CRITICAL FLUCTUATIONS AND CORRELATIONS OF QUARK SPIN IN HOT AND DENSE QCD MATTER

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OUTLINE

- Introduction: rotational effects in heavy ion collisions
- Motivation: a question comes up to our mind
- Technical details: NJL model calculation
- Results: Critical spin correlation

ANGULAR MOMENTUM IN HEAVY-ION COLLISIONS

- Non-central heavy-ion collisions create fireballs with large global orbital angular momenta

$$\boldsymbol{L}_{\rm init} \sim 10^5\hbar$$

F. Becattini, F. Piccinini, J. Rizzo, PRC 77 (2008) 024906

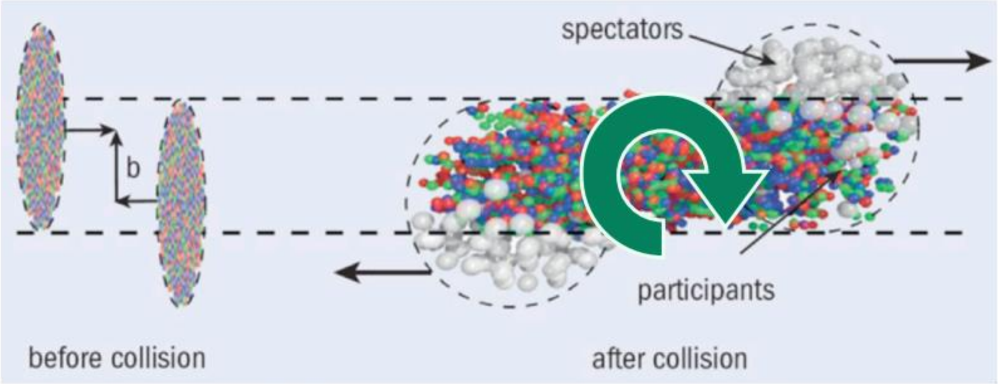
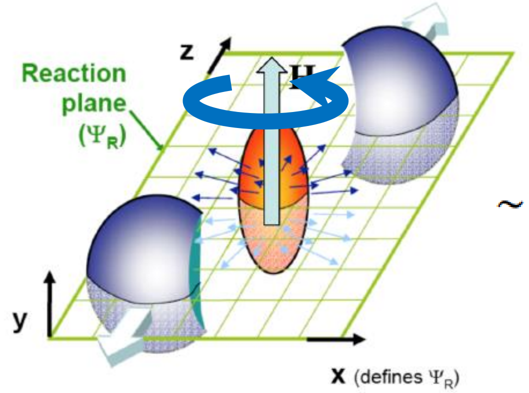


figure: M. Lisa, talk @ “Strangeness in Quark Matter 2016”

- angular momenta → rotation (local/global)
- Rotational effects attract many attentions recently in several aspects

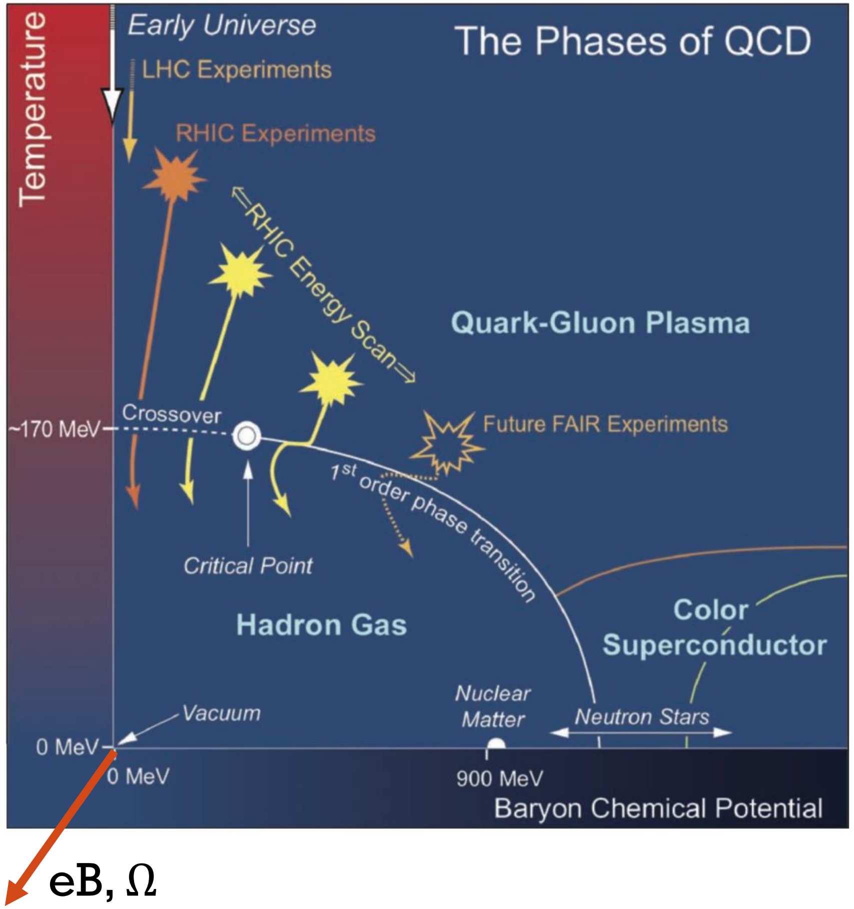
ROTATION RELATED STUDIES



$\sim 10^{21}/s$ at RHIC

- Spin polarization/alignment

Phase transition



Anomalous transport

CVE, Vilenkin (1979)

$$J = \left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12}\right)\omega$$

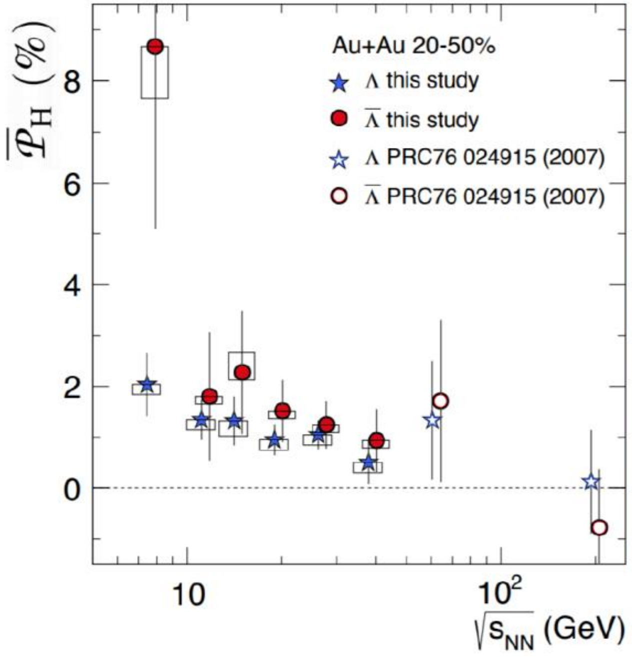
magneto-vortical transport, Hattori-Yin (2016)

$$J^0 = \frac{eB\omega}{4\pi^2}$$

SPIN POLARIZATION AND ALIGNMENT

- Global Λ and $\bar{\Lambda}$ spin polarization

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65



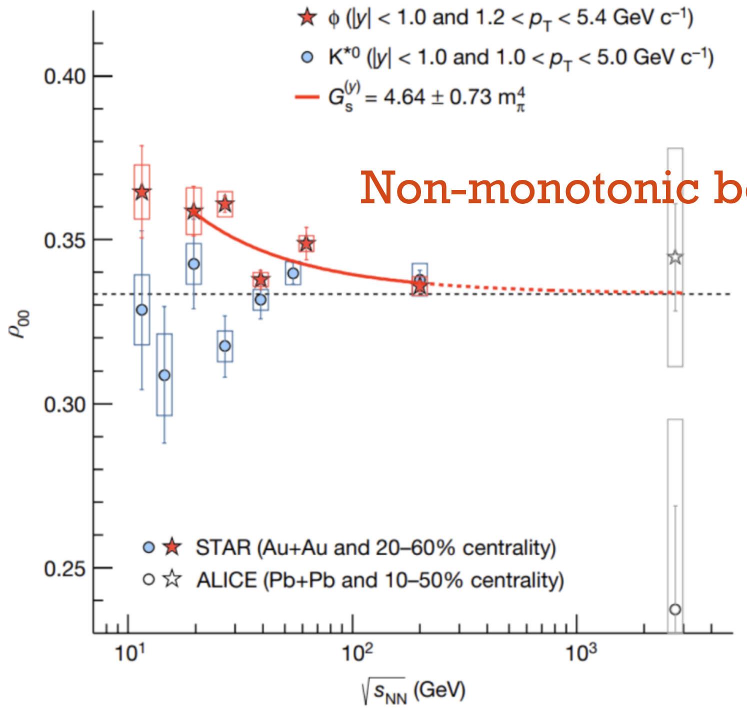
most vortical fluid produced in the laboratory .

$$\omega = (P_\Lambda + P_{\bar{\Lambda}})k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22}\ \mathrm{s}^{-1}$$

Local polarization

- ϕ meson spin alignment

STAR Collaboration, Nature 614 (2023) 7947.

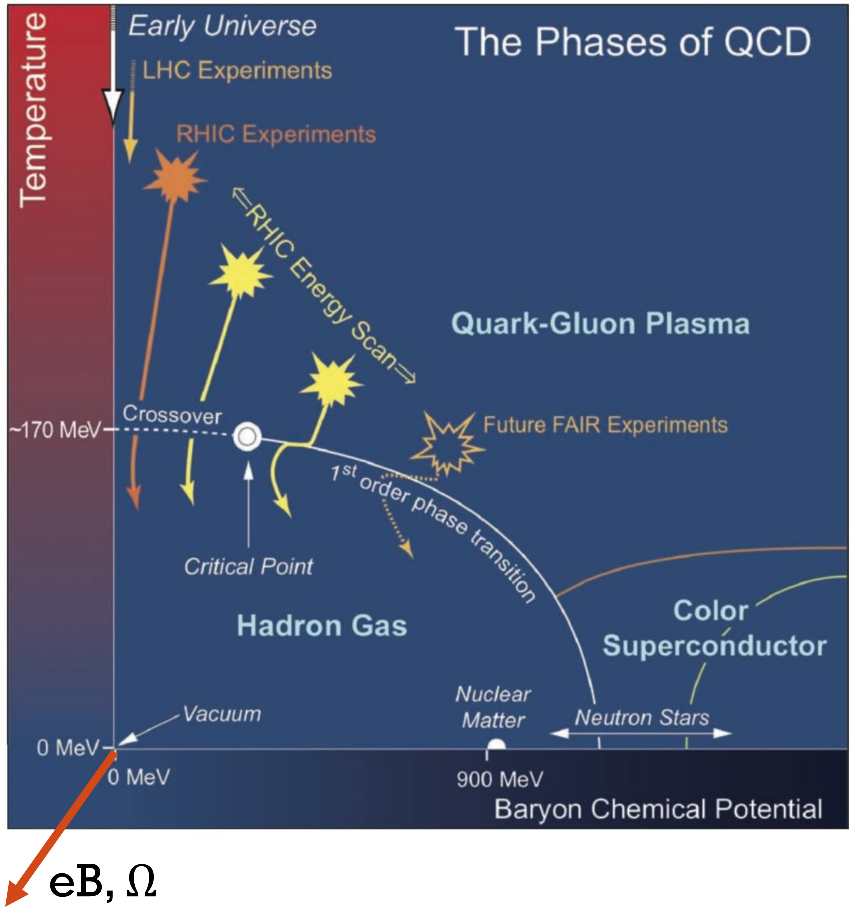


Non-monotonic behavior

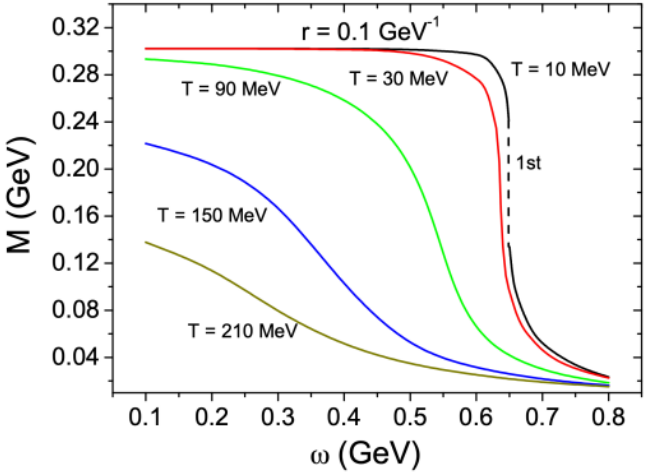
$\rho_{00} \neq 1/3$ indicates spin alignment

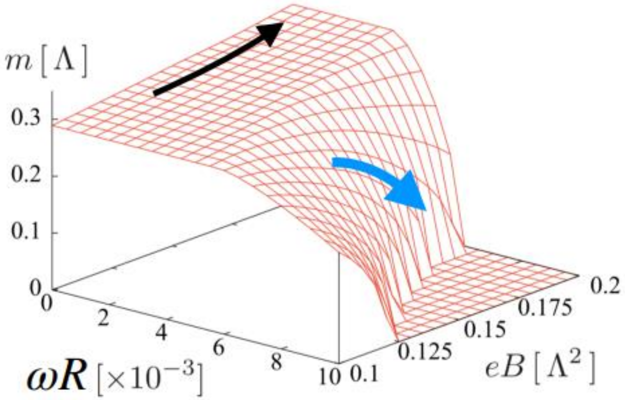
Other vector mesons

QCD PHASE DIAGRAM UNDER ROTATION



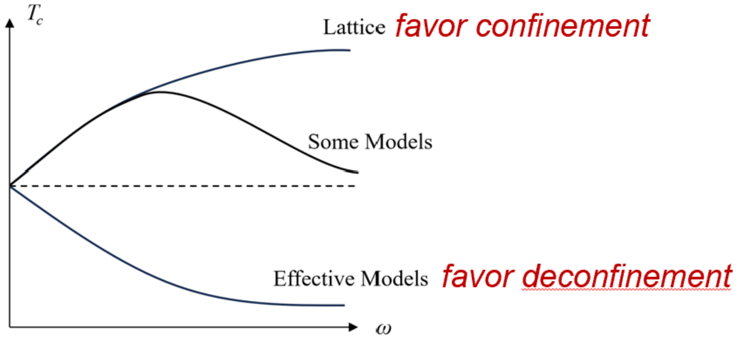
Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016),





HLC, et al, Phys. Rev. D 93, 104052 (2016)

- First studied by NJL model (order parameter: quark mass)
- Rotation behaves like chemical potential
- However, lattice studies give opposite result!



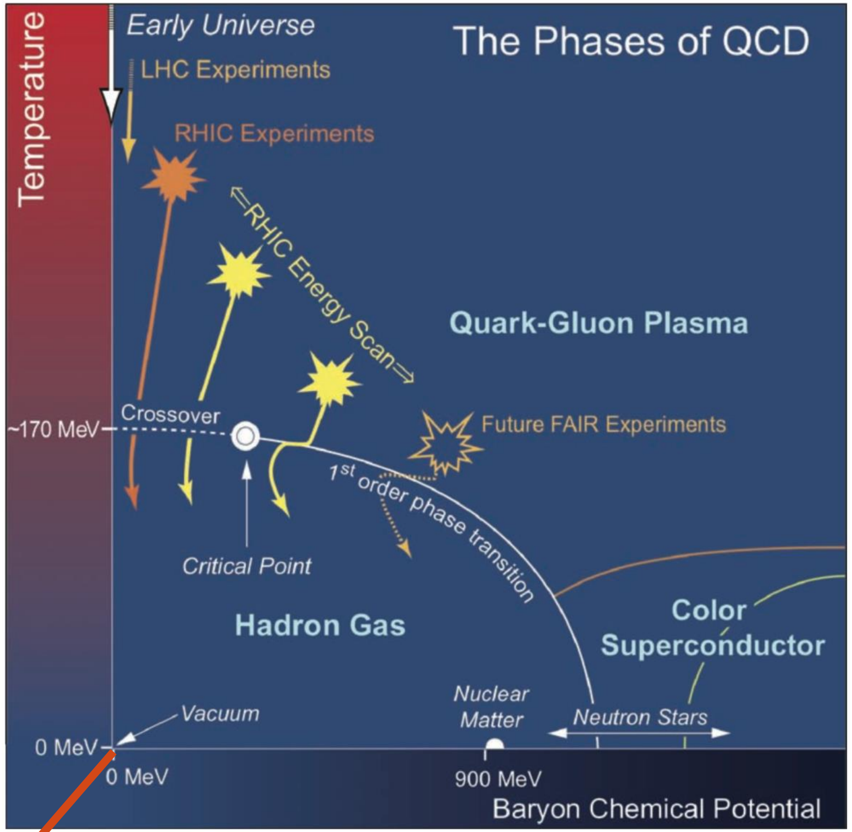
P. Zhuang's talk @PHD2024

MOTIVATION

- Almost studied separately

Spin polarization/alignment

Phase transition



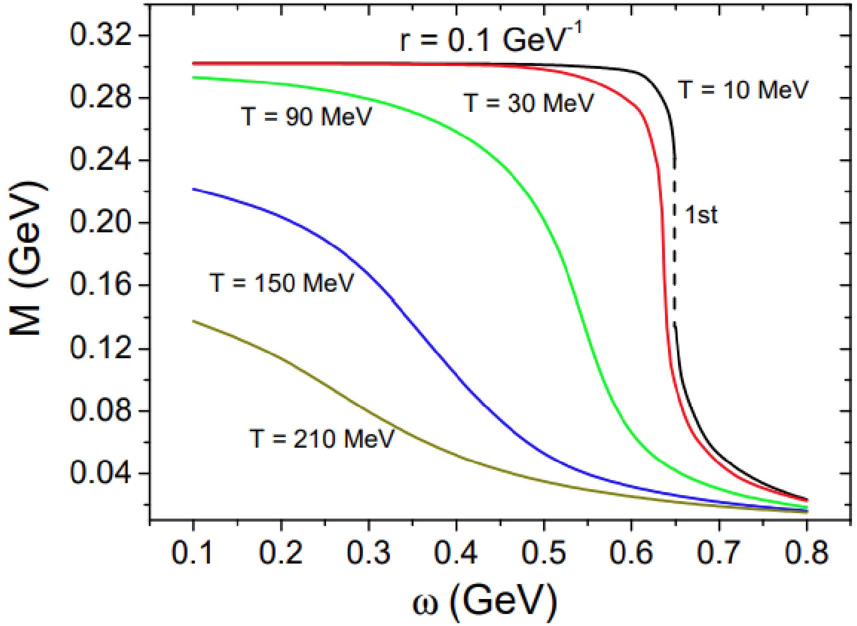
Ω

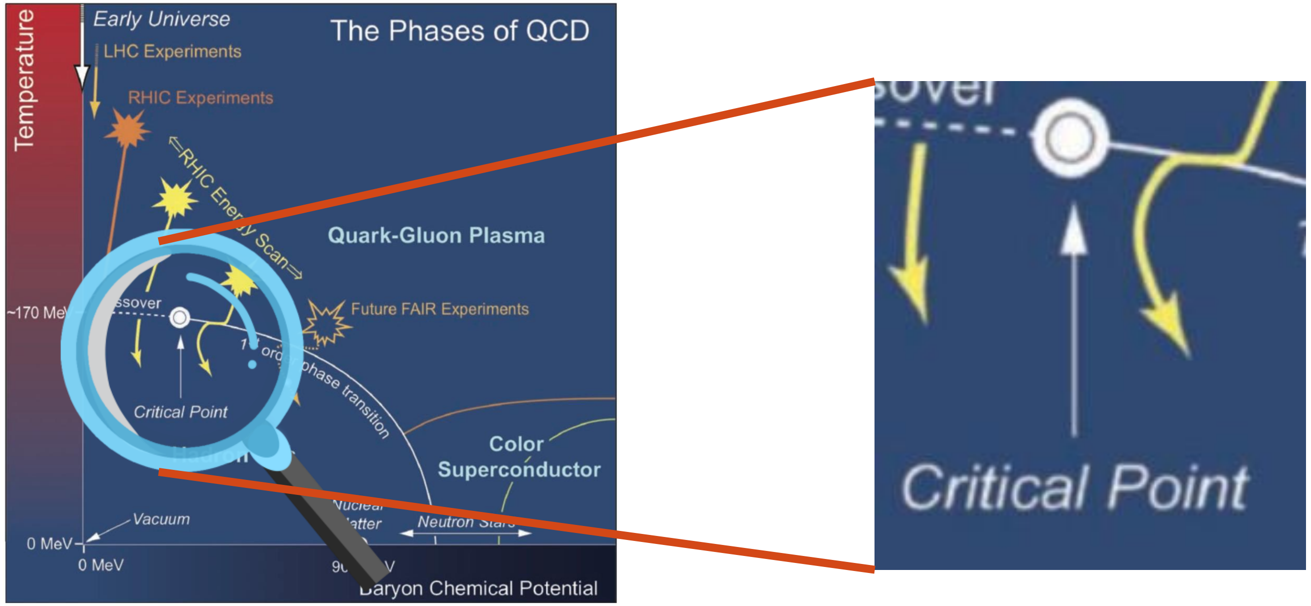
Q: Are these two aspects related?

NAÏVE ESTIMATIONS

- Spin polarization $\sim\Omega$
- Spin alignment $\sim\Omega^2$
- Chiral condensate does not change much at small Ω
- Not that large effect
- But….

Y. Jiang and J. Liao, Phys. Rev. Lett. 117, 192302 (2016),





!!

Spin fluctuation? Just like baryon number?

QUALITATIVE STUDY: NJL MODEL

$$g_{\mu\nu} = \begin{pmatrix} 1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Vierbein formulism (QFT in curved spacetime)

$$\Gamma_\mu = \frac{1}{4}\times\frac{1}{2}[\gamma^a,\gamma^b]\Gamma_{ab\mu} \qquad \Gamma_{ab\mu} = \eta_{ac}\left(e^c_\sigma G^\sigma_{\mu\nu} e^\nu_b - e^\nu_b \partial_\mu e^c_\nu\right) \qquad g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$$

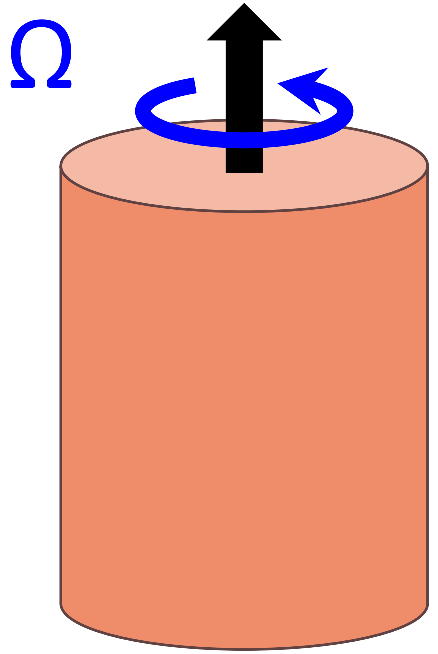
$$e^a_\mu = \delta^a_\mu + \delta^a_i \delta^0_\mu v_i \qquad e^\mu_a = \delta^\mu_a - \delta^0_a \delta^\mu_i v_i$$

- NJL model under rotation

$$\mathcal{L}_{NJL} = \bar\psi i\gamma^\mu \nabla_\mu \psi - m_0 \bar\psi\psi + \mu_B \bar\psi\gamma^0\psi + + \frac{G}{2}[(\bar\psi\psi)^2 + (\bar\psi\gamma^5\vec\tau\psi)^2]$$

$$\nabla_\mu = \partial_\mu + iQA_\mu + \boxed{\Gamma_\mu}$$

spinor connection



QUALITATIVE STUDY: NJL MODEL

- General thermodynamic potential under rotation

$$V_{eff}(r) = \frac{(m-m_0)^2}{4G} - N_c N_f \sum_l \int_0^{\Lambda} \frac{p_t \mathrm{d}p_t \mathrm{d}p_z}{(2\pi)^2} [\varepsilon_p + T\ln(1+\mathrm{e}^{-\beta(\varepsilon_p-\mu-\Omega j)}) + T\ln(1+\mathrm{e}^{-\beta(\varepsilon_p+\mu+\Omega j)})](J_l^2(p_t r) + J_{l+1}^2(p_t r)).$$

- Away from the center, contribution from orbital angular momentum is dominant
- Since we are interested in spin only, we focus on the physics near the center (r=0)

$$\begin{aligned} V_{eff}^0(\Omega,\mu) = &\frac{(m-m_0)^2}{4G} - N_c N_f \int_0^{\Lambda} \frac{\mathrm{d}^3 p}{(2\pi)^2} 2\varepsilon_p \\ &+ N_c N_f \int_0^{\infty} \frac{\mathrm{d}^3 p}{(2\pi)^2} [T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega/2)/T}) + T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega/2)/T}) \\ &+ T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega/2)/T}) + T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega/2)/T})]. \end{aligned}$$

$q,\uparrow$ $\quad$ $q,\downarrow$

$\bar{q},\uparrow$ $\quad$ $\bar{q},\downarrow$

- However, we can only get information about average spin and particle number from this expression

QUALITATIVE STUDY: NJL MODEL

- Calculation from average values

$$V^0_{eff}(\Omega,\mu) = \frac{(m-m_0)^2}{4G} - N_c N_f \int_0^{\Lambda} \frac{\mathrm{d}^3 p}{(2\pi)^2} 2\varepsilon_p$$

$q,\uparrow$ $\quad$ $q,\downarrow$

$$+ N_c N_f \int_0^{\infty} \frac{\mathrm{d}^3 p}{(2\pi)^2} [T \ln(1+\mathrm{e}^{-(\varepsilon_p-\mu-\Omega/2)/T}) + T\ln(1+\mathrm{e}^{-(\varepsilon_p-\mu+\Omega/2)/T})$$

$$+ T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu-\Omega/2)/T}) + T\ln(1+\mathrm{e}^{-(\varepsilon_p+\mu+\Omega/2)/T})].$$

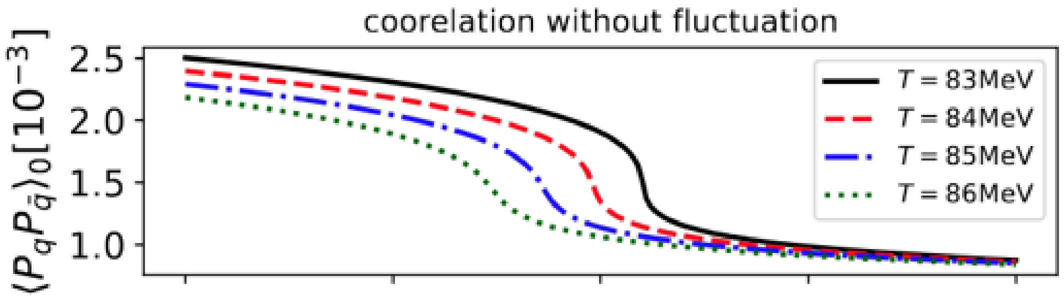
$\bar{q},\uparrow$ $\quad$ $\bar{q},\downarrow$

$$\langle P_q P_{\bar{q}}\rangle_0 = \frac{\int \mathrm{d}^3 p (f_q^{\uparrow} - f_q^{\downarrow})(f_{\bar{q}}^{\uparrow} - f_{\bar{q}}^{\downarrow})}{\int \mathrm{d}^3 p (f_q^{\uparrow} + f_q^{\downarrow})(f_{\bar{q}}^{\uparrow} + f_{\bar{q}}^{\downarrow})}.$$

0 (Gap Eq)

$$\langle S\rangle = -\frac{V}{T}\frac{\mathrm{d}V^0_{eff}}{\mathrm{d}(\frac{\Omega}{T})} = -\frac{V}{T}\frac{\partial V^0_{eff}}{\partial(\frac{\Omega}{T})} - \frac{V}{T}\frac{\partial V^0_{eff}}{\partial m}\frac{\partial m}{\partial(\frac{\Omega}{T})}.$$

$$f_q^{\uparrow/\downarrow} = \frac{1}{\mathrm{e}^{\beta(\epsilon_p-\mu\mp\frac{\Omega}{2})}+1}, \quad f_{\bar{q}}^{\uparrow/\downarrow} = \frac{1}{\mathrm{e}^{\beta(\epsilon_p+\mu\mp\frac{\Omega}{2})}+1}.$$



QUALITATIVE STUDY: NJL MODEL

- Further techniques are needed
  - We focus on the physic near the center (r=0)
  - Introducing rotation and chemical potential only act on quark or antiquark

$$V^0_{eff}(\Omega_q, \Omega_{\bar{q}}, \mu_q, \mu_{\bar{q}}) = \frac{(m-m_0)^2}{4G} - N_c N_f \int_0^{\Lambda} \frac{\mathrm{d}^3 p}{(2\pi)^2} 2\varepsilon_p$$
$$+ N_c N_f \int_0^{\infty} \frac{\mathrm{d}^3 p}{(2\pi)^2} [T \ln(1 + \mathrm{e}^{-(\varepsilon_p - \mu - \Omega_q/2 - \mu_q)/T}) + T \ln(1 + \mathrm{e}^{-(\varepsilon_p - \mu + \Omega_q/2 - \mu_q)/T})$$
$$+ T \ln(1 + \mathrm{e}^{-(\varepsilon_p + \mu - \Omega_{\bar{q}}/2 - \mu_{\bar{q}})/T}) + T \ln(1 + \mathrm{e}^{-(\varepsilon_p + \mu + \Omega_{\bar{q}}/2 - \mu_{\bar{q}})/T})].$$

  - Then by taking derivative, we can get cumulants of spin and particle number

$$\langle \delta S_q^2 \rangle = -\frac{V}{T} \frac{\partial^2 V_{eff}(\Omega_q, \Omega_{\bar{q}})}{\partial(\frac{\Omega_q}{T})^2}\Big|_{\Omega_q = \Omega_{\bar{q}} = \Omega}, \qquad \langle \delta N_q^2 \rangle = -\frac{V}{T} \frac{\partial^2 V_{eff}}{\partial(\frac{\mu_q}{T})^2}\Big|_{\mu_q = \mu_{\bar{q}} = 0},$$
$$\langle \delta S_{\bar{q}}^2 \rangle = -\frac{V}{T} \frac{\partial^2 V_{eff}(\Omega_q, \Omega_{\bar{q}})}{\partial(\frac{\Omega_{\bar{q}}}{T})^2}\Big|_{\Omega_q = \Omega_{\bar{q}} = \Omega}, \qquad \langle \delta N_{\bar{q}}^2 \rangle = -\frac{V}{T} \frac{\partial^2 V_{eff}}{\partial(\frac{\mu_{\bar{q}}}{T})^2}\Big|_{\mu_q = \mu_{\bar{q}} = 0}.$$

THE SPIN CORRELATION OF QUARK-ANTIQUARK

- Separating total spin into quark and antiquark part $S = S_q + S_{\bar{q}}$;
- Easy to prove

$$\langle S_q S_{\bar{q}} \rangle - \langle S_q \rangle \langle S_{\bar{q}} \rangle = \frac{1}{2}(\langle \delta S^2 \rangle - \langle \delta S_q^2 \rangle - \langle \delta S_{\bar{q}}^2 \rangle),$$

- Similarly

$$\langle N_q N_{\bar{q}} \rangle - \langle N_q \rangle \langle N_{\bar{q}} \rangle = \frac{1}{2}(\langle \delta N^2 \rangle - \langle \delta N_q^2 \rangle - \langle \delta N_{\bar{q}}^2 \rangle).$$

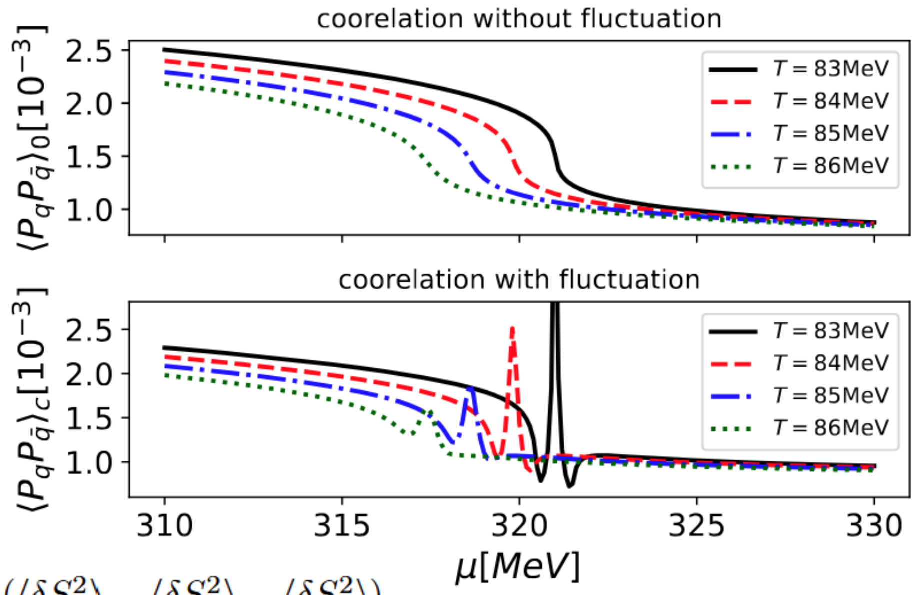
- Then we can define the spin correlation of quark-antiquark as

$$\langle P_q P_{\bar{q}} \rangle_c = \frac{4(\langle \delta S^2 \rangle - \langle \delta S_q^2 \rangle - \langle \delta S_{\bar{q}}^2 \rangle)}{\langle \delta N^2 \rangle - \langle \delta N_q^2 \rangle - \langle \delta N_{\bar{q}}^2 \rangle}.$$

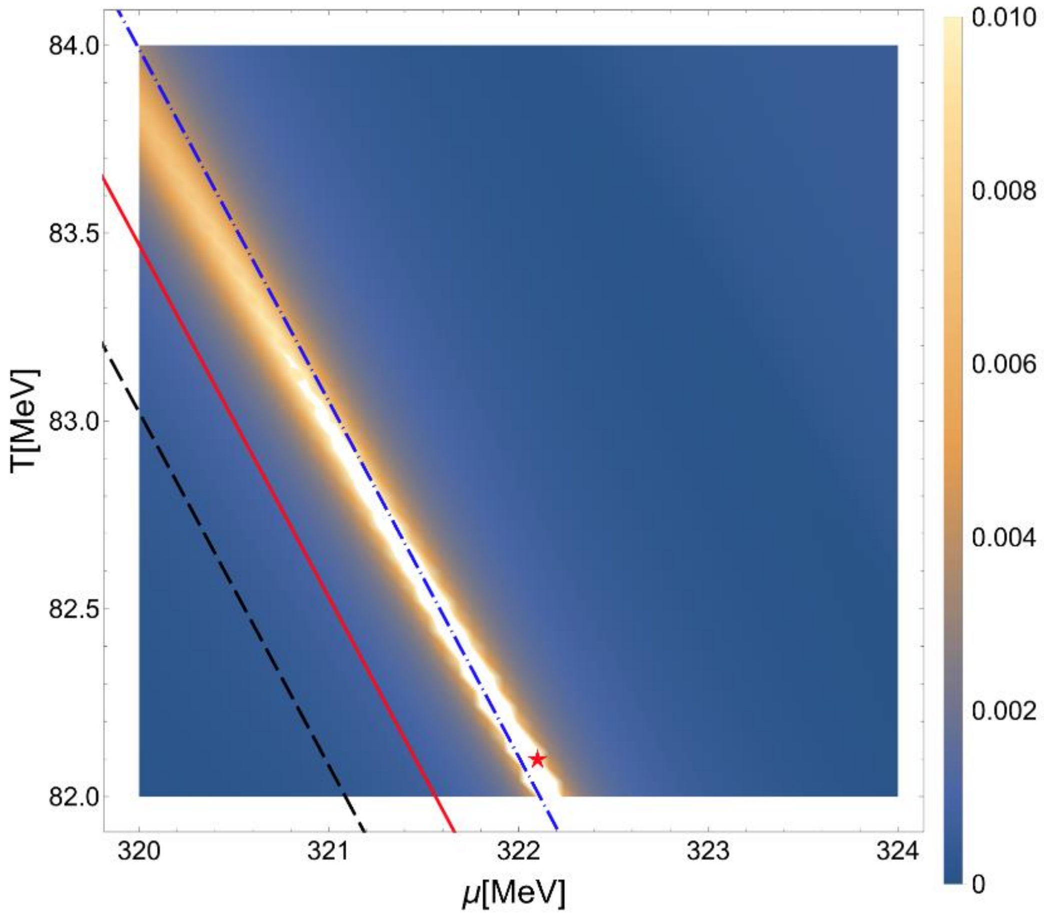
SPIN CORRELATION ENHANCED BY CEP!

- Comparison with the case w/o fluctuation

$$\langle P_q P_{\bar{q}} \rangle_0 = \frac{\int \mathrm{d}^3 p (f_q^{\uparrow} - f_q^{\downarrow})(f_{\bar{q}}^{\uparrow} - f_{\bar{q}}^{\downarrow})}{\int \mathrm{d}^3 p (f_q^{\uparrow} + f_q^{\downarrow})(f_{\bar{q}}^{\uparrow} + f_{\bar{q}}^{\downarrow})}.$$



$$\langle P_q P_{\bar{q}} \rangle_c = \frac{4(\langle \delta S^2 \rangle - \langle \delta S_q^2 \rangle - \langle \delta S_{\bar{q}}^2 \rangle)}{\langle \delta N^2 \rangle - \langle \delta N_q^2 \rangle - \langle \delta N_{\bar{q}}^2 \rangle}.$$

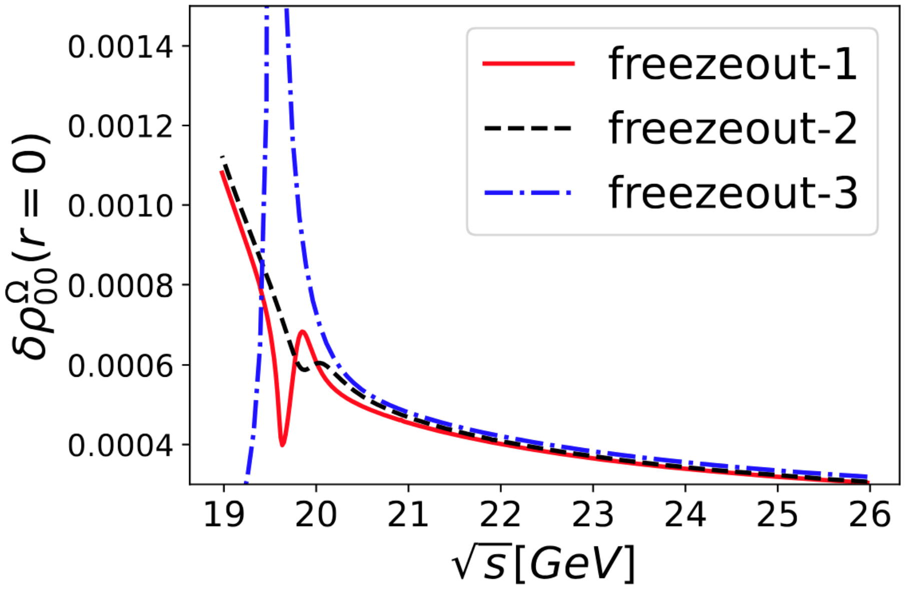


VECTOR MESON SPIN ALIGNMENT

- Along imaginary freezeout lines

$$\rho_{00} = \frac{1 - \langle P_q P_{\bar{q}} \rangle}{3 + \langle P_q P_{\bar{q}} \rangle} \approx \bar{\rho}_{00} - \delta\rho^{\Omega}_{00}.$$

Contribution from critical fluctuation



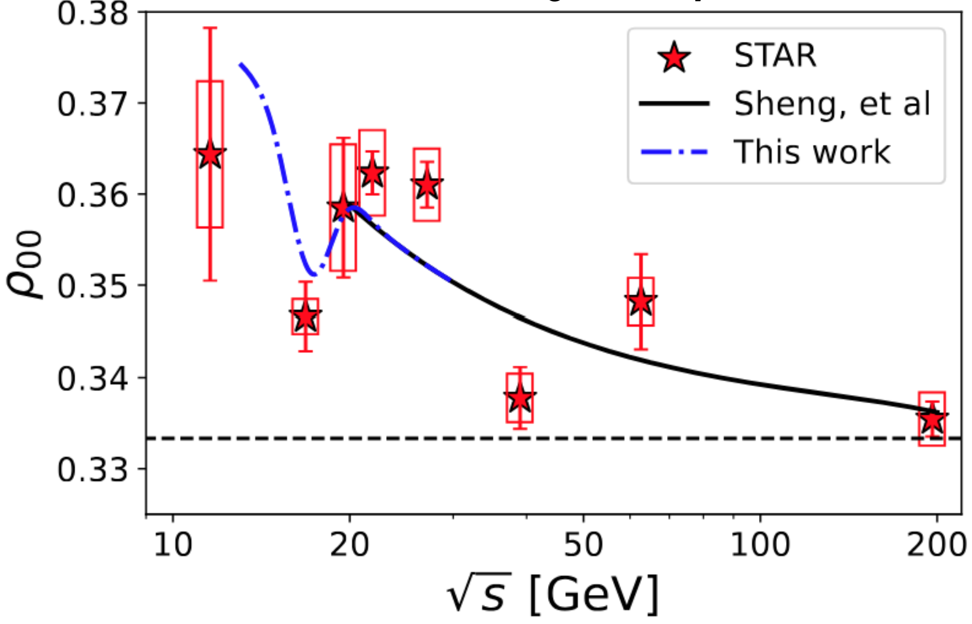
As additional contribution

$$\rho_{00} = \frac{1 - \langle P_q P_{\bar{q}} \rangle}{3 + \langle P_q P_{\bar{q}} \rangle} \approx \bar{\rho}_{00} - \delta\rho^{\Omega}_{00}.$$

A SCHEMATIC FIGURE

G. Wilks' talk@SQM2024

X.-L. Sheng, et al, Phys. Rev. Lett. 131, 042304 (2023)



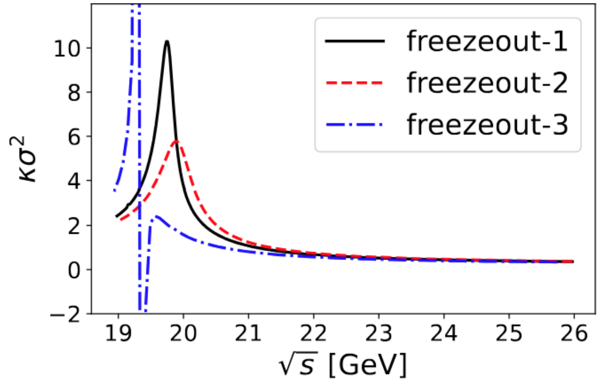
TAKE-HOME MESSAGES

- Quark spin also fluctuates near the CEP
- Critical fluctuation near CEP can lead to non-monotonic behavior of Spin alignment & Hyperon-anti-Hyperon correlation
- Spin alignment & Hyperon-anti-Hyperon correlation can serve as signatures for CEP

$$\frac{N_{H\bar{H}}^{\uparrow\uparrow}+N_{H\bar{H}}^{\downarrow\downarrow}-N_{H\bar{H}}^{\uparrow\downarrow}-N_{H\bar{H}}^{\downarrow\uparrow}}{N_{H\bar{H}}^{\uparrow\uparrow}+N_{H\bar{H}}^{\downarrow\downarrow}+N_{H\bar{H}}^{\uparrow\downarrow}+N_{H\bar{H}}^{\downarrow\uparrow}}$$

- Effect on higher order cumulants will be more significant
- Contribution from gluon field?

Quark spin kurtosis



THANKS！