2025年超级陶粲装置研讨会（7月4日）

Theoretical overview of radiative and semileptonic decays of hyperons

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1

Contents

Contents

Weak decays of hyperons—complicated

Inputs

Non-leptonic hyperon decays:
$B_i \to B_f\,\pi$

- S/P puzzle
- CP violation

Weak radiative hyperon decays (WRHDs):
$B_i \to B_f\,\gamma$

- WRHDs puzzle

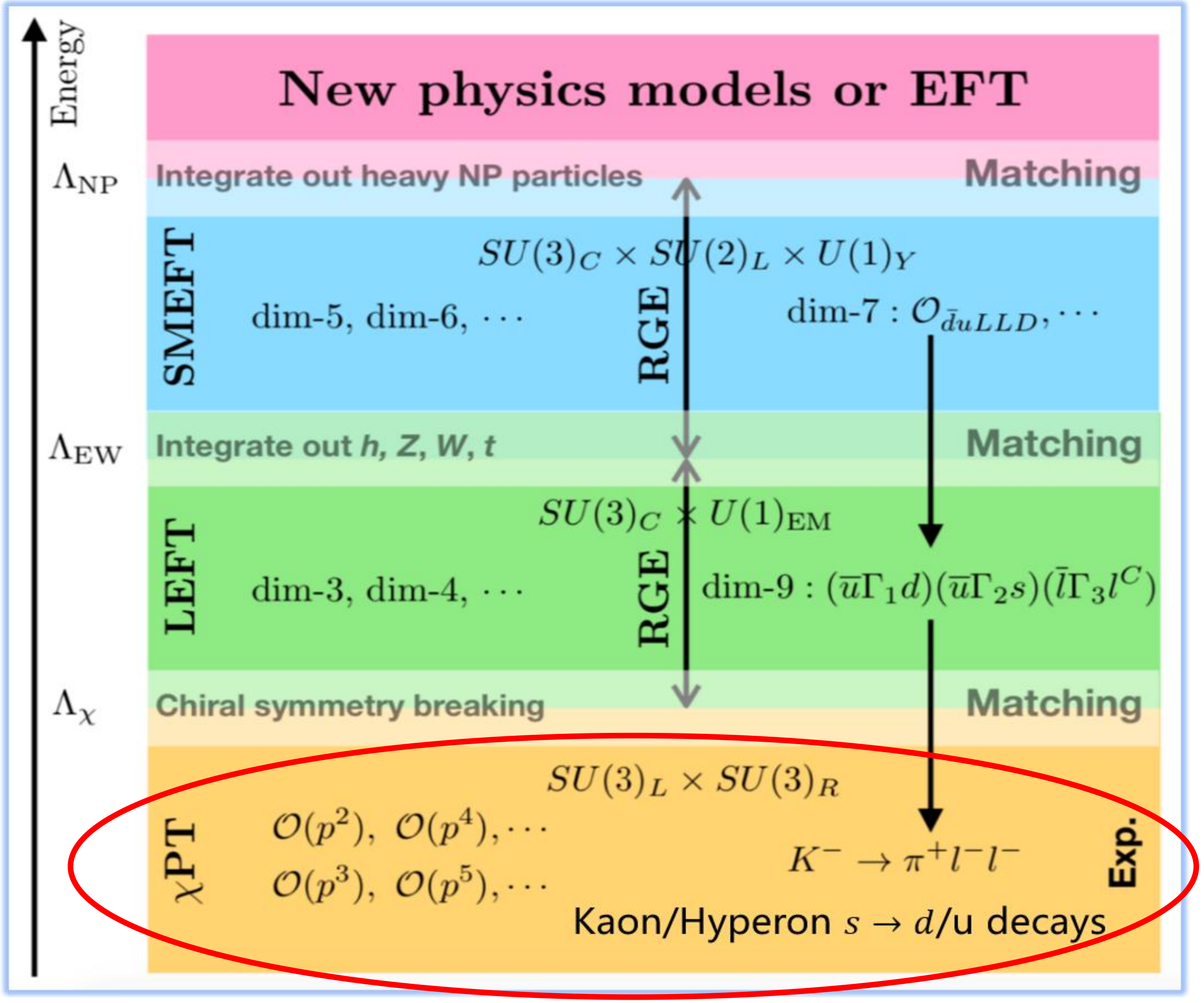
Rare semi-leptonic decays of hyperons:
- $B_i \to B_f \gamma^* \to B_f l\, l$ and $B_i \to B_f \nu\bar{\nu}$
- $B_i \to B_f l \nu$

- NP
- V_{us}

A coherent understanding of all three is compulsory, but remains challenging

Contents

Effective field theories: Bottom-up approach to new physics



- **Low-energy effective Hamiltonian approach at scale $\mathcal{O}(m_s)$**

  $s \to d\nu\bar{\nu}$ **transitions:**

$$\mathcal{H}_{\rm eff} = (C_{\nu_\ell}^{\rm L,SM} + \delta C_{\nu_\ell}^L)\underline{\left(\bar{d}\gamma_\mu(1-\gamma_5)s\right)(\bar{\nu}_\ell\gamma^\mu(1-\gamma_5)\nu_\ell)}$$
$$+ C_{\nu_\ell}^R \underline{\left(\bar{d}\gamma_\mu(1+\gamma_5)s\right)(\bar{\nu}_\ell\gamma^\mu(1-\gamma_5)\nu_\ell)}$$

Perturbation calculation NP χPT

- **Chiral perturbation theory (χPT)** is a powerful tool to deal with the QCD non-perturbative effects

Chiral perturbation theory

□Lagrangian

$$\mathcal{L} = \sum_i c_i\,(Q,\Lambda)\,O_i(\{\psi\})$$

$\boldsymbol{Q}$：软能标，$\boldsymbol{\Lambda}$：硬能标, $\boldsymbol{C_i}$：低能常数， $\boldsymbol{O_i}$：包含场的算符.

□Power counting rule

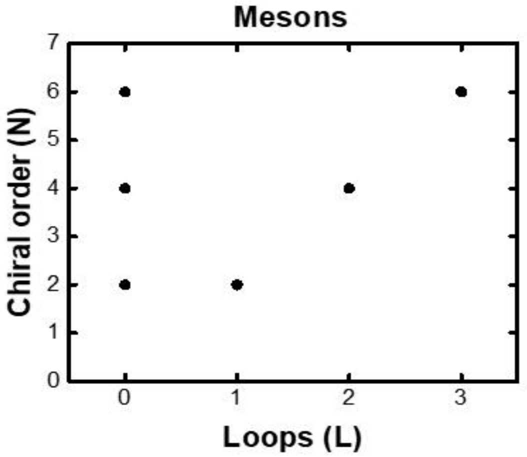
Chiral order: $\boldsymbol{N = 4L - 2N_M - N_B + \sum k\,V_k}$

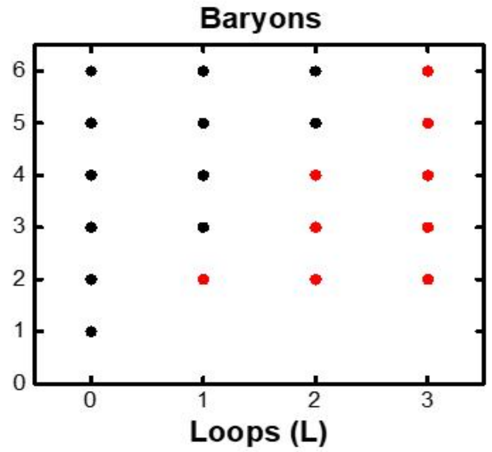
Power counting:

⇩

$O(Q/\Lambda)^n$

□Power-counting-breaking problem





Baryon /Meson system：

Red dots: PCB terms

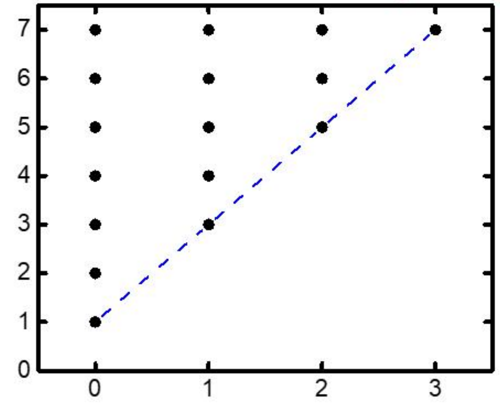
HB vs. Infrared vs. EOMS

LSG, Front. Phys. (Beijing) 8 (2013) 328

Extended-on-mass-shell (EOMS) BChPT

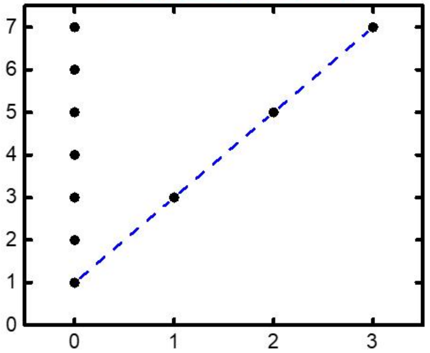
-satisfies all symmetry and analyticity constraints

-converges relatively faster--an appealing feature



Heavy baryon (HB) ChPT

- non-relativistic
- breaks analyticity of loop amplitudes
- converges slowly (particularly in three-flavor sector)
- strict PC and simple nonanalytical results



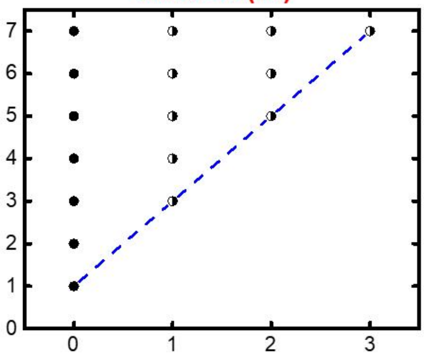
Infrared BChPT

-relativistic

-breaks analyticity of loop amplitudes

-converges slowly (particularly in three-flavor sector)

-analytical terms the same as HBChPT



Successful applications of EOMS

LSG,
Front. Phys. (Beijing) 8 (2013) 328

PRL **101,** 222002 (2008) PHYSICAL REVIEW LETTERS week ending 28 NOVEMBER 2008

Leading SU(3)-Breaking Corrections to the Baryon Magnetic Moments in Chiral Perturbation Theory

Baryon magnetic moments

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(Received 9 May 2008; published 26 November 2008)

We calculate the baryon magnetic moments using covariant chiral perturbation theory (χPT) within the extended-on-mass-shell renormalization scheme. By fitting the two available low-energy constants, we improve the Coleman-Glashow description of the data when we include the leading SU(3)-breaking effects coming from the lowest-order loops. This success is in dramatic contrast with previous attempts at the same order using heavy-baryon χPT and covariant infrared χPT. We also analyze the source of this improvement with particular attention to the comparison between the covariant results.

PHYSICAL REVIEW LETTERS **130,** 071902 (2023)

Cross-Channel Constraints on Resonant Antikaon-Nucleon Scattering

Meson-baryon scattering

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[6]*Institute for Nuclear Studies and Department of Physics, The George Washington University, Washington D.C., 20052, USA*
[7]*Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia, USA*
[8]*Helmholtz-Institut für Strahlen- und Kernphysik (Theorie) and Bethe Center for Theoretical Physics, Universität Bonn, D–53115 Bonn, Germany*

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Chiral perturbation theory and its unitarized versions have played an important role in our understanding of the low-energy strong interaction. Yet, so far, such studies typically deal exclusively with perturbative or nonperturbative channels. In this Letter, we report on the first global study of meson-baryon scattering up to one-loop order. It is shown that covariant baryon chiral perturbation theory, including its unitarization for the negative strangeness sector, can describe meson-baryon scattering data remarkably well. This provides a highly nontrivial check on the validity of this important low-energy effective field theory of QCD. We show that the $\bar{K}N$ related quantities can be better described in comparison with those of lower-order studies, and with reduced uncertainties due to the stringent constraints from the πN and KN phase shifts. In particular, we find that the two-pole structure of $\Lambda(1405)$ persists up to one-loop order reinforcing the existence of two-pole structures in dynamically generated states.

PHYSICAL REVIEW LETTERS **128,** 142002 (2022)

Accurate Relativistic Chiral Nucleon-Nucleon Interaction up to Next-to-Next-to-Leading Order

Nucleon-nucleon scattering

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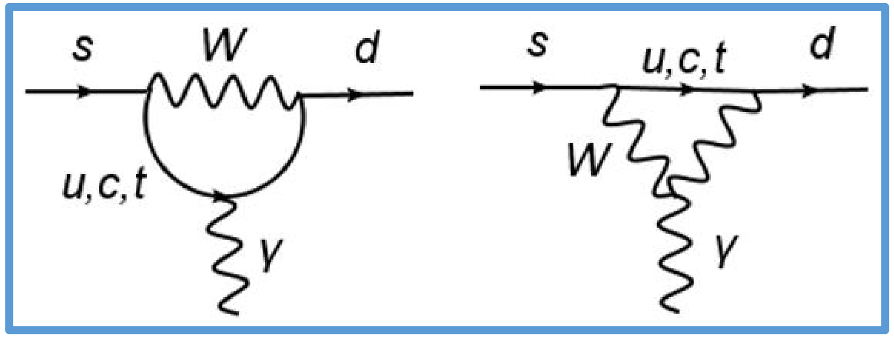
We construct a relativistic chiral nucleon-nucleon interaction up to the next-to-next-to-leading order in covariant baryon chiral perturbation theory. We show that a good description of the np phase shifts up to $T_{\text{lab}} = 200$ MeV and even higher can be achieved with a $\tilde{\chi}^2$/d.o.f. less than 1. Both the next-to-leading-order results and the next-to-next-to-leading-order results describe the phase shifts equally well up to $T_{\text{lab}} = 200$ MeV, but for higher energies, the latter behaves better, showing satisfactory convergence. The relativistic chiral potential provides the most essential inputs for relativistic *ab initio* studies of nuclear structure and reactions, which has been in need for almost two decades.

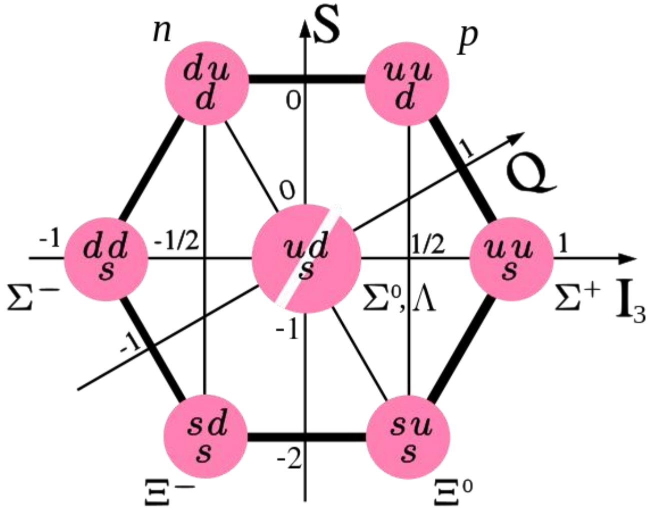
Contents

What are weak radiative hyperon decays

- Weak radiative hyperon decays (WRHDs) are interesting physical processes involving the **electromagnetic, weak, and strong** interactions

- $s \rightarrow d\,\gamma$ transitions at the quark level





- Six WRHDs channels of the ground-state octet baryons

$$\Lambda \rightarrow n\gamma \qquad \Sigma^0 \rightarrow n\gamma \qquad \Xi^0 \rightarrow \Sigma^0\gamma$$
$$\Sigma^+ \rightarrow p\gamma \qquad \Xi^0 \rightarrow \Lambda\gamma \qquad \Xi^- \rightarrow \Sigma^-\gamma$$

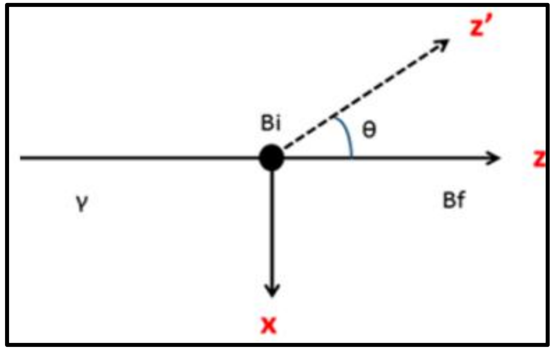
What are weak radiative hyperon decays

- The effective Lagrangian describing the $B_i \to B_f\,\gamma$ WRHDs

$$\mathcal{L} = \frac{eG_F}{2}\bar{B}_f(a + b\gamma_5)\sigma^{\mu\nu}B_iF_{\mu\nu},$$

a: partity-conserving amplitude **b: partity-violating amplitude**

- **Observables** for the WRHDs



$$\frac{d\Gamma}{d\cos\theta} = \frac{e^2G_F^2}{\pi}(|a|^2 + |b|^2)[1 + \frac{2\mathrm{Re}(ab^*)}{|a|^2 + |b|^2}\cos\theta]\cdot|\vec{k}|^3,$$

$$\boxed{\alpha_\gamma = \frac{2\mathrm{Re}(ab^*)}{|a|^2 + |b|^2}, \qquad \Gamma = \frac{e^2G_F^2}{\pi}(|a|^2 + |b|^2)\cdot|\vec{k}|^3,} \qquad |\vec{k}| = \frac{m_i^2 - m_f^2}{2m_i}$$

α_γ: the asymmetry parameter
θ: the angle between the spin of the initial baryon B_i and the 3-momentum $\vec{k}$ of the final baryon B_f

Why study WRHDs: the WRHDs puzzle

Hara's theorem Y. Hara, PRL12, 378 (1964)

- Based on gauge invariance, CP conservation, and U-spin symmetry

- Hara's theorem dictates that the WRHDs $B \to B'\gamma$ and $B' \to B\gamma$ must be identical under the U-spin transformation $s \leftrightarrow d$

$$\mathcal{L}_{\text{p.c.}} = a\left(\bar{p}\sigma^{\mu\nu}\Sigma^{+}F_{\mu\nu} + \bar{\Sigma}^{+}\sigma^{\mu\nu}pF_{\mu\nu}\right)\frac{eG_F}{2},$$

$$\mathcal{L}_{\text{p.v.}} = b\left(\bar{p}\sigma^{\mu\nu}\gamma_5\Sigma^{+}F_{\mu\nu} - \bar{\Sigma}^{+}\sigma^{\mu\nu}\gamma_5 pF_{\mu\nu}\right)\frac{eG_F}{2},$$

leading to

$$b = -b, \quad \text{i.e.} \quad b = 0$$

Why study WRHDs: the WRHDs puzzle

PHYSICAL REVIEW VOLUME 188, NUMBER 5 25 DECEMBER 1969

Asymmetry Parameter and Branching Ratio of $\Sigma^+ \to p\gamma$*

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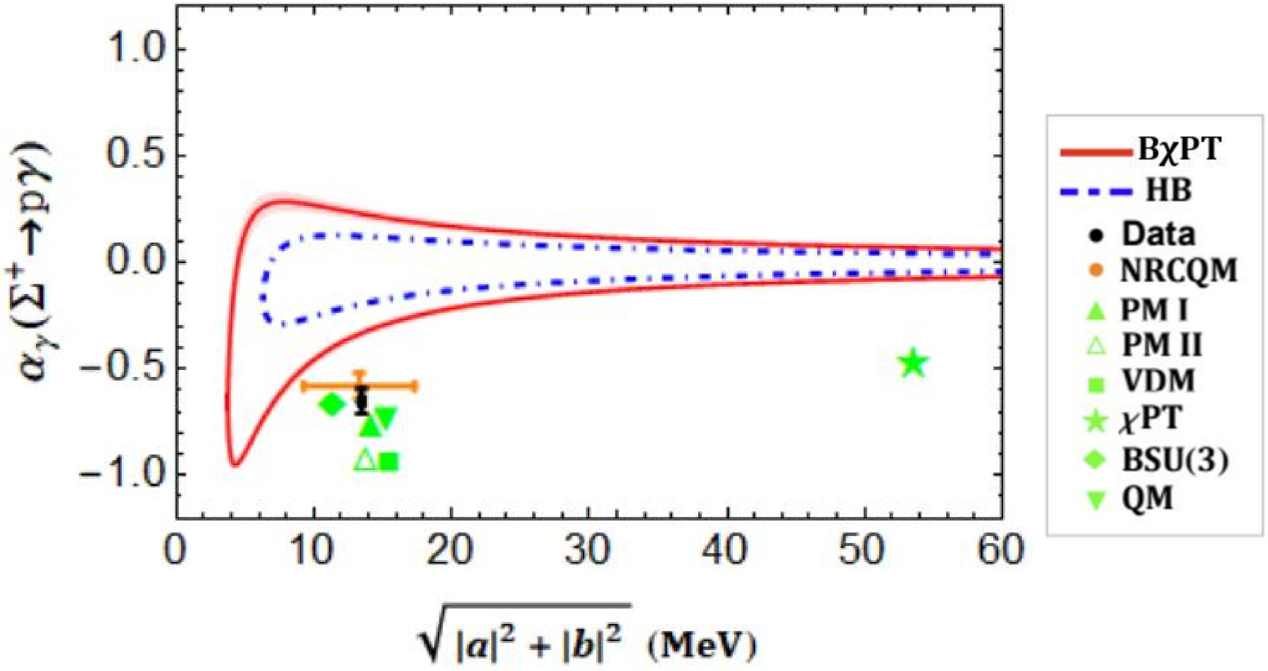
(Received 25 August 1969)

An experiment to study the decay $\Sigma^+ \to p\gamma$ was performed in the Berkeley 25-in. hydrogen bubble chamber. An analysis was made of 48 000 events of the type $K^-p \to \Sigma^+\pi^-$, $\Sigma^+ \to p+$neutral with K^- momenta near 400 MeV/c. The Σ's produced in this momentum region are polarized because of the interference of the Y_0^* (1520) amplitude with the background amplitudes. We have measured the proton asymmetry parameter α for 61 $\Sigma^+ \to p\gamma$ events with an average polarization of 0.4. We found $\alpha = -1.03_{-0.42}^{+0.52}$. $SU(3)$ predicts a value $\alpha = 0$. A more restricted sample of events was used to determine the $\Sigma^+ \to p\gamma$ branching ratio. From 31 $\Sigma^+ \to p\gamma$ events and 11 670 $\Sigma^+ \to p\pi^0$ events, we found $(\Sigma^+ \to p\gamma)/(\Sigma^+ \to p\pi^0) = (2.76 \pm 0.51) \times 10^{-3}$. The result is in agreement with the previous measurements.

Why study WRHDs: the WRHDs puzzle

- The $\Sigma^+ \to p$ asymmetry parameter remains large and negative:

  −0.652±0.056stat±0.020syst.



Data: BESIII, PRL130 (2023) 21, 211901

HB χPT: E. E. Jenkins et al, NPB 397, 84 (1993)

$B\chi PT$: H. Neufeld, Nucl. Phys. B 402, 166 (1993)

NRCQM: Qiang Zhao et al, CPC45, 013101 (2021)

PM1: M. B. Gavela et al, PLB 101, 417 (1981)

PM2: G. Nardulli, PLB 190, 187 (1987)

VDM: P. Zenczykowski, PRD 44, 1485 (1991)

χPT: B. Borasoy et al, PRD 59, 054019 (1999)

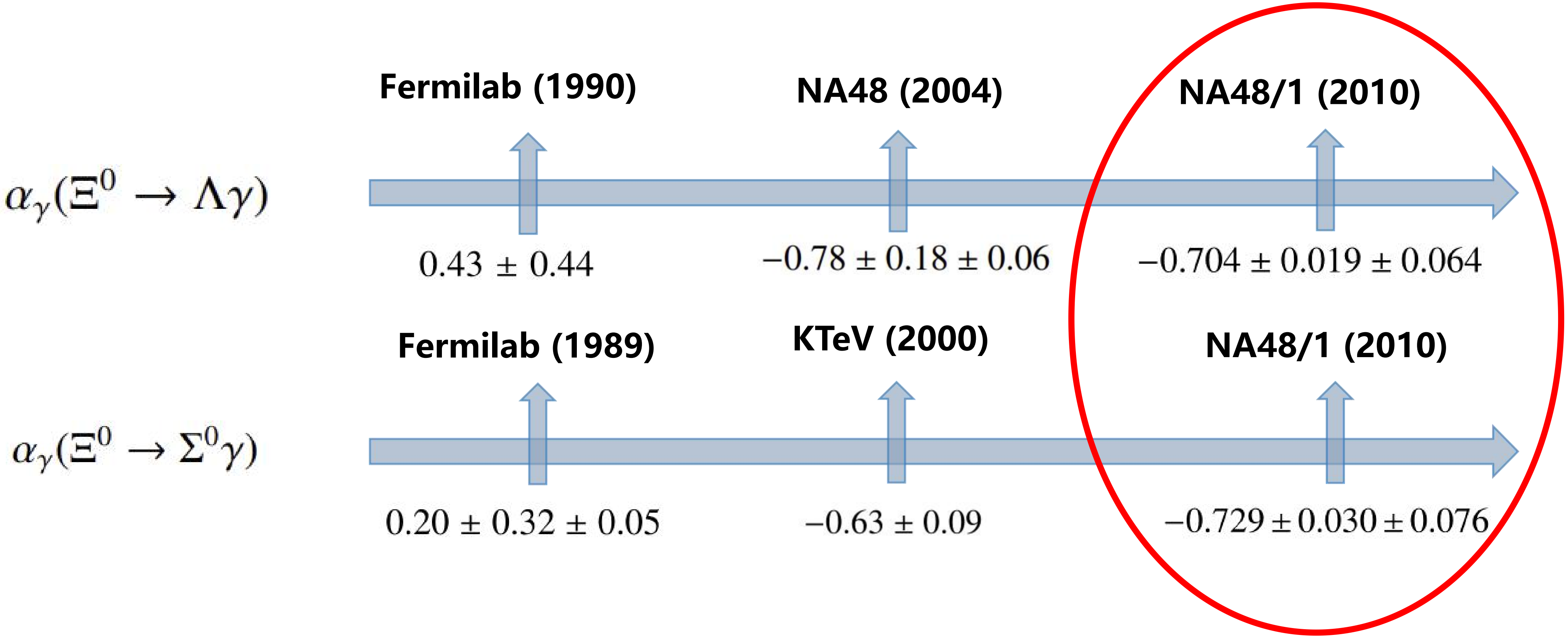
BSU(3): P. Zenczykowski, PRD 73, 076005 (2006)

QM: E. N. Dubovik et al, Phys. Atom. Nucl. 71, 136 (2008)

- Although some models predictions are in agreement with the measurement of the large asymmetry for the $\Sigma^+ \to p\,\gamma$ decay, **they explain poorly the data of other WRHDs**

Why study WRHDs： experimentally challenging

- **Significant changes** in the asymmetry parameters of $\Xi^0 \to \Sigma^0\gamma$ and $\Xi^0 \to \Lambda\gamma$



Why study WRHDs--$\Lambda \rightarrow n\gamma$

- **New BESIII** measurement for the $\Lambda \rightarrow n\,\gamma$ decay (PRL129(2022)21,212002)

Decay Mode	$\Lambda \rightarrow n\gamma$	$\bar{\Lambda} \rightarrow \bar{n}\gamma$
$N_{\rm ST}$ $(\times 10^3)$	6853.2 ± 2.6	7036.2 ± 2.7
$\varepsilon_{\rm ST}$ (%)	51.13 ± 0.01	52.53 ± 0.01
$N_{\rm DT}$	723 ± 40	498 ± 41
$\varepsilon_{\rm DT}$ (%)	6.58 ± 0.04	4.32 ± 0.03
BF $(\times 10^{-3})$	$0.820 \pm 0.045 \pm 0.066$	$0.862 \pm 0.071 \pm 0.084$
	$0.832 \pm 0.038 \pm 0.054$	
α_γ	$-0.13 \pm 0.13 \pm 0.03$	$0.21 \pm 0.15 \pm 0.06$
	$-0.16 \pm 0.10 \pm 0.05$	

PDG2022

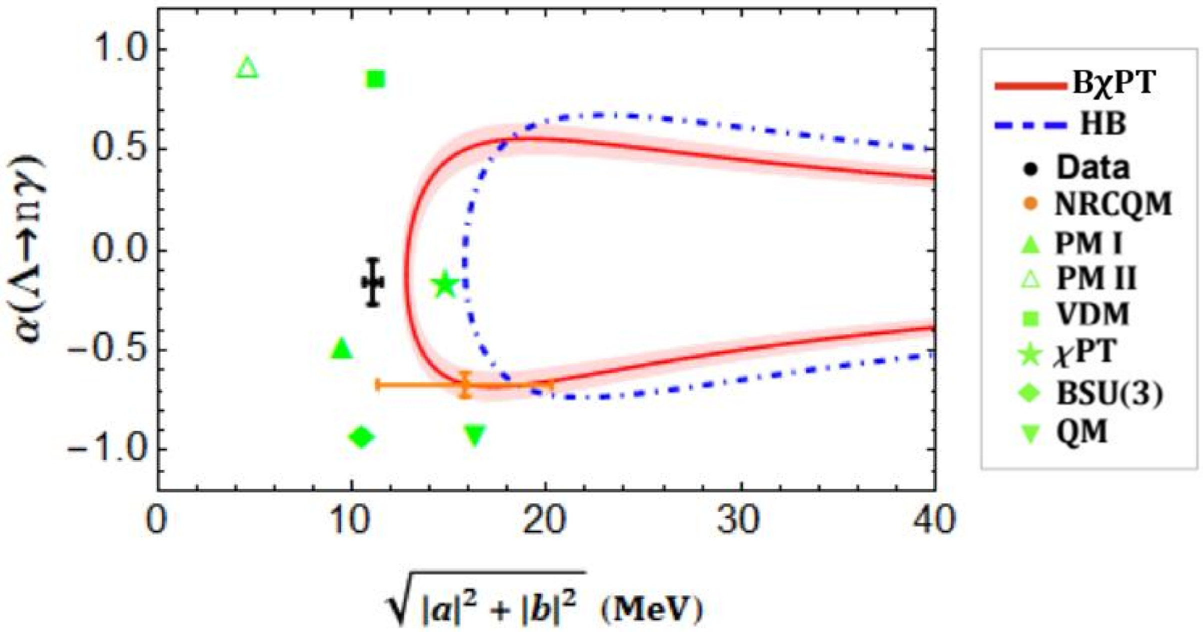
$\Gamma(n\gamma)/\Gamma_{\rm total}$ Γ_3/Γ

VALUE (units 10^{-3})	*EVTS*	*DOCUMENT ID*		*TECN*	*COMMENT*
1.75±0.15 OUR FIT					
1.75±0.15	1816	LARSON	93	SPEC	$K^- p$ at rest
• • • We do not use the following data for averages, fits, limits, etc. • • •					
$1.78 \pm 0.24^{+0.14}_{-0.16}$	287	NOBLE	92	SPEC	See LARSON 93

- The branching fraction is only **about one half** of the current PDG average
- The asymmetry parameter α_γ **is determined for the first time**

Why study WRHDs--$\Lambda \to n\gamma$

- **None of the existing predictions can describe** the new BESIII measurement for the $\Lambda \to n\gamma$ decay



Data: BESIII, PRL129(2022)21,212002

HB χPT: E. E. Jenkins et al, NPB 397, 84 (1993)

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χPT: B. Borasoy et al, PRD 59, 054019 (1999)

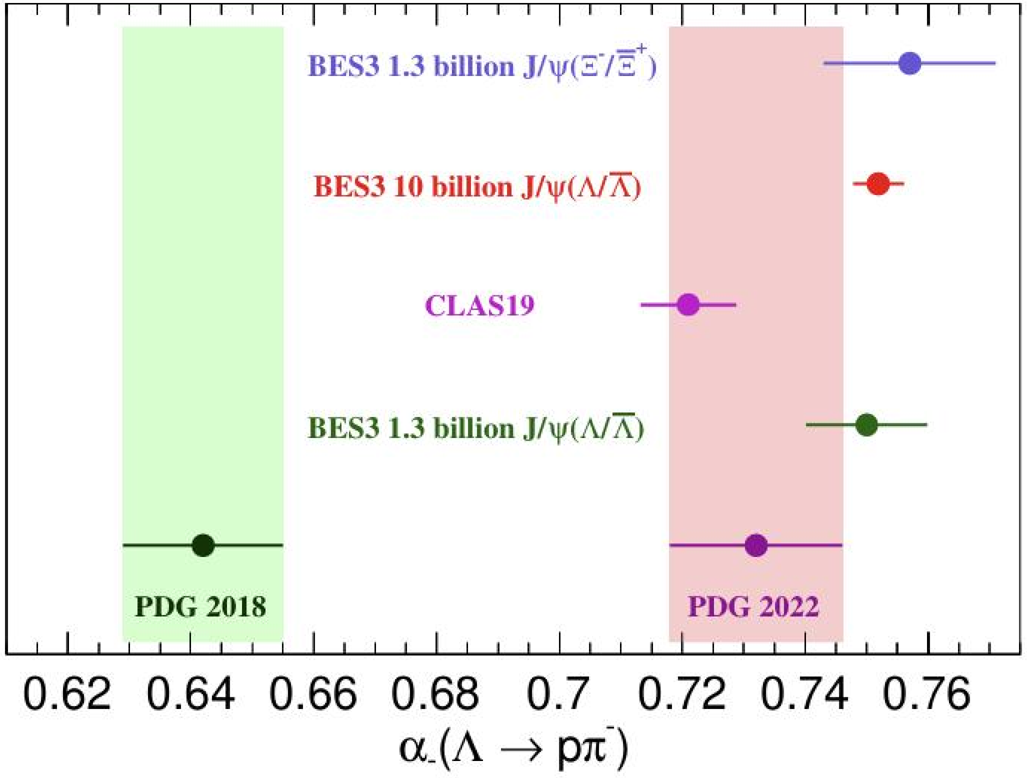
BSU(3): P. Zenczykowski, PRD 73, 076005 (2006)

QM: E. N. Dubovik et al, Phys. Atom. Nucl. 71, 136 (2008)

Why study WRHDs

- **New BESIII and CLAS data for the hyperon non-leptonic decays**

BESIII: Nature Phys. 15, 631 (2019) | CLAS: PRL123,182301 (2019)
BESIII: Nature 606, 64 (2022) | BESIII: PRL129,131801 (2022)



- Decay parameter for the $\Lambda \to p\,\pi^-$ decay

$$\mathcal{M}(B_i \to B_f \pi) = iG_F m_\pi^2 \bar{B}_f \left(A_S - A_P \gamma_5\right) B_i$$

$$\boxed{\alpha_\pi = \frac{2\mathrm{Re}\,(s \cdot p)}{|s|^2 + |p|^2}} \qquad s = A_S \qquad p = A_P|\vec{q}|/(E_f + m_f)$$

- Featured by a **larger statistics** and a **small uncertainty** and very different from previous PDG average

- A significant change for the baryon decay parameter of $\Lambda \to p\,\pi^-$ may **greatly affect the values of LECs hD, hF, and hyperon non-leptonic decay amplitudes as inputs for WRHDs**

Why study WRHDs—theoretical tools

- Theoretically, **two phenomenological models** can explain the current experimental data of WRHDs at least qualitatively **except for the $\Lambda \to n\,\gamma$ decay**
  - *E. N. Dubovik et al, Phys. Atom. Nucl. 71, 136 (2008)*
  - *P. Zenczykowski, PRD 73, 076005 (2006)*

- **Chiral perturbation theory (χPT)** studies on the WRHDs
  - *B. Borasoy et al, PRD 59, 054019 (1999) (Tree level)*
  - *E. E. Jenkins et al, NPB397, 84 (1993)*
  - *J. W. Bos et al, PRD 51, 6308 (1995)*
  - *J. W. Bos et al, PRD 54, 3321 (1996)*
  - *J. W. Bos, et al, PRD 57, 4101 (1998)*

  (Loop level in the heavy baryon formulation)

  - *H. Neufeld, NPB 402, 166 (1993) (Loop level in the covariant formulation)*

Our purpose

Our goal is to study the WRHDs in covariant baryon chiral perturbation theory (BχPT) with the extended-on-mass-shell (EOMS) renormalization scheme

- **The work in the BχPT** *H. Neufeld, NPB 402, 166 (1993)*

✓ The used low energy constants (LECs) and hyperon non-leptonic decay amplitudes are out of date
✓ No efforts were taken to ensure consistent power counting

Update the relevant LECs and hyperon non-leptonic decay amplitudes → Calculate the branching fractions and asymmetry parameters, i.e., **amplitudes a and b**, of the WRHDs order by order → Compare our predictions with those from other approaches/experimental data

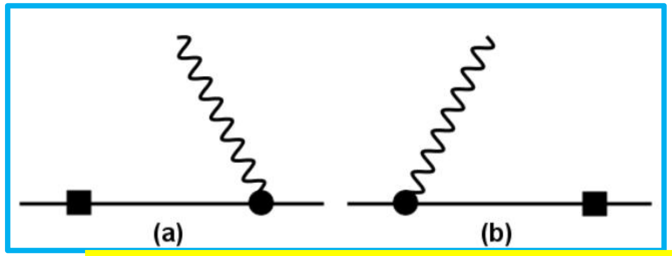
WRHDs in the EOMS BχPT

$$a_{B_iB_f} = a_{B_iB_f}^{(1,\text{tree})} + a_{B_iB_f}^{(2,\text{tree})} + a_{B_iB_f}^{(2,\text{loop})}$$
$$b_{B_iB_f} = b_{B_iB_f}^{(2,\text{tree})} + b_{B_iB_f}^{(2,\text{loop})}$$

Feynman diagrams

Lagrangians

Order contributions



$$\mathcal{L}_{\Delta S=1}^{(0)} = \sqrt{2}G_F m_\pi^2 F_\phi \langle h_D \bar{B}\{u^\dagger \lambda u, B\} + h_F \bar{B}[u^\dagger \lambda u, B]\rangle,$$
$$\mathcal{L}_{MB}^{(2)} = \frac{b_6^D}{8m_B}\langle \bar{B}\sigma^{\mu\nu}\{F_{\mu\nu}^+, B\}\rangle + \frac{b_6^F}{8m_B}\langle \bar{B}\sigma^{\mu\nu}[F_{\mu\nu}^+, B]\rangle,$$

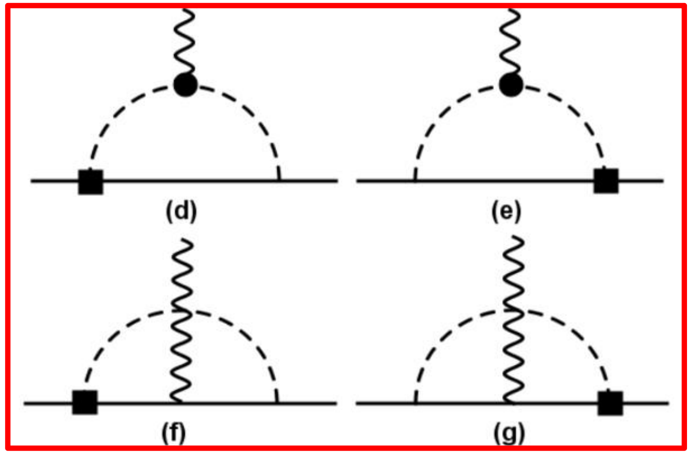
$$a_{B_iB_f}^{(1,\text{tree})}$$

LECs b_6^D and b_6^F :
the experimental data of Octet baryon magnetic moment

- Leading-order LECs hD & hF are determinated by fitting to the latest experimental data on the $B_i \to B_f\, \pi$ decays
- NLO LECs: five C' s

$$b_{B_iB_f}^{(2,\text{tree})}$$

$$\mathcal{L}_\rho^{(2)} = C_\rho \left(\langle \bar{B}\sigma^{\mu\nu}\gamma_5 F_{\mu\nu} Q\rangle\langle B\lambda\rangle - \langle \bar{B}\sigma^{\mu\nu}\gamma_5 F_{\mu\nu}\lambda\rangle\langle BQ\rangle\right)$$



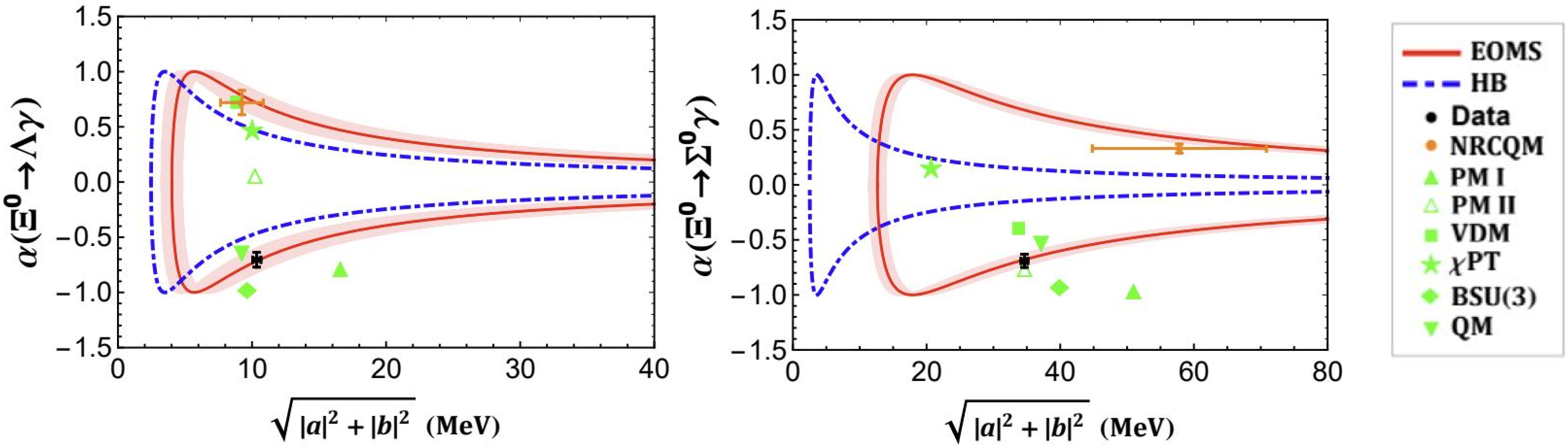
$$\mathcal{L}_{\Delta S=1}^{(0)} = \sqrt{2}G_F m_\pi^2 F_\phi \langle h_D \bar{B}\{u^\dagger \lambda u, B\} + h_F \bar{B}[u^\dagger \lambda u, B]\rangle$$
$$\mathcal{L}_B^{(1)} = \langle \bar{B} i\gamma^\mu D_\mu B - m_0 \bar{B}B\rangle,$$
$$\mathcal{L}_M^{(2)} = \frac{F_\phi^2}{4}\langle u_\mu u^\mu + \chi^+\rangle,$$
$$\mathcal{L}_{MB}^{(1)} = \frac{D}{2}\langle \bar{B}\gamma^\mu\gamma_5\{u_\mu, B\}\rangle + \frac{F}{2}\langle \bar{B}\gamma^\mu\gamma_5[u_\mu, B]\rangle,$$

LECs D and F have been determined in Ref. L. S. Geng et al, PRD 90, 054502 (2014)

$$a_{B_iB_f}^{(2,\text{loop})} \qquad b_{B_iB_f}^{(2,\text{loop})}$$

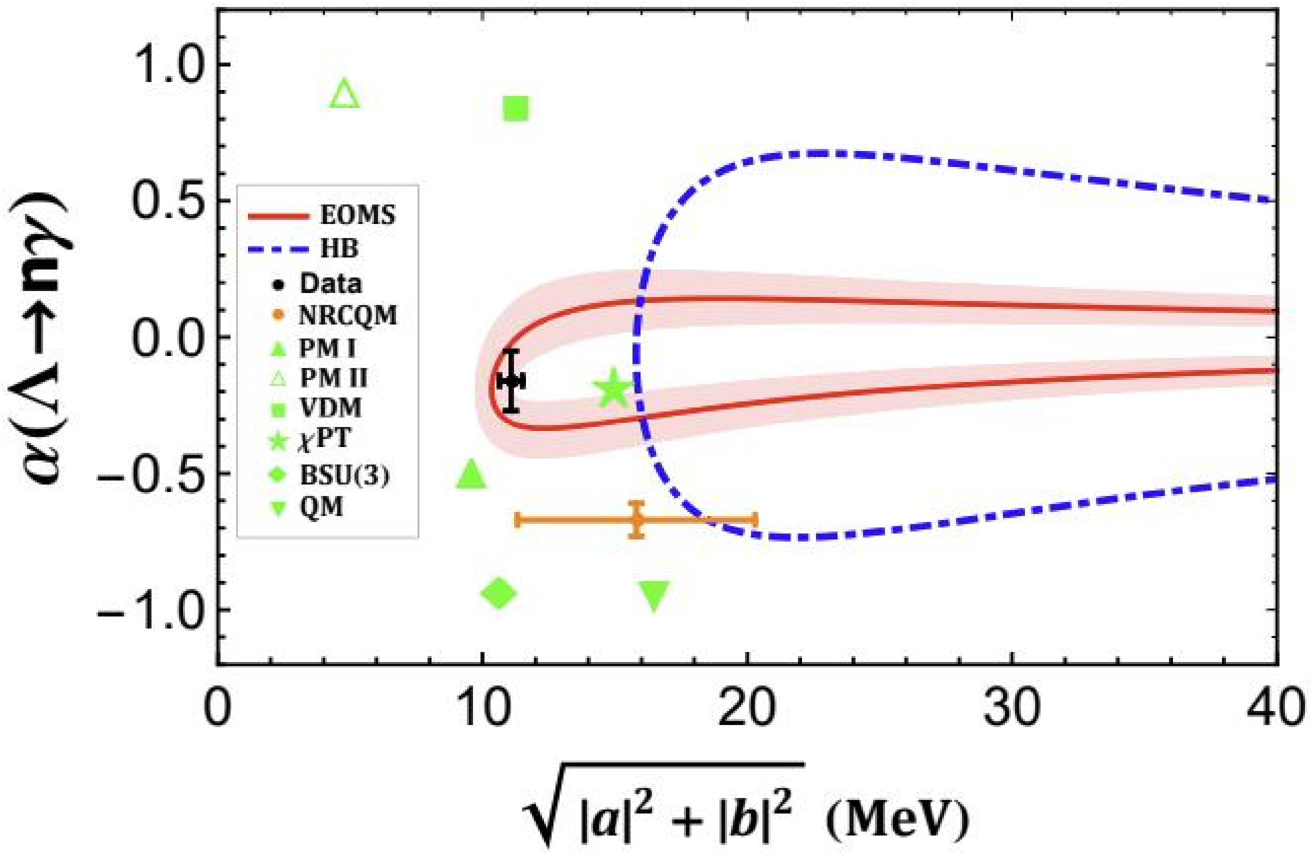
For the amplitude a, weak vertex is γ_5

α_γ of $\Xi^0 \to \Sigma^0\gamma$ and $\Xi^0 \to \Lambda\gamma$ as a function of $\sqrt{|a|^2+|b|^2}$



- $O(p^2)$ counter-term contributions are determined by fitting to $\Xi^0 \to \Sigma^0\gamma$ and $\Xi^0 \to \Lambda\gamma$ for the first time
- The EOMS χPT results manifest as correlations between branching ratios and asymmetry parameters because of the long-standing S/P puzzle in hyperon non-leptonic decays

α_γ of the $\Lambda \to n\,\gamma$ decay as a function of $\sqrt{|a|^2+|b|^2}$



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HB χPT : *E. E. Jenkins et al, NPB 397, 84 (1993)*

NRCQM: *Qiang Zhao et al, CPC45, 013101 (2021)*

PM1: *M. B. Gavela et al, PLB 101, 417 (1981)*

PM2: *G. Nardulli, PLB 190, 187 (1987)*

VDM: *P. Zenczykowski, PRD 44, 1485 (1991)*

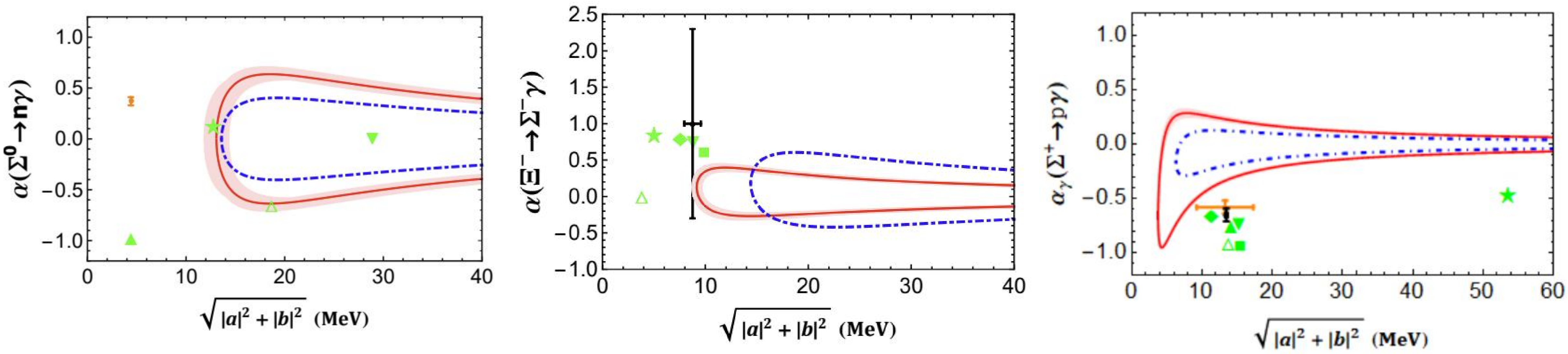
χPT: *B. Borasoy et al, PRD 59, 054019 (1999)*

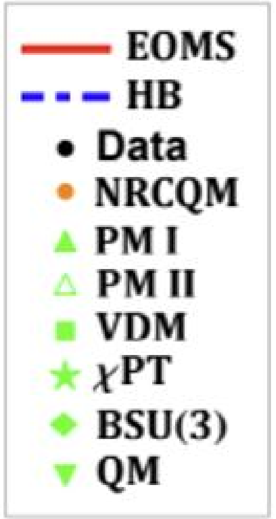
BSU(3): *P. Zenczykowski, PRD 73, 076005 (2006)*

QM: *E. N. Dubovik et al, Phys. Atom. Nucl. 71, 136 (2008)*

- Interestingly, only **EOMS BχPT agrees with** the latest BESIII measurement
- The prediction in the HB χPT **with counter-term contributions** is very close to the BESIII data
- The vector dominance model (VDM) and the pole model (PM II) **are disfavored** by the BESIII data

α_γ of the other WRHDs as a function of $\sqrt{|a|^2+|b|^2}$





- For the $\Sigma^0 \to n\,\gamma$ decay, not yet measured, **our result contradicts** the predictions of PM I and NRCQM
- For the $\Xi^- \to \Sigma^-\gamma$ decay, **our prediction agrees better** with the experimental measurement, and the current PDG data disfavor the results of PM II and tree-level χPT
- **For the $\Sigma^+ \to p\,\gamma$ decay, the results predicted in all the χPT deviate from the PDG average but our prediction is closer**

Hara's theorem: α_γ for $\Xi^- \to \Sigma^-\gamma$ and $\boldsymbol{\Sigma^+ \to p\,\gamma}$ should not be too large.

What is still missing?

- **For the $\Sigma^+ \to p\,\gamma$ decay, the results predicted in all the χPT deviate from the PDG average but our prediction is closer**

- **Could this be somehow rescued?**
  - How about contributions of heavier resonances? Have been tried previously, but the results are a disaster, e.g., B. Borasoy et al, PRD 59, 054019(1999)

$$\alpha^{p\Sigma^+} = -0.49 \quad \alpha^{\Sigma^-\Xi^-} = 0.84$$

$$\alpha^{n\Sigma^0} = 0.12 \quad \alpha^{n\Lambda} = -0.19$$

$$\boxed{\alpha^{\Sigma^0\Xi^0} = 0.15 \quad \alpha^{\Lambda\Xi^0} = 0.46.}$$

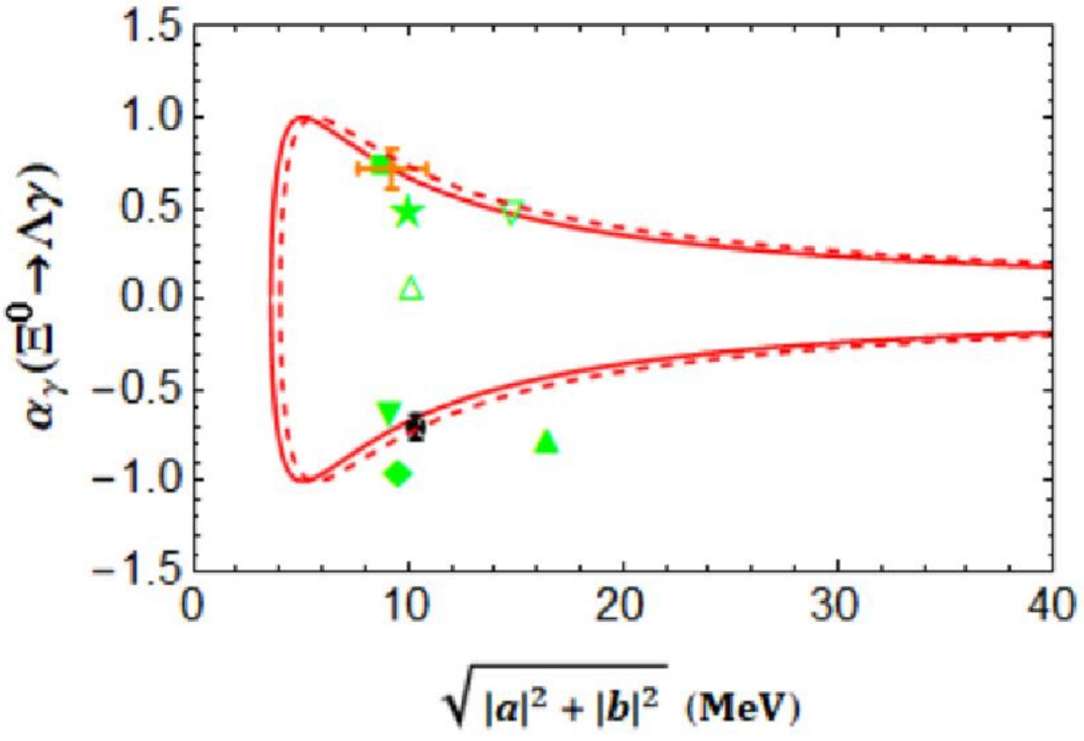
Uncertainties of the relevant LECs are important but remain unstudied.

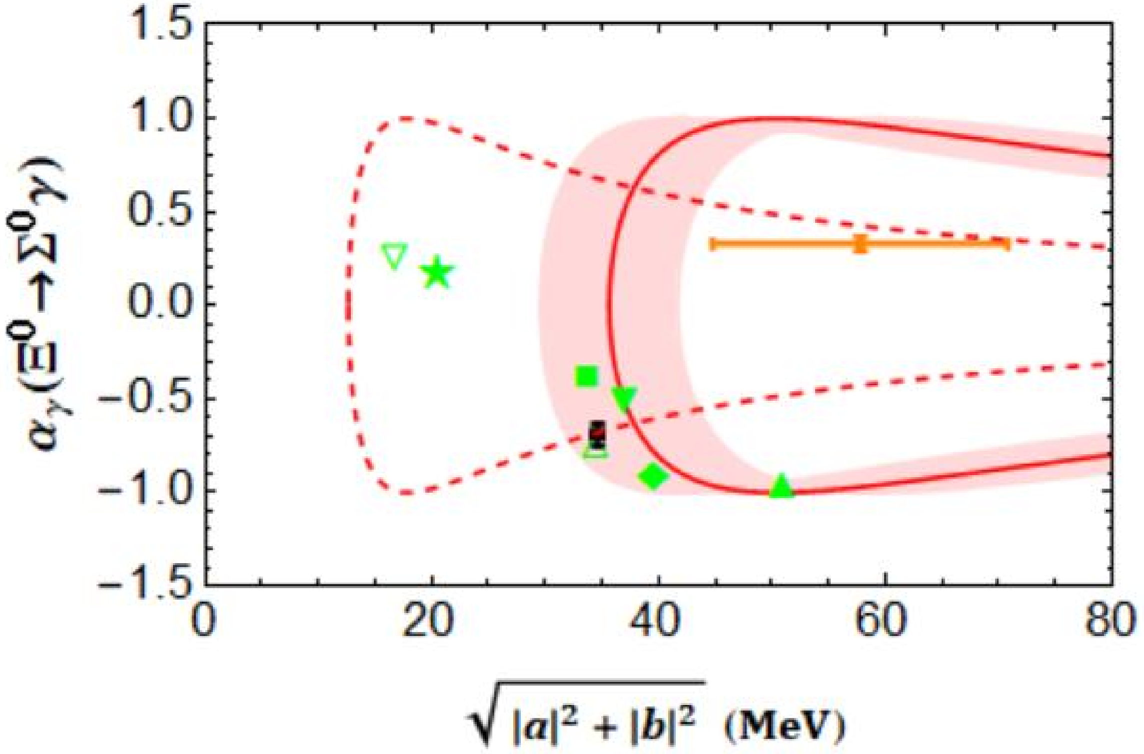
N(1535) DECAY MODES

The following branching fractions are our estimates, not fits or averages.

	Mode	Fraction (Γ_i/Γ)
Γ_1	$N\pi$	32–52 %
Γ_2	$N\eta$	30–55 %
Γ_3	$N\pi\pi$	4–31 %
Γ_4	$\Delta(1232)\pi$, D-wave	1–4 %
Γ_5	$N\rho$	2–17 %
Γ_6	$N\rho$, S=1/2 , S-wave	2–16 %
Γ_7	$N\rho$, S=3/2 , D-wave	<1 %
Γ_8	$N\sigma$	2–10 %
Γ_9	$N(1440)\pi$	5–12 %
Γ_{10}	$p\gamma$, helicity=1/2	0.15–0.30 %
Γ_{11}	$n\gamma$, helicity=1/2	0.01–0.25 %

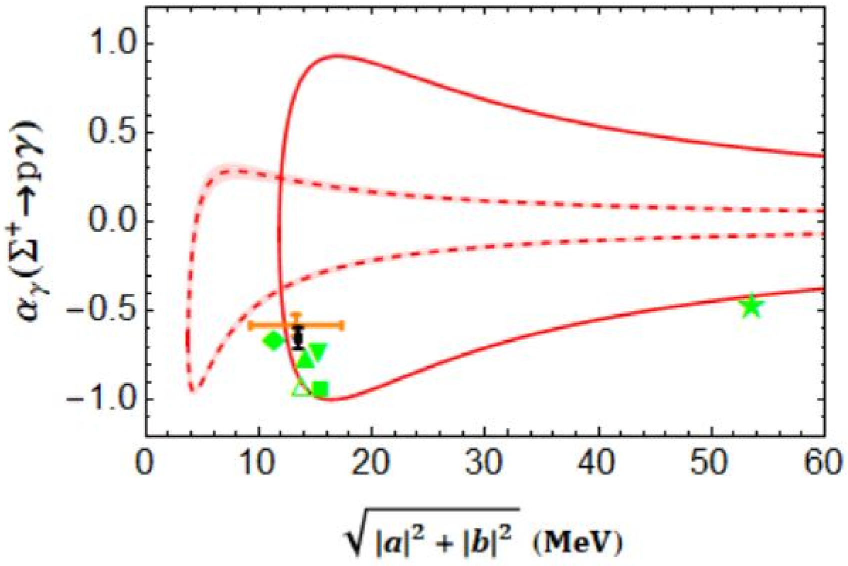
Contributions of heavier resonances

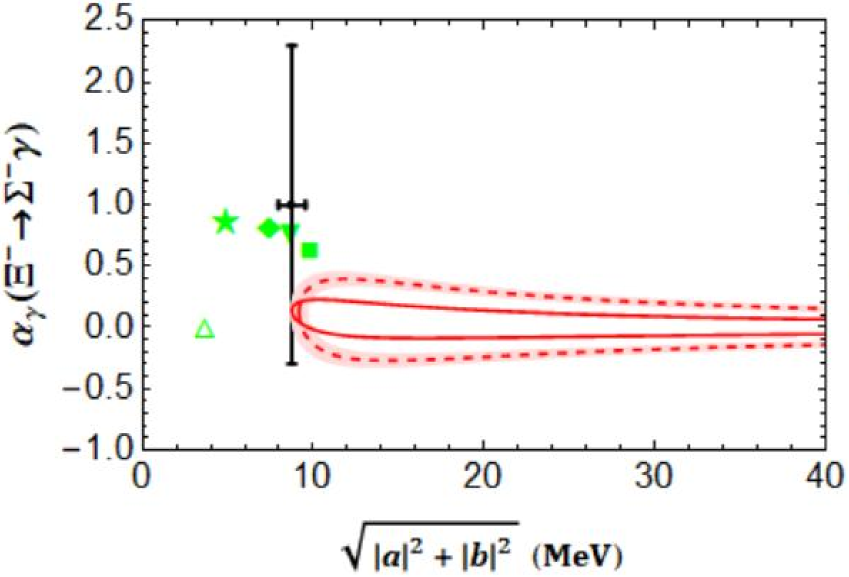


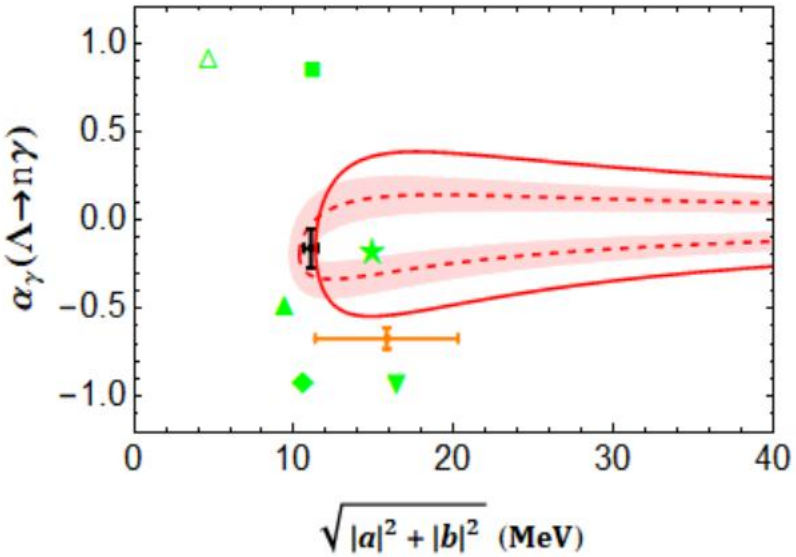


- Solid and dashed lines in red represent the EOMS results with/without heavier resonances, respectively.
- In the figure on the right, we show that after considering the uncertainties of input quantities (LECs), the experimental data can also be well described.

Contributions of heavier resonances





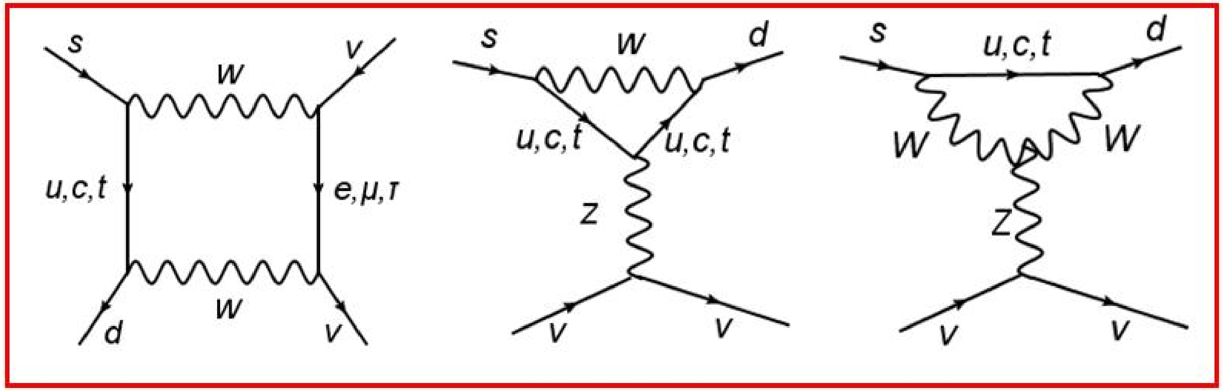


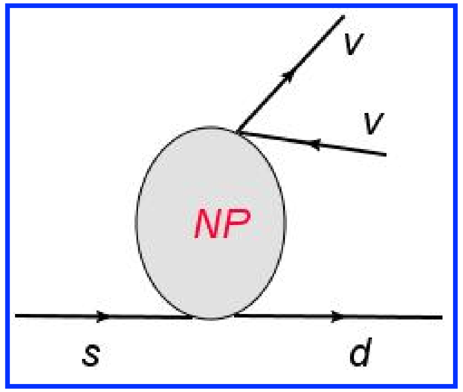
- Contributions of $\frac{1}{2}^-$ states can improve the present EOMS results **(solid lines in red)**
- Uncertainties of resonance contributions are not fully taken into account

Contents

Why to study the rare semileptonic $s \to d$ transitions

- $s \to d$ **transitions are highly suppressed in the SM**





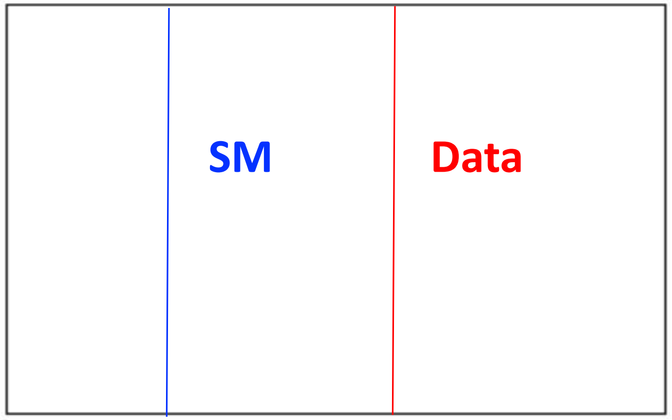
- **As such, they are ideal for tests of the SM and searches for BSM**

  - *G. Buchalla and A. J. Buras,NPB 548 (1999) 309-327*
  - *V. Cirigliano et al., Rev.Mod.Phys. 84 (2012) 399*
  - *Hai-Bo Li, Front.Phys.(Beijing) 12 (2017) 5, 121301*
  - *A. A. Alves Junior et al, JHEP 05 (2019) 048*

$s \to d\nu\bar{\nu}$ transitions: $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$

NA62

@JHEP 06 (2021) 093
and JHEP 11 (2020) 042



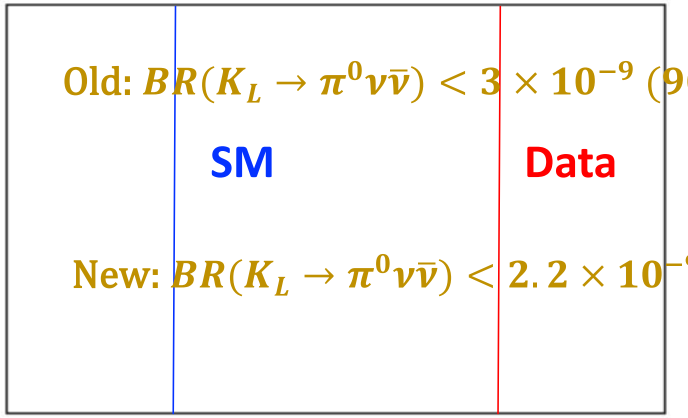
$\mathcal{O}(1)$ $\quad$ $\mathcal{O}(10)$

$\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu}) \times 10^{11}$

New data (in progress)
https://na48.web.cern.ch/Welcome/papers/Overview.html

KOTO

@PRL. 126 (2021) 12,121801t;
PRL. 122 (2019) 021802;
PRL. 134 (2025) 8, 081802 (new data)



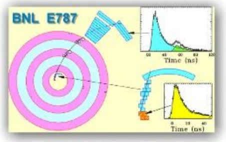
$\mathcal{O}(1)$ $\quad$ $\mathcal{O}(10^2)$

$\mathrm{BR}(K_L \to \pi^0\nu\bar{\nu}) \times 10^{11}$

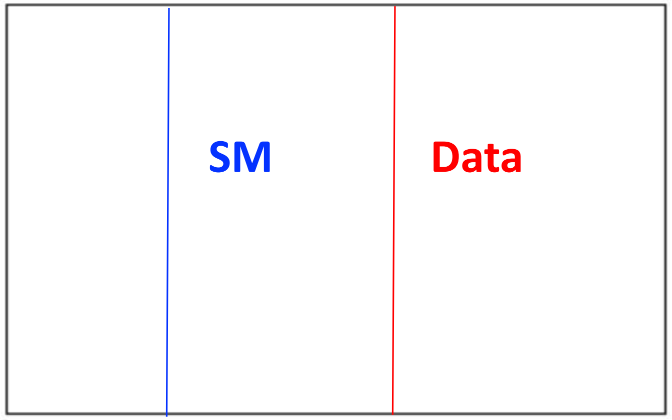
- The $K \to \pi\nu\bar{\nu}$ results imply that there is still room for new physics (NP), but maybe not so much. In addition, they are only sensitive to the vectorial (parity even) couplings of the $s \to d$ currents.

$s \to d\nu\bar{\nu}$ transitions: $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$

- **Latest experimental results**



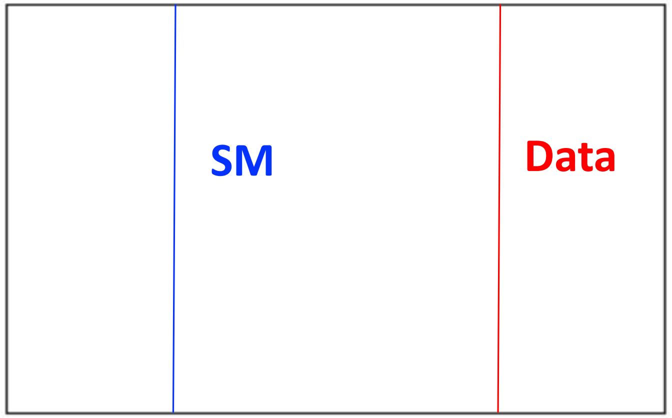
@PRD 63 (2001) 032004



$\mathcal{O}(1)$ $\mathcal{O}(10^9)$

$\mathrm{BR}(K^+ \to \pi^+\pi^0\nu\bar{\nu}) \times 10^{14}$

KEK-PS E391a *@PRD 84 (2011) 052009*



$\mathcal{O}(1)$ $\mathcal{O}(10^6)$

$\mathrm{BR}(K_L \to \pi^0\pi^0\nu\bar{\nu}) \times 10^{13}$

- **Although the $K \to \pi\pi\nu\bar{\nu}$ modes receive contributions from both the vectorial and the axial-vectorial type of NP, the current results provide little constraints on them.**
- **Note that the nonperturbative inputs in previous works are roughly estimated.**

$s \to d\nu\bar{\nu}$ transitions: $B_i \to B_f\nu\bar{\nu}$

◻ Hyperons might be a game changer

- Having spin ½ (instead of spin 0), they lead to different decay modes, observables, as well as sensitivities to the vectorial and the axial-vectorial structure of the $s \to d$ currents

◻ Experimentally and theoretically more challenging, compared to their kaon siblings

- No direct data yet, but promising data from BESIII

  Hai-Bo Li, Front.Phys.(Beijing) 12 (2017) 5, 121301

- On the theory side, the first studies just appeared

  Xiao-Hui Hu et al., CPC43(2019)093104; Jusak Tandean, JHEP04(2019)104;
  Jhih-Ying Su et al.,y, PRD 102 (2020) 075032; Gang Li et al., PRD 100 (2019) 075003

- More theoretical studies are needed

  - ✓ Constraints from/compare with more kaon modes
  - ✓ The state of the art results from covariant baryon chiral perturbation theory for the relevant form factors

  Li-Sheng Geng et al., PRD 79, 094022 (2009) ; T. Ledwig et al., PRD 90, 054502 (2014)

$s \to d\mu^+\mu^-$ transitions: $K_L \to \mu^+\mu^-$ and $K^+ \to \pi^+\mu^+\mu^-$

- $s \to d\mu^+\mu^-$ transitions dominated by long-distance contributions

- The branching ratio of the $\mathrm{K_L} \to \mu^+\mu^-$ decay and the leptonic forward-backward asymmetry ($\mathrm{A_{FB}}$) of the $\mathrm{K}^+ \to \pi^+\mu^+\mu^-$ decay have been measured

$$\mathrm{BR}(\boldsymbol{K_L} \to \boldsymbol{\mu^+\mu^-})_{\mathbf{SM}} = \mathbf{7.64(73)} \times \mathbf{10^{-9}}$$

$$\mathrm{BR}(\boldsymbol{K_L} \to \boldsymbol{\mu^+\mu^-})_{\mathbf{exp}} = \mathbf{6.84(11)} \times \mathbf{10^{-9}}$$

PDG 2024

$$A_{FB}(\boldsymbol{K^+} \to \boldsymbol{\pi^+\mu^+\mu^-})_{\mathbf{SM}} = \mathbf{0}$$

$$|A_{FB}|(\boldsymbol{K^+} \to \boldsymbol{\pi^+\mu^+\mu^-})_{\mathbf{exp}} = \mathbf{2.3} \times \mathbf{10^{-2}}, \ \text{at 90\% CL}$$

NA48/2 collaboration, PLB 697, 107 (2011)

$$|A_{FB}|(\boldsymbol{K^+} \to \boldsymbol{\pi^+\mu^+\mu^-})_{\mathbf{exp}} = (\mathbf{0} \pm \mathbf{0.7}) \times \mathbf{10^{-2}}$$

NA48/2 collaboration, JHEP 11 (2022) 011, JHEP 06 (2023) 040

- They **cannot probe all** the interesting axial-vectorial, scalar operators, and their spin flip structures

$s \to d\mu^+\mu^-$ transitions: $B_i \to B_f\mu^+\mu^-$

- Experimentally, no direct data for the leptonic forward-backward asymmetry (A_{FB}) of the $B_i \to B_f\mu^+\mu^-$ decay yet, but promising measurement from LHCb

LHCb collaboration, JHEP05(2019)048

- On the theory side, only Prof. Xiao-Gang He and his collaborators have studied the rare hyperon decay $\Sigma^+ \to p\mu^+\mu^-$

Xiao-Gang He et al., PRD 72 (2005) 074003
Xiao-Gang He et al., JHEP10 (2018) 040

$\boxed{\langle p|\bar{d}\gamma^\kappa s|\Sigma^+\rangle = -\bar{u}_p\gamma^\kappa u_\Sigma}$ **vs.** $\boxed{\langle B_2(p_2)|\bar{d}\gamma_\mu s|B_1(p_1)\rangle = \bar{u}_2(p_2)\left[f_1(q^2)\gamma_\mu + \frac{f_2(q^2)}{M_1}\sigma_{\mu\nu}q^\nu + \frac{f_3(q^2)}{M_1}q_\mu\right]u_1(p_1)}$

$\boxed{\Sigma^+ \to p\mu^+\mu^-}$ **vs.** $\boxed{K_L \to \mu^+\mu^- \text{ and } K^+ \to \pi^+\mu^+\mu^-}$

Our purpose

- **Study the hyperon rare decays and improve the QCD non-perturbative contributions.**
- **To investigate wthether the anticipated data of hyperon rare decays can better constrain new physics or not, compare with their kaon counterparts.**

□ $s \to d\,\nu\,\bar{\nu}$ **transitions dominated by short−distance contributions**

$\Lambda \to n\nu\bar{\nu}$

$\Sigma^+ \to p\nu\bar{\nu}$

$\Xi^0 \to \Sigma^0\nu\bar{\nu}$

$\Xi^0 \to \Lambda\nu\bar{\nu}$

V.S

$K^+ \to \pi^+\nu\bar{\nu}$

$K_L \to \pi^0\nu\bar{\nu}$

$K^+ \to \pi^+\pi^0\nu\bar{\nu}$

$K_L \to \pi^0\pi^0\nu\bar{\nu}$

Channels	$\delta C^L_{\nu_\ell} + C^R_{\nu_\ell}$	$\delta C^L_{\nu_\ell} - C^R_{\nu_\ell}$
$\Lambda \to n\nu\bar{\nu}$	✔	✔
$\Sigma^+ \to p\nu\bar{\nu}$	✔	✔
$\Xi^0 \to \Sigma^0\nu\bar{\nu}$	✔	✔
$\Xi^0 \to \Lambda\nu\bar{\nu}$	✔	✔

Channels	$\delta C^L_{\nu_\ell} + C^R_{\nu_\ell}$	$\delta C^L_{\nu_\ell} - C^R_{\nu_\ell}$
$K^+ \to \pi^+\nu\bar{\nu}$	✔	✗
$K_L \to \pi^0\nu\bar{\nu}$	✔	✗
$K^+ \to \pi^+\pi^0\nu\bar{\nu}$	✔	✔
$K_L \to \pi^0\pi^0\nu\bar{\nu}$	✗	✔

$\mathrm{BR_{SM}} \sim 10^{-15}$
$\mathrm{BR_{exp}} < 4.3 \times 10^{-5}$
Compared to hyperons,
very weak constraints on NP

Our purpose

□ $s \to d\mu^+\mu^-$ **transitions dominated by long-distance contributions**

$\Sigma^+ \to p\mu^+\mu^-$

V.S

$K_L \to \mu^+\mu^-$

$K^+ \to \pi^+\mu^+\mu^-$

Channels	$\delta C_{10} + C'_{10}$	$\delta C_{10} - C'_{10}$	$C_S + C'_S$	$C_S - C'_S$
$A_{FB}(\Sigma^+ \to p\mu^+\mu^-)$	✔	✔	✔	✔
$\mathrm{BR}(K_L \to \mu^+\mu^-)$	✗	✔	✗	✔
$A_{FB}(K^+ \to \pi^+\mu^+\mu^-)$	✗	✗	✔	✗

Use the low energy effective Hamiltonian approach to derive the relevant physics

Deal with the non-perturbative effects model-independently

Compare hyperon decays with kaon decays to contrain NP

Low-energy effective Hamiltonian approach

☐ In SM

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\lambda_t\left(\sum_{i=1}^{10} C_i O_i + \sum_{\ell=e,\mu,\tau} C^L_{\nu_\ell} O^L_{\nu_\ell}\right) \qquad \lambda_q = V_{qs}V^*_{qd}$$

- $s \to d\,\nu\,\bar{\nu}$ **transitions**

$$C^L_{\nu_\ell} = \frac{1}{2\pi\sin^2\theta_W}\left(\frac{\lambda_c}{\lambda_t}X^\ell_c + X_t\right) \qquad O^L_{\nu_\ell} = \alpha\left(\bar{d}\gamma_\mu(1-\gamma_5)s\right)\left(\bar{\nu}_\ell\gamma^\mu(1-\gamma_5)\nu_\ell\right)$$

- $s \to d\ell^+\ell^-$ **transitions**

Short-distance $O_7 = \frac{e}{4\pi}m_s\bar{d}\sigma_{\mu\nu}(1+\gamma_5)sF^{\mu\nu} \qquad O_9 = \alpha\left(\bar{d}\gamma_\mu(1-\gamma_5)s\right)\left(\bar{\ell}^-\gamma^\mu\ell^+\right) \qquad O_{10} = \alpha\left(\bar{d}\gamma_\mu(1-\gamma_5)s\right)\left(\bar{\ell}^-\gamma^\mu\gamma_5\ell^+\right)$

Long-distance $\mathcal{M}_{\rm LD} = -\frac{e^2G_F}{q^2}\bar{B}_2\sigma_{\mu\nu}q^\nu(a+b\gamma_5)B_1\bar{\ell}^-\gamma^\mu\ell^+ - e^2G_F\bar{B}_2\gamma_\mu(c+d\gamma_5)B_1\bar{\ell}^-\gamma^\mu\ell^+$

☐ In BSM (NP)

- The NP operators can be obtained by a **chiral flip** in the quark current, and one also has scalar, pseudoscalar and their primed operators

$$O_S = \alpha\left(\bar{d}(1+\gamma_5)s\right)\left(\bar{\ell}^-\ell^+\right), \quad O'_S = \alpha\left(\bar{d}(1-\gamma_5)s\right)\left(\bar{\ell}^-\ell^+\right), \quad O_P = \alpha\left(\bar{d}(1+\gamma_5)s\right)\left(\bar{\ell}^-\gamma_5\ell^+\right), \quad O'_P = \alpha\left(\bar{d}(1-\gamma_5)s\right)\left(\bar{\ell}^-\gamma_5\ell^+\right)$$

- Tensor operator does not contribute to $s \to d\ell^+\ell^-$ transitions

J. Martin Camalich et al, PRL.113(2014)241802

Consequence of $B_i \to B_f \mu^+ \mu^-$

Decay width expanded in δ reads

$$\frac{d\Gamma}{d\cos\theta_\ell} = \mathcal{N}\left[k_1 + k_2\cos\theta_\ell + k_3\cos^2\theta_\ell\right],$$

$$\begin{aligned} k_1 = & \left(\frac{137.06}{\Delta^2 f_1(0)^2}\right)\left(1-\frac{3}{2}\delta\right)\left|\frac{a}{\lambda_t}\right|^2 + \left(\frac{58.50}{f_1(0)^2}\right)\left(1-\frac{3}{2}\delta\right)\left|\frac{c}{\lambda_t}\right|^2 \\ & + \left(\frac{1221.67}{\Delta^2 f_1(0)^2}\right)\left(1-\frac{3}{2}\delta\right)\left|\frac{b}{\lambda_t}\right|^2 + \left(\frac{974.60}{f_1(0)^2}\right)\left(1-\frac{3}{2}\delta\right)\left|\frac{d}{\lambda_t}\right|^2 \\ & + \left(\frac{168.52}{\Delta f_1(0)^2}\delta\right)\mathrm{Re}\left[\frac{ac^*}{\lambda_t\lambda_t^*}\right] - \left(\frac{2199.79}{\Delta f_1(0)^2}\right)\left(1-\frac{3}{2}\delta\right)\mathrm{Re}\left[\frac{bd^*}{\lambda_t\lambda_t^*}\right], \end{aligned}$$

$$\begin{aligned} k_3 = & \left(\frac{8.30}{\Delta^2 f_1(0)^2}\right)\left(1-\frac{3}{2}\delta\right)\left|\frac{a}{\lambda_t}\right|^2 - \left(\frac{6.99}{f_1(0)^2}\right)\left(1-\frac{3}{2}\delta\right)\left|\frac{c}{\lambda_t}\right|^2 \\ & + \left(\frac{9.00}{\Delta^2 f_1(0)^2}\right)\left(1-\frac{3}{2}\delta\right)\left|\frac{b}{\lambda_t}\right|^2 - \left(\frac{7.00}{f_1(0)^2}\right)\left(1-\frac{3}{2}\delta\right)\left|\frac{d}{\lambda_t}\right|^2, \end{aligned}$$

$$\begin{aligned} k_2 = & (C_S + C_S')\,\mathrm{Re}\left[\left(\frac{2.32}{\Delta f_1(0)}\delta\right)\frac{f_S(0)}{f_1(0)}\left(\frac{a}{\lambda_t}\right)^* + \left(\frac{4.64}{f_1(0)}\right)\left(1-\frac{3}{2}\delta\right)\frac{f_S(0)}{f_1(0)}\left(\frac{c}{\lambda_t}\right)^*\right] \\ & + (C_S - C_S')\,\mathrm{Re}\left[\left(\frac{2.32}{\Delta f_1(0)}\delta\right)\frac{g_P(0)}{f_1(0)}\left(\frac{b}{\lambda_t}\right)^* - \left(\frac{2.32}{f_1(0)}\delta\right)\frac{g_P(0)}{f_1(0)}\left(\frac{d}{\lambda_t}\right)^*\right] \\ & + (C_{10} + C_{10}')\,\mathrm{Re}\left[\left(\frac{5.51}{\Delta f_1(0)}\delta\right)\left(\frac{b}{\lambda_t}\right)^* - \left(\frac{4.68}{f_1(0)}\delta\right)\left(\frac{d}{\lambda_t}\right)^*\right. \\ & \qquad \left. + \left(\frac{11.02}{\Delta f_1(0)}\delta\right)\frac{\boxed{f_2(0)}}{f_1(0)}\left(\frac{b}{\lambda_t}\right)^* - \left(\frac{9.36}{f_1(0)}\delta\right)\frac{f_2(0)}{f_1(0)}\left(\frac{d}{\lambda_t}\right)^*\right] \\ & + (C_{10} - C_{10}')\,\mathrm{Re}\left[\left(\frac{11.02}{\Delta f_1(0)}\right)\frac{g_1(0)}{f_1(0)}\left(1-\frac{3}{2}\delta\right)\left(\frac{a}{\lambda_t}\right)^* + \left(\frac{4.68}{f_1(0)}\delta\right)\frac{g_1(0)}{f_1(0)}\left(\frac{c}{\lambda_t}\right)^*\right], \end{aligned}$$

where $\Delta = M_1 - M_2$ and $\mathcal{N} = \dfrac{G_F^2 \alpha^2 |\lambda_t|^2 \Delta^5 f_1(0)^2}{2048\pi^3}$.

The branching ratio and forward-backward asymmetry are defined as

$$\boxed{\begin{aligned} \mathrm{BR} &= 2\tau_{B_1}\mathcal{N}\left(k_1 + \frac{1}{3}k_3\right) \\ A_{FB} &= \frac{k_2}{k_1 + \frac{1}{3}k_3} \end{aligned}}$$

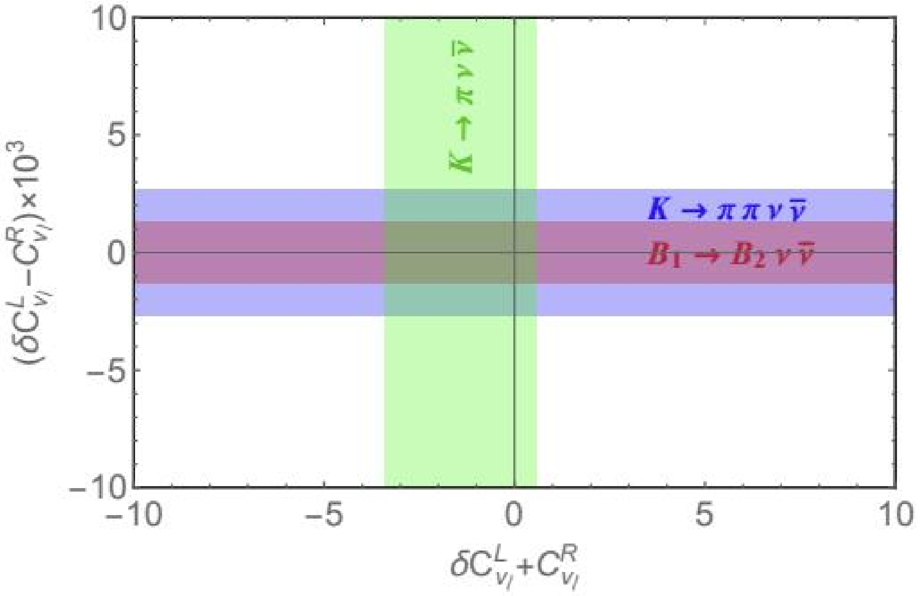
- $f_2(0)$ is relevant for A_{FB}
- Form factors *a,b* and *c,d* are calculated by χPT and vector-meson-dominance model

H. Neufeld, NPB 402 (1993) 166; Xiao-Gang He et al., PRD 72 (2005) 074003

$s \to d\nu\bar{\nu}$ transitions: hyperon vs. kaon

Decay modes	Λn	$\Sigma^+ p$	$\Xi^-\Sigma^-$	$\Xi^0\Sigma^0$	$\Xi^0\Lambda$
$10^{13} \times \mathrm{BR}(B_1 \to B_2\nu\bar{\nu})^{\mathrm{SM}}$	6.26(16)	3.49(16)	1.10(1)	0.89(1)	5.52(13)
$10^{6} \times \mathrm{BR}(B_1 \to B_2\nu\bar{\nu})^{\mathrm{BESIII}}$	< 0.3	< 0.4	–	< 0.9	< 0.8
$10^3 \times \lvert \delta C^L_{\nu_\ell} + C^R_{\nu_\ell} \rvert$	< 1.6	< 1.7	–	< 10	< 1.8
$10^3 \times \lvert \delta C^L_{\nu_\ell} - C^R_{\nu_\ell} \rvert$	< 1.3	< 3.4	–	< 5.2	< 8.6

Decay modes	$K^+\pi^+$	$K_L\pi^0$	$K^+\pi^+\pi^0$	$K_L\pi^0\pi^0$
$\mathrm{BR}^{\mathrm{SM}}$	$8.55(4) \times 10^{-11}$	$2.89(1) \times 10^{-11}$	$8.35(22) \times 10^{-15}$	$2.59(3) \times 10^{-13}$
$\mathrm{BR}^{\mathrm{Expt}}$	$< 1.78 \times 10^{-10}$	$< 3.0 \times 10^{-9}$	$< 4.3 \times 10^{-5}$	$< 8.1 \times 10^{-7}$
$\delta C^L_{\nu_\ell} + C^R_{\nu_\ell}$	$(-3.4, 0.6)$	$(-11.5, 9.4)$	$(-2.2, 2.2) \times 10^6$	–
$\delta C^L_{\nu_\ell} - C^R_{\nu_\ell}$	–	–	$(-1.1, 1.1) \times 10^5$	$(-2.7, 2.7) \times 10^3$



- Branching ratio results predicted in SM for $B_1 \to B_2\nu\bar{\nu}$ decays are $\sim 10^{-13}$, consistent with those predicted in the following Refs:

  Xiao-Hui Hu et al., CPC43(2019)093104; Jusak Tandean, JHEP04(2019)104;
  Jhih-Ying Su et al.,y, PRD 102 (2020) 075032; Gang Li et al., PRD 100 (2019) 075003

- $\boldsymbol{\delta C^L_{\nu_\ell} + C^R_{\nu_\ell}}$ is constrained more stringently by **the kaon modes**
- $\boldsymbol{B_1 \to B_2\nu\bar{\nu}}$ are better than their kaon siblings to constrain $\boldsymbol{\delta C^L_{\nu_\ell} - C^R_{\nu_\ell}}$

$s \to d\mu^+\mu^-$ transitions: hyperon vs. kaon

Essentially, the cases 1 and 2 are caused by the S/P wave puzzle.

	$\mathrm{BR}(K_L \to \mu^+\mu^-)$	$A_{FB}(K^+ \to \pi^+\mu^+\mu^-)$	$A_{FB}(\Sigma^+ \to p\mu^+\mu^-)$ Case 1	$A_{FB}(\Sigma^+ \to p\mu^+\mu^-)$ Case 2
SM predictions	$7.64(73) \times 10^{-9}$	0	-1.4×10^{-5}	0.2×10^{-5}
Data	$6.84(11) \times 10^{-9}$	$(-2.3, 2.3) \times 10^{-2}$	$(-2.3, 2.3) \times 10^{-2}$	
$C_S + C'_S$	–	$(-1.7, 1.7)$	$(-6.8, 6.8) \times 10^2$	$(-9.3, 9.3) \times 10^3$
$C_S - C'_S$	$(-0.12, 0.12)$	–	$(-1.3, 1.3) \times 10^3$	$(-1.8, 1.8) \times 10^3$
$\delta C_{10} + C'_{10}$	–	–	$(-1.2, 1.2) \times 10^3$	$(-1.8, 1.8) \times 10^3$
$\delta C_{10} - C'_{10}$	$(-2.35, 0.59)$	–	$(-5.8, 5.8) \times 10^2$	$(-1.5, 1.5) \times 10^3$

- *Here, we are assuming a hypothetical measurement of* $A_{FB}(\Sigma^+ \to p\mu^+\mu^-)$ *that is identical to* $K^+ \to \pi^+\mu^+\mu^-$.
- Current kaon bounds except for the $\delta C_{10} + C'_{10}$ scenario are a few orders of magnitude better than those of $\Sigma^+ \to p\mu^+\mu^-$ if measured up to the same precision.

Impact of renormalization groups on NP constraints

- **For this purpose, we must work in the SMEFT. The Lagrangian of SMEFT describing the NP contributions to down-quark FCNC semi-leptonic decays is**

$$\mathcal{L}_{\rm NP} = \frac{1}{\Lambda}\sum_i C_i Q_i,$$

$$\begin{aligned}
Q_{\ell q}^{(1),ij\alpha\beta} &= \left(\bar{q}_L^j\gamma^\mu q_L^i\right)\left(\bar{\ell}_L^\beta\gamma_\mu\ell_L^\alpha\right), & Q_{\ell q}^{(3),ij\alpha\beta} &= \left(\bar{q}_L^j\vec{\tau}\gamma^\mu q_L^i\right)\left(\bar{\ell}_L^\beta\vec{\tau}\gamma_\mu\ell_L^\alpha\right),\\
Q_{\ell d}^{ij\alpha\beta} &= \left(\bar{d}_R^j\gamma^\mu d_R^i\right)\left(\bar{\ell}_L^\beta\gamma_\mu\ell_L^\alpha\right), & Q_{qe}^{ij\alpha\beta} &= \left(\bar{q}_L^j\gamma^\mu q_L^i\right)\left(\bar{e}_R^\beta\gamma_\mu e_R^\alpha\right),\\
Q_{ed}^{ij\alpha\beta} &= \left(\bar{d}_R^j\gamma^\mu d_R^i\right)\left(\bar{e}_R^\beta\gamma_\mu e_R^\alpha\right), & Q_{\ell edq}^{ij\alpha\beta} &= \left(\bar{\ell}_L e_R\right)\left(\bar{d}_R q_L\right),\\
Q_{\ell edq}^{\prime ij\alpha\beta} &= \left(\bar{e}_R\ell_L\right)\left(\bar{q}_L d_R\right), & &
\end{aligned}$$

$$\begin{aligned}
q_L &\sim \left(\mathbf{3},\mathbf{2},\tfrac{1}{6}\right)\\
d_R &\sim \left(\mathbf{3},\mathbf{1},-\tfrac{1}{3}\right)\\
\ell_L &\sim \left(\mathbf{1},\mathbf{2},-\tfrac{1}{2}\right)\\
e_R &\sim \left(\mathbf{1},\mathbf{1},-1\right)
\end{aligned}$$

- **We work in the basis where the down type quark mass matrix is diagonal. At the electroweak scale v one has**

$$\begin{aligned}
[\delta C_9]_{sd\mu\mu} &= \frac{2\pi}{e^2\lambda_t}\frac{v^2}{\Lambda^2}\left[C_{\ell q}^{(1)} + C_{\ell q}^{(3)} + C_{qe}\right]_{sd\mu\mu}, & [C_9']_{sd\mu\mu} &= \frac{2\pi}{e^2\lambda_t}\frac{v^2}{\Lambda^2}\left[C_{ed} + C_{\ell d}\right]_{sd\mu\mu},\\
[\delta C_{10}]_{sd\mu\mu} &= \frac{2\pi}{e^2\lambda_t}\frac{v^2}{\Lambda^2}\left[-C_{\ell q}^{(1)} - C_{\ell q}^{(3)} + C_{qe}\right]_{sd\mu\mu}, & [C_{10}']_{sd\mu\mu} &= \frac{2\pi}{e^2\lambda_t}\frac{v^2}{\Lambda^2}\left[C_{ed} - C_{\ell d}\right]_{sd\mu\mu},\\
[C_S]_{sd\mu\mu} = -[C_P]_{sd\mu\mu} &= \frac{2\pi}{e^2\lambda_t}\frac{v^2}{\Lambda^2}\left[C_{ledq}\right]_{sd\mu\mu}, & [C_S']_{sd\mu\mu} = [C_P']_{sd\mu\mu} &= \frac{2\pi}{e^2\lambda_t}\frac{v^2}{\Lambda^2}\left[C_{ledq}'\right]_{sd\mu\mu},\\
[\delta C_{\nu_\ell}^L]_{sd\nu\bar{\nu}} &= \frac{2\pi}{e^2\lambda_t}\frac{v^2}{\Lambda^2}\left[C_{\ell q}^{(1)} - C_{\ell q}^{(3)}\right]_{sd\nu\bar{\nu}}, & [C_{\nu_\ell}^R]_{sd\nu\bar{\nu}} &= \frac{2\pi}{e^2\lambda_t}\frac{v^2}{\Lambda^2}\left[C_{\ell d}\right]_{sd\nu\bar{\nu}}
\end{aligned}$$

Impact of renormalization groups on NP constraints

☐ Example: $s \to d\nu\bar{\nu}$ transitions

We consider a Z' model in which a single $Z' \sim (1,1,0)$ gauge boson couples to left-handed leptons. The Lagrangian for this model is

$$\mathcal{L}_{Z'} = \left(g_L^{ij} \bar{q}_L^j \gamma^\mu q_L^i + g_R^{ij} \bar{d}_R^j \gamma^\mu d_R^i + g_L^{\alpha\beta} \bar{\ell}_L^\beta \gamma^\mu \ell_L^\alpha \right) Z'_\mu,$$

$$[\delta C_{\nu_\ell}^L]_{sd\nu\bar{\nu}} = \frac{2\pi}{e^2 \lambda_t} \frac{v^2}{\Lambda^2} \left[C_{\ell q}^{(1)} - \cancel{C_{\ell q}^{(3)}} \right]_{sd\nu\bar{\nu}}, \qquad [C_{\nu_\ell}^R]_{sd\nu\bar{\nu}} = \frac{2\pi}{e^2 \lambda_t} \frac{v^2}{\Lambda^2} [C_{\ell d}]_{sd\nu\bar{\nu}}$$

$$Q_{\ell q}^{(1),ij\alpha\beta} = \left(\bar{q}_L^j \gamma^\mu q_L^i \right) \left(\bar{\ell}_L^\beta \gamma_\mu \ell_L^\alpha \right) \qquad Q_{\ell d}^{ij\alpha\beta} = \left(\bar{d}_R^j \gamma^\mu d_R^i \right) \left(\bar{\ell}_L^\beta \gamma_\mu \ell_L^\alpha \right)$$

By fine-tuning the couplings $g_L^{sd} g_L^{\nu\bar{\nu}} =- g_R^{sd} g_L^{\nu\bar{\nu}}$, one can obtain

$$\left[\delta C_{\nu_\ell}^L(\Lambda) - C_{\nu_\ell}^R(\Lambda) \right]_{sd\nu\bar{\nu}} \neq 0 \qquad \left[\delta C_{\nu_\ell}^L(\Lambda) + C_{\nu_\ell}^R(\Lambda) \right]_{sd\nu\bar{\nu}} = 0$$

E.E. Jenkins et al., JHEP 10 (2013) 087
E.E. Jenkins et al., JHEP 01 (2014) 035
R. Alonso et al., JHEP 04 (2014) 159

When running the RG equations from the NP scale Λ to the electroweak scale v:

$$16\pi^2 \mu \frac{dC_i(\mu)}{d\mu} = \gamma_{ij} C_j(\Lambda) \equiv \dot{C}_i(\Lambda)$$

Assuming that $\Lambda = 10v$, one obtains

$$\left[\delta C_{\nu_\ell}^L(v) + C_{\nu_\ell}^R(v) \right]_{sd\nu\bar{\nu}} = \frac{2\pi}{e^2 \lambda_t} \frac{v^2}{\Lambda^2} \left[0.02 C_{\ell q}^{(1)}(\Lambda) \right]_{sd\nu\bar{\nu}},$$

$$\left[\delta C_{\nu_\ell}^L(v) - C_{\nu_\ell}^R(v) \right]_{sd\nu\bar{\nu}} = \frac{2\pi}{e^2 \lambda_t} \frac{v^2}{\Lambda^2} \left[2.02 C_{\ell q}^{(1)}(\Lambda) \right]_{sd\nu\bar{\nu}}.$$

Impact of renormalization groups on NP constraints

$$\left[\delta C^L_{\nu_\ell}(v)+C^R_{\nu_\ell}(v)\right]_{sd\nu\bar\nu}=\frac{2\pi}{e^2\lambda_t}\frac{v^2}{\Lambda^2}\left[0.02C^{(1)}_{\ell q}(\Lambda)\right]_{sd\nu\bar\nu},$$

$$\left[\delta C^L_{\nu_\ell}(v)-C^R_{\nu_\ell}(v)\right]_{sd\nu\bar\nu}=\frac{2\pi}{e^2\lambda_t}\frac{v^2}{\Lambda^2}\left[2.02C^{(1)}_{\ell q}(\Lambda)\right]_{sd\nu\bar\nu}.$$

The renormalization group effects lead to an indirect relation between bound of NP, which is

$$(\boldsymbol{\delta C^L_{\nu_\ell}-C^R_{\nu_\ell}})\sim 100(\boldsymbol{\delta C^L_{\nu_\ell}+C^R_{\nu_\ell}})$$

- we see that the loop effects of the RGE generate a non-vanishing vectorial contribution at the scale v, which is about 1% of that of the axial-vectorial contribution.

Decay modes	$K^+\pi^+$	$K_L\pi^0$	$K^+\pi^+\pi^0$	$K_L\pi^0\pi^0$
$\mathrm{BR}^{\mathrm{SM}}$	$8.55(4)\times10^{-11}$	$2.89(1)\times10^{-11}$	$8.35(22)\times10^{-15}$	$2.59(3)\times10^{-13}$
$\mathrm{BR}^{\mathrm{Expt}}$	$<1.78\times10^{-10}$	$<3.0\times10^{-9}$	$<4.3\times10^{-5}$	$<8.1\times10^{-7}$
$\delta C^L_{\nu_\ell}+C^R_{\nu_\ell}$	$(-3.4,0.6)$	$(-11.5,9.4)$	$(-2.2,2.2)\times10^6$	–
$\delta C^L_{\nu_\ell}-C^R_{\nu_\ell}$	–	–	$(-1.1,1.1)\times10^5$	$(-2.7,2.7)\times10^3$

As shown in the left table, $K^+\to\pi^+\nu\bar\nu$ data yields the bound on $\boldsymbol{\delta C^L_{\nu_\ell}+C^R_{\nu_\ell}}$ at $O(1)$.Using the indirect relation of RGE above, one can obtain the bound on $\boldsymbol{\delta C^L_{\nu_\ell}-C^R_{\nu_\ell}}$ at the order of 10^2.

Decay modes	Λn	Σ^+p	$\Xi^-\Sigma^-$	$\Xi^0\Sigma^0$	$\Xi^0\Lambda$
$10^{13}\times\mathrm{BR}(B_1\to B_2\nu\bar\nu)^{\mathrm{SM}}$	6.26(16)	3.49(16)	1.10(1)	0.89(1)	5.52(13)
$10^6\times\mathrm{BR}(B_1\to B_2\nu\bar\nu)^{\mathrm{BESIII}}$	<0.3	<0.4	–	<0.9	<0.8
$10^3\times\lvert\delta C^L_{\nu_\ell}+C^R_{\nu_\ell}\rvert$	<1.6	<1.7	–	<10	<1.8
$10^3\times\lvert\delta C^L_{\nu_\ell}-C^R_{\nu_\ell}\rvert$	<1.3	<3.4	–	<5.2	<8.6

The indirect bound (10^2) on $\boldsymbol{\delta C^L_{\nu_\ell}-C^R_{\nu_\ell}}$ is stronger than the direct bound of 10^3 that could be obtained by the future BESIII data from the hyperon modes shown in the left table.

- **From the perspective of a UV theory, it is important to consider the loop effects from renormalization group evolution when connecting the low-energy EFT to new physics model.**

Impact of renormalization groups on NP constraints

Decay modes	Λn	$\Sigma^+ p$	$\Xi^-\Sigma^-$	$\Xi^0\Sigma^0$	$\Xi^0\Lambda$
$10^{13}\times \mathrm{BR}(B_1\to B_2\nu\bar{\nu})^{\mathrm{SM}}$	6.26(16)	3.49(16)	1.10(1)	0.89(1)	5.52(13)
$10^{6}\times \mathrm{BR}(B_1\to B_2\nu\bar{\nu})^{\mathrm{BESIII}}$	<0.3	<0.4	–	<0.9	<0.8
$10^3 \times \lvert\delta C^L_{\nu_\ell}+C^R_{\nu_\ell}\rvert$	<1.6	<1.7	–	<10	<1.8
$10^3 \times \lvert\delta C^L_{\nu_\ell}-C^R_{\nu_\ell}\rvert$	<1.3	<3.4	–	<5.2	<8.6

For hyperon rare decays, the anticipated BESIII BR$\sim10^{-6}$

- However, when connecting the low-energy EFT to new physics model, hyperons could better constrain some combinations of Wilson coefficients if a sensitivity of 10^{-8} for the branching fractions is achieved by hyperon factories (BESIII, STCF) in the future.

Contents

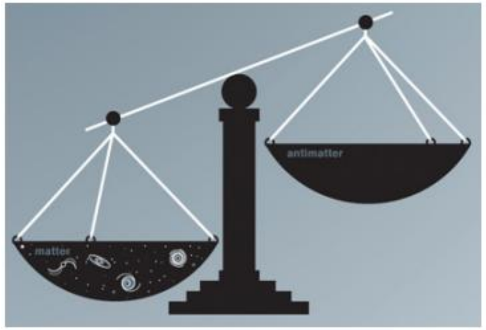
Summary and outlook

- EOMS χPT has improved the previous studies in WRHDs and non-perturbative contributions of the rare hyperon semi-leptonic decay $B_i \to B_f\gamma^* \to B_f l\, l$

- We plan to apply the EOMS χPT to study the S/P puzzle and CP violation in non-leptonic hyperon decays

Why CP violation

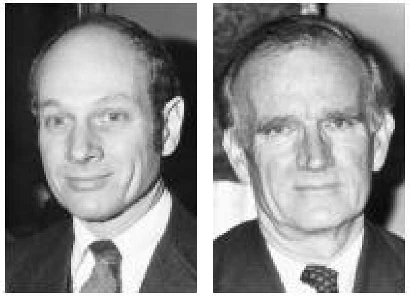
- Explaining the matter-antimatter asymmetry
- Testing SM and searching for NP
- **CPV has been observed in the K[1]**, B[2] and D[3] mesons sequentially during the past 60 years
- **CPV attributed to an irreducible phase in the CKM quark-mixing matrix**
- **The CPV in the baryon system has not yet been established**



[1] Phys. Rev. Lett. 13, 138-140 (1964)
[2] Phys. Rev. Lett. 87, 091801(2001);
Phys. Rev. Lett. 87,091802 (2001)
[3] Phys. Rev. Lett. 122, 211803 (2019)

- **Nobel prizes in physics**

Discovery of CP violation @ 1980



Cronnin Fitch

Mechanism of CP violation @ 2008

Kobayashi Maskawa

Many studies on baryon CPV

Experiment measurements

Beauty baryon

- LHCb,Nature Phys. 13, 391-396 (2017)
- LHCb,JHEP 06, 108 (2017)
- LHCb,JHEP 03, 182 (2018)
- LHCb,Phys. Lett. B 787, 124-133 (2018)
- LHCb,JHEP 08, 039 (2018)
- LHCb,Eur. Phys. J. C 79, no.9, 745 (2019)
- LHCb,Phys. Rev. D 102, no.5, 051101 (2020)

Charm baryon

- LHCb,Eur. Phys. J. C 80, no.10, 986 (2020)
- Belle,Sci. Bull. 68, 583-592 (2023)

Hyperon

- BESIII,Nature Phys. 15, 631 (2019)
- BESIII,Phys. Rev. Lett. 125, no.5, 052004 (2020)
- BESIII,Nature 606, 64 (2022)
- BESIII,Phys. Rev. Lett. 129, 131801(2022)
- BESIII,Phys. Rev. Lett. 129, 212002(2022)
- BESIII,Phys. Rev. Lett. 130, 211901(2023)
- BESIII, arXiv:2408.16654(2024)

Theoretical studies

- Y.K. Hsiao et al, Phys.Rev.D 95 (2017) 9, 093001
- Shibasis Roy et al, Phys.Rev.D 101 (2020) 3, 036018
- Shibasis Roy et al, Phys.Rev.D 102 (2020) 5, 053007
- Ignacio Bediaga et al, Prog.Part.Nucl.Phys. 114 (2020) 103808
- Zhen-Hua Zhang et al, JHEP 07 (2021) 177
- Zhen-Hua Zhang et al, Eur.Phys.J.C 83 (2023) 2, 133
- Jian-Peng Wang et al, Arxiv: 2211.07332
- Yin-Fa Shen et al, Phys.Rev.D 108 (2023) 11, L111901
- Jian-Peng Wang et al, Arxiv: 2411.18323
- Jian-Peng Wang et al, Chin.Phys.C 48 (2024) 10, 101002
- Ji-Xin Yu et al, Arxiv: 2409.02821

- Zhen-Hua Zhang, Phys.Rev.D 107 (2023) 1, L011301
- Cai-Ping Jia et al, JHEP 11 (2024) 072

- Xiao-Gang He et al, Science Bulletin 67 (2022) 1840–1843
- Nora Salone et al, Phys.Rev.D 105 (2022) 11, 116022
- Tandean J et al, Phys.Rev.D 67 (2003) 056001
- Jusak Tandean, Phys.Rev.D 69 (2004) 076008

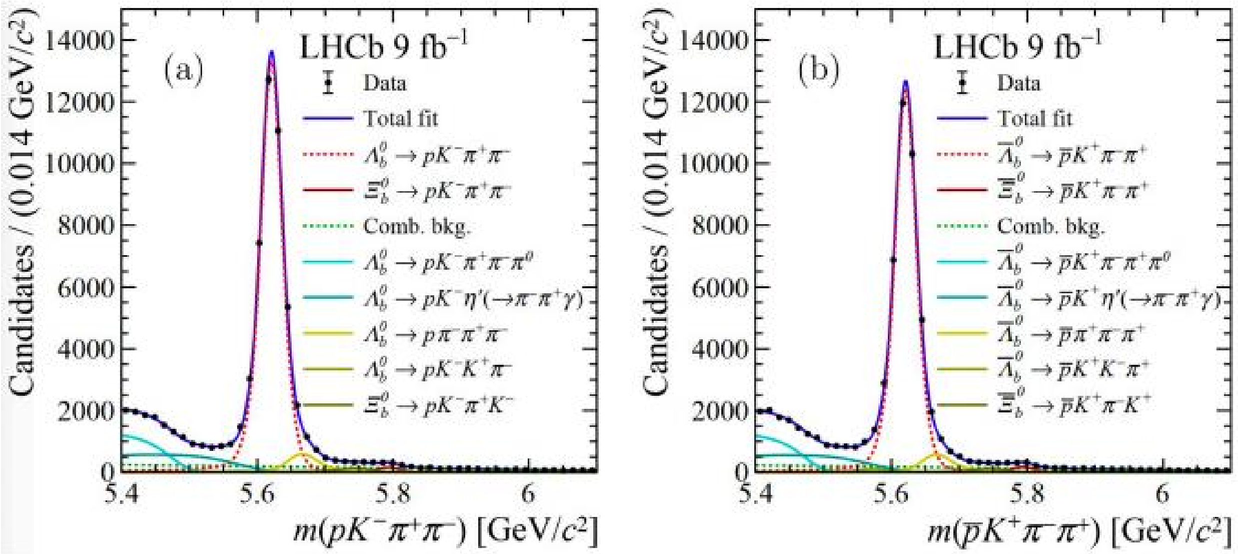
In this presentation, we will focus on the hyperon system.

Discovery of baryonic CP violation

$$\Lambda_b \to pK^-\pi^+\pi^-$$

$$\mathcal{A}_{CP} = (2.45 \pm 0.46 \pm 0.10)\%$$

$$5.2\ \sigma$$



[LHCb, 2503.16954, submitted to Nature]

Hyperon non-leptonic decays

$$B_i \to B_f\, \pi$$

Hyperon non-leptonic decays

Decay amplitudes: $M = G_F m_\pi^2 \cdot \overline{B}_f (A_S - A_P \gamma_5) B_i$

$S = A_S$ and $P = A_P \cdot \frac{|\vec{p}_f|}{E_f + m_f}$

Asymmetry parameters: $\alpha^2 + \beta^2 + \gamma^2 = 1$

$\alpha = \frac{2\mathrm{Re}(S^* P)}{|S|^2+|P|^2}$, $\beta = \frac{2\mathrm{Im}(S^* P)}{|S|^2+|P|^2}$ 和 $\gamma = \frac{|S|^2-|P|^2}{|S|^2+|P|^2}$

CPV observables:

$A_{CP} = \frac{\alpha+\bar{\alpha}}{\alpha-\bar{\alpha}}$ 和 $B_{CP} = \frac{\beta+\bar{\beta}}{\alpha-\bar{\alpha}}$

$$\begin{aligned}
\mathcal{A}_{CP}^{\mathrm{dir}} &= \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \\
&\sim \left(-2(A_1 A_3 \sin(\delta_S^1 - \delta_S^3) \sin(\phi_S^1 - \phi_S^3)\right) \\
&+ \; B_1^r B_3^r \sin(\delta_P^1 - \delta_P^3) \sin(\phi_P^1 - \phi_P^3)) \\
&/ \; |A_1|^2 + |B_1^r|^2
\end{aligned}$$

- The direct CPV is multiplicatively suppressed by both the strong interaction phases δ as well as by the $\Delta I = 3/2$ suppression $A_3/A_1 \sim B_3/B_1 \sim 1/20$
- CPV observables of the largest signal are defined by a combination of the decay asymmetry parameters α and β proposed T.D.Lee and C.N.Yang.

General Partial Wave Analysis of the Decay of a Hyperon of Spin 1/2 #201

T.D. Lee (Princeton, Inst. Advanced Study), Chen-Ning Yang (Princeton, Inst. Advanced Study) (1957)

Published in: *Phys.Rev.* 108 (1957) 1645-1647

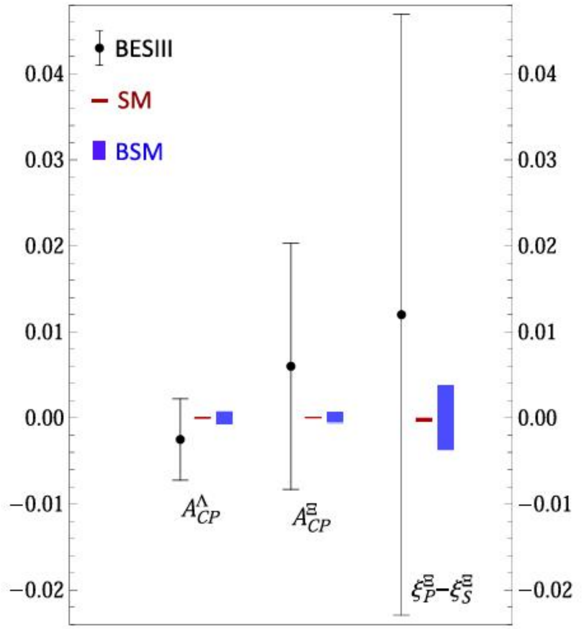
DOI cite claim reference search 214 citations

Why study the hyperon non-leptonic decays

- CP violation has not yet been established in the baryon sector. Hyperons are a good opportunity to observe the CPV in the baryon systems.

  - Jusak Tandean et al , PRD 67 (2003) 056001
  - Salone N et al , PRD 105 (2022) 11, 116022
  - Xiao-Gang He et al, Sci.Bull. 67 (2022) 1840-1843
  - BESIII Collaboration,arXiv:2312.17486

CPV observables	SM predictions	BESIII data
A_{CP}^{Λ}	$(-3\sim3)\times10^{-5}$	$(-2.5\pm4.6\pm1.2)\times10^{-3}$
A_{CP}^{Ξ}	$(0.5\sim6)\times10^{-5}$	$(6\pm13.4\pm5.6)\times10^{-3}$
B_{CP}^{Ξ}	$(-3.8\sim-0.3)\times10^{-4}$	$(1.2\pm3.4\pm0.8)\times10^{-2}$



- BESIII experiment cannot test the CPV in SM. It is hopeful in the future super tau-charm factories.
- **Large theoretical uncertainties are related to the S/P puzzle**

Non-leptonic decay amplitudes—S/P puzzle

- Amplitudes of hyperon non-leptonic decay

$$\mathcal{M}(B_i \to B_f \pi) = i G_F m_\pi^2 \bar{B}_f \left(A_S - A_P \gamma_5\right) B_i$$

Here, both S-wave amplitude A_S and P-wave amplitude A_P are functions of LECs hD and hF

- **The so-called S/P puzzle**: if the two LECs hD and hF can describe well the experimental S-wave amplitudes, they reproduce very poorly the P-wave amplitudes

As a result, we only updated the values of hD and hF by fitting to the experimental S -wave amplitudes for hyperon non-leptonic decays

Why study the hyperon non-leptonic decays

Theories

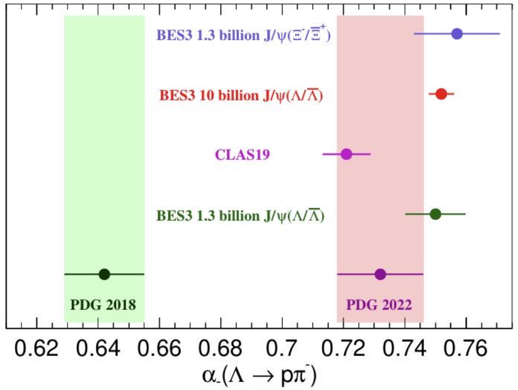
- χPT is a powerful tool for hyperon non-leptonic decays. Previous theoretical studies in HB χPT neglected the contributions of either counterterms or intermediate decuplet-baryons.

- Previous theoretical studies satisfies $\Delta I = 1/2$ rule.

HB χPT: Borasoy B et al, EPJC 6 (1999) 85-107
Abd El-Hady A, PRD 61 (2000) 114014

Experiments

- The recent BESIII measurements of asymmetry parameters associated with the S/P puzzle **deviate from** previous experimental values.



- Ratio of asymmetry parameter reported recently from BESIII **violates $\Delta I = 1/2$ rule** (=1, satisfies)

$$\langle\alpha_{\Lambda 0}\rangle/\langle\alpha_{\Lambda -}\rangle = 0.870 \pm 0.012^{+0.011}_{-0.010}$$

BESIII: PRL 132 (2024) 10, 101801

What to do next

- **Step 1: Investigate the S/P puzzle (ongoing)**
  - ✓ Study the hyperon non-leptonic decays in covariant baryon chiral perturbation theory (BχPT) with the extended-on-mass-shell (EOMS) renormalization scheme
  - ✓ Consider the effects of the $\Delta I = 1/2$ rule violation
  - ✓ Consider the contributions of counterterms, intermediate octet, and decuplet-baryons, even intermediate resonant states
- **Step 2: Revisit CP violation (To be done)**
  - ✓ Taking the S-wave and P-wave amplitudes provided in covariant baryon chiral perturbation theory as inputs to predict the CP violation of hyperon non-leptonic decays.

Thanks for your attention!