Transport Model Study of QCD Phase Transition in Heavy-Ion Collisions

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USTC - Particle and Nuclear Physics

NPMiniWorkshop

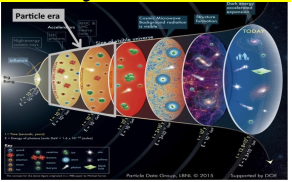
Outline

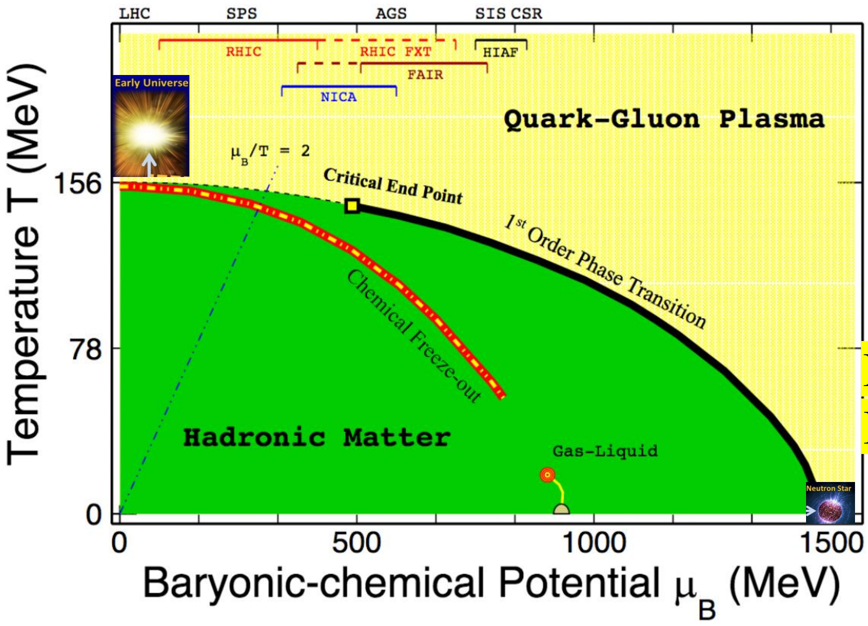
- Introduction
  - QCD Phase Diagram
  - Cumulants of Conserved Quantities
  - Beam Energy Scan (BES) at RHIC
- Method
  - A Multi-Phase Transport Model
- Results
  - Fluctuations of Net-Proton
  - Fluctuations of Net-Kaon
  - Incorporating FRG Into AMPT Model
- Summary and Outlook

Part 1:
Introduction

◆ QCD Phase Diagram

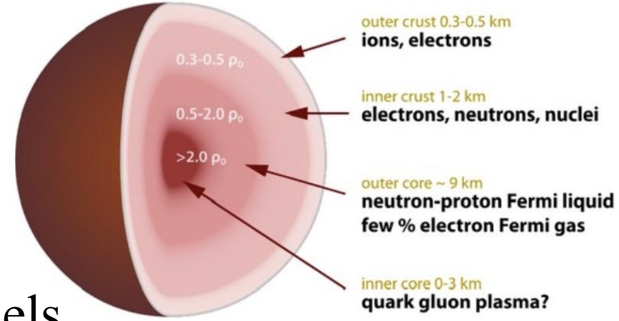
Early Universe





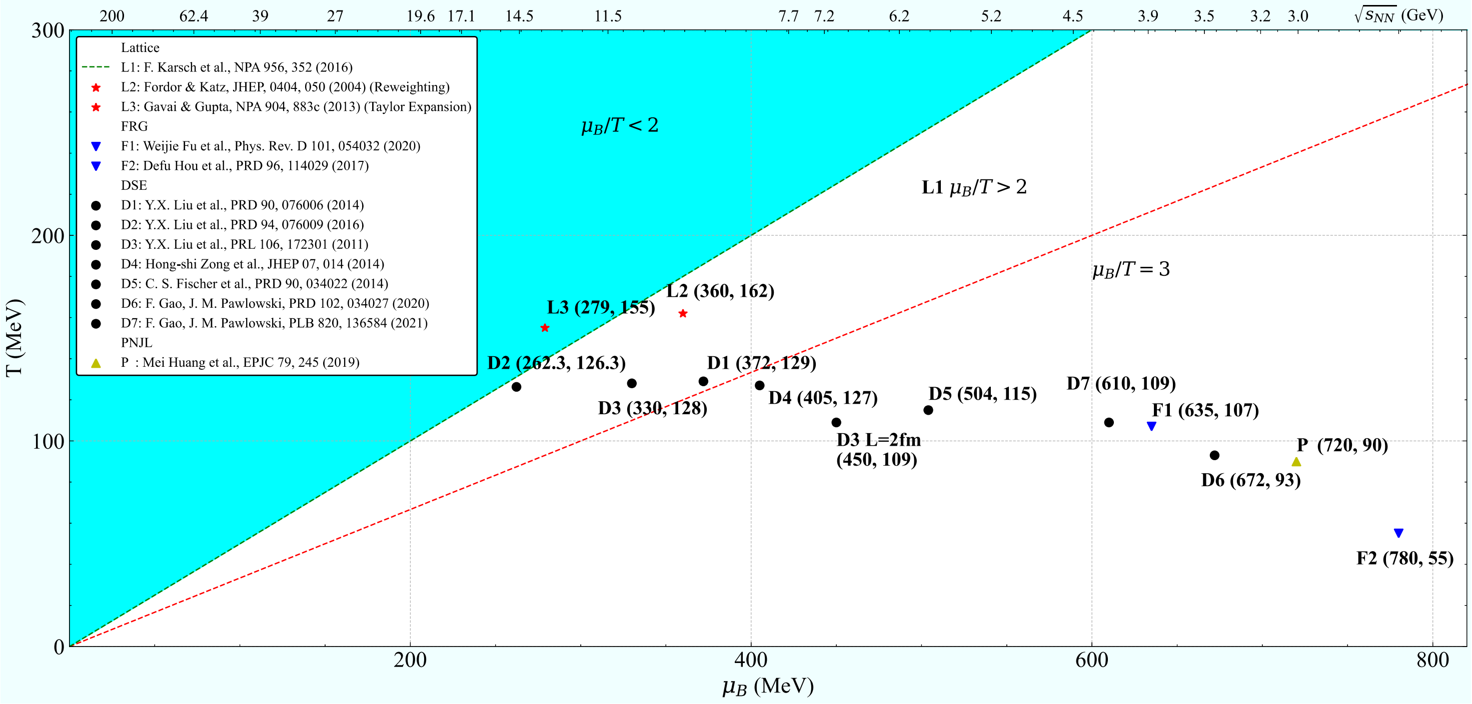
Y. Aoki et al, Nature 443, 675(2006)
A. Bzdak et al, Physics Reports 853,1-87(2020)
X. Luo, N. Xu, Nucl. Sci. Tech. 28, 112 (2017)
X. Luo, Q. Wang, N. Xu, P. F. Zhuang. Properties of QCD Matter at High Baryon Density.
Springer, 2022, doi:10.1007/978-981-19-4441-3
https://drupal.star.bnl.gov/STAR/starnotes/public/sn0598

High baryon density:
Inner structure of compact stars



- Smooth crossover at $\mu_B = 0$ MeV by Lattice QCD
- 1st-order phase transition at large μ_B by various models
- QCD critical point (CP)?

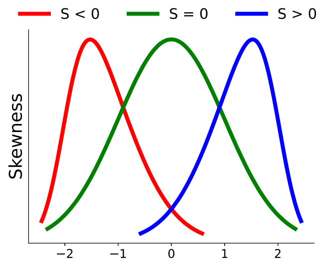
◆ Theoretical Predictions



➢ No consensus on the location of critical point!

◆ Cumulants of Conserved Quantities

Net-baryon (B) (net-proton as proxy)
Net-electric charge (Q)
Net-strangeness (S) (net-kaon as proxy)



◆ Cumulant

$$\delta N = N - \langle N \rangle$$

$$C_1 = \langle N \rangle = M$$

N: event-wise net-particle multiplicity

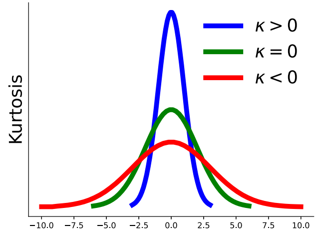
$$C_2 = \langle (\delta N)^2 \rangle = \sigma^2$$

$$C_3 = \langle (\delta N)^3 \rangle$$

$$C_4 = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2$$

$$\frac{C_2}{C_1} = \frac{\sigma^2}{M}, \quad \frac{C_3}{C_2} = S\sigma$$

$$\frac{C_4}{C_2} = \kappa\sigma^2$$



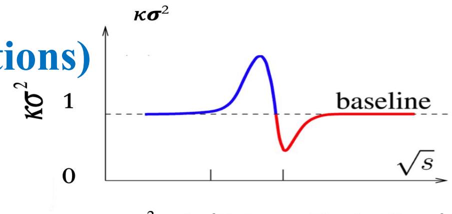
◆ Factorial cumulant (correlation functions)

$$\kappa_1 = C_1$$

$$\kappa_2 = -C_1 + C_2$$

$$\kappa_3 = 2C_1 - 3C_2 + C_3$$

$$\kappa_4 = -6C_1 + 11C_2 - 6C_3 + C_4$$



$\kappa\sigma^2 = 1$ (Poisson Fluctuations)

1. Sensitive to correlation length ξ

Near CP $\rightarrow$ $\xi \uparrow$

$$C_3 = \langle (\delta N)^3 \rangle \sim \xi^{4.5}$$

$$C_4 = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle \sim \xi^7$$

2. Related to susceptibility

$$\frac{\chi_4^q}{\chi_2^q} = \kappa\sigma^2 = \frac{C_4^q}{C_2^q}, \quad \frac{\chi_3^q}{\chi_2^q} = S\sigma = \frac{C_3^q}{C_2^q}$$

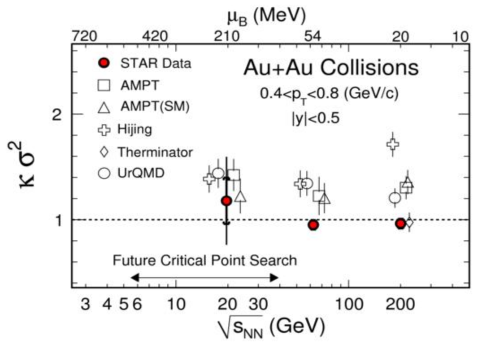
$$\chi_n^q = \frac{1}{VT^3} \cdot C_n^q = \frac{\partial^n (p/T^4)}{\partial(\mu^q)^n}, \quad q = B, Q, S$$

3. Non-monotonic energy dependence of $\kappa\sigma^2$ (C_4/C_2) $\rightarrow$ existence of a critical point

M. A. Stephanov, PRL 102, 032301 (09);
M. Asakawa, S. Ejiri and M. Kitazawa, PRL 103, 262301 (09)
S.Ejiri et al, PLB 633, 275(06);
M. A. Stephanov, PRL 107, 052301 (11);
F. Karsch and K. Redlich, PLB 695, 136 (11)

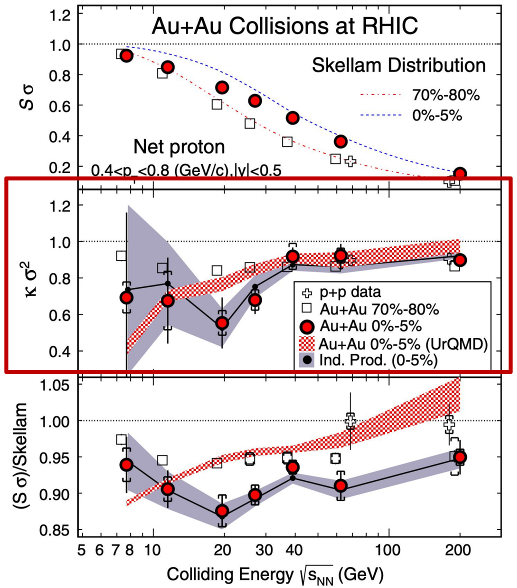
◆ Measurements from STAR

Net-proton
STAR, PRL 105, 022302(2010)



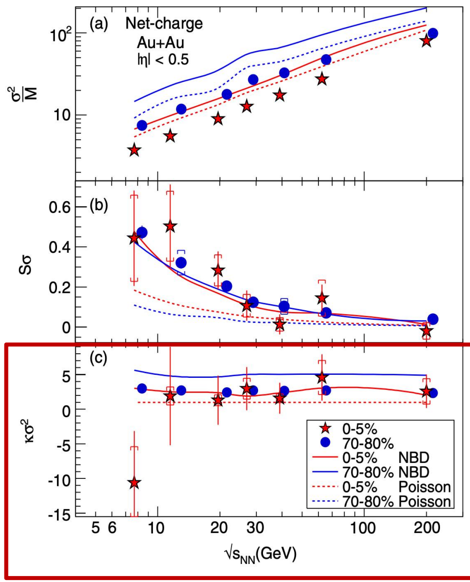
- ➢ First measurement on net-proton cumulant from STAR.
- ➢ Results lack detector efficiency correction.

Net-proton
STAR, PRL 112, 032302 (2014)

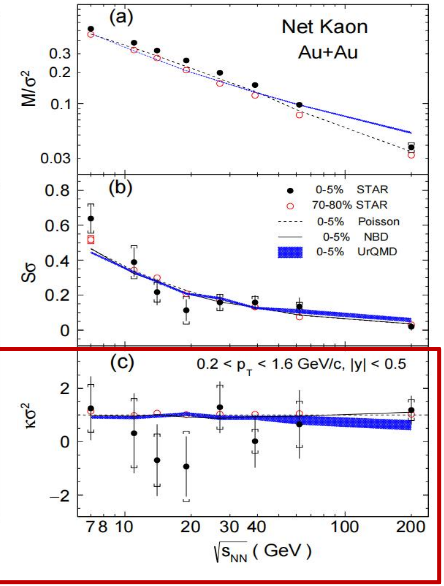


- Measurements within 0.4<p_T<0.8 GeV/c show deviations from Poisson baseline below 39 GeV.

Net-Charge
STAR, PRL113, 092301 (2014)



Net-Kaon
STAR, PLB 785, 551 (2018)



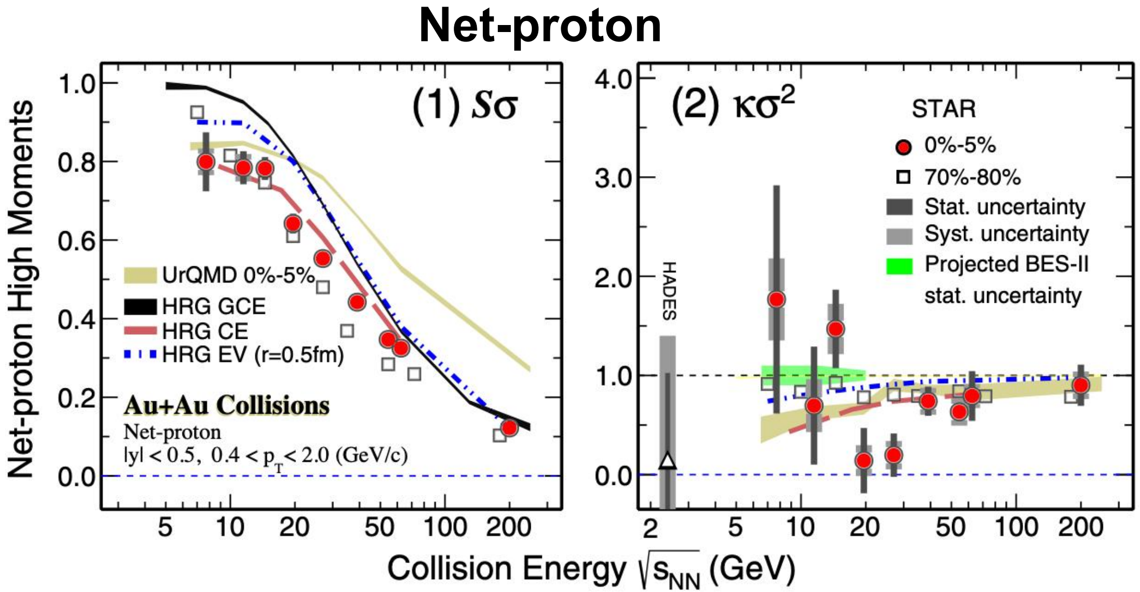
- Net-charge, kaon results of BES-I show weak energy dependence and are consistent with Poisson baseline within uncertainties.

◆ Net-proton Cumulants from STAR BES-I

STAR BES-I Program: Au+Au collisions

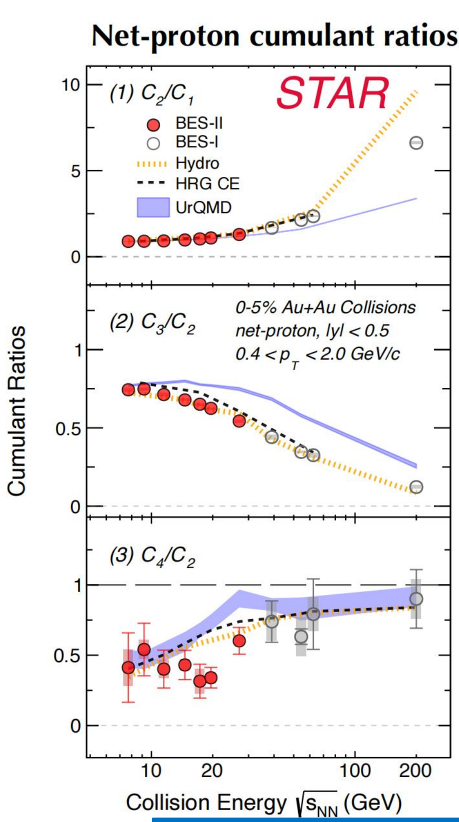
HADES, PRC 102(2020) 024914
STAR, PRL 126 (2021) 092301

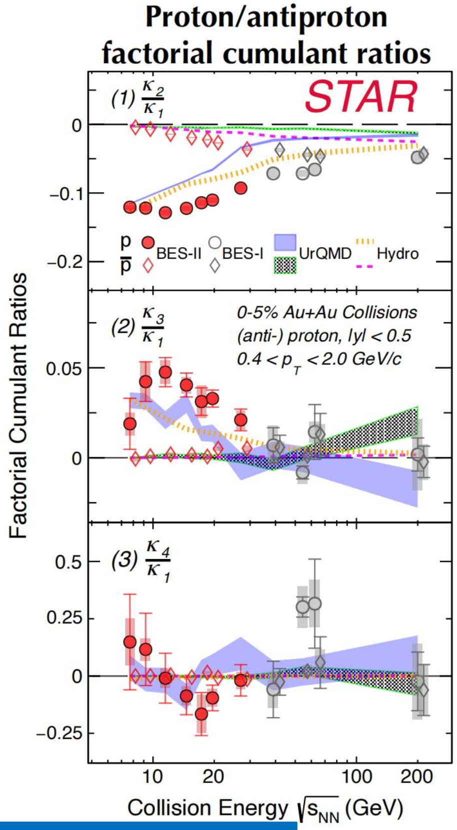
$\sqrt{s_{NN}}$ (GeV)	Events / 10^6	μ_B (Mev)
200	220	25
62.4	43	75
54.4	550	85
39	92	112
27	31	156
19.6	14	206
14.5	14	264
11.5	7	315
7.7	3	420
3.0	140	750

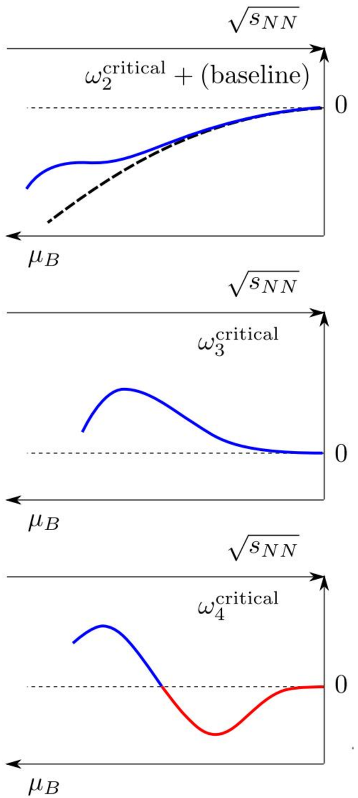


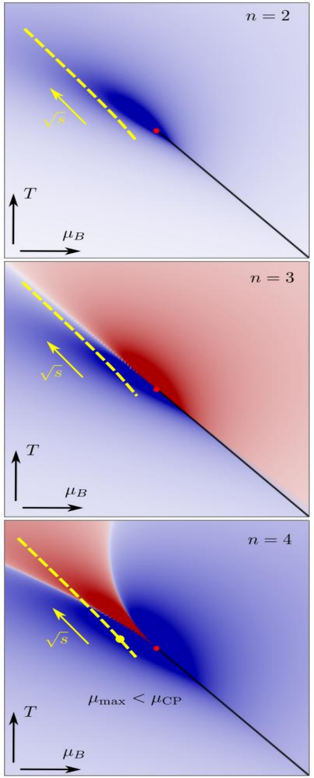
- Full measurement on BES-I datasets
- With TOF detector, p_T coverage is extended to 2.0 GeV/c
- **Non-monotonic energy dependence trend is observed with 3.1σ significance**

◆ Net-proton Cumulants from STAR BES-II







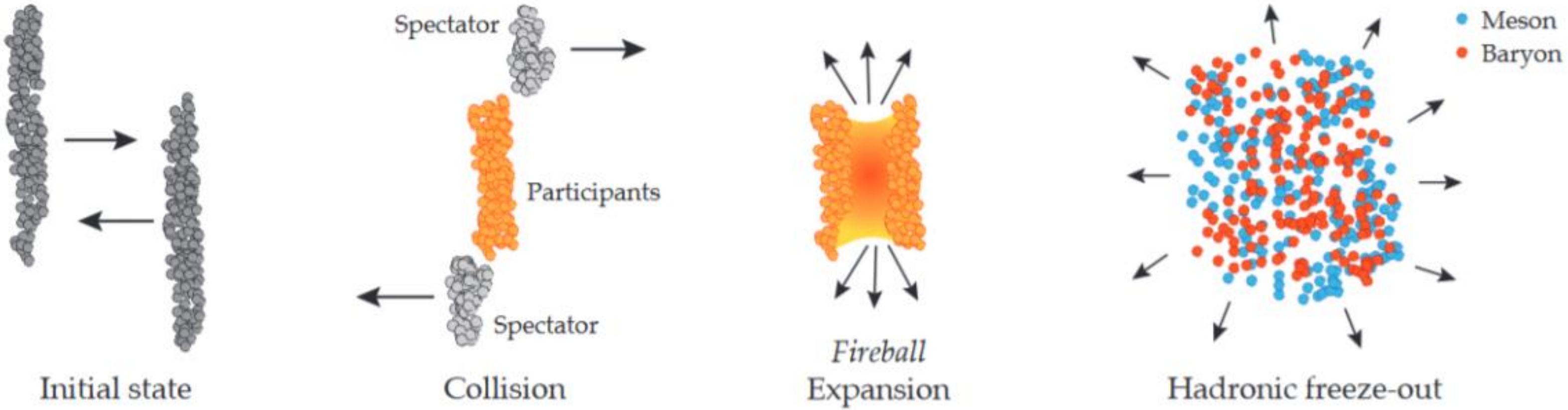


From CPOD2024, SQM2024

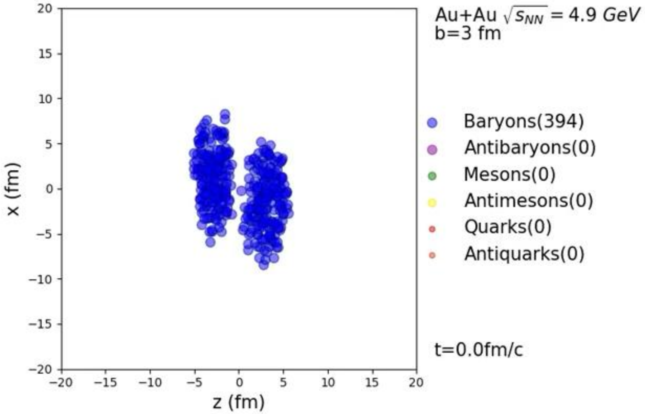
arXiv:2410.02861

Part 2:
A Multi-Phase Transport Model

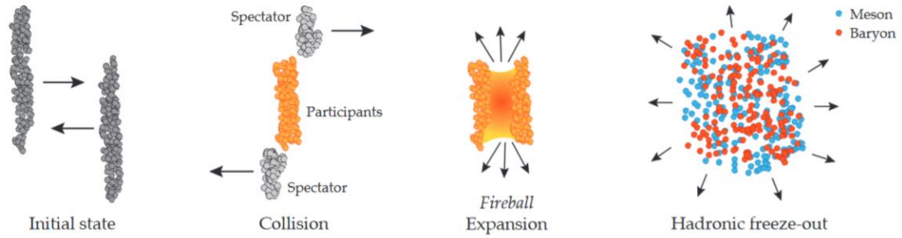
◆ A Multi-Phase Transport Model (AMPT)

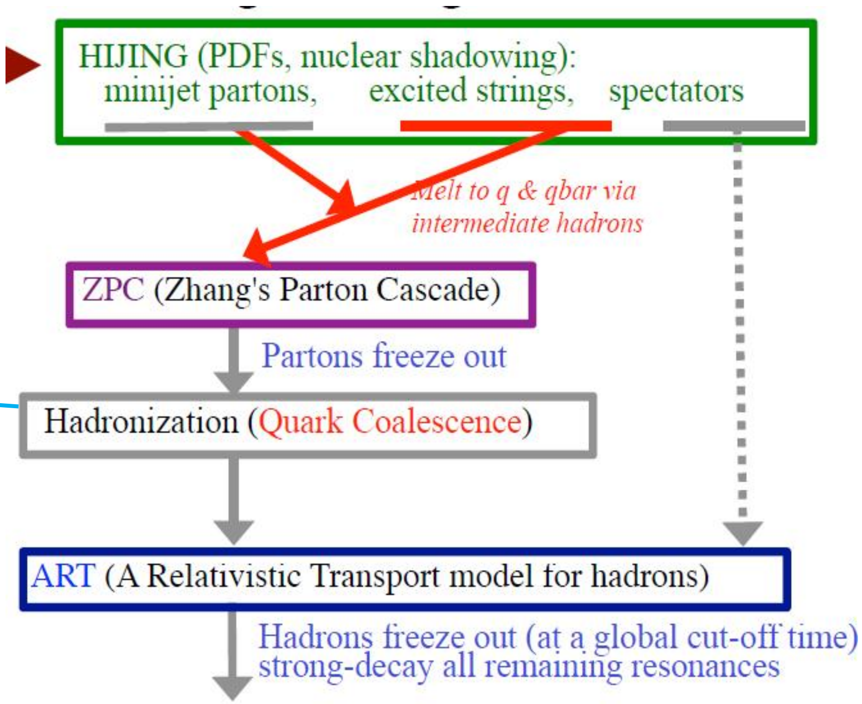


A dynamic simulation of relativistic heavy ion collisions using the **AMPT model:**



1. Lin Z W, Ko C M, Li B A, *et al*. PRC ,2005, **72**(6): 064901.
2. Ma G L, Lin Z W. PRC, 2016, 93(5): 054911.
3. Ma G L, Zhang B. PLB, 2011, 700(1): 39-43.
4. Ma G L. PRC, 2013, 87(6): 064901.
5. Bozek P, Bzdak A, Ma G L. PLB, 2015, 748.
6. Bzdak A, Ma G L. PRL, 2014, 113(25): 252301.





new quark coalescence:

quark to form either a meson or a baryon depending on the distance to its coalescence partner(s)(r_{BM})

$d_B < d_M * r_{BM}$:form a baryon

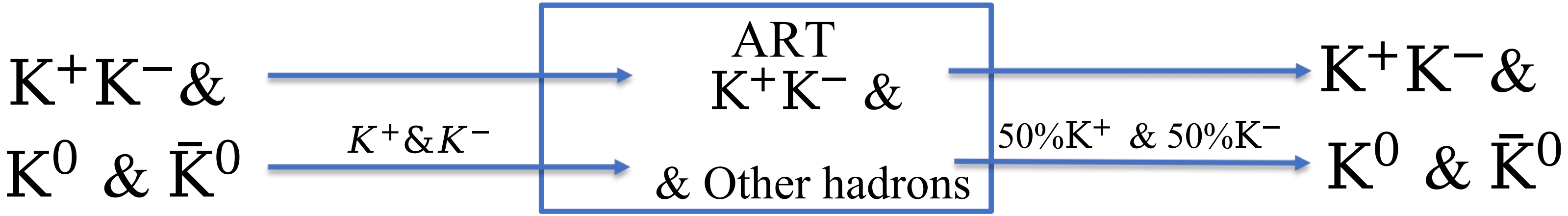
otherwise: form a meson

Y. He and Z.-W. Lin, Phys. Rev. C 96, 014910 (2017).

Extended AMPT model ensures the conservation of various conserved charges (including electric charge, baryon number, and strangeness) for all hadronic reaction channels during the evolution of hadronic phase

◆ A Multi-Phase Transport Model (AMPT)

- In the old version, only K^+ and K^- were introduced in hadron rescatterings as explicit particles, but K^0 and $\bar{K}^0$ were omitted.



- In the old version, some isospin-averaged cross sections were used, and the charge of the final state particles is chosen randomly from all possible charges, independent of the total charge of the initial state.

For example:

1) $\pi^+ + \pi^+ \to \rho^+ + \rho^+$ ✓

2) $\pi^+ + \pi^+ \to \rho^+ + \rho^-$ ×

3) $\pi^+ + \pi^+ \to \rho^- + \rho^-$ ×

Part 3:
Results

Fluctuations of Net-Proton

Cumulants:

$$C_1 = \kappa_1,$$

$$C_2 = \kappa_2 + \kappa_1,$$

$$C_3 = \kappa_3 + 3\kappa_2 + \kappa_1,$$

$$C_4 = \kappa_4 + 6\kappa_3 + 7\kappa_2 + \kappa_1.$$

Only for one particle! ! !

Correlation Functions:

$$\kappa_1 = C_1 = \langle N \rangle,$$

$$\kappa_2 = -C_1 + C_2,$$

$$\kappa_3 = 2C_1 - 3C_2 + C_3,$$

$$\kappa_4 = -6C_1 + 11C_2 - 6C_3 + C_4.$$

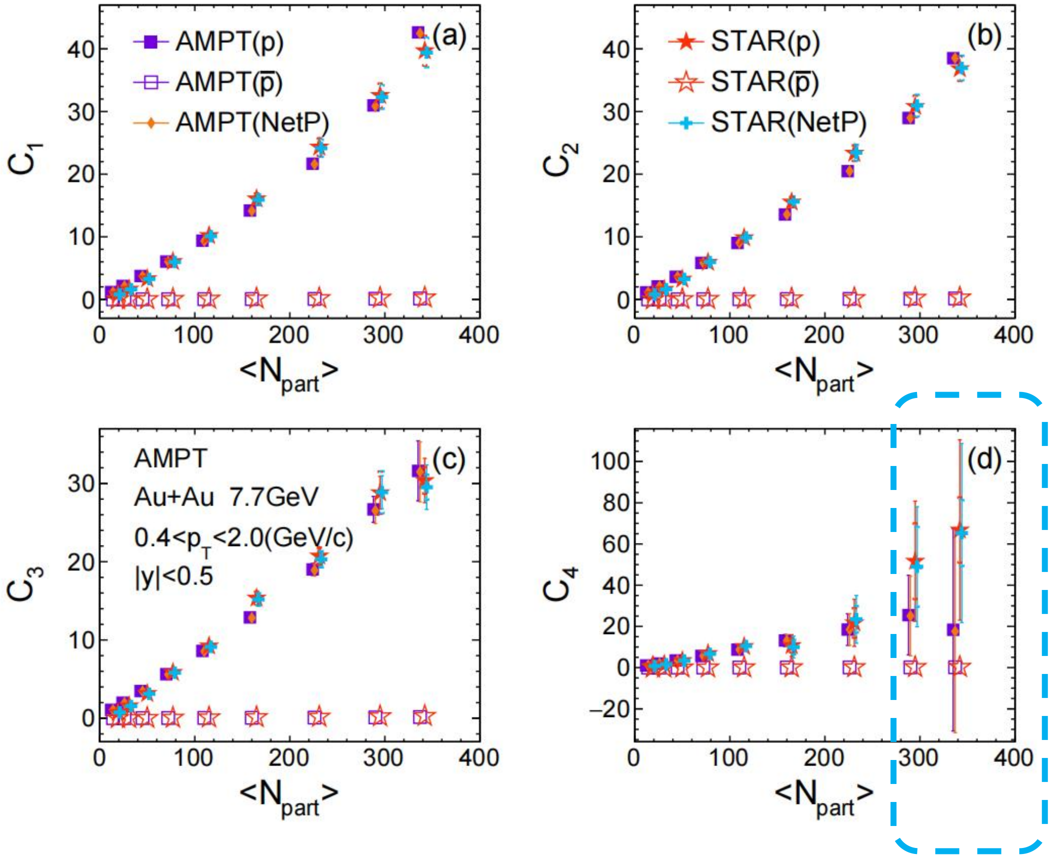
Factorial moments:

$$F_3 = \int dy_1 dy_2 dy_3 \rho_3(y_1, y_2, y_3) = F_1^3 + 3F_1C_2 + C_3$$

$$\rho_3(y_1, y_2, y_3) = \rho_1(y_1)\rho_1(y_2)\rho_1(y_3) + \rho_1(y_1)C_2(y_2, y_3) + \rho_1(y_2)C_2(y_1, y_3) + \rho_1(y_3)C_2(y_1, y_2) + C_3(y_1, y_2, y_3)$$

Bzdak, Adam et al. Phys.Rev. C95 (2017)5,054906.

◆ Fluctuations of Net-Proton



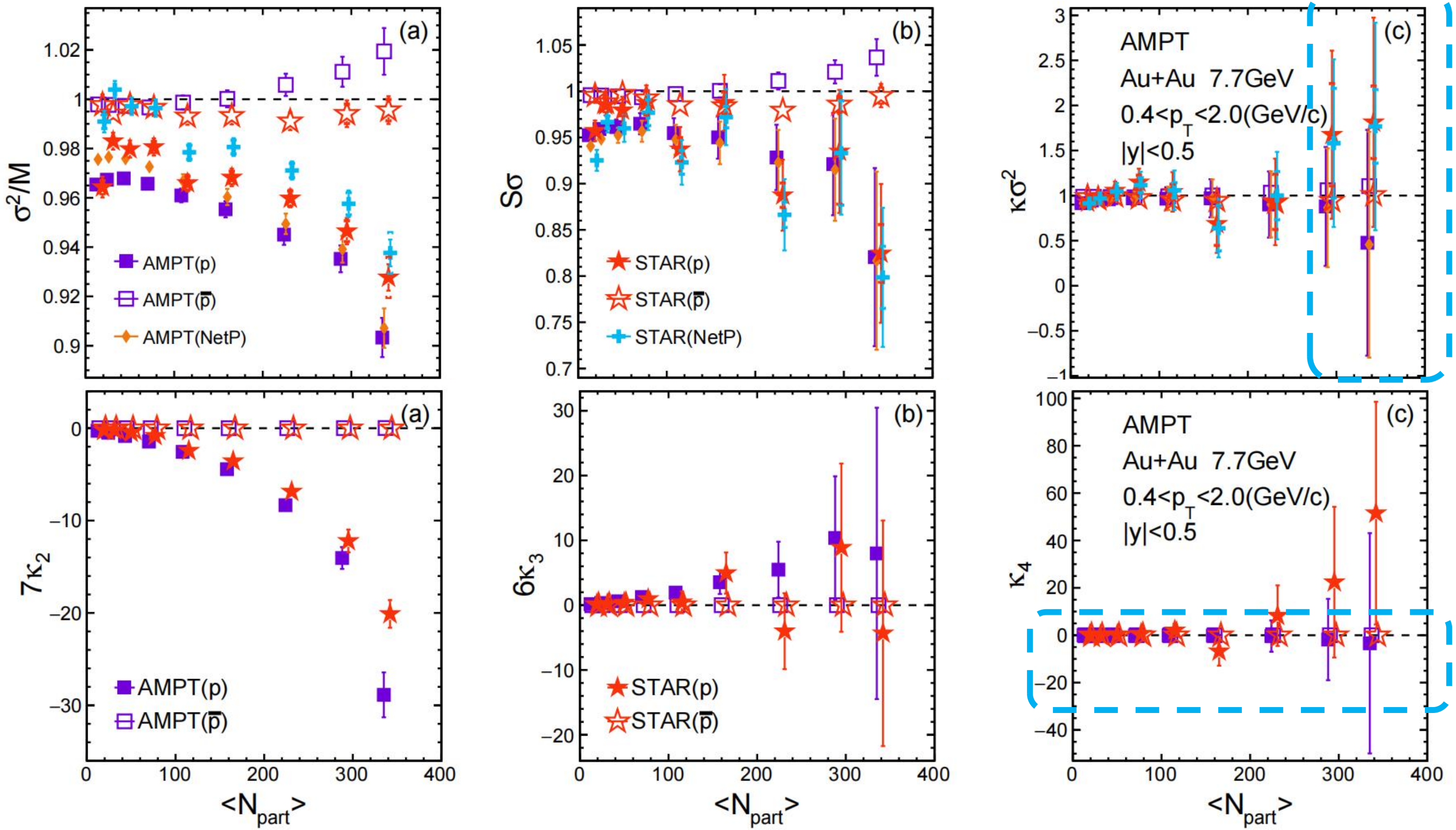
- The cumulants C_n for protons, antiprotons, and net-protons all show a similar increasing dependence on $\langle N_{part} \rangle$

- In the 0-5% and 5-10% centrality ranges, the fourth-order cumulant (C_4) in AMPT notably underestimates STAR's results

Qian Chen, Guo-Liang Ma, Phys.Rev.C 106 (2022) 014907

◆ Fluctuations of Net-Proton

Qian Chen, Guo-Liang Ma, Phys.Rev.C 106 (2022) 014907

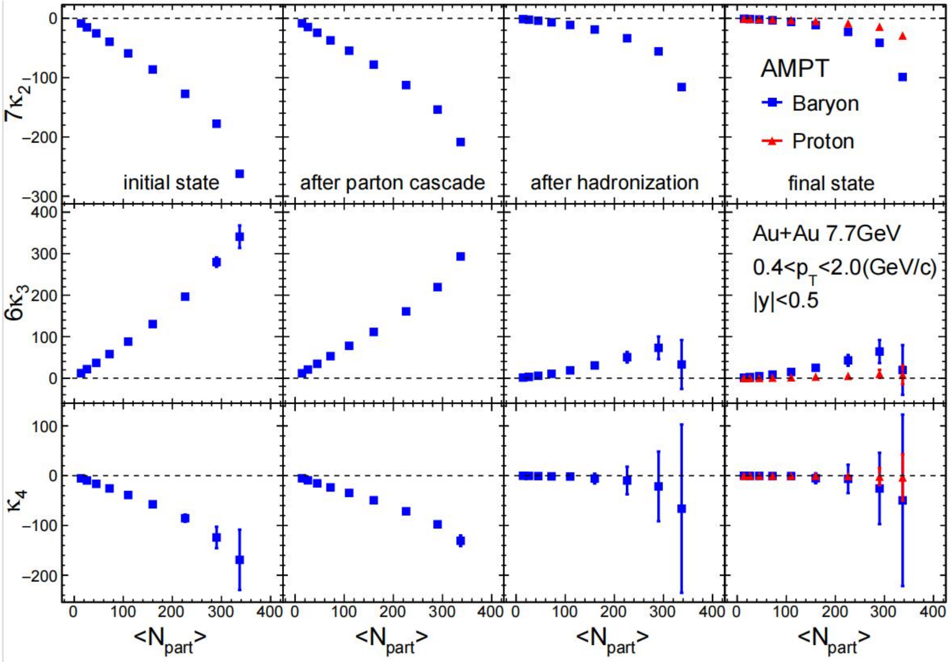


The four-proton correlation from AMPT is very small, consistent with zero.

Fluctuations of Net-Proton

Expectation of baryon number conservation:

$$P(N) = \frac{B!}{N!(B-N)!} p^N (1-p)^{(B-N)}$$



Qian Chen, Guo-Liang Ma, Phys.Rev.C 106 (2022) 014907

n-baryon correlations:

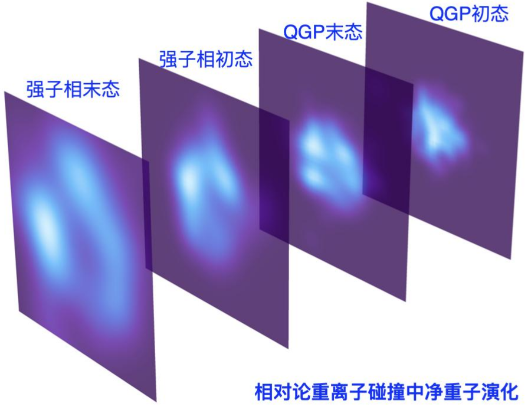
$$\kappa_1 = \langle N \rangle = pB$$

$$\kappa_2 = -\frac{\langle N \rangle^2}{B}$$

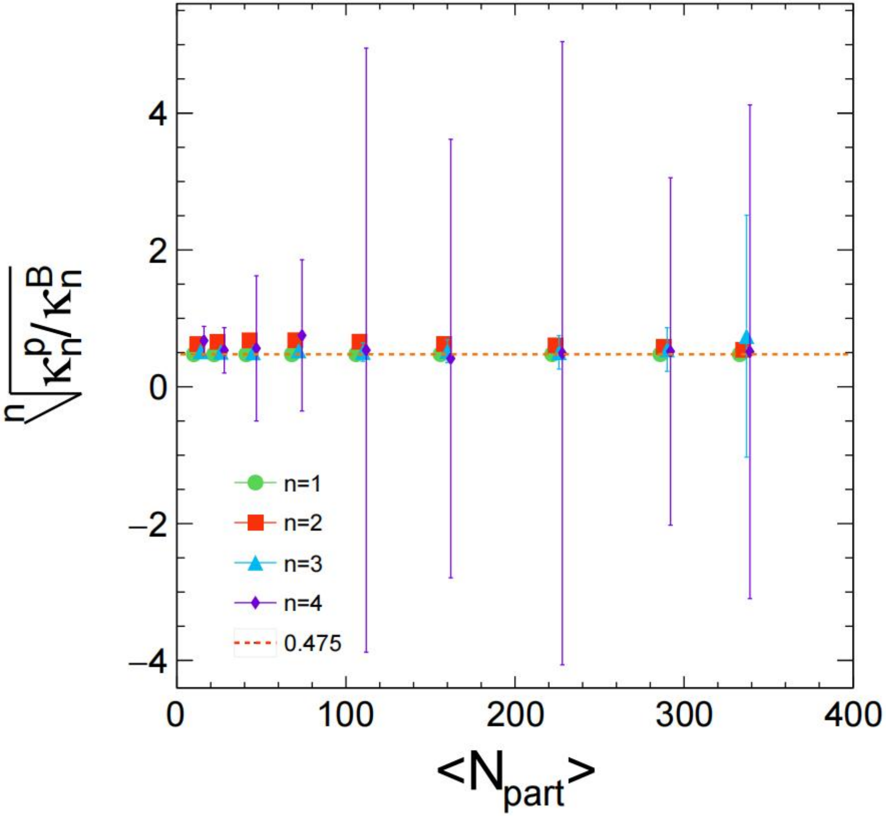
$$\kappa_3 = 2\frac{\langle N \rangle^3}{B^2}$$

$$\kappa_4 = -6\frac{\langle N \rangle^4}{B^3}$$

- Multi-baryon correlations are getting weaker with stage evolution of heavy-ion collisions



◆ Fluctuations of Net-Proton



an effective acceptance factor q:

representing the proton fraction of baryons within limited acceptance and efficiency

$$q = \sqrt[n]{K_n^p / K_n^B}$$

the acceptance factor is almost independent of centrality and is about 0.475 by a constant fitting, which is slightly different from 1/2

[1]M. Kitazawa and M. Asakawa, Phys. Rev. C 86, 024904 (2012);
[2]M. Kitazawa and M. Asakawa, Phys. Rev. C 85, 021901(R)(2012).

◆ Fluctuations of Net-Kaon

Cumulants:

$$C_2 = \langle N \rangle + \langle \overline{N} \rangle + \kappa_2^{(2,0)} + \kappa_2^{(0,2)} - 2\kappa_2^{(1,1)}$$

$$C_3 = \langle N \rangle - \langle \overline{N} \rangle + 3\kappa_2^{(2,0)} - 3\kappa_2^{(0,2)} + \kappa_3^{(3,0)} - \kappa_3^{(0,3)} - 3\kappa_3^{(2,1)} + 3\kappa_3^{(1,2)}$$

$$C_4 = \langle N \rangle + \langle \overline{N} \rangle + 7\kappa_2^{(2,0)} + 7\kappa_2^{(0,2)} - 2\kappa_2^{(1,1)} + 6\kappa_3^{(3,0)} + 6\kappa_3^{(0,3)} - 6\kappa_3^{(2,1)}$$
$$- 6\kappa_3^{(1,2)} + \kappa_4^{(4,0)} + \kappa_4^{(0,4)} - 4\kappa_4^{(3,1)} - 4\kappa_4^{(1,3)} + 6\kappa_4^{(2,2)}$$

Bzdak, Adam et al. Phys.Rev. C86 (2012) 044904

two or more kinds of particles！！！

Factorial moments:

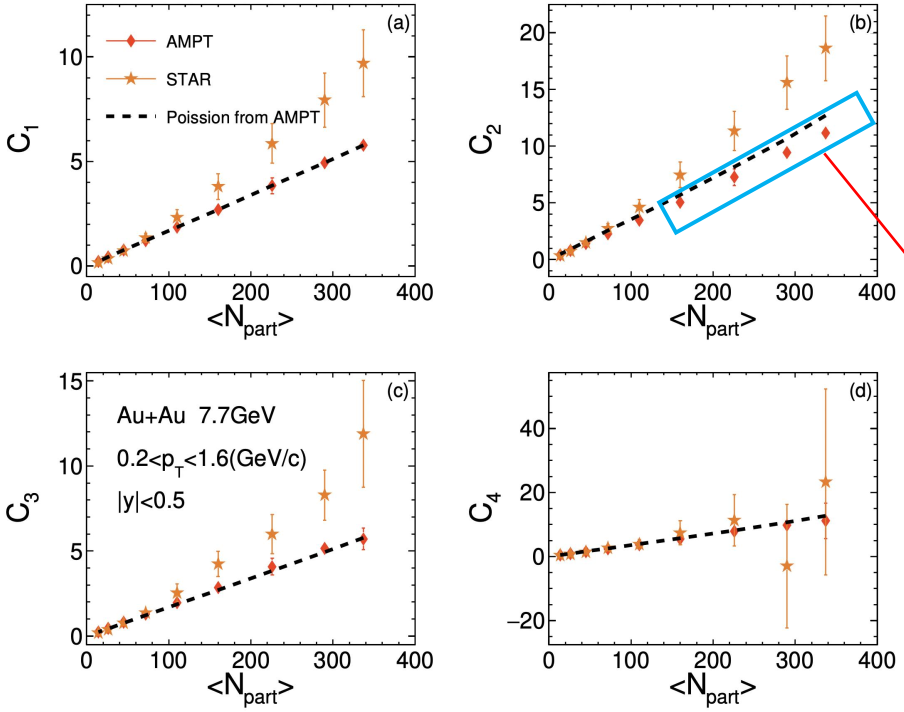
$$F_{i,k} = \left\langle \frac{N!}{(N-i)!} \frac{\overline{N}!}{(\overline{N}-k)!} \right\rangle = \frac{d^i}{dz^i} \frac{d^k}{d\bar{z}^k} H\left(z, \bar{z}\right)\Big|_{z=\bar{z}=1}$$

Correlation Functions:

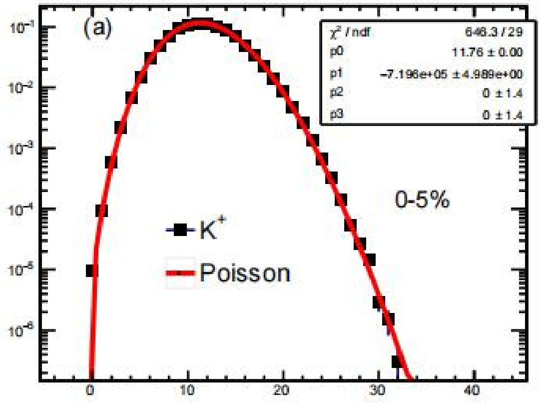
$$\kappa_2^{(2,0)} = -F_{1,0}^2 + F_{2,0},$$

$$\kappa_2^{(1,1)} = -F_{1,0}F_{0,1} + F_{1,1},$$

◆ Fluctuations of Net-Kaon

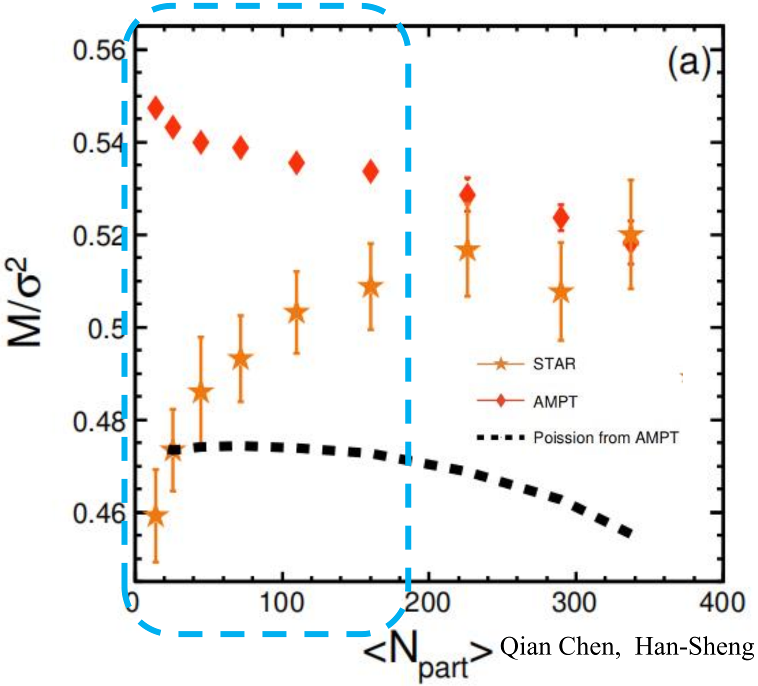


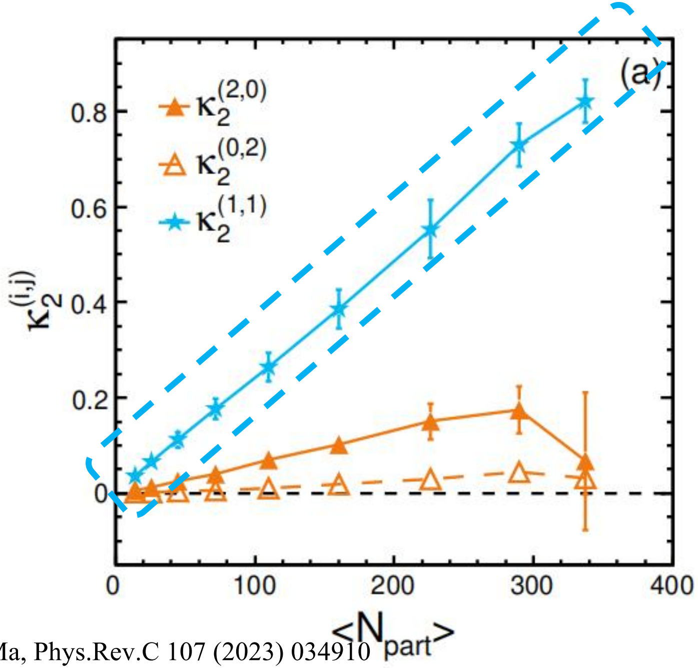
Qian Chen, Han-Sheng Wang, Guo-Liang Ma, Phys.Rev.C 107 (2023) 034910



The C_2 for AMPT is slightly lower than **Poisson baseline** based on its mean multiplicity, suggesting a correlation between K^+ and K^-

◆ Fluctuations of Net-Kaon



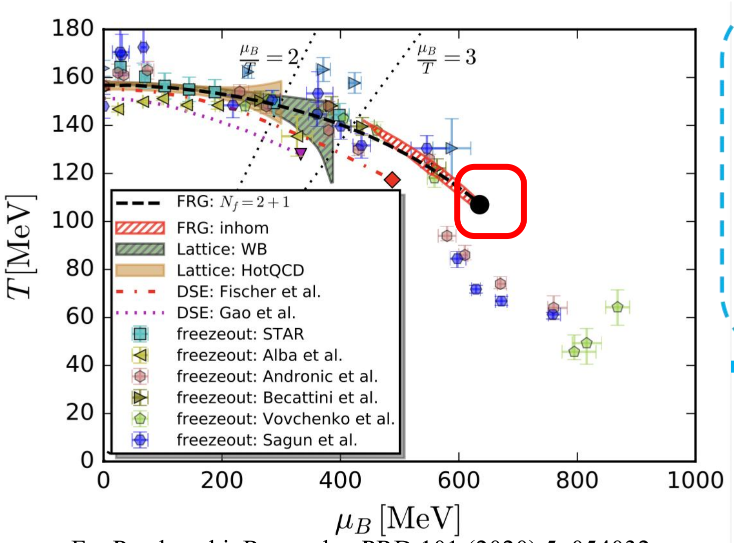


Qian Chen, Han-Sheng Wang, Guo-Liang Ma, Phys.Rev.C 107 (2023) 034910

Caused by the **new quarks coalescence mechanism**

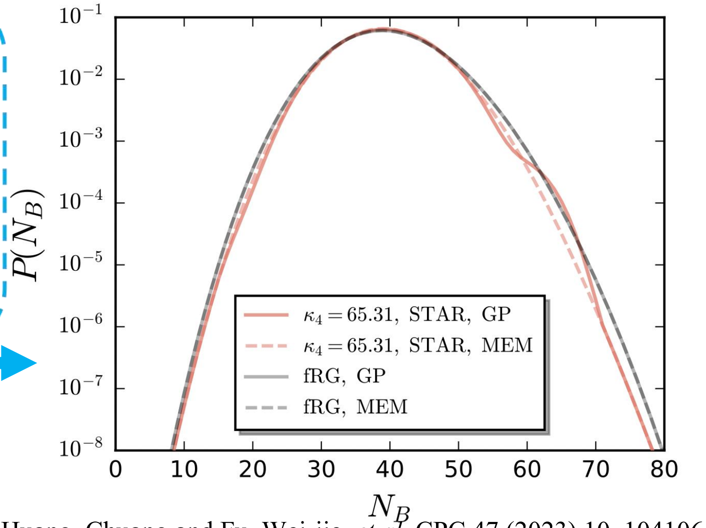
Two-particle correlation function between the K^+ and K^-[$\kappa_2^{(1,1)}$]is dominants——**pair production**

◆ Functional Renormalization Group



Fu, Pawlowski, Rennecke ,PRD 101 (2020) 5, 054032

the net-baryon number distributions are reconstructed from the cumulants of different orders by means of the maximum entropy method



Huang, Chuang and Fu, Wei-jie, *et al*. CPC 47 (2023) 10, 104106

FRG enables the study of equations of state at both high and low baryon chemical potentials.

FRG with critical fluctuations mechanism without interactions between hadrons and decay processes

◆ Incorporating FRG Into AMPT Model

FRG parameter input:

baryon chemical：

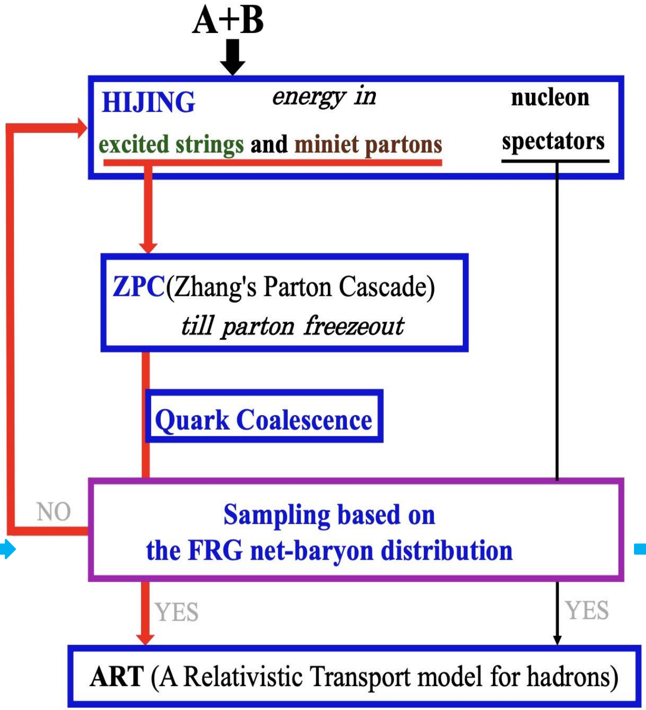
$\mu_B = 399$ MeV;

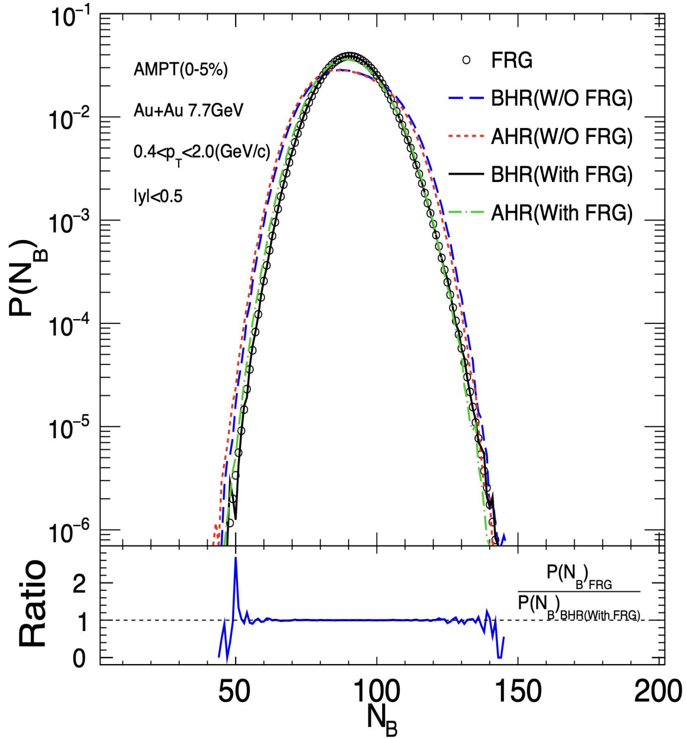
volume of the fire ball：

V = 980 fm^3;

pseudo-critical temperature：

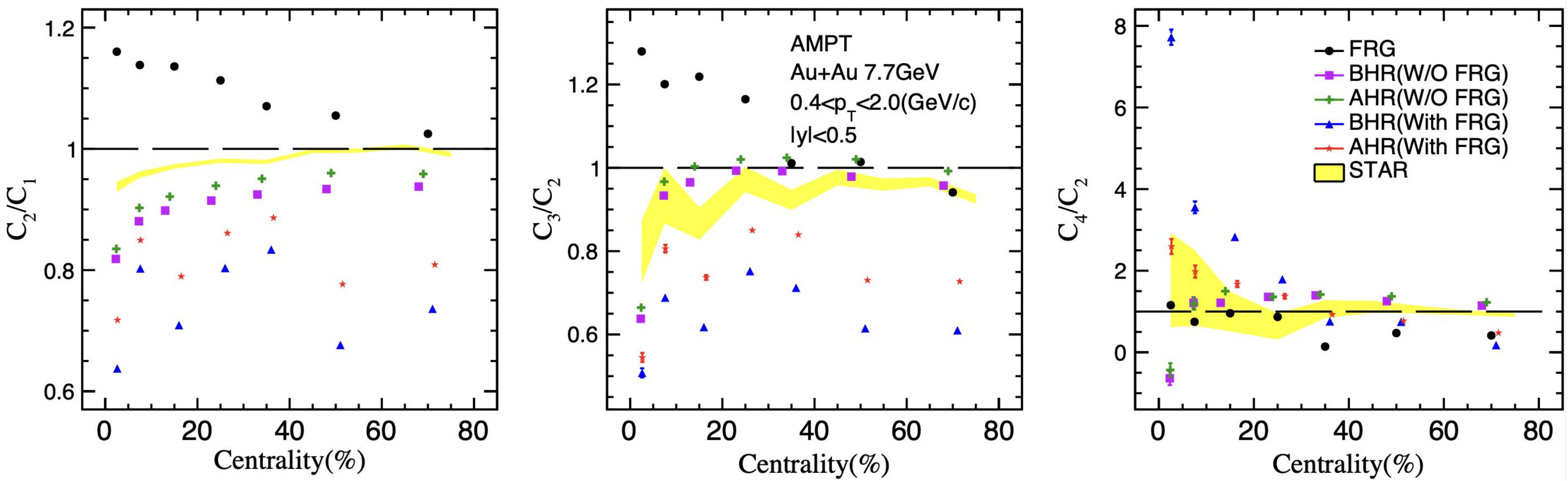
T = 139 MeV;





Qian Chen, Rui Wen, Shi Yin, Wei-jie Fu, Zi-Wei Lin, and Guo-Liang Ma. arXiv:2402.12823.

◆ Incorporating FRG Into AMPT Model



- ➢ The process of hadronic rescatterings exerts a Poissonization effect on fluctuations.
- ➢ The effect of hadronic rescatterings is more significant for **critical fluctuations** than **dynamical fluctuations**.

Qian Chen, Rui Wen, Shi Yin, Wei-jie Fu, Zi-Wei Lin, and Guo-Liang Ma. arXiv:2402.12823.

Part 4:
Summary and Outlook

Summary and Outlook

Summary

- The AMPT results are consistent with the expectation from baryon number conservation.
- By analyzing the cumulants and correlation functions of net-strangeness and net-kaon, we found that they originate from pair production.
- The incorporation of the FRG into the AMPT model reveals that the hadronic rescatterings process affects different orders of net-baryon cumulant ratios.

Outlook

- Incorporation of critical fluctuation physics into AMPT： FRG、density fluctuations.
- nuclear thickness effects, coalescence mechanisms, different collision systems, ...
- Using the extened AMPT model to the analysis of other energy provides a baseline for experimental study.

THANK You for Your Attention!

Back Up

Events used for net-proton cumulants measurements

	$\sqrt{s_{NN}}$ (GeV)	μ_B (GeV)	BES-I Evts (10^6)	BES-II Evts (10^6)
1	27.0	156	30	220
2	19.6	206	15	270
3	17.3	230	-	116
4	14.6	262	20	178
5	11.5	316	7	110
6	9.2	372	-	78
7	7.7	420	3	45

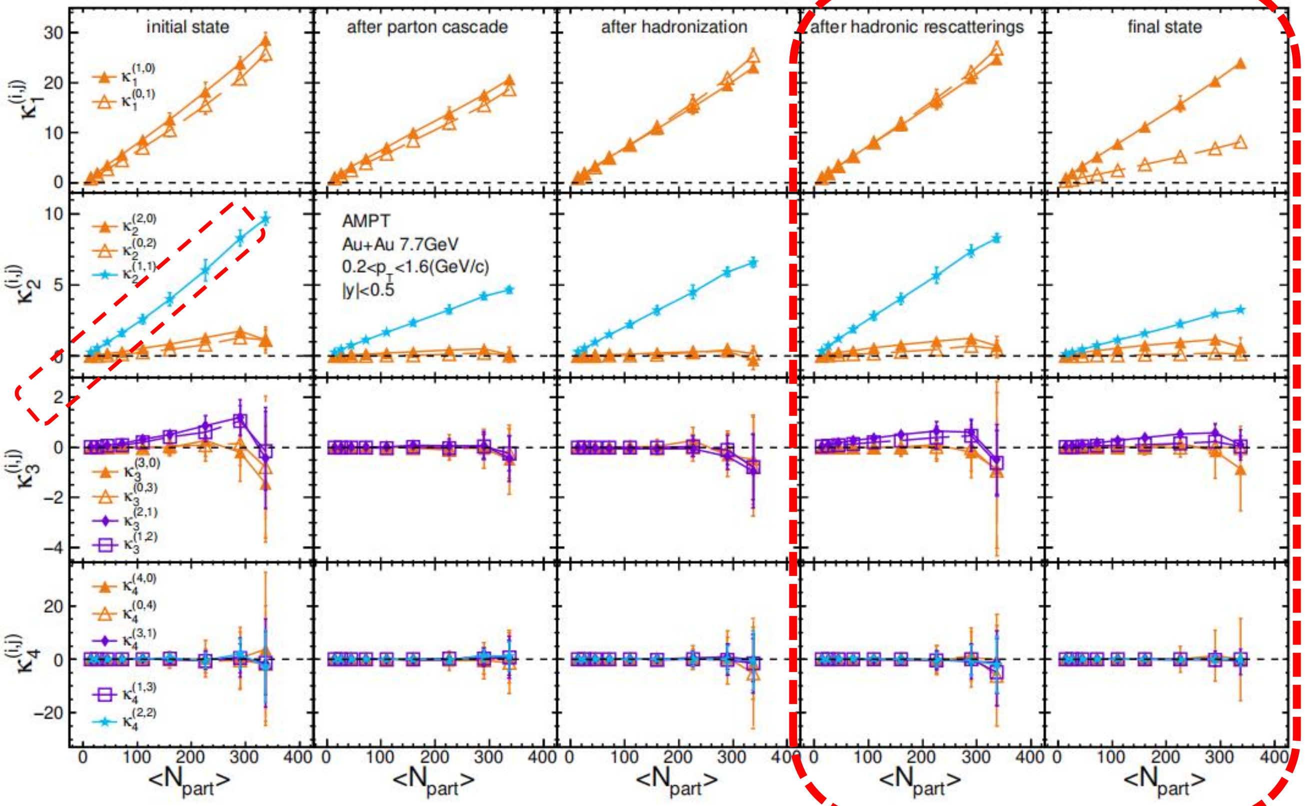
- ~ a factor of 10-18 increase in statistics
- two new energy points: 9.2, 17.3 GeV

From CPOD2024, SQM2024

Back Up

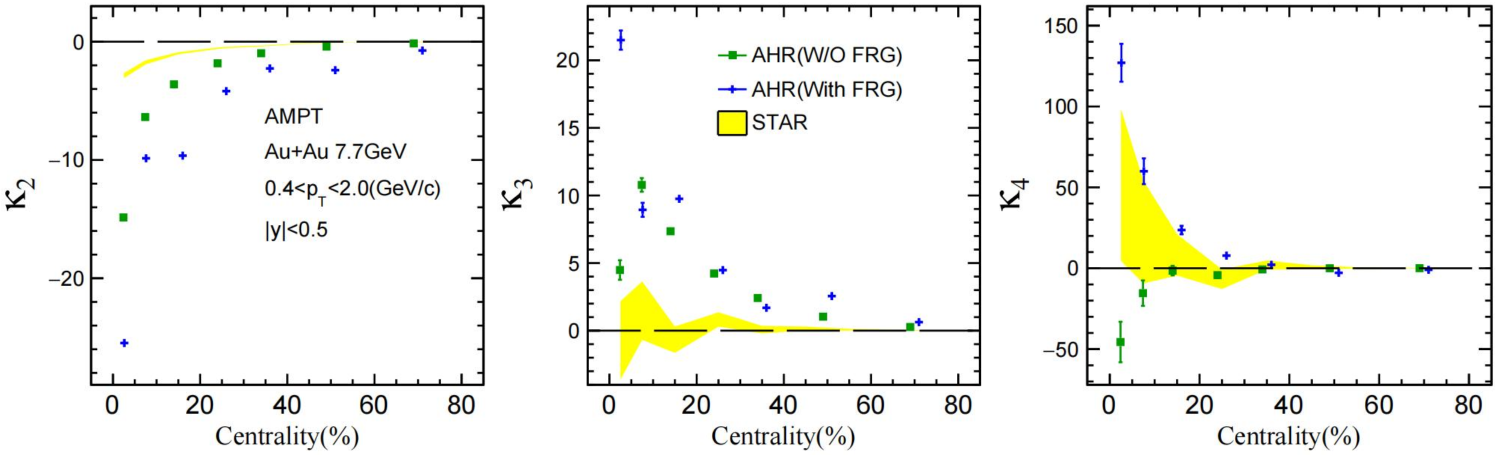
- The fluctuations of strangeness are notably influenced during the weak decay evolution stage

- the two-particle correlation function between the $\bar{s}$ quark and s quark $[\kappa_2^{(1,1)}]$ is dominants



Qian Chen, Han-Sheng Wang, Guo-Liang Ma, Phys.Rev.C 107 (2023) 034910

Back Up



- The strengths of the correlation functions κ_2 and κ_3 in the AMPT model without the FRG sampling are smaller than those in the AMPT model with the FRG sampling.
- The correlation functions κ_4 from negative to positive, which would be more consistent with the current experimental measurement.

Qian Chen, Rui Wen, Shi Yin, Wei-jie Fu, Zi-Wei Lin, and Guo-Liang Ma. arXiv:2402.12823.