Strong interaction in the quantum era

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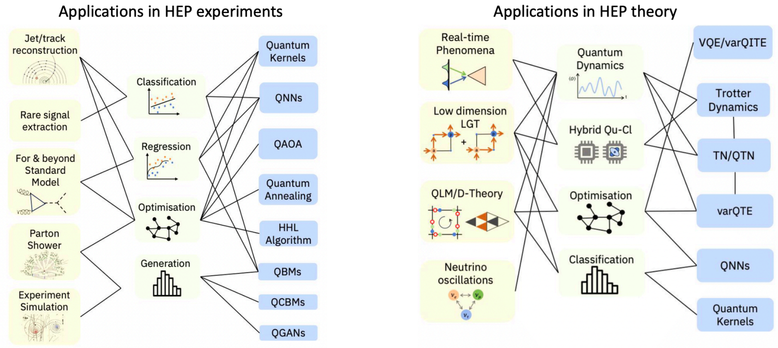
Quantum computing for high-energy physics

[Reviews: 2203.08805, 2307.03236]

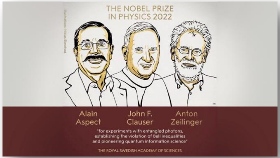
- Quantum algorithms provides theoretical speedup to overcome the large data bottleneck in HEP experiments
- Quantum simulation algorithm provides an ultimate path to overcome the computational complexity in simulating quantum field theories (QFTs)



Bell Inequality Violation & Entanglement in e^+e^- annihilation

➢ Motivation

- In history, the study of quantum information is initially focused on photonic system.
- With the progress of HEP, quantum information can also be investigated in high-energy particles at colliders.



2022 Nobel prize


Article | Open access | Published: 18 September 2024

Observation of quantum entanglement with top quarks at the ATLAS detector

The ATLAS Collaboration

Nature **633**, 542–547 (2024) | Cite this article

See ref. [The ATLAS Collaboration, Nature, 633, 542-547 (2024)].

➢ $e^+e^- \to \gamma^* \to Y\bar{Y}$ at BEPCII

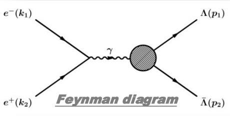
- Bell inequality violation (BIV) and entanglement can be probed in the hyperon-antihyperon system, with $Y\bar{Y} = \Lambda\bar{\Lambda}, \Sigma\bar{\Sigma}, \Xi\bar{\Xi}$.

P^-: spin polarization($\bar{Y}$)

$$\Theta_{\mu\nu} = \frac{1}{1+\alpha_\psi \cos^2\vartheta} \begin{bmatrix} 1+\alpha_\psi\cos^2\vartheta & 0 & \beta_\psi \sin\vartheta\cos\vartheta & 0 \\ 0 & \sin^2\vartheta & 0 & \gamma_\psi\sin\vartheta\cos\vartheta \\ \beta_\psi\sin\vartheta\cos\vartheta & 0 & -\alpha_\psi\sin^2\vartheta & 0 \\ 0 & \gamma_\psi\sin\vartheta\cos\vartheta & 0 & \alpha_\psi+\cos^2\vartheta \end{bmatrix}$$

P^+: spin polarization(Y) C_{ij}: spin correlation($Y\bar{Y}$)

- The investigation is based on the **two-qubit density operator** for final hyperon systems.



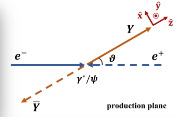


Table I. Some parameters in $e^+e^- \to J/\psi \to Y\bar{Y}$, where $Y\bar{Y}$ is a pair of ground-state octet hyperons.

	$\mathcal{B}(\times 10^{-4})$	α_ψ	$\Delta\Phi$/rad	Ref
$\Lambda\bar{\Lambda}$	19.43(33)	0.475(4)	0.752(8)	[30, 41]
$\Sigma^+\bar{\Sigma}^-$	15.0(24)	−0.508(7)	−0.270(15)	[42, 43]
$\Xi^-\bar{\Xi}^+$	9.7(8)	0.586(16)	1.213(49)	[31, 44]
$\Xi^0\bar{\Xi}^0$	11.65(4)	0.514(16)	1.168(26)	[45, 46]

~~Strong interaction in the quantum era~~

Hamiltonian approach to strongly coupled quantum field theories suitable for quantum computing

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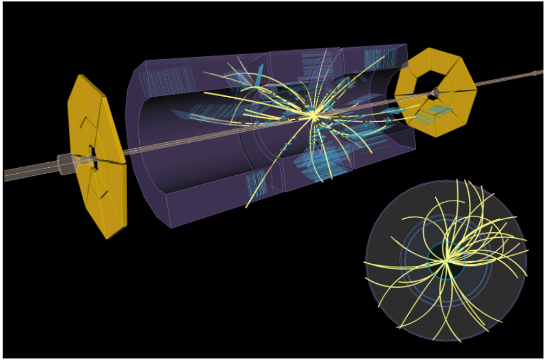
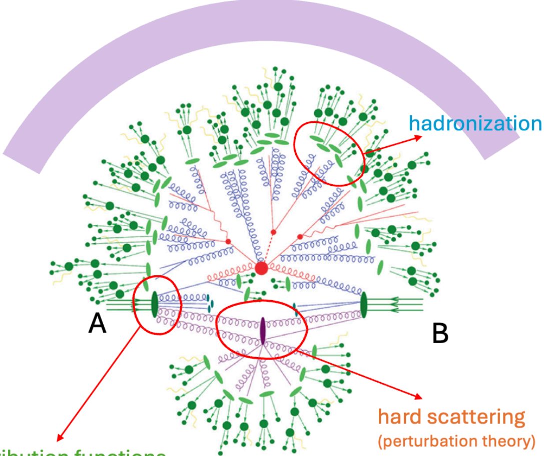


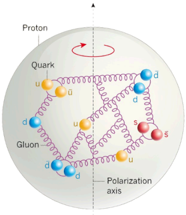


$$\sigma = \sum_{ab} \int_0^1 dx_a dx_b \, f_{a/A}(x_a, \mu_F) f_{b/B}(x_b, \mu_F) \hat{\sigma}_{ab \to X}$$

Parton picture

- Parton distribution function (PDF) $f(x)$ describes the probability density of finding a collinear parton of longitudinal momentum faction x
- Non-perturbative $\rightarrow$ obtained from global fits

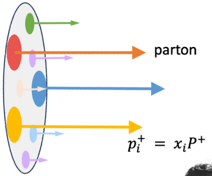
quark model, Gell-Mann 1964



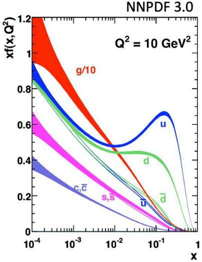
$T_{\text{int}} \sim \Lambda_{QCD}^{-1} = 10^{-23}\text{s}$
quarks are strongly coupled

$P \rightarrow \infty$

parton model, Feynman 1969



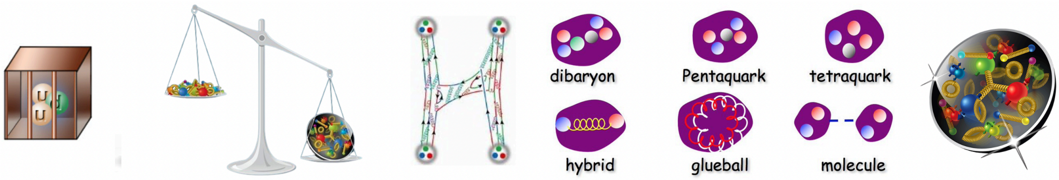
$T_{\text{int}} \sim \gamma\Lambda_{QCD}^{-1} \gg 10^{-23}\text{s}$
partons are free!



Parton entropy

$$S_{part} = -\int_0^1 \mathrm{d}x\, f(x) \log f(x) > 0.$$

- From parton distribution $f(x)$, we can obtain a non-zero parton entropy S_{part}
- But, the proton is a pure state $\rho = |\psi\rangle\langle\psi|$ with vanishing von Neumann entropy
- Entropy = loss of information: some quantum information of the proton is lost in the parton picture
- Related to some of the big puzzles in high-energy physics? confinement, origin of mass, origin of exotic hadrons etc

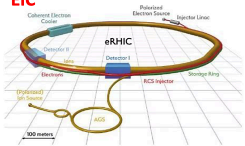


QCD as the next frontier

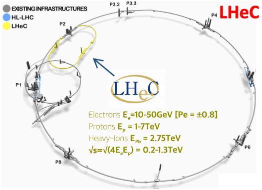
[Reviews: 50 years of QCD, EPJC 2023]

- Electron-ion colliders: eRHIC, EicC, LHeC [AbdulKhalek:2021gbh, Anderle:2021wcy, LHeC:2020van]
- Electron-positron colliders: superKEKB, Super τ-charm facility (STCF) [Peng:2020orp]
- Heavy ion colliders: NICA, HIAF, FAIR

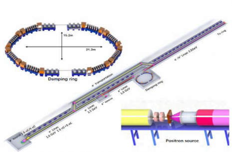
EIC



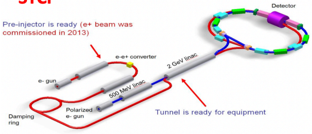
LHeC



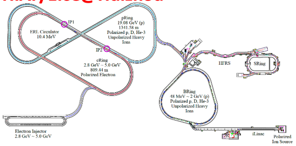
STCF@Hefei



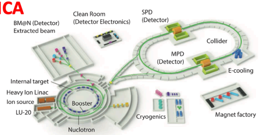
STCF



HIAF/EicC@Huizhou



NICA



Light-Front Hamiltonian formalism

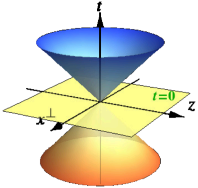
[Reviews: Brodsky '98, Burkardt '02, Bakker '14, Ji '21]

- Full quantum information is contained in the hadronic wave function $|\Psi\rangle$

$$i\frac{\partial}{\partial x^\mu}|\Psi\rangle = P_\mu|\Psi\rangle, \quad (\mu = 0, 1, 2, 3)$$

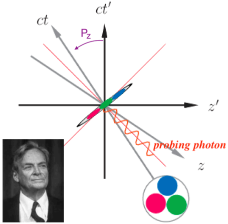
- Relativity allows a free choice of the time variable, e.g. x^0, $x^+ = x^0 + x^3$
- Infinite momentum frame = light-front quantization: retaining the partonic picture

equal-time quantization
$t = 0$

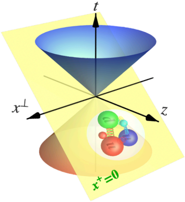


$$i\frac{\partial}{\partial t}|\Psi\rangle = P^0|\Psi\rangle$$

infinite momentum frame
$P_z \to \infty$



light-front quantization
$x^+ = t + z/c = 0$

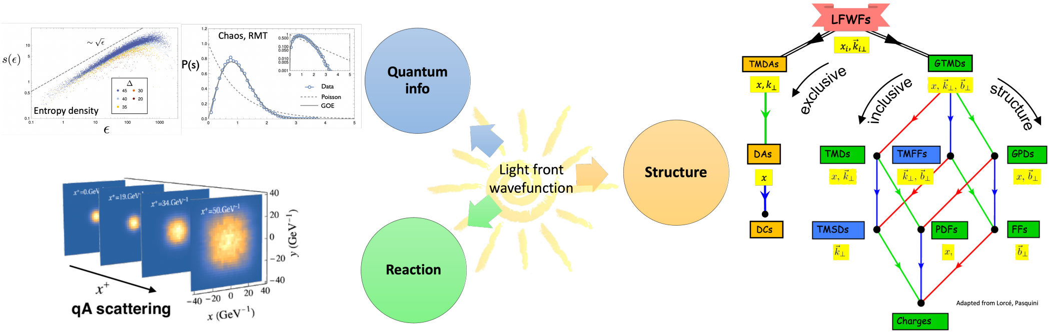


$$i\frac{\partial}{\partial x^+}|\Psi\rangle = P_+|\Psi\rangle$$

Light-front wave functions (LFWFs)

[Reviews: Brodsky:1997de; Anand:2020gnn]

- Light-front physics underlines hadron structures measured in high-energy scattering experiments
- Light-front wave functions provide the full quantum information of hadrons



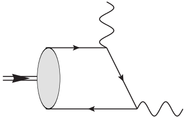
Other approaches to access the LFWFs:

[Chang:2013pq, Yang:2018nqn, dePaula:2022pcb, Eichmann:2021vnj, Frederico:2019noo, Ji:2013dva, Radyushkin:2017cyf, Ma:2021yqx]

Example: diphoton decay $P \to \gamma\gamma^*$

[Lepage:1980fj, Li:2021ejv]

$$\mathcal{M}^{\mu\nu}_{P\to\gamma\gamma^*} = 4\pi\alpha_{\rm em}\varepsilon^{\mu\nu\rho\sigma}q_\rho n_\sigma F_{P\gamma}(-q^2)$$

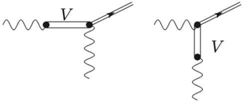


$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_C}\int\frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\psi^*_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp)}{k_\perp^2+m_f^2+x(1-x)Q^2} + \cdots\cdots$$

light-front wave function

$Q \to 0$ | $Q \to \infty$

$$\sum_V \frac{e_f^2 f_V}{1+\frac{M_P}{M_V}}\frac{g_V(0)}{M_V^2+Q^2}$$

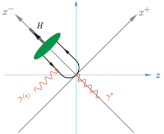


vector meson dominance

(hadron picture)

parton distribution amplitude

$$e_f^2 f_P\int_0^1 \mathrm{d}x\frac{\phi_P(x,\mu)}{x(1-x)Q^2+m_f^2}$$



light-cone dominance

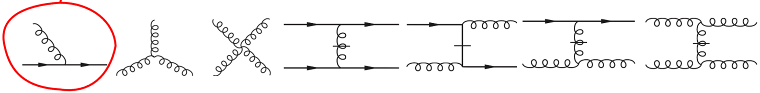
(parton picture)

Light-front wave functions bridge the parton picture and the non-perturbative hadronic picture

Proton as a strongly coupled relativistic quantum many-body system

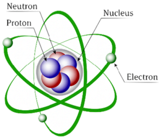
$$H_{\rm LF} = \sum_i \frac{\vec{p}_{i\perp}^{\,2} + m_i^2}{x_i} + \frac{1}{2}\sum_{i,j} V_{ij} + \frac{1}{3!}\sum_{i,j,k} V_{ijk}$$

$$V = \sum_i g_s \bar{\psi}_i \gamma_\mu \psi_i A_i^\mu = \sum_{\alpha\beta\gamma} g_{\alpha\beta\gamma} b_\alpha^\dagger b_\beta a_\gamma$$



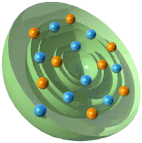
$$H|\Psi\rangle = E|\Psi\rangle$$

Atoms



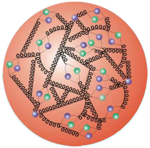
non-relativistic
weakly coupled

Nuclei



non-relativistic
strongly coupled

Hadrons

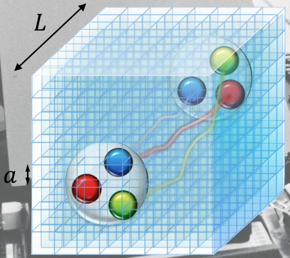


relativistic
strongly coupled

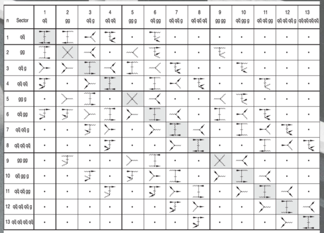
a hard problem!

Wilson's big idea:
put QCD in big computers

First transistor: 1947 (Bell Lab)
First qubit: 1998 (LANL)



Lattice gauge theory, 1974



LF Hamiltonian theory, 1989

QCD$_{1+1}$ at large N_c

$$\mathcal{L} = \overline{\psi}(i\not{D} - m)\psi - \frac{1}{4}F^{\mu\nu a}F^{a}_{\mu\nu}, \qquad (a = 1, 2, \cdots, N_c^2 - 1)$$

- Confinement and chiral symmetry breaking
- 't Hooft quantized the theory on the light front $x^+ = 0$:

$$H_{\text{LF}} = \sum_i \int_{\epsilon}^{+\infty} \frac{\mathrm{d}k^+}{2\pi}\left(\frac{m^2 - 2\lambda}{2k^+} + \frac{\lambda}{\epsilon}\right)\left[b_i^\dagger(k^+)b_i(k^+) + d_i^\dagger(k^+)d_i(k^+)\right]$$
$$- \frac{\lambda}{8\pi^2 N_c}\sum_{i,j}\int \mathrm{d}k_1^+\mathrm{d}k_2^+\mathrm{d}k_3^+ \frac{1}{(k_1^+ - k_2^+)^2} b_i^\dagger(k_2^+ + k_3^+ - k_1^+)d_i^\dagger(k_1^+)d_j(k_2^+)b_j(k_3^+)$$

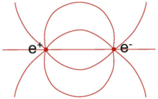
where, $\lambda = g_s^2 N_c/4\pi$ is known as the 't Hooft coupling.

- Hadrons as eigenstates of the light-front Hamiltonian,

$$H_{\text{LF}}|\psi_n(p^+)\rangle = \frac{M_n^2}{p^+}|\psi_n(p^+)\rangle$$

(1+1)D

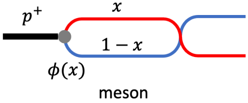
(3+1)D



QCD$_{1+1}$ at large N_c

- Meson sector:

$$|M_n(p^+)\rangle = \sqrt{\frac{p^+}{2\pi N_c}} \sum_i \int_0^1 \mathrm{d}x\, \phi_n(x) b_i^\dagger(xp^+) d_i^\dagger((1-x)p^+)|0\rangle$$



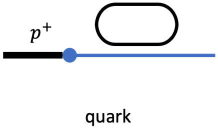
where, the meson light-front wave function $\phi_n(x)$ satisfies the 't Hooft equation,

$$\Big(\frac{m^2-2\lambda}{x} + \frac{m^2-2\lambda}{1-x}\Big)\phi_n^2(x) - 2\lambda \int_0^1 \frac{dy}{(x-y)^2}\phi_n(y) = M_n^2 \phi_n(x)$$

- Quark sector: the light-front energy of a quark diverges -- confinement!

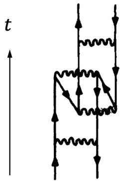
$$|Q(p^+, i)\rangle = b_i^\dagger(p^+)|0\rangle,$$

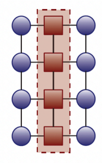
$$\Rightarrow \quad M_Q^2 = \lim_{\epsilon\to 0}\Big(\frac{m^2-2\lambda}{2p^+} + \frac{\lambda}{\epsilon}\Big) \to \infty$$



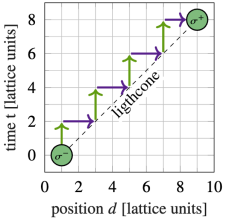
QCD$_{1+1}$ at large N_c

- Instant-from ($x^0 = 0$) Hamiltonian: involving forward and backward propagating modes -- complicated (& no probabilistic interpretation)
- Lattice Hamiltonian (Schwinger model): TN/VQE, zig-zag light cone [Nature Comm. 12, 6499 (2021),]
- Approach the light-front Hamiltonian in the infinite momentum limit

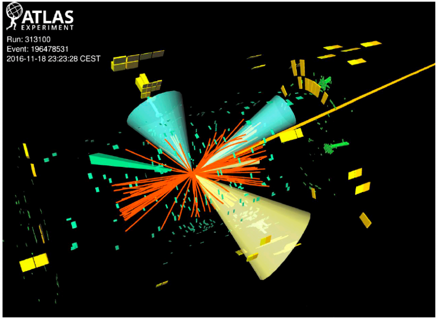




$E[\Psi] = \langle \Psi | H | \Psi \rangle$



Quantum Jet simulation: single parton evolution in QCD_{3+1}



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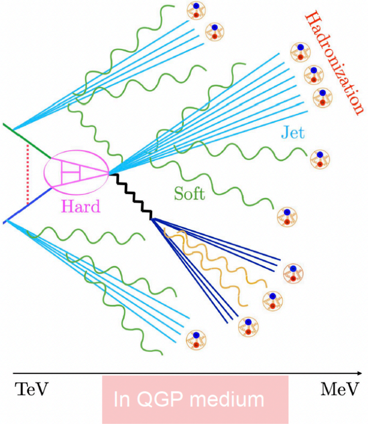
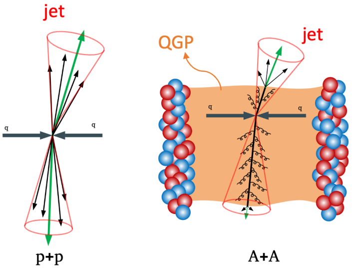


Jet evolution in QGP medium

- In relativistic heavy-ion collision, quark-gluon plasma (QGP) is formed
- We describe the QGP as a classical external field, and investigatethe evolution of a parton within the medium

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu}{}_a F^a_{\mu\nu} + \overline{\Psi}(i\gamma^\mu D_\mu - m_q)\Psi$$

$$D^\mu \equiv \partial_\mu + ig(A^\mu + \mathcal{A}^\mu)$$



Classical simulation

Electron in laser field Zhao et al, 1303.3273

Ultrarelativistic quark-nucleus scattering Li et al, 2002.09757

Scattering and gluon emission in a color field Li, Lappi, Zhao, 2107.02225

Jet in Glasma field Li et al, 2305.12490

… Ongoing work by Avramescu et al

Quantum simulation

Nuclear inelastic scattering Du et al, 2006.01369

Strategy to Jet quenching parameter Barata, Salgado, 2104.04661

Medium-induced QCD jet Barata et al, 2208.06750, 2307.01792

Yao, 2205.07902

Wu et al, 2404.00819

WQ et al, 2411.09762

…

Adapted from W. Qian

Digital quantum simulation on the light-front

➢ **Quantum algorithms:**
- Static: ground state
- **Dynamic** : time evolution√

➢ **Truncated Taylor Series** $H = \sum_{\ell=0}^{L-1} \alpha_\ell H_\ell$

$$e^{-itH} = \sum_{k=0}^{\infty} \frac{(-it)^k}{k!} H^k$$

Error

$$= \sum_{k=0}^{K} \frac{(-it)^k}{k!} H^k + \mathcal{O}\left(\frac{\|tH\|^{K+1}}{(K+1)!}\right)$$

$$\approx \sum_{k=0}^{K} \sum_{\ell_1,\dots,\ell_k=0}^{L-1} \frac{(-it)^k}{k!} \alpha_{\ell_1} \cdots \alpha_{\ell_k} H_{\ell_1} \cdots H_{\ell_k}$$

A post-Trotter algorithm based on linear combination of unitaries , see ref. [Dominic W. Berry, et al. Phys. Rev. Lett., 114, 090502].

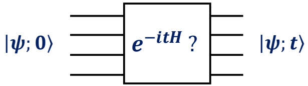
➢ **Time complexity**

$$\text{Complexity}_{\text{TTS}} \sim T \frac{\log \frac{T}{\epsilon}}{\log \log \frac{T}{\epsilon}}$$

Trotter algorithm: $\frac{T^2}{\epsilon}$

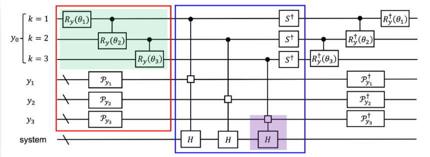
better than trotter's

➢ **Quantum circuit design**



Dynamic of the quantum system.

The truncation makes the evolution non-unitary.

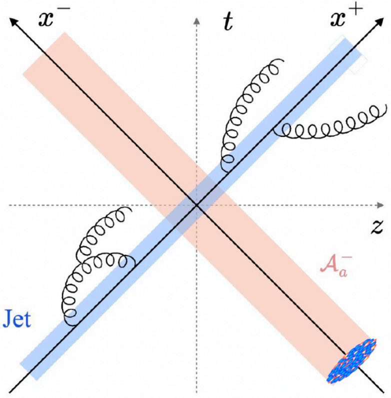


➢ **Block-encoding**

$$H = \sum_{\ell=0}^{L-1} \alpha_\ell H_\ell \longrightarrow \begin{bmatrix} H/\Lambda & * \\ * & * \end{bmatrix} \xrightarrow{\text{TTS}} \begin{bmatrix} \frac{1}{e^{t\Lambda}} e^{-itH} & * \\ * & * \end{bmatrix} \xrightarrow{\text{OAA}} \begin{bmatrix} e^{-itH} & * \\ * & * \end{bmatrix}$$

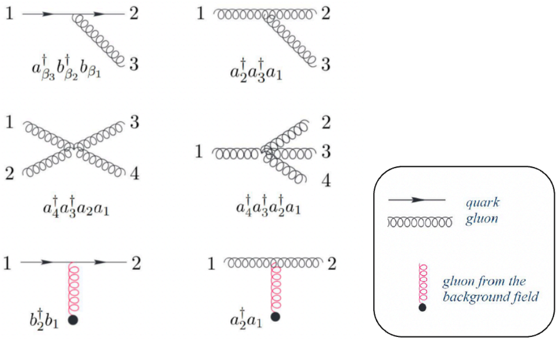
Embed the non-unitary into a larger system.

Efficient quantum simulation of quark/gluon jet



$$i\frac{\partial}{\partial x^+}|\Psi\rangle = H_{LF}|\Psi\rangle$$

where, $H_{LF} = H_{LFQCD} + J_\mu \mathcal{A}^\mu$



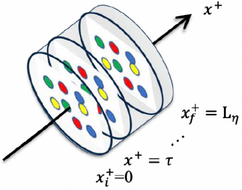
Medium and Evolution

Classical stochastic background field (to reduce problem complexity)

$$\langle\langle \rho_a(x^+, \boldsymbol{x})\rho_b(y^+, \boldsymbol{y})\rangle\rangle = g^2\mu^2\delta_{ab}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\delta(x^+ - y^+)$$

$$(m_g^2 - \nabla_\perp^2)\mathcal{A}_a^-(x^+, \boldsymbol{x}) = \rho_a(x^+, \boldsymbol{x}) \qquad Q_s^2 \equiv \frac{C_F g^4\mu^2 L_\eta}{2\pi}$$

saturation scales



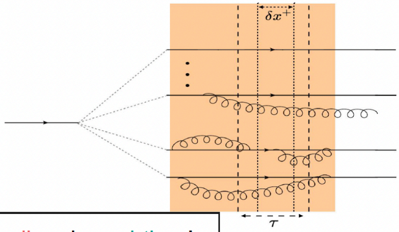
McLerran and Venugopalan, 9309289 (1993)

Jet probe evolution, decomposed as sequence of unitary operators

$$|\psi_{L_\eta}\rangle = U(L_\eta; 0)\,|\psi_0\rangle \equiv \mathcal{T}_+ e^{-i\int_0^{L_\eta} \mathrm{d}x^+\, P^-(x^+)}\,|\psi_0\rangle$$

$$U(L_\eta; 0) = \prod_{k=1}^{N_t} U(x_k^+; x_{k-1}^+)$$

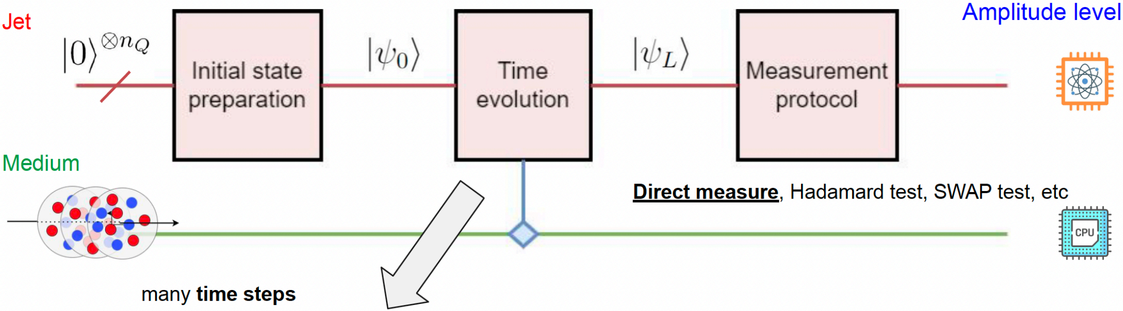
non-perturbative

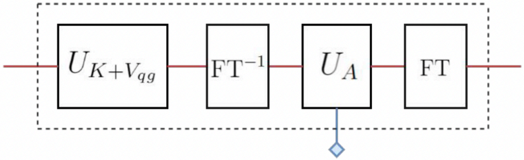


Universal framework to simulate (3+1)-d QCD jet probe evolution in medium in real-time!

Quantum simulation algorithm

Wiesner, 9603028 (1996); Zalka, 9603026 (1996)





$$U_A(x^+, \delta x^+)\,|\boldsymbol{r}\rangle = \exp(-iV_A\delta x^+)\,|\boldsymbol{r}\rangle$$
$$U_K(\delta x^+)\,|\boldsymbol{p}\rangle = \exp(-iP^-_{\mathrm{KE}}\delta x^+)\,|\boldsymbol{p}\rangle$$
$$U_{V_{qg}}(\delta x^+)\,|\boldsymbol{p}\rangle = \exp(-iV_{qg}\delta x^+)\,|\boldsymbol{p}\rangle$$

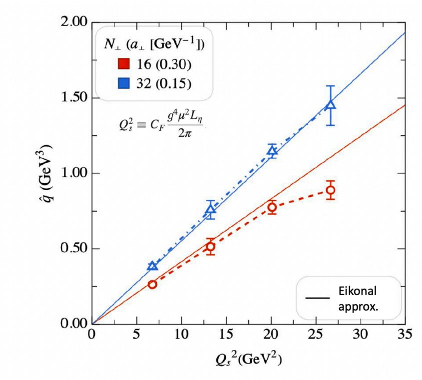
FT allows efficient/sparse simulation in the respective basis

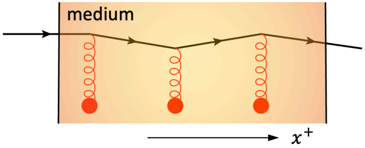
Sparse H => fewer pauli strings => less quantum gates

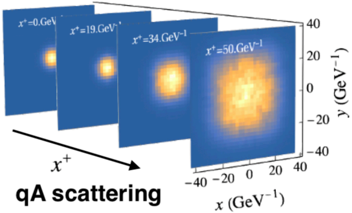
Jet quenching

Quark momentum broadening: $\hat{q} = \frac{\Delta \langle p_\perp^2(x^+) \rangle}{\Delta x^+}$

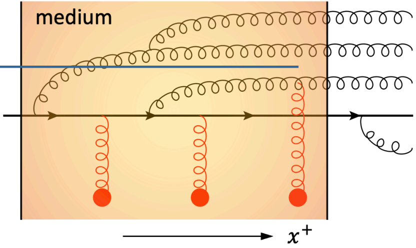
Eikonal approximation: $\hat{q} = \frac{Q_s^2}{L_\eta}$



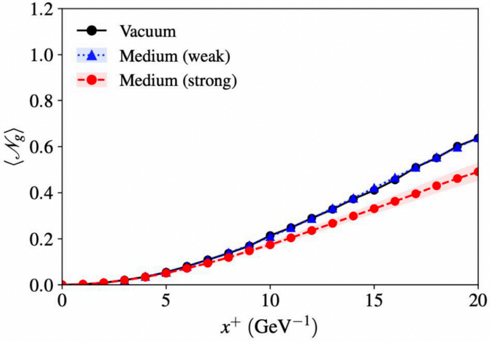




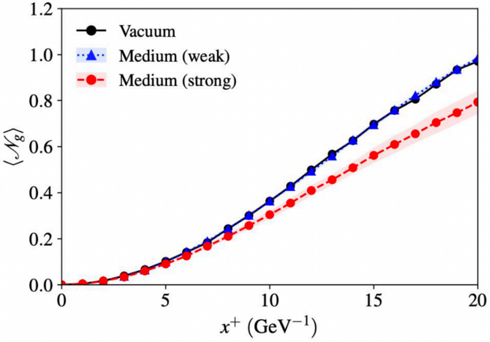
Medium modification of gluon radiation



Gluon number operator $\mathcal{N}_g \equiv \sum_{\beta} a_{\beta}^{\dagger} a_{\beta}$



(a) $K = 1.5$, $|q\rangle + |qg\rangle$

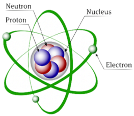


(b) $K = 2.5$, $|q\rangle + |qg\rangle + |qgg\rangle$

QCD$_{3+1}$: strongly coupled relativistic quantum many-body problem

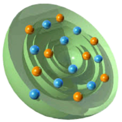
$$H|\Psi\rangle = E|\Psi\rangle$$

Atoms



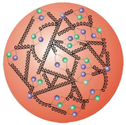
non-relativistic
weakly coupled

Nuclei



non-relativistic
strongly coupled

Hadrons



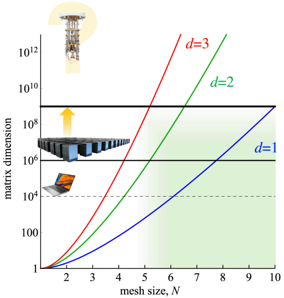
relativistic
strongly coupled

- The quantum Hamiltonian is not fully known
  -- need non-perturbative renormalization:

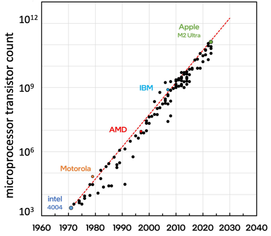
$$H_R = \underbrace{H_{\text{can}}}_{\text{divergent}} + \underbrace{H_{\text{ct}}}_{\text{divergent}} = \underbrace{H_{\text{can}}}_{\text{known}} + \underbrace{\sum_i c_i O_i}_{\text{not known}}$$

- Exponential wall

$$\dim \mathcal{H} = N^{dA}, \qquad (A \sim N)$$



Moore's law: computational power doubles every 2 years



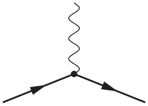
Strongly coupled scalar theory with Yukawa interaction [Li:2015iaw, Karmanov:2016yzu]

$$\mathcal{L} = \partial_\mu N^\dagger \partial^\mu N - m^2 N^\dagger N + \frac{1}{2}\partial_\mu \pi \partial^\mu \pi - \frac{1}{2}\mu^2\pi^2 + gN^\dagger N\pi,$$

$m = 0.94\,\mathrm{GeV}$, $\mu = 0.14\,\mathrm{GeV}$, $\alpha = g^2/(16\pi m^2) \sim \mathrm{O}(1)$

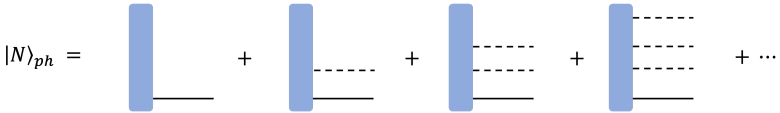
- Simplest strongly coupled QFT in (3+1)D, a rudimentary model for $N\pi$ interaction

$$\Rightarrow \quad H_{\text{int}} = -g\int \mathrm{d}^3x\, N^\dagger N\pi, \qquad \rightarrow \qquad U(r) = -\alpha\frac{e^{-\mu r}}{r}$$



- Systematic Fock sector expansion and truncation:

$$|N\rangle_{\text{ph}} = |N\rangle + |N\pi\rangle + |N\pi\pi\rangle + |N\pi\pi\pi\rangle + \cdots$$



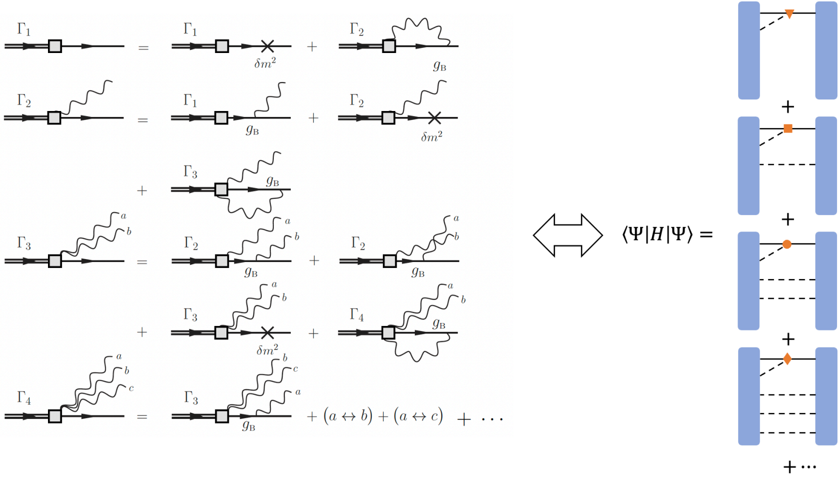
Fock sector dependent renormalization (FSDR)

[Perry:1990mz, Karmanov:2008br]

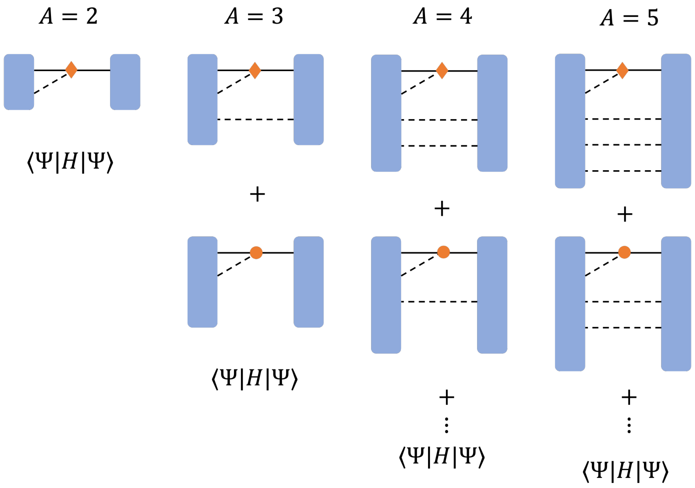
$$H_R = \underbrace{H_{\text{can}}}_{\text{divergence}} + \underbrace{H_{\text{ct}}}_{\text{divergence}} = H_{\text{can}} + \sum_i c_i O_i$$

- Weinbergian renormalization: symmetry + power counting
  - Breaking of Poincare symmetry due to truncation requires an infinite number of counterterms
  - Power counting insufficient to cancel out all divergences
- Fock sector dependent renormalization [Karmanov:2008br]
  - Continuum regularization: introducing Pauli-Villars (PV) particles whose mass μ_{PV} serves as a regulator
  - Kinematical symmetry + parton number counting [Li:2015iaw, Karmanov:2016yzu]

Diagrammatic representation of eigenvalue equation



Fock sector dependent renormalization

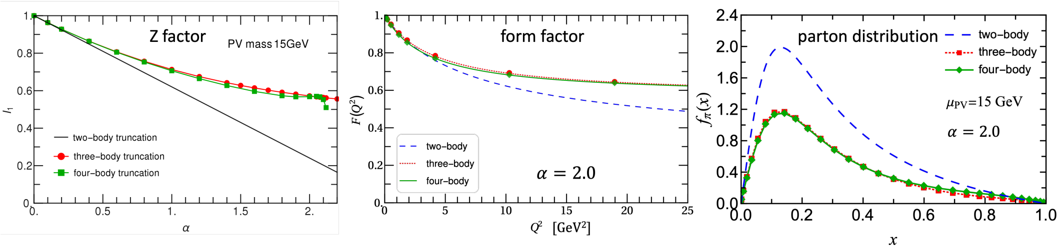


Numerical solution and Fock sector convergence

- Work with momentum space and Fock sector (parton number) truncation $A = 1, 2, 3, 4, 5\cdots$
- Adopt Lagrange mesh as the momentum-space grid
- Adopt standard linear solver on parallel machines

$$\dim \mathcal{H}_{A=4} = N^8 \to N^5 \sim 10^7$$

- Scalar theory is numerically well converged up to three- and four-body truncations



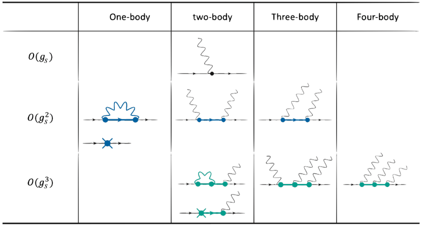
Similarity renormalization group (SRG)

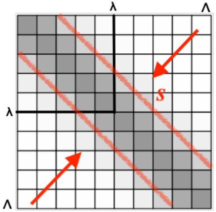
- Effective Hamiltonian at a finite scale s, built from infinitesimal similarity transformations:

$$H_s = \underbrace{\mathcal{U}_s^{-1} H_\infty \mathcal{U}_s}_{\text{divergence free}} \quad \Rightarrow \quad \frac{\mathrm{d}H_s}{\mathrm{d}s} = [G_s, H_s]$$

- Perturbative solution of SRG at scale $s \gg \Lambda_{\text{QCD}}$ thanks to asymptotic freedom [Glazek:2012qj]

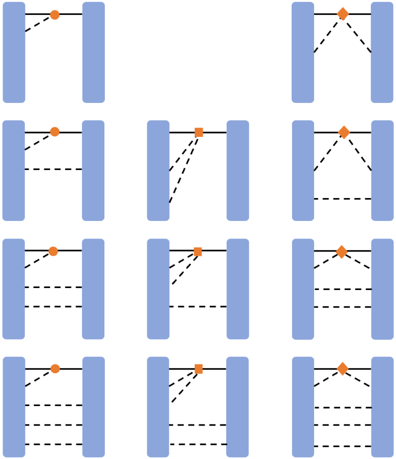
$$H_s = T + g_s V_{1s} + g_s^2 V_{2s} + O(g_s^3)$$





Similarity renormalization group (SRG)

$$\langle\Psi|H|\Psi\rangle =$$

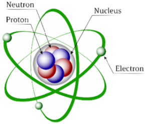
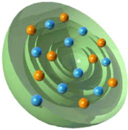
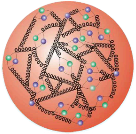


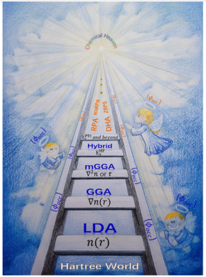
Lessons from quantum chemistry

AB INITIO QUANTUM CHEMISTRY:
A SOURCE OF IDEAS FOR LATTICE GAUGE THEORISTS

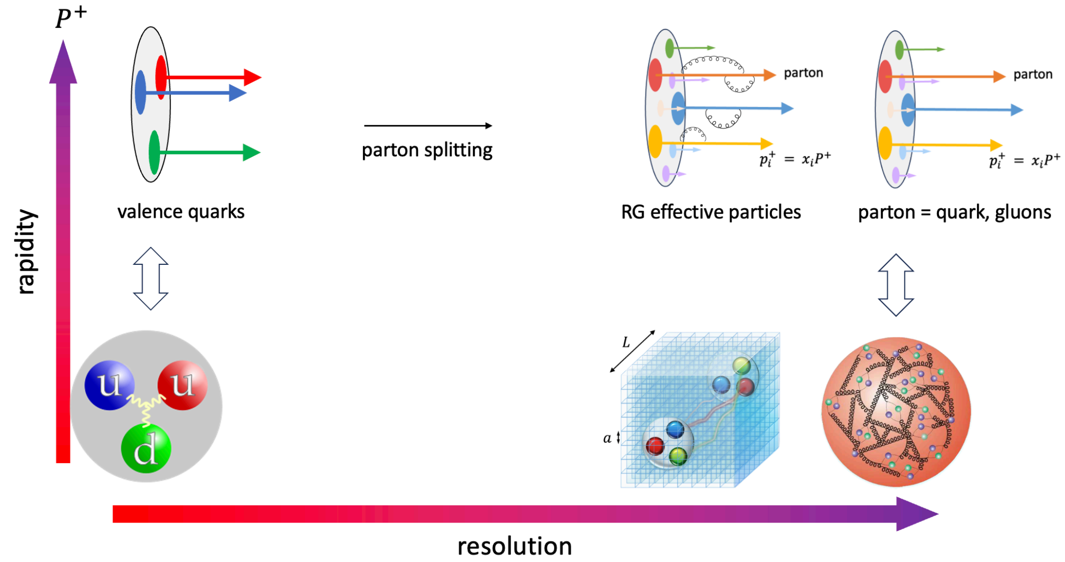
Kenneth G. WILSON

The Ohio State University, Department of Physics, 174 W. 18th Avenue, Columbus, OH 43210 USA

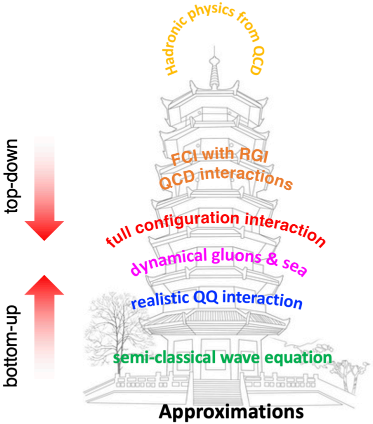
Atoms	Nuclei	Hadrons
		
non-relativistic weakly coupled	non-relativistic strongly coupled	relativistic strongly coupled
Bohr Model 	Shell Model 	?



Jacob's ladder in quantum chemistry


P^+
rapidity
valence quarks
parton splitting
parton
$p_i^+ = x_i P^+$
parton
$p_i^+ = x_i P^+$
RG effective particles
parton = quark, gluons
u
u
d
L
a
resolution

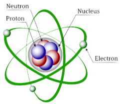
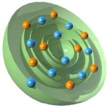
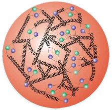
Jacob's tower for light-front QCD

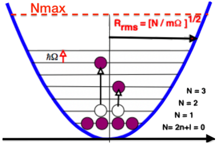


Approaches	Applications
FBLFQ	Quantum Info
BLFQ	PDF, FF, decay
AdS/QCD, LF QM	Spectroscopy

Basis light-front quantization (BLFQ)

[Vary:2009gt]

Atoms	Nuclei	Hadrons
		
Coulomb interaction	NN, NNN interactions	QCD interactions
↓	↓	↓
Bohr Model	Shell Model	Light Front Holography
$\left(\frac{\vec{p}^2}{2m} - \frac{Z\alpha}{r}\right)\psi = E\psi$	$\left(\frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2}r^2\right)\psi = E\psi$	$\left(\frac{k_\perp^2 + m_q^2}{x(1-x)} + \kappa^4\zeta_\perp^2 + U_\parallel\right)\psi = M^2\psi$



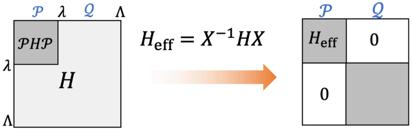
Ab initio no-core shell model

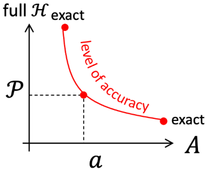
[Barrett:2013nh]

- Work with small basis space ($N \sim 8$) through Hamiltonian renormalization: $\dim \mathcal{H} = N^{dA}$

$$H_{\text{eff}} = \mathcal{P}X^{-1}H_0X\mathcal{P} \approx T + V_1 + V_2 + \cdots + V_a + \cancel{V_{a+1} + \cdots + V_A}$$

- Retain semi-classical approximation & all symmetries in basis space





Basis light-front quantization I

- Configuration interaction: [Vary:2009gt]

$$H|\psi\rangle = M^2|\psi\rangle \quad \Rightarrow \quad \sum_j H_{ij}c_j = M^2 c_i$$

where, $H_{ij} = \langle\phi_i|H|\phi_j\rangle$, $c_i = \langle\phi_i|\psi\rangle$.

- Single-particle basis $\{|\phi_i\rangle\}$ is chosen to preserve all kinematical symmetries of QCD
  - 3 Boosts + 1 rotation:

$$\phi_{nml}(\vec{x}_\perp, x^-) = \mathcal{N} e^{\frac{i\pi l x^-}{L} + im\theta_\perp - \frac{\rho^2}{2}} \rho^{|m|} L_n^{|m|}(\rho), \quad (\rho = \Omega\sqrt{l/K}x_\perp)$$

  - Longitudinal direction: discretized momentum basis, Jacobi polynomials on a k-simplex [Chabysheva:2013oka, Li:2017mlw]
  - Other symmetries of QCD
- Consistent with holographic LFQCD
- Large sparse matrix eigenvalue problem: suitable for modern HPC (& quantum computing)

Basis light-front quantization II

- Energy cutoffs/regularization:

$$\sum_i [2n_i + |m_i| + 1] \le N_{\max}, \quad \sum_i m_i + s_i = m_J, \quad \sum_i l_i = K.$$

- UV & IR regulators:

$$\Lambda_{\text{UV}} = \Omega\sqrt{N_{\text{max}}}, \qquad \Lambda_{\text{IR}} = \Omega/\sqrt{N_{\text{max}}}$$

- Truncation on the many-body basis:
  - Fock sector truncation
  - $N_{\max}$-truncation, K-truncation
  - Coupled cluster/coherent basis [Hiller:2016itl, More:2014rna]
  - DMRG, matrix product states, tensor network, ...
- Variational theorem
- Continuum limit (need a proof):

$$N_{\max} \to \infty, \quad K \to \infty, \quad A \to \infty$$

Effective hadron Hamiltonian

One may also consider effective (3+1)d Hamiltonian on the light front.

Valence $|q\bar{q}\rangle$ for light mesons

WQ, Jia, Li, Vary, 2005.13806

$$H_{\text{eff},\gamma_5} = \overbrace{\underbrace{\frac{\mathbf{k}_\perp^2 + m_q^2}{x} + \frac{\mathbf{k}_\perp^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 x(1-x)\mathbf{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\frac{\partial}{\partial x}(x(1-x)\frac{\partial}{\partial x})}_{\text{confinement}} + V_g}^{H_{\text{eff}}} + H_{\gamma_5}$$

For given set of basis states, the Hamiltonian operator can be written in creation and annihilation operators for those modes:

$$\hat{H} = \hat{H}_1 + \hat{H}_2 + \cdots = \sum_{ij} h_{ij}\hat{a}_i^\dagger \hat{a}_j + \frac{1}{4}\sum_{ijkl} h_{ijkl}\hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_l + \cdots$$

Qubit encoding: One-hot encoding and Binary encoding $(N, N) = (2^n, 2^n) \rightarrow H_q = \sum_\alpha c_\alpha P_\alpha$

$O(N)$ $\qquad$ $O(\log N)$

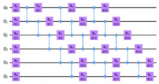
Jordan & Wigner (1928);
Kreshchuk et al, 2002.04016

Variational approaches are used to solve the hadronic mass spectrum

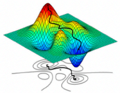
Variational quantum algorithms

Peruzzo, et al, 1304.3061 (2014)
Nakanishi, Mitarai, Fujii, 1810.09434 (2019)

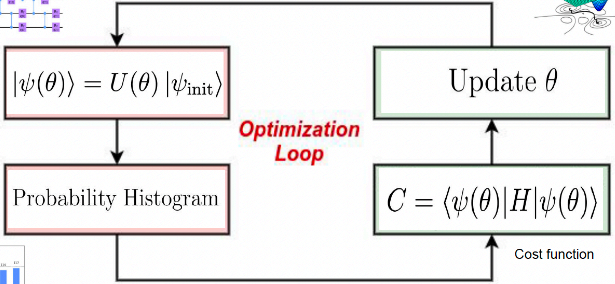
Based on variational principle, build parameterized wave functions $\langle\psi(\theta)|H|\psi(\theta)\rangle \geq \langle E_0\rangle = \langle\psi_0|H|\psi_0\rangle$



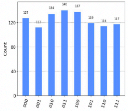
Problem-inspired
Heuristic ansatz



Quantum simulation



Classical optimization



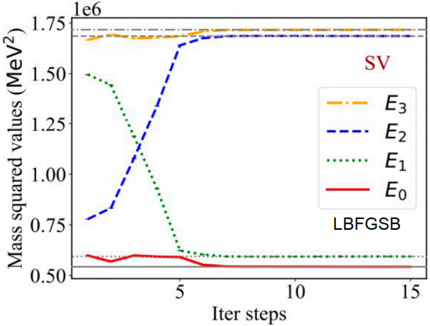
Hybrid approach:
Complementary Advantage for QC and CC

Variational quantum eigensolver (VQE) for ground state
Subspace-search VQE (SSVQE) for full spectrum

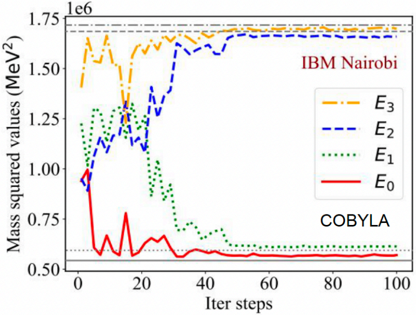
Hadron spectrum

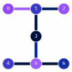
WQ, Basili, Pal, Luecke, Vary; 2112.01927

Noiseless simulation (2 qubits)



IBM Quantum devices **with Noise-Mitigation**

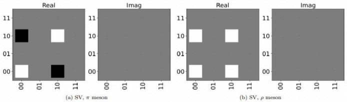


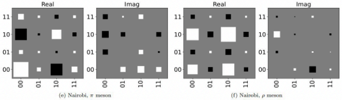


IBM Nairobi:

7 qubit
32 QV

Qubit representation (density matrix) for lowest two states: $D_{ij} = |\psi_i\rangle \langle\psi_j|$





Qiskit

Hadronic observables

Li, Maris, Vary, 1704.06968
WQ, Basili, Pal, Luecke, Vary, 2112.01927

Using light-front wave functions (LFWF) on qubits to compute various observables with projection ops

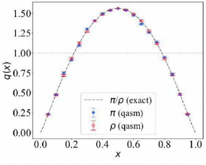
$$f_{\mathrm{P,V}} = 2\sqrt{2N_c}\int_0^1 \frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int \frac{\mathrm{d}^2\boldsymbol{k}_\perp}{(2\pi)^3}\psi^{(m_j=0)}_{\uparrow\downarrow\mp\downarrow\uparrow}(x,\boldsymbol{k}_\perp)$$
$$\equiv \frac{\kappa\sqrt{N_c}}{\pi}\sum_{nl}(-1)^n C_l(m_q,\kappa)\left(\tilde{\psi}^{(m_j=0)}_{\uparrow\downarrow}(n,0,l)\mp\tilde{\psi}^{(m_j=0)}_{\downarrow\uparrow}(n,0,l)\right)$$

$$f_{\mathrm{P,V}} \propto |\langle \nu_{\mathrm{P,V}}|\psi(\vec{\theta})\rangle|$$

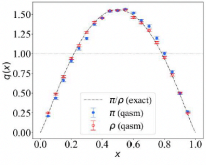
$$q(x;\mu) = \frac{1}{x(1-x)}\sum_{s\bar{s}}\int\frac{\mathrm{d}^2\boldsymbol{k}_\perp}{2(2\pi)^3}|\psi^{(m_j=0)}_{s\bar{s}}(x,\boldsymbol{k}_\perp)|^2$$
$$\equiv \frac{1}{4\pi}\sum_{s\bar{s}}\sum_{nm}\sum_{l\bar{l}}\tilde{\psi}^{*(m_j=0)}_{s\bar{s}}(n,m,\bar{l})\tilde{\psi}^{(m_j=0)}_{s\bar{s}}(n,m,l)\chi_l(x)\chi_{\bar{l}}(x)$$

$$q(x) = \sum_{s\bar{s}}\sum_{nm}\sum_{l\bar{l}}\langle\psi(\vec{\theta})|\hat{O}_{\mathrm{pdf}}(x)|\psi(\vec{\theta})\rangle$$

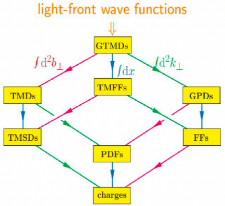
$N_{\max}$	$L_{\max}$	Decay constants	Exact result (MeV)	qasm sim (MeV)
1	1	f_π	178.18	178.17 ± 1.97
		f_ρ	178.18	178.17 ± 1.97
4	1	f_π	193.71	194.28 ± 15.49
		f_ρ	231.00	225.72 ± 13.44



(a) PDFs at $N_{\max} = L_{\max} = 1$



(b) PDFs at $N_{\max} = 4, L_{\max} = 1$

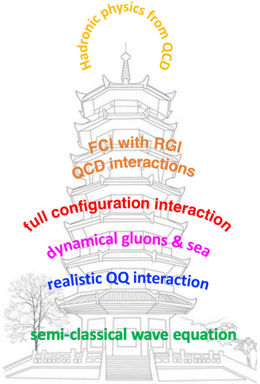


Qiskit

Applications

[Review: 50 years of QCD Sect. 5.3, 2212.11107]

- Semi-classical LF wave equations: [Li:2021jqb, Li:2022izo, Li:2022ytx, Li:2023izn]
- Realistic QQ interaction:
  - QED: [Honkanen:2010rc, Zhao:2014xaa, Wiecki:2014ola, Hu:2020arv, Nair:2022evk, Nair:2023lir]
  - heavy flavors [Li:2015zda, Li:2017mlw, Leitao:2017esb, Li:2018uif, Adhikari:2018umb, Tang:2018myz, Lan:2019img, Tang:2019gvn, Tang:2020org, Li:2021ejv, Li:2021cwv, Wang:2023nhb]
  - light mesons [Jia:2018ary, Lan:2019vui, Lan:2019rba, Qian:2020utg, Mondal:2021czk, Adhikari:2021jrh, Li:2022mlg, Zhu:2023lst]
  - nucleons & baryons [Mondal:2019jdg, Xu:2021wwj, Liu:2022fvl, Hu:2022ctr, Peng:2022lte, Zhu:2023lst, Zhang:2023xfe, Zhu:2023nhl]
  - tetraquarks [Kuang:2022vdy]
- Dynamical gluons & sea: [Lan:2021wok, Xu:2022abw, Xu:2023nqv, Lin:2023ezw, Kaur:2024iwn]
- Time-dependent problems (tBLFQ): [Zhao:2013cma, Zhao:2013jia, Chen:2017uuq, Li:2020uhl, Li:2023jeh]
- Quantum computing: [Kreshchuk:2020kcz, Kreshchuk:2020aiq, Qian:2021jxp, Kreshchuk:2023btr, Wu:2024adk, Du:2024zvr]

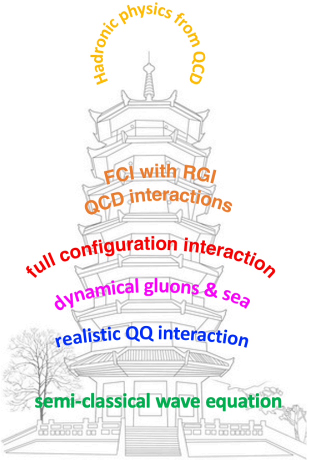


Similar methods: [deTeramond:2021yyi, Ahmady:2021lsh Lyubovitskij:2022rod, Ahmady:2022dfv, Shuryak:2021fsu, Shuryak:2021hng, Shuryak:2021mlh, Shuryak:2022thi, Shuryak:2022wtk, Shuryak:2023siq, Liu:2023yuj, Liu:2023fpj, Miesch:2023hvl]

Summary

- Light-front Hamiltonian formalism is a natural framework to describe hadrons as relativistic quantum many-body problems
- Quantum computers are possible powerful platform to enable the description of strong interaction on the amplitude level
- Basis light-front quantization provides an avenue to solving QCD from first principles on classical and quantum machines

Thank you!



backup

Positronium

[Wiecki:2014ola]

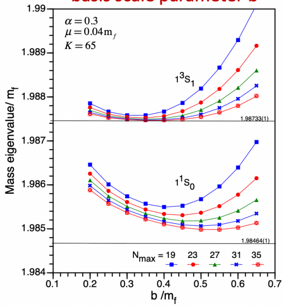
- Bloch-Wilson interaction: perturbative solution to the OSL effective Hamiltonian [Krautgartner:1991xz]

$$V = \frac{4\pi\alpha}{Q^2}\bar{u}_{s'}(k')\gamma_\mu u_s(k)\bar{v}_{\bar{s}}(\bar{k})\gamma^\mu v_{\bar{s}'}(\bar{k}') + O(\alpha^2)$$

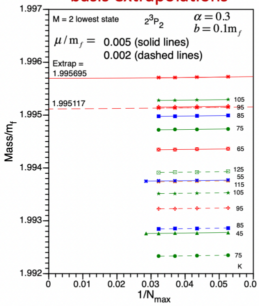
- Comparison with perturbative QED: [Bethe & Salpeter, 1977 Springer; cf. Lamm:2016djr]

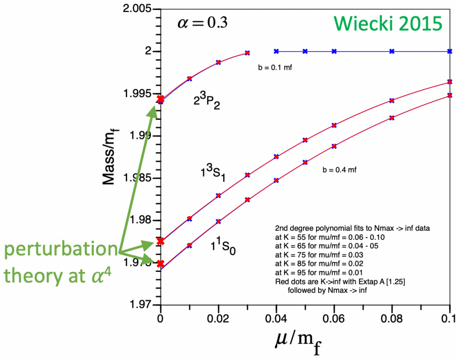
$$H = T + H_{\rm Col} + H_{\rm Dar} + H_{\rm rel} + H_{\rm LS} + H_{\rm SS}$$

variational minima wrt basis scale parameter b



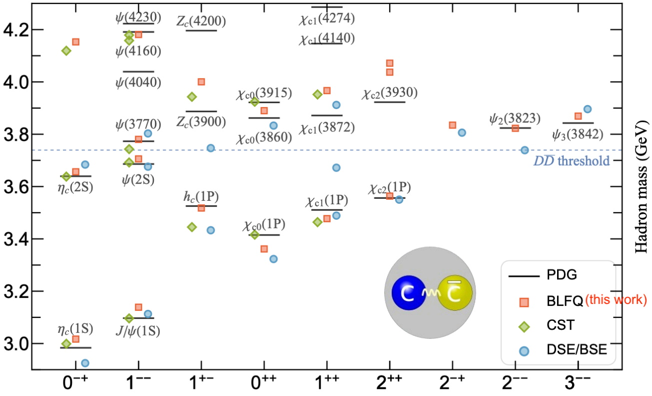
basis extrapolations

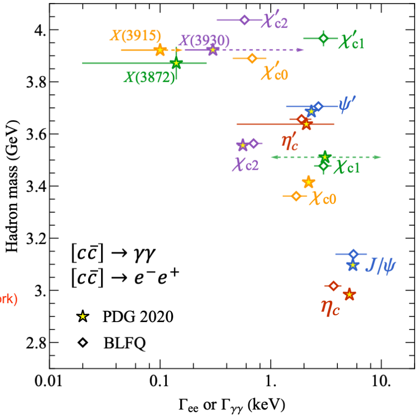




Charmonium: hydrogen atom of QCD

[Li:2015zda, Li:2017mlw, Li:2021ejv]



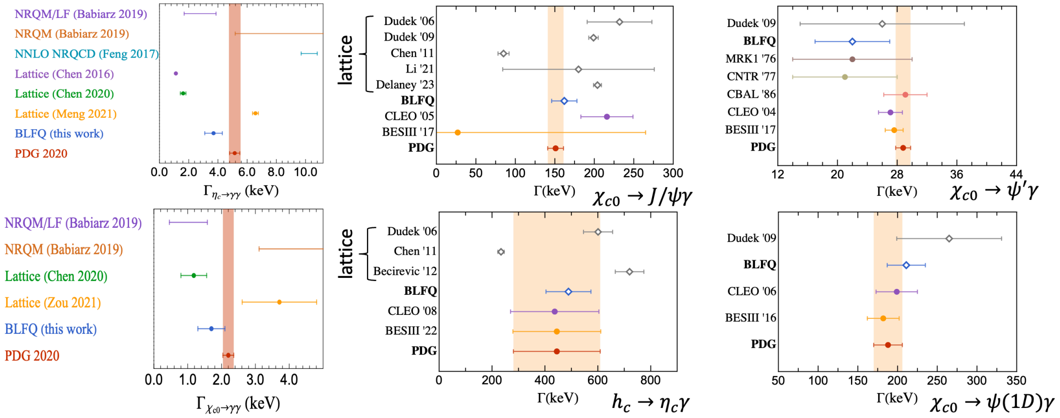


- Two free parameters (m_c, κ), rms deviation: 30 MeV [Gross:2022hyw]
- Good agreement with the PDG data for both the masses and the widths

[LFWFs published on Mendeley Data, doi: 10.17632/cjs4ykv8cv.2]

Parameter-free prediction of radiative widths

[Li:2018uif, Li:2021ejv, Wang:2023nhb]



$$\langle V(p',\lambda')|J^\mu(0)|S(p)\rangle = E_1(Q^2)\Big[e^{\mu *}_{\lambda'}(p') - \frac{e^*_{\lambda'}\cdot p}{(p\cdot p')^2 - M_S^2 M_V^2}\big(p'^\mu(p\cdot p') - M_V^2 p^\mu\big)\Big]$$

$$+ C_1(Q^2)\frac{M_V}{Q(p\cdot p')^2 - M_S^2 M_V^2}(e^*_{\lambda'}\cdot p)\Big[(p\cdot p')(p+p')^\mu - M_S^2 p'^\mu - M_V^2 p^\mu\Big],$$

All radiative transitions

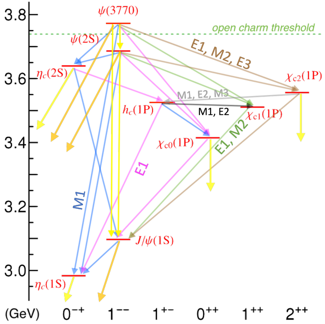
[Li:2015zda, Li:2017mlw, Li:2018uif, Li:2021ejv, Wang:2023nhb]

Leptonic and radiative transitions probe the fundamental structure of the hadrons:

[Review: Barnes & Yuan, Int. J. Mod. Phys. A 2009]

decay width (keV)		Γ_{ee}	$\Gamma_{\gamma\gamma}$	$\Gamma_{\eta_c\gamma}$	$\Gamma_{J/\psi\gamma}$	$\Gamma_{\chi_{c0}\gamma}$	$\Gamma_{\chi_{c1}\gamma}$	$\Gamma_{h_c\gamma}$	$\Gamma_{\chi_{c2}\gamma}$	$\Gamma_{\eta_c'\gamma}$	$\Gamma_{\psi'\gamma}$
η_c	PDG	-	5.15(35)								
	BLFQ	-	3.7(6)								
J/ψ	PDG	5.53(10)	-	1.6(4)							
	BLFQ	5.7(1.9)	-	2.72(5)							
χ_{c0}	PDG	-	2.20(16)	-	151(10)						
	BLFQ	-	1.7(4)	-	162(16)						
χ_{c1}	PDG	-	-	-	288(16)	-					
	BLFQ	-	-	-	in progress	-					
h_c	PDG	-	-	445(164)	-						
	BLFQ	-	-	489(85)		in progress	in progress				
χ_{c2}	PDG	-	0.56(5)	-	374(20)	-	-				
	BLFQ	-	0.70(13)	-	in progress	-	-	in progress			
η_c'	PDG	-	2.1(1.6)	-	<195	-	-		-		
	BLFQ	-	1.9(4)	-	0.16(23)	-	-	26.4(1.1)	-		
ψ'	PDG	2.33(81)	-	1.00(16)	-	28.8(1.0)	28.7(1.1)	-	28(1)	0.23(17)	
	BLFQ	2.7(1.3)	-	2.9(2.0)	-	22(5)	in progress	-	in progress	0.165(12)	
$\psi(1D)$	PDG	0.261(21)	-	<19	-	188(18)	80(7)	-	<17	<24.9	-
	BLFQ	0.051(19)	-	0.136(2)	-	211(24)	in progress	-	in progress	0.0017(24)	-

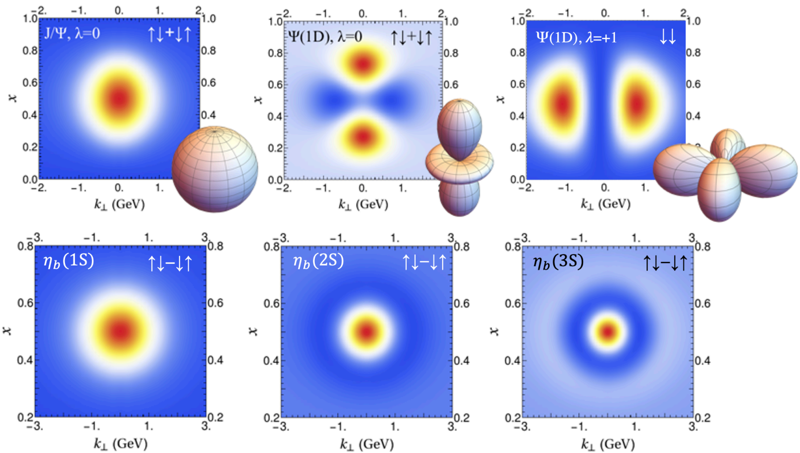
- M1
- E1
- E1, M2
- M1, E2
- E1,M2,E3
- M1,E2,M3



Quarkonium light-front wave functions

angular excitations

radial excitations

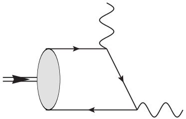


[LFWFs published on Mendeley Data, doi: 10.17632/cjs4ykv8cv.2]

LFWF representation of radiative transition form factors [Lepage:1980fj, Li:2021ejv]

Amplitude of single-tag two-photon of a pseudo-scalar meson ($n^2 = 0$):

$$\mathcal{M}^{\mu\nu}_{P\to\gamma\gamma^*} = 4\pi\alpha_{\rm em}\varepsilon^{\mu\nu\rho\sigma}q_\rho n_\sigma F_{P\gamma}(-q^2)$$



$$F_{P\gamma}(Q^2) = e_f^2 2\sqrt{2N_C}\int\frac{\mathrm{d}x}{2\sqrt{x(1-x)}}\int\frac{\mathrm{d}^2k_\perp}{(2\pi)^3}\frac{\psi^*_{\uparrow\downarrow-\downarrow\uparrow/P}(x,\vec{k}_\perp)}{k_\perp^2+m_f^2+x(1-x)Q^2} + \cdots\cdots$$

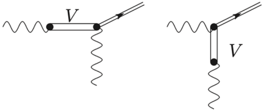
higher Fock sector contributions

$Q\to 0$

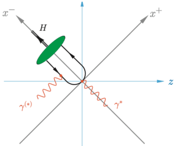
$Q\to\infty$

$$\sum_V \frac{e_f^2 f_V}{1+\frac{M_P}{M_V}}\frac{g_V(0)}{M_V^2+Q^2}$$

$$e_f^2 f_P\int_0^1 \mathrm{d}x\frac{\phi_P(x,\mu)}{x(1-x)Q^2+m_f^2}$$



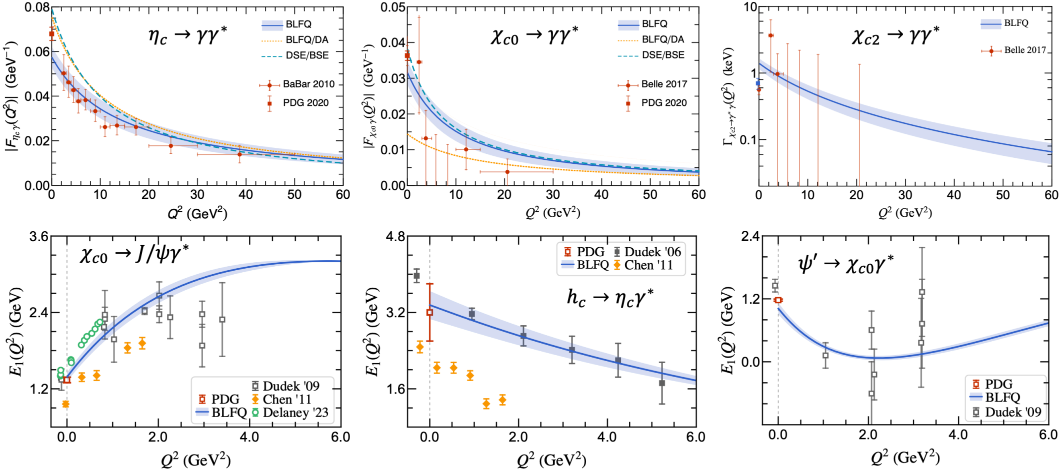
vector meson dominance



light-cone dominance

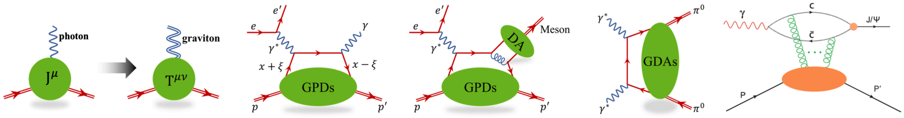
Parameter-free prediction of transition form factors

[Li:2018uif, Li:2021ejv, Wang:2023nhb]



Gravitational form factors (GFFs)

[Polyakov:2018zvc, Burkert:2023wzr]



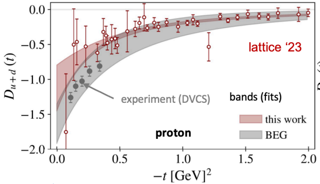
$$\langle p'|T_i^{\mu\nu}(0)|p\rangle = P^\mu P^\nu A_i(-q^2) + \frac{1}{4}(q^\mu q^\nu - g^{\mu\nu}q^2)D_i(-q^2) + g^{\mu\nu}\bar{c}_i(-q^2)$$

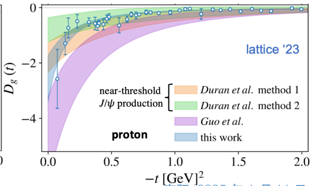
- Hadronic matrix element of the stress-energy tensor, which encodes the energy and stress distributions within the system
  - Mass decomposition and the anomalous mass of the proton
  - Mechanical properties, equilibrium and stability
- Experimentally accessible through GPDs -- tremendous attention [Ji:1996nm, Polyakov:2002yz]
- Physical interpretation of GFFs are under tense debate

[Lattice '23: Hackett:2023nkr]

electromagnetic	$G_E(0) = Q = 1.602176487(40)\times 10^{-19}\mathrm{C}$ $G_M(0) = \mu = 2.792847356(23)\mu_N$
weak	$G_A(0) = g_A = 1.2694(28)$ $G_P(0) = g_p = 8.06(55)$
gravitational	$mA(0) = m = 938.272013(23)\ \mathrm{MeV}/c^2$ $J(0) = J = \frac{1}{2}$ $D(0) = D = ?$

"The last global unknown of the proton"



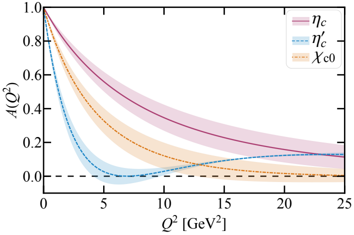


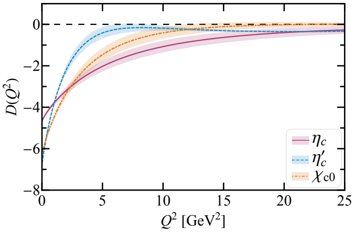
Charmonium gravitational form factors

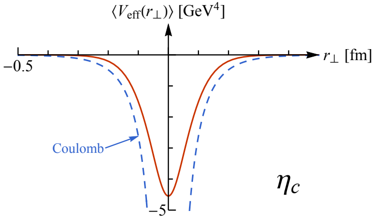
[Xu:2024cfa; Hu:2024edc]

- Adopt charmonium wave functions from basis light-front quantization (BLFQ) [Li:2017mlw]
- Alternative charmonium wave functions from Dyson-Schwinger equations [Cao, in progress]
- Effective one-body $c\bar{c}$ potential:

$$\mathcal{P}^-_{\text{int}}(r_\perp) \equiv \frac{1}{2}\mathcal{T}^{+-}(r_\perp) - \Big\langle \sum_j \delta^2(r_\perp - r_{j\perp}) \frac{-\frac{1}{4}\overleftrightarrow{\nabla}^2_{j\perp} + m_j^2 + \frac{1}{4}\nabla^2_\perp}{x_j P^+} \Big\rangle = \frac{1}{P^+}\langle V_{\text{eff}}(r_\perp)\rangle \sim \psi^*(r)V(r)\psi(r)$$







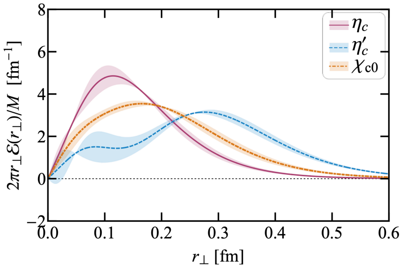
Energy density vs invariant mass squared density

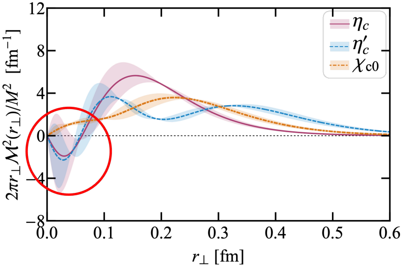
Energy density $\mathcal{E}(r_\perp)$ vs the invariant mass squared density $\mathcal{M}^2(r_\perp)$:

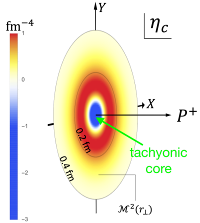
$$\mathcal{E}(r_\perp) = M \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} \Big\{ \Big(1 + \frac{q_\perp^2}{4M^2}\Big) A(q_\perp^2) + \frac{q_\perp^2}{4M^2} D(q_\perp^2) \Big\},$$

$$\mathcal{M}^2(r_\perp) = M^2 \int \frac{\mathrm{d}^2 q_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp\cdot\vec{r}_\perp} \Big\{ \Big(1 + \frac{q_\perp^2}{4M^2}\Big) A(q_\perp^2) + \frac{q_\perp^2}{2M^2} D(q_\perp^2) \Big\} = M\Big[\mathcal{E}(r_\perp) - \frac{3}{2}\mathcal{P}(r_\perp)\Big]$$

- Energy density is positive
- Invariant mass squared density becomes negative at small $r_\perp$: tachyonic core within charmonium?





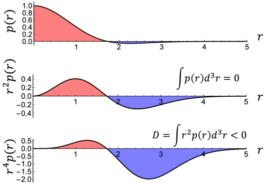


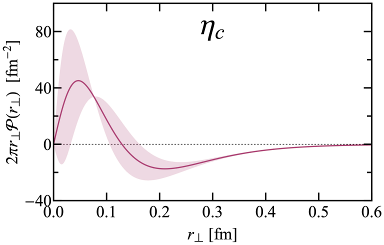
Mechanical stability conjecture $D < 0$

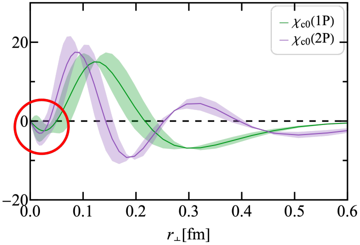
[Hu:2024edc]

$$D = \int \mathrm{d}^3 r\, r^2 \mathcal{P}(r) \overset{???}{<} 0$$

- Speculation: a mechanically stable system must have a repulsive core and an attractive edge
- We find that while η_c has a repulsive core, χ_{c0} has an attractive cores, and both have negative D!



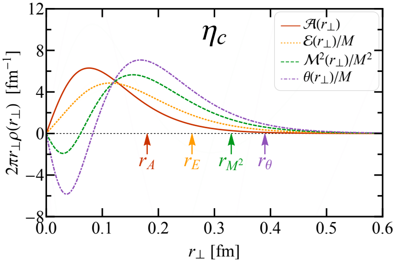


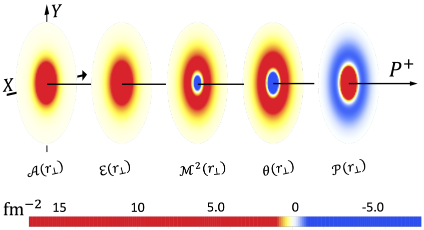


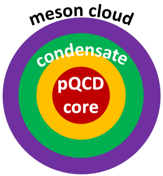
Physical densities

[Xu:2024cfa]

Matter density $\mathcal{A}(r_\perp)$, energy density $\mathcal{E}(r_\perp)$, invariant mass squared density $\mathcal{M}^2(r_\perp)$ and trace scalar density $\theta(r_\perp)$







Application to nucleon structures

[Mondal:2019jdg, Xu:2021wwj, Xu:2022abw, Xu:2023nqv, Xu:2024sjt]