & 0 \\ 0 & -i\frac{\mu_n}{2\pi}k_n(E) \end{pmatrix}$$

Backup

Cut off dependence of the scattering lengths