Charm mixing and indirect CPV studies at STCF

Ying-Hao Wang[1] , Qiang Lan[2], Yu Zhang[2], Pei-Rong Li[1], Ying-Rui Hou[3]
Xiao-Rui Lyu[3] , Yang-Heng Zheng[3]

[1]LZU [2]USC [3]UCAS

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Outline

- Charm mixing and CPV
- Time-integrated measurement
- Sensitivity study with STCF
- Summary and future plan

Charm mixing

- Mass eigenstates of D^0 and $\overline{D}^0$ mesons can be written as superpositions of flavor eigenstates:

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle$$

- Time evolution of a initially flavor eigenstates of D^0 and $\overline{D}^0$ mesons:

$$|D^0_{phys}(t)\rangle = g_+(t)|D^0\rangle + \frac{q}{p}g_-(t)|\overline{D}^0\rangle \qquad g_+(t) = \exp\left(-(im+\frac{\Gamma}{2})t\right)\cosh\left((i\Delta m - \frac{\Delta\Gamma}{2})\frac{t}{2}\right) \qquad m \equiv \frac{m_1+m_2}{2}, \Delta m \equiv m_2 - m_1$$

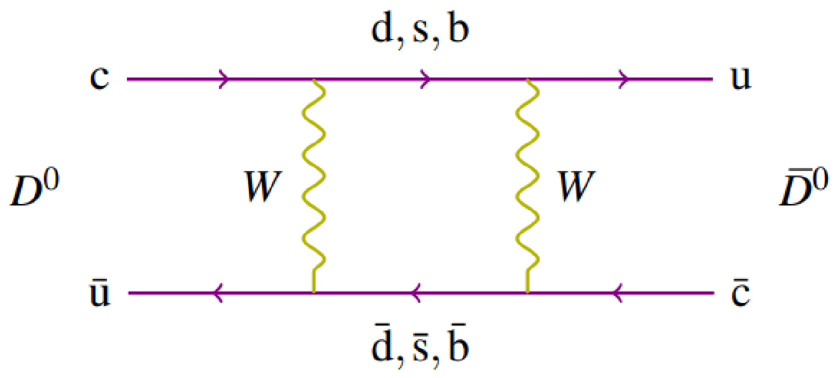
$$|\overline{D}^0_{phys}(t)\rangle = g_+(t)|\overline{D}^0\rangle + \frac{q}{p}g_-(t)|D^0\rangle \qquad g_-(t) = \exp\left(-(im+\frac{\Gamma}{2})t\right)\sinh\left((i\Delta m - \frac{\Delta\Gamma}{2})\frac{t}{2}\right) \qquad \Gamma \equiv \frac{\Gamma_1+\Gamma_2}{2}, \Delta\Gamma \equiv \Gamma_1 - \Gamma_2$$

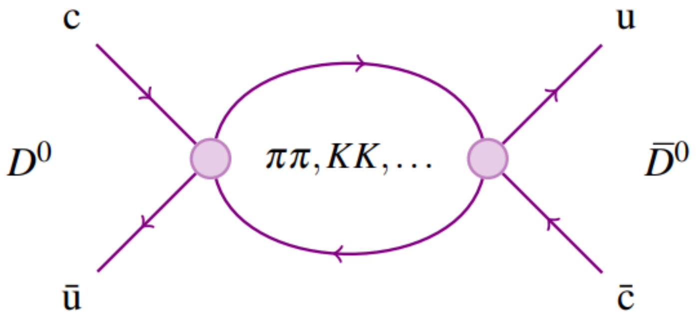
- Practically, it is more popular to use the following two dimensionless parameters for describing D^0-$\overline{D}^0$ mixing:

$$\boxed{x \equiv \frac{\Delta m}{\Gamma}, y \equiv \frac{\Delta\Gamma}{2\Gamma}}$$

Charm mixing

- Mixing amplitudes governed by two contributions:
  - Short distance:
    - ✓ Via box diagrams, $(x, y) \sim 10^{-7}$
    - ✓ Suppressed by GIM cancellation





  - Long distance:
    - ✓ re-scattering diagram, $(x, y) \sim 10^{-3}$
    - ✓ Theoretical prediction of x and y very challenging

Plots from arXiv:1503.00032

Charm mixing

- Today charm global average values largely dominated by the B factory, specifically LHCb, which exploit the largest flavor tag D^0 dataset.
- $D^0\bar{D}^0$ pairs produced by e^+e^- annihilations near threshold are in quantum correlated (QC) state with $\mathcal{C} = \pm 1$.
- Exploring the QC in $D^0\bar{D}^0$ allows to extract the mixing parameters with time-integrated decay rates.
- It is a novel approach to study of the $D^0\bar{D}^0$ mixing parameters using C-even $D\bar{D}$ samples.

Experiment	**Decay**	$\boldsymbol{x}(\times \mathbf{10^{-3}})$	$\boldsymbol{y}(\times \mathbf{10^{-3}})$
Belle (2024)	$K_S\pi\pi$	$4.0 \pm 1.7 \pm 0.4$	$2.9 \pm 1.4 \pm 0.3$
LHCb (2024)	Simultaneous fitting	4.01 ± 0.43	6.10 ± 0.17
LHCb (2023)	$K_S\pi\pi$ $(B\ tag)$	$4.29 \pm 1.48 \pm 0.26$	$12.61 \pm 3.12 \pm 0.83$
LHCb (2021)	$K_S\pi\pi$ $(D^*\ tag)$	$3.97 \pm 0.46 \pm 0.29$	$4.59 \pm 1.20 \pm 0.85$
LHCb (2016)	$K_S\pi\pi$ $(D^*\ tag)$	$-8.6 \pm 5.3 \pm 1.7$	$0.3 \pm 4.6 \pm 1.3$
Belle (2014)	$K_S\pi\pi$	$5.6 \pm 1.9^{+0.3+0.6}_{-0.9-0.9}$	$3.0 \pm 1.5^{+0.4+0.3}_{-0.5-0.6}$

Charm CPV

- Direct CPV:

$$|A(D \to f)| \neq |\bar{A}(\bar{D} \to \bar{f})|.$$

- CPV in mixing:

Probability of $|D^0_{phy}(t)\rangle \to |\bar{D}^0\rangle \neq |\bar{D}^0_{phy}(t)\rangle \to |D^0\rangle$: $|q/p| \neq 1$.

- CPV in the interference of mixing and decay:

Non-vanishing ϕ for $\frac{q}{p} = \left|\frac{q}{p}\right| e^{i\phi}$.

Decay rates: $$R(D^0_{\rm phys}(t)\to f)\propto|A_f|^2\exp(-\Gamma t)[1+\tfrac{1}{4}(x_D^2+y_D^2)|\lambda_f|^2\Gamma^2t^2 - \tfrac{1}{4}(x_D^2-y_D^2)\Gamma^2t^2-(y_D\mathrm{Re}\lambda_f+x_D\mathrm{Im}\lambda_f)\Gamma t]$$

$$\lambda_f = r_f\left|\frac{q}{p}\right| e^{-i(\delta_f+\phi)}$$

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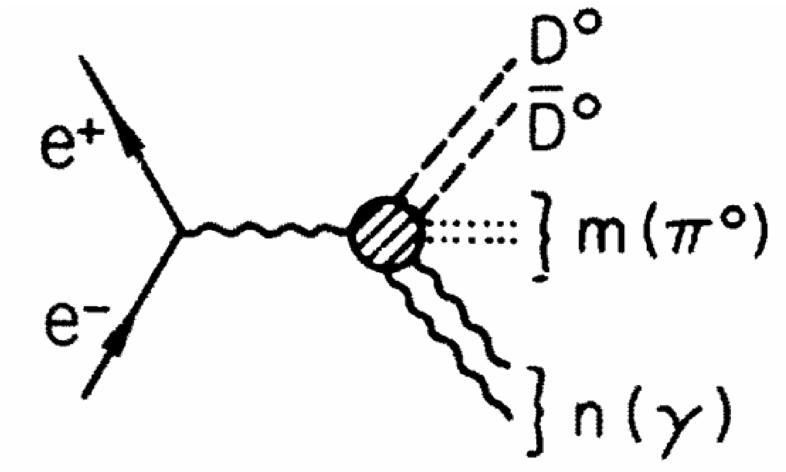
- Currently, the mixing and CPV parameters are usually measured with the time-dependent analysis in B factory.

Quantum correlated $D^0\bar{D}^0$



Phys. Rev. D 15 (1977) 1254

- $D^0\bar{D}^0$ pairs produced by e^+e^- annihilations near threshold are in quantum correlated state with $C = (-1)^{n+1}$.
- @3770 MeV
  - $C = -1, e^+e^- \to D^0\bar{D}^0$
- @>4009 MeV
  - $C = +1$ for $e^+e^- \to D^{*0}\bar{D}^0 + c.c, D^{*0} \to \gamma D^0$
  - $C = -1$ for $e^+e^- \to D^{*0}\bar{D}^0 + c.c, D^{*0} \to \pi^0 D^0$

Exploring the QC in $D^0\bar{D}^0$ allows to extract the mixing and CPV parameters with time-integrated decay rates.

Quantum correlated decay rates

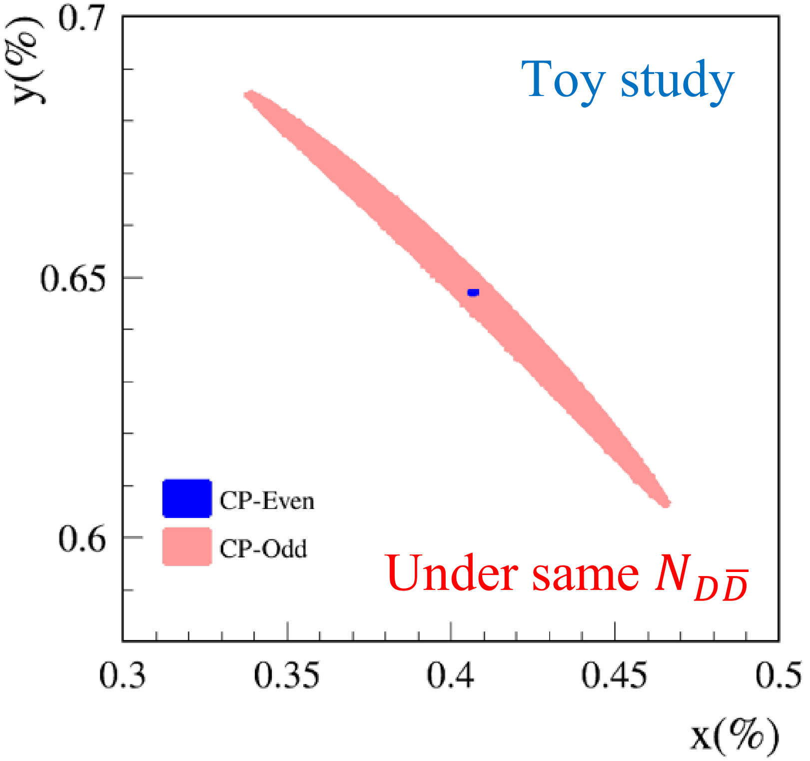
$C = +1$

$$
\begin{aligned}
W(f_1, f_2) = {} & 3(x^2+y^2)(|\lambda_{D^0}|^2 + |\lambda_{\bar{D}^0}|^2 + 2R_{D^0}R_{\bar{D}^0}\lambda_{D^0}\lambda_{\bar{D}^0}) \\
& + (2 - 3(x^2 - y^2))(1 + 2R_{D^0}R_{\bar{D}^0}\mathrm{Re}(\lambda_{D^0}\lambda_{\bar{D}^0}) + |\lambda_{D^0}\lambda_{\bar{D}^0}|^2) \\
& \underline{-4y}[R_{\bar{D}^0}(1+|\lambda_{D^0}|^2)\mathrm{Re}(\lambda_{\bar{D}^0}) + R_{D^0}(1+|\lambda_{\bar{D}^0}|^2)\mathrm{Re}(\lambda_{D^0})] \\
& \underline{-4x}[R_{\bar{D}^0}(1-|\lambda_{D^0}|^2)\mathrm{Im}(\lambda_{\bar{D}^0}) + R_{D^0}(1-|\lambda_{\bar{D}^0}|^2)\mathrm{Im}(\lambda_{D^0})]
\end{aligned}
$$

$C = -1$

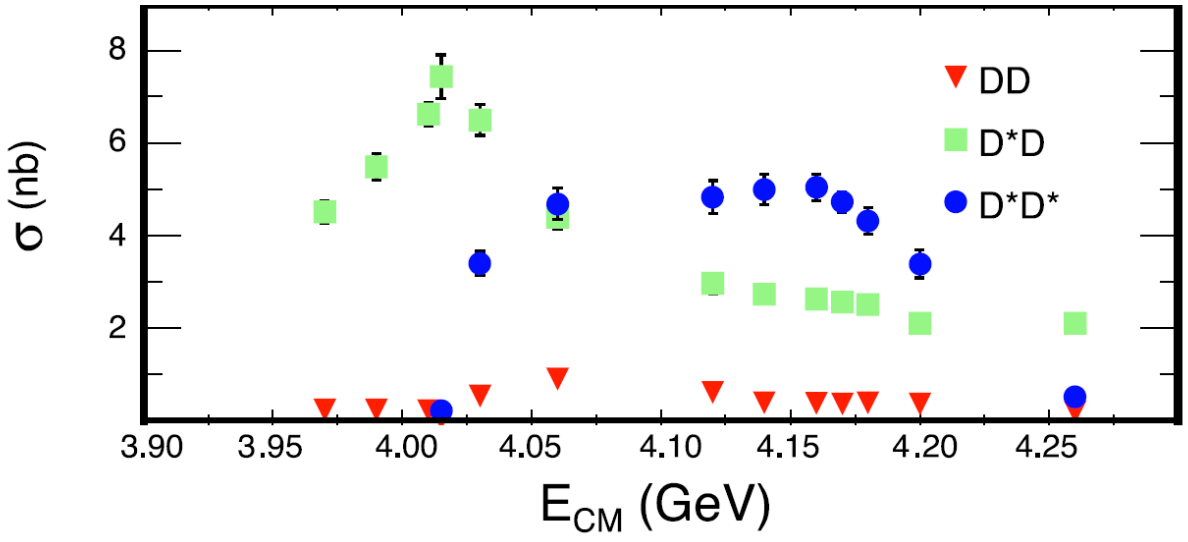
$$
\begin{aligned}
W(f_1, f_2) = {} & (x^2+y^2)(|\lambda_{D^0}|^2 + |\lambda_{\bar{D}^0}|^2 - 2R_{D^0}R_{\bar{D}^0}\mathrm{Re}(\lambda_{D^0}\lambda_{\bar{D}^0})) \\
& + (2 - (x^2 - y^2))(1 - 2R_{D^0}R_{\bar{D}^0}\mathrm{Re}(\lambda_{D^0}\lambda_{\bar{D}^0}) + |\lambda_{D^0}\lambda_{\bar{D}^0}|^2)
\end{aligned}
$$

- C-even quantum correlation samples are better at measuring mixing and CPV parameters.



Fake data study at STCF

- $E_{cms} = 4030\ \mathrm{MeV}, \int L = 1\mathrm{ab}^{-1}$.
- Coherent samples: $e^+e^- \to D^{*0}\bar{D}^0 + c.c.$ and $e^+e^- \to D^{*0}\bar{D}^{*0}$.
- Incoherent samples: $e^+e^- \to D^{*+}D^- + c.c.$ and $e^+e^- \to D^{*+}D^{*-}$.
- Simulation and reconstruction framework is based on OSCAR (2.6.2_pre2) software.



	Component	Final State	$\mathcal{C}(D^0\bar{D}^0)$
Coherent	$D^0\bar{D}^0$	$D^0\bar{D}^0$	Odd
	$D^{*0}\bar{D}^0$	$\gamma D^0\bar{D}^0$	Even
		$\pi^0 D^0\bar{D}^0$	Odd
	$D^{*0}\bar{D}^{*0}$	$\gamma\pi^0 D^0\bar{D}^0$	Even
		$\gamma\gamma(\pi^0\pi^0)D^0\bar{D}^0$	Odd
Incoherent	$D^{*+}D^{*-}$	$\pi^+\pi^- D^0\bar{D}^0$	-
	$D^{*+}D^-$	$\pi^+ D^0 D^-$	-

Analysis strategy: signal modes

- Decay modes: $K\pi\pi^0$.
- For the coherent samples, the double tag method is performed.
  - Flavor tag: $K\pi\pi\pi$, $K\pi\pi^0$, $K\pi\pi\pi$ (like-sign and opposite sign for $Kn\pi$).
  - CP-even tag: KK, $\pi\pi$, $\pi\pi\pi^0$.
  - CP-odd tag: $K_S\pi^0$, $K_S\eta$.
  - Self-conjugate tag: $K_S\pi\pi$.

Event selection

- **Charged track**
  - ✓ $|V_r| < 1.0cm, |V_z| < 10.0cm, |\cos\theta| < 0.93.$
- **PID**
  - ✓ $K: prob(K) > 0 \,\&\&\, prob(K) > prob(\pi).$
  - ✓ $\pi: prob(\pi) > 0 \,\&\&\, prob(\pi) > prob(K).$
- $\boldsymbol{K_S^0}$
  - ✓ π^+ and π^- with $|V_z| < 20cm, |\cos\theta| < 0.93$, and no PID are performed for π.
  - ✓ Primary vertex fit, $\chi^2 < 100$.
  - ✓ Second vertex fit, $L/\sigma > 2$.
  - ✓ $0.487 < M(\pi^+\pi^-) < 0.511$ GeV$/c^2$.
- $\boldsymbol{\pi^0/\eta}$
  - ✓ $E_{barrel} > 0.025$ GeV ($\cos\theta < 0.8$), $E_{encap} > 0.05$ GeV ($0.86 < \cos\theta < 0.92$).
  - ✓ $0.115 < M(\gamma\gamma) < 0.150$ GeV$/c^2$ for π^0.
  - ✓ $0.480 < M(\gamma\gamma) < 0.580$ GeV$/c^2$ for η.
  - ✓ 1C kinematic fit, $\chi^2 < 200$.

Analysis strategy: fitting method of coherent sample

- The charm mixing parameters are extracted in a ratio between C-even and C-odd $D^0\bar{D}$ production processes.
- In quantum correlations:

$$N_{sig}^{C-even} = N_{D^{*0}\bar{D}^0} \cdot R^{C-even} \cdot \mathcal{B}(D^{*0} \to D^0\gamma) \cdot \varepsilon_{sig}$$

decay rate

$$N_{sig}^{C-odd} = N_{D^{*0}\bar{D}^0} \cdot R^{C-odd} \cdot \mathcal{B}(D^{*0} \to D^0\pi^0) \cdot \varepsilon_{sig}$$

$$\frac{N_{sig}^{C-even}}{N_{sig}^{C-odd}} = \frac{R^{C-even} \cdot \mathcal{B}(D^{*0} \to D^0\gamma)}{R^{C-odd} \cdot \mathcal{B}(D^{*0} \to D^0\pi^0)}$$

- This will cancel the number of $D^0\bar{D}^0$ pairs.
- Cancel out some systematic uncertainty.

$$\begin{aligned}
R^{C-even}(K^-\pi^+\pi^-\pi^+;K^+\pi^-) \propto A_{K3\pi}^2 A_{K\pi}^2 \{ & 3(x^2+y^2)[K_i(r_{CP}^{-1})^2(r_D^{K\pi})^2 + \bar{K}_i(r_{CP})^2(r_D^{K3\pi})^2 \\
& + 2\sqrt{K_i\bar{K}_i} R_{K3\pi}^i r_D^{K3\pi} r_D^{K\pi} \cos(\delta_D^{i,K3\pi} - \delta_D^{K\pi} - 2\phi)] \\
& + [2 - 3(x^2 - y^2)][K_i + \bar{K}_i(r_D^{K\pi})^2(r_D^{K3\pi})^2 \\
& + 2\sqrt{K_i\bar{K}_i} r_D^{K\pi} R_{K3\pi}^i r_D^{K3\pi} \cos(\delta_D^{i,K3\pi} + \delta_D^{K\pi})] \\
& - 4y[r_D^{K\pi}(K_i r_{CP}^{-1} + \bar{K}_i r_{CP}(r_D^{K3\pi})^2)\cos(\delta_D^{K\pi} + \phi) \\
& + \sqrt{K_i\bar{K}_i} R_{K3\pi}^i r_D^{K3\pi}(r_{CP} + r_{CP}^{-1}(r_D^{K\pi})^2)\cos(\delta_D^{i,K3\pi} - \phi)] \\
& - 4x[r_D^{K\pi}(K_i r_{CP}^{-1} - \bar{K}_i r_{CP}(r_D^{K3\pi})^2)\sin(\delta_D^{K\pi} + \phi) \\
& + \sqrt{K_i\bar{K}_i} R_{K3\pi}^i r_D^{K3\pi}(r_{CP} - r_{CP}^{-1}(r_D^{K\pi})^2)\sin(\delta_D^{i,K3\pi} - \phi)]\}
\end{aligned}$$

$$\begin{aligned}
R^{C-odd}(K^-\pi^+\pi^-\pi^+;K^+\pi^-) \propto A_{K3\pi}^2 A_{K\pi}^2 \{ & (x^2+y^2)[K_i(r_{CP}^{-1})^2(r_D^{K\pi})^2 + \bar{K}_i(r_{CP})^2(r_D^{K3\pi})^2 \\
& - 2\sqrt{K_i\bar{K}_i} R_{K3\pi}^i r_D^{K3\pi} r_D^{K\pi} \cos(\delta_D^{i,K3\pi} - \delta_D^{K\pi} - 2\phi)] \\
& + [2 - (x^2 - y^2)][K_i + \bar{K}_i(r_D^{K\pi})^2(r_D^{K3\pi})^2 \\
& - 2\sqrt{K_i\bar{K}_i} r_D^{K\pi} R_{K3\pi}^i r_D^{K3\pi} \cos(\delta_D^{i,K3\pi} + \delta_D^{K\pi})]
\end{aligned}$$

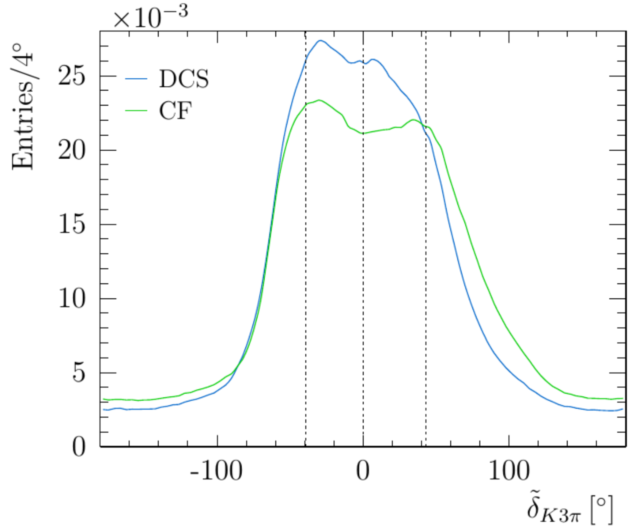
Strong phase parameters are fixed.

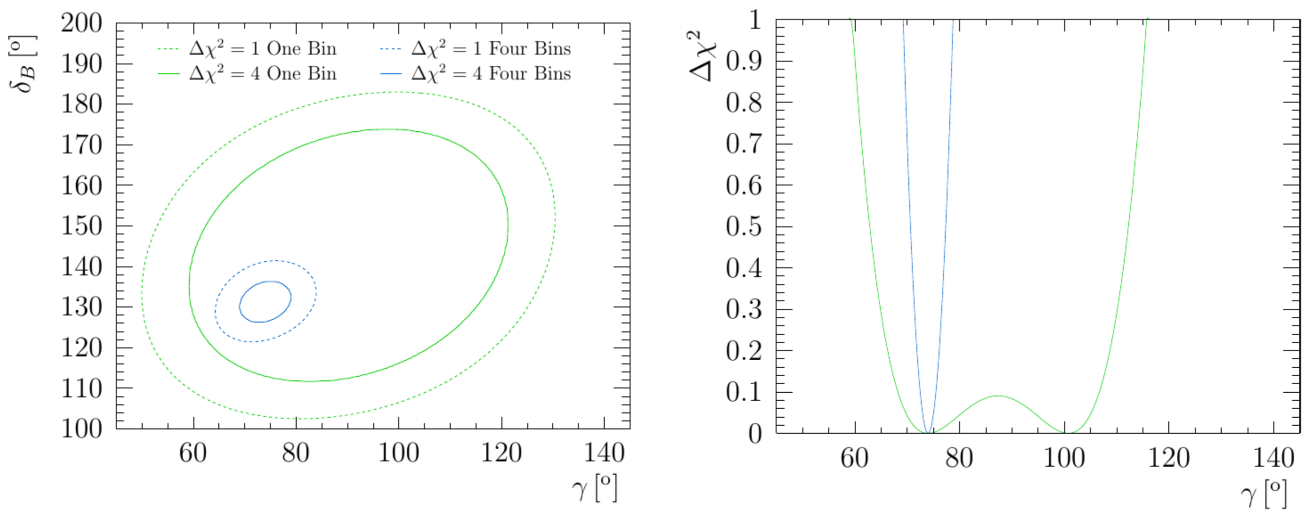
Binning scheme of $K^-\pi^+\pi^+\pi^-$

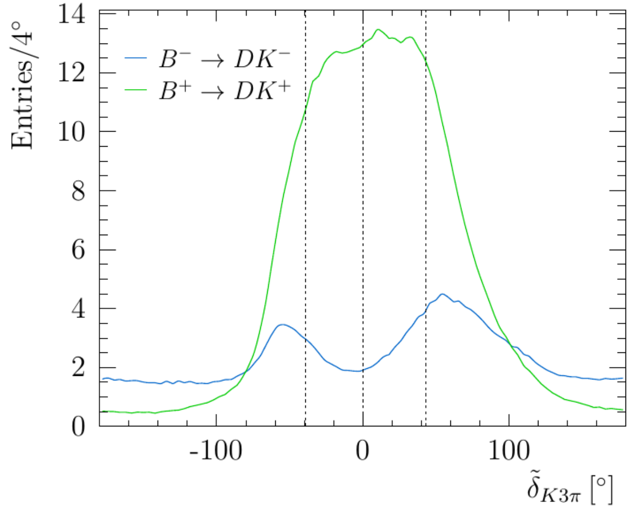
- Local strong phase has been divided into 4 bins with approximately equal population of CF decay events.

$$\tilde{\delta}_D^{K3\pi} = \arg(A_{\bar{D}^0\to K^+3\pi}(\mathbf{x})A^*_{D^0\to K^+3\pi}(\mathbf{x})) - \arg(\int A_{\bar{D}^0\to K^+3\pi}(\mathbf{x}')A^*_{D^0\to K^+3\pi}(\mathbf{x}')\mathrm{d}\mathbf{x}')$$

- Binned parameters $R_{K3\pi,i}$ and $\delta_D^{K3\pi,i}$ in each bin has been measured, while the $r_D^{K3\pi,i}$ can be written as $\sqrt{\frac{\overline{K_i}}{K_i}}\, r_D^{K3\pi}$.







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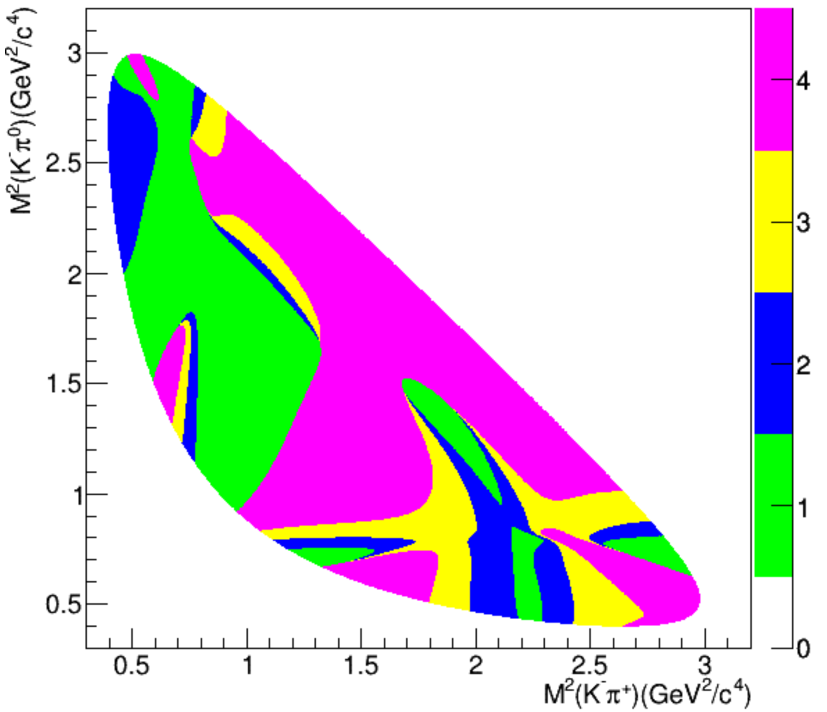
Binning scheme of $K^-\pi^+\pi^0$

- The phase space of the D meson decay is divided into disjoint regions using the model predictions for the strong-phase difference between the BaBar CF and DCS amplitudes.

$$\Delta\delta_{K\pi\pi^0} = \arg(A_{\bar{D}^0\to K^+\pi^-\pi^0} A^*_{D^0\to K^+\pi^-\pi^0})$$

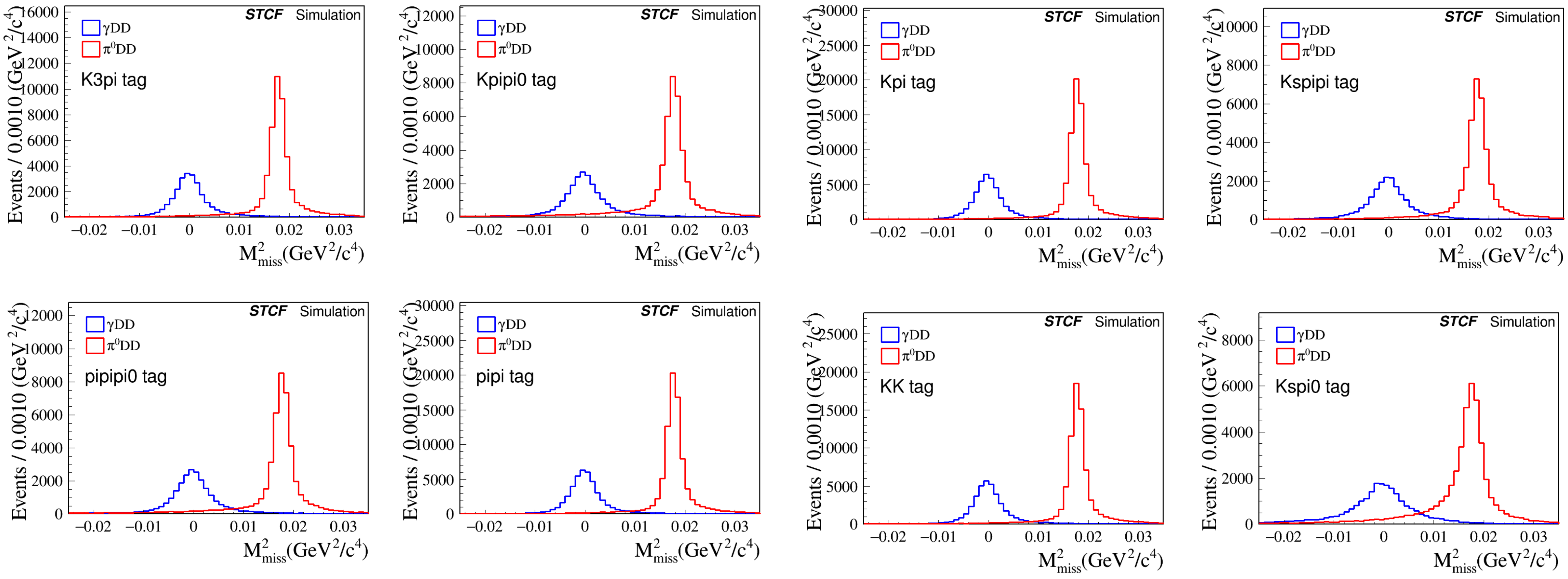
- Similar to the binning scheme for the decay $D^0 \to K\pi\pi\pi$, we initially categorize the decay $D^0 \to K\pi\pi^0$ into four bins.

Bin	Range
1	$-180^\circ < \Delta\delta_{K\pi\pi^0} < -18^\circ$
2	$-18^\circ < \Delta\delta_{K\pi\pi^0} < 0^\circ$
3	$0^\circ < \Delta\delta_{K\pi\pi^0} < 22^\circ$
4	$22^\circ < \Delta\delta_{K\pi\pi^0} < 180^\circ$



M_{miss}^2 distributions

- The signal yields will be extracted in the M_{miss}^2 distribution.



Signal MC samples without quantum correction modifier.

χ^2 fitting method

- The $\chi^2_{+/-}$ can be defined as: $\chi^2_{+/-} = \Delta_{FL} V_{FL}^{-1} \Delta_{FL}^T + \Delta_{CP} V_{CP}^{-1} \Delta_{CP}^T + \Delta_{K_S\pi\pi} V_{K_S\pi\pi}^{-1} \Delta_{K_S\pi\pi}^T$.
- At present, the covariance only considers statistical uncertainty with $V_{ij} = \sigma^2_{stat.,ij}$.
- The strong phase parameters are fixed to the nominal value in the sensitivity estimation.

Input parameters	Value	Reference
$\mathcal{B}_{D^{*0}\to\gamma D^0}[\%]$	35.3 ± 0.9	PDG
$\mathcal{B}_{D^{*0}\to\pi^0 D^0}[\%]$	64.7 ± 0.9	
$\mathcal{B}_{\pi^0\to\gamma\gamma}$	98.823 ± 0.034	
$(r_D^{K\pi})^2[\%]$	0.344 ± 0.002	arXiv: 2409.06449
$\delta_D^{K\pi}[^\circ]$	190.2 ± 2.8	
$F^+_{\pi\pi\pi^0}$	0.9406 ± 0.0042	PRD 111 (2025) 1, 012007
c_i, s_i, K_i, K_{-i}		arXiv: 2503.22126
$R_{K3\pi}, r_D^{K3\pi}, \delta_D^{K3\pi}$		JHEP 05 (2021) 164
$R_{K\pi\pi^0}, r_D^{K\pi\pi^0}, \delta_D^{K\pi\pi^0}$		

Sensitivity study of charm mixing

- The results of the charm mixing and CPV parameters are summarized in the following table, where the uncertainty is only statistical.

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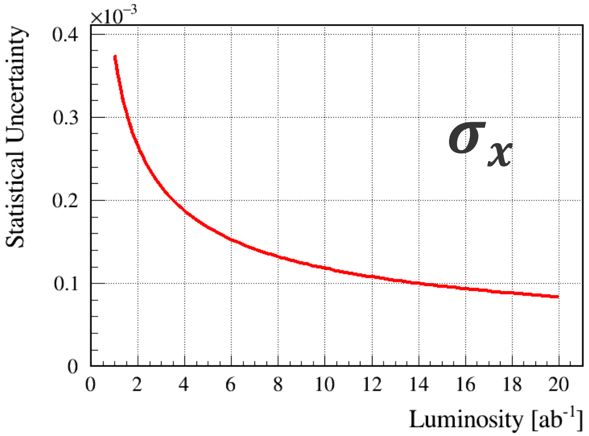
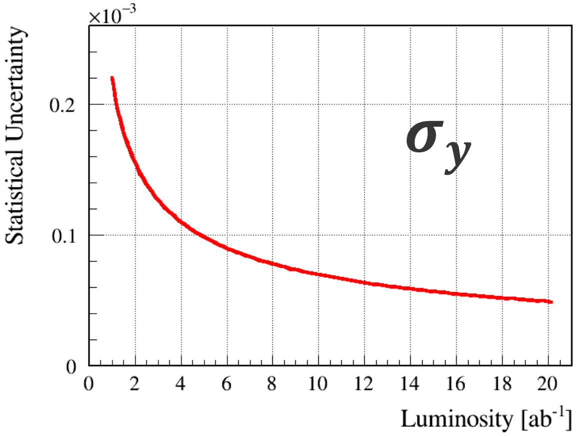
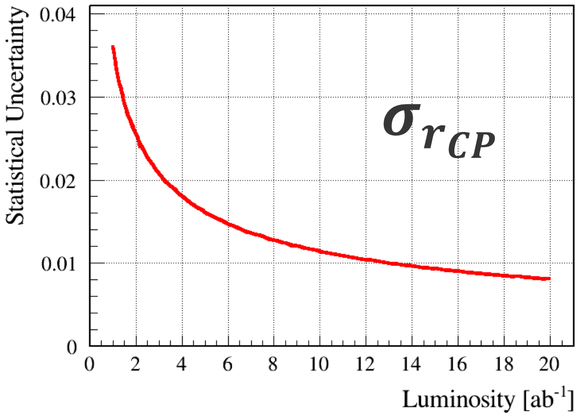
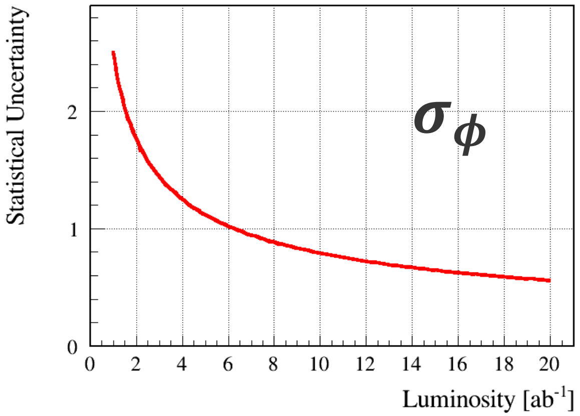
$K^-\pi^+\pi^0$	Unbinned results	Binned results	Belle II $50\ \mathrm{ab}^{-1}$
$\sigma_x(\%)$	0.062	0.037	0.057
$\sigma_y(\%)$	0.021	0.021	0.049
$\sigma_{r_{CP}}$	0.060	0.036	-
$\sigma_\phi(°)$	3.31	2.50	-









Sensitivity study of strong phase

- The published results of hadronic parameters for the decay $D \to K\pi\pi^0$ are summarized in the table below.

$R_{K\pi\pi^0}$	$\delta_D^{K\pi\pi^0}$	$r_D^{K\pi\pi^0}$
0.80 ± 0.04	$(200 \pm 11)^{\circ}$	$(4.41 \pm 0.11)\%$

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2.93fb^{-1}

- The uncertainty of strong phase difference will directly affect the measurement of the mixed charm mixing.

$\boldsymbol{K^-\pi^+\pi^0}$	I	II	III	Belle II **50 ab^{-1}**
$\sigma_x(\%)$	0.062	0.200	0.080	0.057
$\sigma_y(\%)$	0.021	0.067	0.030	0.049
$\sigma_{r_{CP}}$	0.060	0.060	0.060	-
$\sigma_\phi(^{\circ})$	3.31	3.34	3.31	-
$\sigma_r(\%)$	-	0.045	-	-
σ_R	-	0.014	-	-
$\sigma_\delta(^{\circ})$	-	2.43	-	-

- I: Unbinned results of fixed strong phase parameters.
- II: Unbinned results of floated strong phase parameters.
- III: Unbinned results of consider the uncertainties (1ab^{-1}) of strong phase parameters as one Gaussian constrain.

Summary and to do list

Summary:

- Precision measurement of charm mixing is an important goal in heavy flavor physics for the next decade.
- Time-integrated measurements with C-even $D\bar{D}$ at STCF are essential contributions.
- The most accurate results will be obtained at $E_{cms} = 4030$ MeV, as more samples are available at this energy point.

Plan:

- Study of more double tag channels, such as $K^-\pi^+\pi^-\pi^+$ and $K_S^0\pi^+\pi^-$.
- Finish the quantum correlation correction algorithm package.

Thanks!

Backup

量子关联

$$\psi(3770) \to D^0\bar{D}^0$$

$\psi(3770)$： $I^G(J^{PC}) = 0^-(1^{--})$

角动量守恒 $\longrightarrow$ $D^0\bar{D}^0$轨道角动量$L=1$， $D^0\bar{D}^0$可看作全同玻色子。

D^0： $I(J^P) = \frac{1}{2}(0^-)$

两玻色子体系的总波函数： $\psi = \psi(r_1, r_2) \cdot \chi_{s,s_z} \cdot \chi_{I,I_3} \cdot \psi(D^0, \bar{D}^0)$

空间波函数，自旋波函数，同位旋波函数，态函数

$L=1$	空间波函数满足反对称关系
两末态粒子总自旋为0	自旋波函数是对称的
两末态粒子总同位旋为0	同位旋波函数是对称的
母粒子为玻色子，满足玻色对称关系	态函数必须满足反对称关系

量子关联

$$\psi(4030) \to D^{*0}\bar{D}^0 \to \gamma D^0\bar{D}^0$$

$\psi(4030)$ ： $I^G(J^{PC}) = 0^-(1^{--})$

γ： $J = 1$ $\xrightarrow{\text{角动量守恒}}$ $D^0\bar{D}^0$轨道角动量$L = 0$。

D^0： $I(J^P) = \frac{1}{2}(0^-)$

两玻色子体系的总波函数： $\psi = \psi(r_1, r_2) \cdot \chi_{s,s_z} \cdot \chi_{I,I_3} \cdot \psi(D^0, \bar{D}^0)$

空间波函数，自旋波函数，同位旋波函数，态函数

$L = 0$	空间波函数满足对称关系
两末态粒子总自旋为0	自旋波函数是对称的
两末态粒子总同位旋为0	同位旋波函数是对称的
母粒子为玻色子，满足玻色对称关系	态函数必须满足对称关系

$n\gamma m\pi^0 D^0\bar{D}^0$，当n为奇数时，态函数满足交换对称关系，n为偶数时，态函数满足交换反对称关系。