Prospects of τ EDM Measurements at STCF

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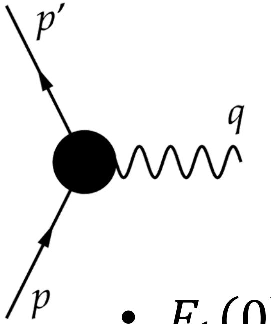
Xulei Sun, Y. Wu, Xiaorong Zhou; arXiv: 2411.19469

Introduction

- Electric Dipole Moment
  - Current Strongest CPV test
    - **Matter-antimatter asymmetry**
- Electromagnetic form factors



$$= eQ_f\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_\ell}F_2(q^2) + \frac{\sigma^{\mu\nu}q_\nu\gamma_5}{2m_\ell}F_3(q^2) + \left(\gamma^\mu - \frac{2m_\ell q^\mu}{q^2}\right)\gamma^5 F_4(q^2)\right]u(p)$$

- $F_1(0) = 1,\quad F_2(0) = a_f = \left(\frac{g-2}{2}\right)_f,\quad F_3(0) = \frac{2m_f}{eQ_f}d_f,\quad F_4(0)$

Total Charge **Anomalous Magnetic Moment** **EDM** **Anapole Moment**

- BSM Contributions
  - Lepton Universality
  - Flavor Structure

$$\frac{\delta d_\ell}{\delta d_e} \sim \frac{m_\ell}{m_e}$$

$$\delta a_\ell \sim \frac{m_\ell^2}{\Lambda^2}$$

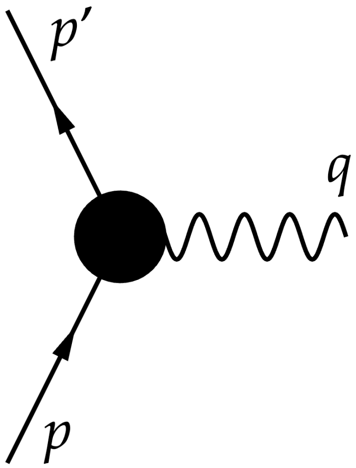
$$\frac{m_\tau}{m_e} \sim 3460$$

$$\left(\frac{m_\tau}{m_\mu}\right)^2 \sim 280$$

Mod.Phys.Lett.A22(2007)159
Phys.Lett.B255(1991)611
Phys.Lett.B395(1997)369
Rev.Mod.Phys.87(2015)531

Introduction

- Electromagnetic form factors



$$= e\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_\ell}F_2(q^2) + \frac{\sigma^{\mu\nu}q_\nu\gamma_5}{2m_\ell}F_3(q^2) + \left(\gamma^\mu - \frac{2m_\ell q^\mu}{q^2}\right)\gamma^5 F_4(q^2)\right]u(p)$$

- $F_2(q^2), F_3(q^2)$: Measured at corresponding energy scale
  - Can develop imaginary part

$$\frac{e}{2m_\ell}\Re\left(F_3(q^2)\right) \to \Re d_\ell \qquad \frac{e}{2m_\ell}\Im\left(F_3(q^2)\right) \to \Im d_\ell$$

$$\Re\left(F_3(q^2)\right) = \Re\left(F_3^{SM}(q^2)\right) + \Re\left(F_3^{BSM}(q^2)\right) \approx \Re\left(F_3^{BSM}(q^2)\right)$$

Current Limits for d_τ

- Current Results

Citation: S. Navas *et al.* (Particle Data Group), Phys. Rev. D **110**, 030001 (2024) and 2025 update

τ ELECTRIC DIPOLE MOMENT (d_τ)

A nonzero value is forbidden by both T invariance and P invariance.

The q^2 dependence is expected to be small providing no thresholds are nearby.

Re(d_τ)

VALUE (10^{-16} e cm)	*CL%*	*DOCUMENT ID*		*TECN*	*COMMENT*
− 0.185 to 0.061	95	[1] INAMI	22	BELL	E^{ee}_{cm} = 10.6 GeV
• • • We do not use the following data for averages, fits, limits, etc. • • •					
< 2.3	90	[2] GROZIN	09A	RVUE	From e EDM limit
< 3.7	95	[3] ABDALLAH	04K	DLPH	$e^+e^- \to e^+e^-\tau^+\tau^-$ at LEP2
< 11.4	95	[4] ACHARD	04G	L3	$e^+e^- \to e^+e^-\tau^+\tau^-$ at LEP2
− 0.22 to 0.45	95	[5] INAMI	03	BELL	E^{ee}_{cm} = 10.6 GeV
< 4.6	95	[6] ALBRECHT	00	ARG	E^{ee}_{cm} = 10.4 GeV
> −3.1 and < 3.1	95	ACCIARRI	98E	L3	1991–1995 LEP runs
> −3.8 and < 3.6	95	[7] ACKERSTAFF	98N	OPAL	1990–1995 LEP runs
< 0.11	95	[8,9] ESCRIBANO	97	RVUE	$Z \to \tau^+\tau^-$ at LEP
< 0.5	95	[10] ESCRIBANO	93	RVUE	$Z \to \tau^+\tau^-$ at LEP
< 7	90	GRIFOLS	91	RVUE	$Z \to \tau\tau\gamma$ at LEP
< 1.6	90	DELAGUILA	90	RVUE	$e^+e^- \to \tau^+\tau^-$ E^{ee}_{cm} = 35 GeV

Belle – Spin Correlation

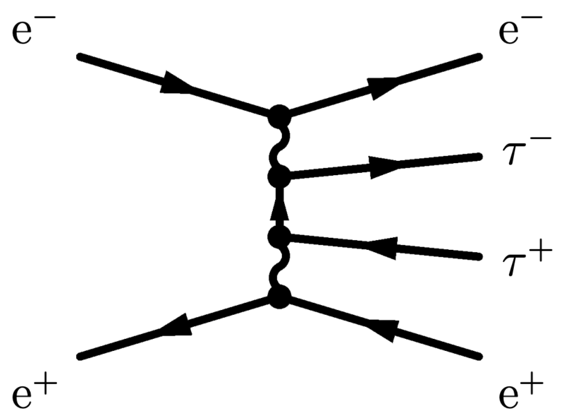
LEP – Cross Section

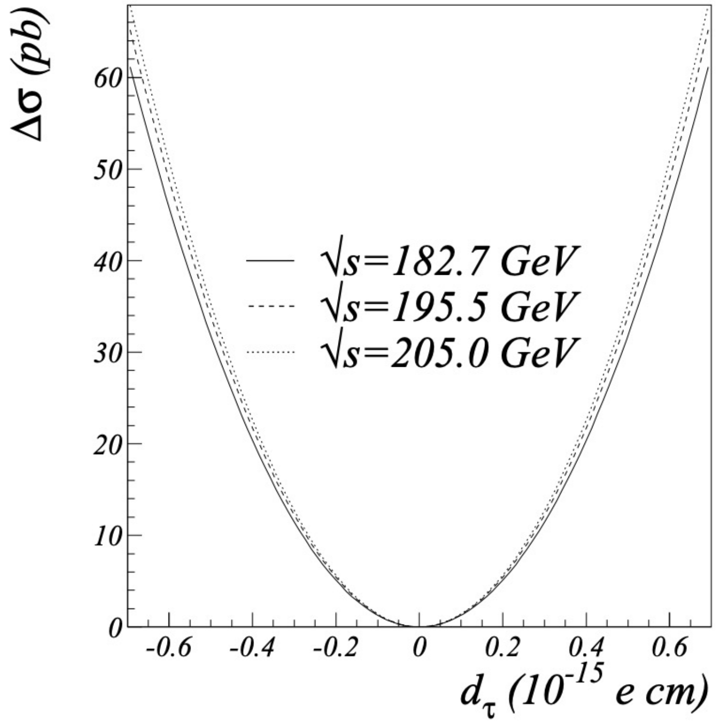
d_e – Indirect

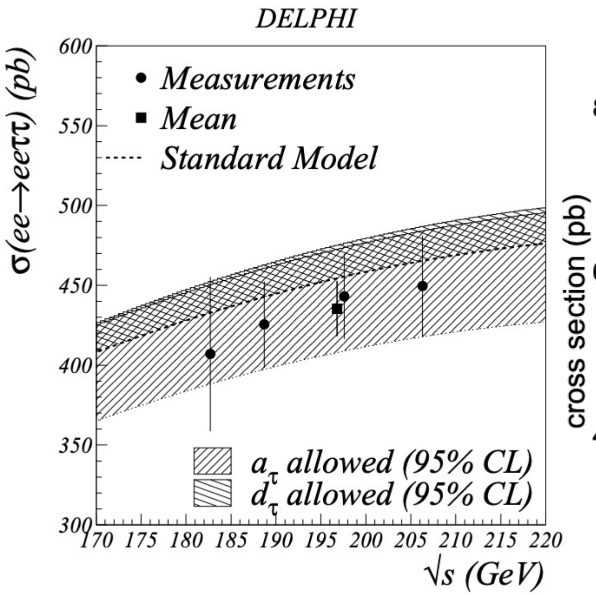
Current Limits for d_τ

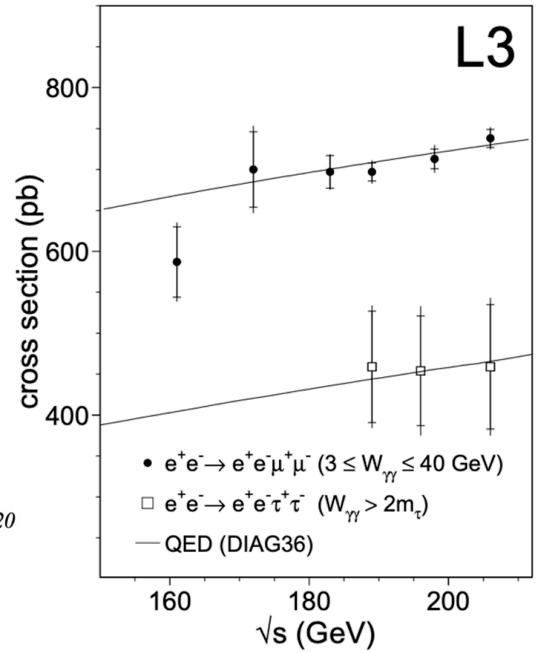
- Direct Measurement
  - Production cross section
    - LEP
      - $e^+e^- \to e^+e^-\tau^+\tau^-$

Phys.Rev.D 53 (1996) 1181
Eur.Phys.J.C 35 (2004) 159 - DELPHI
Phys.Lett.B 585 (2005) 53 – L3







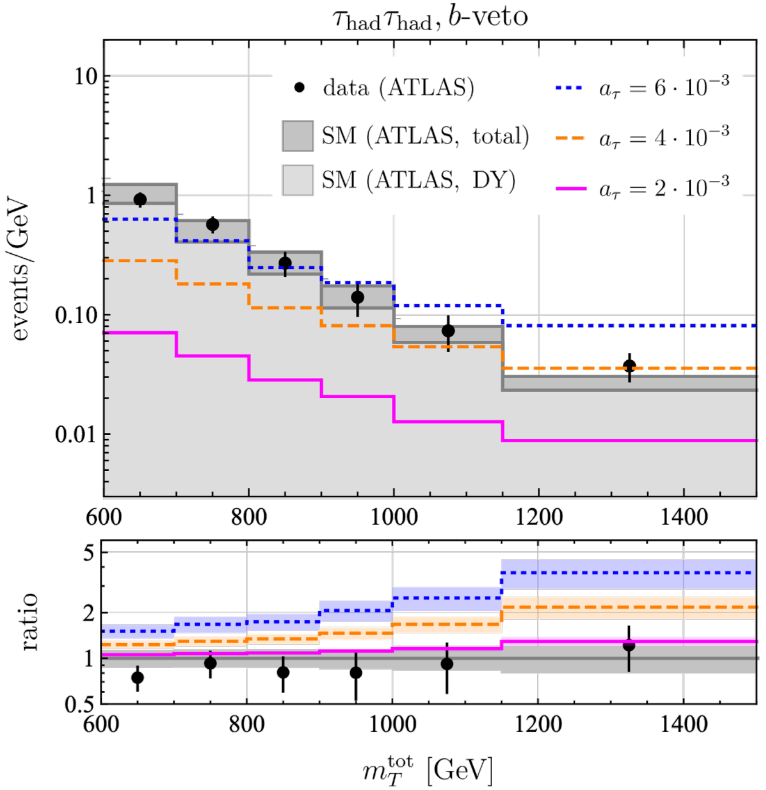


L3 $|d_\tau| \leq 1.14\times10^{-15}\, e\cdot\mathrm{cm}$

DELPHI $|d_\tau| \leq 3.7\times10^{-16}\, e\cdot\mathrm{cm}$

Current Limits for d_τ

- Direct Measurement
  - Production cross section
    - LHC
      - $q\bar{q} \to \tau^+\tau^-$
      - Contribution from d_τ/a_τ enhance at high energies



***SciPost Phys.* 16 (2024) 2, 048**

Based on ATLAS measurements: Phys.Rev.Lett. 125 (2020) 051801

$$|d_\tau| < 1.0\times10^{-17} e\cdot \mathrm{cm}$$

Comparable with Belle measurements

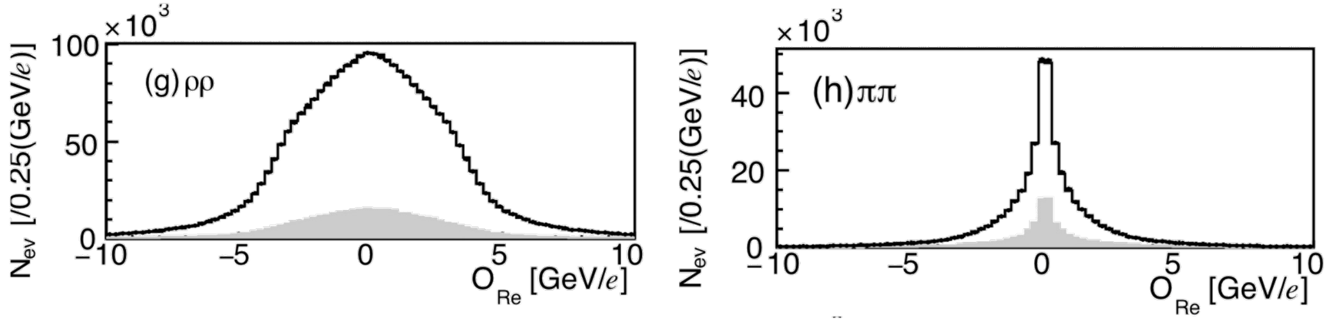
Current Limits for d_τ

- Direct Measurement - Belle
  - Spin correlation at low energy lepton collider
    - $e^+e^- \to \gamma^* \to \tau^+\tau^-$
  - 8 different channels:
    - $(e\nu\bar{\nu})(\mu\nu\bar{\nu}), (e\nu\bar{\nu})(\pi\nu), (\mu\nu\bar{\nu})(\pi\nu), (e\nu\bar{\nu})(\rho\nu),$
      $(\mu\nu\bar{\nu})(\rho\nu), (\pi\nu)(\rho\nu), (\rho\nu)(\rho\bar{\nu}), (\pi\nu)(\pi\bar{\nu})$
  - Matrix Element

$$\chi_{\rm prod} = \chi_{\rm SM} + {\rm Re}(d_\tau)\chi_{\rm Re} + {\rm Im}(d_\tau)\chi_{\rm Im} + |d_\tau|^2\chi_{d^2},$$

  - Optimal Observable

JHEP 04 (2022) 110
Phys.Lett.B 551 (2003) 16



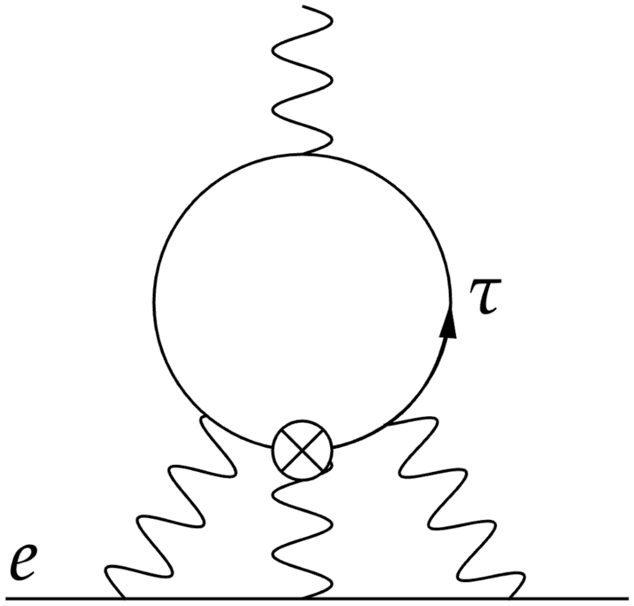
$$\Re(d_\tau) \in [-1.85, 0.61]\times 10^{-17} e\cdot{\rm cm}$$

$$\Im(d_\tau) \in [-1.03, 0.23]\times 10^{-17} e\cdot{\rm cm}$$

Current Limits for d_τ

- Indirect Constraints from d_e
  - d_τ contributes to d_e through 3-loop diagram

A. Grozin, I. Khriplovich, A. Rudenko; Nucl.Phys.B 821 (2009) 285; Phys.Atom.Nucl. 72 (2009) 1203



$$d_e \approx \left(\left(\frac{15}{4}\zeta(3) - \frac{31}{12}\right)\frac{m_e}{m_\tau}\left(\frac{\alpha}{\pi}\right)^3\right) d_\tau \approx 6.9\times10^{-12} d_\tau$$

  - Current Measurement (Prospects) of d_e

$|d_e| < 4.1\times10^{-30} e\cdot cm$ @ 90% CL

Roussy et al. Science 381 (2023) 6653, 46

$|d_e| \lesssim 4\times10^{-31} e\cdot cm$

Next generation of ACME

$\Rightarrow$

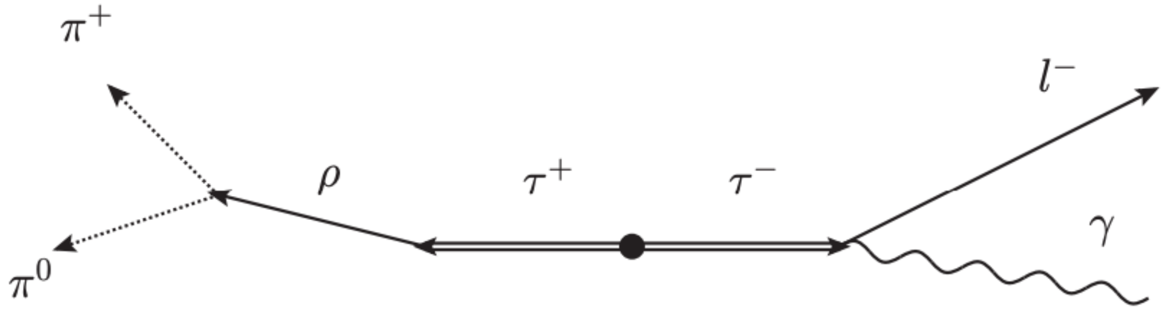
$|d_\tau| < 5.9\times10^{-19} e\cdot cm$

$|d_\tau| < 5.80\times10^{-20} e\cdot cm$

Model Dependent

Current Studies

- Radiative Decay **JHEP 03 (2016) 140**
  - $\tau^{\pm} \rightarrow \ell^{\pm}\nu_{\tau}\nu_{\ell}\gamma$



Prospects | **Current Limits**

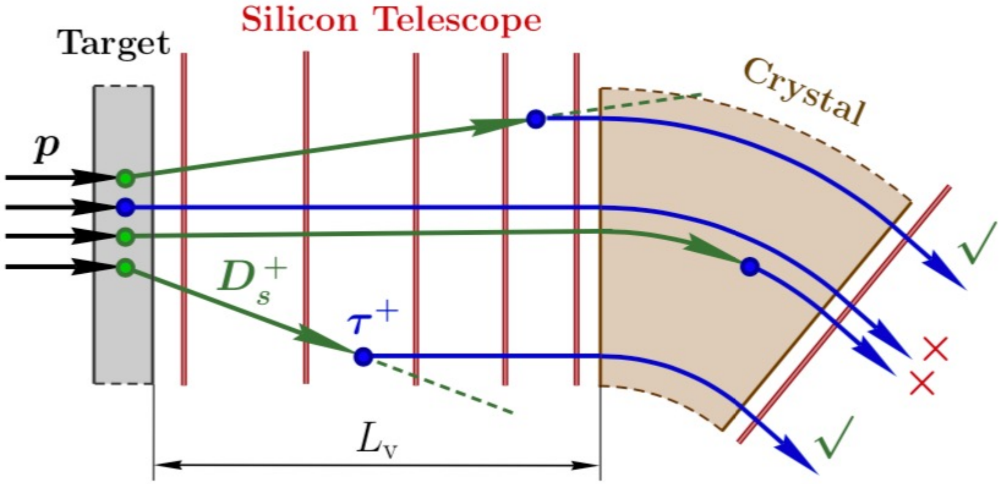
	Belle (ρ)	Belle II (ρ)	Belle (full)	Belle II (full)	DELPHI [2]	Belle [52]
$\tilde{a}_{\tau}$	0.16	0.023	0.085	0.012	0.017	—
$(m_{\tau}/e)\,\tilde{d}_{\tau}$	0.15	0.021	0.080	0.011	—	0.0015

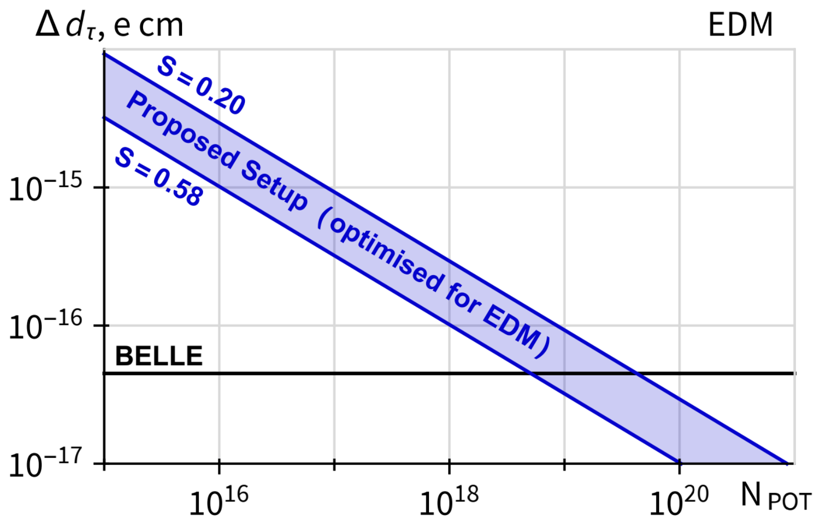
Current Studies

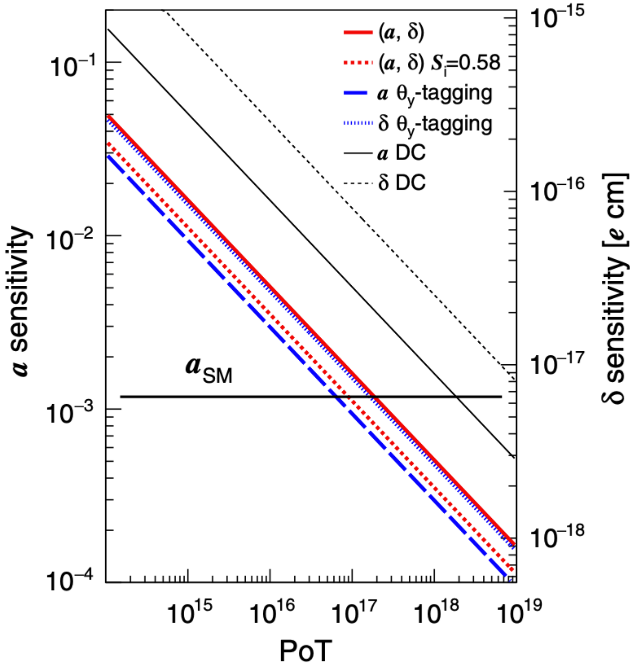
- Bent Crystal

JHEP 03 (2019) 156
Phys. Rev. Lett 123 (2019) 011801

$$pp \to D_s^+ X, \quad D_s^+ \to \tau^+ \nu_\tau, \quad \tau^+ \to \pi^+ \pi^+ \pi^- \bar{\nu}_\tau .$$

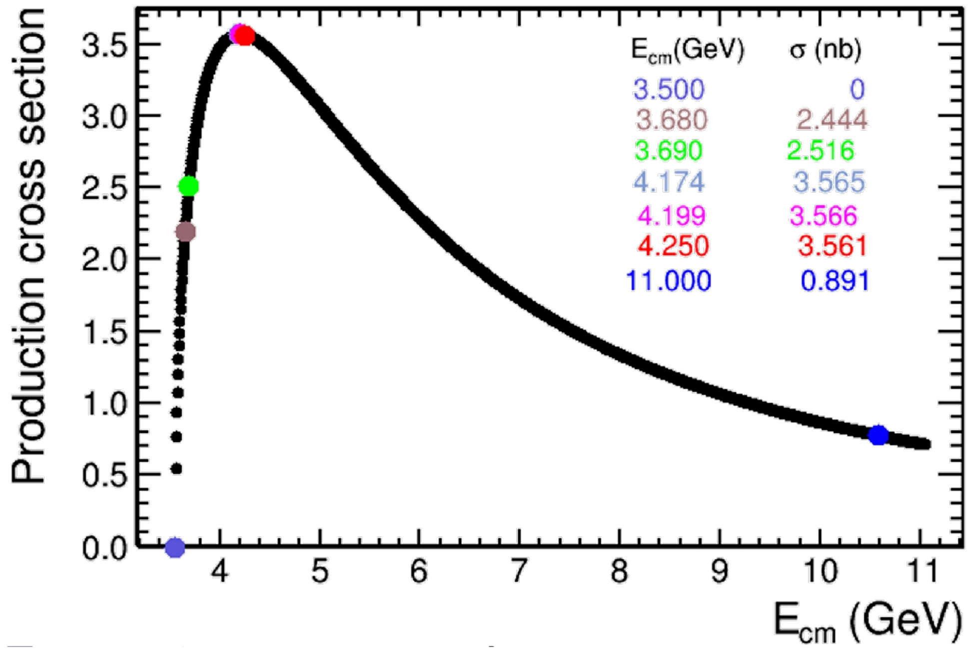






Super Tau-Charm Facility (STCF)

- Large amount of tau-pair $e^+e^- \to \gamma^* \to \tau^+\tau^-$
  - 3.5×10^9 tau pair per year @ $\sqrt{s} = 4.26$ GeV
  - 1.7×10^9 @ $\sqrt{s} = 7.0$ GeV



- STCF can significantly improve the sensitivity for tau-physics

τ EDM at STCF

- Cross Section
  - Summing over decay phase space

$$e^+e^- \to \gamma^* \to \tau^+\tau^-$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta}{4s}\left((2-\beta^2\sin^2\theta)(|F_1|^2 - \gamma^2|F_2|^2) + 2(1+\gamma^2)|F_2|^2 + \beta^2\gamma^2\sin^2\theta\,|F_3|^2 + 4\Re(F_1F_2^*)\right)$$

- $F_1(0) = 1,\ \ F_2(0) = a_f = \left(\frac{g-2}{2}\right)_f,\ \ F_3(0) = \frac{2m_f}{eQ_f}d_f$
- $\beta = \sqrt{1-4m_\tau^2/s},\ \gamma = 1/\sqrt{1-\beta^2} = \sqrt{s}/(2m_\tau)$

  - Weak Sensitivity on d_τ

- Decay phase space matters – Spin Correlations

τ EDM at STCF

- Squared spin density matrix

$$e^+(\vec{p})e^-(-\vec{p}) \to \tau^+(\vec{k},\vec{s}_+)\tau^-(-\vec{k},\vec{s}_-)$$

$$|\mathcal{M}|^2 = |\mathcal{M}_{SM}|^2 + \Re(d_\tau)v|\mathcal{M}_{Re}|^2 + \Im(d_\tau)v|\mathcal{M}_{Im}|^2 + |d_\tau|^2v^2|\mathcal{M}_{d^2}|^2$$

$$|\mathcal{M}_{Re}|^2 = \frac{4e^3}{k_0 v}|\vec{k}|\left[-\left(m_\tau + (k_0 - m_\tau)(\hat{k}\cdot\hat{p})^2\right)(\vec{s}_+\times\vec{s}_-)\cdot\hat{k} + k_0(\hat{k}\cdot\hat{p})(\vec{s}_+\times\vec{s}_-)\cdot\hat{p}\right]$$

CP-odd

- Key factor: the momentum $\vec{k}$ and spin $\vec{s}$ of the τ leptons

The spin of τ

- For different τ decay modes:

$\tau \to \ell\nu_\ell\nu_\tau$

$$\vec{s}_\pm = \frac{4c_\pm - m_\tau^2 - 3m_\ell^2}{3m_\tau^2 c_\pm - 4c_\pm^2 - 2m_\ell^2 m_\tau^2 + 3c_\pm m_\ell^2}\left(\pm m_\tau \vec{p}_\ell - \frac{c_\pm + E_\ell m_\tau}{k_0 + m_\tau}\vec{k}\right) \qquad c_\pm = k_0 E_\ell \mp \vec{k}\cdot\vec{p}_\ell$$

$\tau \to \pi\nu_\tau$

$$\vec{s}_\pm = \frac{2}{m_\tau^2 - m_\pi^2}\left(\pm m_\tau \vec{p}_\pi + \frac{m_\tau^2 + m_\pi^2 + 2m_\tau E_\pi}{2(E_\tau + m_\tau)}\vec{k}\right)$$

$\tau \to \pi\pi^0\nu_\tau$

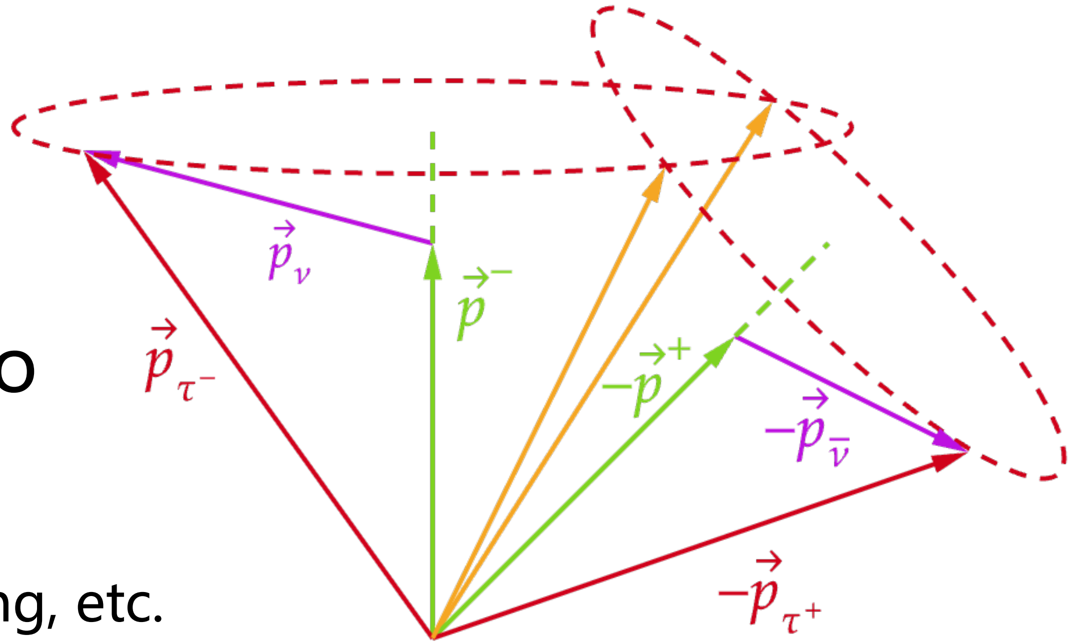
$$\vec{s}_\pm = \mp\frac{1}{k_\pm \cdot H_\pm - m_\tau^2 (p_{\pi^\pm} - p_{\pi^0})^2}\left(\mp H_0^\pm \vec{k} + m_\tau \vec{H}^\pm + \frac{\vec{k}(\vec{k}\cdot\vec{H}^\pm)}{E_\tau + m_\tau}\right)$$

$$H_\pm^\mu = 2\left(k_\pm \cdot (p_{\pi^\pm} - p_{\pi^0})\right)(p_{\pi^\pm} - p_{\pi^0})^\mu + (p_{\pi^\pm} - p_{\pi^0})^2 (p_{\pi^\pm} + p_{\pi^0})^\mu$$

- Reconstruction of the τ momentum
  - Decay products – Neutrinos
  - Two-fold ambiguity (Even worse for leptonic decay)

τ Momentum

- Reconstruction (semi-leptonic)
  - The visible part (π) can be measured (and well paired)
    - $\vec{p}^-$ and $\vec{p}^+$
  - $2 \rightarrow 2$ system with known E_{cm} and m_τ
    - $|\vec{p}_\nu|$ and $|\vec{p}_{\bar{\nu}}|$
    - $\vec{p}_\nu \cdot \vec{p}^-$ and $\vec{p}_{\bar{\nu}} \cdot \vec{p}^+$
  - $\vec{p}_{\tau^-}$ and $\vec{p}_{\tau^+}$ are back-to-back
  - Two solutions (orange arrows)
    - One solution/No solution
- More information about neutrino
  - Decay vertex detection
    - Global fitting
    - Remove ambiguity, improve the pairing, etc.



Xin Chen, Y. Wu; JHEP10(2019)089, 1803.00501
Xin Chen, Y. Wu; Eur.Phys.J. C77(2017)697, 1703.04855
Xin Chen, Y. Wu; Phys.Lett.B790(2019)332, 1708.02882

τ EDM at STCF

- Optimal Observable (OO)

$$|\mathcal{M}|^2 = |\mathcal{M}_{SM}|^2 + \Re(d_\tau) v |\mathcal{M}_{Re}|^2$$

$$\mathcal{O}_{Re} = \frac{|\mathcal{M}_{Re}|^2}{|\mathcal{M}_{SM}|^2}$$

$$\langle \mathcal{O}_{Re} \rangle = \int \mathcal{O}_{Re} |\mathcal{M}|^2 d\Pi = a_{Re} \cdot \Re(d_\tau) + b_{Re}$$

- Procedure:
  - Reconstruct the final state momentum
  - τ momentum and spin
  - Matrix elements – Optimal observable - EDM

Event Reconstruction

- Signal:

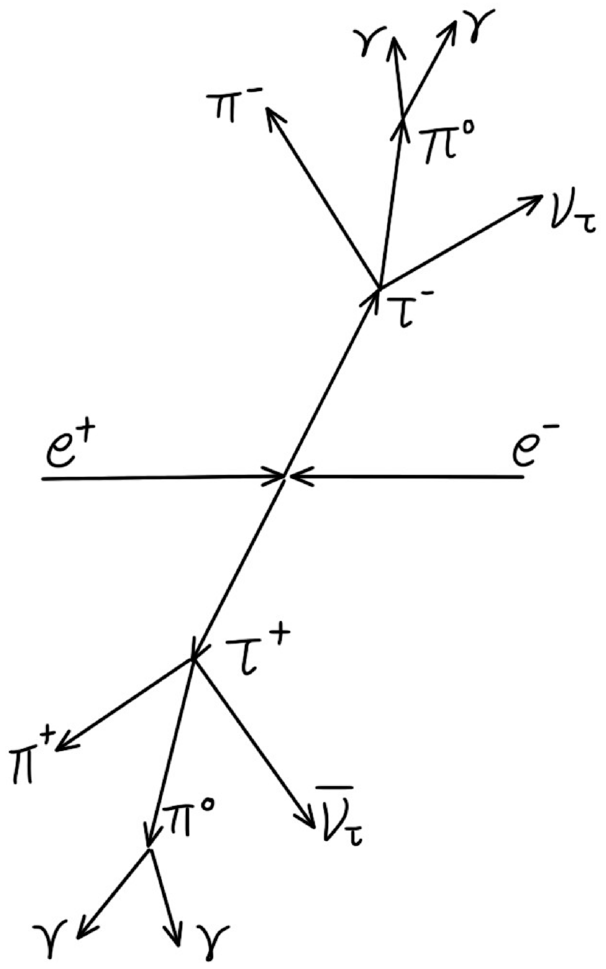
$$e^+e^- \to \tau^+\tau^-(\tau^\pm \to \pi^\pm\pi^0\nu_\tau, \pi^0 \to \gamma\gamma)$$

$$(\mathcal{B}(\tau \to \pi\pi^0\nu) \approx 25\%)^2 \approx 6.2\%$$

- Backgrounds:
  - Contamination from other decays

$$\tau^\pm \xrightarrow{17.8\%} \nu_\tau e^\pm \nu_e \qquad \tau^\pm \xrightarrow{17.4\%} \nu_\tau \mu^\pm \nu_\mu$$

$$\tau^\pm \xrightarrow{10.8\%} \nu_\tau \pi^\pm \qquad \tau^\pm \xrightarrow{9.3\%} \nu_\tau \pi^\pm \pi^0 \pi^0$$



Event Reconstruction

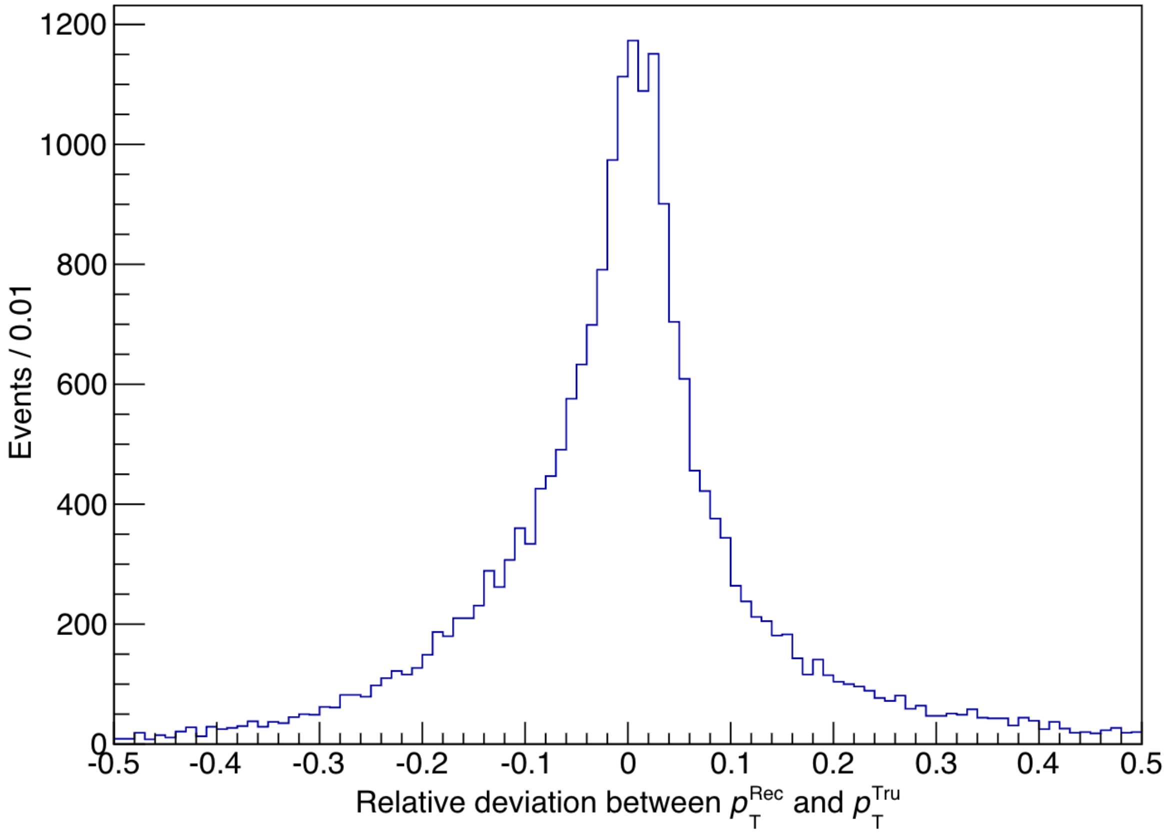
- Selections

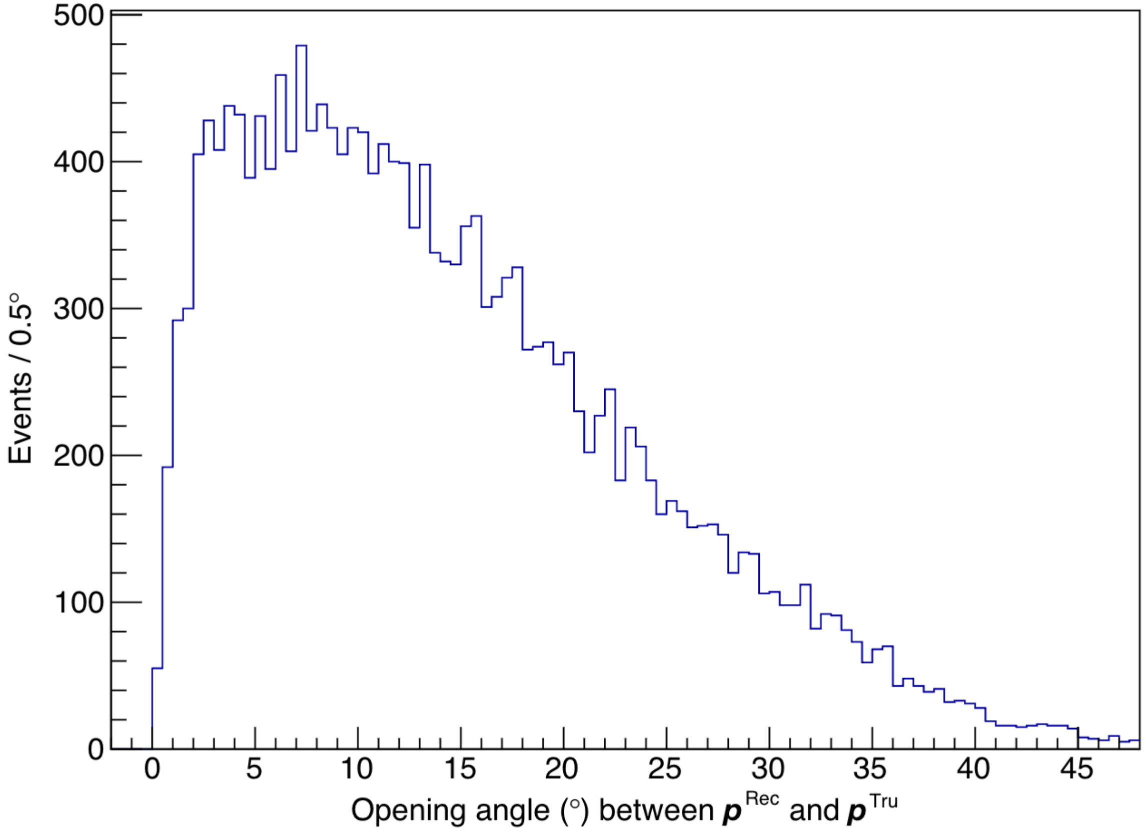
No.	Step	Percentage of previous step		Signal purity
		Inclusive events	Signal events	
0	Total events	-	-	**6.2%**
1	Number of charged tracks = 2, total charge = 0	58.3%	76.5%	8.1%
2	Number of photons = 4	7.2%	23.7%	26.7%
3	Number of $\pi^+ = 1$, Number of $\pi^- = 1$	81.8%	92.1%	30.0%
4	Passed the particle pairing	25.2%	52.3%	62.5%
5	Passed event-level machine learning selection	57.9%	73.4%	79.3%
6	Passed the τ momentum reconstruction	97.0%	97.7%	**80.0%**
	Overall	0.49%	6.3%	

Main background: multi-π^0 processes, mainly $\tau^\pm \to \pi^\pm\pi^0\pi^0\nu_\tau$ (about 14%)

Tau Reconstruction Results

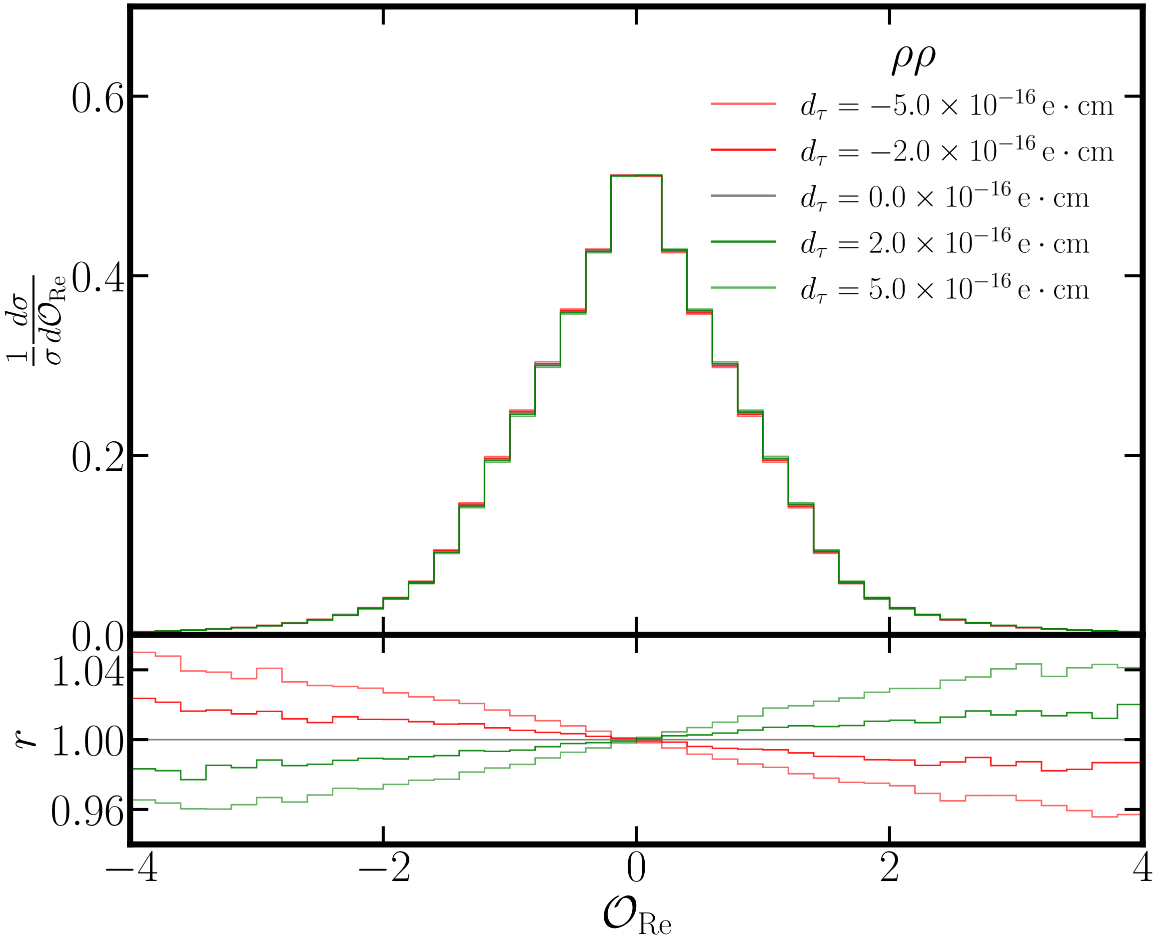
- Constructed vs. Truth level information





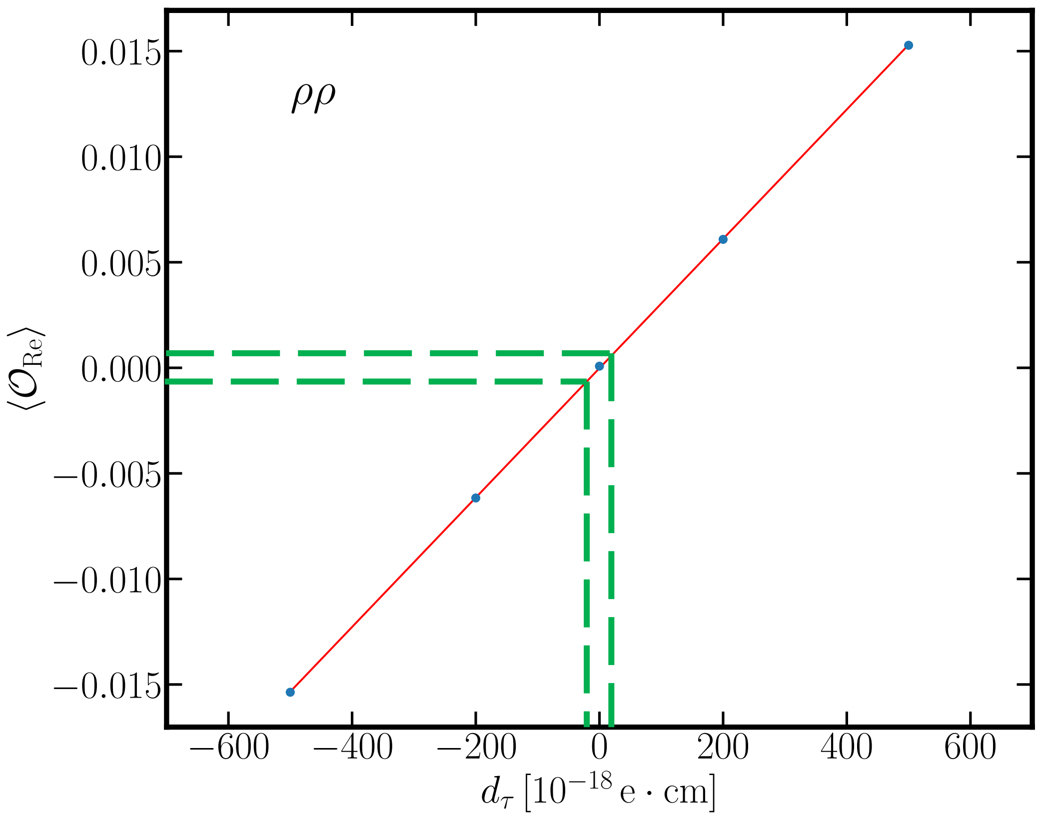
The distribution of OO

- Distribution of OO for $\rho\rho$ channel
  - Average over the two solutions of τ momentum



The distribution of OO

- $\langle \mathcal{O}_{Re} \rangle$ vs. $\Re(d_\tau)$ in $\rho\rho$ channel



$$\langle \mathcal{O}_{Re} \rangle = a_{Re} \cdot \Re(d_\tau) + b_{Re}$$

Uncertainties of $\langle \mathcal{O}_{Re} \rangle$: 1.09×10^{-4}

$$a_{Re} \approx 3.06\times10^{-5}\ /(10^{-18}\ e\cdot cm)$$

$\pm1.43\times10^{-7}$

$$b_{Re} \approx -1.85\times10^{-5}$$

$\pm4.87\times10^{-5}$

$$|\Re(d_\tau)| < 3.89\times10^{-18} e\cdot cm$$

3.55×10^{-18}

If neutrino can be reconstructed by using impact parameters
Another factor of 4 improvement can be achieved

$$|\Re(d_\tau)| \lesssim 9.73\times10^{-19} e\cdot cm$$

8.88×10^{-19}

Depends on the resolution of impact parameters

Summary and Outlook

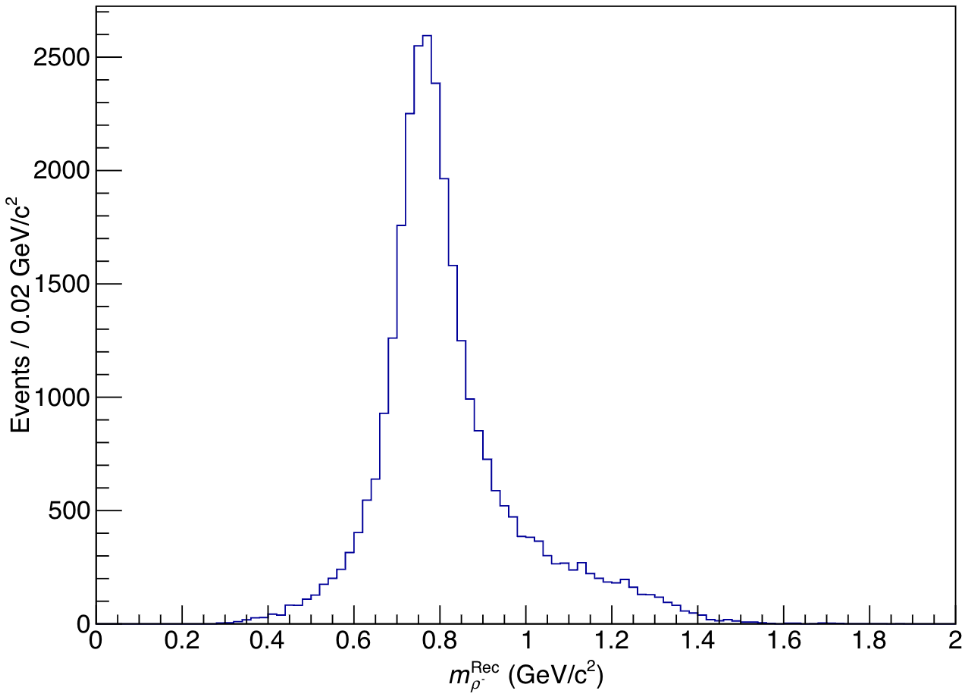
- STCF produce tremendous τ pair events
  - 3.5×10^9 per year
  - From $\rho\rho$ channel: $|\Re(d_\tau)| < 3.89\times10^{-18} e\cdot cm$
- Extension to other channels:
  - $\pi\pi, \pi\rho, \cdots$
- The polarization of the beams
- Reconstruction of the neutrino - vertex detector
  - Removing the ambiguity
    - Factor of 4 improvement - How good do we need for the vertex detection?
- Application to $(g-2)_\tau$

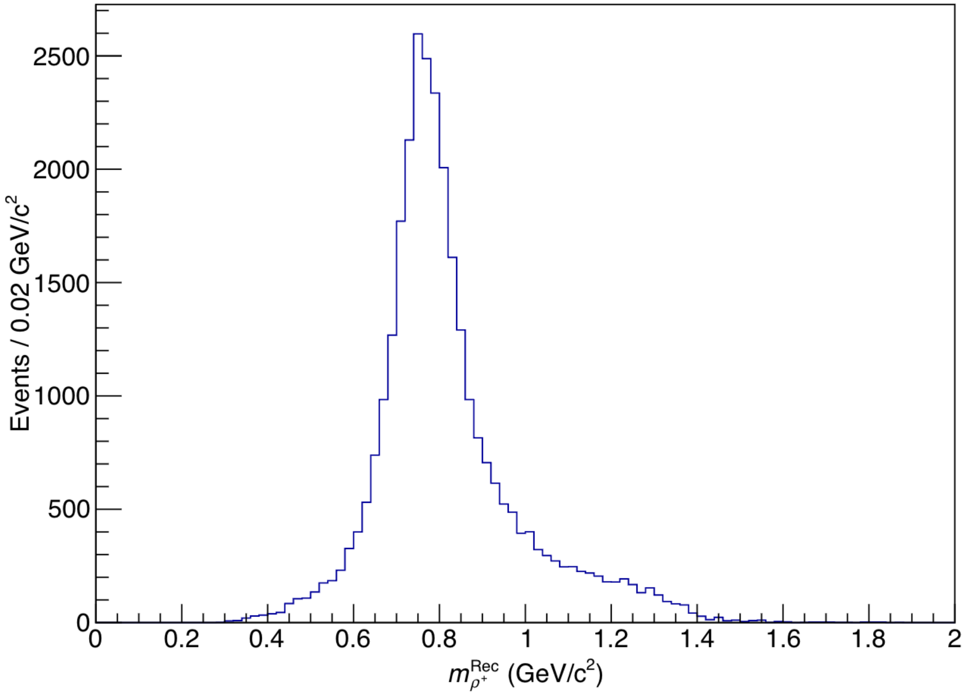
Thanks for your attention!

Backups

Particle Pairing

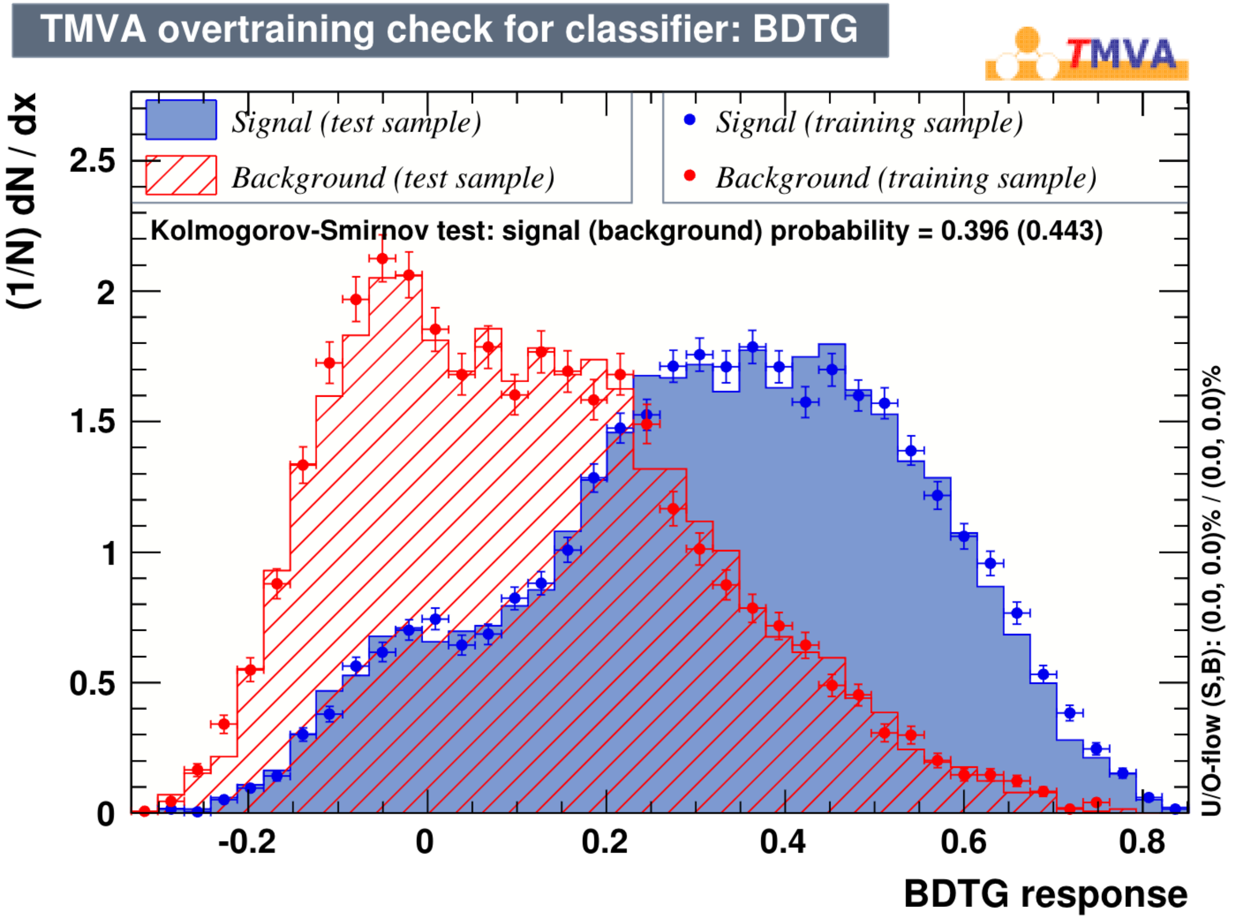
- Pairing the photons with the corresponding charged pions
  - Using kinematic constraints
    - Energy-momentum conservation
    - Mass constraints of the intermediate states

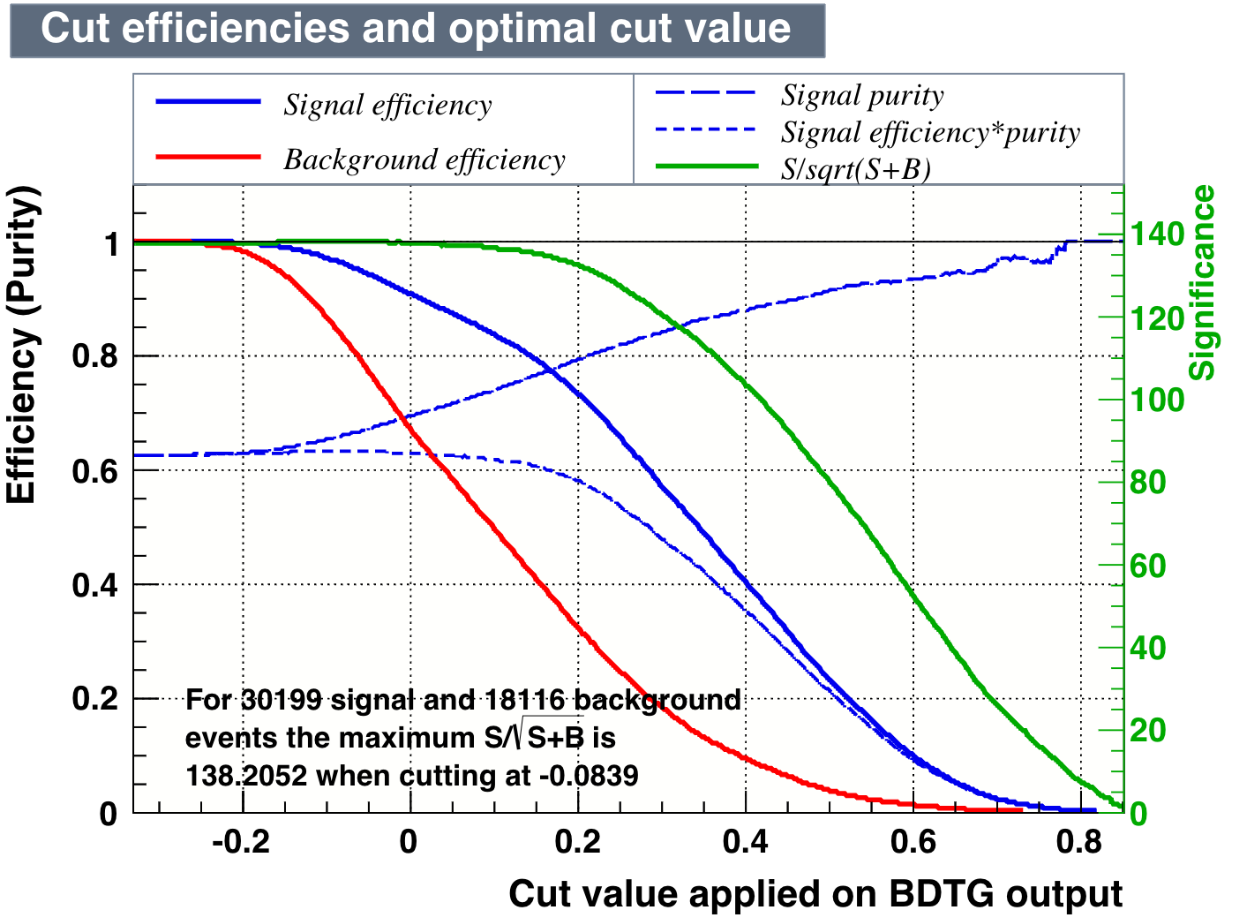




Event-level ML Selection

- BDTG
  - Inputs: momenta of $\pi^{\pm}$ and four photons





Different Decay Channels

- Comparison among different channels

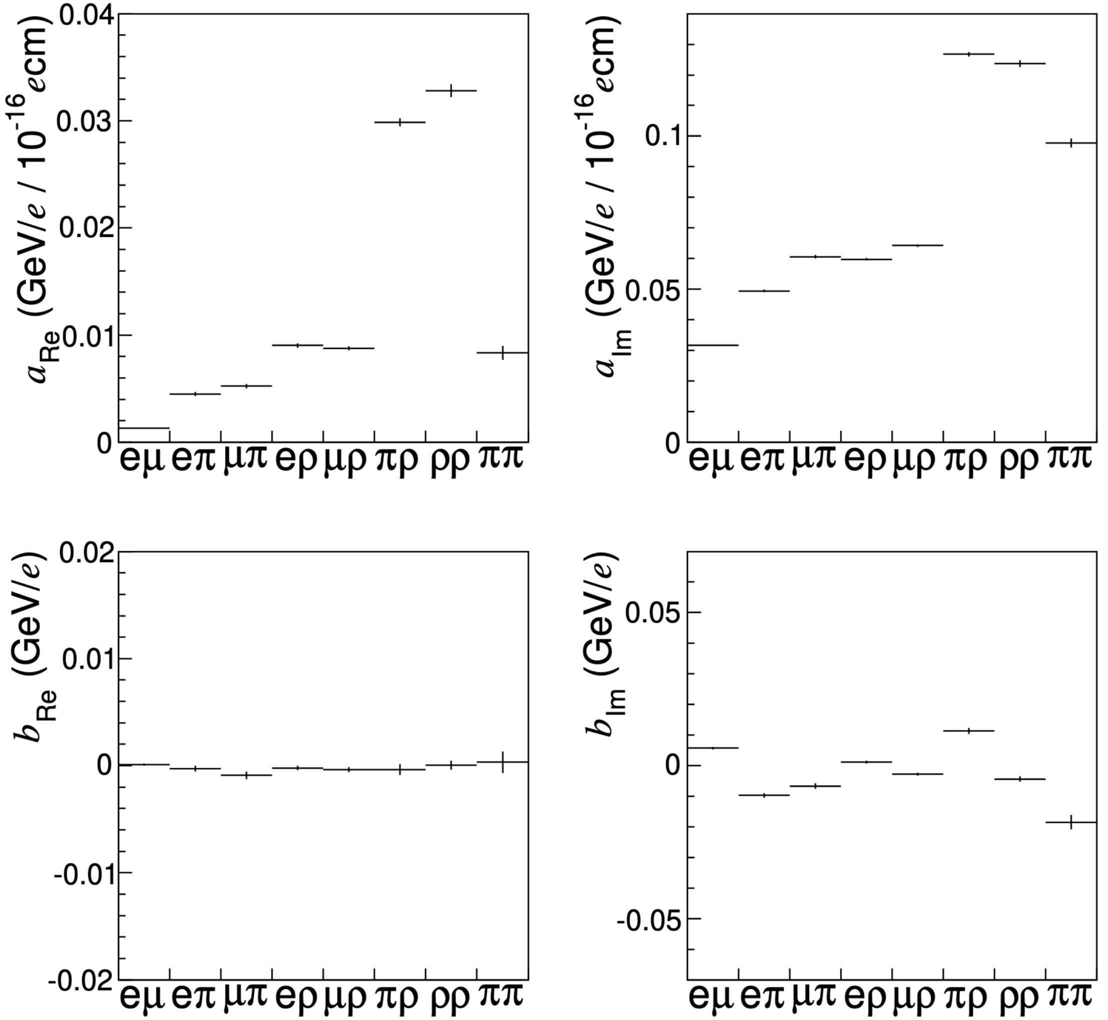
$\tau^{\pm}$ Decay Channels:

$\tau^{\pm} \to \pi^{\pm}\nu$ (~10.8%)

$\tau^{\pm} \to \pi^{\pm}\pi^{0}\nu$ $(\rho^{\pm})$ (~25.4%)

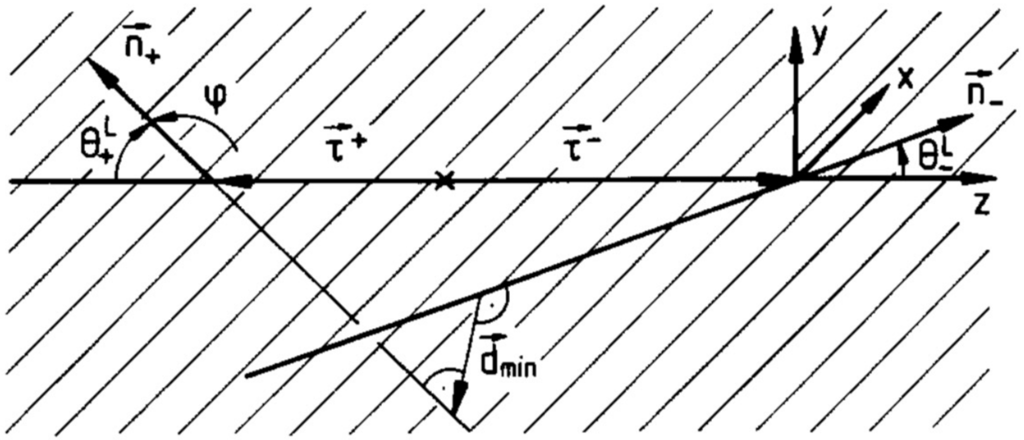
$\tau^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}\nu$ $(a^{\pm})$ (~9.3%)

$\tau^{\pm} \to \ell^{\pm}\nu_{\ell}\nu$ (~17% × 2)



Impact Parameters

- The ambiguity can be resolved by using the impact parameters



Phys. Lett. B 313 (1993) 458

- Advancement in silicon trackers
  - Much better resolutions

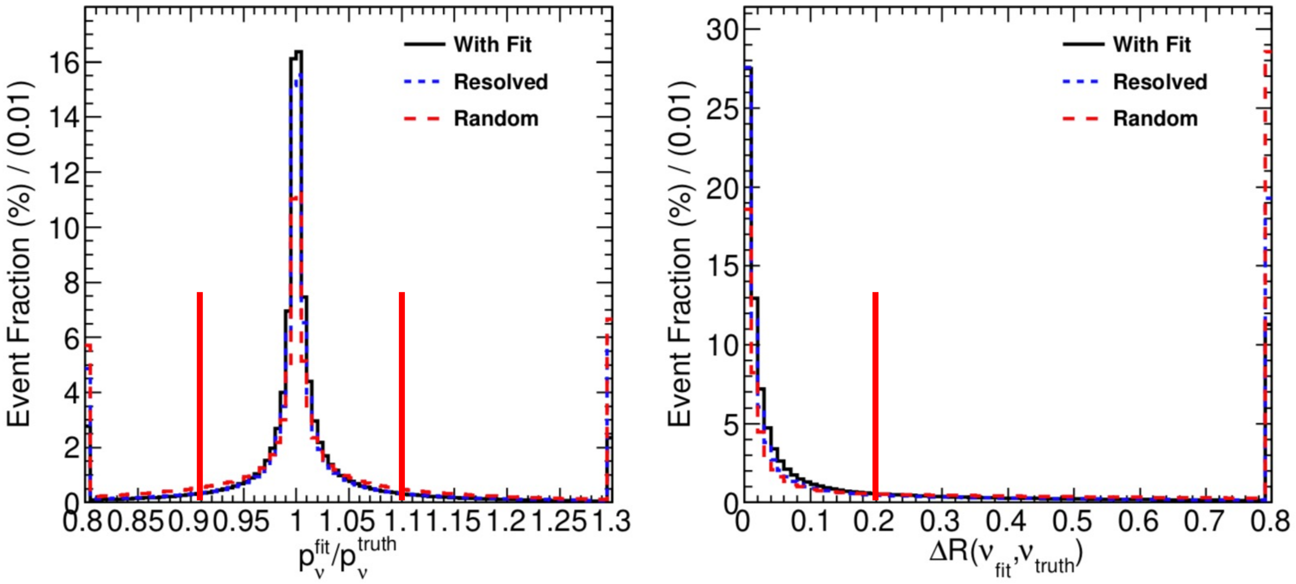
$$\sigma = a \oplus b/(p_T \sin^{1/2}\theta)$$

In mm	a	b/GeV
d_0	0.015	0.007
Belle z_0	0.020	0.010

Belle II TDR - 1011.0352

Performance

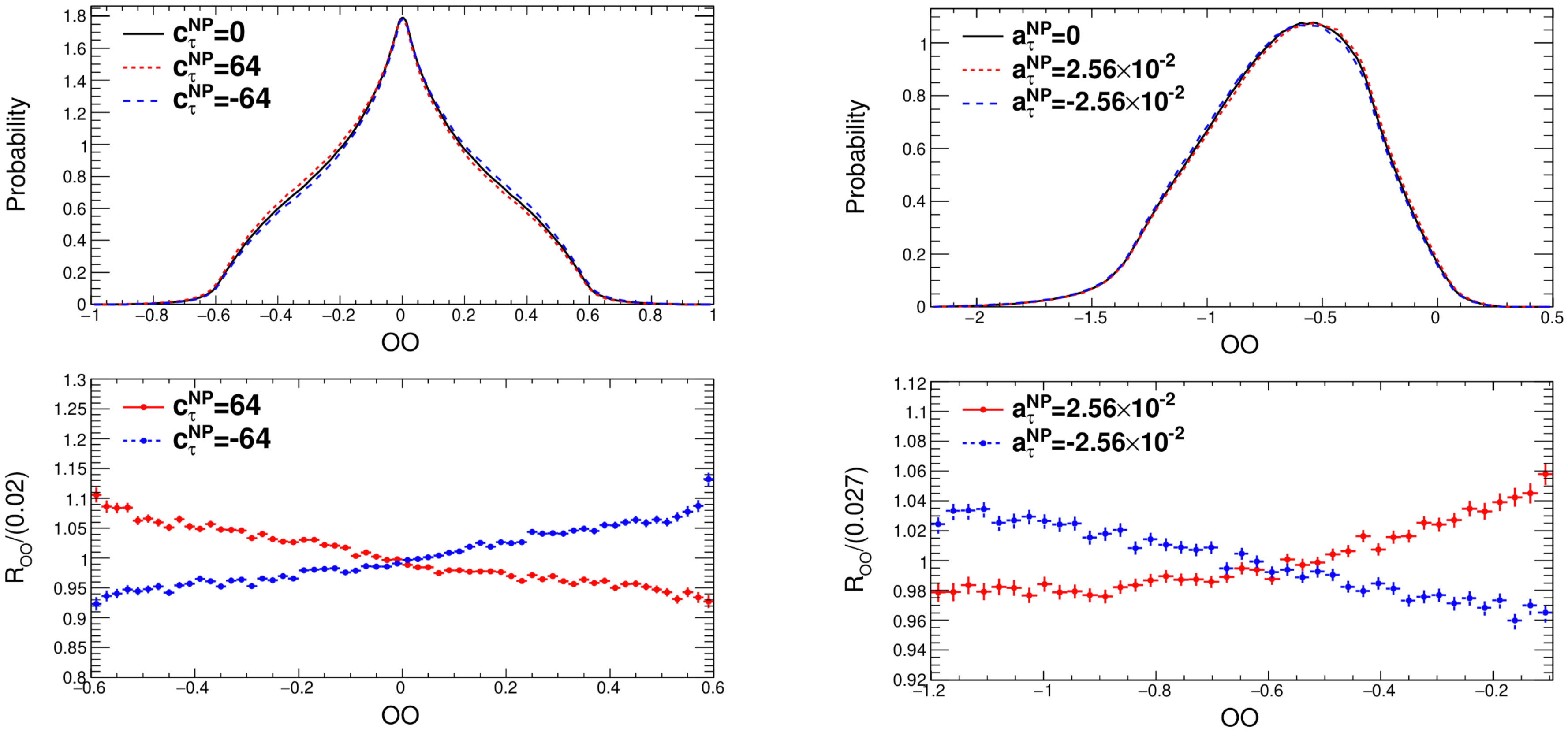
- The tau/neutrino momentum can be reconstructed by minimizing χ^2 event by event
- Compared with two other cases:
  - Random: Choose one of the two solutions randomly (PLB 313, 458)
  - Resolved: no fitting, but using the impact parameter to resolve the two-fold ambiguity



- Fractions of good reconstructed neutrinos: 42% → 59% → 65%

Xin Chen, Y. Wu; JHEP10(2019)089, 1803.00501
Xin Chen, Y. Wu; Eur.Phys.J. C77(2017)697, 1703.04855
Xin Chen, Y. Wu; Phys.Lett.B790(2019)332, 1708.02882

Sensitivities: OO distributions with neutrino reconstructed



At 50 fb^{-1}

- For small c_τ and a_τ, the ratio of the distribution of different value can be parameterized as
  - $R_{OO} = 1 + b(OO - x_0)$

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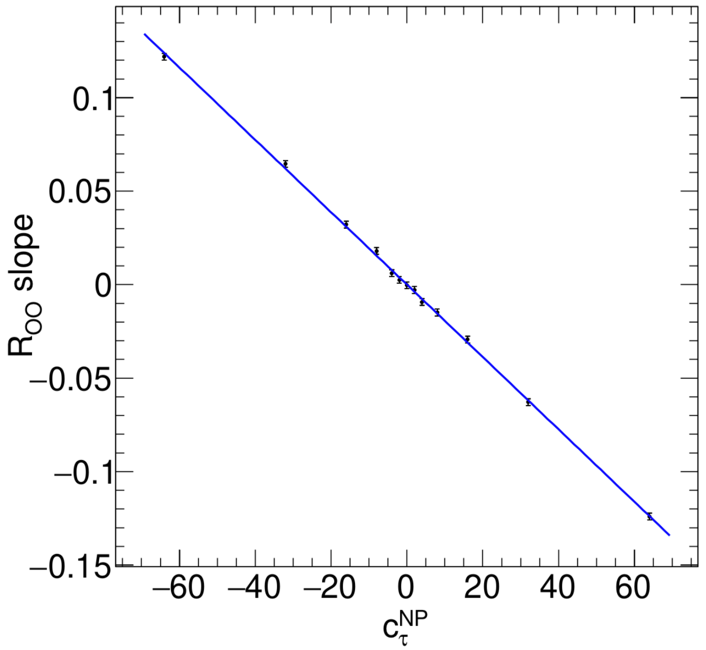
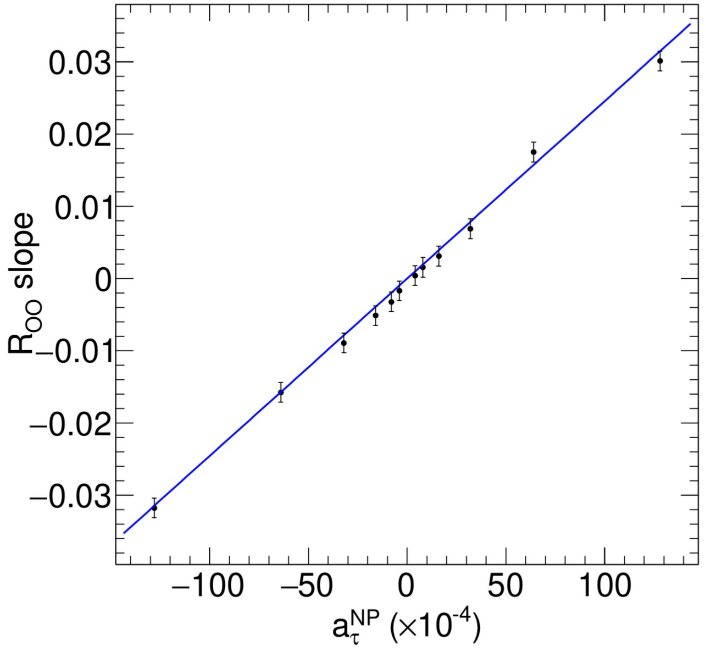
Sensitivities with neutrino reconstructed





$\mathcal{L}$	1 ab^{-1}	10 ab^{-1}	50 ab^{-1}
$\|d_\tau^{NP}\|$ (e·cm)	1.44×10^{-18}	4.56×10^{-19}	2.04×10^{-19}
$\|a_\tau^{NP}\|$	1.24×10^{-4}	3.92×10^{-5}	1.75×10^{-5}

$a_\tau^{SM} = 0.00117721(5)$

Comparison with Belle's method is also performed, factor of 4 better

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