Preliminary Study of Quantum Entanglement in $e^+e^- \to \tau^+\tau^-$ under $\sqrt{s}$ =6 GeV at STCF

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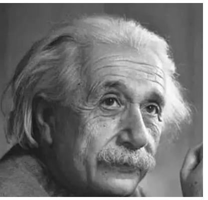
Outline

Introduction: Theory

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)



Albert Einstein, Boris Podolsky, and Nathan Rosen questioned whether QM gives a complete description of reality. They introduced a thought experiment (a.k.a EPR paradox)

1935

VOLUME 47, NUMBER 7 PHYSICAL REVIEW LETTERS 17 AUGUST 1981

Experimental Tests of Realistic Local Theories via Bell's Theorem

Alain Aspect, Philippe Grangier, and Gérard Roger
Institut d'Optique Théorique et Appliquée, Université Paris-Sud, F-91406 Orsay, France
(Received 30 March 1981)

2022

Alain Aspect`s Bell test experiments (1981-1982)
Used polarized photon pairs to test Bell's inequalities

1981-1982

1964

J.S. Bells`s Theorem Formulated Bell's Inequality:
predictions of local hidden variable theories must obey specific statistical limits Provided a testable framework

ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

J. S. BELL†
Department of Physics, University of Wisconsin, Madison, Wisconsin

(*Received* 4 *November* 1964)

Today

ATLAS, CMS, BelleII test quantum entanglement in high energy particle pairs ($\bar{t}t$, $\tau^+\tau^-$)

Article | Open access | Published: 18 September 2024

Observation of quantum entanglement with top quarks at the ATLAS detector

The ATLAS Collaboration

Nature **633**, 542–547 (2024) | Cite this article

nature
2024

Introduction: Theory

- **Bit**

  In classical computing, a bit can only be in one state, either 0 or 1

- **Qubit**

  Unlike classical bits, qubits can exist in a superposition.

  They can represent both 0 and 1 simultaneously.

  $$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \ (|\alpha|^2 + |\beta|^2 = 1).$$

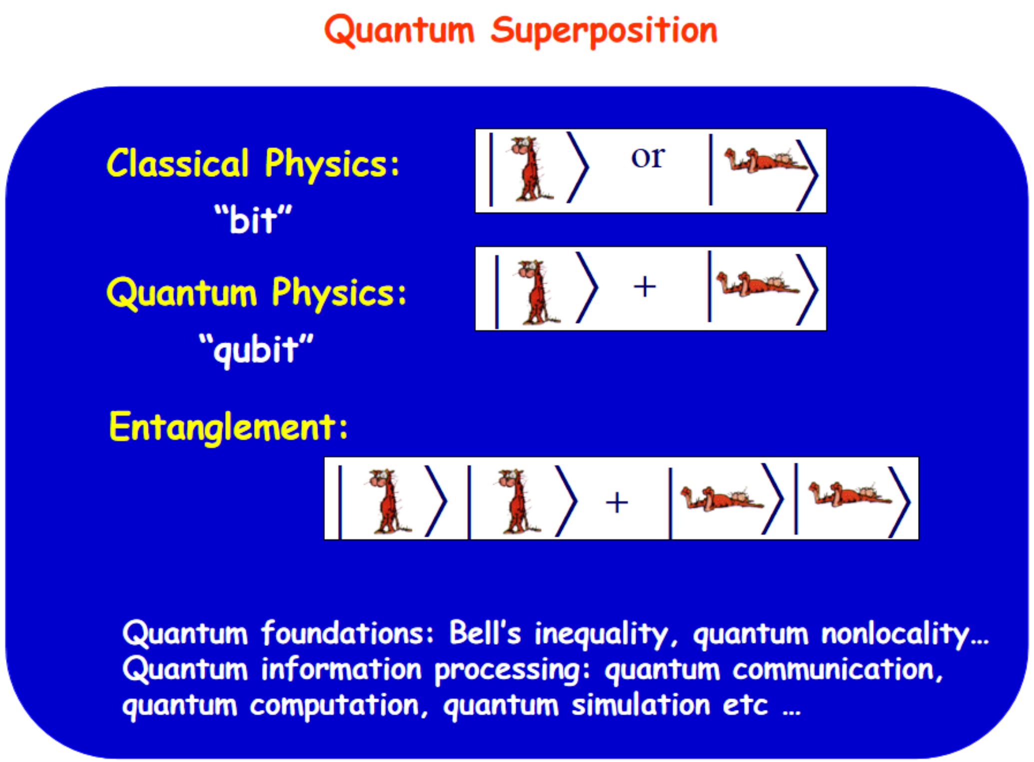
- **Quantum mixed state**

  Quantum mixed state can be described by a density matrix

  $$\rho = \sum p_i |\phi_i\rangle\langle\phi_i|$$

- **General density matrix for a qubit**

  For single bit, $\rho = \frac{I_2 + \sum B_i \sigma^i \otimes I_2}{2}$, where ***B*** is a spin polarization and are Pauli matrices



Introduction: Entanglement observables

- **Spin Density Matrix (SDM) for Two-qubit system ($\tau^+\tau^-$)**
  - Spin quantum state of a τ-pair is described by the spin density matrix

$$\rho = \frac{1}{4}[I\otimes I + \sum_i B_i^+(\sigma_i\otimes I) + \sum_j B_j^-(I\otimes\sigma_j) + \sum_{i,j} C_{ij}(\sigma_i\otimes\sigma_j)]$$

where B_i^+ / B_j^- is spin polarization of τ^+/τ^-, C_{ij} is correlation matrix connecting spin of τ^+/τ^-

- **QE observables based on SDM:**
  - **Concurrence $\mathcal{C}[\rho]$** :

For a bipartite qubit system, an entanglement monotone can be defined as:

no entanglement $0 \leq \mathcal{C}[\rho] = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\} \leq 1$ maximally entangled

where λ_i are the eigenvalues, in decreasing order, of the matrix

$$R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}, with \tilde{\rho} = (\sigma_2\otimes\sigma_2)\rho^*(\sigma_2\otimes\sigma_2)$$

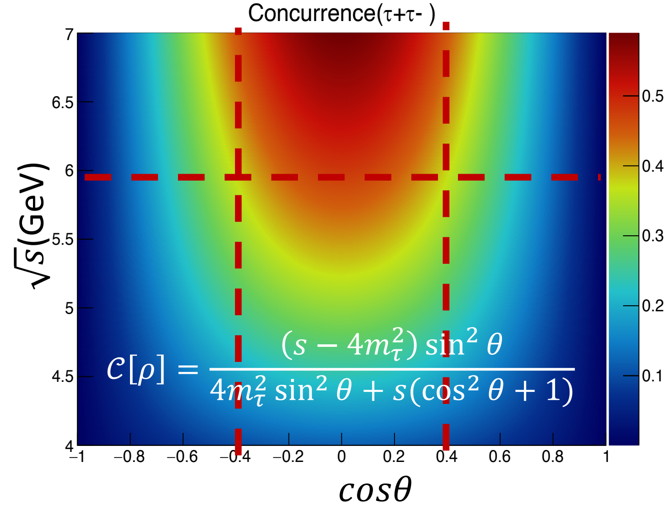
  - **Bell-inequality**: an optimized operator **$m12[C]$**:

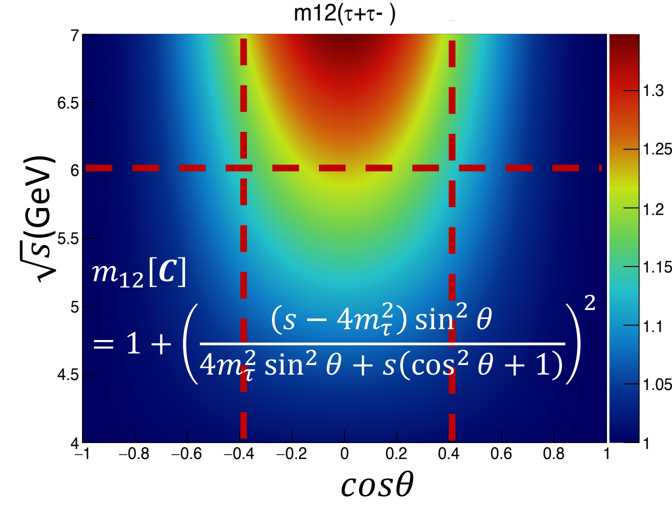
$$m_{12}[C] = m_1 + m_2$$

$m_{12}[C] > 1$ Bell-inequality is violated

where $m_1 \geq m_2 \geq m_3$ are the eigenvalues of the positive semi-definite matrix $M = C^T C$

theoretical signal region





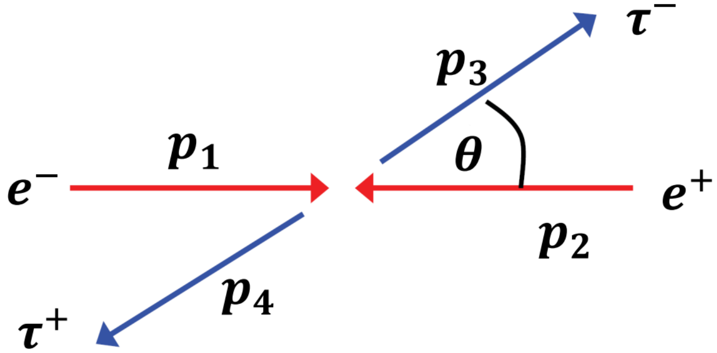
Introduction: Entanglement observables

❑ B, C

- $\rho = \frac{1}{4}[I\otimes I + \sum_i B_i^+ (\sigma_i \otimes I) + \sum_j B_j^- (I \otimes \sigma_j) + \sum_{i,j} C_{ij}(\sigma_i \otimes \sigma_j)$, where:

$$B_i^\pm = \frac{3}{\kappa_\pm}\frac{1}{\sigma}\int d\Omega^\pm \frac{d\sigma}{d\Omega^\pm}(\vec{h}^\pm \cdot \hat{e}_i)$$

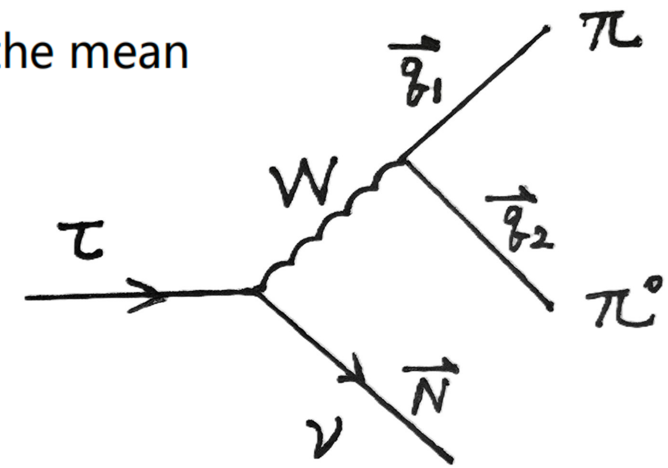
$$C_{ij} = \frac{9}{\kappa_+\kappa_-}\frac{1}{\sigma}\int d\Omega^+ d\Omega^- \frac{d\sigma}{d\Omega^+ d\Omega^-}(\vec{h}^+ \cdot \hat{e}_i)(\vec{h}^- \cdot \hat{e}_j)$$



- $\kappa_\pm = \pm 1.0$, $\vec{h}^\pm$ are polarimetric vectors for $\tau^\pm$.

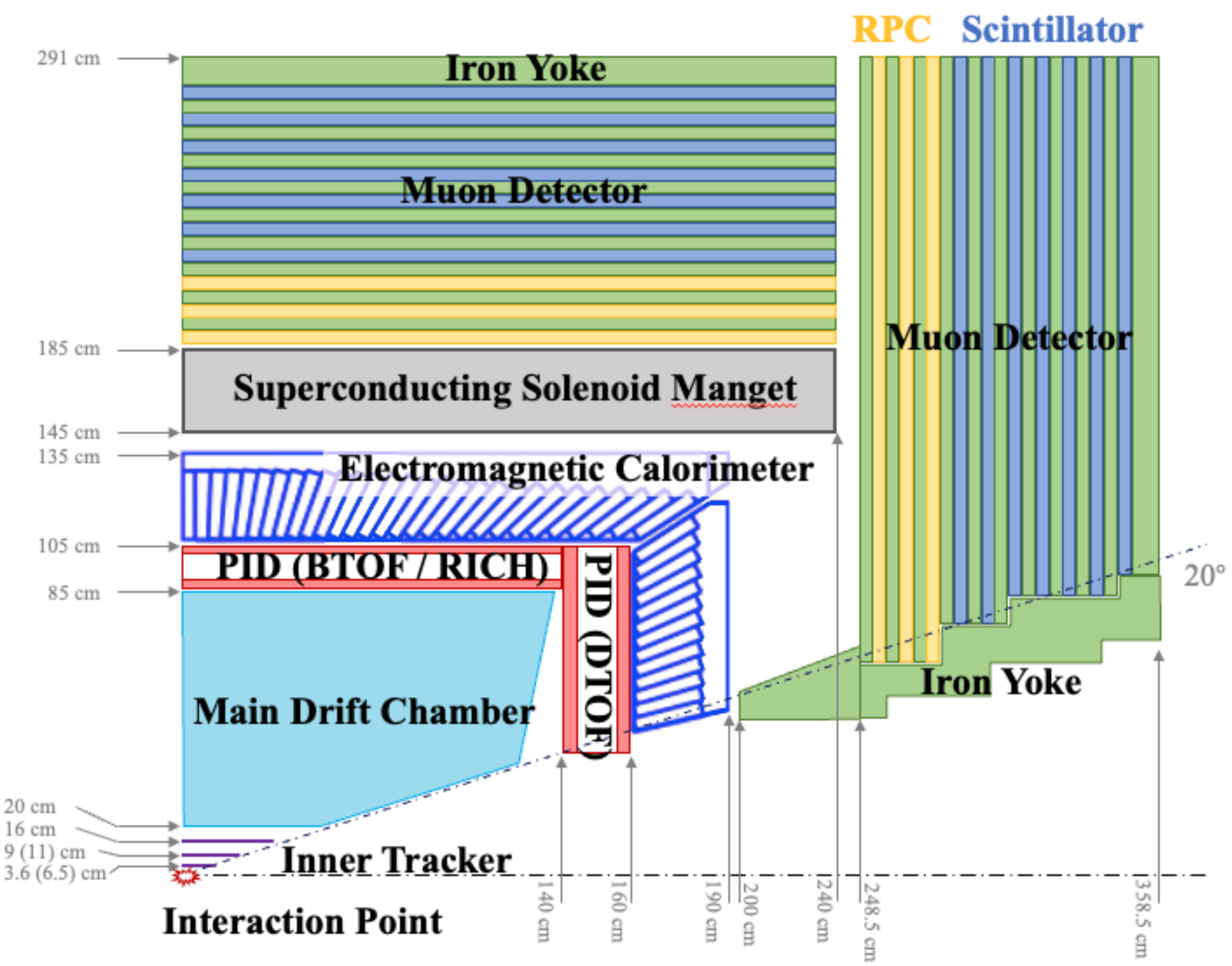
- So the key point is to retrieve $\vec{h}^\pm$, and B, C coefficients correspond to the mean value of $\vec{h}^\pm$ in different directions of $\{\hat{e}_i\}(i = 1,2,3)$, usually $\{\hat{n}, \hat{r}, \hat{k}\}$.

- $\vec{\boldsymbol{h}} \propto -[2(q^\mu N_\mu)\vec{\boldsymbol{q}} - (q^\mu q_\mu)\vec{\boldsymbol{N}}]$



Outline

STCF Detector



- ITK: Inner Tracker
  - ITKW: cylindrical MPGD
  - ITKM: CMOS M-MAPS
- MDC: Main Drift Chamber, Central tracker
  - π^+/π^- track Reconstruction
- PIDE: Particle Identification-Endcap
  - DTOF: DIRC-like TOF
  - PIDB: Particle Identification-Barrel
  - RICH: Ring Imaging Cherenkov detectors CsI-MPGD
  - BTOF: Barrel-TOF
  - π^+/π^- Identificaton
- ECAL(EMC): Electromagnetic Calorimeter pure CsI + APD
  - π^0 Reconstruction
- MUD: Muon Detector RPC + scintillator

τ-pair Production on STCF

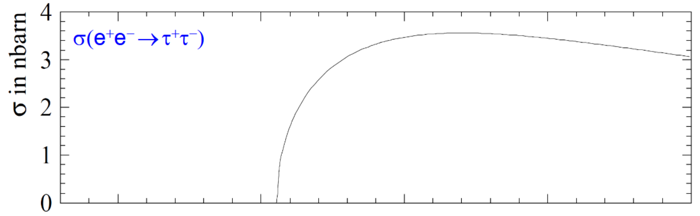
❑ τ-pair Production on STCF

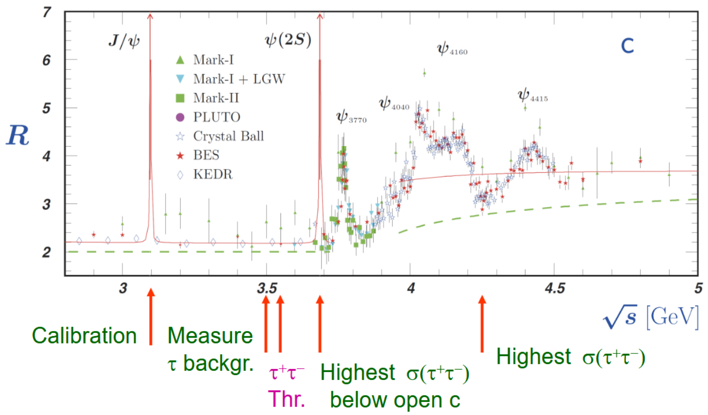
- STCF: 2-7GeV event rate: $3.5\times10^{9}/y(4.26GeV)$
- E_{cm}=4.26GeV: highest $\sigma(\tau^{+}\tau^{-})$
- E_{cm}=6.0-7.0GeV: higher significance of Quantum Entanglement
- Decay channel: $\tau \to \pi\nu_{\tau}, \tau \to \rho\nu_{\tau}, \tau \to a_1\nu_{\tau}, \tau \to e\nu_e\nu_{\tau}, \tau \to \mu\nu_{\mu}\nu_{\tau}$

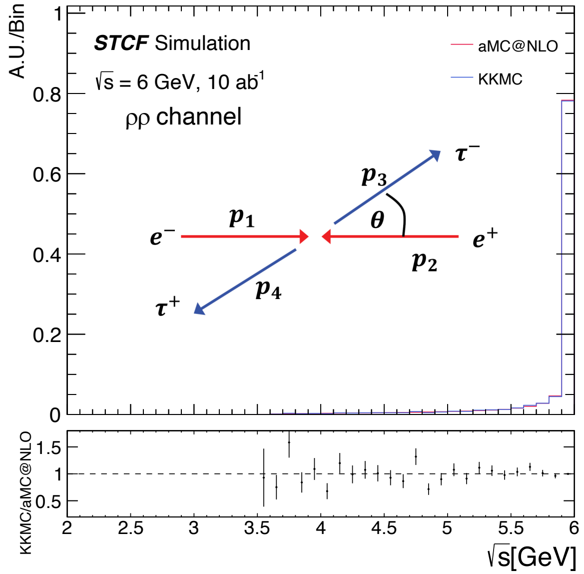
❑ Tau pair production on **Oscar 2.6.2** (Fullsim + digi + reco):

- ✓ $\sqrt{s} = 6GeV, 10\ million\ events \sim 0.0042ab^{-1}$with ISR
- ✓ Matrix element (LHE) simulated via aMC@NLO
- ✓ Tau decay simulated via aMC@NLO(taudecay_UFO)
- ✓ Spin correlations between tau and their decay products are fully considered

❑ Background Simulation: Considered different tau decay channel





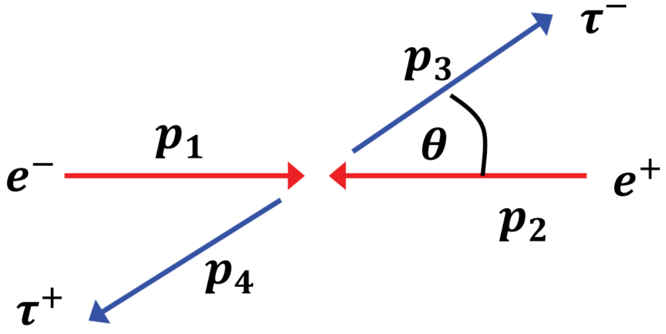


Outline

Reconstruction

- Signal decay: $\tau^+\tau^- \to \rho^+\rho^-\nu\,\bar{\nu}$

  main background: $\tau^+\tau^- \to \rho\pi\pi^0\pi^0, \tau^+\tau^- \to \pi\pi\pi^0\pi^0, \tau^+\tau^- \to \rho K^*$,etc



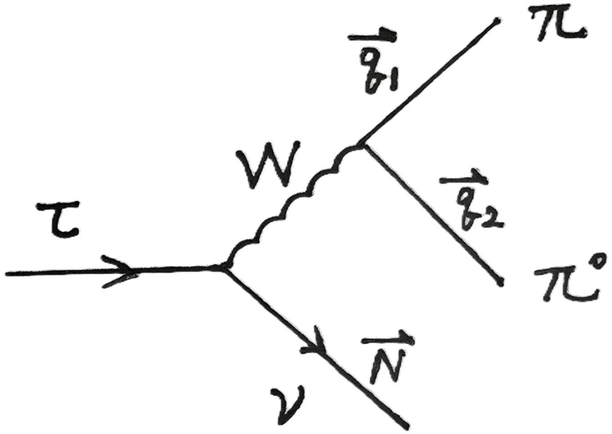
- Selection criteria:

  γ energy $E > 0.05\ GeV$(endcap), $E > 0.025\ GeV$(barrel)

  geometric acceptance: $20^\circ < \theta < 160^\circ$

- Reconstruction step:

  0. Number of π^+ = 1, Number of π^- = 1
  1. Passed γ-level machine learning selection, Number of γ = 4(separate signal from bkg)
  2. Passed pairing of γ, Passed pairing of π^0 and $\pi^+/\ \pi^-$
  3. Passed event-level machine learning selection



Step1: π^0 gamma Reconstruction

- Beam background gamma: cut by ECAL(~200ps)
- ISR feature: low energy, relatively forward
- Use BDTG to Select π^0 gamma from other gamma(ISR gamma)
- Input train variable

  E_{seed}:energy deposited in the center crystal of the shower

  E_{total} hits $\cos(\theta)$

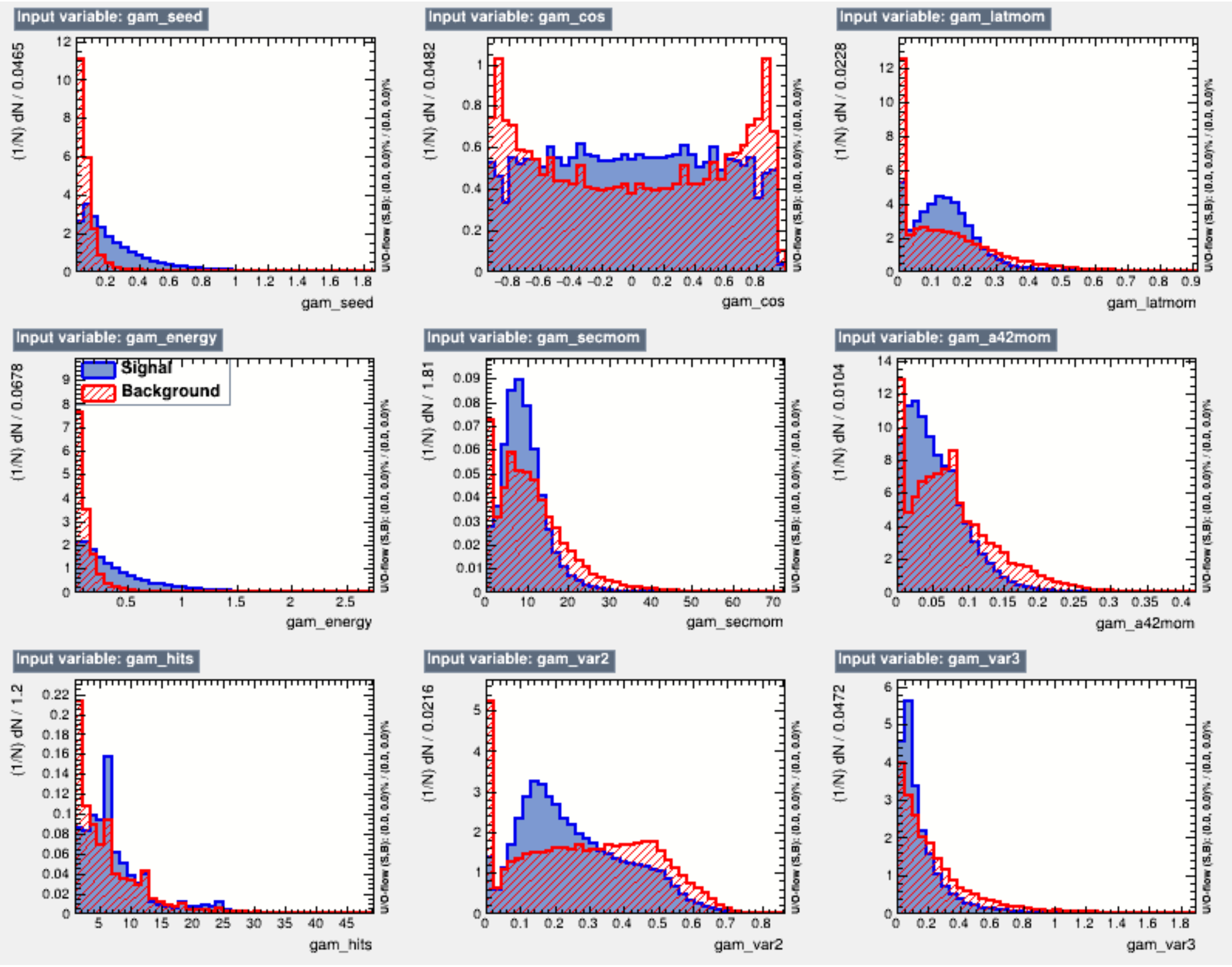
  secondary moment:$\sum E_i r_i^2 / \sum E_i$

  Lateral moment:$\sum_{i=3}^{n} E_i r_i^2 / (E_1 r_0^2 + E_2 r_0^2 + \sum_{i=3}^{n} E_i r_i^2)$

  A_{42}moment: $\sum \frac{E_i}{E_{tot}} f_{4,2}\left(\frac{r_i}{R_0}\right) e^{im\phi}$

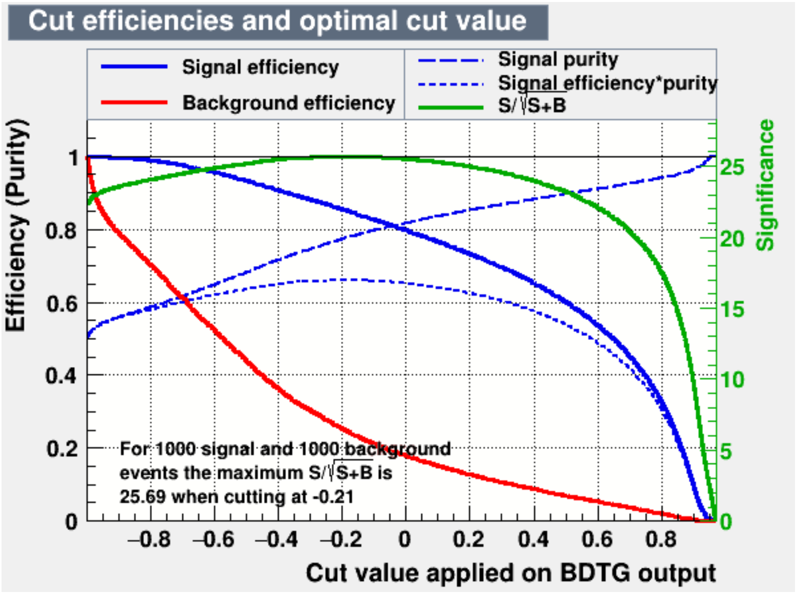
  Variable2$= 1 - \frac{E_{seed}}{E_{3x3}}$

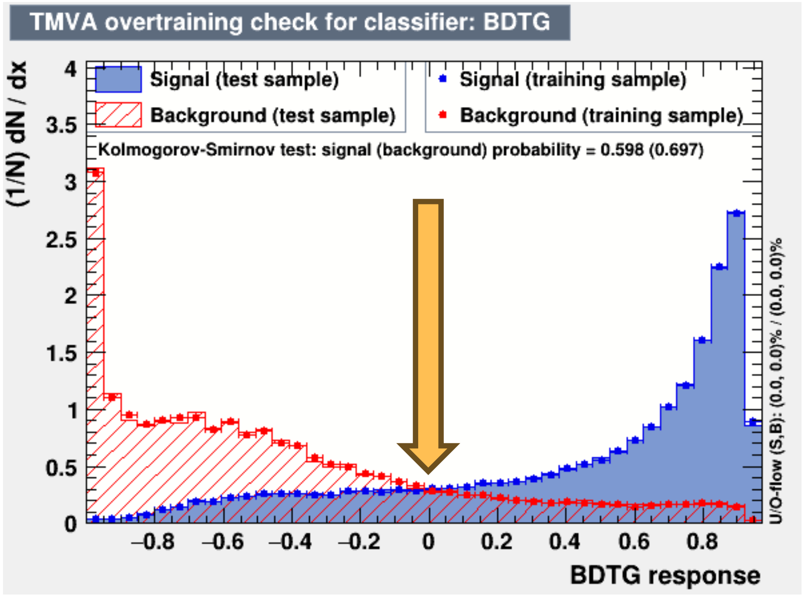
  Variable3$= \begin{cases} 0, & hits = 1 \\ (\frac{E_{tot}}{E_{seed}} - 1)/(hits - 1), & hits > 1 \end{cases}$

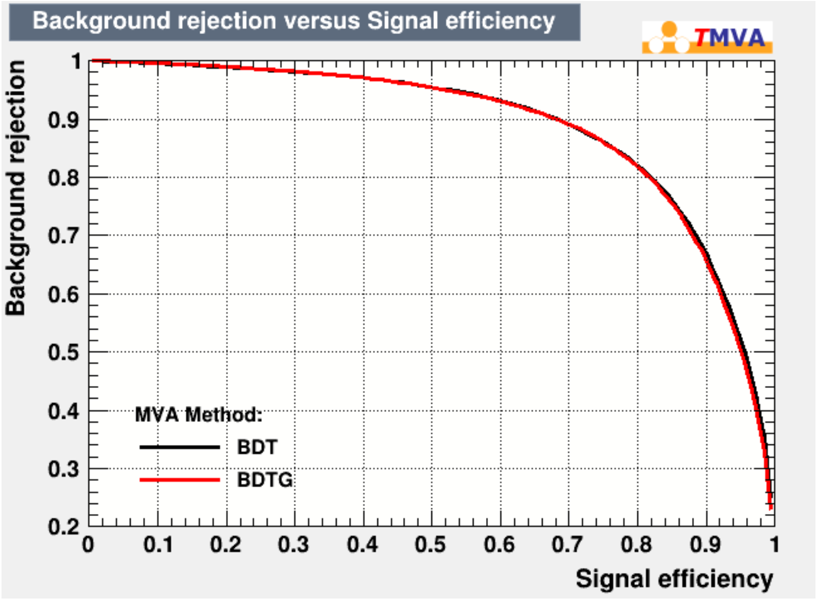


Step1: π^0 gamma Reconstruction

- Main background: ISR gamma
- Set BDTG cut = 0
- retaining about 80% pi0 gamma and 20% other gamma(ISR)
- Efficiency : $\rho\rho$ decay channel(25.2%), all channel(7.3%)







Step2: Event Pairing

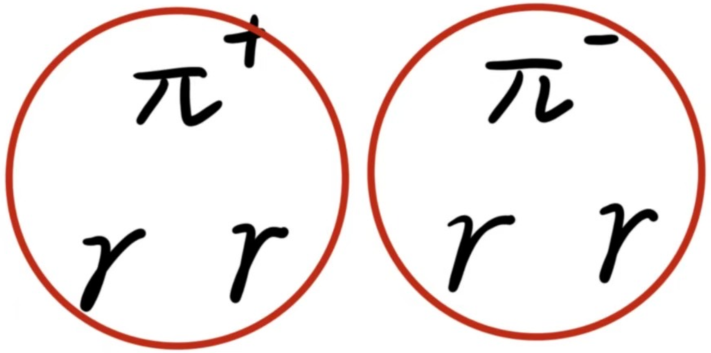
- ❑ Loop all pairing schemes and select
- ❑ Kalman Kinematic Fit (BES)

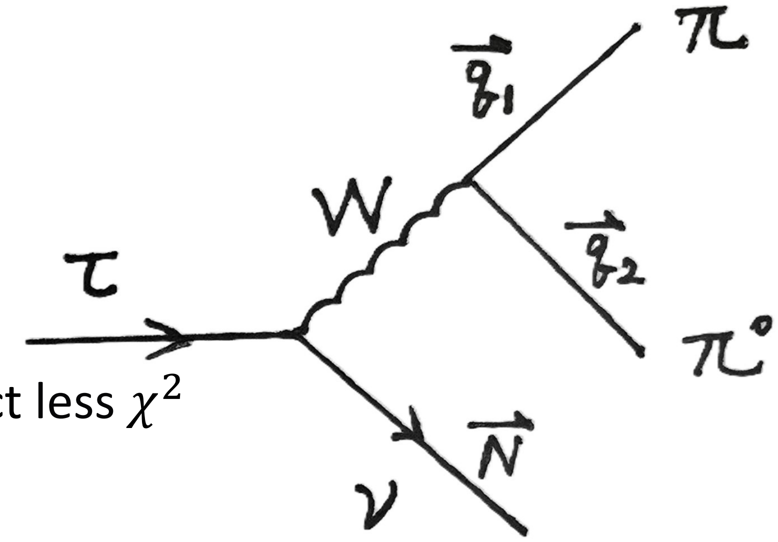
  Pairing gamma

  - Using mass constraints of π^0

  Pairing π^0 with π^+/π^- , Solve the ν energy-momentum $p_{\nu(1)}, p_{\nu(2)}$

  - Add Missing Momentum ν_1, ν_2 (6 unknown quantities)
  - Add Energy-momentum conservation (4 equation)
  - Add Mass constraints of τ (2 equation)

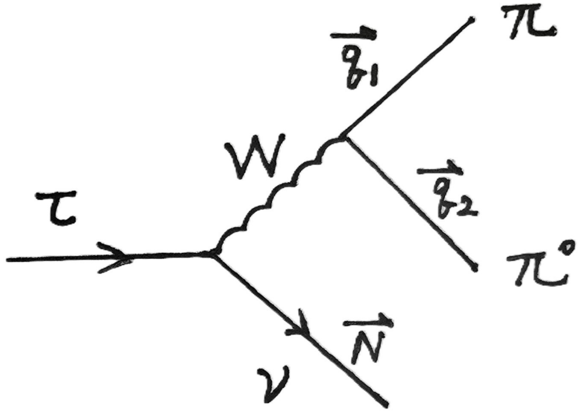


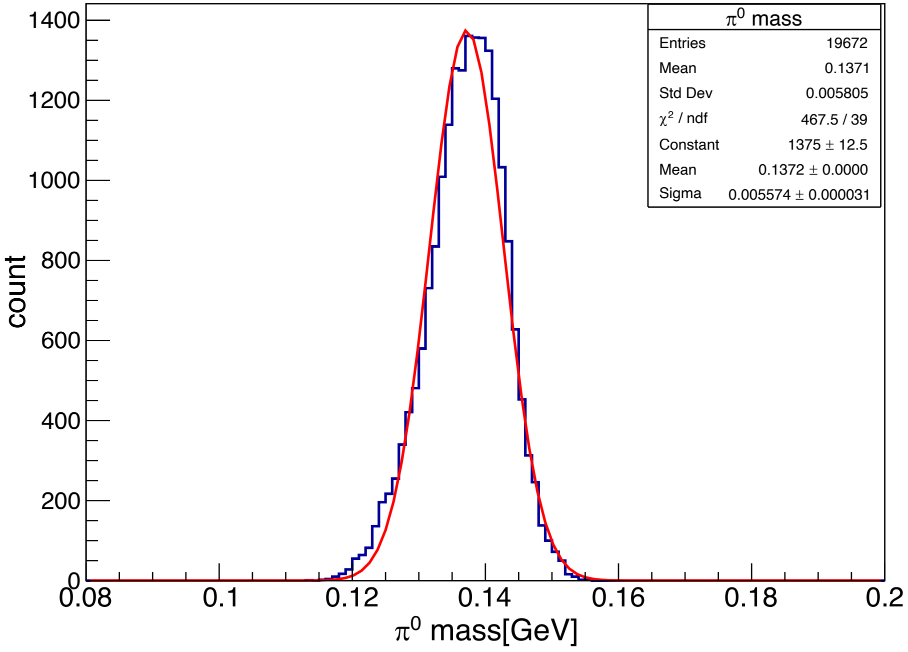


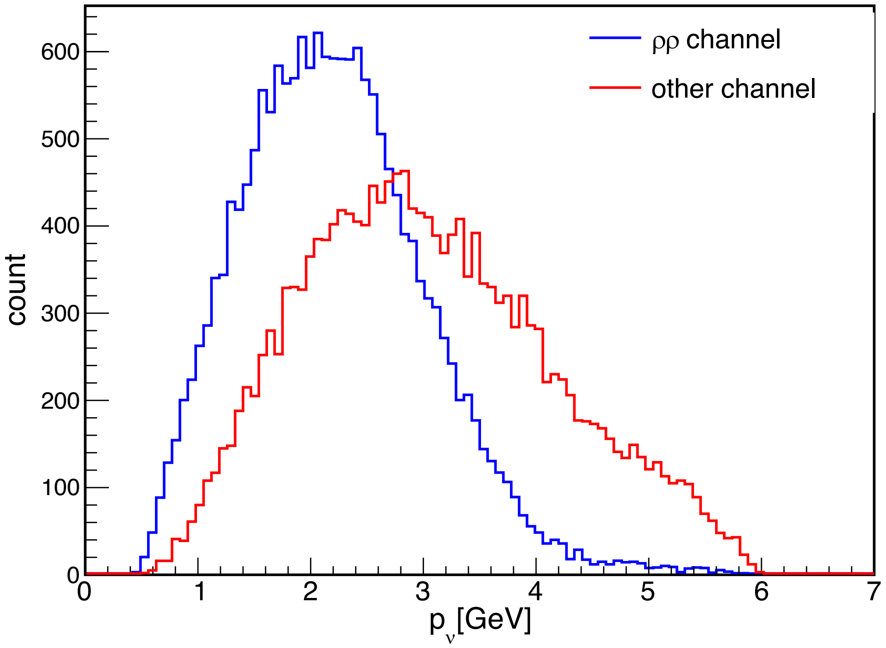
- ❑ passed the pairing cut: $\chi^2 < 10$ & $p_\nu = p_{\nu(1)} + p_{\nu(2)} < \frac{E_{total}}{2}$ and select less χ^2
- ❑ Efficiency : $\rho\rho$ decay channel(45.9%), all channel(22.4%)
- ❑ For $\rho\rho$ decay channel: true pairing: 98.25% , wrong pairing: 0.92% , include ISR: 0.83%

Step2: Event Pairing

- ❑ mass constraints of π^0
- ❑ pairing cut $p_\nu = p_{\nu(1)} + p_{\nu(2)} < \frac{E_{total}}{2}$
  - Ensure $p_{\nu(1)} < E_\tau$
  - Input p_ν to Event-level ML selection to select $\rho\rho$ channel







Step3: $\rho\rho$ event Reconstruction

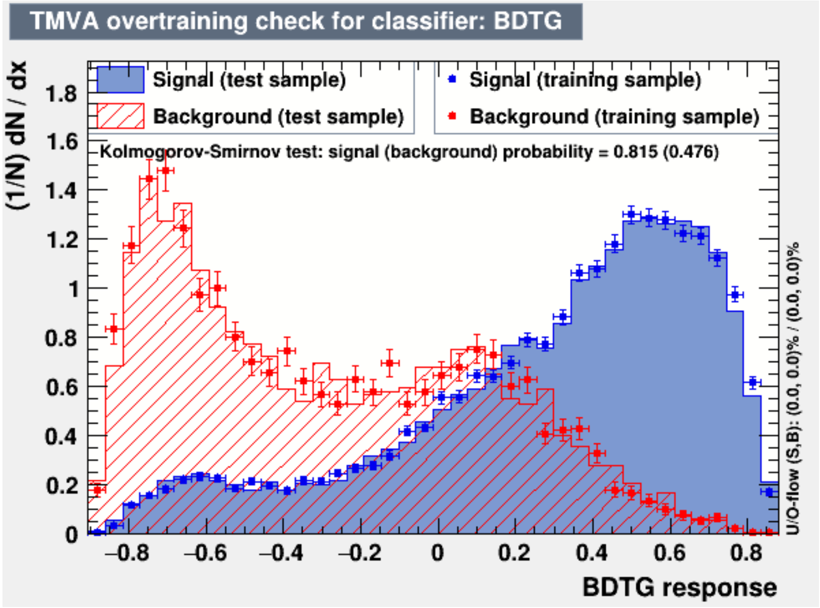
- Main background event: $\rho a(\rho \to \pi\pi^0, a \to \pi\pi^0\pi^0)$,etc
- Use BDTG
- Input train variable

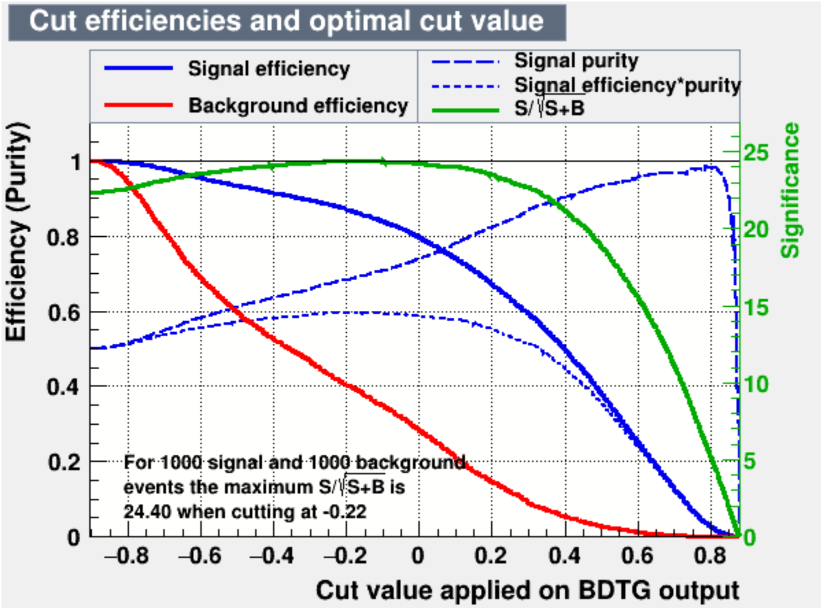
  momentum $p_{\nu(1)}, p_{\nu(2)}, p_{\pi^+}, p_{\pi^-}, 4\ p_\gamma$

  Pairing χ^2, p_ν

  PID: probPi, probMu, probK

  nGamma before gamma-level BDTG cut
- Set BDTG cut = -0.2， retaining about 87% signal event and 40% other event
- Efficiency : $\rho\rho$ decay channel(87.5%), all channel(78.8%)





Reconstruction: cutflow

- $\rho\rho$ decay channel (1000W events test)

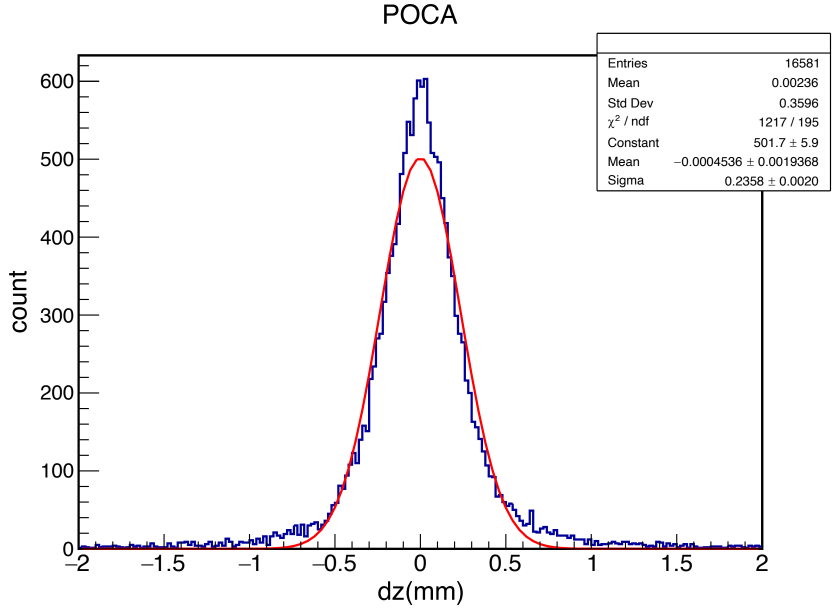
step	Percentage of previous step		Signal purity
	Signal + BKG	Signal	
Total events	-	-	6.9%
Number of charged tracks = 2, total charge = 0	65.4%	78.9%	8.3%
Number of photons = 4	7.3%	25.2%	28.8%
Number of π^+ = 1, Number of π^- = 1	51.8%	72.3%	40.1%
Passed the particle pairing	22.4%	45.9%	82.0%
Passed event-level machine learning selection	78.8%	87.5%	91.1%
Passed the τ momentum reconstruction	85.3%	85.4%	91.2%

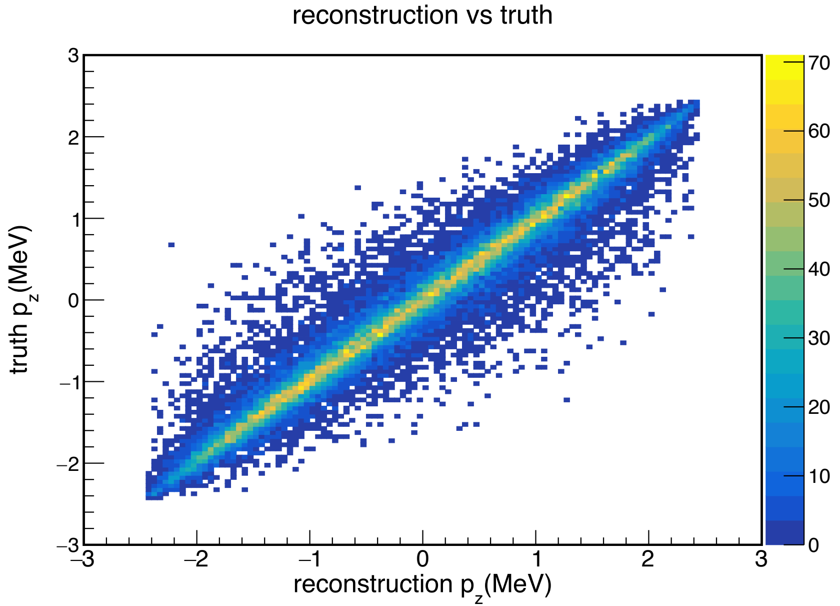
Overall efficiency: 0.37% , Signal efficiency: 4.9% , Signal purity: 91.2%

Signal region: true pairing: 89.56% , $\rho\rho$ wrong pairing: 0.84% , include ISR: 0.76% , Background event: 8.83%

Reconstruction: P_τ

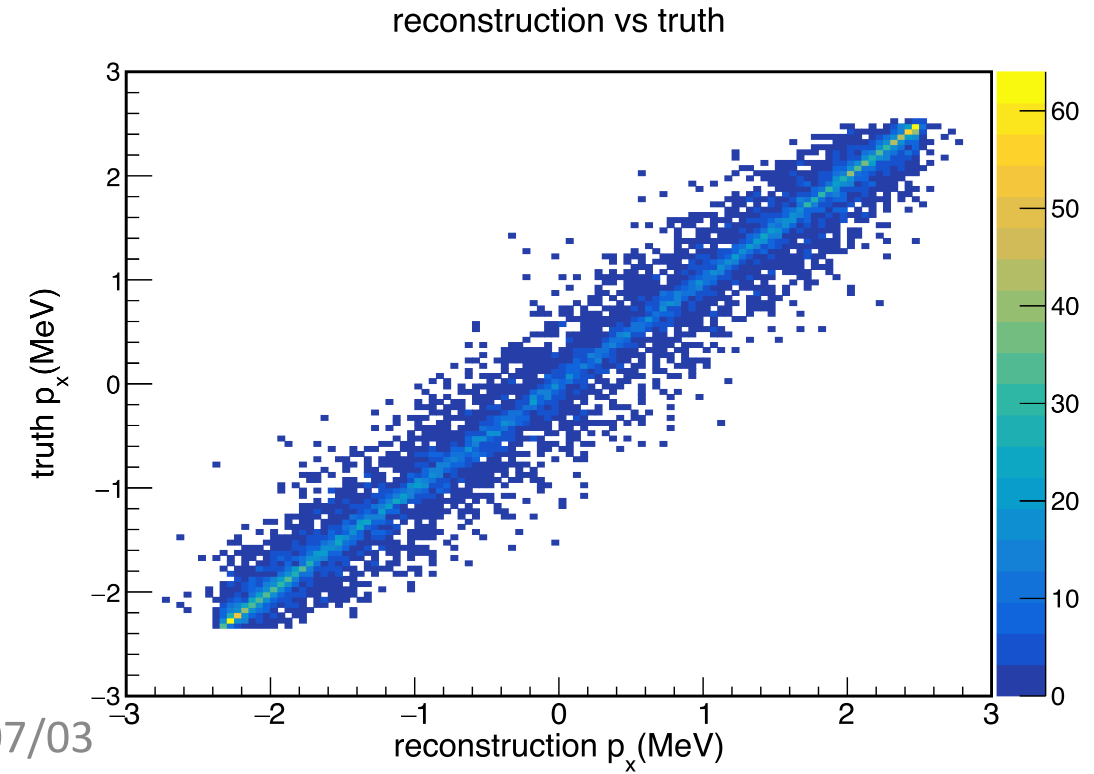
- P_τ reconstruction: solving a set of analytic equations
- Existing problem:
  - Because of two missing ν , the τ flight direction is calculated with a two-fold ambiguity
  - Vertex resolution/POCA point resolution($\sigma_z \sim 235um, \sigma_T \sim 80um$)
- Assuming higher resolution($\sigma_z' \sim 80um, \sigma_T' \sim 27um$)
  - Replace reconstruction vertex by smearing vertex
  - We can solve the two-fold ambiguity and reconstruction P_τ

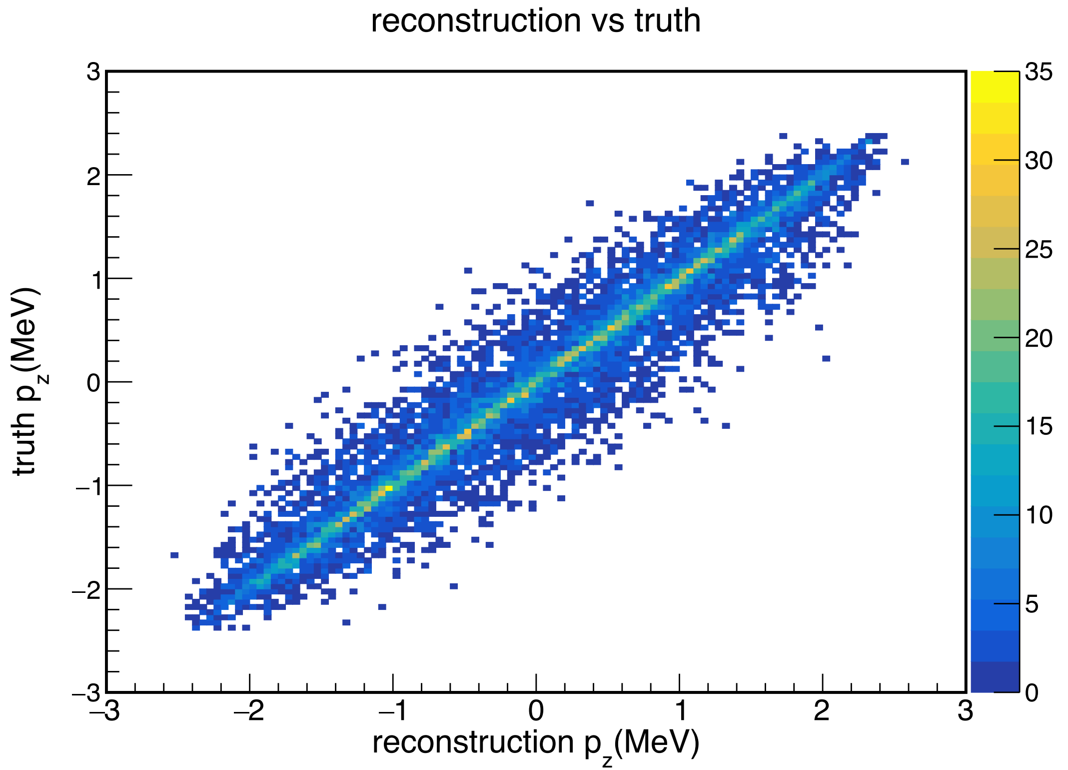




Reconstruction validation: based on truth P_τ

- ❑ P_τ reconstruction: Kalman Kinematic Fit (BES)
- ❑ Event cut:
  - Because of two missing ν , the τ flight direction is calculated with a two-fold ambiguity
  - Select event which two τ flight direction is similar





Outline

1 Introduction and Theoretical Background

2 STCF Detector and τ-pair Production

3 Reconstruction and Analysis

4 **Results and Systematics**

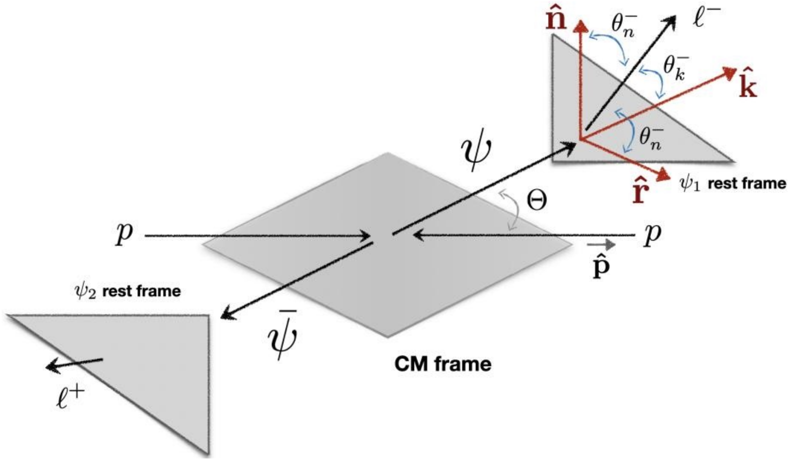
5 Summary and Prospect

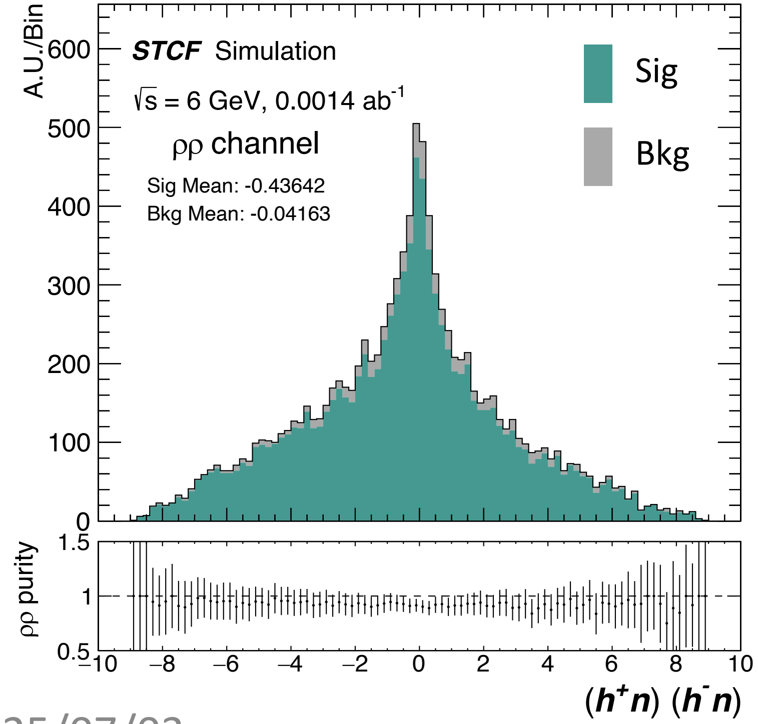
SDM Reconstruction

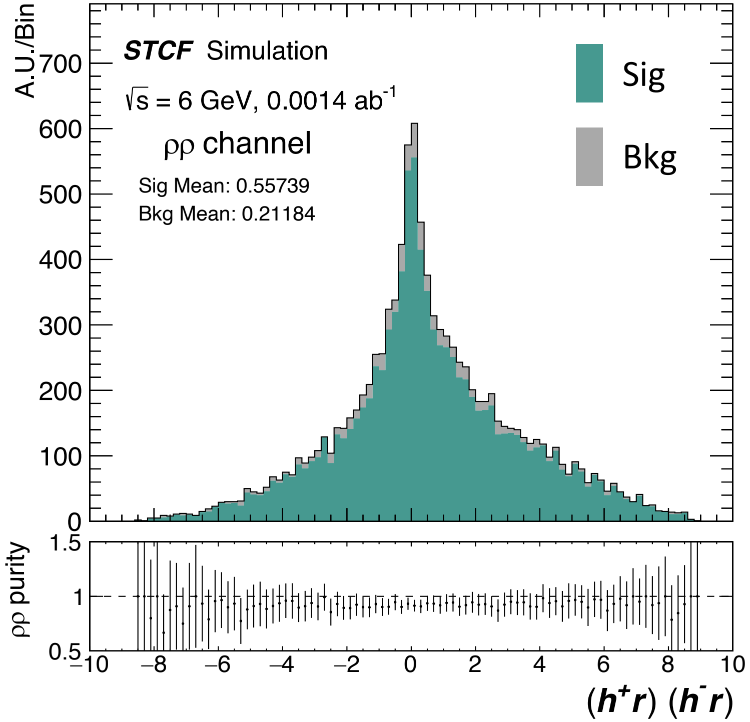
- Spin quantum state of a τ-pair is described by the spin density matrix

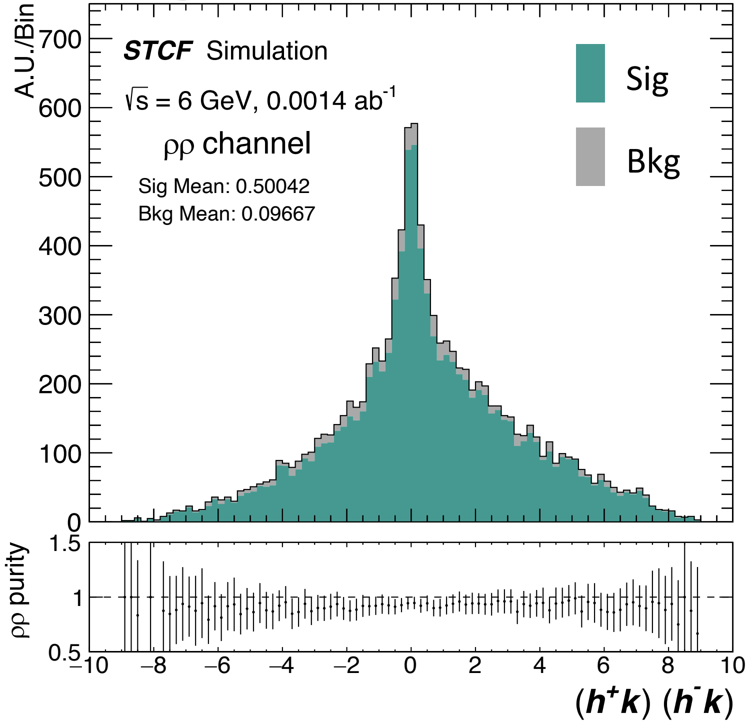
$$\rho = \frac{1}{4}\left[I\otimes I + \sum_i B_i^+(\sigma_i\otimes I) + \sum_j B_j^-(I\otimes\sigma_j) + \sum_{i,j} C_{ij}(\sigma_i\otimes\sigma_j)\right]$$

B_i^+ / B_j^- is spin polarization of $\tau^+\tau^-$, C_{ij} is spin correlation matrix



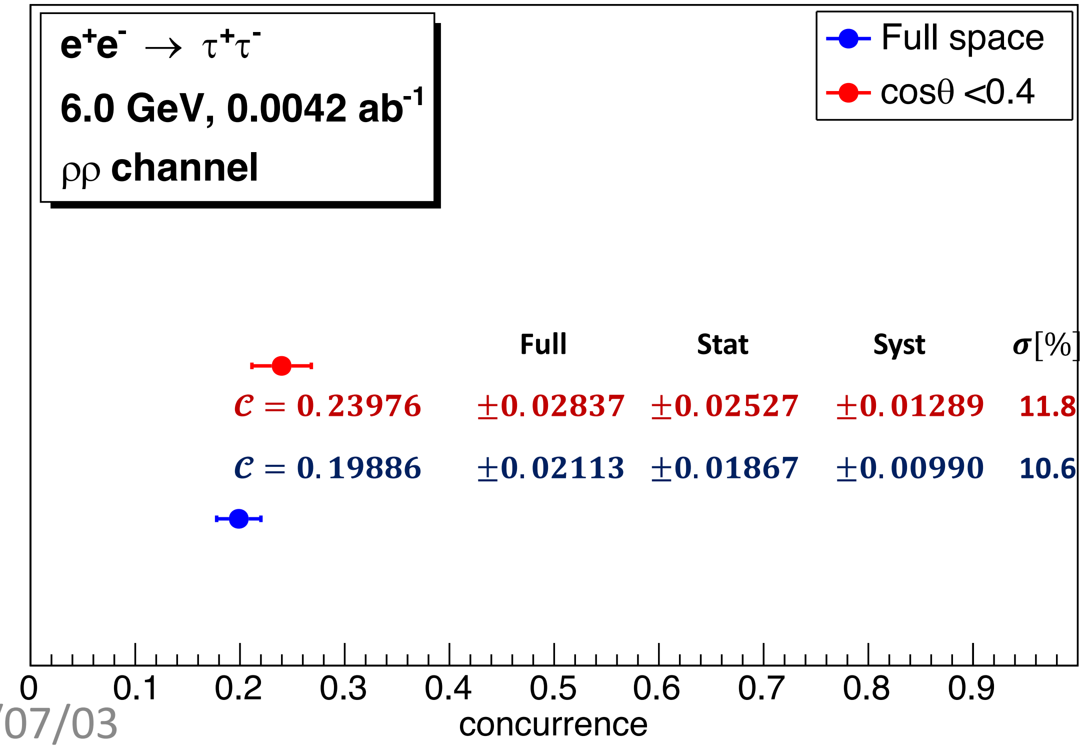


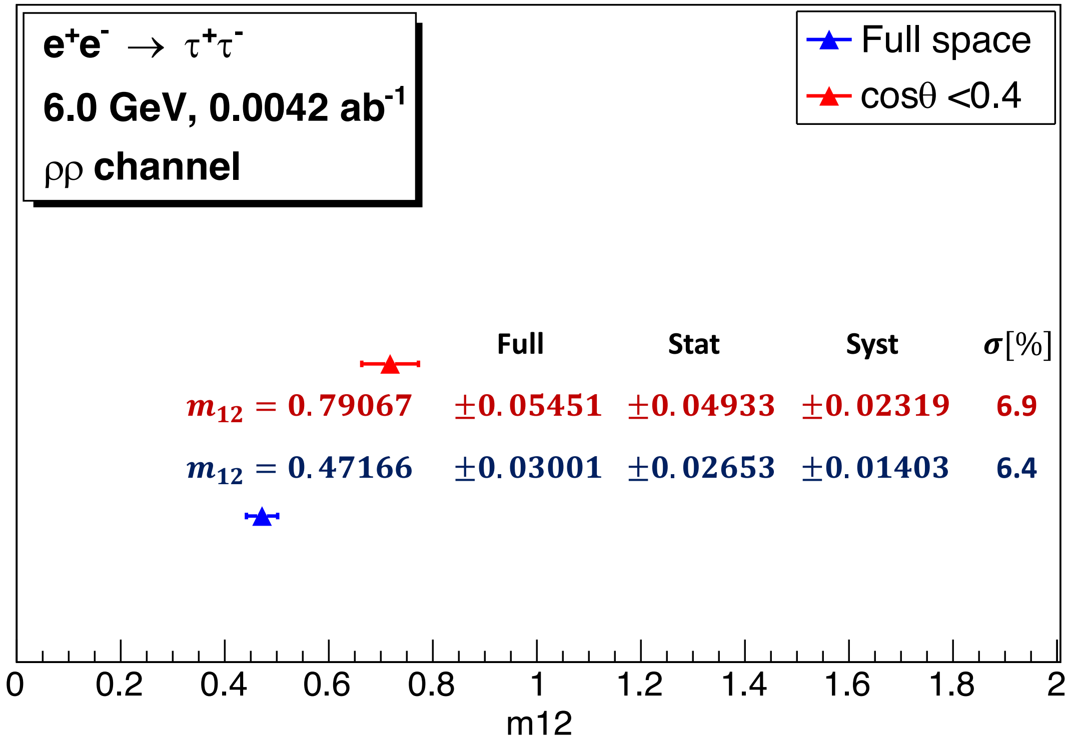




Uncertainties and Systematics

- A sample of 10 million MC events with ISR under **Oscar** full detector simulation
- For systematics, we simulate the experimental resolution by randomly varying ("**smearing**") the four-vectors of the charged and neutral particles produced in the τ decays before applying kinematic cut and fit
- We evaluated the statistical uncertainty via bootstrap method. Extend the sample to 100 million, the uncertainties $\boldsymbol{\sigma}$ will reaches 5%





Results and Prospect

- The prospective value for tau pair concurrence under the STCF luminosity :

$$\mathcal{C} = 0.23976 \pm 0.02527[stat.] \pm 0.01289[syst.]$$

Promising for an witness (>5σ) of the entanglement

- For the **witness m_{12}**

$$\mathbf{m_{12}} = 0.79067 \pm 0.04933[stat.] \pm 0.02319[syst.]$$

Still not enough to support violation of Bell's Inequality, we need:

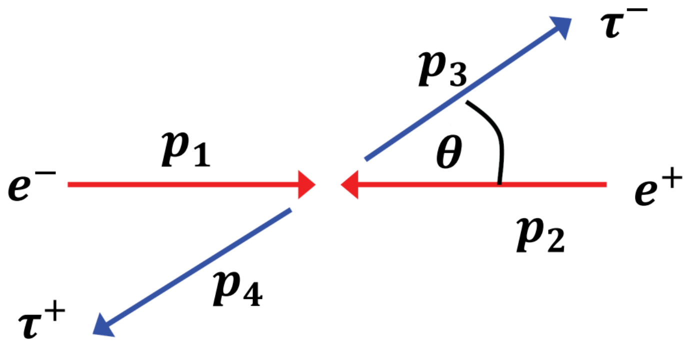
  - Higher event vertex resolution
  - Higher reconstruction accuracy

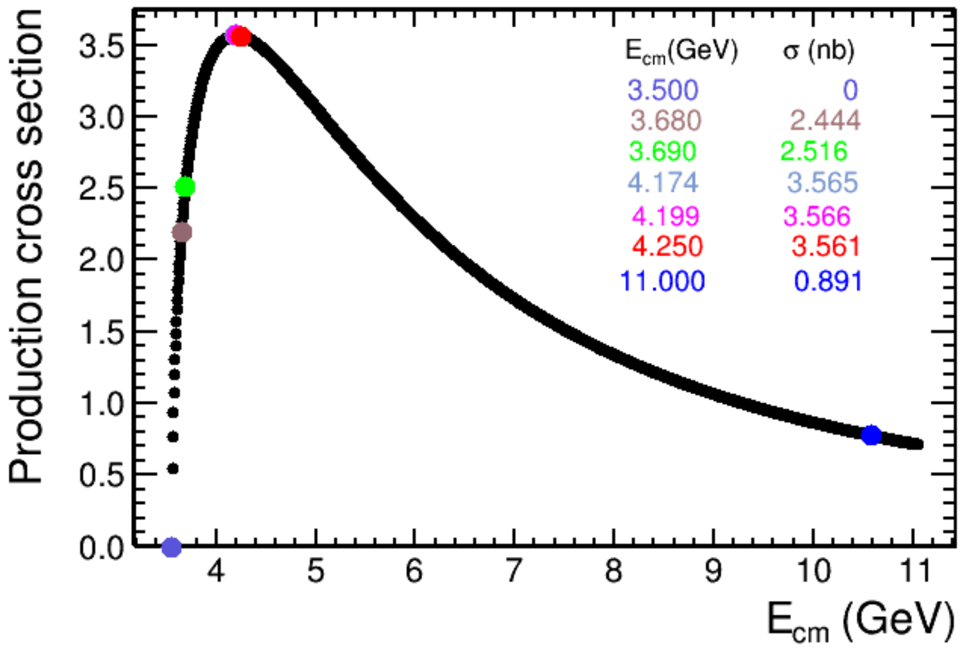
- **Use entanglement witness for search of tauonium**

BACKUP

τ-pair Production on STCF

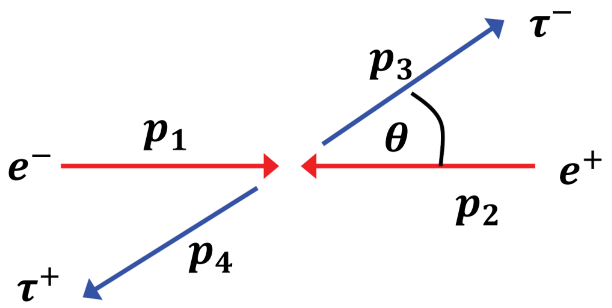
- **Cross section** of τ-pair Production
- STCF: 2-7GeV event rate: $3.5\times10^9/y(4.26GeV)$
- E_{cm}
  - 4.26GeV: highest $\sigma(\tau^+\tau^-)$
  - 6.0-7.0GeV: higher significance of Quantum Entanglement
- Decay channel: $\tau\rightarrow\pi\nu_\tau, \tau\rightarrow\rho\nu_\tau, \tau\rightarrow a_1\nu_\tau, \tau\rightarrow e\nu_e\nu_\tau, \tau\rightarrow\mu\nu_\mu\nu_\tau$



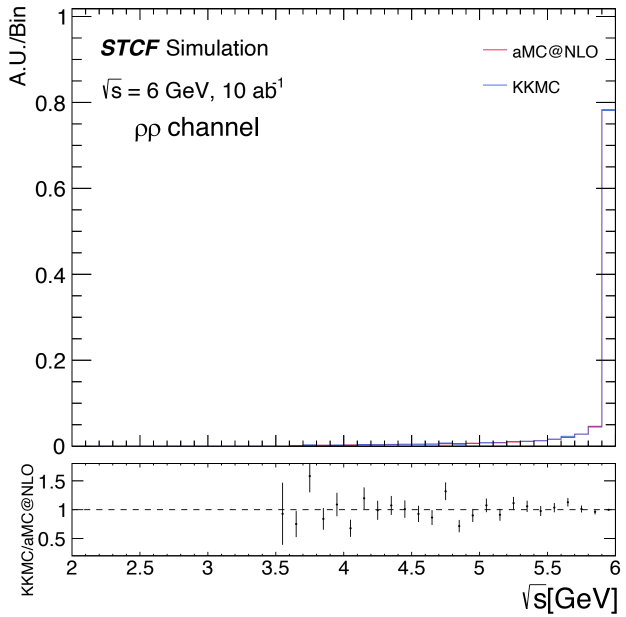


τ-pair MC simulation

- Tau pair production on STCF
  - ✓ Matrix element (LHE) simulated via aMC@NLO
  - ✓ Tau decay simulated via aMC@NLO(taudecay_UFO)
  - ✓ Considered ISR
  - ✓ Spin correlations between tau and their decay products are fully considered
  - ✓ $\sqrt{s} = 6GeV, 10\ million\ events \sim 0.0042ab^{-1}$
- Detector simulation (Fullsim + digi + reco)
  - **Oscar 2.6.2**
- Background Simulation
  - Considered different tau decay channel



$\tau^+\tau^-$ CME with ISR



τ-pair Production

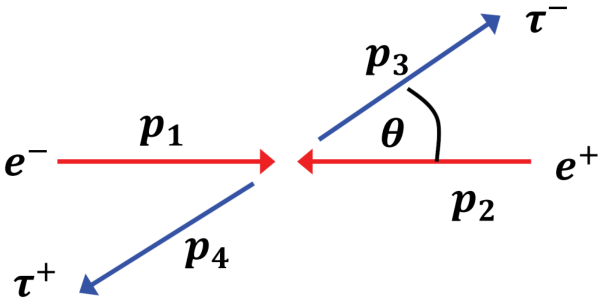
- Tau pair production on STCF
  - ✓ Matrix element (LHE) simulated via aMC@NLO
  - ✓ Tau decay simulated via aMC@NLO
  - ✓ Considered ISR
  - ✓ $\sqrt{s} = 6GeV, 0.0042ab^{-1}$
- Considered ISR

  $\sqrt{s}$(KKMC generator) Ratio: KKMC / aMC@NLO
- Concurrence:

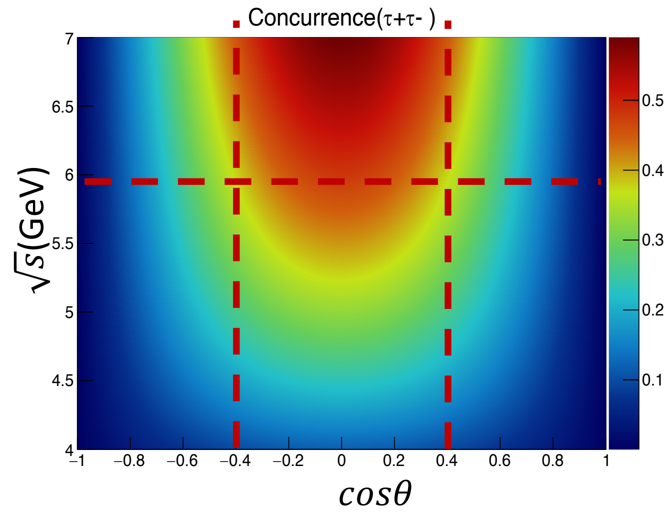
$$\mathcal{C}[\rho] = \frac{(s-4m_\tau^2)\sin^2\theta}{4m_\tau^2\sin^2\theta+s(\cos^2\theta+1)}$$

- $m12[C]$:

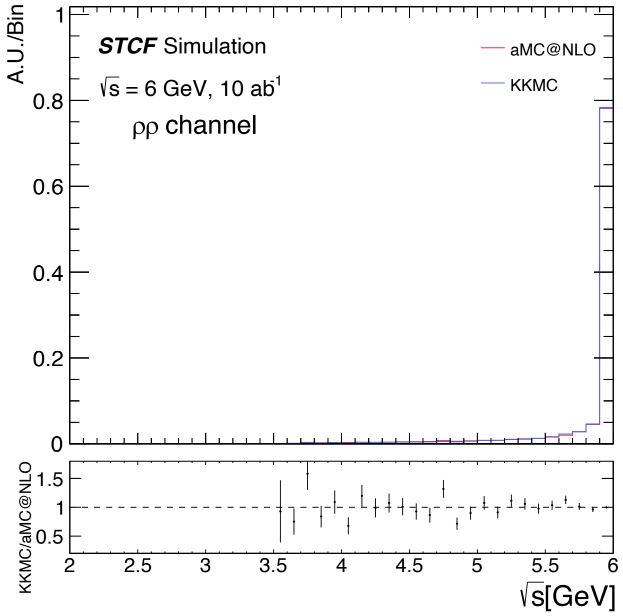
$$m_{12}[\boldsymbol{C}] = 1 + \left(\frac{(s-4m_\tau^2)\sin^2\theta}{4m_\tau^2\sin^2\theta+s(\cos^2\theta+1)}\right)^2$$

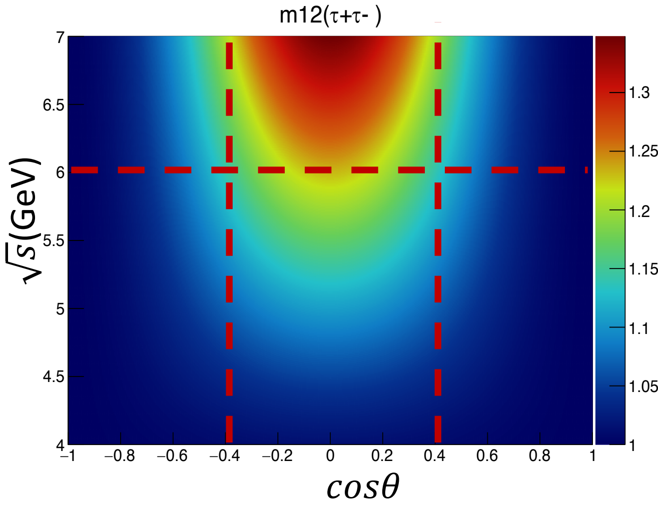


theoretical signal region



$\tau^+\tau^-$ CME with ISR





Variable from ECAL

- From Bo Wang

E.1 Input Variables

The input variables considered to distinguish γ from K_L^0 are listed:

- N_{hit}: the number of hitting crystals in EMC.
- E_{seed}/E_{3x3}: the ratio of energy deposited in the center crystal of the shower and energy deposited in the 3x3 crystal around the center of the shower.
- E_{3x3}/E_{5x5}: the ratio of energy deposited in the 3x3 crystal and 5x5 crystal around the center of the shower.
- A_{20} moment and A_{42} moment: the Zernike moment A_{nm} is defined as:

$$A_{n,m} = |\sum_i \frac{E_i}{E_{tot}} f_{n,m}(r_i/R_0)e^{im\phi}| \quad (3)$$

  with $f_{2,0} = 2x^2 - 1$ and $f_{4,2} = 4x^4 - 3x^2$, i denotes the different crystals, E_i is the energy deposited in the crystal and r_i is its distance from the shower center.
- secondary moment, which is defined as: $\sum_i E_i r_i^2 / \sum_i E_i$
- lateral moment, which is defined as: $\sum_{i=3}^n E_i r_i^2/(E_1 r_0^2 + E_2 r_0^2 + \sum_{i=3}^n E_i r_i^2)$

The variables with low correlation coefficient as shown in Fig. 83 are choosen as the input parameters such as N_{hit}, E_{seed}/E_{3x3}, E_{3x3}/E_{5x5} and A_{42} moment.