Single and Double Quarkonium Production at STCF
Within the framework of NRQCD Factorization

Xiang-Peng Wang

PRD 105 (2022) 11, L111503; JHEP 03 (2023) 242 &
2411.16384 (PRD Letters (2025))

Super Tau-Charm Facility Workshop 2025

Xiangtan, Hunan, 02-06/07/2025

Outlines

- Review of Inclusive Single Quarkonium Production
- Recent Progress on Inclusive Single Quarkonium Production
- Single and Double Quarkonium Production at STCF
- Summary and Outlook

Nonrelativistic QCD (NRQCD) factorization

- NRQCD factorization is the most prominent approach to describe quarkonium production
  Bodwin, Braaten & Lepage, PRD 51, 1125 (1995), $3000+$ citations.

$$\sigma_{\mathcal{Q}+X} = \sum_n \hat{\sigma}\left(ij \to Q\bar{Q}(n) + X\right) \langle \mathcal{O}^{\mathcal{Q}}(n) \rangle, \tag{1}$$

  with $n = {}^{2S+1}L_J^{[1/8]}$, representing the quantum state of the heavy quark anti-quark pair.
- $\hat{\sigma}$, short-distance-coefficients (SDCs), α_s expansion,
- $\langle \mathcal{O}^{\mathcal{Q}}(n) \rangle$, long-distance-matrix-elements (LDMEs), supposed to be universal, v^2 expansion.
- Typically, the ${}^3S_1^{[1]}$ ${}^3S_1^{[8]}$, ${}^1S_0^{[8]}$, ${}^3P_J^{[8]}$ channels are considered in perturbative computations.
- NRQCD factorization for inclusive quarkonium production is a conjecture, not proved yet.
- Violations of NRQCD factorization for inclusive processes were found in specific cases, for instance, processes involving two or more quarkonia. He, Kniehl & **XPW**, PRL 121, (2018) 17, 172001
- For exclusive processes, only color-singlet channels contribute.

NRQCD long-distance-matrix elements (LDMEs)

The LDMEs definitions of the spin-1 S-wave quarkonium V are given by

$$\langle \mathcal{O}^V({}^3S_1^{[1]})\rangle = \langle\Omega|\chi^\dagger\sigma^i\psi\mathcal{P}_{V(\boldsymbol{P}=\boldsymbol{0})}\psi^\dagger\sigma^i\chi|\Omega\rangle, \quad (2a)$$

$$\langle \mathcal{O}^V({}^3S_1^{[8]})\rangle = \langle\Omega|\chi^\dagger\sigma^i T^a\psi\Phi_\ell^{\dagger ab}\mathcal{P}_{V(\boldsymbol{P}=\boldsymbol{0})}\Phi_\ell^{bc}\psi^\dagger\sigma^i T^c\chi|\Omega\rangle, \quad (2b)$$

$$\langle \mathcal{O}^V({}^1S_0^{[8]})\rangle = \langle\Omega|\chi^\dagger T^a\psi\Phi_\ell^{\dagger ab}\mathcal{P}_{V(\boldsymbol{P}=\boldsymbol{0})}\Phi_\ell^{bc}\psi^\dagger T^c\chi|\Omega\rangle, \quad (2c)$$

$$\begin{aligned}\langle \mathcal{O}^V({}^3P_0^{[8]})\rangle = {}& \frac{1}{3}\langle\Omega|\chi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma})T^a\psi\Phi_\ell^{\dagger ab}\mathcal{P}_{V(\boldsymbol{P}=\boldsymbol{0})} \\ & \times\Phi_\ell^{bc}\psi^\dagger(-\tfrac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma})T^c\chi|\Omega\rangle, \end{aligned} \quad (2d)$$

here $\mathcal{P}_{V(\boldsymbol{P})} = \sum_X |V+X\rangle\langle V+X|$, $\Phi_\ell = P\exp[-ig\int_0^\infty d\lambda\,\ell\cdot A^{\rm adj}(\ell\lambda)]$ is the path-ordered Wilson line that ensures the gauge invariance.

- CS LDMEs can be related to quarkonium nonrelativistic wavefunction squared at the origin $|R(0)|^2$.
- Unclear how to calculate CO LDMEs using lattice QCD.
  CO LDMEs are determined through fitting with experimental data.

Heavy quark spin symmetry (HQSS)

- For the spin-1 S-wave quarkonium V ($J/\psi, \Upsilon$...), based on HQSS, we have

$$\langle \mathcal{O}^V({}^3P_J^{[8]})\rangle = (2J+1)\langle \mathcal{O}^V({}^3P_0^{[8]})\rangle(1+\mathcal{O}(v^2)). \tag{3}$$

  So, for each spin-1 S-wave quarkonium V, we have 3 independent frequently used color-octet LDMEs $\langle \mathcal{O}^V({}^3S_1^{[8]})\rangle$, $\langle \mathcal{O}^V({}^1S_0^{[8]})\rangle$, $\langle \mathcal{O}^V({}^3P_0^{[8]})\rangle$.

- Relations between the LDMEs of η_c and J/ψ due to HQSS,

$$\langle \mathcal{O}^{\eta_c}({}^1S_0^{[1]}/{}^1S_0^{[8]})\rangle = \frac{1}{3}\langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}/{}^3S_1^{[8]})\rangle(1+\mathcal{O}(v^2)), \tag{4}$$

$$\langle \mathcal{O}^{\eta_c}({}^3S_1^{[8]})\rangle = \langle \mathcal{O}^{J/\psi}({}^1S_0^{[8]})\rangle(1+\mathcal{O}(v^2)), \tag{5}$$

$$\langle \mathcal{O}^{\eta_c}({}^1P_1^{[8]})\rangle = 3\langle \mathcal{O}^{J/\psi}({}^3P_0^{[8]})\rangle(1+\mathcal{O}(v^2)). \tag{6}$$

  We will use the above relations between J/ψ and η_c to perform $J/\psi, \eta_c$ combined fit to constrain the 3 J/ψ color-octet LDMEs later.

J/ψ LDMEs $\langle\mathcal{O}^V(^3S_1^{[8]})\rangle$, $\langle\mathcal{O}^V(^1S_0^{[8]})\rangle$, $\langle\mathcal{O}^V(^3P_0^{[8]})\rangle$ fittings

- Chao et al. : $p_T > 7$GeV hadroproduction, two linear combinations (of the 3 CO LDMEs) are constrained, but the best fit gives large $\langle\mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$. Ma, Wang & Chao, PRL 106, 042002 (2011)

- Butenschön et al. : $p_T > 3$GeV , global fit $(pp,\ p\bar{p},\ \gamma p,\ \gamma\gamma,\ e^+e^-)$. Butenschön & Kniehl, PRD 84, 051501 (2011)

- Zhang et al. : $p_T > 7$GeV, combine J/ψ and η_c hadron production data based on HQSS, constrains $\langle\mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$ to be small. Zhang, Sun, Sang & Li, PRL 114, 092006 (2015); Butenschön, He & Kniehl, PRL 114, 092004 (2015); Han, Ma, Meng, Shao & Chao, PRL 114, 092005 (2015).

- Bodwin et al. : $p_T > 10$GeV hadroproduction, combine leading-power resummation with NLO fixed-order calculation. Bodwin, Chao, Chung, Kim, Lee & Ma, PRD 93, 034041 (2016)

- Feng et al. : $p_T > 7$Gev, fit both J/ψ hadron production and polarization data.
  Feng, Gong, Chang & Wang, PRD 99, 014044 (2019)

J/ψ LDMEs fittings – up to 2022

Table: Selected representative fitting results in units of 10^{-2} GeV^3.

Group	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$	$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$	$\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle / m_c^2$
Chao et al. set 1	0.05	7.4	0
Chao et al. set 2	1.11	0	1.89
Butenschön et al.	0.168 ± 0.046	3.04 ± 0.35	-0.404 ± 0.072
Zhang et al.	1.0 ± 0.3	0.74 ± 0.3	1.7 ± 0.5
Bodwin et al.	-0.713 ± 0.364	11 ± 1.4	-0.312 ± 0.151
Feng et al.	0.117 ± 0.058	5.66 ± 0.47	0.054 ± 0.005

- Fittings are based on NLO calculations, which are complicated, NNLO are infeasible in near future.
- High p_T J/ψ hadroproduction data can only well constrain 2 linear combinations of the 3 CO LDMEs.
- Dramatically different LDME sets are fitted, but none of them can well describe all the data, challenging the LDME universality.

Score card of fittings

Table: Tests of the J/ψ LDMEs fits from high p_T pp, and low p_T γp, $e^+e^-, \gamma\gamma$ data.

Group	pp (p_T in fit)	pp (pol.)	pp (η_c)	$J/\psi + Z$	e^+e^-	γp	$\gamma\gamma$
Chao et al. set 1	✓ ($p_T > 7$GeV)	✓	✗	-	✗	✗	-
Chao et al. set 2	✓ ($p_T > 7$GeV)	✓	✓	-	✗	✗	-
Butenschön et al.	✓ ($p_T > 3$GeV)	✗	✗	✗	✓	✓	✗
Zhang et al. $+\eta_c$	✓ ($p_T > 7$GeV)	✓	✓	-	✗	✗	-
Bodwin et al.	✓ ($p_T > 10$GeV)	✓	✗	✗	✗	✗	-
Feng et al.	✓ ($p_T > 7$GeV)	✓	✗	-	✗	✗	-

- All the fits fail to describe the hadroproduction data with $p_T < 7$ GeV, except for Butenschön et al.
- Simplest explanation: NRQCD factorization fails at relatively low p_T (< 7GeV).
- Another point of view: it has been pointed out, NRQCD factorization fails for $\gamma p, e^+e^-$ at the end-point regions, $z \to 1$, $E_{J/\psi} \to E_{J/\psi}^{\max}$, respectively.

  Beneke, Rothstein & Wise, PLB 408, 373 (1997)

Spin-1 S-wave LDMEs in pNRQCD

- Based on potential NRQCD (pNRQCD), we have (up to $\mathcal{O}(1/N_c^2, v^2)$ corrections),
  Brambilla, Chung, Vairo & **XPW**, PRD105, L111503 (2022); JHEP 03 (2023) 242

$$\langle \mathcal{O}^V(^3S_1^{[1]}) \rangle = 2N_c \times \frac{3|R_V^{(0)}(0)|^2}{4\pi}, \tag{7a}$$

$$\langle \mathcal{O}^V(^3S_1^{[8]}) \rangle = \frac{1}{2N_c m^2} \frac{3|R_V^{(0)}(0)|^2}{4\pi} \mathcal{E}_{10;10}, \tag{7b}$$

$$\langle \mathcal{O}^V(^1S_0^{[8]}) \rangle = \frac{1}{6N_c m^2} \frac{3|R_V^{(0)}(0)|^2}{4\pi} c_F^2 \mathcal{B}_{00}, \tag{7c}$$

$$\langle \mathcal{O}^V(^3P_0^{[8]}) \rangle = \frac{1}{18N_c} \frac{3|R_V^{(0)}(0)|^2}{4\pi} \mathcal{E}_{00}, \tag{7d}$$

- c_F is the NRQCD (HQET) matching coefficient,
- $R_V^{(0)}(0)$ is the wave-function at the origin,
- $\mathcal{E}_{10;10}$, $\mathcal{B}_{00}$, and $\mathcal{E}_{00}$ are universal gluonic correlators of mass dimension 2.

Gluonic correlators and their scale evolutions

$$\mathcal{E}_{10;10} = \left| d^{dac} \int_0^\infty dt_1\, t_1 \int_{t_1}^\infty dt_2\, gE^{b,i}(t_2)\Phi_0^{bc}(t_1;t_2) gE^{a,i}(t_1)\Phi_0^{df}(0;t_1)\Phi_\ell^{ef}|\Omega\rangle \right|^2, \tag{8a}$$

$$\mathcal{B}_{00} = \left| \int_0^\infty dt\, gB^{a,i}(t)\Phi_0^{ac}(0;t)\Phi_\ell^{bc}|\Omega\rangle \right|^2, \tag{8b}$$

$$\mathcal{E}_{00} = \left| \int_0^\infty dt\, gE^{a,i}(t)\Phi_0^{ac}(0;t)\Phi_\ell^{bc}|\Omega\rangle \right|^2, \tag{8c}$$

$$\mathcal{B}_{00}(m_b) = \mathcal{B}_{00}(m_c)\left(1 - 2N_c/\beta_0 \ln\left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right)\right), \tag{8d}$$

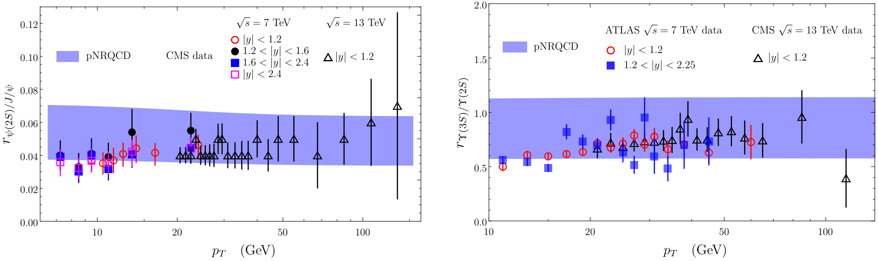
$$\mathcal{E}_{10;10}(m_b) = \mathcal{E}_{10;10}(m_c) + 4/(3\beta_0)\frac{N_c^2-4}{N_c}\mathcal{E}_{00}\ln\left(\frac{\alpha_s(m_c)}{\alpha_s(m_b)}\right), \tag{8e}$$

where $\Phi_0(t,t') = \mathcal{P}\exp[-ig\int_t^{t'} d\tau\, A_0^{\rm adj}(\tau,\mathbf{0})]$ is a Schwinger line.

- Gluonic correlators can be calculated using lattice QCD unlike CO LDMEs.
- By evolving the scale of $\mathcal{E}_{10;10}$, $\mathcal{B}_{00}$, and $\mathcal{E}_{00}$ (does not evolve at one-loop) from charm mass scale m_c to bottom mass scale m_b, we can related CO LDMEs between $\psi(nS)$ and $\Upsilon(nS)$.

pNRQCD predictive power

- Significantly reduces the number of independent CO LDMEs ($15 \to 3$).
- J/ψ and $\psi(2S)$ share the same $\mathcal{E}_{10;10}$, $\mathcal{B}_{00}$, and $\mathcal{E}_{00}$, thus their cross sections (without feeddown) ratio equals the ratio of $|R^{(0)}_{J/\psi}(0)|^2$ and $|R^{(0)}_{\psi(2S)}(0)|^2$ (same for $\Upsilon(nS)$ states).



Figures from Brambilla, Chung, Vairo & **XPW**, JHEP 03 (2023) 242

The prediction is based on NRQCD factorization and pNRQCD relations of the LDMEs without explicit perturbative calculations!

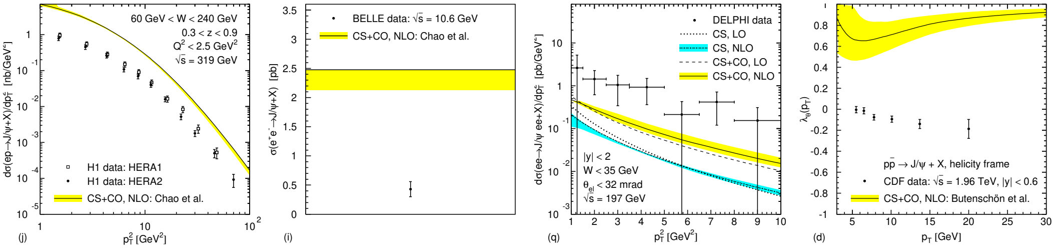
Score card of fittings – up to Nov. 2024

Table: Tests of the J/ψ LDMEs fits from high p_T pp, and low p_T γp, $e^+e^-, \gamma\gamma$ data.

Group	pp (p_T in fit)	pp (pol.)	pp (η_c)	$J/\psi + Z$	e^+e^-	γp	$\gamma\gamma$
Chao et al. set 1	✓ ($p_T > 7\,\mathrm{GeV}$)	✓	✗	-	✗	✗	-
Chao et al. set 2	✓ ($p_T > 7\,\mathrm{GeV}$)	✓	✓	-	✗	✗	-
Butenschön et al.	✓ ($p_T > 3\,\mathrm{GeV}$)	✗	✗	✗	✓	✓	✗
Zhang et al. $+\eta_c$	✓ ($p_T > 7\,\mathrm{GeV}$)	✓	✓	-	✗	✗	-
Bodwin et al.	✓ ($p_T > 10\,\mathrm{GeV}$)	✓	✗	✗	✗	✗	-
Feng et al.	✓ ($p_T > 7\,\mathrm{GeV}$)	✓	✗	-	✗	✗	-
pNRQCD	✓ ($p_T > 3 \times 2m_Q$)	✓	✗	✓(?)	✗	✗	-
pNRQCD	✓ ($p_T > 5 \times 2m_Q$)	✓	✓	✓(?)	✗	✗	-

- pNRQCD: fit the 3 gluonic correlators to high p_T hadroproduction data of $J/\psi, \psi(2S), \Upsilon(2S), \Upsilon(3S)$, constrains $\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$ to be small negative (η_c data also constrain $\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]})\rangle$ to be small).
- Conflicts between descriptions for the low p_T and high p_T data still remain.

The remaining main conflicts/puzzles – up to Nov. 2024



Figures from Butenschön & Kniehl, Mod.Phys.Lett. A 28 (2013) 1350027.

- All high $p_T > 7$GeV fittings overshoot the low p_T $\gamma p, e^+e^-$ data by a factor of $\sim 5-10$, see left two figures (taking Chao et al. as an example).
- Global fit cannot describe the low p_T $\gamma\gamma$ data and the J/ψ polarization data, see right two figures.
- Conflict between low p_T and high p_T fittings and descriptions.

Observations & Discussions

- It has been argued that NRQCD factorization may only hold at $p_T \gg 2m_Q$.
- NRQCD factorization fails for $\gamma p, e^+e^-$ at the end-point regions, $z \to 1$, $E_{J/\psi} \to E_{J/\psi}^{\max}$, respectively.
  Beneke & Wise, PLB 408 (1997) 373
- Ongoing debate: The conflicts are due to factorization breaking at the end-point regions or relatively low p_T regions? or both?
- Observation 1: There is no NLO prediction using high p_T fit for J/ψ p_T distribution from γp collision in the region $1 \gg z$ (existing predictions at $0.3 < z < 0.9$), although the data exist long time ago (surprising!).
- Observation 2: There is no NLO prediction using high p_T fit for the low p_T LEP data (surprising!), while the global fit cannot describe the LEP data.
- Revisit the low p_T and high p_T S-wave inclusive quarkonium production data from different experiments ($pp, \gamma p, e^+e^-, \gamma\gamma$), and check how well does NRQCD factorization at NLO describe the data.

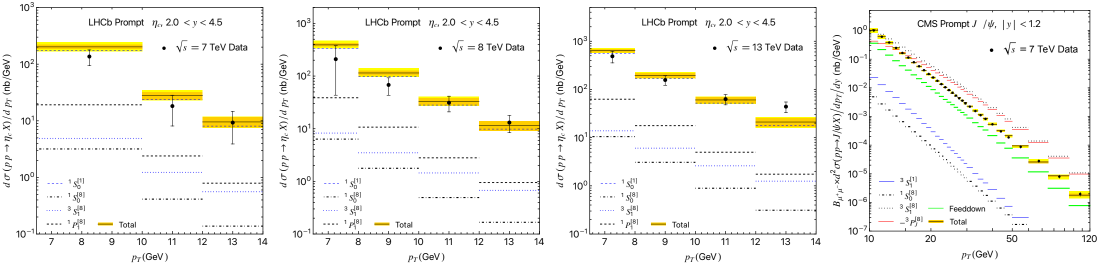
Our new fitting strategies and fitting results

- We combine LHC η_c and J/ψ data (42 data points) to fit 3 J/ψ CO LDMEs based on HQSS.
- We choose three different scale choices, $\mu_r = \mu_f = [\frac{1}{2}, 1, 2]m_T$, with the default scale choice $\mu_r = \mu_f = m_T$, where $m_T = \sqrt{4m_Q^2 + p_T^2}$;
- Systematically taking scale variations into account for the first time.
- $\psi(2S), \Upsilon(nS)$ CO LDMEs are related to those of J/ψ through pNRQCD relations. Feeddown from χ_{QJ} states are fitted from the measured data.

We obtain (in units of 10^{-2} GeV^3),

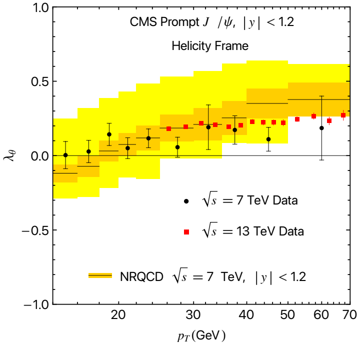
$\mu_r = \mu_f$	$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle$	$\langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle$	$\frac{\langle \mathcal{O}^{J/\psi}(^3P_0^{[8]}) \rangle}{m_c^2}$	$\frac{\chi^2_{\text{min}}}{\text{d.o.f}}$
$m_T/2$	0.592 ± 0.057	-0.205 ± 0.196	0.697 ± 0.089	0.34
m_T	1.050 ± 0.121	0.068 ± 0.2489	1.879 ± 0.261	0.22
$2m_T$	1.382 ± 0.189	0.358 ± 0.303	3.270 ± 0.533	0.21

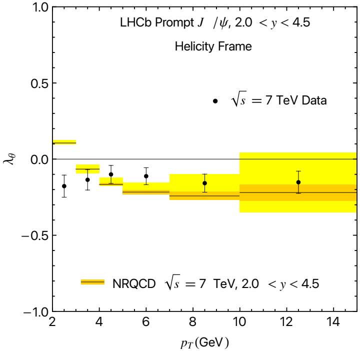
Fitting results – LHCb η_c & CMS J/ψ production



- Fit the LHCb η_c & CMS J/ψ production data to the 3 J/ψ CO LDMEs: $\langle \mathcal{O}^V(^3S_1^{[8]})\rangle$, $\langle \mathcal{O}^V(^1S_0^{[8]})\rangle$, $\langle \mathcal{O}^V(^3P_0^{[8]})\rangle$, the CO LDMEs of η_c are related to those of J/ψ based on HQSS.
- Inner bands (orange) correspond to the default scale choice (both for SDCs and fitted LDMEs), the outer bands (yellow) encompass the uncertainties coming from the two other scale choices.

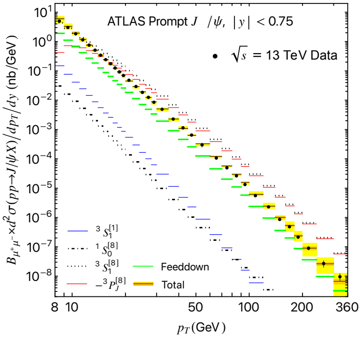
Prediction – J/ψ polarization

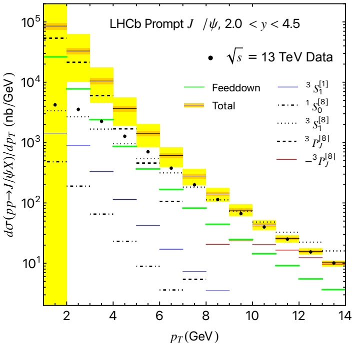




- In good agreement with the measurements and match the pattern that λ_θ turns from slightly negative at relatively low p_T to positive and converges to $\lambda_\theta \sim 0.3$ at high p_T.
- No polarization puzzle appears.

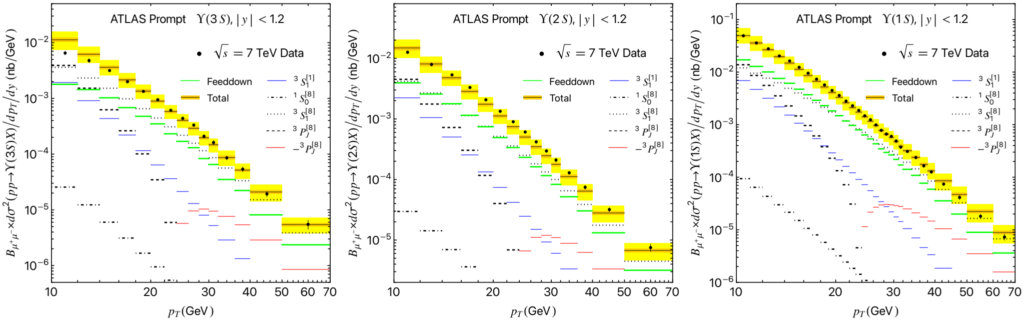
Prediction – J/ψ production at very high p_T & low p_T





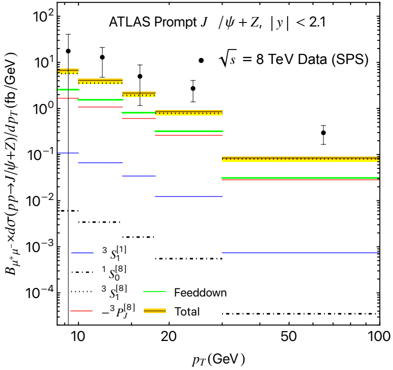
- Excellent description up to the highest measured p_T $(360\,\text{GeV})$, supprising!
- Data with $p_T < 7$ GeV are not well described. Small-x resummation needed?

Prediction – ATLAS $\Upsilon(nS)$ production in pNRQCD



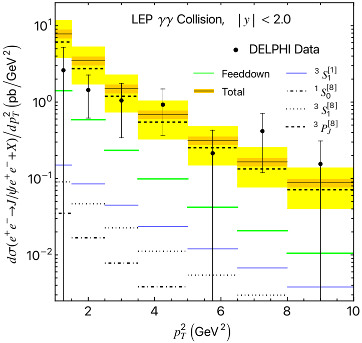
- $\Upsilon(nS)$ data are very well reproduced using the fitting results of the $3\ J/\psi$ CO LDMEs and pNRQCD expressions of LDMEs, Brambilla, Chung, Vairo & **XPW**, PRD105, L111503 (2022); JHEP 03 (2023) 242
highly nontrivial test of the pNRQCD relations.

Prediction – ATLAS $J/\psi + Z$, single parton scattering (SPS)



- For the two highest p_T bins, predictions lie $\sim 2\sigma$ deviations below data.
  Underestimated DPS contributions, unlikely? or?

Prediction – LEP $\gamma\gamma \to J/\psi + X$



- The cross section is exclusively dominated by single-resolved photon contributions. CS contribution is far below the data.

Prediction – HERA $\gamma p \to J/\psi + X$ $(0.1 < z < 0.6)$

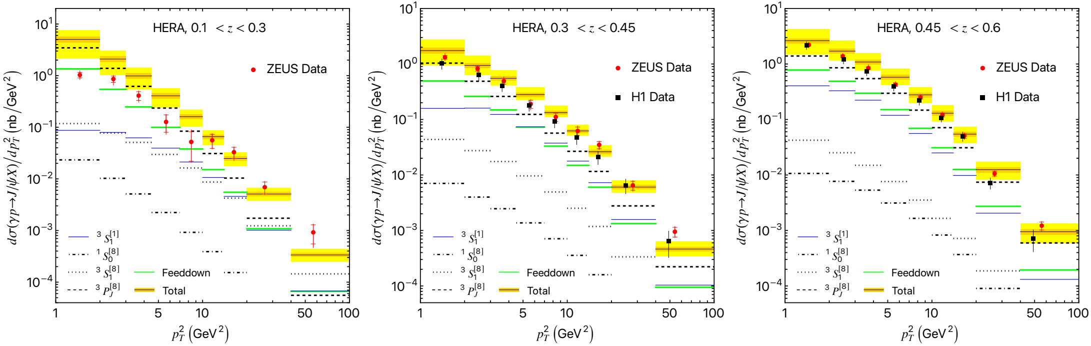
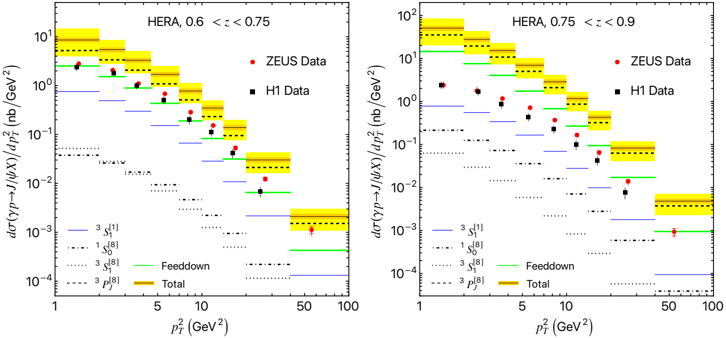


Figure: Prediction with divided z bins. Inelasticity $z = E_{J/\psi}/E_\gamma$ in the proton rest frame.

- For $0.1 < z < 0.3$, good description for the data except for a few lowest p_T bins, where resolved photon $(gg \to J/\psi + X)$ contribution dominates.
- The data can be well described in the whole measured p_T range, $[1, 10]$GeV.

Prediction – HERA $\gamma p \to J/\psi + X$ $(0.6 < z < 0.9)$



- Obviously overshoot the data, regardless of p_T. For $0.75 < z < 0.9$, predictions overshoot the data by factors of 5.2 to 20.
- The region $z \to 1$ corresponds to the endpoint region, where the NRQCD factorization may not be valid, v^2 expansion becomes $v^2/(1-z)$ expansion. Beneke, Rothstein & Wise, PLB 408, 373 (1997).

Score card of fittings – up to now

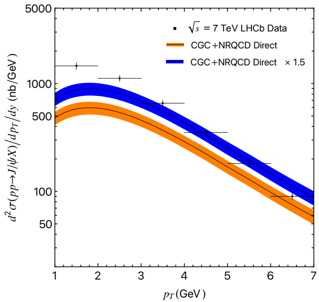
Table: Tests of the J/ψ LDMEs fits from high p_T pp, and low p_T γp, $e^+e^-, \gamma\gamma$ data.

Group	pp (p_T in fit)	pp (pol.)	pp (η_c)	$J/\psi + Z$	e^+e^-	γp	$\gamma\gamma$
Chao et al. set 1	✓ ($p_T > 7\,\mathrm{GeV}$)	✓	✗	-	✗	✗	-
Chao et al. set 2	✓ ($p_T > 7\,\mathrm{GeV}$)	✓	✓	-	✗	✗	-
Butenschön et al.	✓ ($p_T > 3\,\mathrm{GeV}$)	✗	✗	✗	✓	✓	✗
Zhang et al. $+\eta_c$	✓ ($p_T > 7\,\mathrm{GeV}$)	✓	✓	-	✗	✗	-
Bodwin et al.	✓ ($p_T > 10\,\mathrm{GeV}$)	✓	✗	✗	✗	✗	-
Feng et al.	✓ ($p_T > 7\,\mathrm{GeV}$)	✓	✗	-	✗	✗	-
pNRQCD	✓ ($p_T > 3 \times 2m_Q$)	✓	✗	✓(?)	✗	✗	-
pNRQCD	✓ ($p_T > 5 \times 2m_Q$)	✓	✓	✓(?)	✗	✗	-
2411.16384	✓ ($p_T > 7\,\mathrm{GeV}$)	✓	✓	✓(?)	✗	✓($z < 0.6$)	✓

- Less likely, the conflicts result from NRQCD factorization violations at relatively low p_T.

The remaining puzzles and possible solutions

- Observables still evade a consistent description: coincide with "extensions" of endpoint regions.
- Low p_T hadroproduction ✗ Possible solution: small-x resummation
- J/ψ photoproduction $(z > 0.6)$, J/ψ from Belle ✗ Possible solution: quarkonium shape function



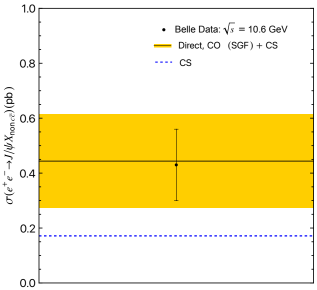


Figure: Plots unpublished. SDCs from Ma & Venugopalan, PRL 113, 192301 (2014); Chen, Jin, Ma & Meng, JHEP 03 (2022), 202

$e^+e^- \to J/\psi + \text{light hadrons}$

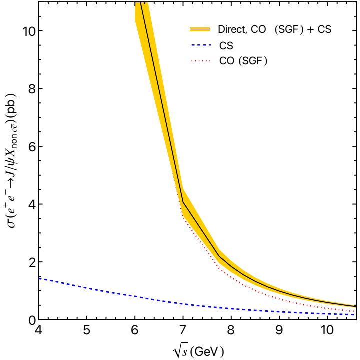


Figure: Predictions for $e^+e^- \to J/\psi + \text{light hadrons}$. CO (SGF) SDCs provided by Anping Chen.

- Soft-Gluon Factorization (SGF) may not hold at small $\sqrt{s}$, and we may estimate the cross section only by using the CS contributions.
- At $\sqrt{s} = 7\,\text{GeV}$, $\sigma_{\text{CS}} = 0.545\,\text{pb}$ and the total cross section $\sigma_{\text{tot}} = 4.07\,\text{pb}$.

$e^+e^- \to J/\psi + \eta_c$

- Helicity flipped process: an ideal process to study next-to-leading power factorization.
- Belle measurement (2004): $\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{>2} = 25.6 \pm 2.8 \pm 3.4$ fb, where $\mathcal{B}_{>n}$ denotes the branching fraction for the η_c into n charged tracks.
- Babar measurement (2005): $\sigma[e^+e^- \to J/\psi + \eta_c] \times \mathcal{B}_{>2} = 17.6 \pm 2.8^{+1.5}_{-2.1}$ fb.
- At $\sqrt{s} = 10.58$GeV, the NNLO predictions are: $15.2^{+4.1+7.6}_{-3.6-4.6}$ fb (Pole mass), $13.7^{+5.7}_{-4.7}$ fb ($\overline{\mathrm{MS}}$ mass).

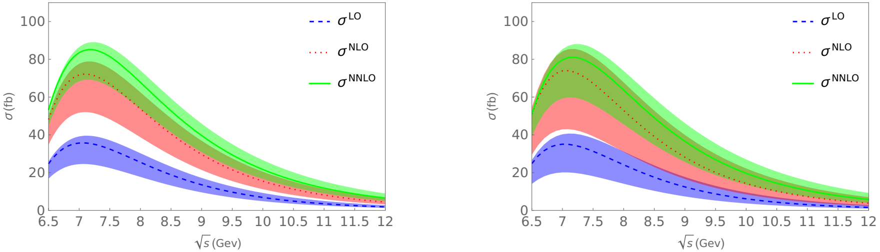


Figure: Predictions from Li, Huang & Sang, arXiv: 2506.16317, figures provided by Wen-long Sang. Left panel: pole mass with $m_c = 1.5$ GeV, right panel: $\overline{\mathrm{MS}}$ mass with $m_c = 1.273$ GeV. Renormalization scale $\mu = \sqrt{s}/2$, uncertainty bands coming from scale variation from 3 GeV to $\sqrt{s}$.

$e^+e^- \to J/\psi + J/\psi$

- Belle measurement (2003): $\sigma[e^+e^- \to J/\psi + J/\psi] \times \mathcal{B}_{>2} < 9.1$ fb (90% confidence level).
- At $\sqrt{s} = 10.58$GeV, the predictions given by Sang, Feng, Jia, Mo, Pan & Zhang, PRL 131 (2023) 16, 161904 :

σ (fb)	Fragmentation	LO	NLO	NNLO
Optimized NRQCD	2.52	1.85	$1.93^{+0.05}_{-0.01}$	$2.13^{+0.30}_{-0.06}$
Traditional NRQCD		6.12	$1.56^{+0.73}_{-2.95}$	$-2.38^{+1.27}_{-5.35}$

TABLE II: Integrated cross section of $e^+e^- \to J/\psi J/\psi$ at various perturbative accuracy. The uncertainties are estimated by varying μ_R from m_c to $\sqrt{s}$.

- Fragmentation dominates, which is close to the optimized NRQCD NNLO prediction.
- Based on fragmentation dominant scenario, we give the prediction at $\sqrt{s} = 7$ GeV:

$$\sigma(e^+e^- \to J/\psi + J/\psi) \simeq \sigma_{\rm fr} = 4.2\,\text{fb}, \quad (9)$$

with the fragmentation contribution given by $(\beta = \sqrt{1 - 4M^2_{J/\psi}/s})$

$$\sigma_{\rm fr}(e^+e^- \to J/\psi + J/\psi) = \frac{32\pi^3 e_c^4 \alpha^4 f^4_{J/\psi}}{M^4_{J/\psi}} \frac{1}{s} \left[\frac{4 + (1-\beta^2)^2}{1+\beta^2} \ln\left(\frac{1+\beta}{1-\beta}\right) - 2\beta \right]. \quad (10)$$

Summary and outlook

Summary:

- NRQCD factorization works pretty well except for end-point regions, where resummations are needed.
- Good descriptions for $\Upsilon(nS)$ production, highly nontrivial tests of the pNRQCD relations for LDMEs.
- We have provided predictions for single and double charmonium production at STCF energy.

pp High p_T $(J/\psi, \eta_c, \Upsilon(nS)$, pol.)	$J/\psi + Z$	e^+e^-	γp	$\gamma\gamma$	pp (Low p_T, J/ψ)
✓	✓(?)	✓(SGF)	✓$(z < 0.6)$	✓	✓ (small-x resum)

Outlook:

- Further study is needed to confirm or disprove: conflicts are not due to NRQCD factorization violations at relatively low p_T, but end-point regions.
- $z > 0.6$ for $\gamma p \to J/\psi X$, quarkonium shape functions, soft gluon factorization (SGF) ...
- First lattice calculation of CO decay LDMEs in pNRQCD.
- Better factorization formalism for J/ψ inclusive production at STCF energy.