scib.2025.05.045

Baryon ~~CPV~~ at colliders

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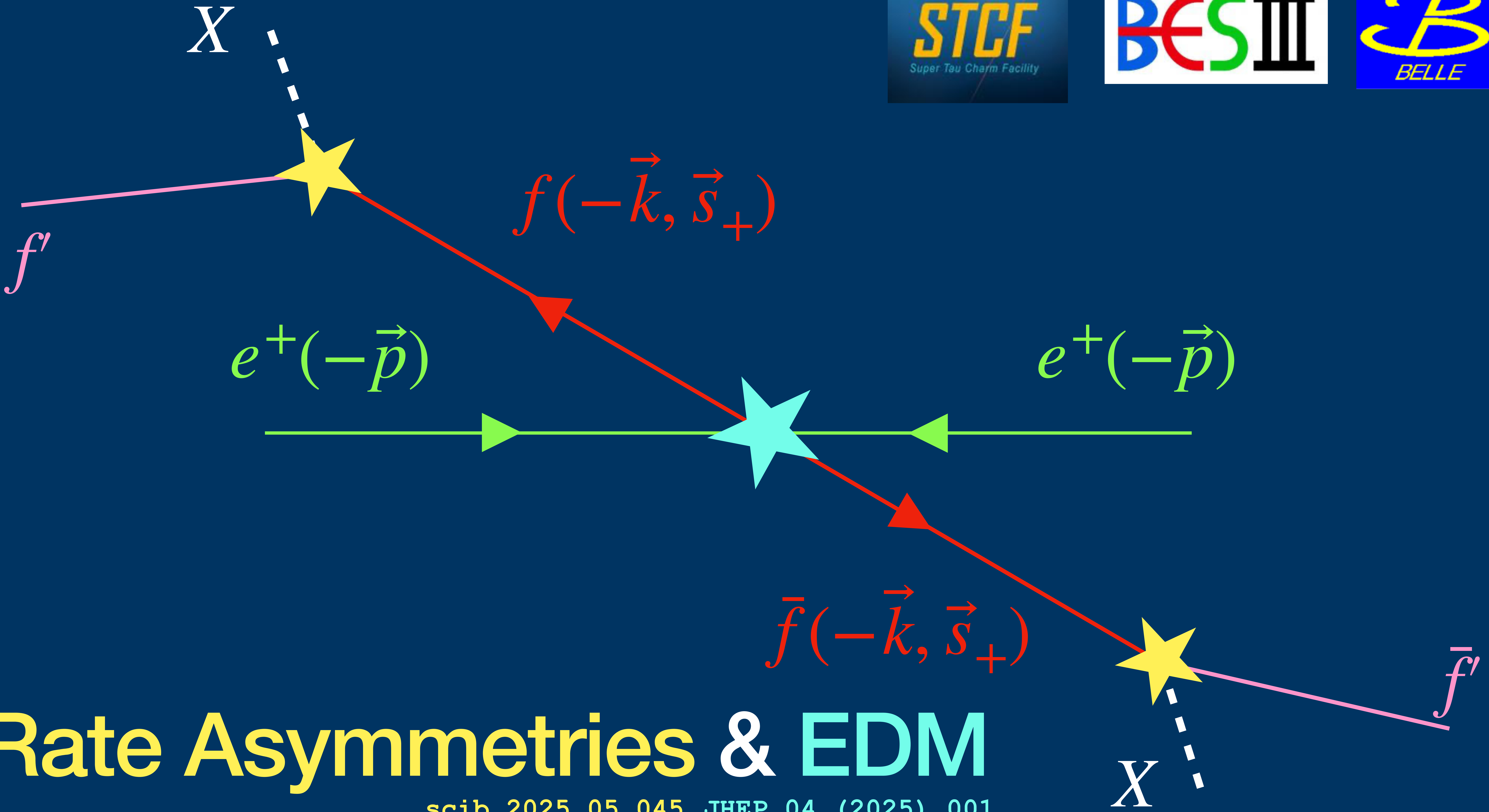


Rate Asymmetries & EDM

scib.2025.05.045 JHEP 04 (2025) 001

Charming physics - CP violation

$$a_{CP}(D^0 \to K^+K^-) - a_{CP}(D^0 \to \pi^+\pi^-) = (-1.54 \pm 0.29) \times 10^{-3}$$

$$a_{CP}^{KK} = (7.7 \pm 5.7) \times 10^{-4}, \quad a_{CP}^{\pi\pi} = (23.2 \pm 6.1) \times 10^{-4}$$

PRL **122**, 211803 (2019); PRL **131**, 091802 (2023)

- Short distance predictions are **an order smaller**!
  Data driven approach:

  Factorization with fitted hadron matrix element.

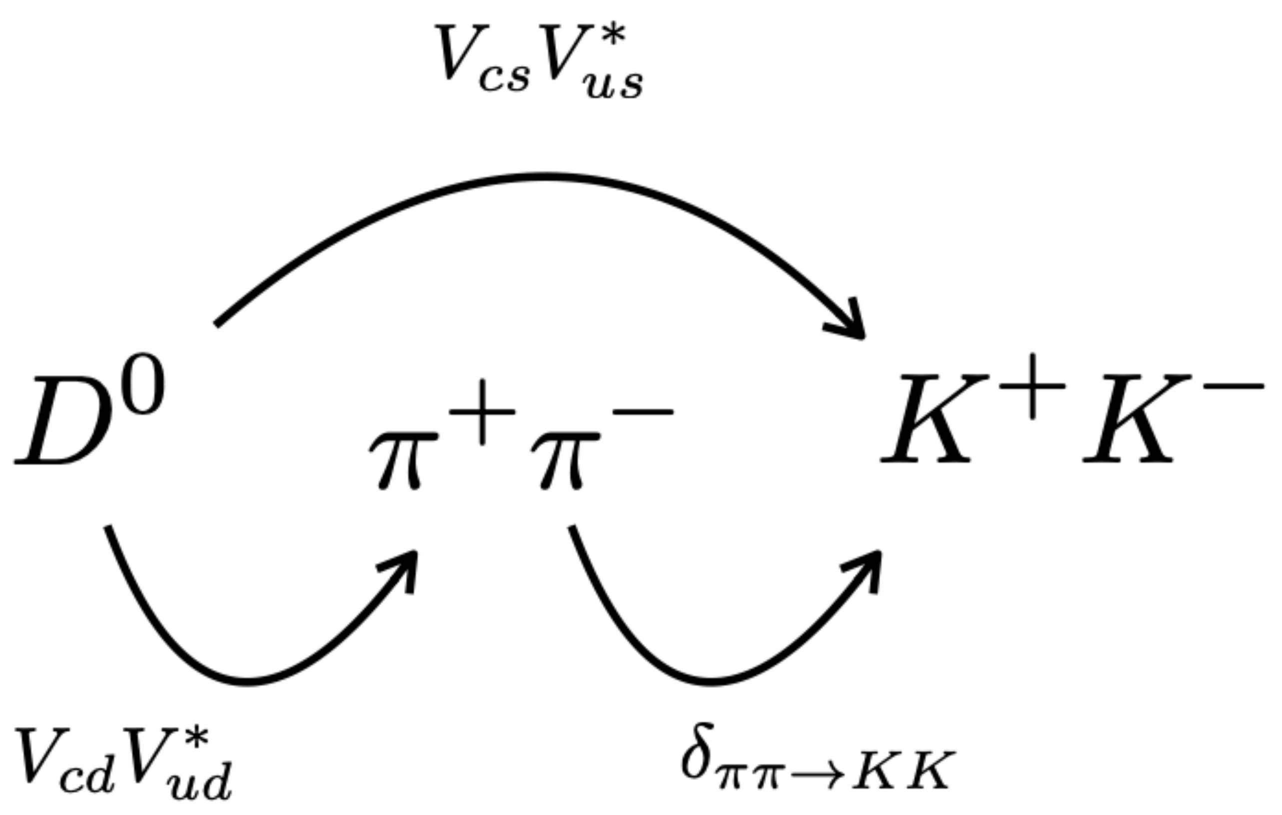
  PRD **86**, 036012 (2012).

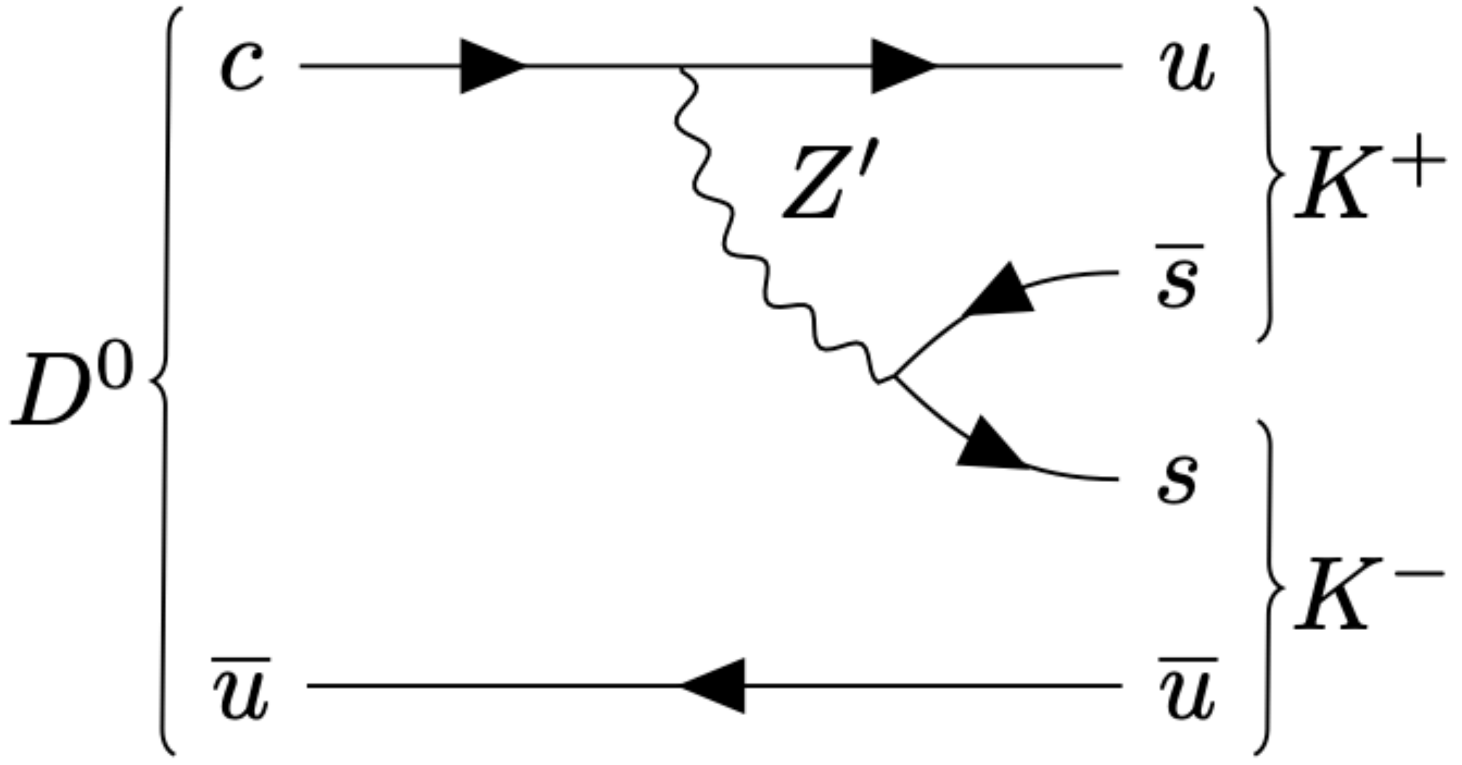
  Use the relations of final state interactions; $P^{LD} = E$.

  PRD **86**, 014014 (2012); PRD **109**, 073008 (2024).

  Consider the re-scattering of $\pi\pi \to KK$.

  PRL **131**, 051802 (2023).





PRD **108**, 035005 (2023)

- SM ***naively*** predicts $a_{CP}^{\pi\pi} = -a_{CP}^{KK}$ but data found opposite!

• Charming physics - CP violation

Reasons to go **beyond** charmed mesons:

$$a_{CP}^{KK} = (7.7 \pm 5.7) \times 10^{-4}, \quad a_{CP}^{\pi\pi} = (23.2 \pm 6.1) \times 10^{-4}$$

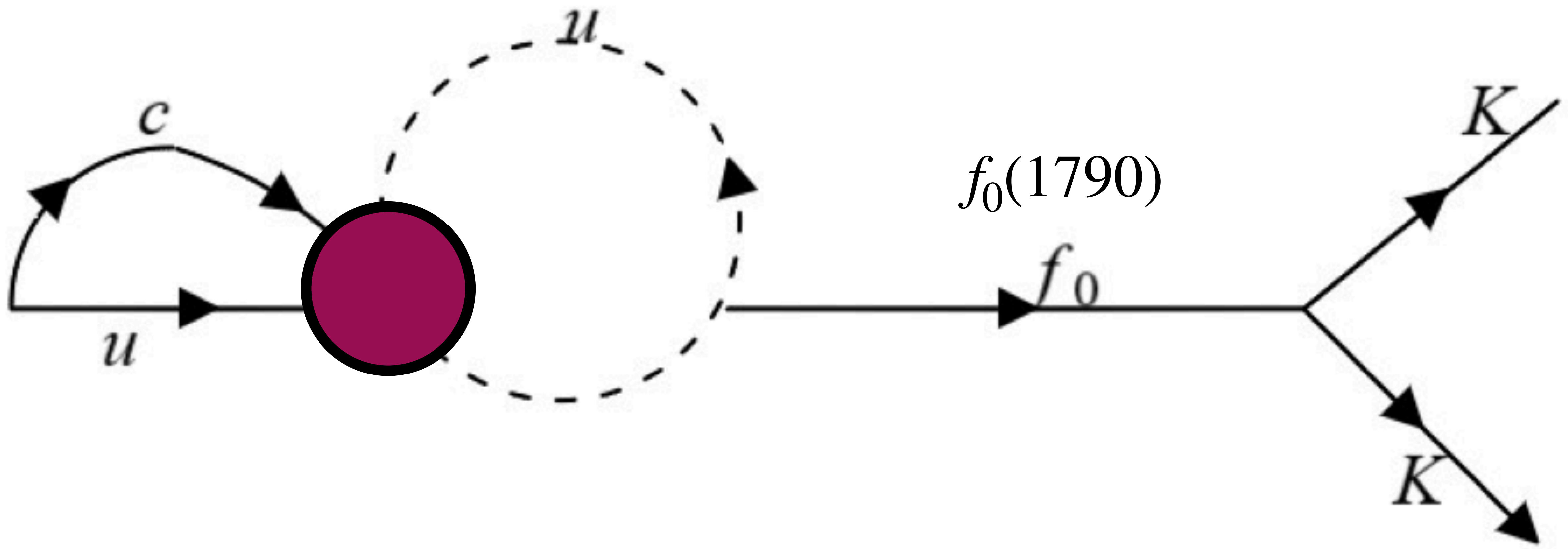
PHYSICAL REVIEW D **81,** 074021 (2010)

Two-body hadronic charmed meson decays

Hai-Yang Cheng[1,2] and Cheng-Wei Chiang[1,3]

Enhancement of charm CP violation due to nearby resonances

Stefan Schacht[a,*], Amarjit Soni[b] PLB **825,** 136855 (2022)



1. f_0 might be a **glueball** which mainly decays to kaons. Leading order amplitude $\propto m_s$.

2. Its mass is too close to D meson, enhancing **SU(3) breaking** effects from mass splitting.

3. Unlike $D^0 \to h^+h^-$, CP-even **phase shifts** in baryon decays can be directly measured.

For D CPV see: PRD **86,** 036012 (2012); PRD **86,** 014014 (2012); PRD **109,** 073008 (2024); PRL **131,** 051802 (2023).

Experimental status of charmed baryon decays

2023: The ***first*** measurement of CP violation in charmed baryon two-body decays

Sci. Bull. **68**, 583-592 (2023)

$$A_{CP}(\Lambda_c^+ \to \Lambda K^+) = 0.021 \pm 0.026$$

* The most precise CP asymmetries in branching fractions by far in charmed baryons.

2024: Measurements of the ***strong phase*** in $\Lambda_c^+ \to \Xi^0 K^+$

PRL **132**, 031801 (2024)

$$\delta_P - \delta_S = -1.55 \pm 0.27 (+\pi) \,, \quad \alpha = 0.01 \pm 0.16$$

* CP even and Cabibbo-favored, but very important to studies of ***CP violation***!

2024: Measurements of ***strong phases*** in $\Lambda_c^+ \to \Lambda\pi^+, \Lambda K^+$

PRL **133**, 261804 (2024)

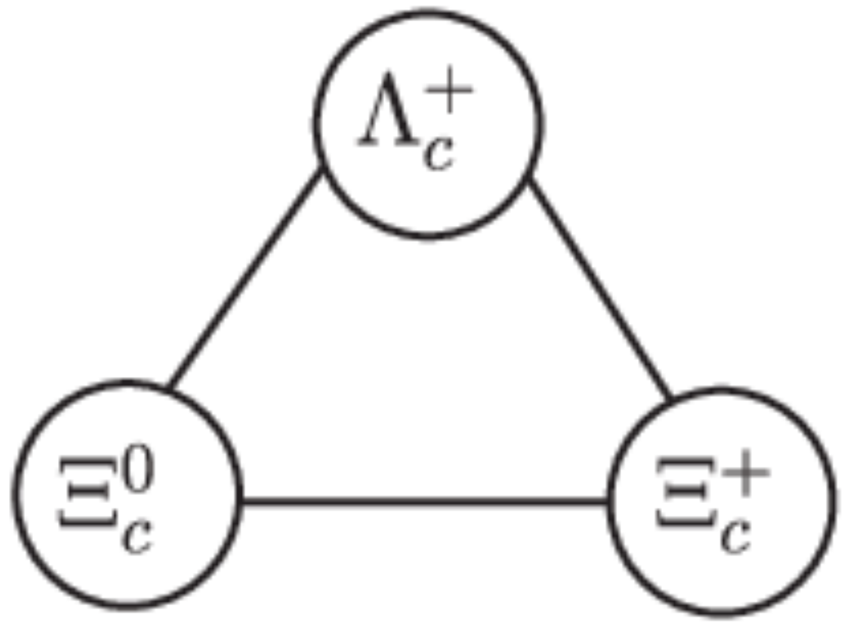
$$(\beta_\pi, \beta_K) = (0.368 \pm 0.019 \pm 0.008 \,, \, 0.35 \pm 0.12 \pm 0.04) \,.$$

* Confirmed the discovery of large strong phases in charmed baryon decays.

SU(3) flavor perspective of charmed baryon decays

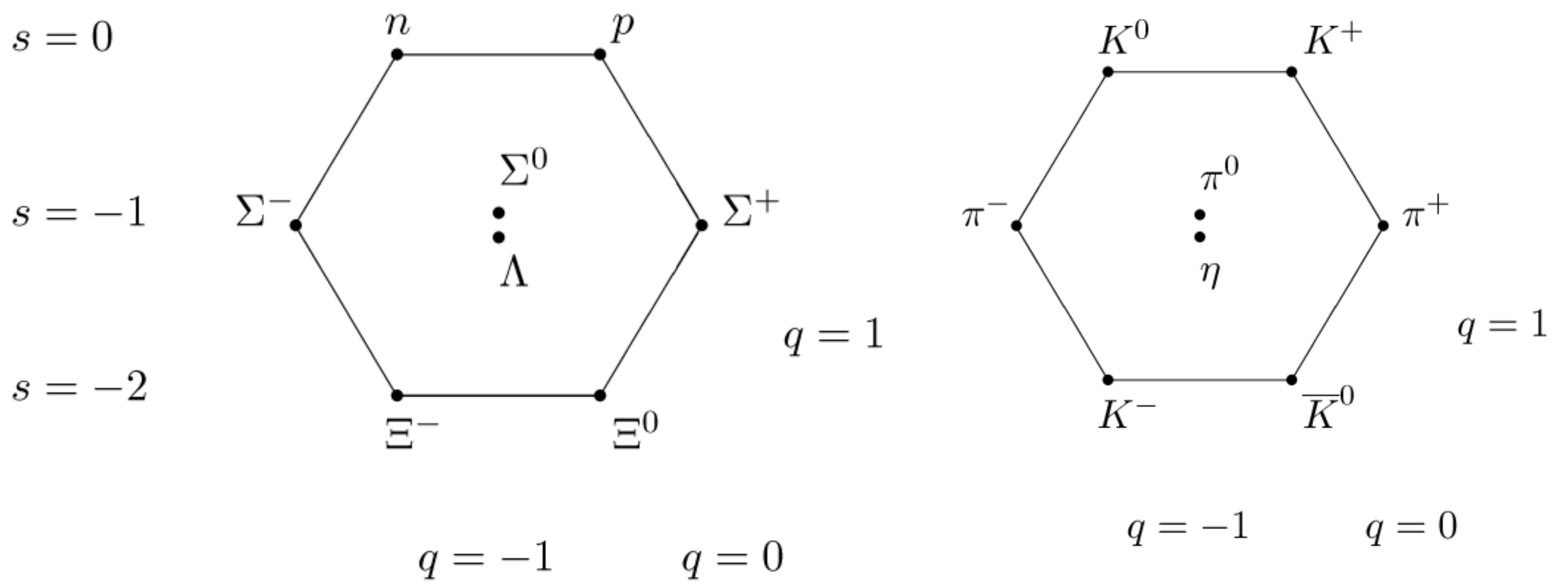
By far, the only ***reliable*** **(?)** way is the $SU(3)_F$ **symmetry**.

PRD **93**, 056008 (2016), NPB **956**, 115048 (2020)
JHEP **09**, 035 (2022), JHEP **03**, 143 (2022) …



Weak interactions

$\mathbf{B}_c \to \mathbf{B}P$



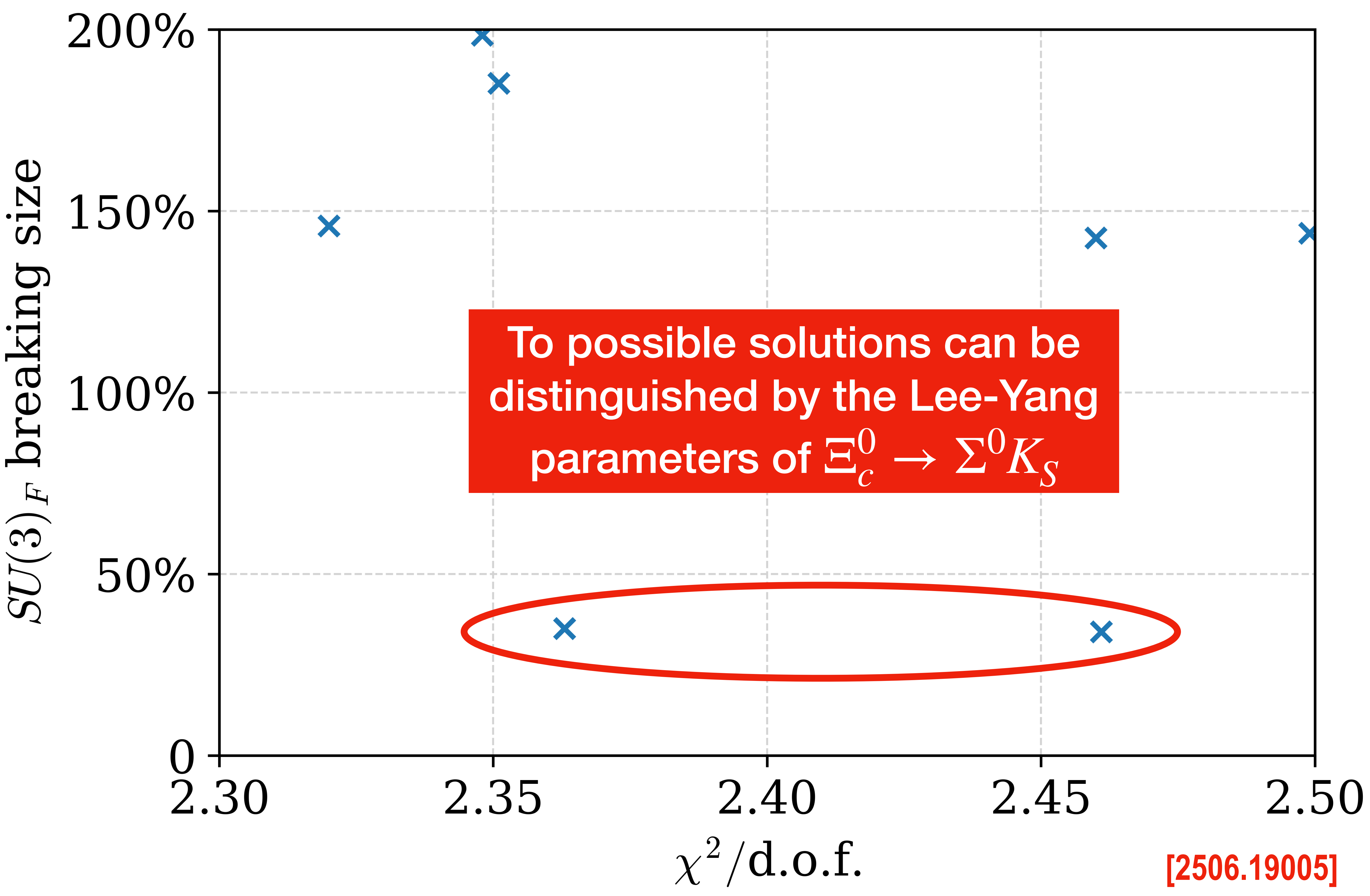
	PDG (2023)	Theory (2023)	Data (2024)	
$\alpha(\Lambda_c^+ \to pK_S^0)$	0.18 ± 0.45	-0.40 ± 0.49	-0.744 ± 0.015	LHCb
$10^4\mathscr{B}(\Lambda_c^+ \to p\pi^0)$	< 0.8	1.6 ± 0.2	1.79 ± 0.41	BESIII
$10^3\mathscr{B}(\Lambda_c^+ \to \Lambda K_S^0\pi^+)$	None	1.97 ± 0.38	1.73 ± 0.28	BESIII
$10^3\mathscr{B}(\Xi_c^0 \to \Xi^0\eta)$	None	2.94 ± 0.97	1.6 ± 0.5	BELLE
$10^3\mathscr{B}(\Xi_c^0 \to \Xi^0\eta')$	None	5.66 ± 0.93	1.2 ± 0.4	BELLE

There are some ***shortcomings*** in $SU(3)_F$ symmetry approach.

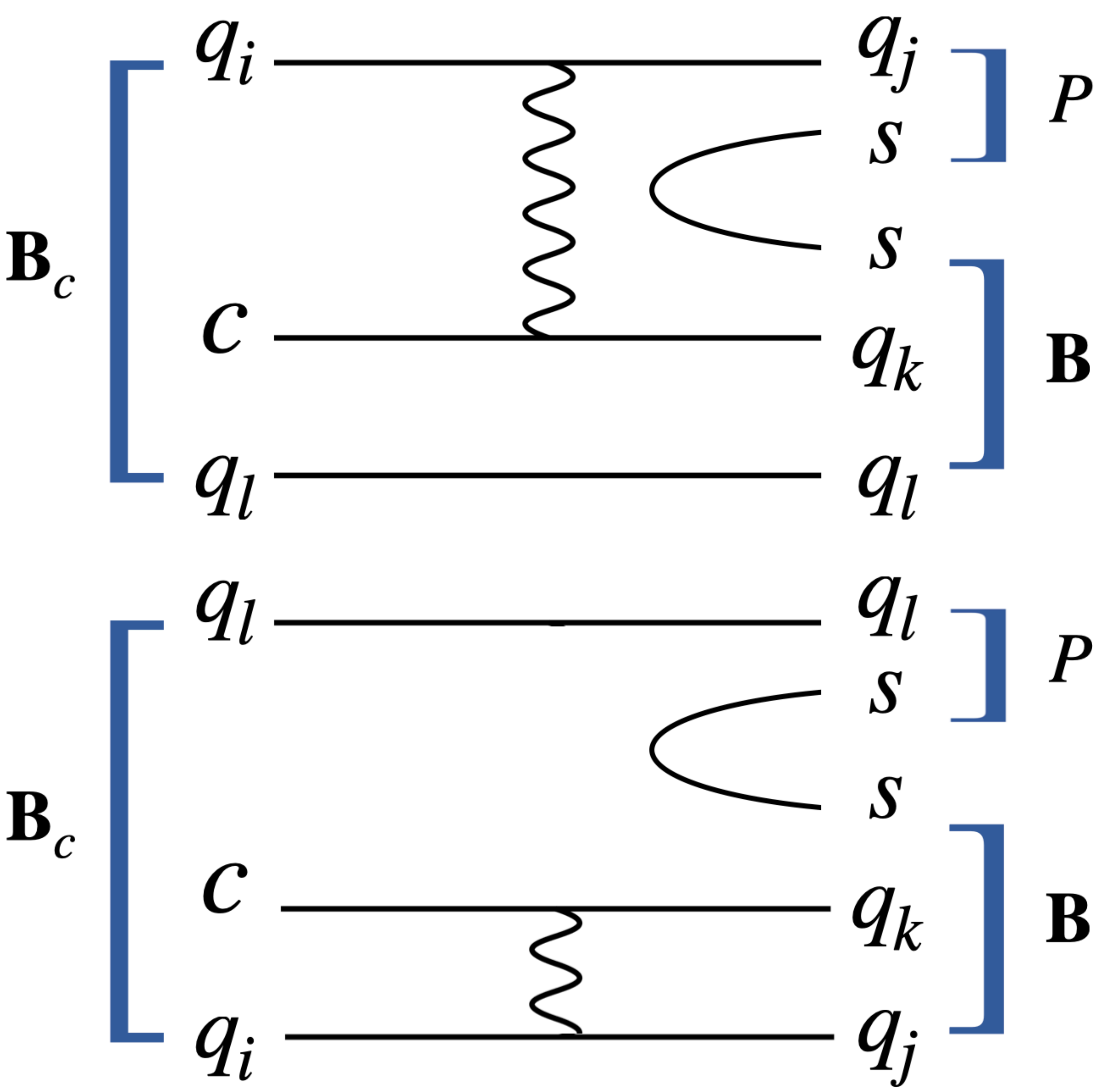
• SU(3) flavor perspective of charmed baryon decays

The $SU(3)_F$ is an approximate symmetry with **errors** in 10^{-1}.

We propose a new scenario that incorporates the $SU(3)_F$ **breaking** of strange quark pair production from the vacuum.



[2506.19005]



$m_s^{-2} : m_d^{-2} \approx 1 : 2$

(constituent quark masses)

SU(3) flavor perspective of charmed baryon decays

The large χ^2 is mainly contributed by two channels:

	PDG	$SU(3)_F$ conserved	[2506.19005] $SU(3)_F$ broken
$10^2\mathscr{B}(\Xi_c^0 \to \Xi^-\pi^+)$	1.43 ± 0.32	2.72 ± 0.09	2.9 ± 0.1
$10^2\mathscr{B}(\Xi_c^+ \to \Xi^-\pi^+\pi^+)$	2.9 ± 1.3	6.82 ± 0.36	6.0 ± 0.4

Both of them are the normalized channels in $\Xi_c^{0,+}$, indicating an possible **underestimation of factor two** in the experimental side.

Same **underestimations** occurs in $\Xi_c^0 \to \Xi^-\ell^+\nu_\ell$.

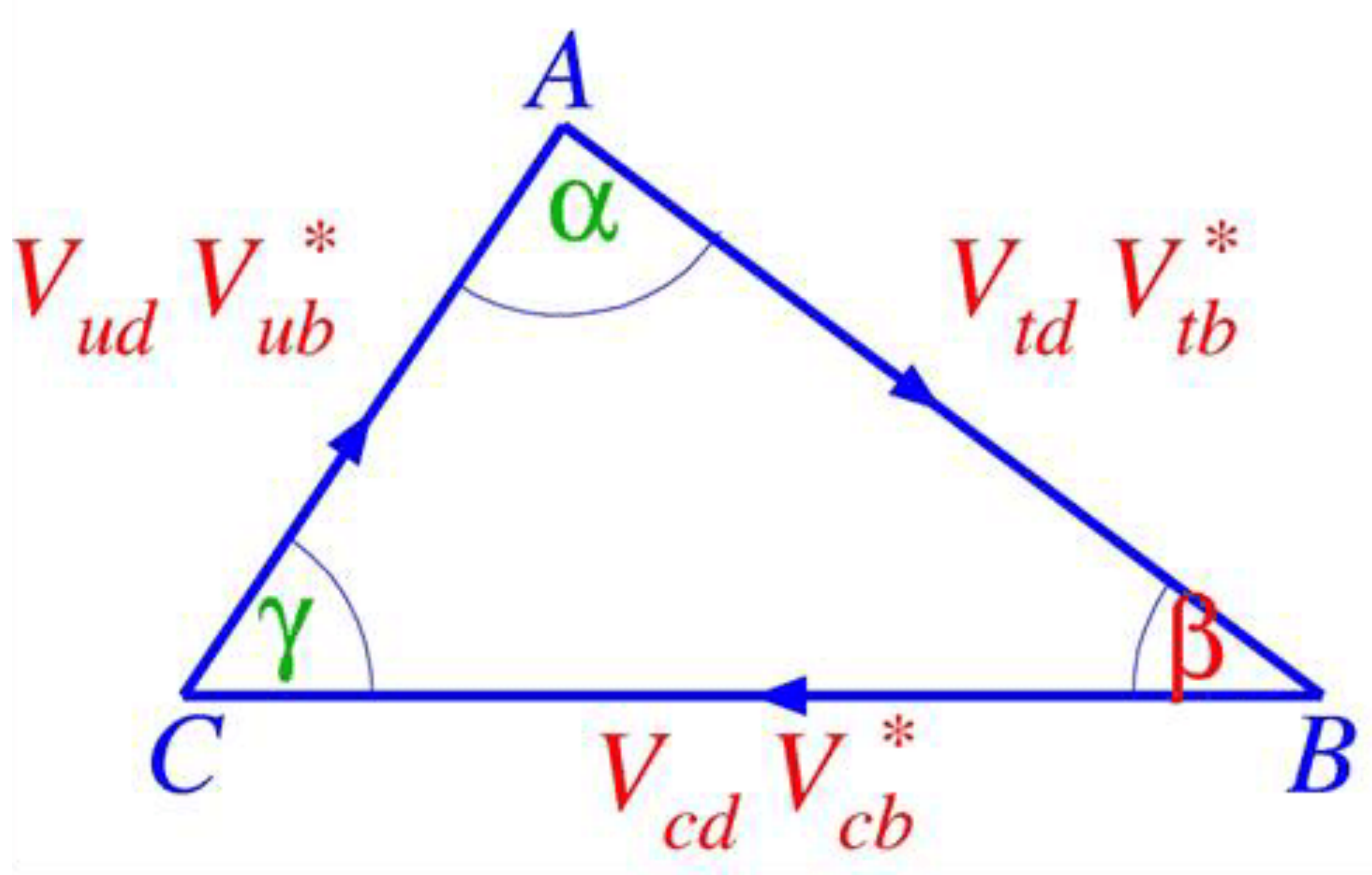
	PDG	$SU(3)_F$	Lattice	Lattice
$10^2\mathscr{B}(\Xi_c^0 \to \Xi^-e^+\nu_e)$	1.05 ± 0.20 2.12 ± 0.13*	4.10 ± 0.46	2.38 ± 0.44	3.58 ± 0.12
$10^2\mathscr{B}(\Xi_c^0 \to \Xi^-\mu^+\nu_\mu)$	1.02 ± 0.21 2.05 ± 0.19*	3.98 ± 0.57	2.29 ± 0.42	3.47 ± 0.12
		[2110.04179]	[2103.07064]	[2504.07302]

*Using $\mathscr{B}(\Xi_c^0 \to \Xi^-\pi^+) = (2.9 \pm 0.1)\,\%$

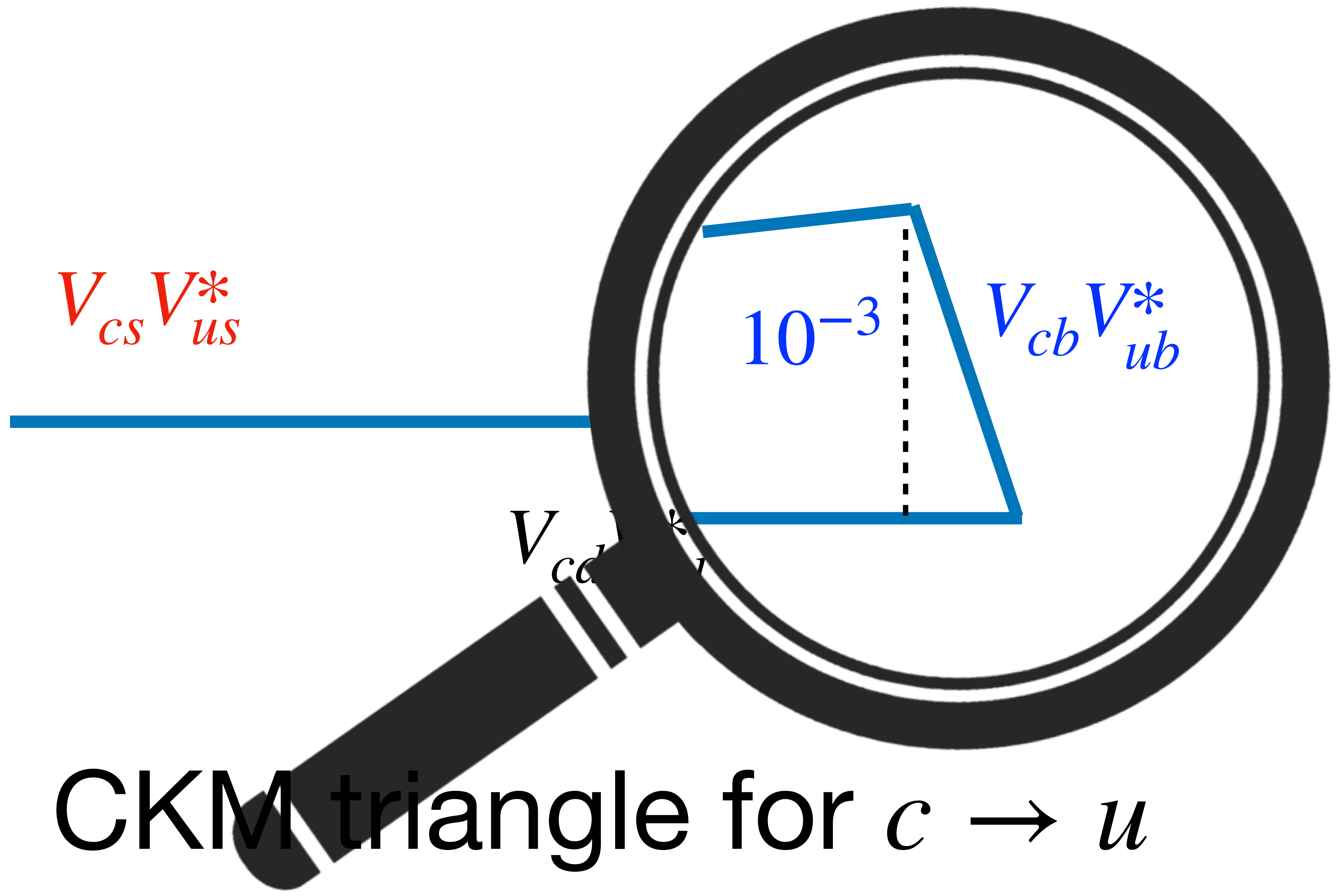
• SU(3) flavor perspective of charmed baryon decays

$$\text{Amplitude}: V_{cs}V_{us}^{*} \underbrace{F^{s-d}}_{\text{4 parameters}} + V_{cb}V_{ub}^{*} \underbrace{F^{b}}_{\text{3 parameters}}$$

Do not need to consider F^b in studying CP-even quantities. ⟷ F^b cannot be determined with CP-even quantities.



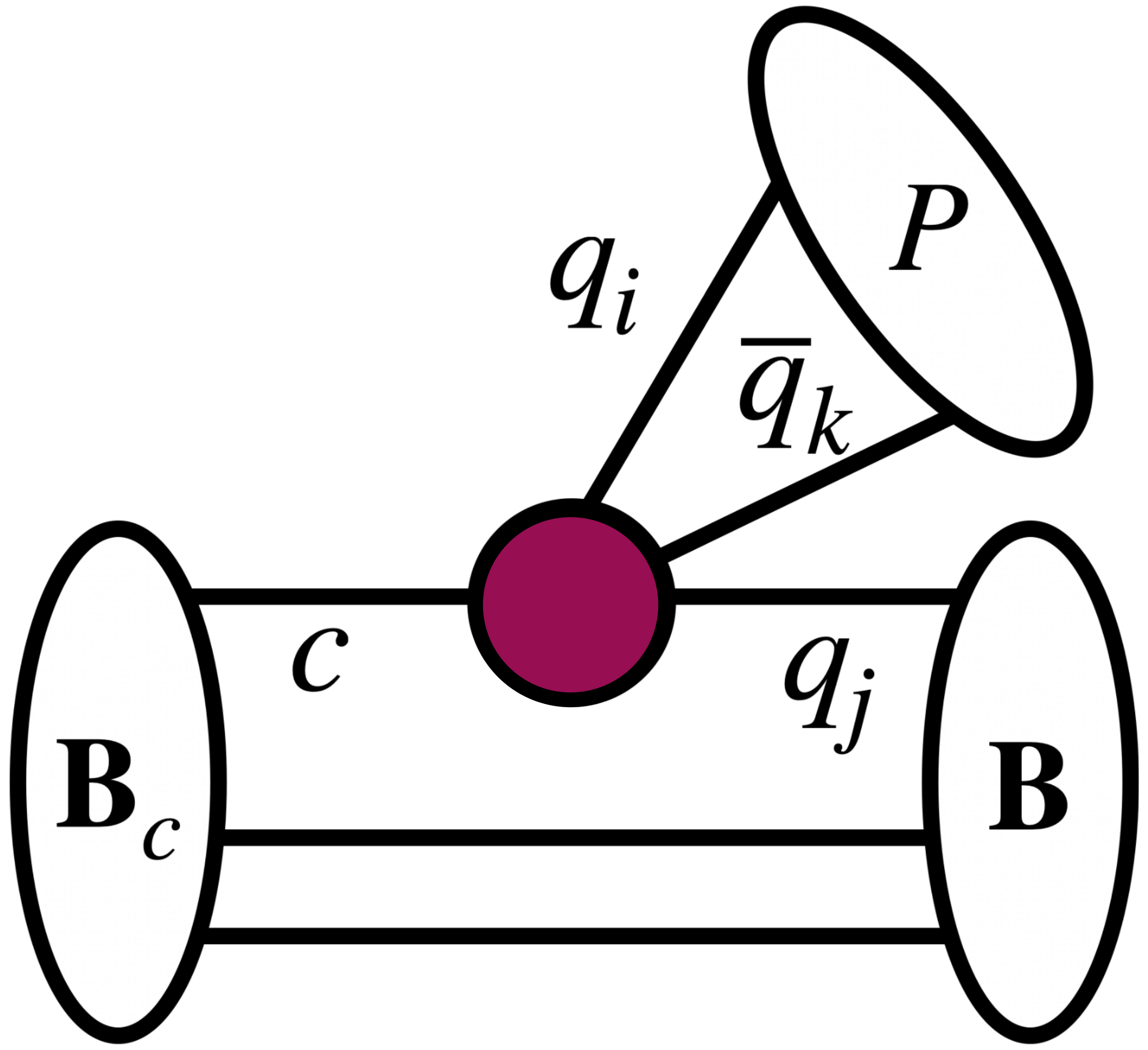
CKM triangle for $b \to d$

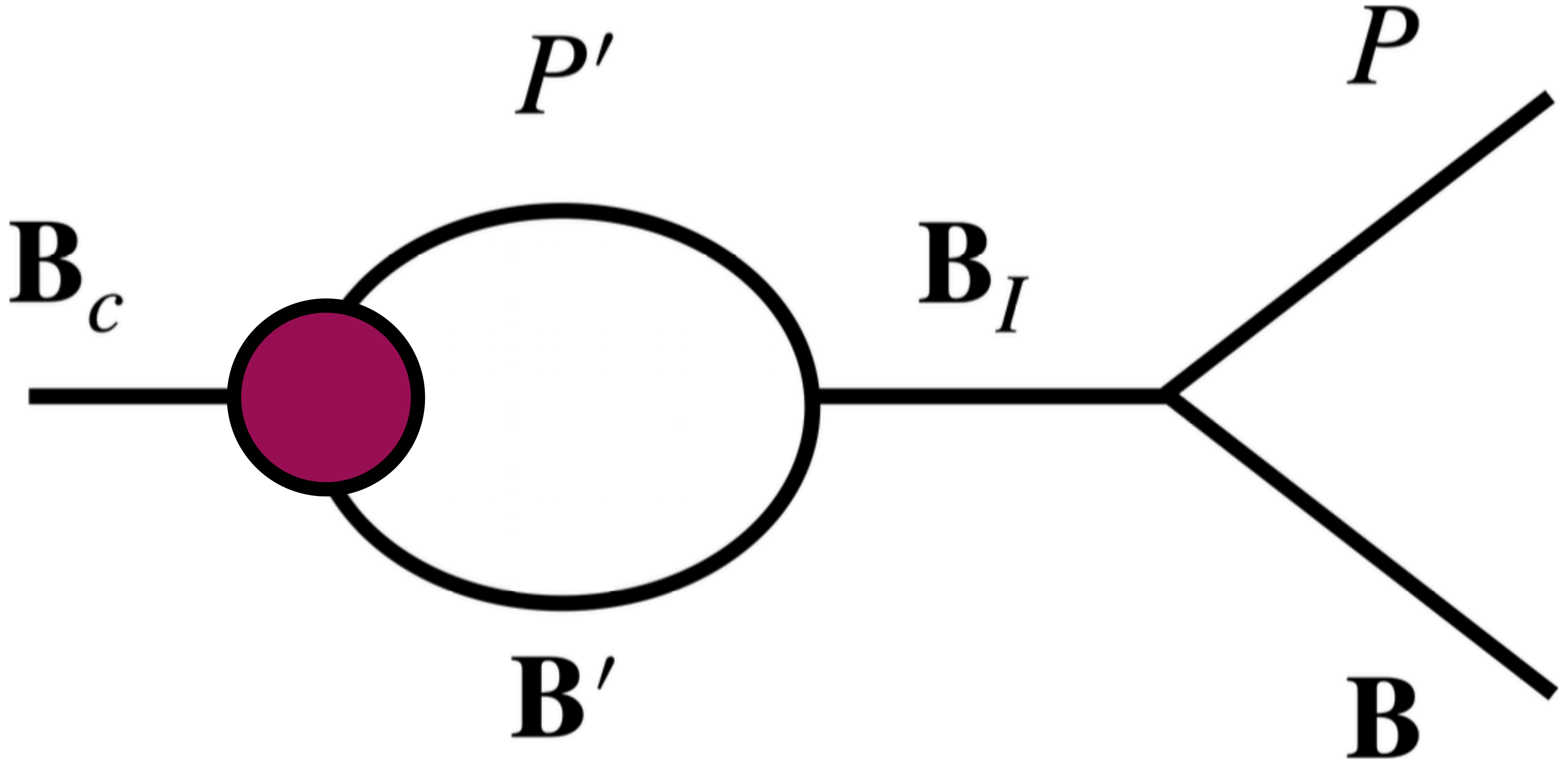


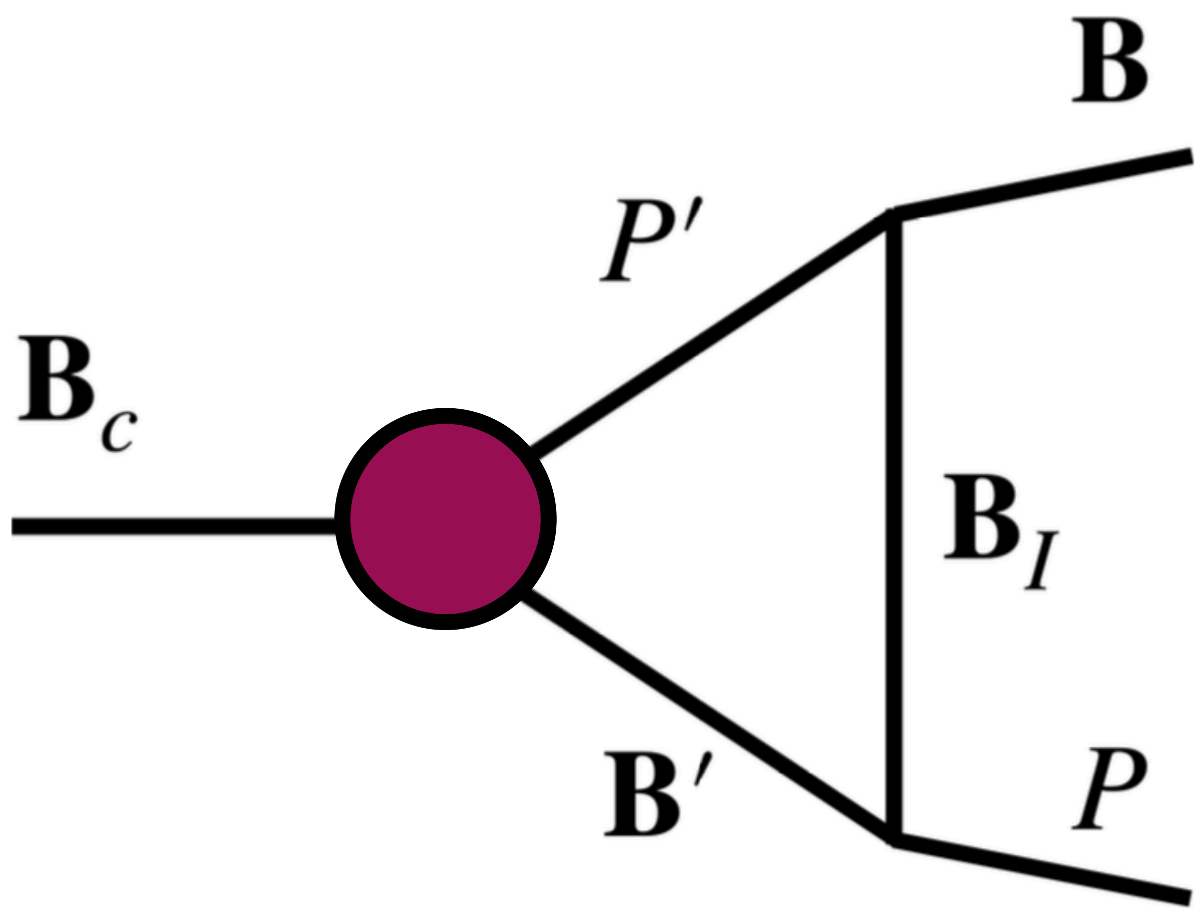
CKM triangle for $c \to u$

Rescattering, solving penguin/tree

$$\mathcal{L}_{\mathbf{B}_c\mathbf{B}P} = \mathcal{L}^{\text{Tree}}_{\mathbf{B}_c\mathbf{B}P} + \mathcal{L}^{\text{FSR}-\text{s}}_{\mathbf{B}_c\mathbf{B}P} + \mathcal{L}^{\text{FSR}-\text{t}}_{\mathbf{B}_c\mathbf{B}P}$$







Induce two parameters:

$F_V^{\pm}$, including effective color number and form factors.

Induce one parameter:

$\tilde{S}^-$, containing the q^2 dependencies of couplings.

Induce one parameter:

$\tilde{T}^-$, containing the q^2 dependencies of couplings.

Described by 4 complex parameters, having the same number of parameters with the $SU(3)_F$ analysis !

Rescattering, numerical results

The sizes of CP violation are of the order $\mathcal{O}(10^{-4})$, in accordance with naive expectations.

Channels	$\mathcal{B}(10^{-3})$	$A_{CP}(10^{-3})$	$\alpha_{CP}(10^{-3})$
$\Lambda_c^+ \to \Sigma^+ K_S$	0.37(3)	0.29(3)	−0.22(4)
$\Lambda_c^+ \to \Sigma^0 K^+$	0.37(3)	0.29(3)	−0.22(4)
$\Lambda_c^+ \to p\pi^0$	0.20(3)	0.97(28)	0.99(13)
$\Lambda_c^+ \to n\pi^+$	0.72(7)	−0.21(13)	−0.43(16)
$\Lambda_c^+ \to \Lambda^0 K^+$	0.66(3)	−0.42(12)	0.29(8)
$\Xi_c^+ \to \Sigma^+ \pi^0$	2.34(13)	0.45(6)	−0.02(10)
$\Xi_c^+ \to \Sigma^0 \pi^+$	2.34(18)	0.28(6)	−0.38(10)
$\Xi_c^+ \to \Xi^0 K^+$	1.20(18)	1.11(17)	−0.08(22)
$\Xi_c^+ \to p K_S$	1.61(9)	−0.23(2)	0.19(3)
$\Xi_c^+ \to \Lambda^0 \pi^+$	0.95(12)	−0.35(5)	0.22(6)

Channels	$\mathcal{B}(10^{-3})$	$A_{CP}(10^{-3})$	$\alpha_{CP}(10^{-3})$
$\Xi_c^0 \to \Sigma^+ \pi^-$	0.23(2)	1.78(25)	−0.45(46)
$\Xi_c^0 \to \Sigma^0 \pi^0$	0.46(2)	0.36(9)	−0.39(7)
$\Xi_c^0 \to \Sigma^- \pi^+$	1.62(6)	0.36(3)	−0.09(3)
$\Xi_c^0 \to \Xi^0 K_S$	0.33(5)	0.31(7)	0.43(6)
$\Xi_c^0 \to \Xi^- K^+$	0.86(8)	−0.46(4)	0.03(3)
$\Xi_c^0 \to p K^-$	0.27(3)	−1.50(25)	0.66(39)
$\Xi_c^0 \to n K_S$	0.50(2)	−0.43(4)	0.28(8)
$\Xi_c^0 \to \Lambda^0 \pi^0$	0.16(2)	−0.02(5)	0.40(4)

Large CP violation is found !

$$A_{CP} = \frac{\Gamma - \overline{\Gamma}}{\Gamma + \overline{\Gamma}}, \qquad \alpha_{CP} = \frac{1}{2}(\alpha + \overline{\alpha}).$$

Rescattering, numerical results

- A_{CP} in the same size with the ones in *D* meson!

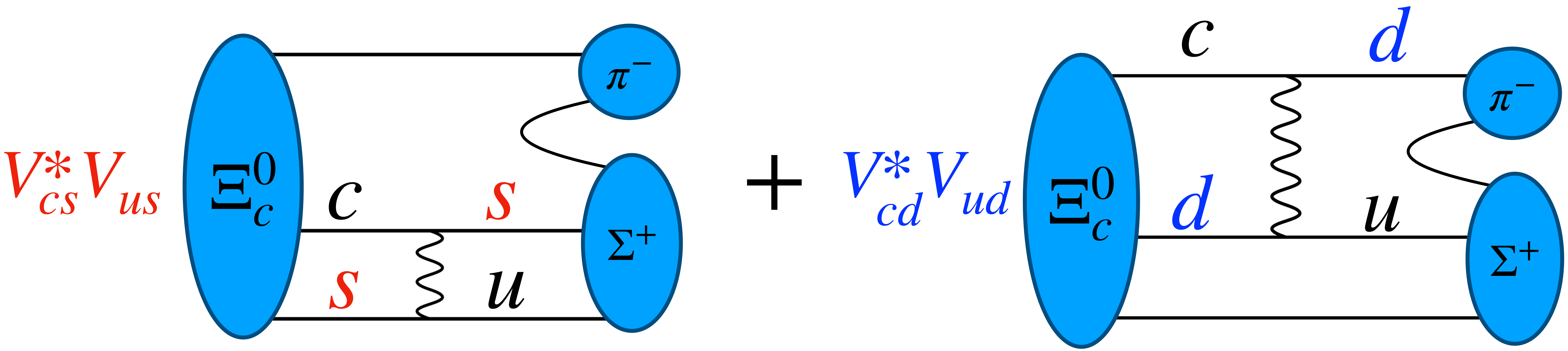
$$A_{CP}\left(\Xi_c^0 \to \Sigma^+\pi^-\right) = (1.78 \pm 0.25) \times 10^{-3}$$

$$A_{CP}\left(\Xi_c^0 \to pK^-\right) = (-1.50 \pm 0.25) \times 10^{-3}$$

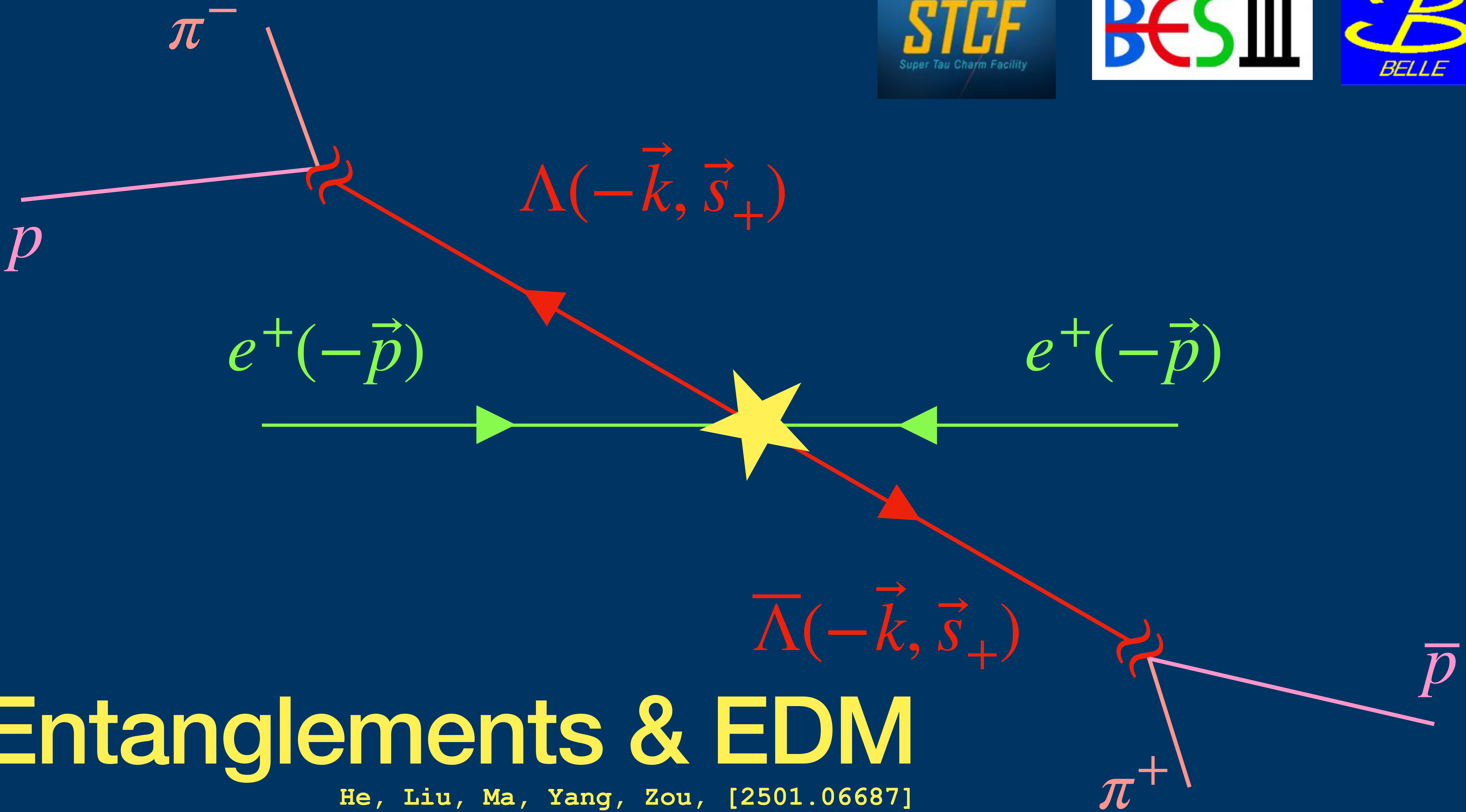
- In the U-spin limit, we have that

$$A_{CP}\left(\Xi_c^0 \to \Sigma^+\pi^-\right) = -A_{CP}\left(\Xi_c^0 \to pK^-\right).$$

EPJC 79, 429 (2019)



Two topological diagrams are in the same size, leads to $A_{CP} \sim \left|2\mathrm{Im}(V_{cs}^*V_{us}/V_{cd}^*V_{ud})\right| \sim 10^{-3}$.



Entanglements & EDM

He, Liu, Ma, Yang, Zou, [2501.06687]

Timelike EDM

- The decay distributions are obtained by squaring the amplitudes:

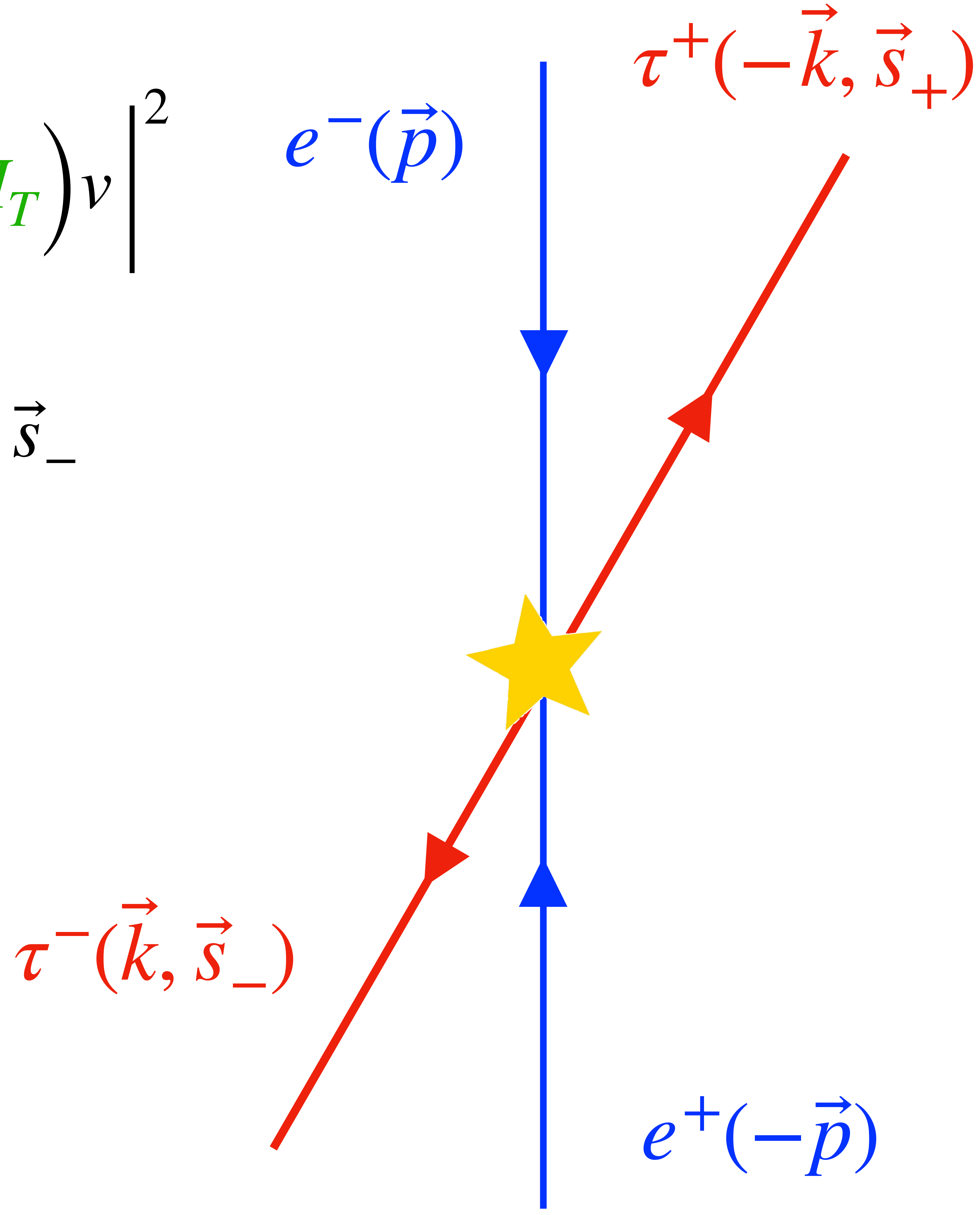
Polarization fraction

$$\frac{\partial\Gamma}{\partial\vec{\Omega}} = \sum_{\epsilon} P_\epsilon \left| \epsilon_\mu \, \bar{u}\left(\gamma^\mu F_V + \frac{i}{2m}\sigma^{\mu q} H_\sigma + \gamma^\mu\gamma^5 F_A + \sigma^{\mu q}\gamma^5 H_T\right) v \right|^2$$

$$\propto 1 + \vec{B}_+ \cdot (\vec{s}_- + \vec{s}_+) + \vec{B}_- \cdot (\vec{s}_- - \vec{s}_+) + \vec{s}_+ \cdot \overleftrightarrow{C} \cdot \vec{s}_-$$

$$\vec{B}_-(\vec{p},\vec{k}) = \left(b_p \hat{p} + b_k \hat{k}\right) \mathrm{Im}\left(H_T\right)$$

$$C^{ij}(\vec{p},\vec{k}) = \underbrace{\delta^{ij} c_0 \cdots}_{\text{CP-even}} + \underbrace{\epsilon^{ijk}\left(\hat{p}^k c_1 + \hat{k}^k c_2\right)}_{\text{CP-odd}} \mathrm{Re}\left(H_T\right)$$



Timelike EDM

- Net results of the EDM formula:

$$\mathrm{Im}\left(d_\tau\right) = -\frac{3}{4}\frac{e\left(s+2m_\tau^2\right)}{m_\tau\sqrt{s}\sqrt{s-4m_\tau^2}}\left(\left\langle \hat{p}_{\pi^-}\cdot\hat{k}\right\rangle + \left\langle \hat{p}_{\pi^+}\cdot\hat{k}\right\rangle\right)$$

$\left\langle \hat{p}_{\pi^-}\cdot\hat{k}\right\rangle$ → Polarization fraction of τ^-

$\left\langle \hat{p}_{\pi^+}\cdot\hat{k}\right\rangle$ → Polarization fraction of τ^+

No need for simultaneous detection of $\tau^- \to \pi^-\nu_\tau$ and $\tau^+ \to \pi^+\overline{\nu}_\tau$.

$$\mathrm{Re}\left(d_\tau\right) = e\frac{9}{4}\frac{s+2m_\tau^2}{m_\tau\sqrt{s^2-4sm_\tau^2}}\left\langle\left(\hat{p}_{\pi^-}\times\hat{p}_{\pi^+}\right)\cdot\hat{k}\right\rangle$$

Need for simultaneous detection of $\tau^- \to \pi^-\nu_\tau$ and $\tau^+ \to \pi^+\overline{\nu}_\tau$.

- ***Or*** one can adopt simple observables: [2506.19557]

$$\mathrm{Re}\left(d_\tau\right) \propto \left\langle\left(\hat{p}_{\pi^+}-\hat{p}_{\pi^-}\right)\cdot\hat{p}_e\ \left(\hat{p}_{\pi^+}\times\hat{p}_{\pi^-}\right)\cdot\hat{p}_e\right\rangle$$

No need detection of $\hat{k}$!

$\mathrm{Re}\left(d_\tau\right)$	Simple	Optimal
$\sqrt{s}$=3.686 GeV ($N_{\tau^+\tau^-} = 3.5\times10^6$)	4.11×10^{-16}	4.87×10^{-17}
$\sqrt{s}$=5.6 GeV ($N_{\tau^+\tau^-} = 5.5\times10^7$)	1.78×10^{-17}	2.93×10^{-18}

- It is important to **measure** $\hat{k}$!

• τ EDM

$$\sigma_{xy} = 130\ \mu\text{m} \longrightarrow 30\ \mu\text{m}$$

- We urge the addition of **silicon pixel detectors** at STCF to filter fast decay events.

$$P_\tau = 1 - \underbrace{\left(\int_0^{D/D_0} \exp(-x)dx \right)^2}_{\text{Probability of } not \text{ being detected}}$$

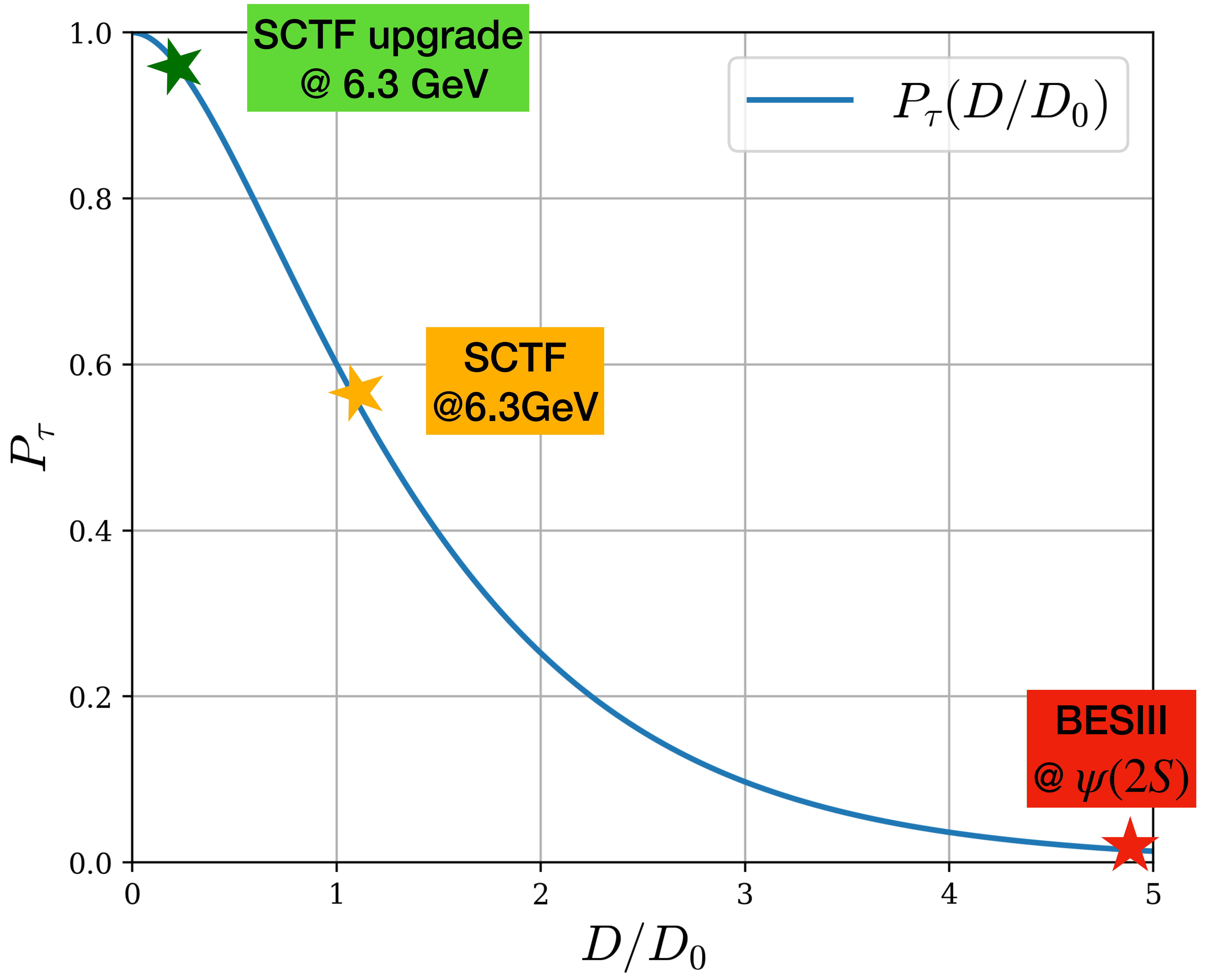
Probability of being detected (P_τ)

D : the detector resolution

D_0 : the average flight distance.

- We have to ***sacrifice*** some statistics when $\hat{k}$ cannot be detected.

- $P = 2\,\%$, nearly impossible to probe $\mathrm{Re}(d_\tau)$ @ **BESIII** but excellent at **SCTF**.



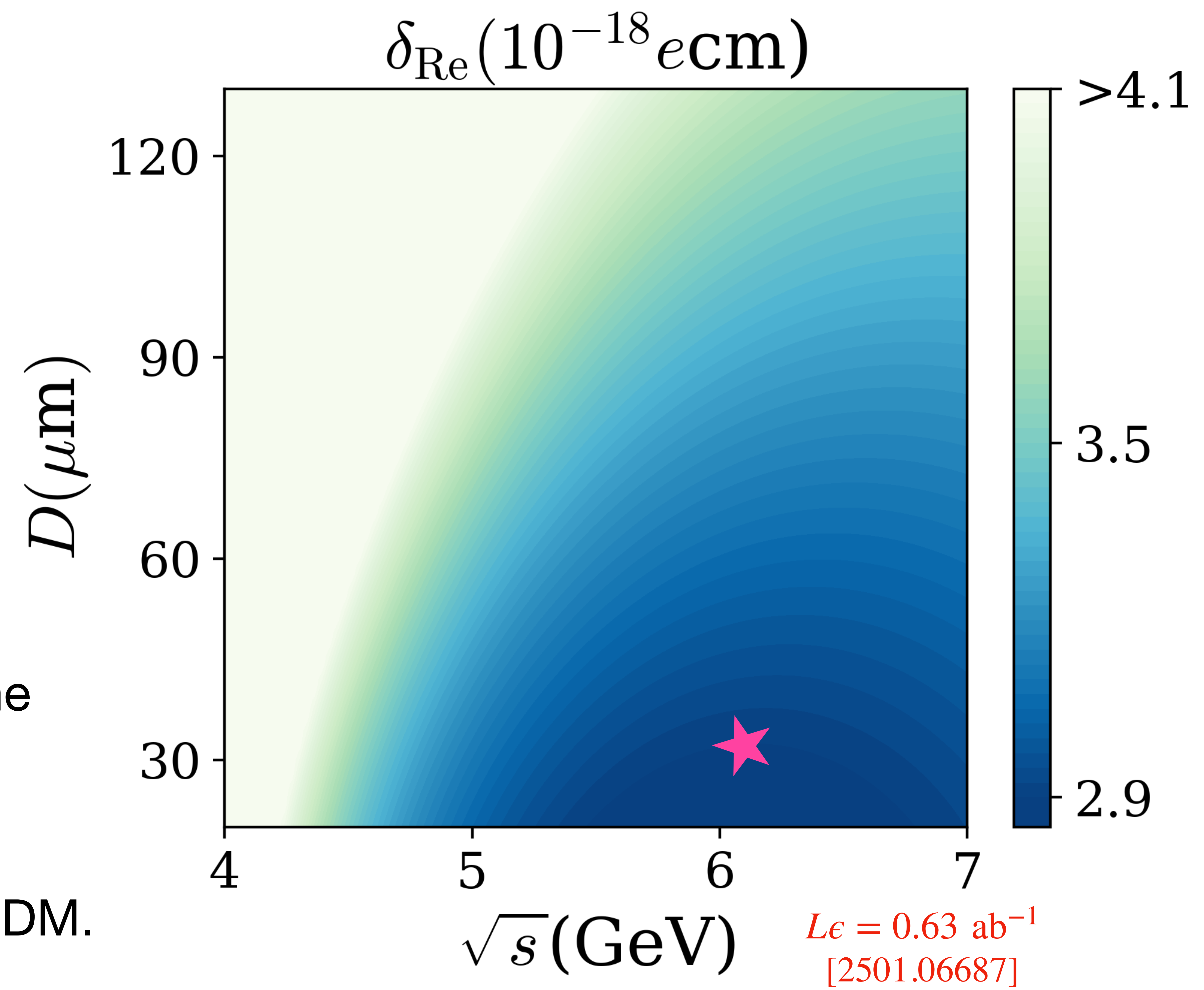
• τ EDM

$\sigma_{xy} = 130\ \mu m \longrightarrow 30\ \mu m$

- We urge the addition of **silicon pixel detectors** at STCF to filter fast decay events.

$\sqrt{s}$	$m_{\psi(2S)}$	5.6 GeV	6.3 GeV
$\delta_{\rm Im}$	1.8	0.7	0.7
$\delta_{\rm Re}(180)$	235	4.9	4.2
$\delta_{\rm Re}(130)$	83	4.0	3.6
$\delta_{\rm Re}(80)$	29	3.3	3.1
$\delta_{\rm Re}(30)$	11	2.9	2.8

Table. Precision of d_τ with $D = 180,\ 130\ldots$



$L\epsilon = 0.63\ {\rm ab}^{-1}$
[2501.06687]

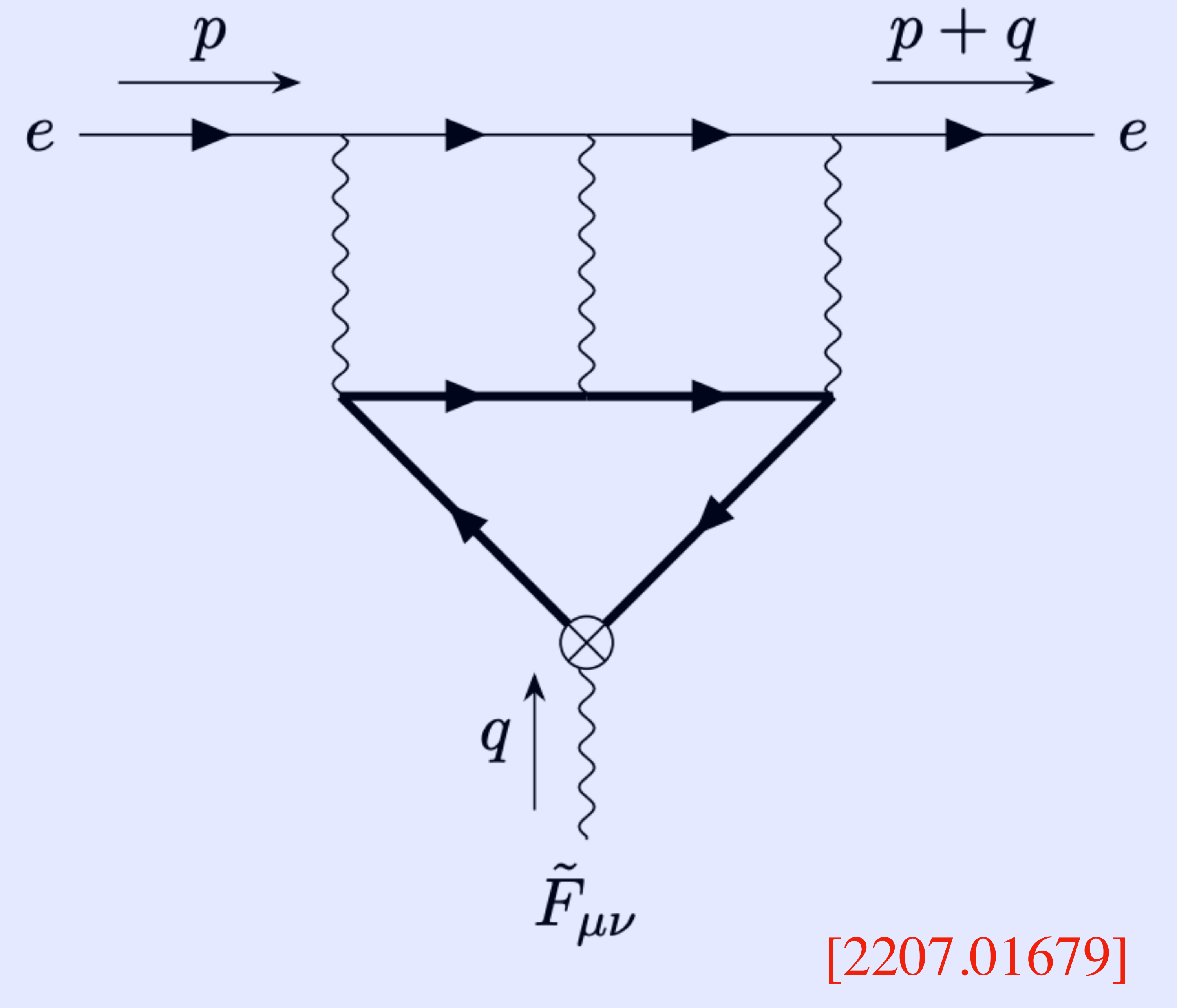
- ★ sweet spot @$\sqrt{s} = 6.3$ GeV, pushing the upper bound to $10^{-18}\ e$cm.
- See the next speaker for more details on τEDM.

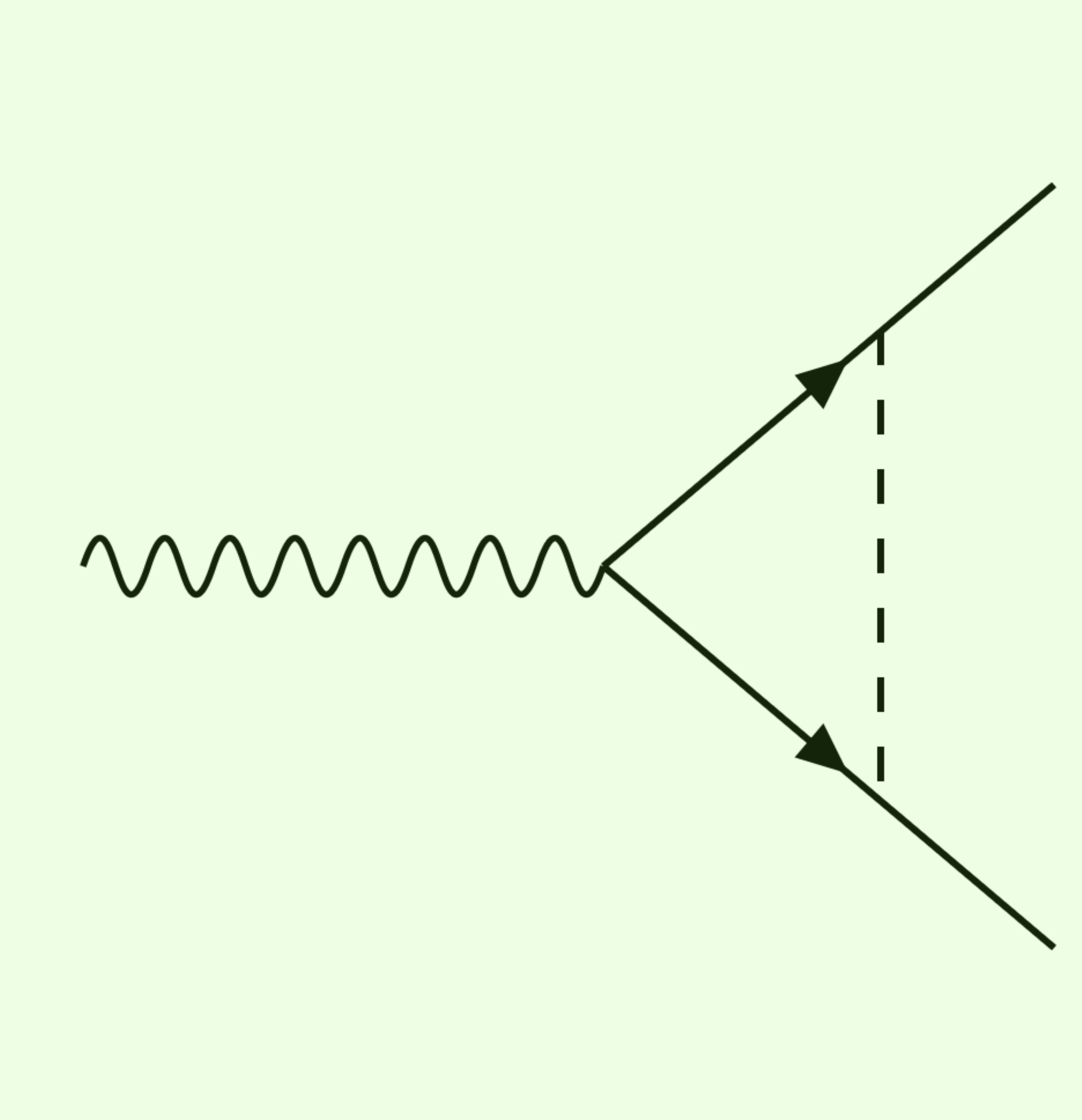
• EDM experiments

- τ and hyperons have **short lifetimes**. Traditional EDM measurement techniques are **not feasible**.
- Can be probed directly at **colliders**.
- May **induce electron/nucleons** EDM.

$$\left(\frac{d_n}{d_\Lambda}\right)^* = (2.7 \pm 1.6) \times 10^{-3},$$

* assuming $\bar{\theta}, d_u, d_d = 0$.





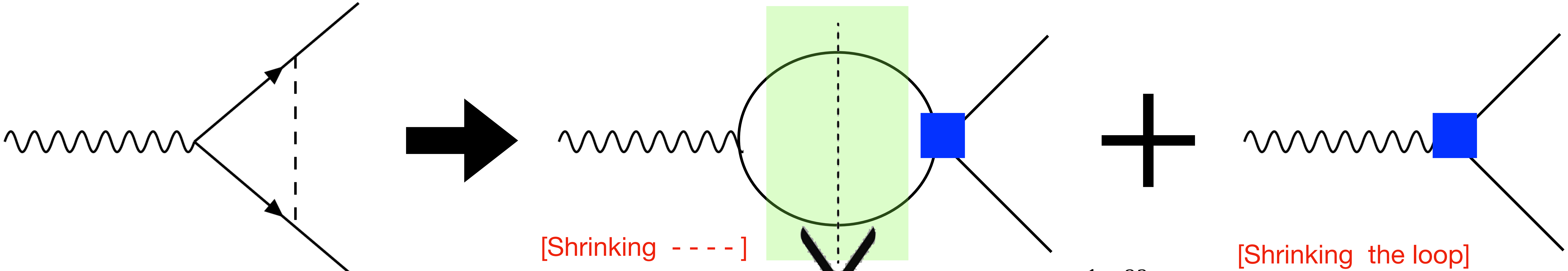
Particle	Method	Upper limit	Particle	Method	Upper limit
e^-	Ion trap	4.1×10^{-30} e·cm	neutron	Hg*	1.4×10^{-26} e·cm
μ^-	(g-2) storage ring	1.5×10^{-19} e·cm	proton	Hg*	1.7×10^{-25} e·cm
τ^-	From eEDM	4.1×10^{-19} e·cm	Λ	From nEDM	2×10^{-22} e·cm
τ^-	e^+e^- colliders	1.9×10^{-17} e·cm	Λ	e^+e^- colliders	5.5×10^{-19} e·cm

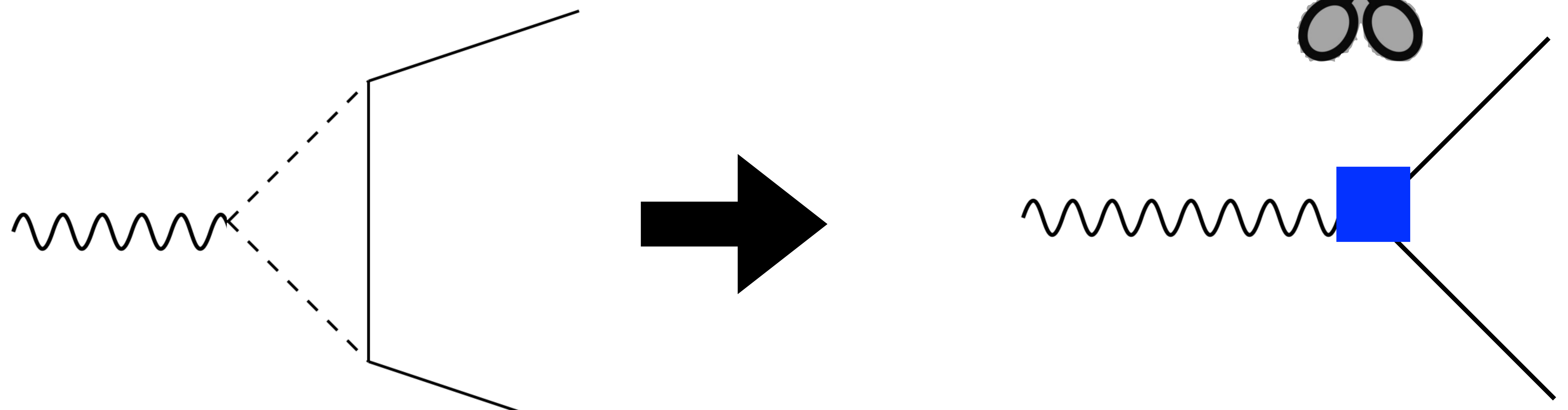
@ BELLE — @ BESIII

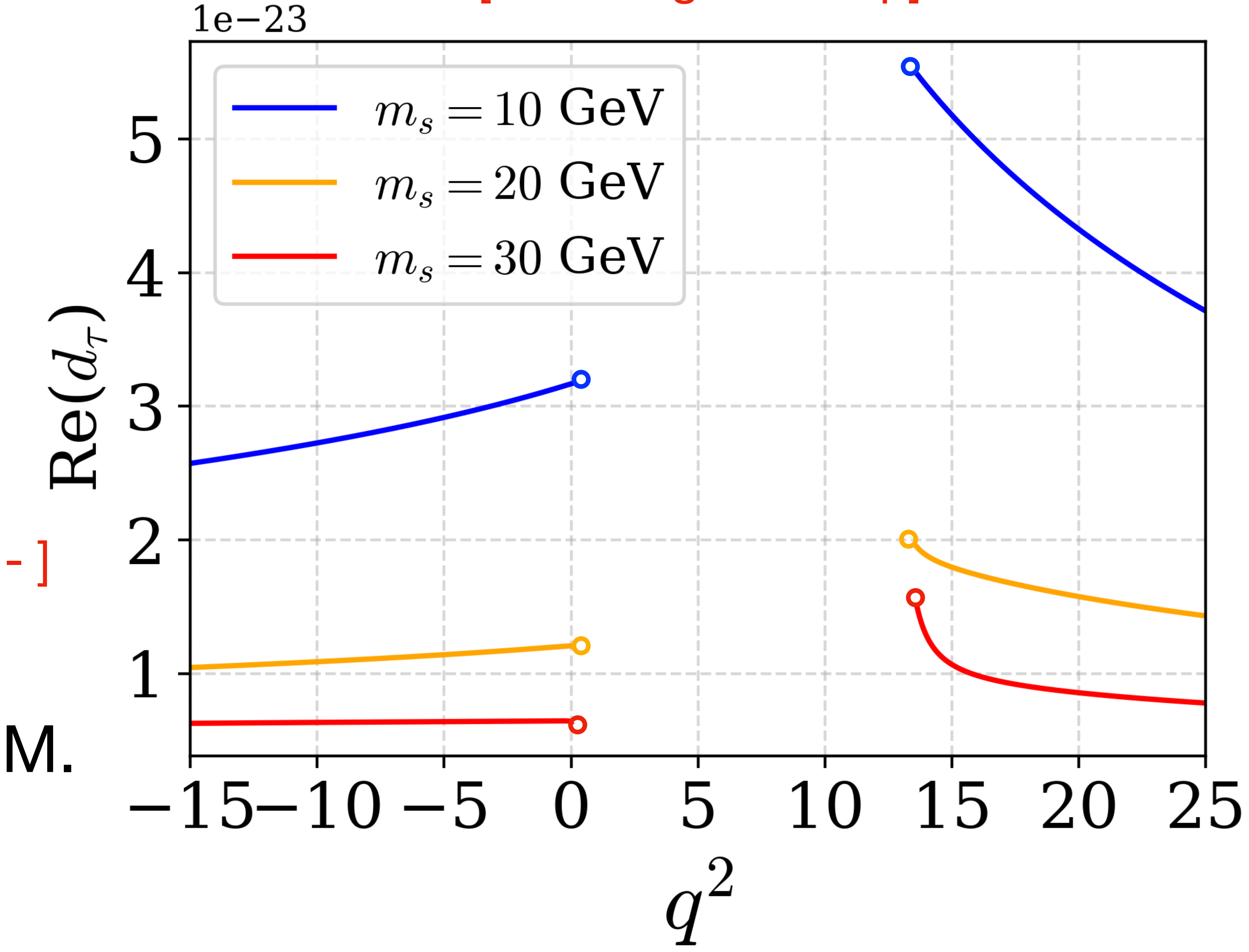
State-of-the-art upper limits of $|d_f|$ at 90% confidence level

EDM numerical results

- Why shall we **consider** **EDM** at colliders? For UV NP: $\mathscr{L}_{eff} = c_1 \frac{g_e m_\tau}{\Lambda^2} F^{\mu\nu}(\bar{\tau}\sigma_{\mu\nu}\gamma_5\tau) + \frac{c_2}{\Lambda^2}(\bar{\tau}\gamma_5\tau)(\bar{\tau}\tau)$





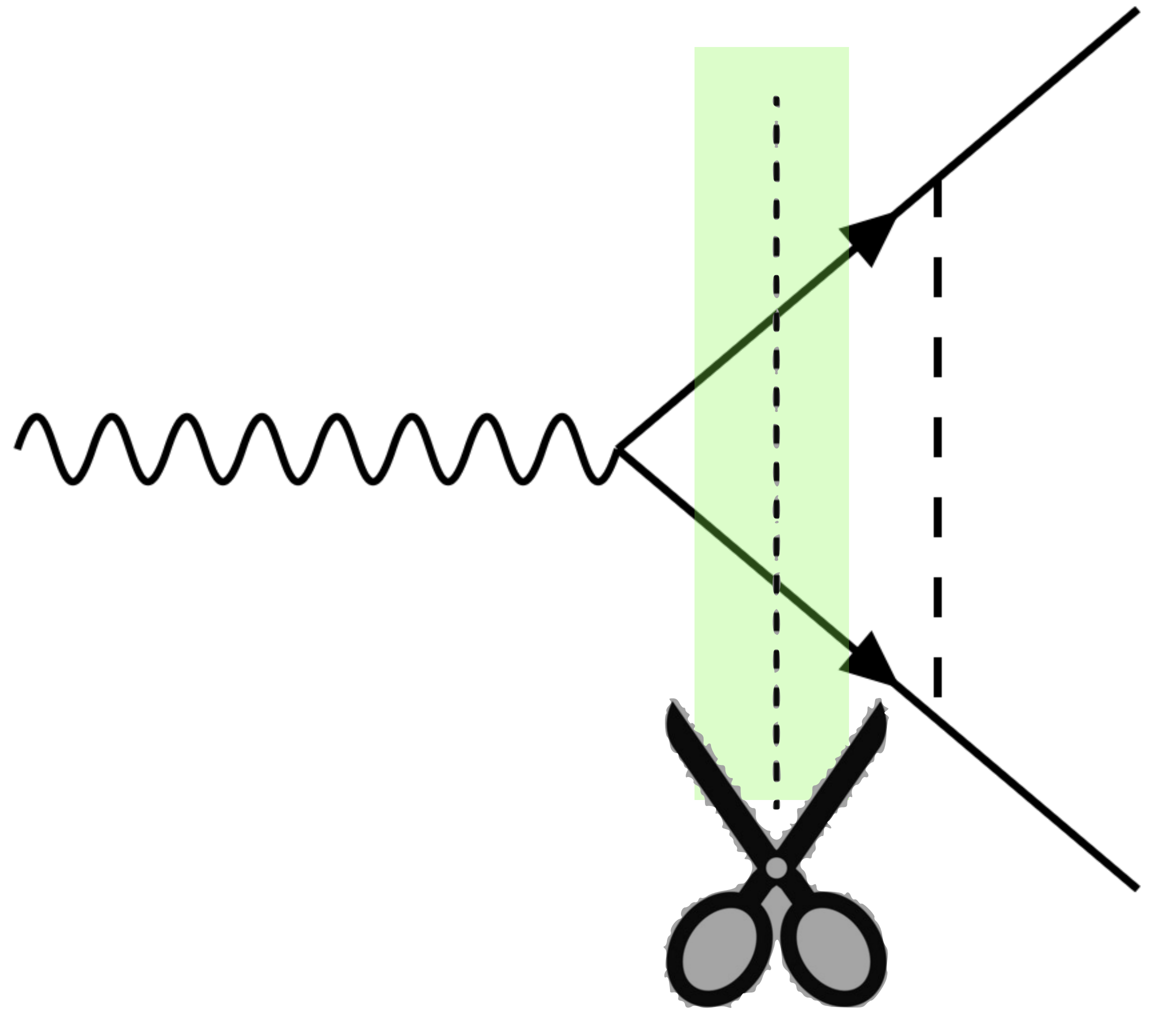


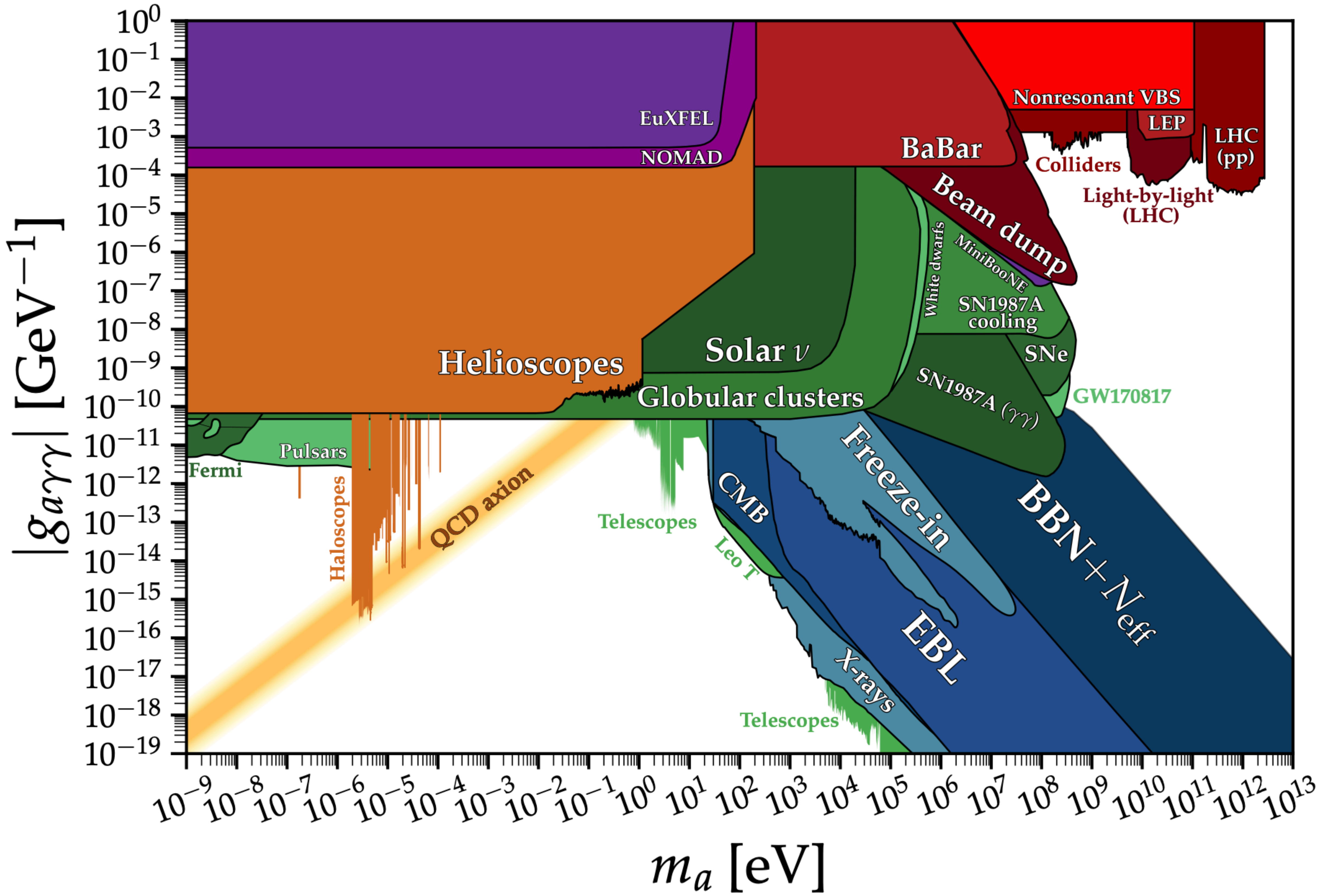
[Shrinking the loop] = [Shrinking - - - -]

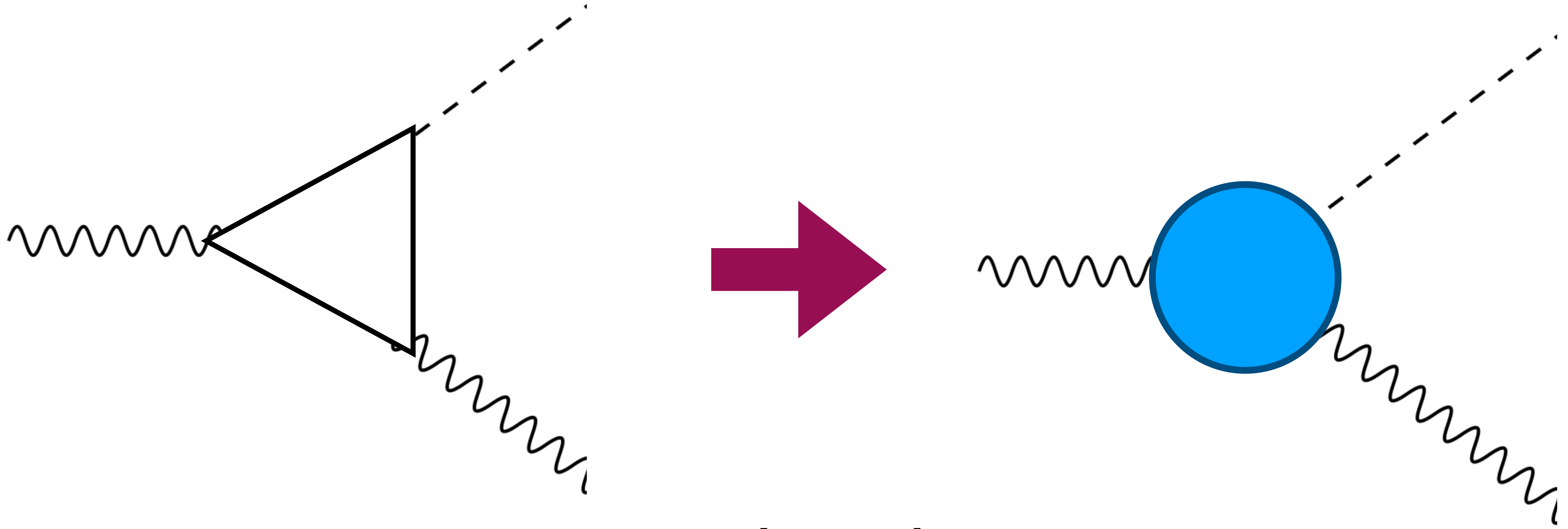
- Only $c_1 - c_2 \frac{g_e}{8\pi^2} \log\left(\frac{m_\tau^2}{m_s^2}\right)$ ***is*** constrained by conventional EDM.

EDM numerical results

- Axion light particles **enhance** imaginary parts of **EDM.**
- The couplings are proportional to m_f. EDMs are **enhanced** at $m_a \sim m_f \sim \sqrt{s}$, providing natural reasons for EDM to be absent in light fermions.





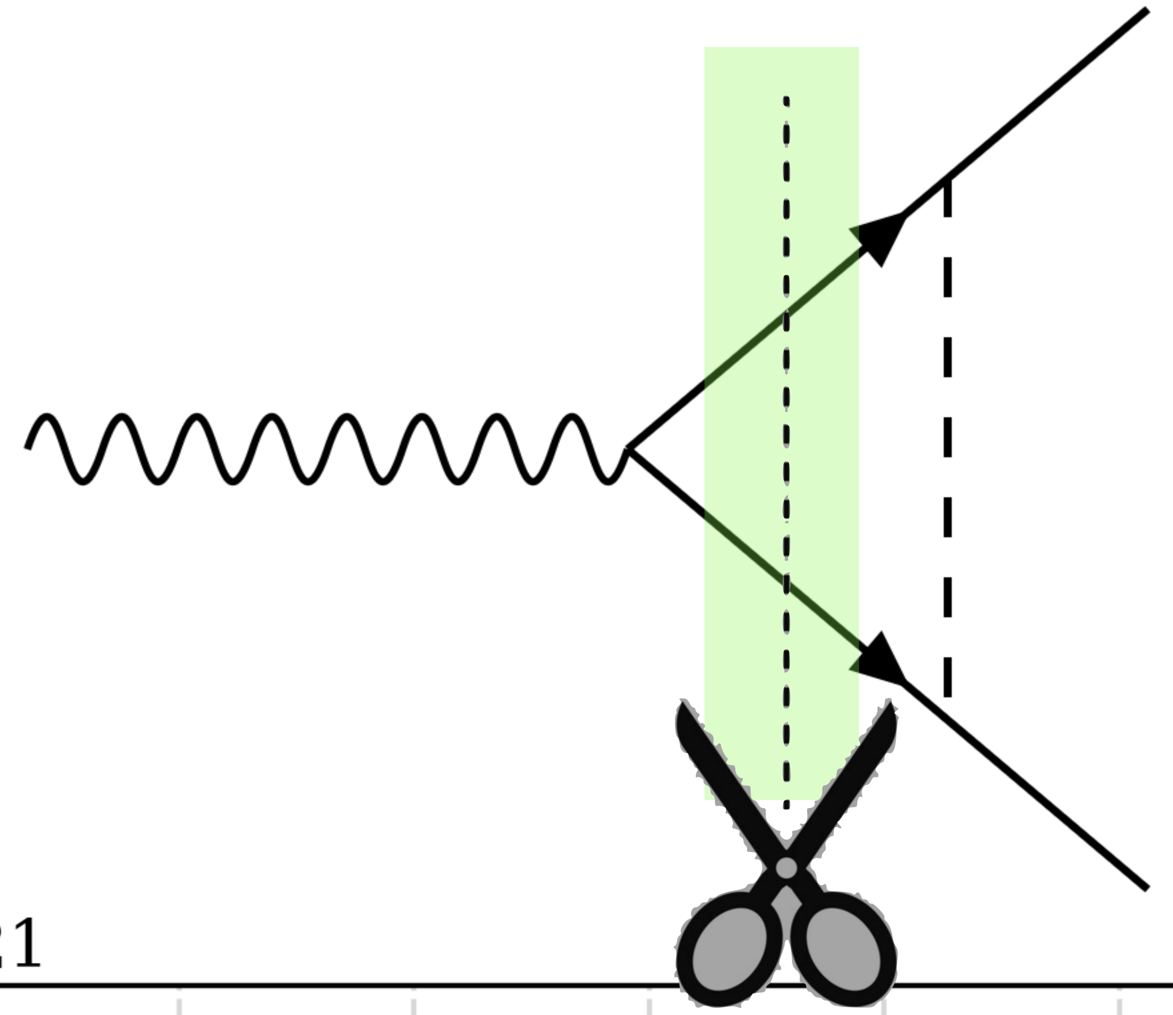


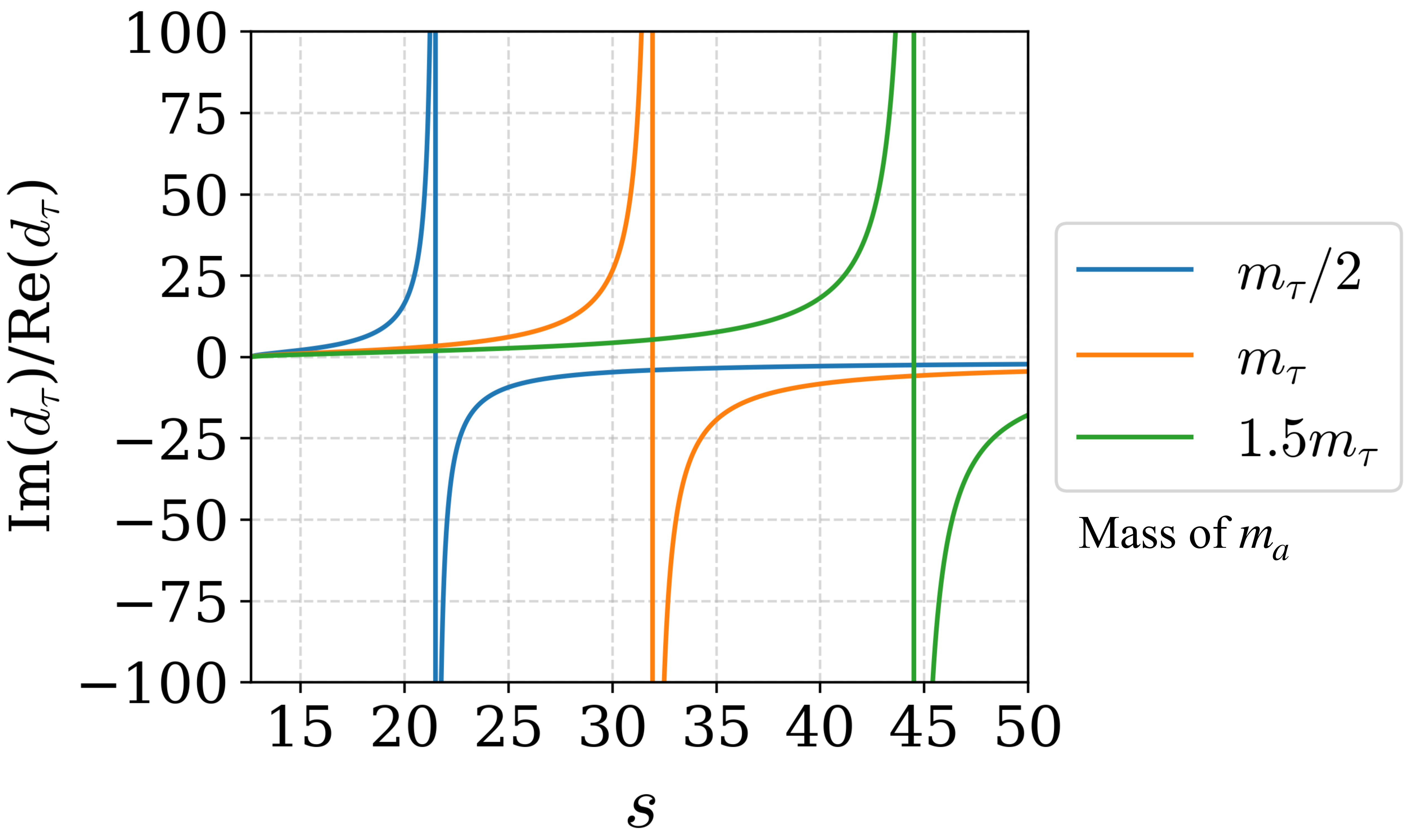
$$g_{a\gamma\gamma} \sim \frac{|g_{af}|}{4\pi^2 m_f} \le 10^{-3}$$

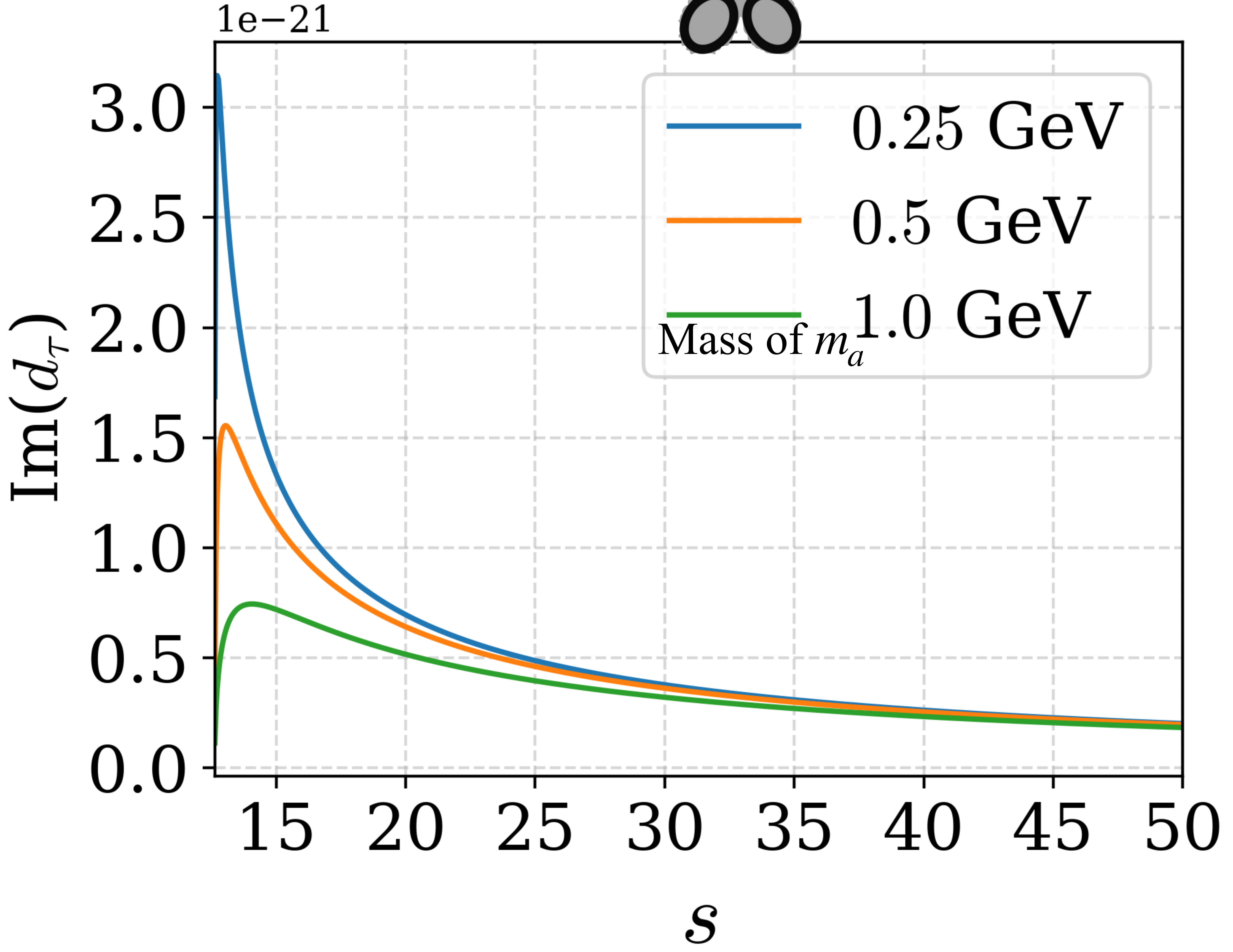
- We use conservative value of $g_{af} = 10^{-3}$.

EDM numerical results

- Axion light particles **enhance** imaginary parts of **EDM.**
- The couplings are proportional to m_f. EDMs are **enhanced** at $m_a \sim m_f \sim \sqrt{s}$, providing natural reasons for EDM to be absent in light fermions.







EDM numerical results

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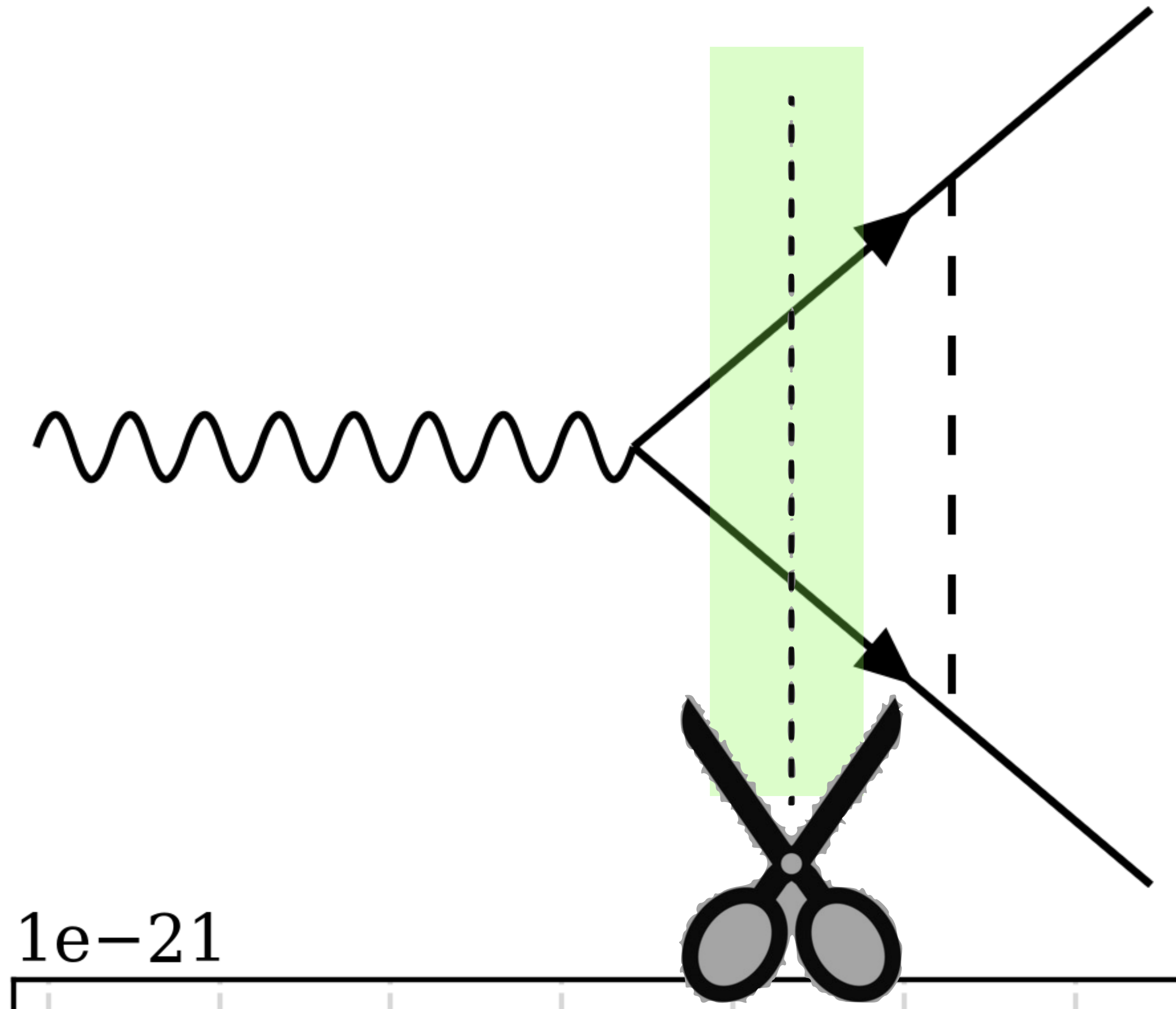

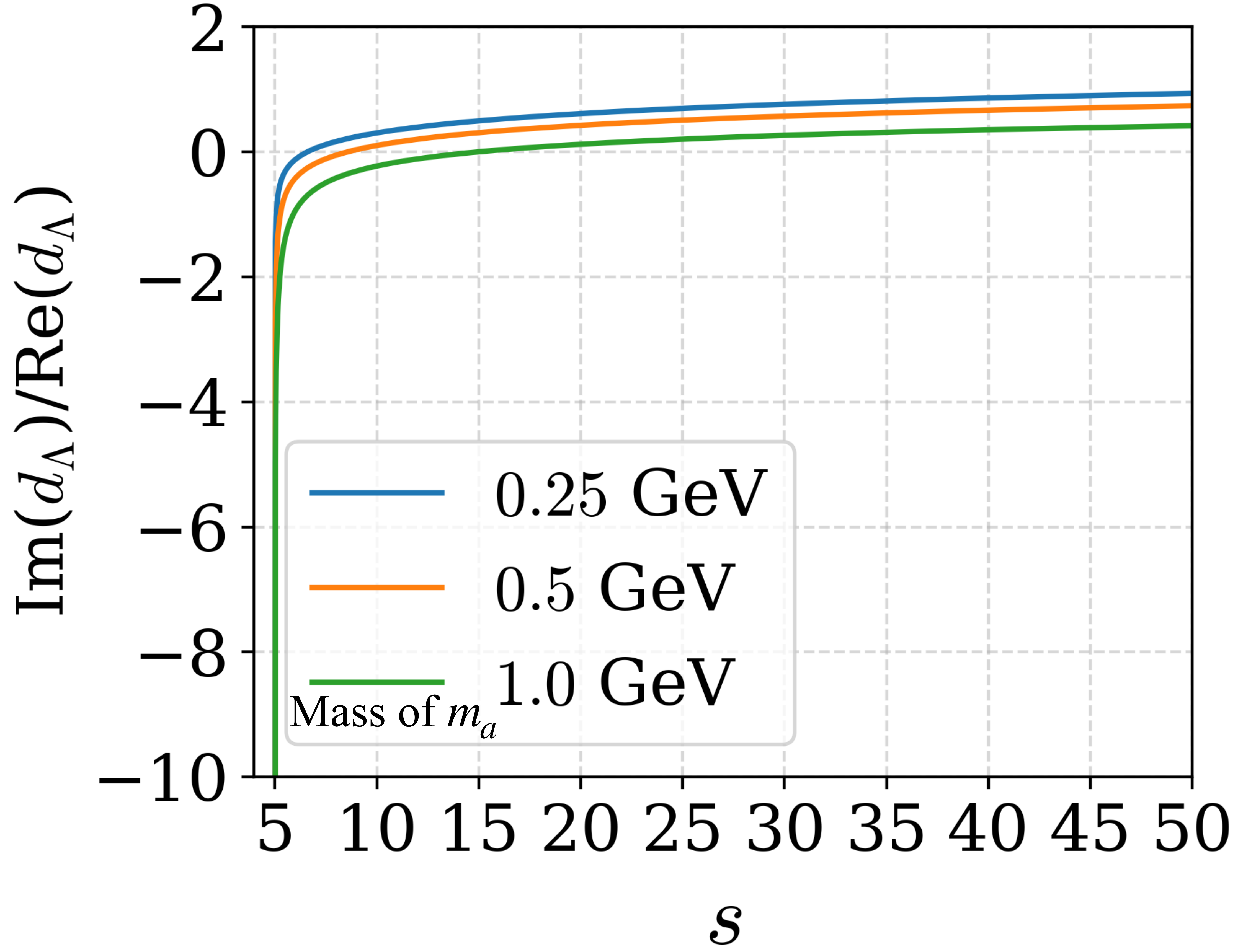

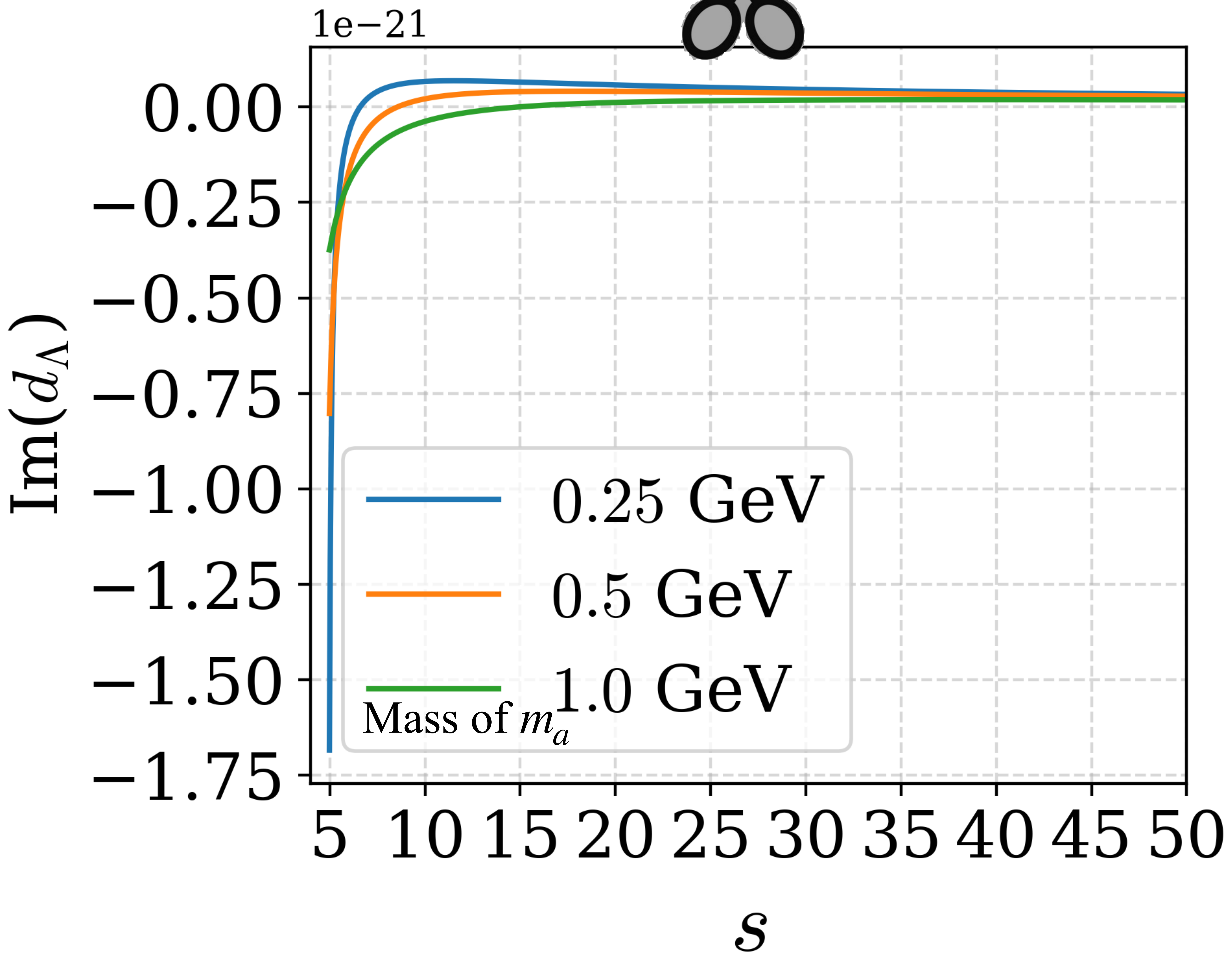

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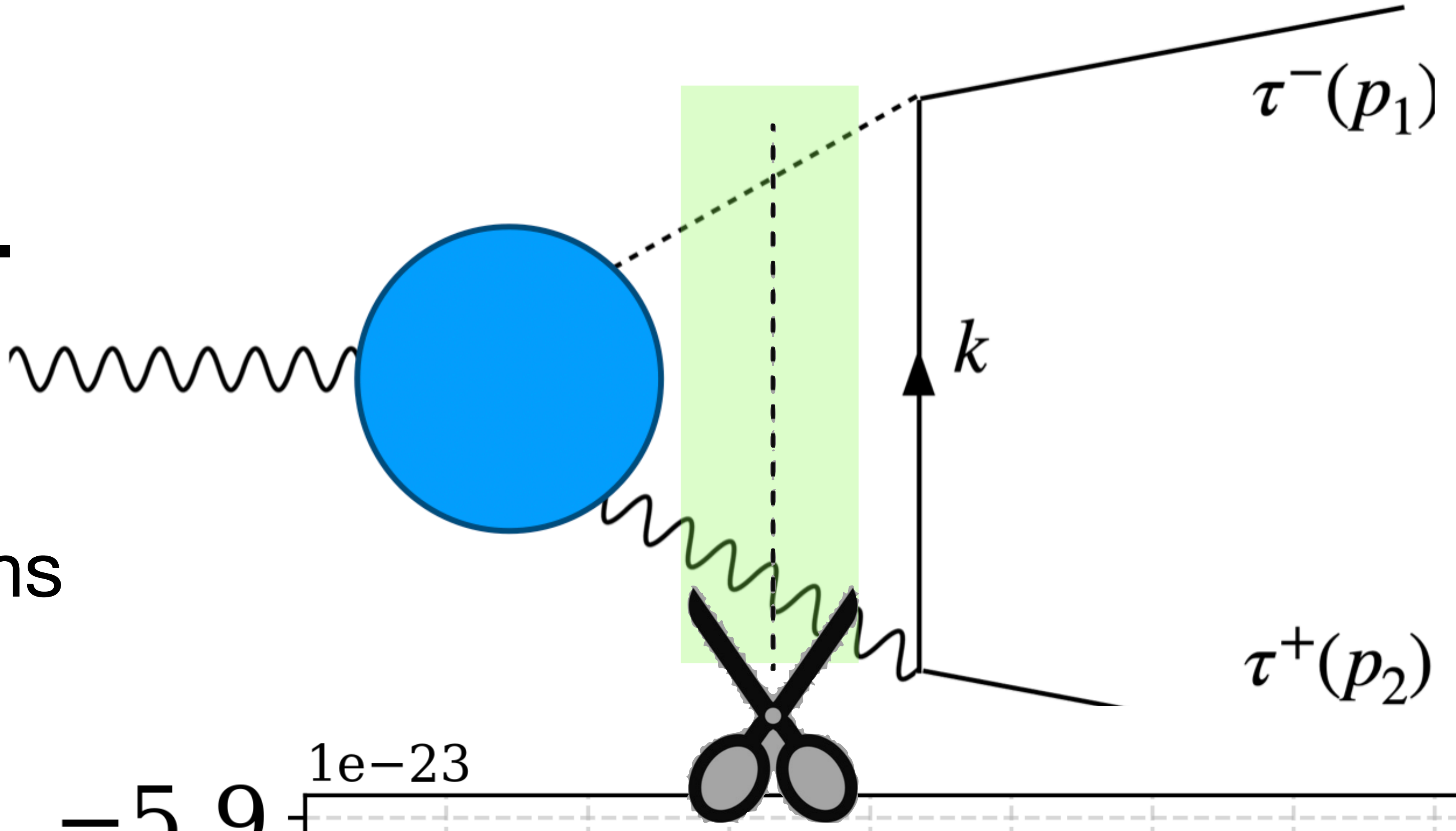
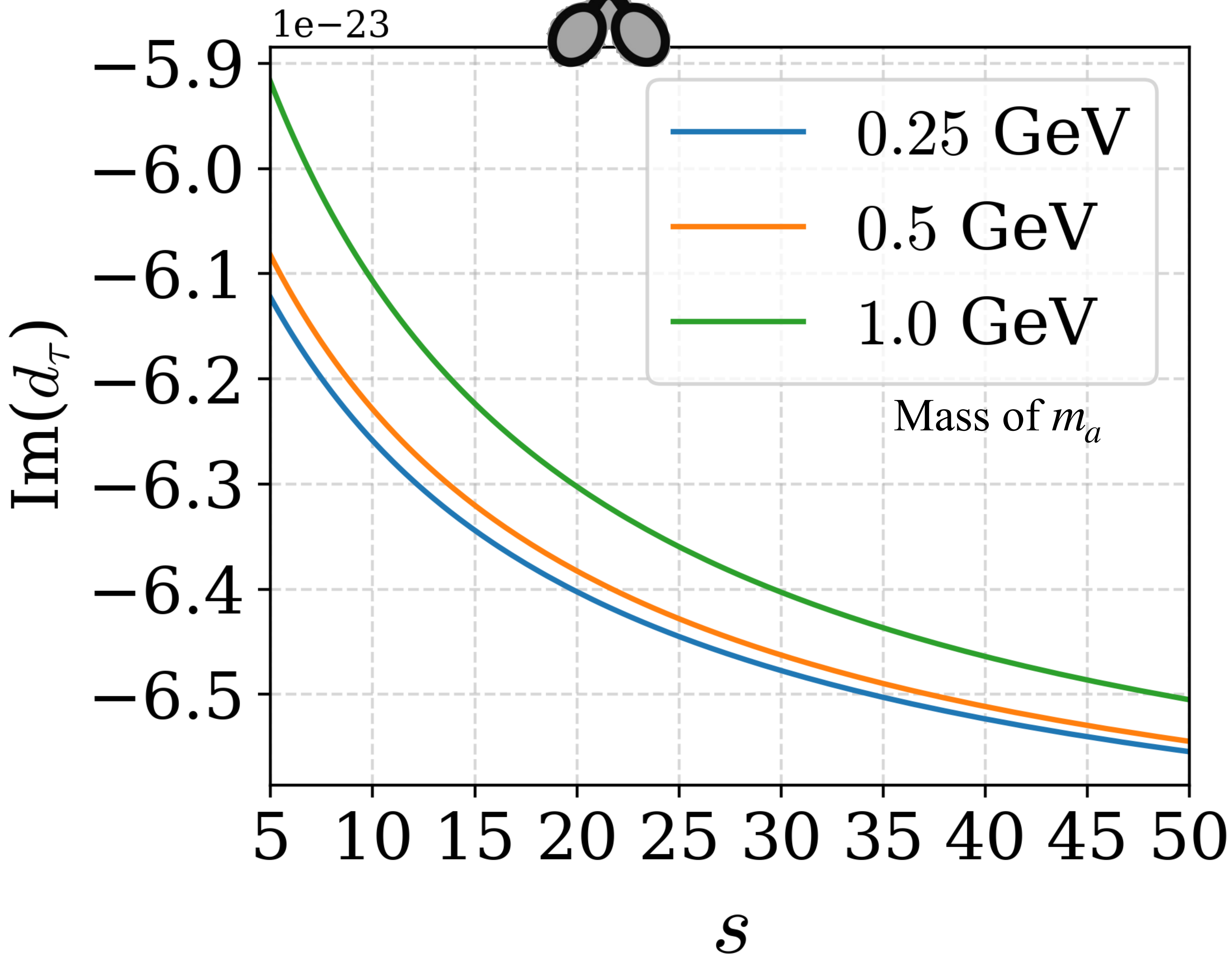
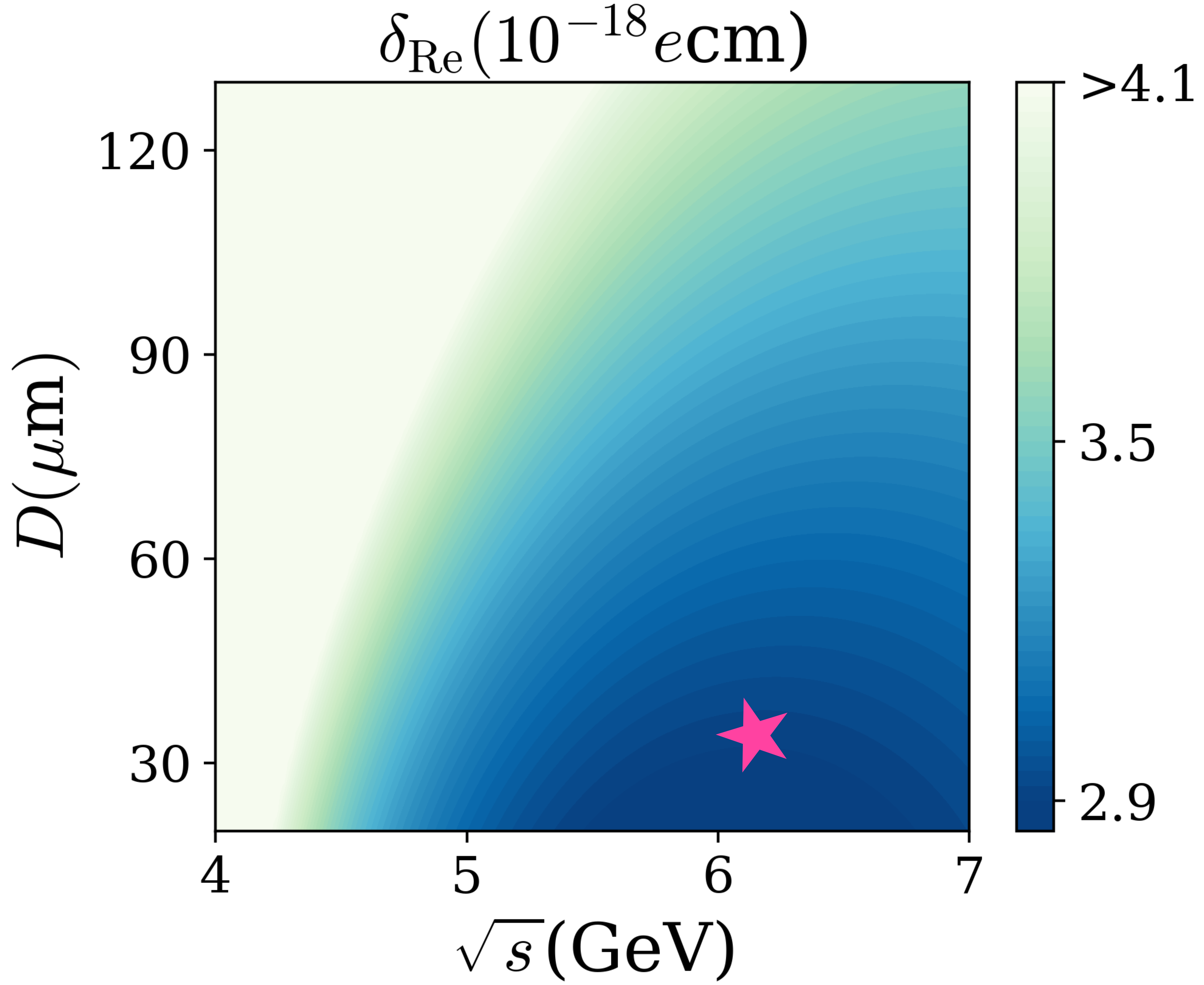


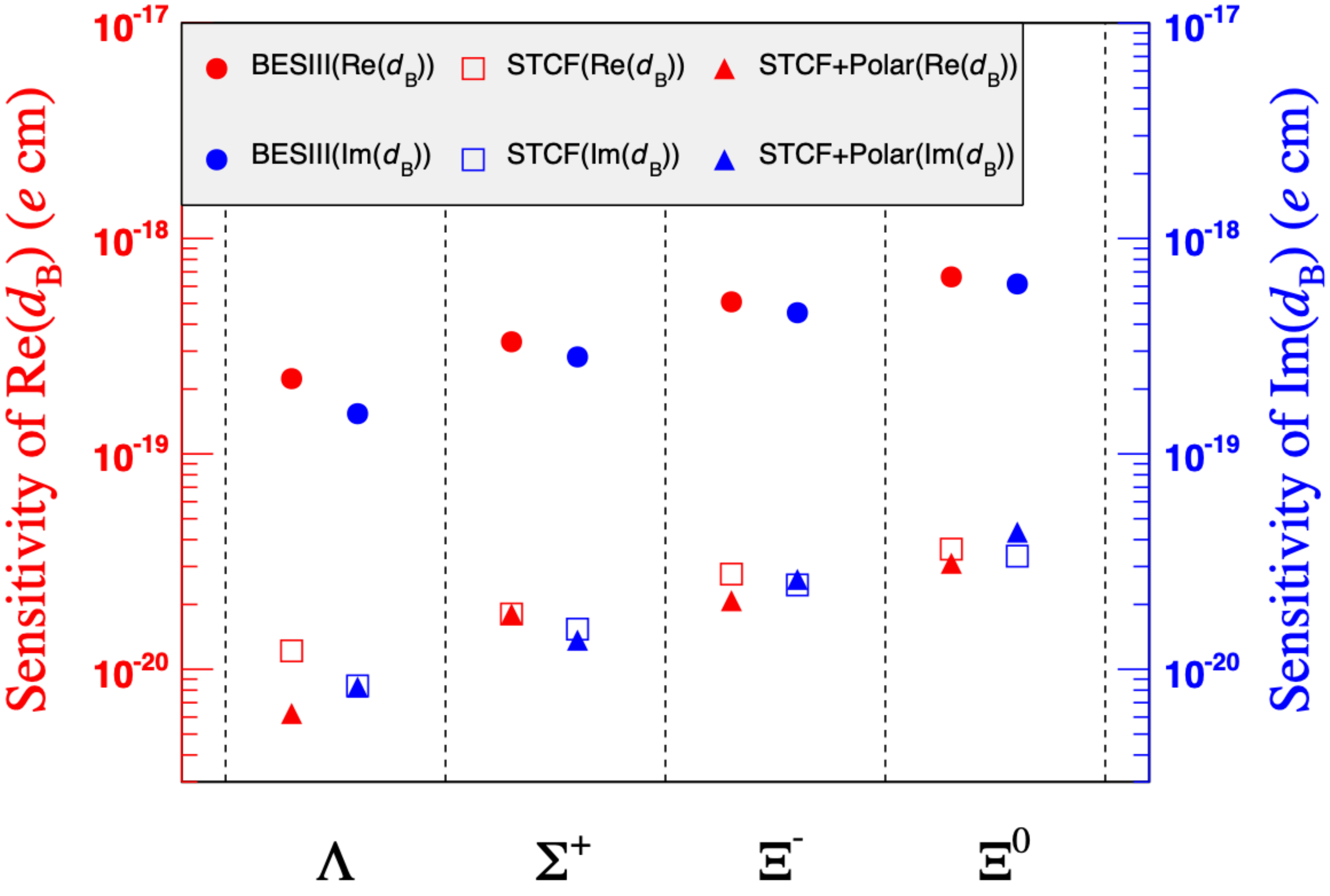
- Real part diverge! Only leading log is trustworthy.
- The chiral **enhancements** in electrons and muons are not found here.



Future aspect

- The STCF will improve sensitivity to the hyperon and τ EDM by a **factor** of **ten**, significantly enhancing tests of the Standard Model and searches for NP.



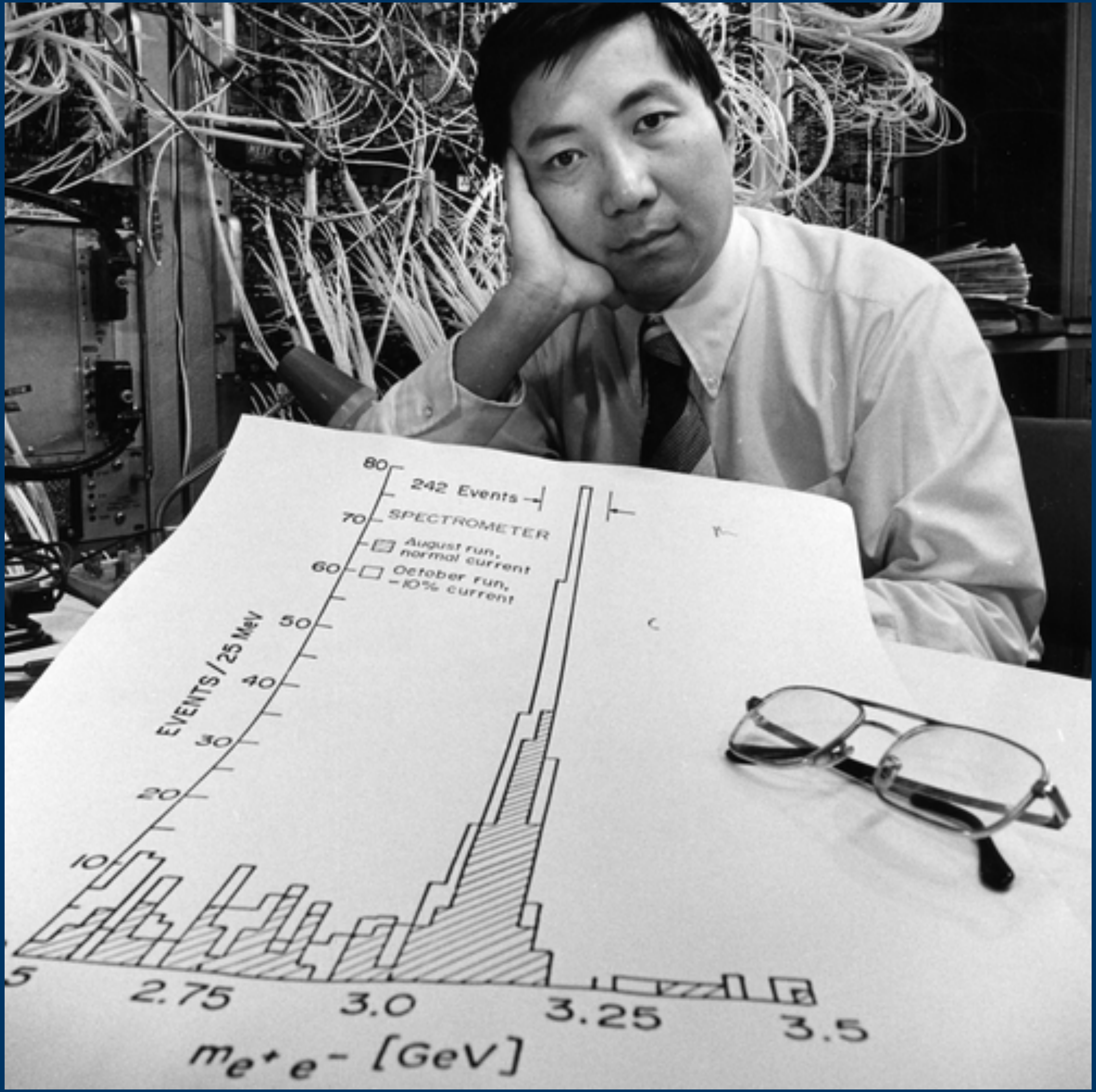


Fu, Li, Wang, Yu, Zhang, [2307.04364]

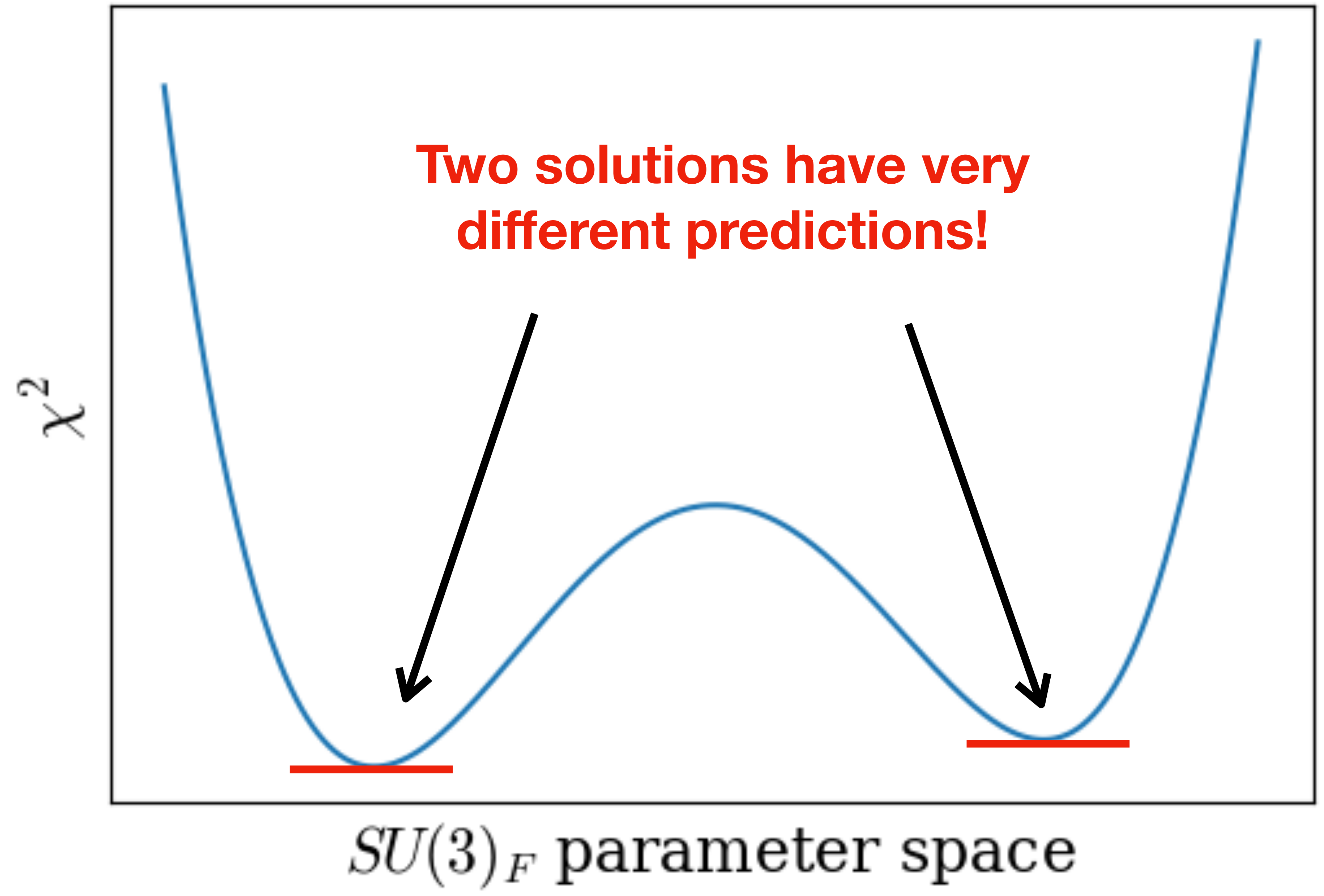
CP violation	Im(d_B) ($\times 10^{-18}$ e cm)		Re(d_B) ($\times 10^{-18}$ e cm)	
	BESIII	STCF	BESIII	STCF
Λ ($\epsilon = 0.4$)	2.62	0.14	8.64	0.47
Σ^+ ($\epsilon = 0.2$)	1.47	0.08	18.4	1.00
Ξ^0 ($\epsilon = 0.2$)	6.12	0.33	82.6	4.41
Ξ^- ($\epsilon = 0.2$)	6.79	0.37	95.9	5.20

With 10^{10} J/ψ, Du[2], He, Ma, [2405.09625]

Backup slides



• SU(3) flavor perspective of charmed baryon decays



The $SU(3)_F$ is an approximate symmetry with **errors** in 10^{-1}.

There exhibits Z_2 **ambiguities**:

$$\Gamma \propto |F^2| + \kappa^2 |G^2|, \quad \alpha = \frac{2\kappa \mathrm{Re}(F^* G)}{|F^2| + \kappa^2 |G^2|}, \quad \beta = \frac{2\kappa \mathrm{Im}(F^* G)}{|F^2| + \kappa^2 |G^2|}, \quad \gamma = \frac{|F^2| - \kappa^2 |G^2|}{|F^2| + \kappa^2 |G^2|}.$$

Γ and α are **invariant** under $(F, G) \to (F^*, G^*)$ and $F \leftrightarrow \kappa G^*$ but β and γ **flip signs**.
In general, the amplitudes cannot be fully reconstructed without β and γ as input.

Precise β and γ data can break the ambiguities, highlighting the importance of LHCb work!

Nevertheless, there are ***still*** a few **ambiguities**.

Measurement of Λ_b^0, Λ_c^+, and Λ Decay Parameters Using $\Lambda_b^0 \to \Lambda_c^+ h^-$ Decays
PRL **133**, 261804 (2024)

- **Rescattering, solving penguin/tree**

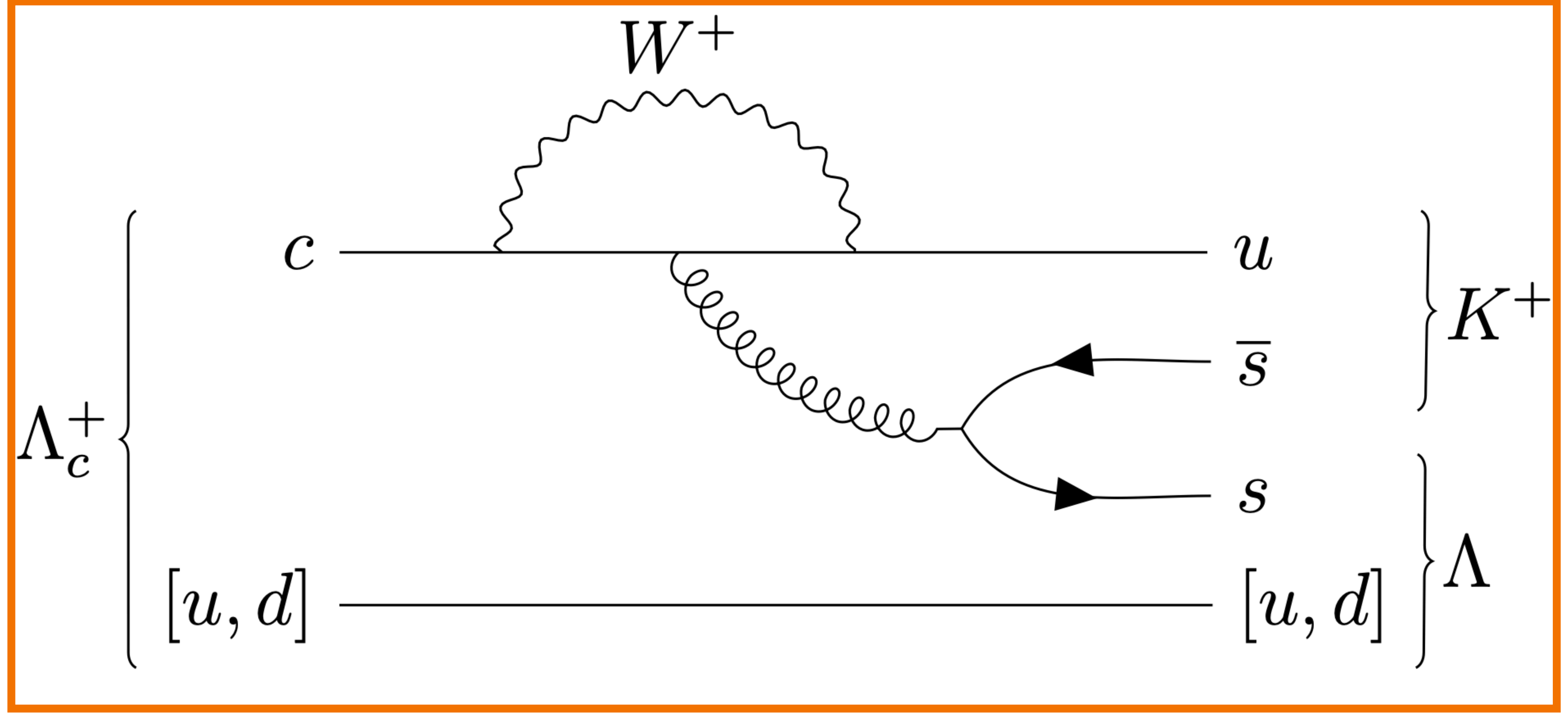
Amplitudes : $\frac{\lambda_s - \lambda_d}{2} F^{s-d} + \lambda_b\, F^b$

$$\tilde f^b = \tilde F_V^- + \tilde S^- - \sum_{\lambda=\pm}(2r_\lambda^2 - r_\lambda)\tilde T_\lambda^- ,$$

$$\tilde f^c = r_- \tilde S^- - \sum_{\lambda=\pm}(r_\lambda^2 - 2r_\lambda + 3)\tilde T_\lambda^- ,$$

$$\tilde f^d = \tilde F_V^- - \sum_{\lambda=\pm}(2r_\lambda^2 - 2r_\lambda - 4)\tilde T_\lambda^- , \quad \tilde f^e = \tilde F_V^+ ,$$

Corrections to A_{CP} are around 10%



$$\tilde f_{\mathbf 3}^b = \frac{7r_- - 2}{8 + 2r_-}\tilde S^- - \sum_{\lambda=\pm}(r_\lambda^2 - 5r_\lambda/2 + 1)\tilde T_\lambda^- ,$$

$$\tilde f_{\mathbf 3}^c = \frac{(r_- + 1)(2 - 7r_-)}{24 + 6r_-}\tilde S^- + \sum_{\lambda=\pm}\frac{1}{6}(r_\lambda^2 + 11r_\lambda + 1)\tilde T_\lambda^- ,$$

$$\tilde f_{\mathbf 3}^d = \frac{r_-(7r_- - 2)}{8 + 2r_-}\tilde S^- - \sum_{\lambda=\pm}\frac{1}{2}(r_\lambda + 1)^2\tilde T_\lambda^- - \frac{1}{4}\left(\tilde F_V^+ + 2\tilde F_V^-\right)$$

$$\left(1 + \frac{(3C_4 + C_3)\, m_c - \frac{2m_K^2}{m_s + m_u}(3C_6 + C_5)}{(C_+ + C_-)m_c}\right)$$

$$\left(\tilde f^b, \tilde f^c, \tilde f^d, \tilde f^e\right) \longleftrightarrow \left(\tilde F_V^+, \tilde F_V^-, \tilde S^-, \tilde T^-\right) \longrightarrow \left(\tilde f_{\mathbf 3}^b, \tilde f_{\mathbf 3}^c, \tilde f_{\mathbf 3}^d\right)$$

PRD **100**, 093002 (2019)

Much more complicated compared to $P^{LD} = E$ in D mesons !

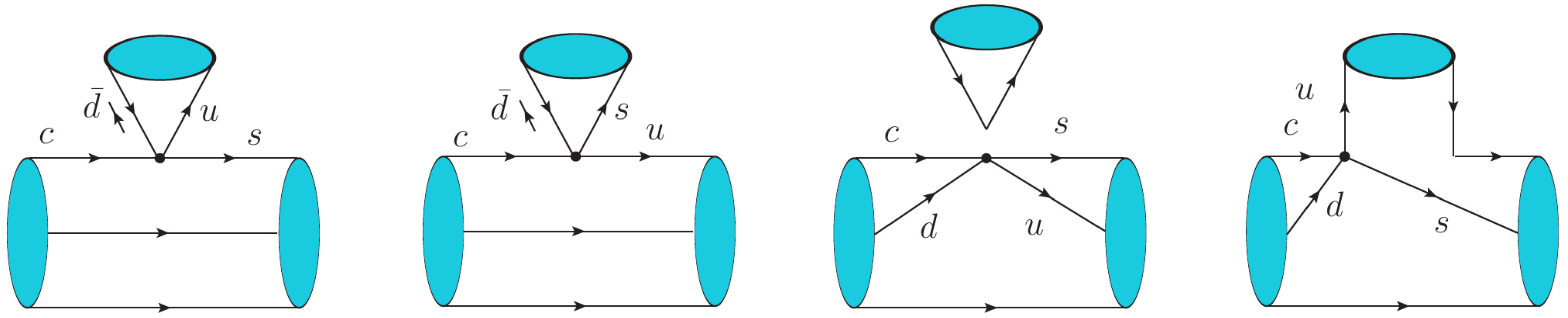
Rescattering, solving penguin/tree

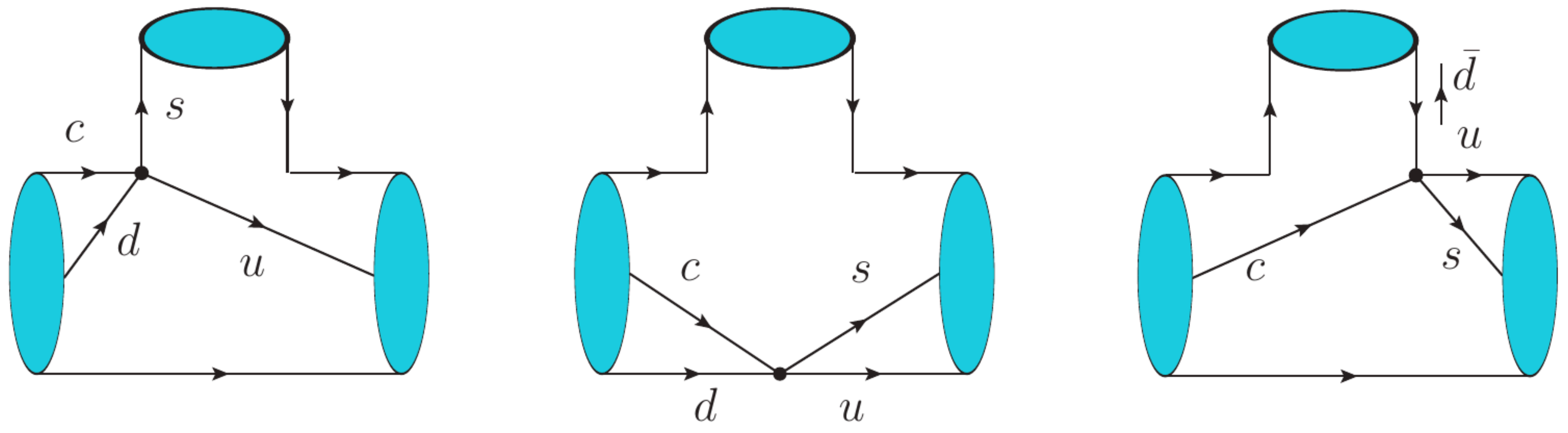
$$\text{Amplitudes}: \frac{\lambda_s-\lambda_d}{2}\tilde{f}^{b,c,d,e}+\lambda_b\,\tilde{f}_{\mathbf{3}}^{b,c,d}$$

$$\tilde{f}^b = \tilde{F}_V^- - (r_- + 4)\tilde{S}^- + \sum_{\lambda=\pm}(2r_\lambda^2 - r_\lambda)\tilde{T}_\lambda^- \,,$$

$$\tilde{f}^c = -r_-(r_- + 4)\tilde{S}^- + \sum_{\lambda=\pm}(r_\lambda^2 - 2r_\lambda + 3)\tilde{T}_\lambda^- \,,$$

$$\tilde{f}^d = \tilde{F}_V^- + \sum(2r_\lambda^2 - 2r_\lambda - 4)\tilde{T}_\lambda^- \,, \quad \tilde{f}^e = \tilde{F}_V^+$$

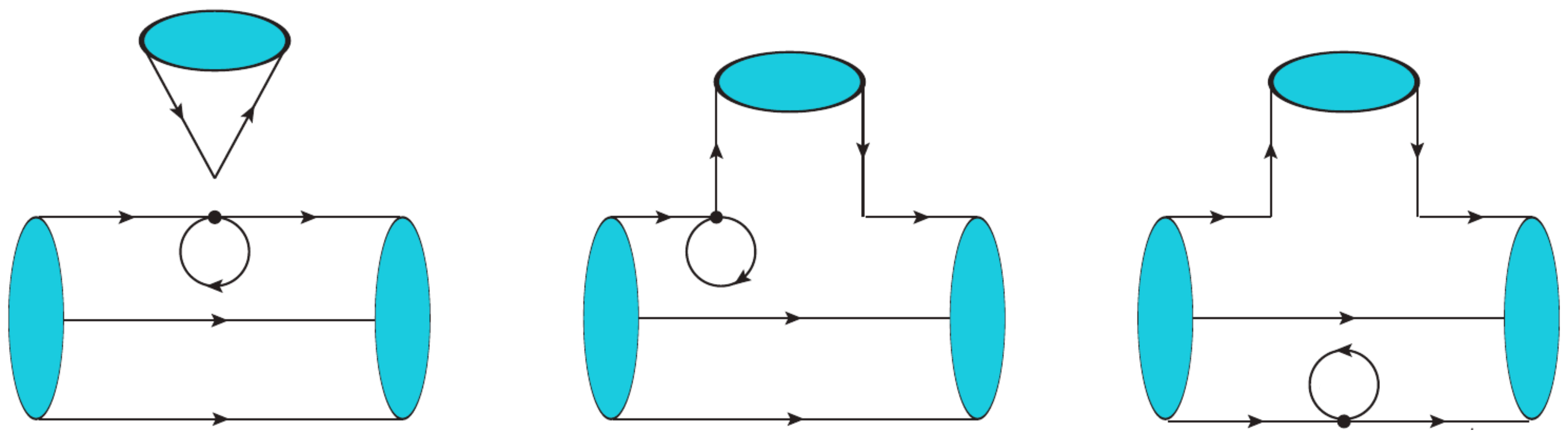




$$\tilde{f}_{\mathbf{3}}^b = (1 - \frac{7r_-}{2})\tilde{S}^- + \sum_{\lambda=\pm}(r_\lambda^2 - 5r_\lambda/2 + 1)\tilde{T}_\lambda^- \,,$$

$$\tilde{f}_{\mathbf{3}}^c = \frac{(r_- + 1)(7r_- - 2)}{6}\tilde{S}^- - \sum_{\lambda=\pm}\frac{r_\lambda^2 + 11r_\lambda + 1}{6}\tilde{T}_\lambda^- \,,$$

$$\tilde{f}_{\mathbf{3}}^d = \frac{2r_- - 7r_-^2}{2}\tilde{S}^- + \sum_{\lambda=\pm}\frac{(r_\lambda + 1)^2}{2}\tilde{T}_\lambda^- - \frac{\tilde{F}_V^+ + 2\tilde{F}_V^-}{4}.$$



$$\left(\tilde{f}^b, \tilde{f}^c, \tilde{f}^d, \tilde{f}^e\right) \longleftrightarrow \left(\tilde{F}_V^+, \tilde{F}_V^-, \tilde{S}^-, \tilde{T}^-\right) \longrightarrow \left(\tilde{f}_{\mathbf{3}}^b, \tilde{f}_{\mathbf{3}}^c, \tilde{f}_{\mathbf{3}}^d\right)$$

PRD **100**, 093002 (2019)

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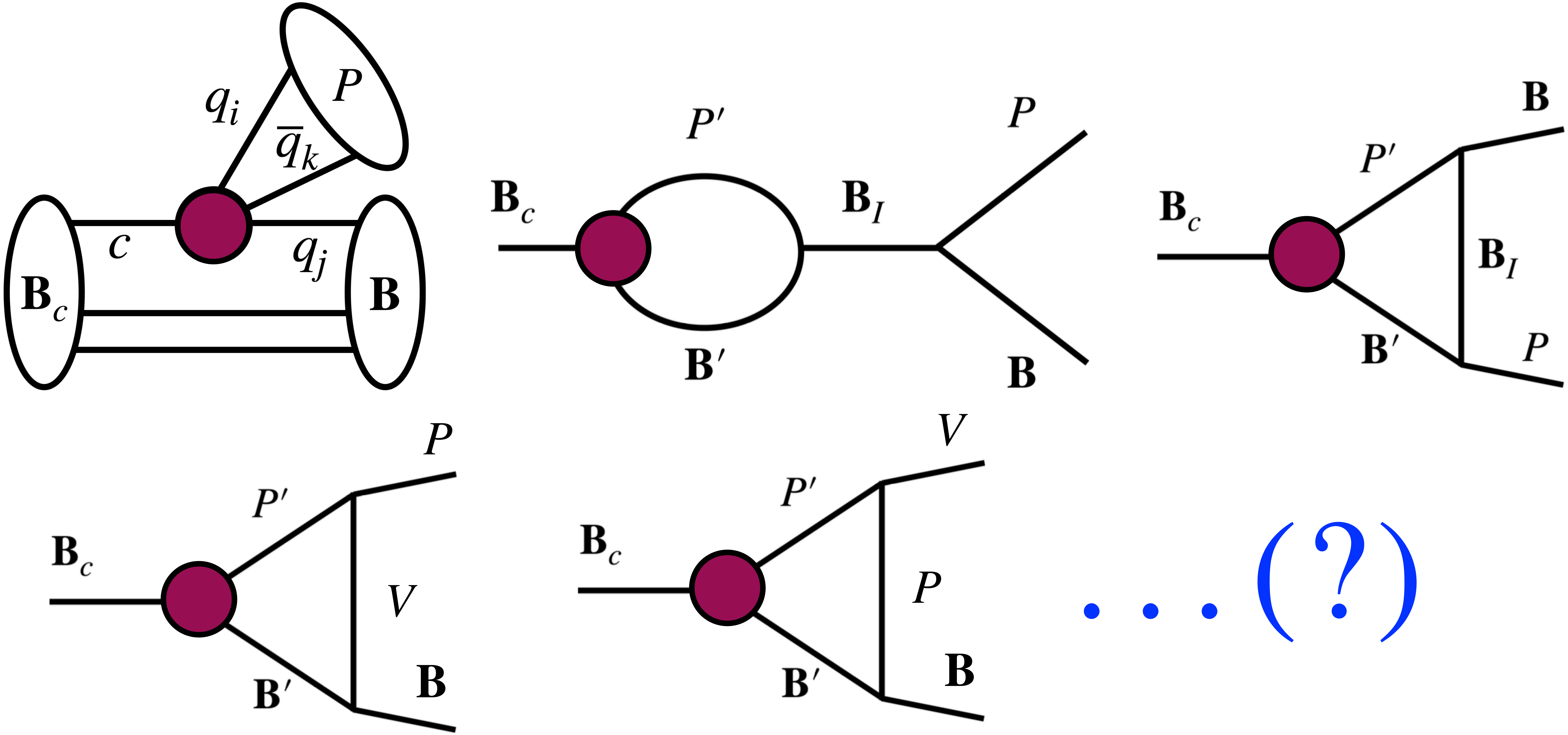
- **Rescattering, solving penguin/tree**

$$\mathscr{L}_{\mathbf{B}_c\mathbf{B}P} = \mathscr{L}^{\mathrm{Tree}}_{\mathbf{B}_c\mathbf{B}P} + \mathscr{L}^{\mathrm{FSR-s}}_{\mathbf{B}_c\mathbf{B}P} + \mathscr{L}^{\mathrm{FSR-t}}_{\mathbf{B}_c\mathbf{B}P} + \mathscr{L}^{\mathrm{FSR-u}}_{\mathbf{B}_c\mathbf{B}P} + \ldots(?)$$



$\ldots(?)$