Theoretical overview of charm mixing and CPV

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$D^0 - \bar{D}^0$ mixing

◉ Neutral meson mixing: Mass eigenstates ≠ Flavor eigenstates ⟶ oscillation of flavors

◉ $\Delta F = 2$ flavor changing neutral current (FCNC) ⟶ Rare process in the SM

◉ It imposes constraints on beyond the standard model (BSM) scenarios.

◉ Difficulty specific to charmed mesons: the GIM mechanism gives rise to extreme suppression.

⟶ Dominance of long-distance effect

◉ It requires dedicated understanding in strong interaction, especially its non-perturbative aspects.

Mixing parameters

off-diagonal entry $\bar{D}^0 \to D^0$

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix} = H \begin{pmatrix} D^0(t) \\ \bar{D}^0(t) \end{pmatrix}$$

$$H = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}$$

Mass eigenstates

$$\begin{cases} |D_1\rangle = p\,|D^0\rangle - q\,|\bar{D}^0\rangle \\ |D_2\rangle = p\,|D^0\rangle + q\,|\bar{D}^0\rangle \end{cases}$$

$$\begin{cases} \Delta m = m_1 - m_2 \\ \Delta\Gamma = \Gamma_1 - \Gamma_2 \\ q/p \neq \text{unity} \end{cases}$$

Observable quantities

CP violation

Dimensionless observables

$$\begin{cases} x = \Delta m/\Gamma \\ y = \Delta\Gamma/(2\Gamma) \end{cases}$$

Total width $\Gamma = (\Gamma_1 + \Gamma_2)/2$

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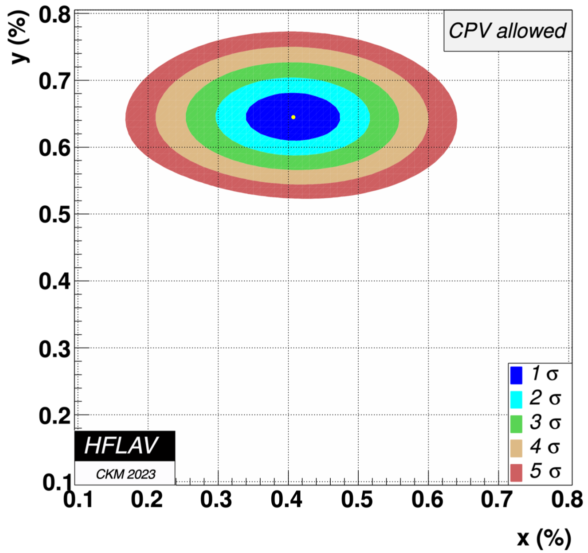
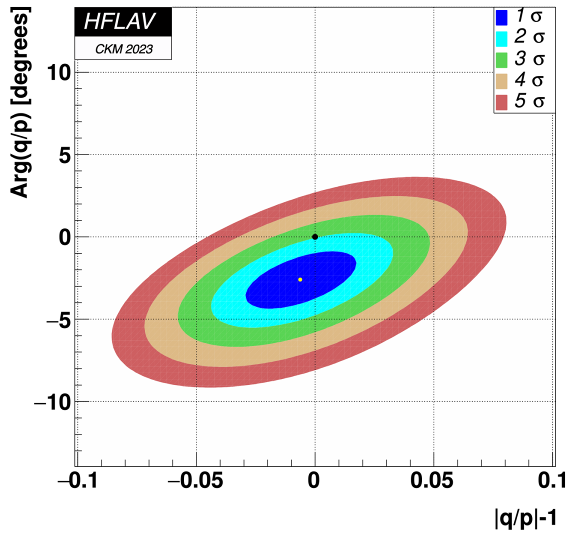
x vs. *y*



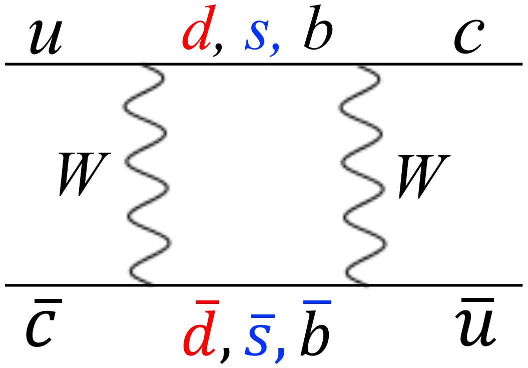
|*q*/*p*|-1 vs. Arg(*q*/*p*)



◉ No mixing, (*x*, *y*) = (0, 0), is excluded by significance greater than 11.5σ.

◉ No CPV, (0, 0), is currently at 2.1σ.

Naive evaluation of box diagrams



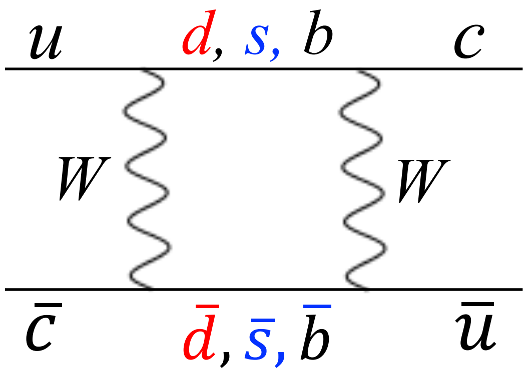
CKM product: $\lambda_i = V_{ci} V_{ui}^*$

Unitarity: $\lambda_d + \lambda_s + \lambda_b = 0$

◉ CKM suppression for λ_b: $(\lambda_b/\lambda_s) = (2.9 + 6.4i) \times 10^{-4}$ (CKM values from 2007.03022)

$(\lambda_b = 0)$ + Unitarity

Naive evaluation of box diagrams



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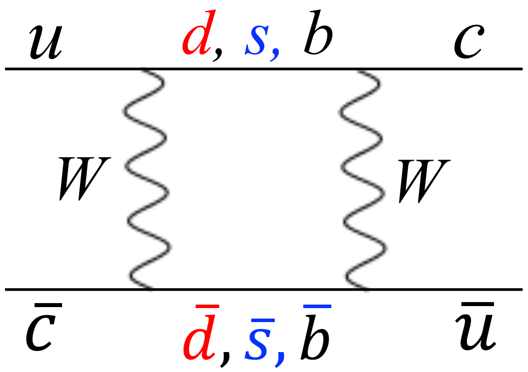
$$M_{12} = \lambda_s^2 \underbrace{[M_{12}^{(dd)} + M_{12}^{(ss)} - 2M_{12}^{(ds)}]}_{\text{vanish for } s = d} \propto \left(\frac{m_s^2 - m_d^2}{m_c^2}\right)^2$$

vanish for $s = d$
GIM cancellation

Suppression factor
(for box diagrams)

→ Sensitivity to SU(3) breaking

Naive evaluation of box diagrams



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Theory Short-distance estimate (box diagrams + NLO corrections)

Golowich and Petrov [0506185]

$$\begin{cases} x = 6 \times 10^{-7} \\ y = 6 \times 10^{-7} \end{cases}$$

Experiment Heavy flavor averaging group (HFLAV)

(all CPV allowed)

$$\begin{cases} x = (4.07 \pm 0.44) \times 10^{-3} \\ y = 6.45^{+0.24}_{-0.23} \times 10^{-3} \end{cases}$$

Gap in four orders of magnitude → Charm mixing is more challenging than $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixings.

Outline: Theoretical methods for $D^0 - \bar{D}^0$ mixing

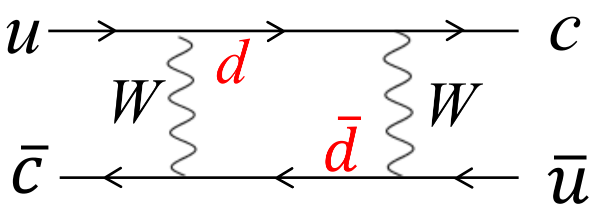
(1) Inclusive approach

(2) Dispersive approach

(3) Dyson-Schwinger-based approach

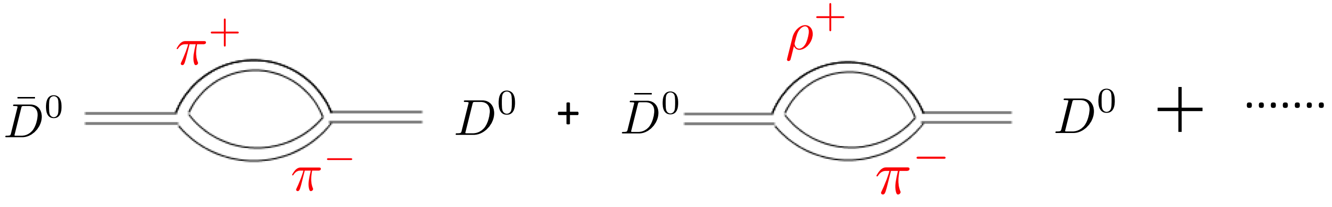
Inclusive

Quark-gluon diagram



Exclusive

Hadronic diagram



$$\Gamma_{12}[\bar{D}^0 \to d\bar{d} \to D^0] = \Gamma_{12}[\bar{D}^0 \to \pi^+\pi^- \to D^0] + \Gamma_{12}[\bar{D}^0 \to \rho^+\pi^- \to D^0] + \Gamma_{12}[\bar{D}^0 \to \rho^-\pi^+ \to D^0] + \cdots$$

and similar discussion for *ds* and *ss* flavors

Quark-hadron duality: Inclusive rate = Sum of exclusive rates

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- The inclusive approach tacitly assumes duality, while duality violation is hard to quantify.

  → Difficulty analogous to $B \to X_c l\, \nu$ decays and B/D-meson lifetimes.

- The estimate of duality violation requires modelling.

Duality violation in $D^0 - \bar{D}^0$ mixing

◉ A simple model: Jubb, Kirk, Lenz and Tetlalmatzi-Xolocotzi [1603.07770]

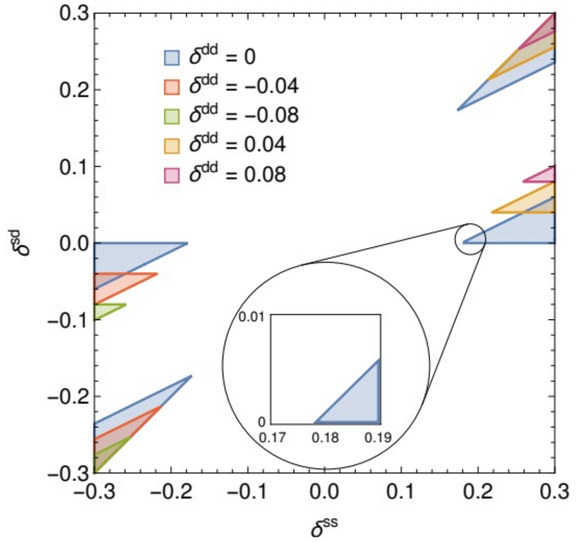
(Taking account of $\lambda_b \neq 0$)

$$\Gamma_{12} = -\lambda_s^2(\Gamma_{dd} + \Gamma_{ss} - 2\Gamma_{dd}) + 2\lambda_s\lambda_b(\Gamma_{sd} - \Gamma_{dd}) - \lambda_b^2\Gamma_{dd}$$

Replacement: $\Gamma_{12}^{ss} \to \Gamma_{12}^{ss}(1+\delta^{ss})$ $\Gamma_{12}^{sd} \to \Gamma_{12}^{sd}(1+\delta^{sd})$ $\Gamma_{12}^{dd} \to \Gamma_{12}^{dd}(1+\delta^{dd})$

⟶ $\mathcal{O}(20\%)$ duality violation explains data δ^{ij} : free-parameter

δ^{ss} vs. δ^{sd}



◉ Test of duality in the 't Hooft model: two-dimensional QCD in the large N_c limit

$$\Gamma_{12}^{\text{incl.}} \quad \text{vs.} \quad \sum_{P^{(\prime)}=\pi,K} \Gamma_{12}^{\text{excl.}}[D \to PP']$$

(solvable)

The result

Umeeda [2106.06215]

$$\frac{\Gamma_{12}^{\text{incl.}}|_*}{\sum \Gamma_{12}^{\text{excl.}}} = \mathcal{O}(10^{-3})$$

Practical theo./data : $\dfrac{\Gamma_{12}^{\text{incl.}}}{\Gamma_{12}^{\text{exp.}}} = \mathcal{O}(10^{-4})$

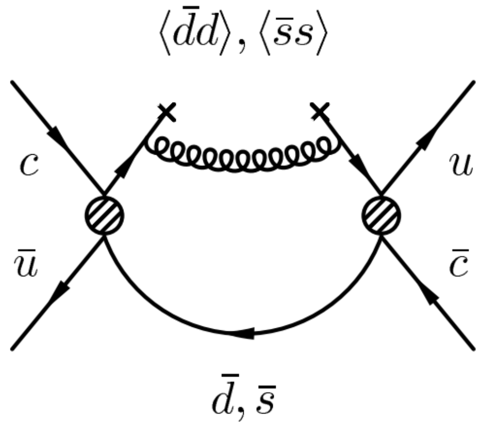
(OPE vs. HFLAV)

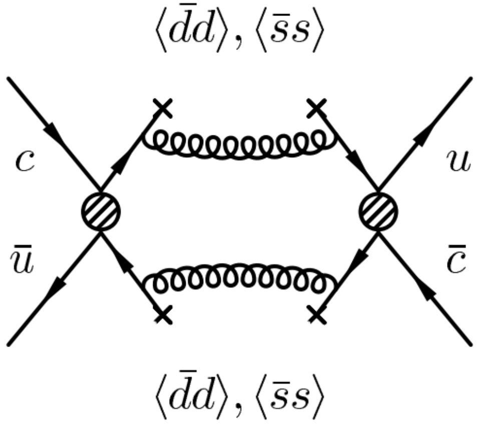
The situation that the exclusive sum of y is more enhanced is possible in general.

$*$: 4D-like phase space

Higher order corrections suppressed by powers of $1/m_c$

◉ Six-quark and eight-quark operators for possible enhancements





Bobrowski, Lenz, Riedl and Rohrwild [1002.4794]

also Bigi and Uraltsev [0005089]

⟶ It in principle (at least partially) avoids extremely small SU(3) breaking suppression of the form, $(m_s/m_c)^n$.

◉ A number of non-perturbative matrix elements are required for quantitative prediction, resulting in difficulty.

(some assumptions) Estimate: $x \sim y \lesssim 10^{-3}$ Falk, Grossman Ligeti and Petrov [0110317]

Non-local chiral condensate

Melić, Dulibić and Petrov, [2410.14382] PoS ICHEP2024 (2025) 413

$$\langle \bar q(x)^a_\alpha q(0)^b_\beta\rangle = \frac{\langle\bar q q\rangle}{4N_C}\delta^{ab}\left[\delta_{\alpha\beta}\left(1-\frac{x^2}{4}\left(\frac{m^2}{2}-\frac{\langle\bar q ig\sigma G q\rangle}{4\langle\bar q q\rangle}\right)\dots\right)+ +i(\not x)_{\beta\alpha}\left(\frac{m}{4}-\frac{x^2}{4}\left(\frac{m^3}{12}-\frac{m}{12}\frac{\langle\bar q ig\sigma Gq\rangle}{2\langle\bar qq\rangle}+\frac{2}{81}\pi\alpha_s\frac{\langle\bar qq\rangle^2}{\langle\bar qq\rangle}\right)\dots\right)\right]$$

expanded by local condensates

$$x^{NP} = (1.27 \mp 0.18)10^{-5}$$ update in CHARM2025

◉ Still two orders of magnitude below the experimental data.

(2) Dispersive approach

Li, Umeeda, Xu and Yu [2001.04079]

Dispersive approach

Li, Umeeda, Xu and Yu [2001.04079]

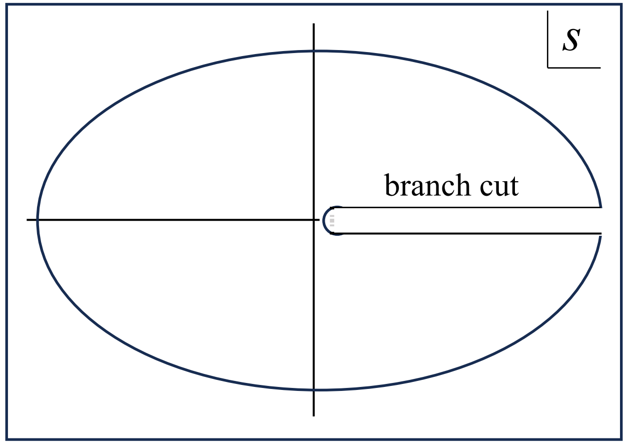
Dispersion relation: $x(s) = \frac{1}{\pi}\int_0^\infty ds' \frac{y(s')}{s-s'}$

fictitious quark mass squared

$s = m_h^2$

observables

$x(m_c^2), \quad y(m_c^2)$



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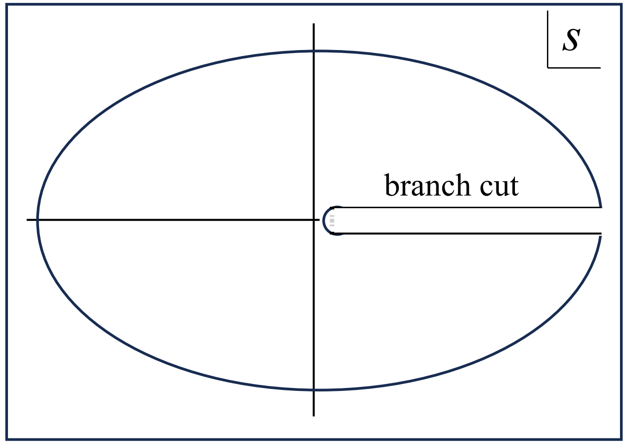


Procedure

(a) For large (-*s*), *x*(*s*) and *y*(*s*) are computed in perturbation theory.

(the OPE in deep Euclidean domain, asymptotic freedom)

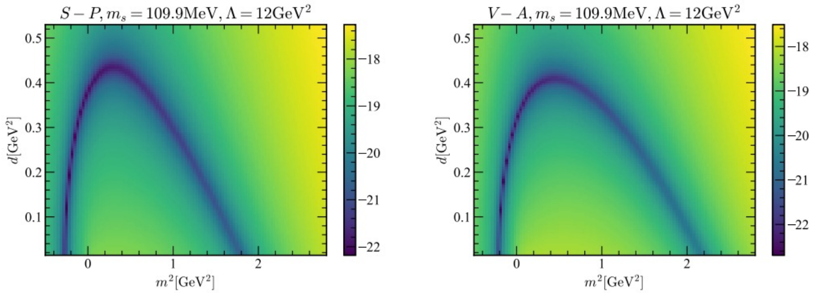
(b) The dispersion relation is rewritten as, $(\Lambda:$ large scale$)$

$$\int_0^\Lambda ds' \frac{y(s')}{s-s'} = \pi x(s) - \int_\Lambda^\infty ds' \frac{y(s')}{s-s'} \quad \cdots (*)$$

unknown: $y(s')$

calculable from (a): $\pi x(s) - \int_\Lambda^\infty ds' \frac{y(s')}{s-s'}$

parameter scan over $m^2 - d$ plot



arc $\leftrightarrow$ rhs=lhs is almost satisfied.

(c) The unknow function is determined so as to satisfy Eq. (*).

Parametrization: $y(s) = \frac{Ns[b_0 + b_1(s-m^2) + b_2(s-m^2)^2]}{[(s-m^2)^2+d^2]^2}$ $\qquad b_i, m^2, d^2$: parameters

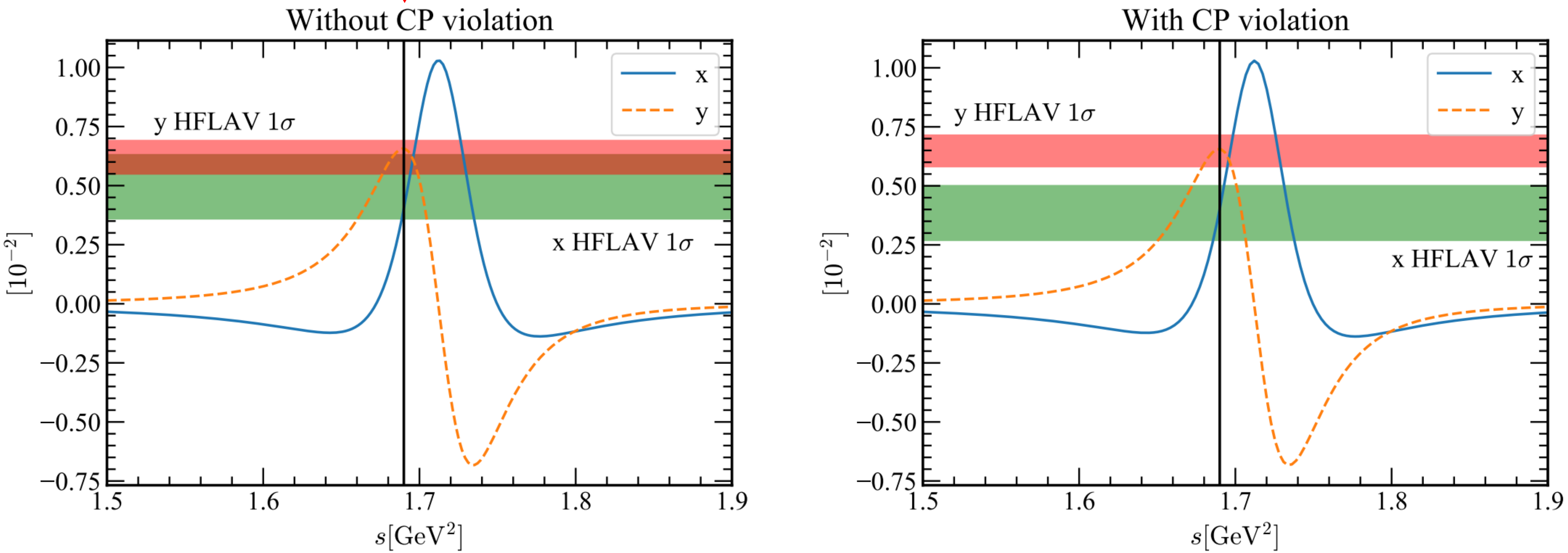
(d) $x(m_c^2)$ is fixed from the original dispersion relation. CP violation is also analyzed via,

$$\frac{q}{p} = \sqrt{\frac{2M_{12}^* - i\Gamma_{12}^*}{2M_{12} - i\Gamma_{12}}}$$

Li, Umeeda, Xu, Yu [2001.04079]

Numerical result: Dispersive approach

corresponding to observables



$s = m_h^2$

This work $\begin{cases} |q/p| \approx 1.0002 \\ \phi \approx 0.006 \text{ degrees} \end{cases}$

HFLAV as of 2023 $\begin{cases} |q/p| = 0.994^{+0.016}_{-0.015} \\ \phi = -2.6^{+1.1}_{-1.2} \text{ degrees} \end{cases}$

(3) Dyson-Schwinger-based approach

Xie, Umeeda and Zhu [2504.11745]

SU(3) breaking from quark masses

$$M_{12} \propto \left(\frac{m_s^2 - m_d^2}{m_c^2}\right)^2 = 5.2 \times 10^{-5}$$

extreme GIM suppression

current quark masses

$$\begin{cases} m_s = 0.108 \text{ GeV} \\ m_d = 0.0055 \text{ GeV} \end{cases}$$

$\overline{\text{MS}}$ scheme @ m_c

RunDec [0004189]

$$|x| = 2.5 \times 10^{-6}$$

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SU(3) breaking from quark masses

$$M_{12} \propto \left(\frac{m_s^2 - m_d^2}{m_c^2}\right)^2 = 5.2 \times 10^{-5}$$

extreme GIM suppression

replacement →

$$M_{12} \propto \left[\frac{(m_s^{\rm con})^2 - (m_d^{\rm con})^2}{m_c^2}\right]^2 = 4.6 \times 10^{-3}$$

Enhancement

current quark masses

$$\begin{cases} m_s = 0.108\ \text{GeV} \\ m_d = 0.0055\ \text{GeV} \end{cases}$$

$\overline{\text{MS}}$ scheme @ m_c

RunDec [0004189]

$$|x| = 2.5 \times 10^{-6}$$

constituent quark masses

$$\begin{cases} m_s^{\rm con} = 0.49\ \text{GeV} \\ m_d^{\rm con} = 0.36\ \text{GeV} \end{cases}$$

Ivanov, Kalinovsky and Roberts, [9812063]

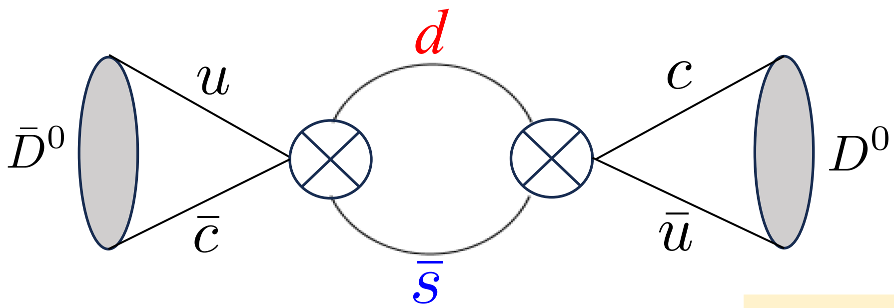
$$|x| = 2.3 \times 10^{-4}$$

- Importance of chiral symmetry breaking (χSB) is seen by the above discussion.
- Its significance is also indicated in the inclusive analysis as non-local condensates improve the result.
  Melić, Dulibić and Petrov CHARM 2025 — condensates: order parameter of χSB
- More consistent treatment of momentum dependence → The Dyson-Schwinger-based approach

Xie, Umeeda and Zhu [2504.11745]

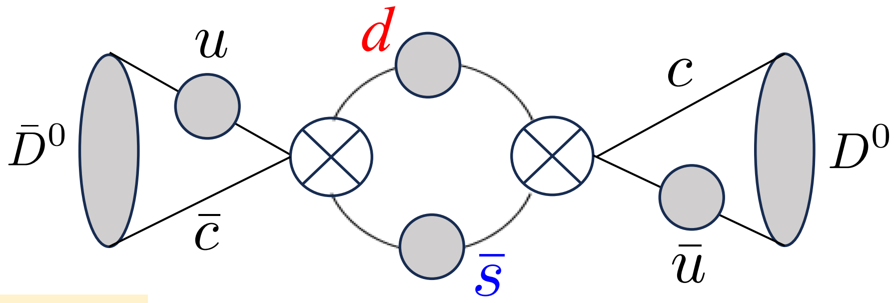
Dyson Schwinger based approach

Conventional discussion
(after integrating out W boson)



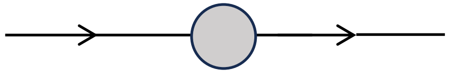
replacement →

Use of dressed quark propagators



Modelling of DSE quark propagators

Ivanov, Kalinovsky and Roberts, [9812063]

$S =$  $= -ip\cdot\gamma\sigma_V^i(p_E^2) + \sigma_S^i(p_E^2)$

$$x = p_E^2/(2D)$$

$$\bar{m}_i = \tilde{m}_i/\sqrt{2D}$$

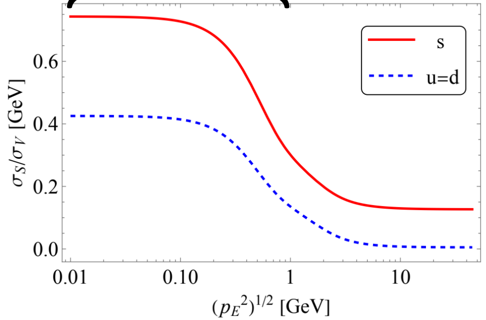
Vector part: $\sigma_V^i(p_E^2) = \dfrac{\bar{\sigma}_V^i(x)}{2D}$ $\quad \bar{\sigma}_V^i(x) = \dfrac{2(x+\bar{m}_i^2) - 1 + e^{-2(x+\bar{m}_i^2)}}{2(x+\bar{m}_i^2)^2}$ $\quad \mathcal{F}(z) = (1-e^{-z})/z$

Scalar part: $\sigma_S^i(p_E^2) = \dfrac{\bar{\sigma}_S^i(x)}{\sqrt{2D}}$ $\quad \bar{\sigma}_S^i(x) = 2\bar{m}_i\mathcal{F}(2(x+\bar{m}_i^2)) + \mathcal{F}(b_1^i x)\mathcal{F}(b_3^i x)[b_0^i + b_2^i\mathcal{F}(\epsilon x)]$

$D, b_n, \tilde{m}, \epsilon$: parameters

- Dynamical chiral symmetry breaking (DCSB): parameters from [9812063]
- Primary SU(3) breaking from DCSB is included.

large @ low momenta



Numerical results: Dyson Schwinger based approach

This work	$\lvert x\rvert = 2.0 \times 10^{-3}$	DSE	
OPE-based approaches	$\lvert x\rvert = (5.6 - 22) \times 10^{-6}$	HQET	Ohl, Ricciardi and Simmons [9301212]
	$\lvert x\rvert = 6 \times 10^{-7}$	NLO	Golowich and Petrov [0506185]
	$x^{NP} = (1.27 \mp 0.18) 10^{-5}$	Non-local condensate	Melić, Dulibić and Petrov CHARM2025
Experiment	$x = (4.07 \pm 0.44) \times 10^{-3}$	HFLAV	

◉ The Dyson-Schwinger-based approach gives the order of magnitude comparable to the HFLAV value, and therefore offering an improved theoretical formulation.

Our result: $\lvert x\rvert = (1.3 - 2.9) \times 10^{-3}$

Range: dependence on decay constant, Wilson coefficients and D-meson mass

Summary

- Charm mixing is a challenging process for the theoretical side, since the accuracy is uncontrollable due to the GIM suppression.
- The order of magnitude is *not* reproduced in the inclusive approach, even in the case including $1/m_c$ power-suppressed corrections.
  -- *Non-local condensates* Melić, Dulibić and Petrov (CHARM2025)
- Violation of quark-hadron duality is hard to quantify and thus relies on specific modelling. -- *The simple model* Jubb et al. [1603.07770] -- *The 't Hooft model* Umeeda [2106.06215]
- The recent works alternative to the inclusive approach:
  -- *Dispersive analysis from OPE input* Li, Umeeda, Xu and Yu [2001.04079] Li [2208.14798]
  -- *Dyson-Schwinger-based method* Xie, Umeeda and Zhu [2504.11745] (preprint)
  -- *Lattice QCD + spectral reconstruction* Di Carlo, Erben and Hansen [2504.16189] (preprint)

Back up

CP violation

How large can the SM contribution to CP violation in $D^0 - \bar{D}^0$ mixing be?

M. Bobrowski[a], A. Lenz[b,a], J. Riedl[a] and J. Rohrwild[a,c]

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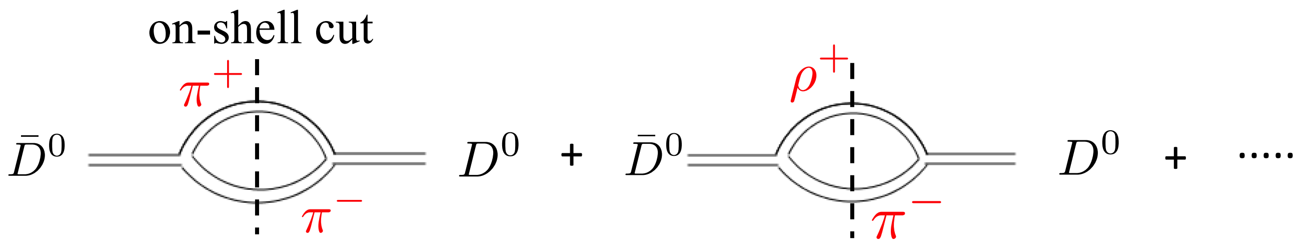
Exclusive approach

Exclusive approach

Γ_{12}:



$$y \propto \frac{\Gamma_{12}}{\Gamma}$$

In CP-conserving limit

$$y = \sum_n \eta_{\rm CKM}(n)\eta_{\rm CP}(n)\cos\delta_n \sqrt{\mathcal{B}(D^0 \to n)\mathcal{B}(D^0 \to \bar{n})} = y_{PP} + y_{PV} + y_{VV} + \cdots$$

n: all hadronic final states

Falk, Grossman and Ligeti and Petrov [0110317]

$D \to PP$, $D \to PV$ and $D \to VV$

$\eta_{(\rm CKM,\ CP)}$: sign coming from CKM, CP eigenvalue

δ_n : strong phase difference

Two-pseudoscalar channel as an example: Cheng and Chiang [2401.06316]

$$\begin{aligned} y_{PP} = {} & \mathcal{B}(\pi^+\pi^-) + \mathcal{B}(\pi^0\pi^0) + \mathcal{B}(\pi^0\eta) + \mathcal{B}(\pi^0\eta') + \mathcal{B}(\eta\eta) + \mathcal{B}(\eta\eta') + \mathcal{B}(K^+K^-) + \mathcal{B}(K^0\overline{K}^0) \\ & -2\cos\delta_{K^-\pi^+}\sqrt{\mathcal{B}(K^-\pi^+)\mathcal{B}(K^+\pi^-)} - 2\cos\delta_{\bar{K}^0\pi^0}\sqrt{\mathcal{B}(\overline{K}^0\pi^0)\mathcal{B}(K^0\pi^0)} \\ & -2\cos\delta_{\bar{K}^0\eta}\sqrt{\mathcal{B}(\overline{K}^0\eta)\mathcal{B}(K^0\eta)} - 2\cos\delta_{\bar{K}^0\eta'}\sqrt{\mathcal{B}(\overline{K}^0\eta')\mathcal{B}(K^0\eta')}. \end{aligned}$$

Experimental data are used for branching ratios, with strong phases properly extracted by topological amplitudes.

Exclusive methods

Topological diagram approach (TDA) Cheng and Chiang [2401.06316]

PP channel: $y_{PP} \sim (0.110 \pm 0.011)\%$. (multiple solutions, dependence on $\eta - \eta'$ mixing angle)

VP channel: $y_{VP} \gtrsim (0.220 \pm 0.071)\%$ (multiple solutions) (detail in the reference)

VV channel: naive factorization $y_{VV,S} = -0.167\%$, $y_{VV,P} = 1.61 \times 10^{-4}$, $y_{VV,D} = 4.71 \times 10^{-5}$

Data-driven + naive fact for unmeasured $y_{VV,S} = 0.0271\%$, $y_{VV,P} = -0.0958\%$, $y_{VV,D} = 0.361\%$.

Some Br are not measured yet.

S, P, D : partial wave contributions

Factorization-assisted topological (FAT) approach Jiang, Yu, Qin, Li and Lu [1705.07335]

PP+PV channel: $y_{PP+PV} = (0.21 \pm 0.07)\ \%$ smaller than data

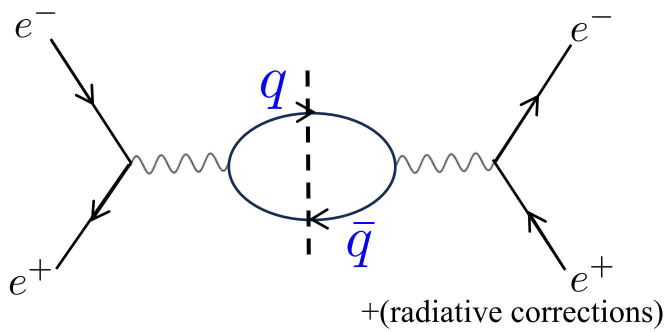
HFLAV: $y = (0.645^{+0.024}_{-0.023})\ \%$ all CPV allowed

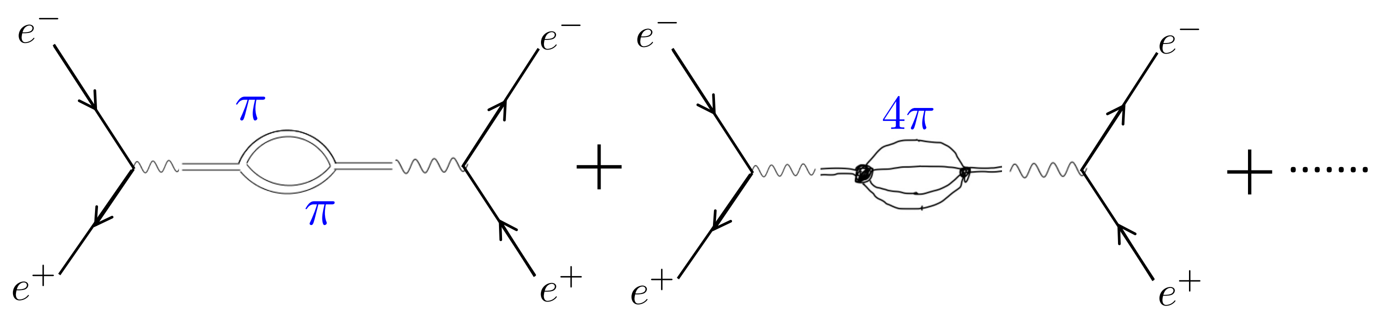
◉ The approaches reproduce (at least) half of experimental data.

Quark/gluon diagrams and hadronic diagrams

Inclusive | **Exclusive**

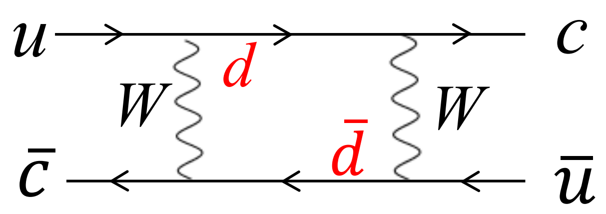
$e^+e^- \to$ hadrons
(example)

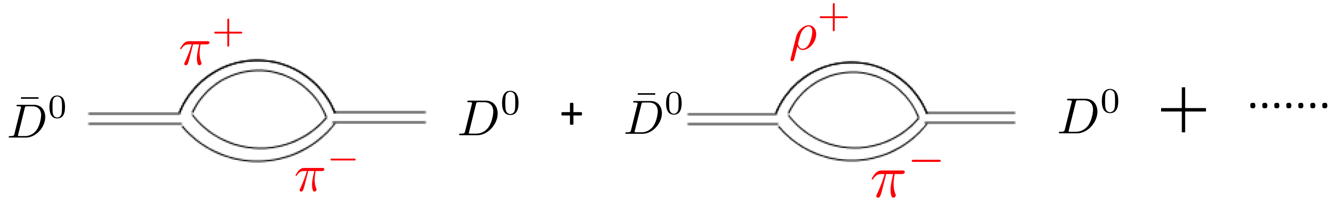




$$\sigma(e^+e^- \to \text{hadrons}) = \sigma(e^+e^- \to 2\pi) + \sigma(e^+e^- \to 4\pi) + \cdots$$

$\bar{D}^0 - \bar{D}^0$ mixing





$$\Gamma_{12}[\bar{D}^0 \to d\bar{d} \to D^0] = \Gamma_{12}[\bar{D}^0 \to \pi^+\pi^- \to D^0] + \Gamma_{12}[\bar{D}^0 \to \rho^+\pi^- \to D^0] + \Gamma_{12}[\bar{D}^0 \to \rho^-\pi^+ \to D^0] + \cdots$$

↑ Quark-hadron duality

and similar discussion for *ds* and *ss* flavors

- The inclusive approach tacitly assumes duality, while duality violation is hard to quantify.

→ Difficulty analogous to $B \to X_c l \nu$ decays and B/D-meson lifetimes.