Gibbsian hydrodynamics

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2307.07021 , 2309.05154 2007.09224 (JHEP),
2109.06389 (Annals of Physics, With T.Dore,M.Shokri,L.Gavassino,D.Montenegro)
Answers somewhat speculative... but I think I am asking good questions!

What is ideal hydrodynamics?

Hydrodynamics is an "effective theory", built around coarse-graining and "fast thermalization". Fast w.r.t. Gradients of coarse-grained variables
If thermalization instantaneus, then isotropy,EoS enough to close evolution

$$T_{\mu\nu} = (e + P(e))u_\mu u_\nu + P(e) g_{\mu\nu}$$

In rest-frame at rest w.r.t. u^μ

$$T_{\mu\nu} = \mathrm{Diag}\left(e(p), p, p, p\right)$$

(NB: For simplicity we assume no conserved charges, $\mu_B = 0$)

This makes sysem solvable: $\partial_\mu T^{\mu\nu} = 0, p = p(e)$

A beautiful, rigorous theory with a direct connection to statistical mechanics, i.e. fundamental physics and maths. Exciting that HIC can be described by it!

If thermalization not instantaneous,

$$T_{\mu\nu} = T_{\mu\nu}^{eq} + \Pi_{\mu\nu} \quad , \quad u_\mu \Pi^{\mu\nu} = 0$$

$$\sum_n \tau_{n\Pi} \partial_\tau^n \Pi_{\mu\nu} = -\Pi_{\mu\nu} + \mathcal{O}\left(\partial u\right) + \mathcal{O}\left((\partial u)^2\right) + ...$$

A series whose "small parameter" (Barring phase transitions/critical points/... all of these these same order):

$$K \sim \frac{l_{micro}}{l_{macro}} \sim \frac{\eta}{sT} \nabla u \sim \frac{\mathrm{Det}\Pi_{\mu\nu}}{\mathrm{Det}T_{\mu\nu}} \sim ...$$

and the transport coefficients calculable from asymptotic correlators of microscopic theory Navier-Stokes $\sim K$, Israel-Stewart $\sim K^2$ etc.
Non-relativistic version still considered beautiful and profound, but with relativity...

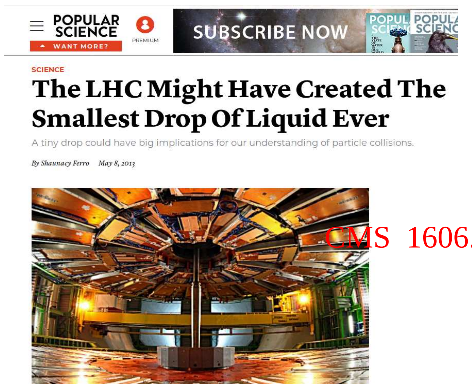
What's wrong with this?

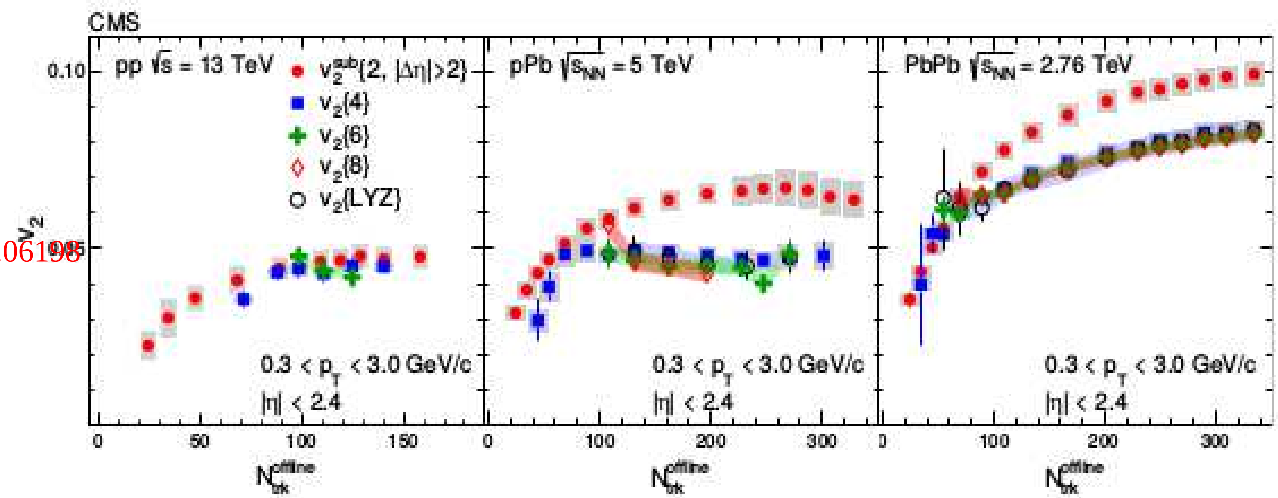
u_μ **ambiguus** many definitions (Landau, Eckart,BDNK...)
We think flow is "clear", so this is a bit strange . choices supposed to be field redefinitions but give slightly different dynamics

$\Pi_{\mu\nu}$ **ambiguus** can even be eliminated as a DOF ($\sim \partial u$ by carefully choosing u_μ (BDNK)

Fluctuations ...

- Defined linearly, whith a Langevin-like fluctuation-dissipation relation which contradicts experiment!





CMS 1606.06198

- Exact theory strongly depends on u_μ convention! Also on pseudogauge! but if field redefinition, does "everything" fluctuate? What if fluctuation of $u_\mu, T, \Pi_{\mu\nu}$ leave $T_{\mu\nu}$ invariant?

More concretely

A theorist will say that fluctuations of e.g. $\delta\Pi_{\mu\nu}, \delta f(x,p)$ produce "non-hydrodynamic modes", sensitive to underlying theries, and hydrodynamics is easy to break down

An experimentalist measures neither $\Pi_{\mu\nu}$ nor f but rather, e.g.

$$\frac{dN}{dyp_T dp_T d\phi} \equiv \frac{dN}{dyp_T dp_T}\left[1 + 2v_n(p_T, y)\cos\left(n\left(\phi - \phi_{0n}\right)\right)\right]$$

i.e. gradients of $T_{\mu\nu}$,entropy and finds hydro everywhere they look!
in a fluctuating medium are "non-hydrodynamic modes" detectable in principle? Can your non-hydro mode be my fluctuating sound-wave?

The two are in a very complicated correspondence which is not $1 \leftrightarrow 1$

Hydrodynamics from microscopic theories

QFT transport coefficients plagued by divergences, need truncation (Schwinger-Keldysh separates "fast", "slow", Kadanoff-Baym needs truncation)

Boltzmann equation Sequential scattering and molecular chaos. Weak coupling, Lose microscopic correlations

AdS/CFT strong coupling and large N_c, lose microscopic correlations

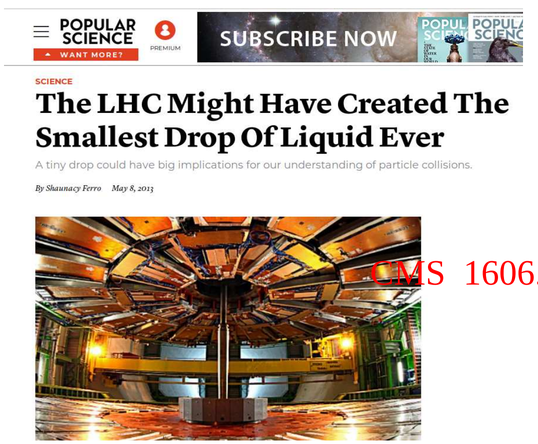
Molecular dynamics keeps microscopic correlations, lose Lorentz invariance (in practice not a problem)

Basic problem with either Lorentz invariance or correlations on scale of gradients! Ambiguity in flow, $\Pi_{\mu\nu}$ comes from here!

In brief most microscopic approaches to EFT hydrodynamics assume that

$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

But this seems falsified by hydrodynamics in small systems!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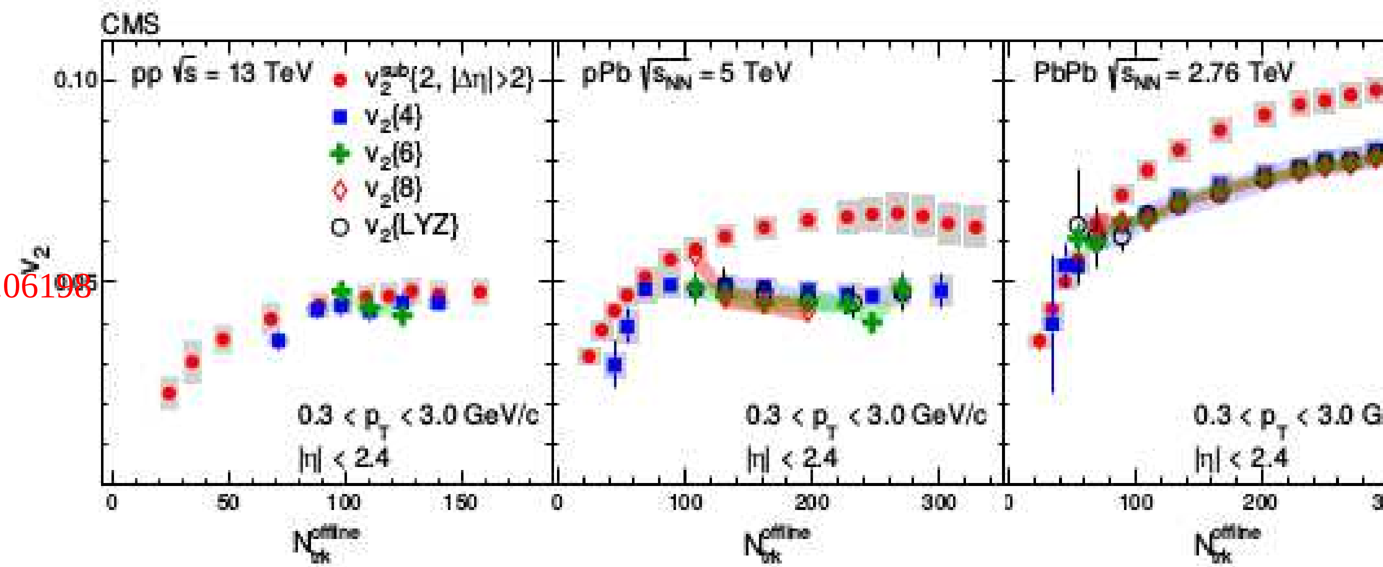
The LHC Might Have Created The Smallest Drop Of Liquid Ever

A tiny drop could have big implications for our understanding of particle collisions.

By Shaunacy Ferro May 8, 2013



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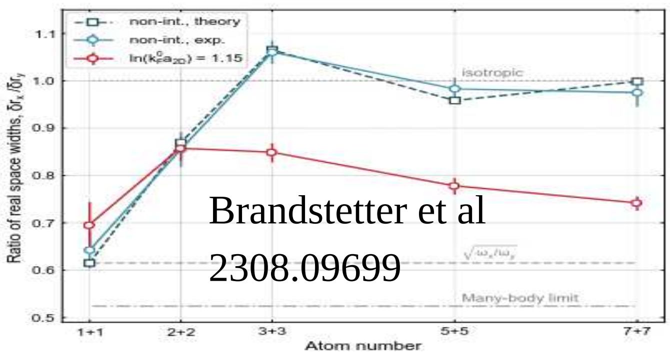
Not just in heavy ions



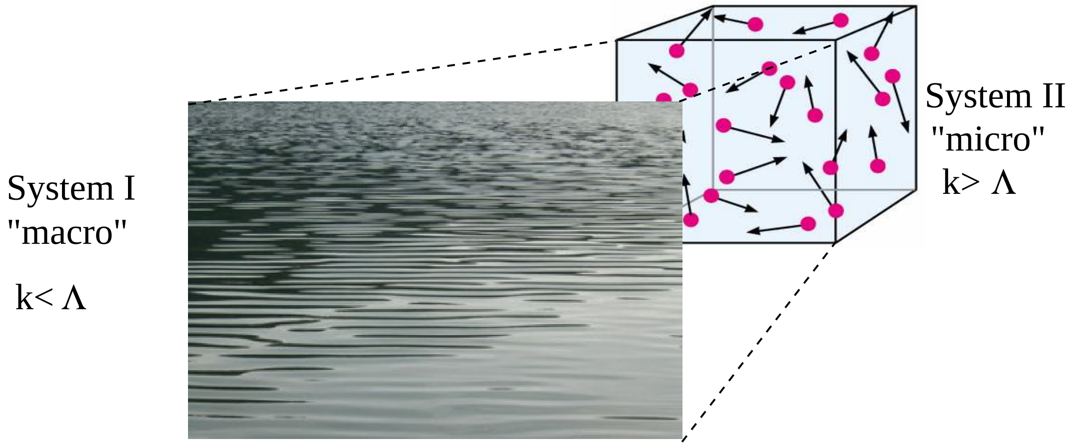
Brandstetter et al
2308.09699

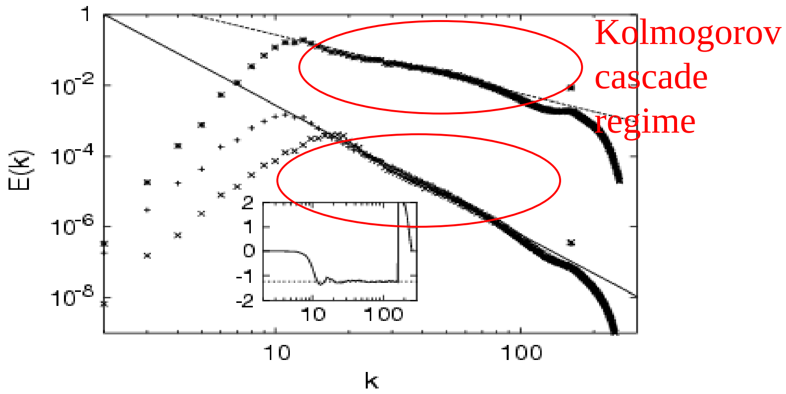
The Brazil nut effect

Empirically, strongly coupled systems with enough thermal energy seem to be "fluid" even with a small number of DoFs. EFT does not explain this!

The role of fluctuations in hydrodynamics, and of the exast relation of statistical physics and hydrodynamics, are still ambiguous and this is related to experimental puzzles

A final issue: Entropy current not clearly connected to energy-momentum current, need microscopic theory to "select good EFT" (2nd law)





At best related to stability (sound waves don't explode) and causality (sound waves $dw/dk \leq c$)

Hydrodynamics and statistical mechanics

Equation of state $p(E)$ comes from basic statistical mechanics

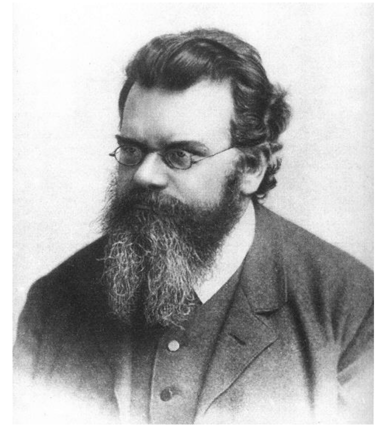
$$p = T \ln \mathcal{Z} \quad , \quad \frac{dP}{dT} = \frac{dS}{dV} = \frac{p + e - \mu n}{T}$$

But the same partition function also predicts fluctuations

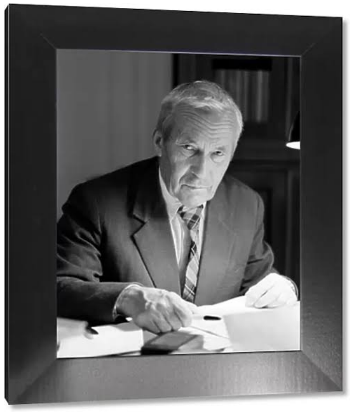
$$\left\langle (\Delta E)^2 \right\rangle = \frac{\partial \ln \mathcal{Z}}{\partial \beta^2} \sim \frac{1}{(\Delta V) \times s}$$

which in a deterministic theory are completely neglected. could this have something to do with the above ambiguity?

the battle of the entropies



Boltzmann entropy (associated with frequentist probability) a property of the "DoF", and is "kinetic" subject to the H-theorem which is really a consequence of the not-so-justified molecular chaos assumption. Gibbsian entropy (more Bayesian) is the log of the area of phase space, and is justified from coarse-graining and ergodicity . The two are different even in equilibrium, with interactions! (Khinchin,stat.mech.) Note, Von Neumann $\langle ln\hat{\rho} \rangle$ Gibbsian . Gibbs is more general, but...

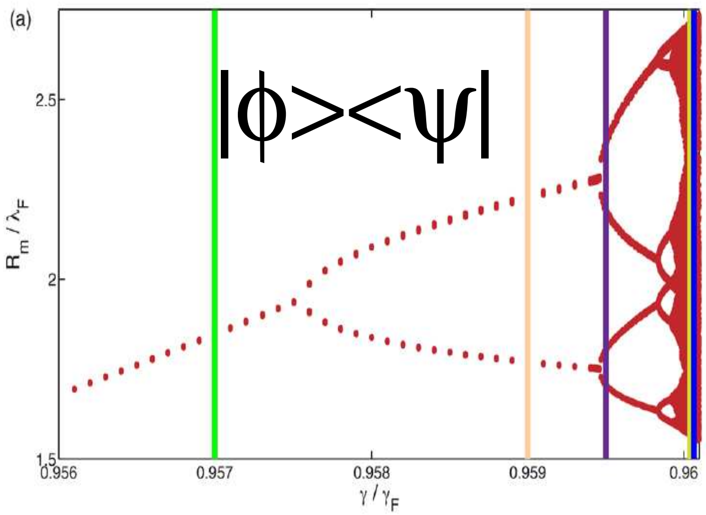


the unreasonable effectiveness of stat mech

Non-ideal hydrodynamics is based around approximate local equilibrium . Boltzmannian global and local equilibrium are defined, but they depend on Boltzmannian physics Only Global equilibrium well defined in Gibbs (what is "approximate maxiumum" Gibbsian entropy?)

Khinchin's "failed" PhD: Stat Mech just seems wrong but seems to apply everywhere! Just like hydro?

QM to rescue? Berry/Bohigas/Eigenstate thermalization

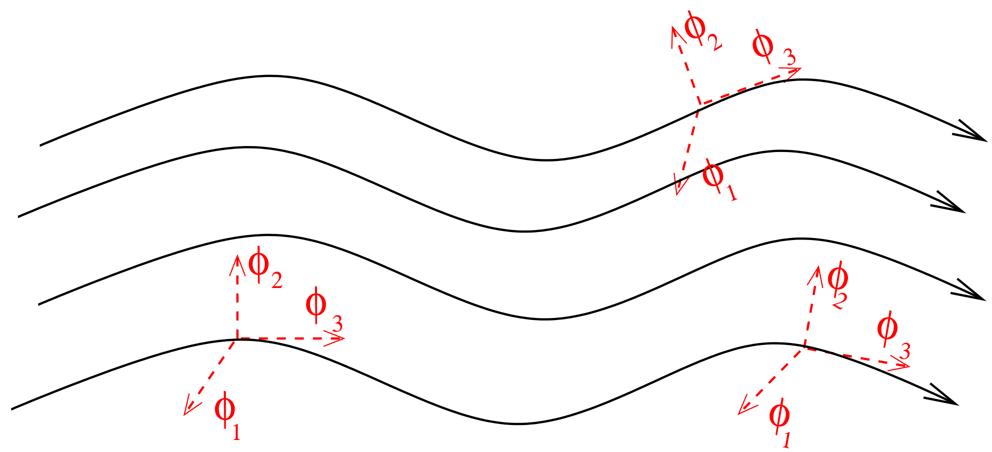


<u>$E_{n>>1}$</u> of quantum systems whose classical correspondent is <u>chaotic</u> have density matrices that look like pseudo-random. If off-diagonal elements oscillate <u>fast</u> or observables <u>simple</u>, <u>indistinguishable</u> from MCE!

But need to coarse-grain, impose causality, and build hydro-like EFT out of this. could be very different from usual EFT expansion!

Let's look at this ambiguity a bit deeper: Lagrangian and Eulerian hydrodynamics Hydro as fields: (Nicolis et al,1011.6396 (JHEP))

Continuus mechanics (fluids, solids, jellies,...) is written in terms of 3-coordinates $\phi_I(x^\mu), I = 1...3$ of the position of a fluid cell originally at $\phi_I(t = 0, x^i), I = 1...3$. (Lagrangian hydro . NB: no conserved charges)



The system is a Fluid if it's Lagrangian obeys some symmetries (Ideal hydrodynamics ↔ Isotropy in comoving frame) Excitations (Sound waves, vortices etc) can be thought of as "Goldstone bosons"

Translation invariance at Lagrangian level $\leftrightarrow$ Lagrangian can only be a function of $B^{IJ} = \partial_\mu \phi^I \partial^\mu \phi^J$ Now we have a "continuus material"!

Homogeneity/Isotropy means the Lagrangian can only be a function of $B = \mathrm{det} B^{IJ}, \mathrm{diag} B^{IJ}$
The comoving fluid cell must not see a "preferred" direction $\Leftarrow SO(3)$ invariance

Invariance under Volume-preserving diffeomorphisms means the Lagrangian can only be a function of B
In all fluids a cell can be infinitesimally deformed
(with this, we have a fluid. If this last requirement is not met, Nicolis et all call this a "Jelly")

A few exercises for the bored public Check that L = -F(B) leads to

$$T_{\mu\nu} = (P+\rho)u_\mu u_\nu - P g_{\mu\nu}$$

provided that

$$\rho = F(B)\ , \qquad p = F(B) - 2F'(B)B\ , \qquad u^\mu = \frac{1}{6\sqrt{B}}\epsilon^{\mu\alpha\beta\gamma}\epsilon_{IJK}\,\partial_\alpha\phi^I\partial_\beta\phi^J\partial_\gamma\phi^K$$

Equation of state chosen by specifying $F(B)$. "Ideal": $\Leftrightarrow F(B) = B^{4/3}$ $\sqrt{B}$ is identified with the entropy and $\sqrt{B}\frac{dF(B)}{dB}$ with the microscopic temperature. u^μ fixed by $u^\mu\partial_\mu\phi^{\forall I} = 0$

Conserved charges (Dubovsky et al, 1107.0731(PRD))

Within Lagrangian field theory a scalar chemical potential is added by adding a $U(1)$ symmetry to system.

$$\phi_I \to \phi_I e^{i\alpha} \quad , \quad L(\phi_I,\alpha) = L(\phi_I,\alpha+y) \quad , \quad J^\mu = \frac{dL}{d\partial_\mu \alpha}$$

generally flow of b and of J not in same direction. Can impose a well-defined u^μ by adding chemical shift symmetry

$$L(\phi_I,\alpha) = L(\phi_I,\alpha+y(\phi_I)) \to L = L\left(b, y = u_\mu \partial^\mu \alpha\right)$$

A comparison with the usual thermodynamics gives us

$$\mu = y \quad , \quad n = dF/dy$$

obviously can generalize to more complicated groups

This looks a bit like GR and this is not a coincidence!

4D local Lorentz invariance becomes local SO(3) invariance

Vierbein $g_{\mu\nu} = \eta^{\alpha\beta} e_\mu^\alpha e_\nu^\beta$ is

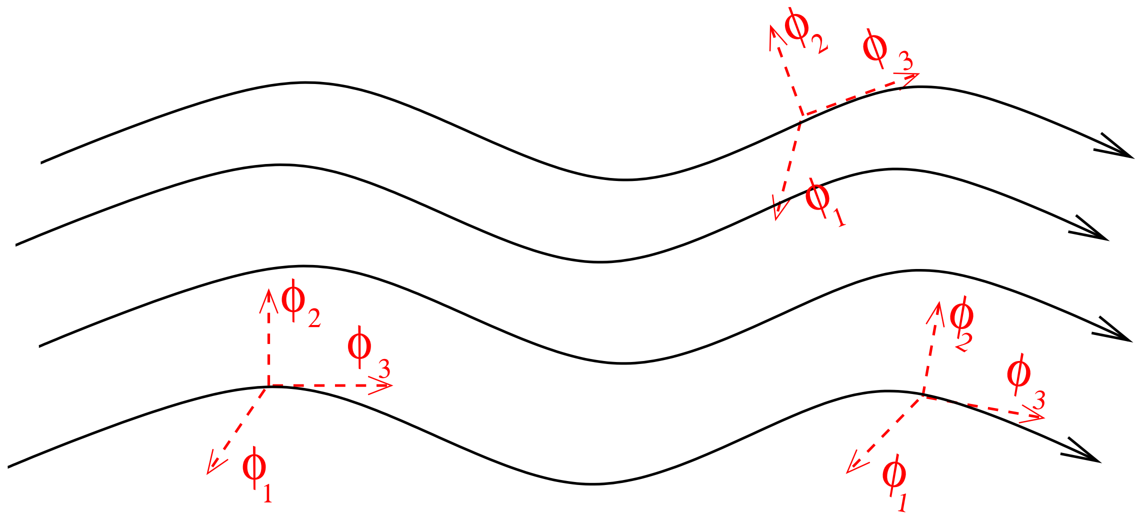
$\frac{\partial x_I^{comoving}}{\partial x_\mu} = \partial_\mu \phi_I$ (with Gauge phase for chemical potential)

Killing vector becomes u_μ

$\mathcal{L} \sim \sqrt{-g}\,(\Lambda + R + ...)$ becomes $\mathcal{L} \sim F(B) \equiv f(\sqrt{-g})$ Just cosmological constant, expanding fluid $\equiv$ dS space

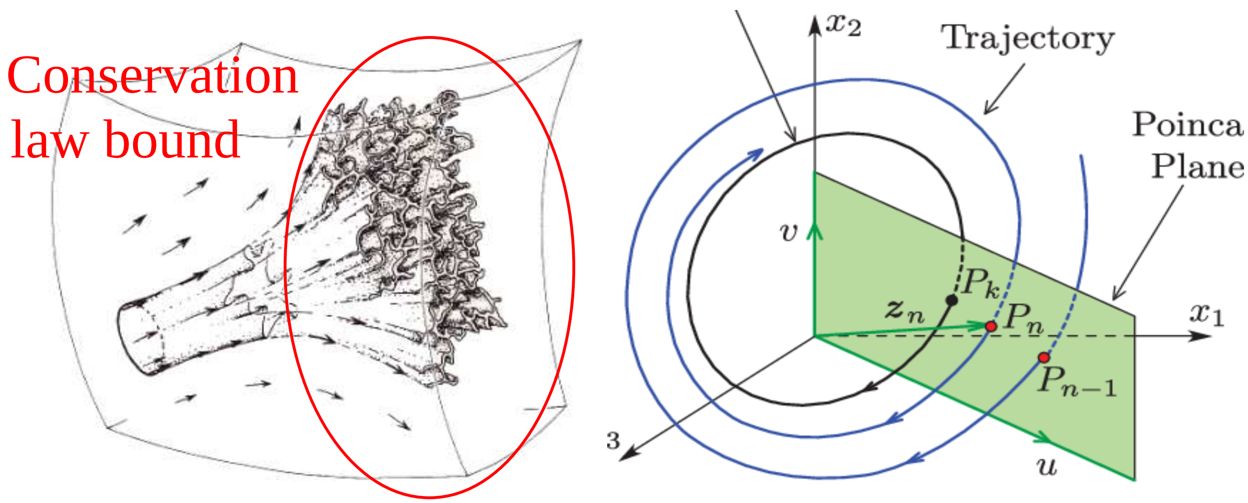
Very nice... but the ambiguities beyond ideal hydro generally break this .
Who cares? Should beyond idel hydrodynamics have this general covariance?

The poor people's quantum gravity: How can fluctuations and dissipation keep hydrodynamic's diffeomorphism invariance?



First step: Lagrangian hydrodynamics very elegant, but where is the connection to local thermalization? Statistical mechanics? Transport? Hint from D.T.Son: it is the largest group of diffeomorphisms <u>where time plays no role!</u>

Where does statistical mechanics come from? Ergodicity



Classical evolution via Hamilton's equations

$$\dot{x} = \frac{\partial H}{\partial p} \quad , \quad \dot{p} = -\frac{\partial H}{\partial x} \quad , \quad \dot{O} = \{O, H\}$$

"Chaos",conservation laws$\rightarrow$ phase space more "fractal", recurring

"After some time", for any observable ergodic limit applies

$$\int_0^{(large)\ \ T} \dot{O}(p,q)dt = \int P(O(p,q))dqdp$$

where $P(...)$ probability independent of time. This probability can only be given by conservation laws

$$P(O) = \frac{\left(\sum_i O_i\right)\delta^4\left(\sum_i P_i^\mu - P^\mu\right)\delta\left(\sum_i Q_i - Q\right)}{N} \quad , \quad N = \int P(O)dO = 1$$

this is the microcanonicanal ensemble. In thermodynamic limit

$$P(O) \to \delta(O - \langle O \rangle)$$

Hydrodynamics is "thermodynamics in every cell

$$\int_0^{(large)\ T} \dot{O}(p,q)dt \to \frac{\Delta \phi}{\Delta t}$$

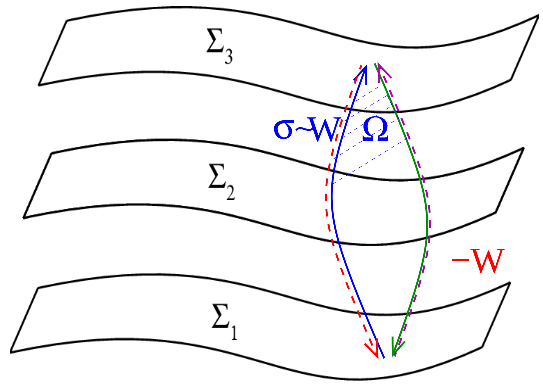
where ϕ is some local observable.

$$\left.\frac{\Delta \phi}{\Delta t}\right|_{t-t'=\Delta} \simeq \frac{1}{d\Omega(Q,E)}\times$$

$$\times \sum \delta^4_{P^\mu, P^\mu_{macro}(t)} \delta_{Q, Q_{macro}(t)} \delta\left(\sum_j^\infty p_j^\mu - P^\mu\right) \delta\left(\sum_j^\infty Q_j - Q\right)$$

Problem: This is not relativistically covariant!

Solution: Foliation!



$$t \to \Sigma_0 \quad , \quad x_\mu \to \Sigma_\mu \quad , \quad \Delta \to \text{``}smooth'' \ \frac{\partial \Sigma_\mu}{\partial \Sigma_\nu}$$

Smooth: $R_{curvature}$ of metric change smaller than "cell size" (New l_{mfp})

$$\frac{\Delta \phi}{\Delta \Sigma_0} = \int P(\phi, \Sigma_\mu) d\Sigma_i \quad , \quad \Sigma_\mu \to \Sigma'_\mu \quad , \quad \frac{\Delta \phi}{\Delta \Sigma'_0} = \frac{\Delta \phi}{\Delta \Sigma_0}$$

What kind of effective lagrangian would enforce

$$\frac{\Delta\phi}{\Delta\Sigma_0} = \int P(\phi, \Sigma_\mu) d\Sigma_i \quad , \quad \frac{\Delta\phi}{\Delta\Sigma_0'} = \frac{\Delta\phi}{\Delta\Sigma_0}$$

with

$$P(...) \sim \delta(\sum_i P_i^\mu - P)\delta(\sum_i Q_i - Q)$$

Now Remember Noether's theorem!

$$p_\mu = \int d^3\Sigma^\nu T_{\mu\nu} \quad , \quad T_{\mu\nu} = \frac{\partial L}{\partial\partial^\mu\phi}\Delta_\nu\phi - g_{\mu\nu}L \quad , \quad \Delta_\nu\phi(x_\mu) = \phi(x_\mu + dx_\nu)$$

$$Q = \int d^3\Sigma^\nu j_\nu \quad , \quad j_\nu = \frac{\partial L}{\partial\partial^\mu\phi}\Delta_\psi\phi \quad , \quad \Delta_\psi\phi = |\phi(x)|e^{i(\psi(x)+\delta\psi(x))}$$

momentum generates spatial translations, conserved charges generate complex rotations!

Space-like foliations decompose

$$d\Sigma_\mu = \epsilon_{\mu\nu\alpha\beta}\frac{\partial\Sigma^\nu}{\partial\Phi_1}\frac{\partial\Sigma^\alpha}{\partial\Phi_2}\frac{\partial\Sigma^\beta}{\partial\Phi_3}d\Phi_1 d\Phi_2 d\Phi_3$$

where the determinant (needed for integrating out $\delta - functions$ is only in the volume part

$$\frac{\partial\Sigma'_\mu}{\partial\Sigma_\nu} = \Lambda_\mu^\nu \det\frac{d\Phi'_I}{d\Phi_J} \quad , \quad \det\Lambda_\mu^\nu = 1$$

Physically, Λ_μ^ν moves between the frame $d\Sigma^\mu_{rest} = d\Phi_1 d\Phi_2 d\Phi_3 (1,\vec{0})$

so lets try

$$\underbrace{L(\phi)}_{microscopic\ \ DoFs} \simeq L_{eff}(\Phi_{1,2,3})$$

with

$$\frac{\Delta\phi}{\Delta\Sigma_0} = \int P(\phi,\Sigma_\mu) d\Sigma_i \quad , \quad P(...) = \delta(...)\delta(...)$$

the general covariance requirement of $\frac{\Delta\phi}{\Delta\Sigma_0} = \frac{\Delta\phi}{\Delta\Sigma_0'}$ means the invariance of the RHS

$$\frac{d\Omega(dP_\mu', dQ', \Sigma_0')}{d\Omega(dP_\mu, dQ, \Sigma_0)} =$$

$$= \frac{d\Sigma_0'}{d\Sigma_0} \frac{\int da_\mu d\psi \delta^4 \left(d\Sigma^\nu a_\alpha \partial^\alpha \left(\delta_\nu^\mu L\right) - dP^\mu(\Sigma_0)\right) \delta\left(d\Sigma^\mu \psi \partial_\mu L - dQ(\Sigma_0)\right)}{\int da_\mu' d\psi' \delta^4 \left(d\Sigma_\nu' a_\alpha' \partial^\alpha \left(\delta_\nu^\mu L\right) - dP_\mu'(\Sigma_0')\right) \delta\left(d\Sigma_\mu' \psi' \partial^\mu L - dQ'(\Sigma_0')\right)}$$

It is then easy to see,via

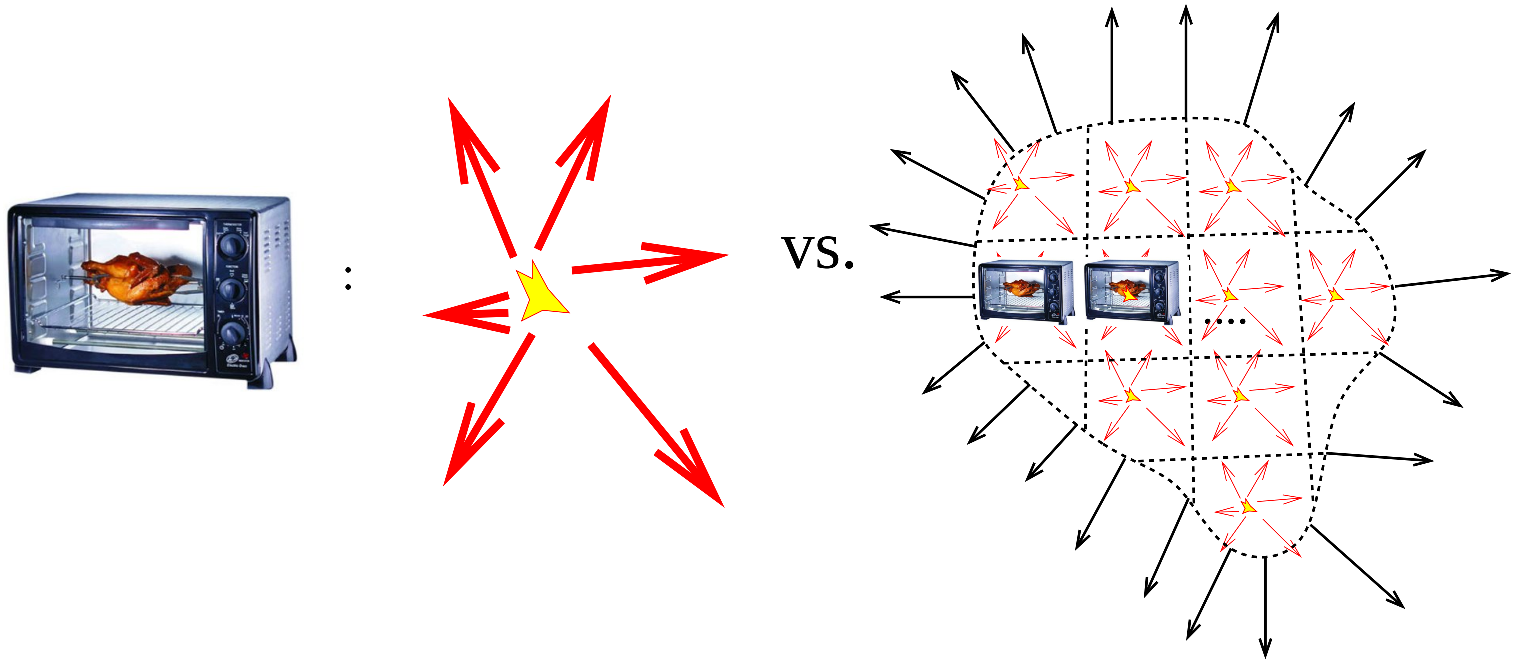
$$\delta((f(x_i))) = \sum_i \frac{\delta(x_i - a_i)}{\underbrace{f'(x_i = a_i)}_{f(a_i)=0}} \quad , \quad \phi'_I = \frac{\partial_\alpha \Sigma'_I}{\partial^\alpha \Sigma^J} \Phi_J \quad , \quad \delta^4(\Sigma_\mu) = \det \left| \frac{\partial \Sigma^\mu}{\partial \Sigma^\nu} \right| \delta^4$$

that for general covariance to hold

$$L(\Phi_I, \psi) = L(\Phi'_I, \psi') \quad , \quad \det \frac{\partial \phi_I}{\partial \phi_J} = 1 \quad , \quad \psi' = \psi + f(\phi_I)$$

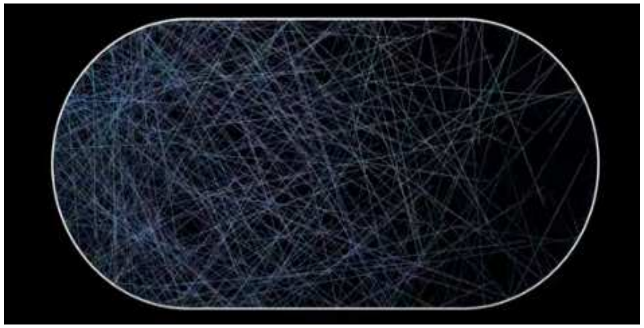
the symmetries of perfect fluid dynamics are equivalent to requiring the ergodic hypothesys to hold for generally covariant causal spacetime foliations!!!!

Classical to quantum F.Becattini, 0901.3643



Berry's conjecture: quantum systems with Chaotic classical counterparts and Above ground state $E_{n \gg 1}$

Density matrix pseudorandom , indistinguishable from microcanonical ensemble. born in equilibrium



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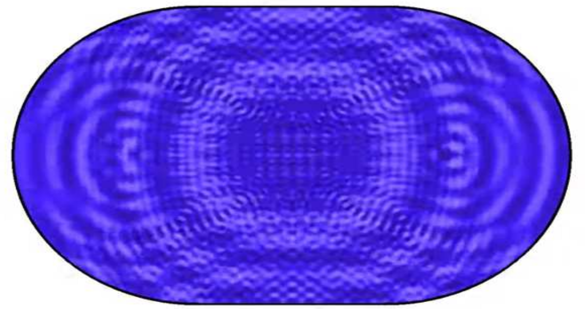
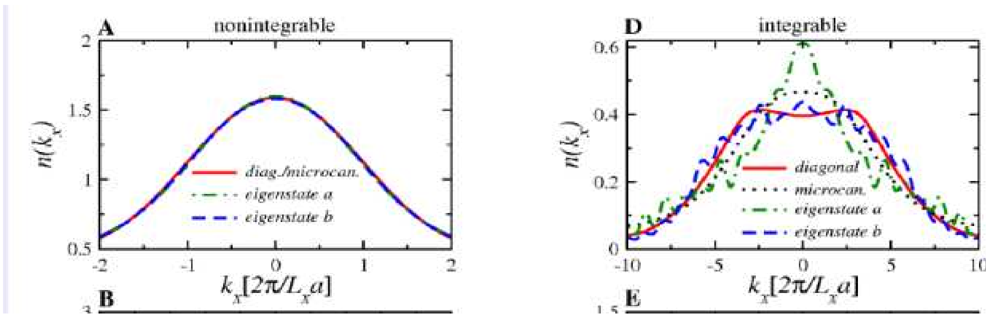




M. Rigol, V. Dunjko, and M. Olshanii,Nature 452, 854 (2008)
Quantum billiard balls very different from classical and semi-classical ones! Any "non-integrability" modifies "initial state" which already "looks thermal". All evolution does is randomize phase . Related to divergences in finite temperature QFT? "loop" corrections to transport hard!

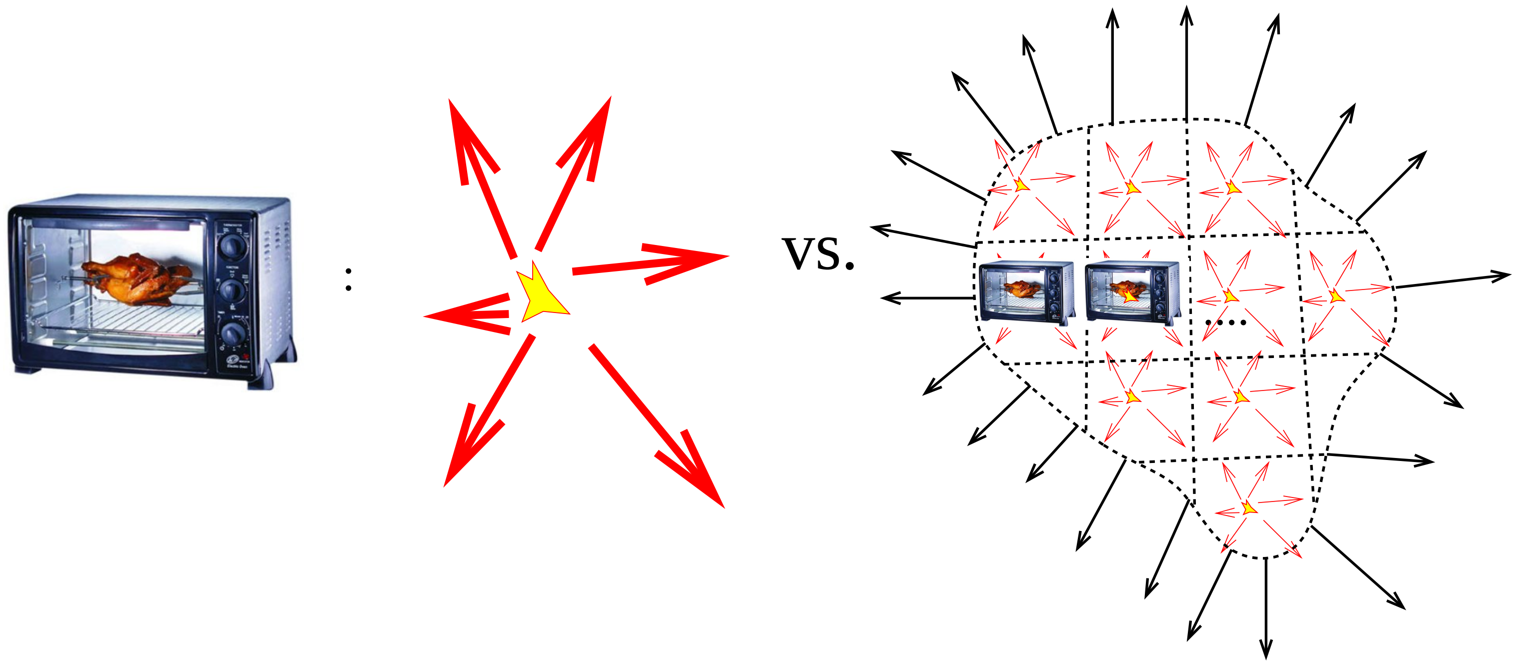
Applying the Eigenstate thermalization hypothesis to every cell in every foliation is equivalent to promoting $J_{\mu\nu}, \theta, P, Q$ to functions of x_μ and imposing foliation independence on the "pseudo-randomness" of $\hat{\rho}$.

$$\left.\frac{d\hat{\rho}}{d\Sigma_0}\right|_{\Sigma_0-\Sigma_0'\simeq\Delta} = 0 \quad , \quad \hat{\rho} \simeq \frac{1}{d\Sigma}\hat{\delta}_{E,E'}\hat{\delta}_{Q,Q'}$$

$$\hat{U}^{-1}(x)\hat{\rho}\hat{U}(x) \simeq \hat{\rho} \quad , \quad \hat{U}(x) = \exp\left[i\hat{T}^{\mu\nu}d^3\Sigma_\mu \delta x_\nu\right]\exp\left[i\partial_\alpha\theta d^3\Sigma^\alpha\delta\hat{Q}\right]$$

for arbitrary $d^3\Sigma_\mu$. Above derivation follows.

So one expects hydro together with statistical hadronization!



So the symmetries of ideal hydrodynamics are equivalent to ideal local ergodicity . So what? turns out one might be able to extend this "close" to equilibrium while retaining these symmetries!

The crucial question: Does this extend to non-ideal hydrodynamics?

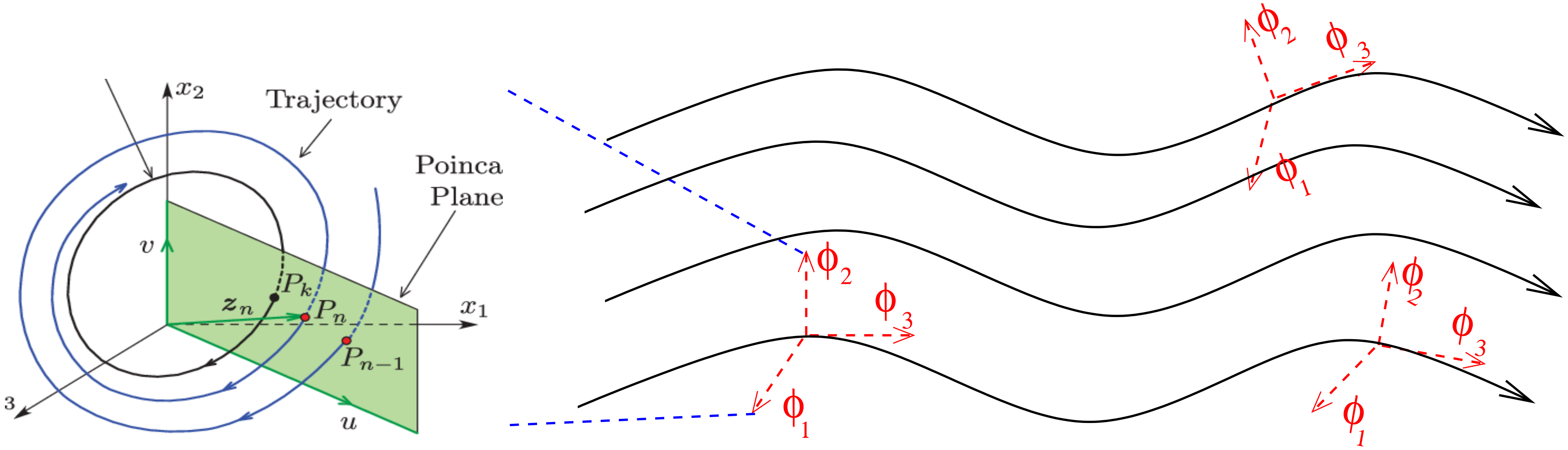
Close to local equilibrium is not on gradient expansion but the approximate applicability of fluctuation-dissipation
These are not automatically the same!

For smaller fluctuating systems many equivalent definitions of $e, u_\mu, J^\mu, \Pi^{\mu\nu}, ...$ leaving $T_{\mu\nu}$ invariant!
Different Boltzmannian entropy but all counted as Gibbsian entropy

If many equivalent choices of $e, u_\mu, J^\mu, \Pi^{\mu\nu}, ...$ likely in one its "small"!
Ideal hydro behavior.

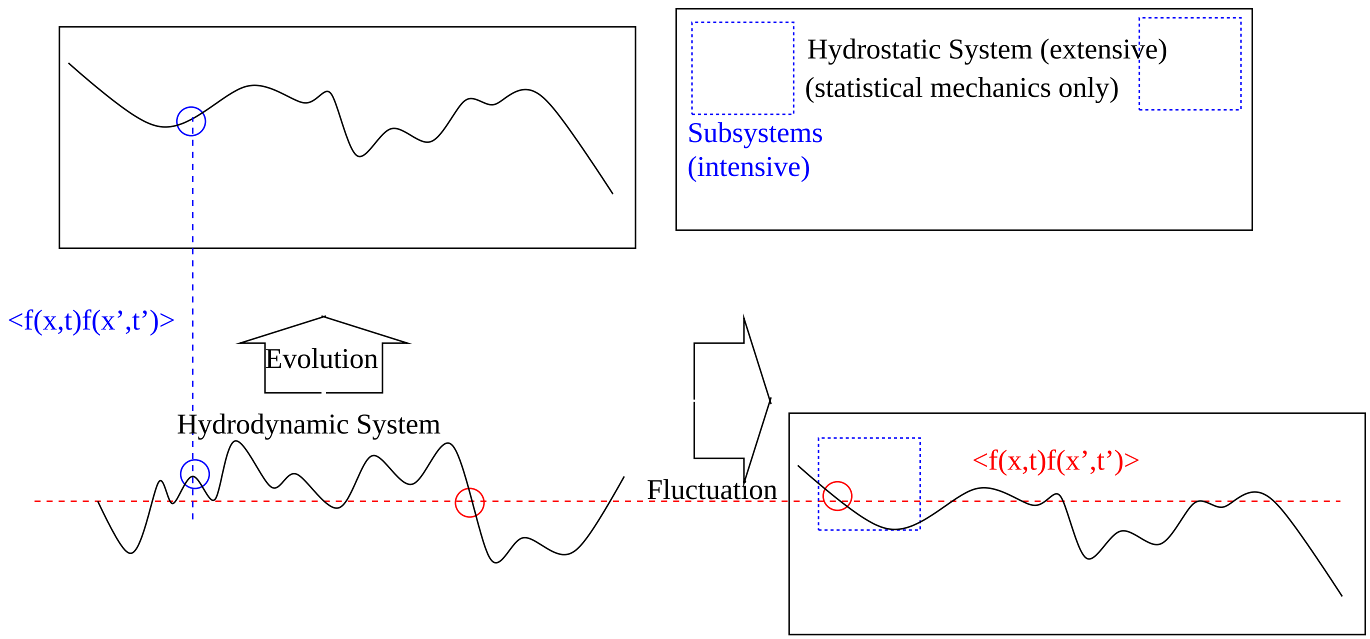
So indeed Ambiguity from fluctuations makes system look like a fluid.

The physical intuition Ergodicity/Poncaire cycles meet relativity slightly away from equilibrium!

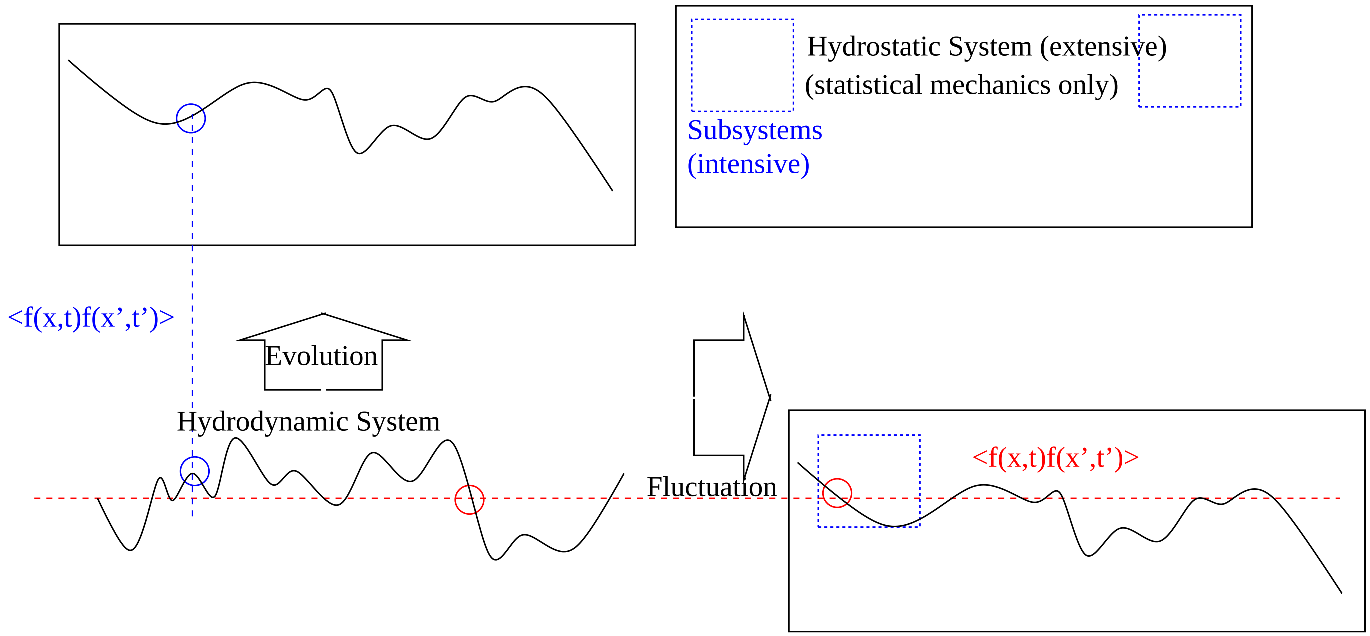


Gibbs entropy level+relativity : Lack of equilibrium is equivalent to "loss of phase" of Poncaire cycles. one can see a slightly out of equilibrium cell <u>either</u> as a "mismatched u_μ" (fluctuation) or as lack of genuine equilibrium (dissipation)

How to make physics fully "gauge"-invariant?



Fluctuation-dissipation at the cell level could do it! We don't know if a "step" is fluctuation ($T_0^{\mu\nu}$ or evolution ($\Pi_{\mu\nu}$)-driven!



But in hydro $T_0^{\mu\nu}, \Pi_{\mu\nu}$ treated very differently! "Sound-wave" $u \sim \exp[ik_\mu x^\mu]$ or "non-hydrodynamic Israel-Stewart mode?"
$D\Pi_{\mu\nu} + \Pi_{\mu\nu} = \partial u$
Only in EFT $1/T \ll l_{mfp}$ they are truly different!

What kind of expansion is this? remember that usual hydro

$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

if $l_{micro} \sim l_{mfp}$ can rotate between them, rotation mimics evolution, microstates and sound-waves not separated. Will not be visible in

- Boltzmann and Wigner function expansion
- Large N_c (or any other microscopic degeneracy)

As there micro and macro cleanly separate. but will be important for small systems

Stability and causality: A new "paradigm"

Stability: for small enough cells ergodicity means system "maximally unstable", stability should emerge in the infrared

Causality: Fluctuations acausal. Criterion for causality should be time-ordered correlations

$$G(x_1,x_2) \equiv \theta(x_1^0 - x_2^0)\,\langle [T_{\mu\nu}(x_1) T_{\alpha\beta}(x_2)] \rangle - x_1 \leftrightarrow x_2$$

all stability and causality analysis would need to be revised

What is a gauge theory,exactly?

$$\mathcal{Z} = \int \mathcal{D}A^\mu \exp\left[S[F_{\mu\nu}]\right] \equiv \int \mathcal{D}A_1^\mu \mathcal{D}A_2^\mu \exp\left[S[A_1^\mu]\right]$$

$A^\mu_{1,2}$ can be separated since physics sensitive to derivatives of $\ln \mathcal{Z}$

$$\ln \mathcal{Z} = \Lambda + \ln \mathcal{Z}_G \quad , \quad Z_G = \int \mathcal{D}\mathcal{A}^\mu \delta\left(G(A^\mu)\right) \exp\left[S(A_\mu)\right]$$

Ghosts come from expanding $\delta(...)$ term. In KMS condition/Zubarev

$$Z = \int \mathcal{D}\phi \quad , \quad "S" \rightarrow d\Sigma_\nu \beta_\mu T^{\mu\nu}$$

Multiple $T_{\mu\nu}(\phi) \rightarrow$ Gauge-like configuration . Related to Phase space fluctuations of ϕ

In summary,what we need is a hydrodynamics...

Manifestly in terms of observable quantities

Diffeomorphism-invariant at the level of fluctuations

Entropy content a scalar

Zubarev partition function for local equilibrium: think of Eigenstate thermalization...

Let us generalize the GC ensemble to a co-moving frame $E/T \to \beta_\mu T_\nu^\mu$

$$\hat{\rho}(T_0^{\mu\nu}(x), \Sigma_\mu, \beta_\mu) = \frac{1}{Z(\Sigma_\mu, \beta_\mu)} \exp\left[- \int_{\Sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}_0^{\mu\nu} \right]$$

Z is a partition function with a field of Lagrange multiplies β_μ , with microscopic and quantum fluctuations included.

Effective action from $\ln[Z]$. Correction to Lagrangian picture?

All normalizations diverge but hey, it's QFT! (Later we resolve this!)

This is perfect global equilibrium. What about imperfect local?

- Two vectors, $d\Sigma_\mu u_\mu T_0^{\mu\nu}$ $d\Sigma_\mu$ foliation choice not clear (with vorticity it can't be parallel to flow everywhere). Physics should be choice independent. If $d\Sigma_\mu$ close to β_μ , $d\Sigma_\mu$ non-inertial

- Dynamics is not clear. Naively partition function can not depend on time (Adiabatically wrt microscopic scale however it could!) Becattini et al, 1902.01089: Gradient expansion in β_μ . Reproduces Euler and Navier-Stokes, but...

  - 2nd order Gradient expansion (Navier stokes) non-causal perhaps...
  - Use Israel-Stewart, $\Pi_{\mu\nu}$ arbitrary perhaps...
  - Foliation $d\Sigma_\mu$ arbitrary but not clear how to link to Arbitrary $\Pi_{\mu\nu}$

- What about fluctuations? Coarse-graining and fluctuations mix? How does one truncate?

An operator formulation $\hat{T}^{\mu\nu} = \hat{T}_0^{\mu\nu} + \hat{\Pi}_{\mu\nu}$
and $\hat{T}_0^{\mu\nu}$ truly in equilibrium! Each microscopic particle "does not know" if it "belongs" to $\hat{T}_0^{\mu\nu}, \hat{\Pi}_{\mu\nu}$

$$\hat{\rho}(T_0^{\mu\nu}(x), \Sigma_\mu, \beta_\mu) = \frac{1}{Z(\Sigma_\mu, \beta_\mu)} \exp\left[- \int_{\Sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}_0^{\mu\nu} \right]$$

describes all cumulants and probabilities

$$\langle T_0^{\mu\nu}(x_1) T_0^{\mu\nu}(x_2) ... T_0^{\mu\nu}(x_n) \rangle = \prod_i \frac{\delta^n}{\delta \beta_\mu(x_i)} \ln Z$$

Equilibrium at "probabilistic" level and KMS Condition obeyed by "part of density matrix" in equilibrium, "expand" around that! An operator constrained by KMS condition is still an operator! $\equiv$ time dependence in interaction picture

Does this make sense? Nishioka, 1801.10352 $\langle x|\,\rho\,|x'\rangle =$

$$= \frac{1}{Z}\int_{\tau=-\infty}^{\tau=\infty}\int \left[\mathcal{D}\phi, \mathcal{D}y(\tau)\,\mathcal{D}y'(\tau)\right]\, e^{-iS(\phi y, y')} \cdot \underbrace{\delta\left[y(0^+)-x'\right]\,\delta\left[y'(0^-)-x\right]}_{\frac{\delta J_i(y(0^+))}{\delta J_i(x')}\frac{\delta J_j(y(0^-))}{\delta J_j(x)}}$$

$$\Rightarrow \frac{\delta^2}{\delta J_i(x)\delta J_j(x')}\ln\left[Z_{T_0}(T^{\mu\nu}, J)\times Z_{\Pi}(J)\right]_{J=J_1(x)+J_2(x')}$$

$J_1(x) + J_2(x')$ chosen to respect Matsubara conditions!

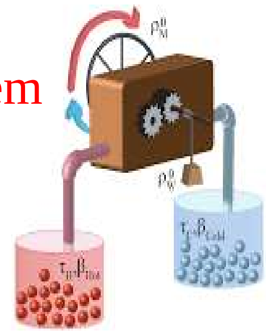
Any ρ can be separated like this for <u>any</u> β_μ . The question is, is this a good approximation? "Close enough to equilibrium"

The source J related to the smearing in "weak solutions". Pure maths angle?

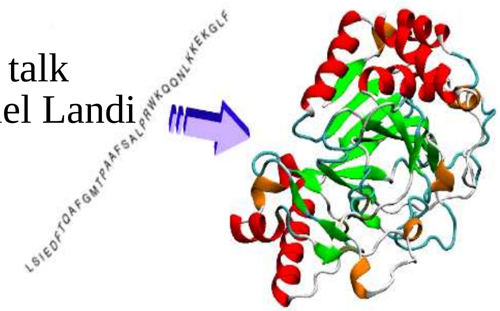
How to go forward... Crooks fluctuation theorem

Crooks fluctuation theorem

$P(W)/P(-W) = e^{\Delta s}$



From talk
Gabriel Landi



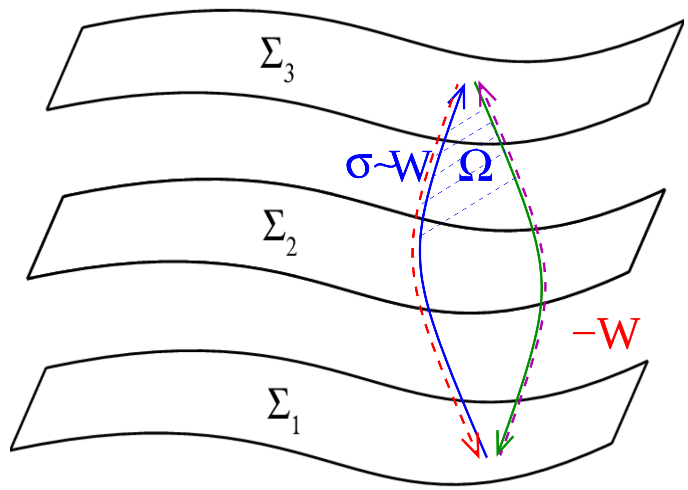
Relates fluctuations, entropy in <u>small</u> <u>fluctuating</u> systems (Nano,proteins)

P(W) Probability system doing work in its usual thermal evolution

P(-W) Probability of the same system “running in reverse” and decreasing entropy due to a <u>thermal fluctuation</u>

ΔS Entropy produced by $P(W)$

How to go forward... Crooks fluctuation theorem redtextApplying Crooks theorem to Zubarev hydrodynamics: Stokes theorem



$$-\int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu} \beta_\nu \right) = -\int_{\Sigma(\tau')} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu} \beta_\nu \right) + \int_\Omega \mathrm{d}\Omega \left(\widehat{T}^{\mu\nu} \nabla_\mu \beta_\nu \right),$$

In a past work (2007.09224,JHEP) I have shown Zubarev+Crooks fluctuation theorem has right limits and symmetries But highly non-local and non-linear, "lattice" .

A simpler EFT: A Gaussian approximation

General covariance via the Gravitational Ward identity

Gaussian approximation from Zubarev hydrodynamics

Kramers-Konig to enforce fluctuation-dissipation

The gravitational ward identity $\nabla \mathcal{W} = 0$

$$\mathcal{W} = G^{\mu\nu,\alpha\beta}\left(\Sigma_\mu, \Sigma'_\nu\right) - \frac{1}{\sqrt{g}}\delta\left(\Sigma' - \Sigma\right) \times$$

$$\times \left(g^{\beta\mu} \left\langle \hat{T}^{\alpha\nu}\left(x'\right) \right\rangle_\Sigma + g^{\beta\nu} \left\langle \hat{T}^{\alpha\mu}\left(x'\right) \right\rangle_\Sigma - g^{\beta\alpha} \left\langle \hat{T}^{\mu\nu}\left(x'\right) \right\rangle_\Sigma \right)$$

Fancy name and complicated but consequence of elementary properties of the metric and energy conservation

$$\partial_\mu T^{\mu\nu} + \Gamma_{\nu\alpha\beta} T^{\alpha\beta} = 0 \quad , \quad \left\langle T^n_{\mu\nu} \right\rangle = \frac{\delta^n}{\sqrt{-g}\delta g^{\mu\nu(n)}} \ln \mathcal{Z}$$

Cumulant expansion: An possibly diffeomorphism invariant alternative to gradient expansion which isn't't!

$$\ln \mathcal{Z} \simeq \ln \mathcal{Z}|_0 - \left.\frac{\partial^2 \ln \mathcal{Z}}{\partial \beta_\mu \partial \beta_\nu}\right|_0 [\ d\Sigma_\alpha d\Sigma'_\tau \left(T^{\mu\alpha}(\Sigma) - \langle T^{\mu\alpha}(\Sigma')\rangle\right)\left(T^{\nu\tau}(\Sigma) - \langle T^{\nu\tau}(\Sigma')\rangle\right)]$$

A covariantization of

$$\left\langle E^2 \right\rangle - \left\langle E \right\rangle^2 \equiv C_V T \Rightarrow\Rightarrow C_{\alpha\beta\mu\nu} \sim \left.\frac{\partial \ln \mathcal{Z}}{\partial \beta_\mu \partial \beta_\nu}\right|_0 F(\Sigma)_{\alpha\beta}$$

This way metric tensor propagator can be modelled as a Gaussian

$$f(...) \sim \prod_{\Sigma(x),\Sigma(x')} \exp\left[-\frac{1}{2}\left(T_{\mu\nu}(\Sigma(x')) - \langle T_{\mu\nu}(\Sigma(x'))\rangle\right) C^{\mu\nu\alpha\beta}(\Sigma(x),\Sigma(x'))\left(T_{\alpha\beta}(\Sigma\right.$$

and Ward identity imposed on width $C_{\alpha\beta\gamma\nu}$.

fluctuation-dissipation relation From Kramers-Konig relations

$$\mathrm{Im}\left[\tilde{\mathcal{G}}^{\mu\nu,\alpha\beta}(\omega,k)\right] = -\frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{\mathrm{Re}\left[\tilde{\mathcal{G}}^{\mu\nu,\alpha\beta}(\omega,k)\right]}{\omega'-\omega}d\omega'$$

$$\mathrm{Re}\left[\tilde{\mathcal{G}}^{\mu\nu,\alpha\beta}(\omega,k)\right] = \frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\frac{\mathrm{Im}\left[\tilde{\mathcal{G}}^{\mu\nu,\alpha\beta}(\omega,k)\right]}{\omega'-\omega}d\omega'$$

Direct consequence of causality, relate the real and imaginary part of the response function in momentum space But non-local in frequency, generally invalidates gradient expansion! (inherently breaks fluctuation-dissipation)

Apply on the linear response function of energy-momentum tensor

$$T_{\mu\nu}(\Sigma) = \int e^{\epsilon\Sigma_0} G^{\mu\nu,\alpha\beta}(\Sigma_0' - \Sigma_0)\delta g_{\alpha\beta}(\Sigma_0')d\Sigma_0$$

$$\tilde{\mathcal{G}}^{\mu\nu\alpha\beta} = \frac{1}{2i}\left(\frac{\tilde{G}^{\alpha\beta\mu\nu}(\Sigma_0, k)}{\tilde{G}^{\alpha\beta\mu\nu}(-i\epsilon\Sigma_0, k)} - 1\right)$$

These equations together should do it!

Only in terms of $T_{\mu\nu}, J_\mu, \Sigma_\mu$ "observables" and a "gauge" reditemSecond law imposed via fluctuation dissipation (redundances, fluctuations of observables)

Conclusions

Fluctuations in non-ideal hydrodynamics not well understood

Intimately related to entropy current, double counting of DoFs
Could alter fluctuation-dissipation expectation, ”fluctuations help dissipate”, in analogy to Gauge theory

Approximate local equilibrium not understood in Gibbsian picture
My proposal: applicability of fluctuation-dissipation

Need a covariant description purely in terms of observable quantities
Ergodicity works in ideal hydro, Crooks theorem/K-K beyond it?

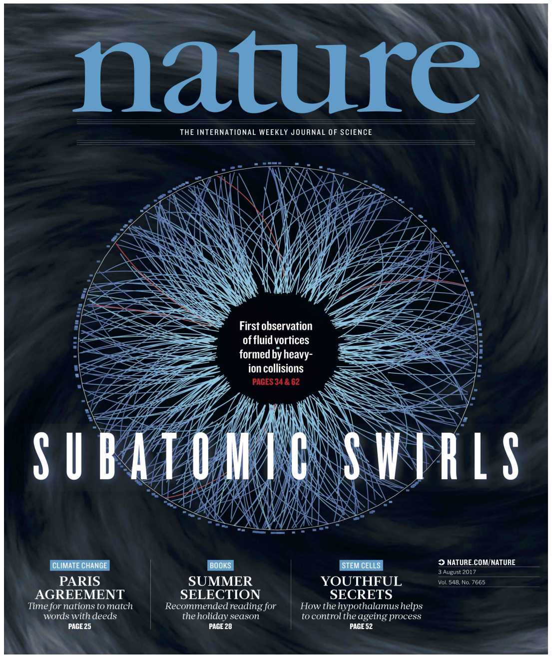
Could be relevant for hydro in small systems

PS: Onto spin hydrodynamics

STAR collaboration
1701.06657

NATURE
August 2017

Polarization by vorticity
in heavy ion collisions



Could give new talk about this, but will mention hydro with spin not developed and a lot of conceptual debates.

- "at what order in Gradient" are spin-vorticity interactions? Causality constraints ,"minimum viscosity", "same order as fluctuations" (microstates).
  Spin hydrodynamics is transfer of micro to macro DoFs

- Transport description inherently "non-local" (violation of ensemble average/molecular chaos)

- Pseudogauge! Spin part of angular momentum not uniquely defined!

- "trivial" in a sense: Let $\Phi^{\alpha\beta\gamma}$ be fully antisymmetric

$$T_{\mu\nu} \to T'_{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda,\mu\nu} + \Phi^{\mu,\nu\lambda} + \Phi^{\nu,\mu\lambda}\right) \quad , \quad \partial^\mu T_{\mu\nu} = \partial^\mu T'_{\mu\nu} = 0$$

- Can move around spin and angular momentum
- Can symmetrize $T_{\mu\nu}$ (good for gravity, bad for equilibrium spin-orbit)
- For particles $\vec{S} = \sum_i \vec{S}_i$ but remember, spin violation of molecolar chaos
- Not clear if dynamics should depend on it! Most approaches pseudo-gauge covariant but Entropy usually does, hence fluctuations!
- Spin 1: Pseudogauge $\to$ Gauge symmetry "ghosts"? GT,1810.12468

Pseudo-gauge symmetries physical interpretation: T.Brauner, 1910.12224

$$x^\mu \to x^\mu + \epsilon \zeta^\mu(x) \quad , \quad \psi_a \to \psi_a + \epsilon \psi'_a \quad , \quad \mathcal{L} \to \mathcal{L}$$

For particles field redefinition "observable", but what about for fluctuating fields?
Entropy depends on pseudogauge as spin-orbit interactions mix entropyless vortices with entropyful spin microstates

Previous picture offers a way out! Pseudo-gauge transformations could be exactly the sort of equations that produce redundancies!
$\ln \mathcal{Z}|_{class}$ not invariant but full $\ln \mathcal{Z}$ should be! Spin $\leftrightarrow$ fluctuation, need equivalent of DSE equations!

Basic idea: Define ensemble via $\ln \mathcal{Z}$ and "gauge constraints" so that pseudo-gauge transformations "move aorund" the ensemble

How to see it: Grossi,Floerchinger, 2102.11098 (PRD) Let us define a J co-moving with u_μ and use the "exact" (before coarse-graining) partition function to build

$$\Gamma(\phi) = \mathrm{Sup_J}\left(\int J(x)\phi(x) - i\ln \mathcal{Z}[J]\right)$$

$u_\mu \rightarrow u'_\mu$ non-inertial and does not change $\langle T_{\mu\nu}\rangle$, so one can define

$$J_{\mu\nu\gamma} = \frac{1}{\sqrt{g}}\frac{\delta \ln \mathcal{Z}[J']}{\delta \Gamma^{\alpha\nu\gamma}} \quad , \quad D_\mu J^{\mu\nu\gamma} = 0$$

Setting the gauge at the level of the microscopic approximately thermalized partition function equivalent adding auxiliary field $D_\mu M_{\alpha\beta} = 0$ to

$$\mathcal{Z}[J_{\alpha\beta\gamma}] = \int \mathcal{D}\phi\mathcal{D}M_{\alpha\beta}\exp\left[\int det[M] d^4x \mathcal{L}\left(\phi, \partial_\mu + \Gamma ...\right) + \int d\Sigma^\gamma M^{\alpha\beta} J_{\alpha\beta\gamma}\right]$$

SPARE SLIDES