

# A brief Overview on CP violation in Heavy Quark



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# Outline

- History of CP violation in Quarks
- Theoretical foundations of CP violations
- Recent Progress (biased)
- Prospect

(3)  $J_\mu$  has “unit length,” i.e.,  $a^2 + b^2 = 1$ .

We then rewrite  $J_\mu$  as<sup>4</sup>

$$J_\mu = \cos\theta(j_\mu^{(0)} + g_\mu^{(0)}) + \sin\theta(j_\mu^{(1)} + g_\mu^{(1)}), \quad (2)$$

where  $\tan\theta = b/a$ . Since  $J_\mu$ , as well as the baryons and the pseudoscalar mesons, belongs to the octet representation of  $SU_3$ , we have relations (in which  $\theta$  enters as a parameter) between processes with  $\Delta S = 0$  and processes with  $\Delta S = 1$ .

To determine  $\theta$ , let us compare the rates for  $K^+ \rightarrow \mu^+ + \nu$  and  $\pi^+ \rightarrow \mu^+ + \nu$ ; we find

$$\Gamma(K^+ \rightarrow \mu\nu)/\Gamma(\pi^+ \rightarrow \mu\nu)$$

$$= \tan^2\theta M_K^2 (1 - M_\mu^2/M_K^2)^2 / M_\pi^2 (1 - M_\mu^2/M_\pi^2)^2. \quad (3)$$

From the experimental data, we then get<sup>5,6</sup>

$$\theta = 0.257. \quad (4)$$

Cabibbo  
1963

Cabibbo, PRL 10, 531(1963)

$(u)$   
 $(d)$

$(s)$

N. Cabibbo 1963

(Cabibbo angle)

$W$ -coupling:

$$u \longleftrightarrow c \cos\theta_c + s \sin\theta_c$$

$$\theta_c: \sim 13.04^\circ$$

$$0.974$$

$$0.226$$

EVIDENCE FOR THE  $2\pi$  DECAY OF THE  $K_2^0$  MESON\*†

J. H. Christenson, J. W. Cronin, ‡ V. L. Fitch, ‡ and R. Turlay §  
 Princeton University, Princeton, New Jersey  
 (Received 10 July 1964)

PRL13,138(1964)

We would conclude therefore that  $K_2^0$  decays to two pions with a branching ratio  $R = (K_2^0 \rightarrow \pi^+ + \pi^-) / (K_2^0 \rightarrow \text{all charged modes}) = (2.0 \pm 0.4) \times 10^{-3}$  where the error is the standard deviation. As emphasized above, any alternate explanation of the effect requires highly nonphysical behavior of the three-body decays of the  $K_2^0$ . The presence of a two-pion decay mode implies that the  $K_2^0$  meson is not a pure eigenstate of  $CP$ . Expressed as  $K_2^0 = 2^{-1/2}[(K_0 - \bar{K}_0) + \epsilon(K_0 + \bar{K}_0)]$  then  $|\epsilon|^2 \cong R_T \tau_1 \tau_2$  where  $\tau_1$  and  $\tau_2$  are the  $K_1^0$  and  $K_2^0$  mean lives and  $R_T$  is the branching ratio including decay to two  $\pi^0$ . Using  $R_T = \frac{3}{2}R$  and the branching ratio quoted above,  $|\epsilon| \cong 2.3 \times 10^{-3}$ .

Cabibbo  
1963



K CPV  
1964

# Theoretical foundations

Strong eigenstates:  $K^0 = d\bar{s}$ ,  $\overline{K^0} = s\bar{d}$ ,

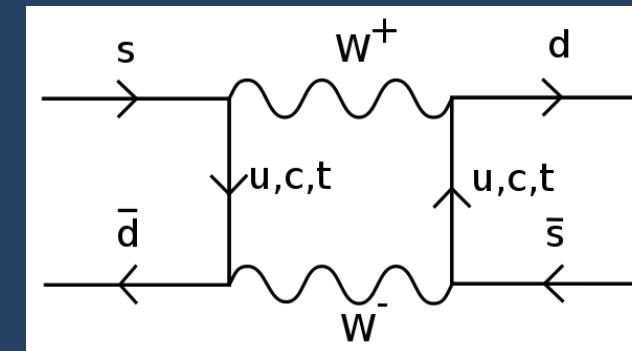
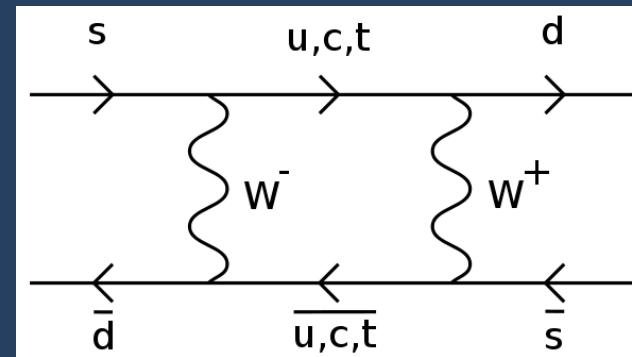
Weak eigenstates:

CP eigenstates:  $K_1 = \frac{1}{\sqrt{2}}(K^0 + \overline{K}^0)$

$$K_S = \frac{K_1 + \varepsilon \cdot K_2}{\sqrt{1 + |\varepsilon|^2}}$$

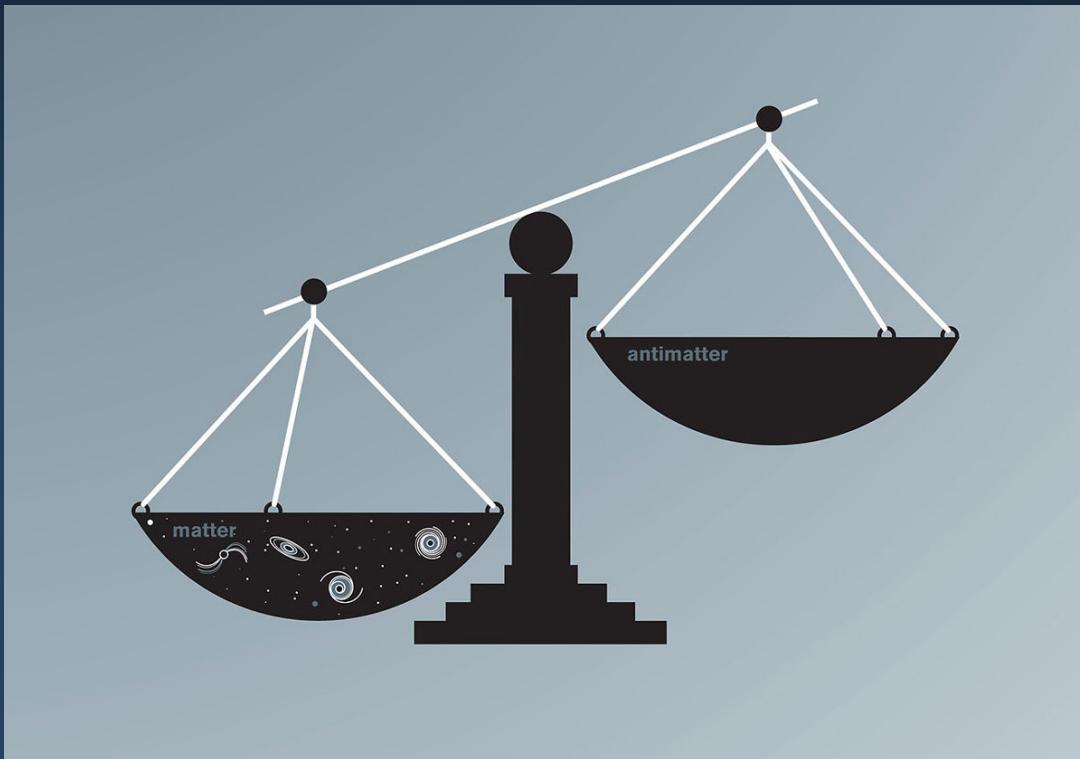
$$K_2 = \frac{1}{\sqrt{2}}(K^0 - \overline{K}^0)$$

$$K_L = \frac{K_2 + \varepsilon \cdot K_1}{\sqrt{1 + |\varepsilon|^2}}$$



Mixing CPV

$$A_{\text{CP}}^B \equiv \frac{\Gamma(\bar{B}^0(t) \rightarrow l^+ \nu_l X) - \Gamma(B^0(t) \rightarrow l^- \bar{\nu}_l X)}{\Gamma(\bar{B}^0(t) \rightarrow l^+ \nu_l X) + \Gamma(B^0(t) \rightarrow l^- \bar{\nu}_l X)}.$$



### Violation of $CP$ invariance, Casymmetry, and baryon asymmetry of the universe

A. D. Sakharov

(Submitted 23 September 1966)

Pis'ma Zh. Eksp. Teor. Fiz. **5**, 32–35 (1967) [JETP Lett. **5**, 24–27 (1967)].

Also S7, pp. 85–88]

Usp. Fiz. Nauk **161**, 61–64 (May 1991)

Мы здорова С. Окубо  
нам добавил нечестивые  
глаза Всемирной сущности  
но ее хрупкую спутник

Literal translation: *Out of S. Okubo's effect  
At high temperature  
A fur coat is sewed for the Universe  
Shaped for its crooked figure.*

3 conditions required to generate a baryon asymmetry:  
Period of departure from thermal equilibrium in the early  
universe.  
Baryon number violation.  
 $C$  and  $CP$  violation.

Cabibbo  
1963

Sakharov  
1967

K CPV  
1967

$$\begin{aligned}
V_{\text{CKM}} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}
\end{aligned}$$

Prog. Theor. Phys. 49, 652 (1973)

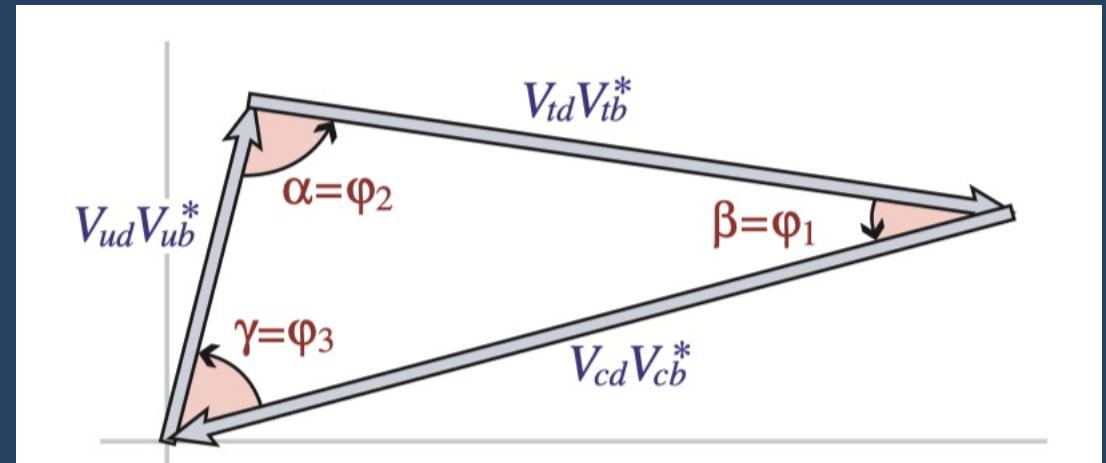


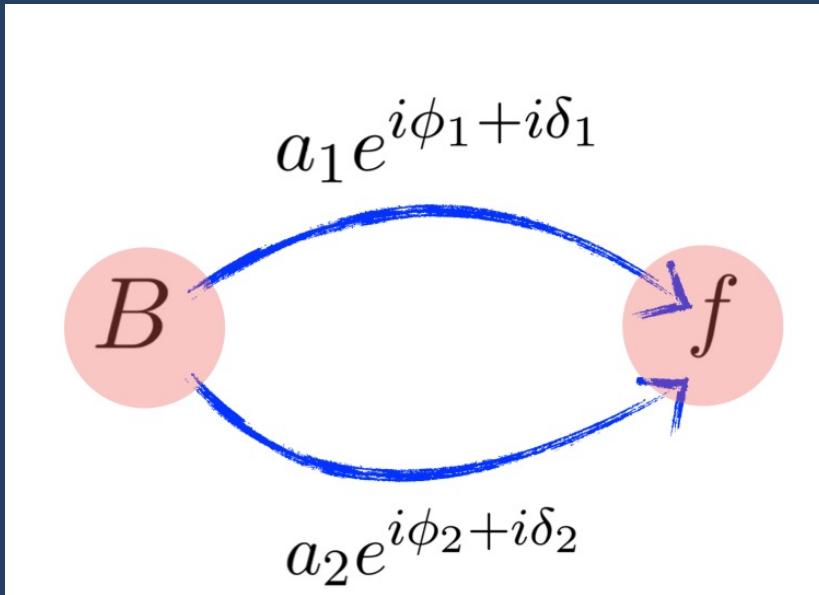
# Theoretical foundations

8

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$|V_{\text{CKM}}| =$$
$$\begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$





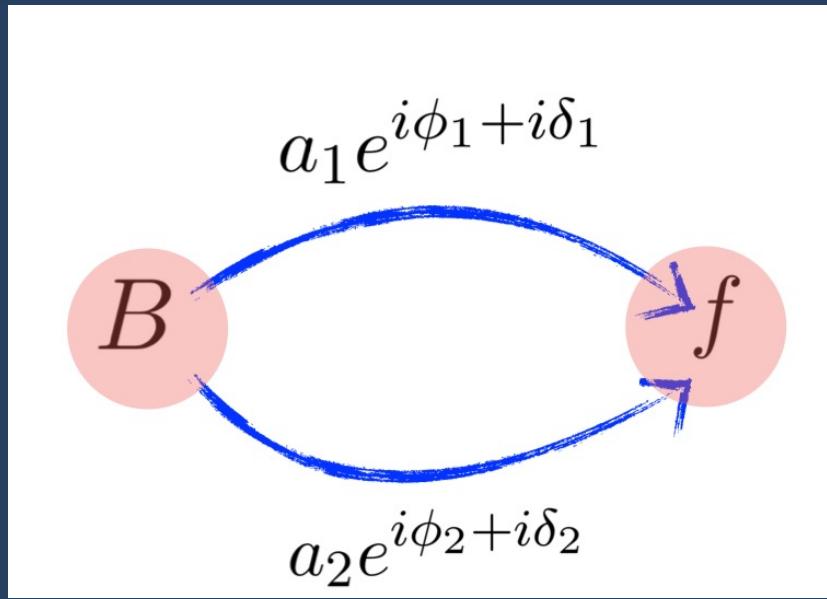
Direct CPV

$$\mathcal{A}_f \equiv \frac{\Gamma(\bar{B} \rightarrow \bar{f}) - \Gamma(B \rightarrow f)}{\Gamma(\bar{B} \rightarrow \bar{f}) + \Gamma(B \rightarrow f)} = \frac{1 - |A_f/\bar{A}_f|^2}{1 - |A_f/\bar{A}_f|^2},$$

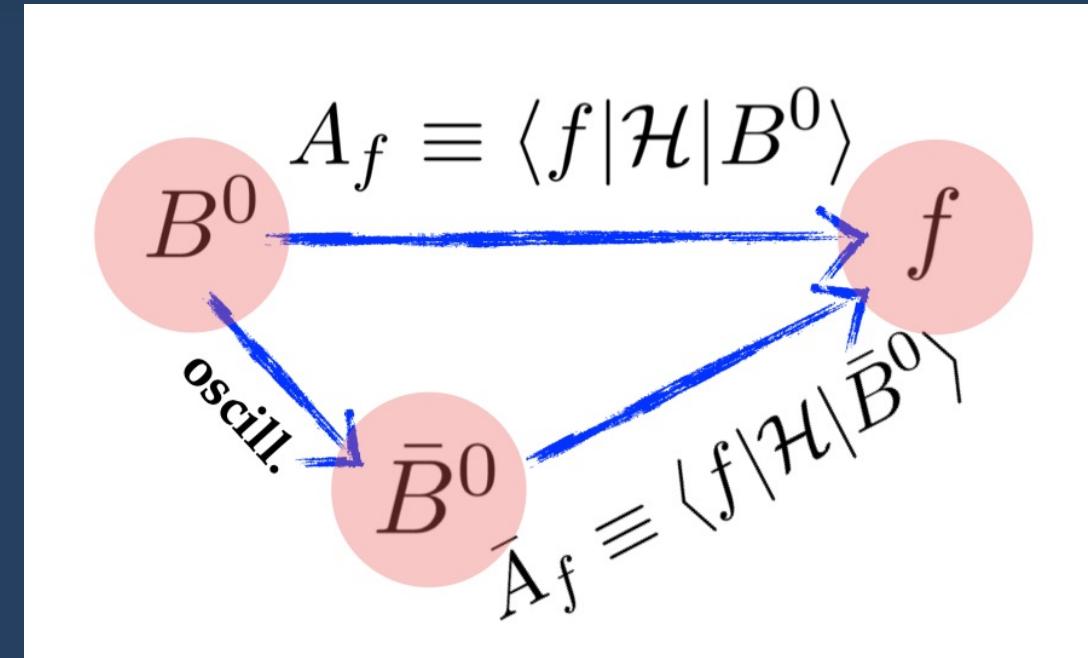
$$A_f = a_1 e^{i\phi_1 + i\delta_1} + a_2 e^{i\phi_2 + i\delta_2},$$

$$\bar{A}_f = a_1 e^{-i\phi_1 + i\delta_1} + a_2 e^{-i\phi_2 + i\delta_2}.$$

$$\mathcal{A}_f = \frac{a_2}{a_1} \sin(\phi_2 - \phi_1) \sin(\delta_2 - \delta_1) + \mathcal{O}(a_2^2/a_1^2).$$



Direct CPV



CPV in interference between decays with and without mixing

$$|B_{L,H}\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle$$

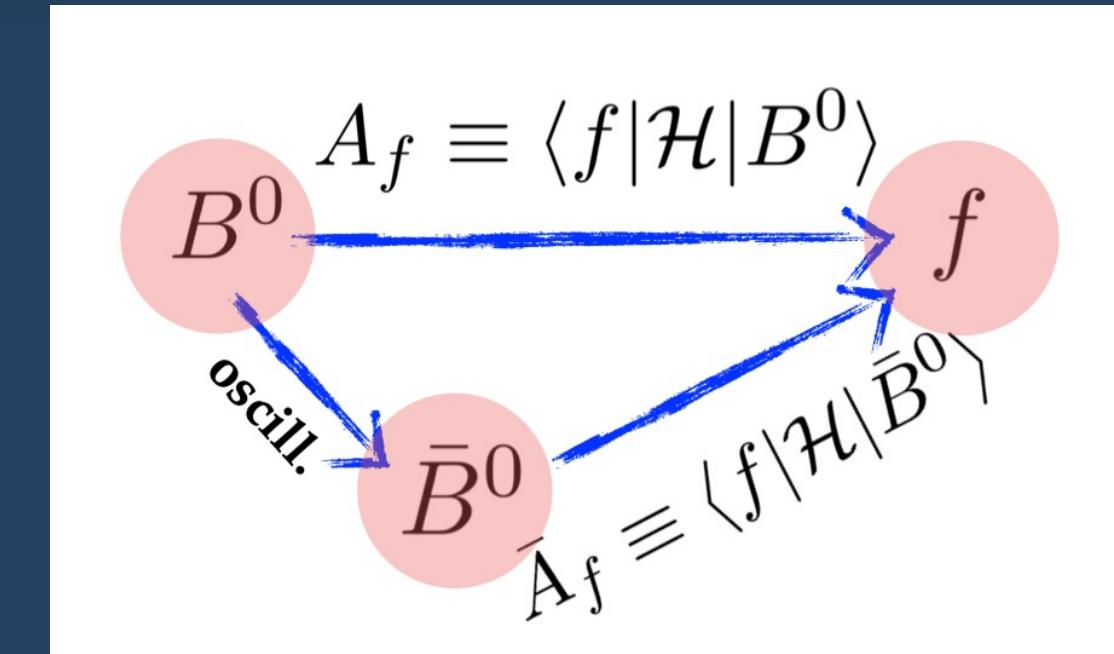
$$i\frac{d}{dt} \begin{pmatrix} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{pmatrix} = \mathcal{H} \begin{pmatrix} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{pmatrix} = \begin{pmatrix} M_{11} + i\Gamma_{11}, & M_{12} + i\Gamma_{12} \\ M_{21} + i\Gamma_{21}, & M_{22} + i\Gamma_{22} \end{pmatrix} \cdot \begin{pmatrix} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{pmatrix}$$

$$\frac{d}{dt} \Gamma(\bar{B}^0(t)[B^0(t)] \rightarrow f_{CP}) \propto e^{-\Gamma t} \left[ \frac{1}{2} (1 + |\lambda_f|^2) \pm S_f \sin(\Delta m t) \mp C_f \cos(\Delta m t) \right]$$

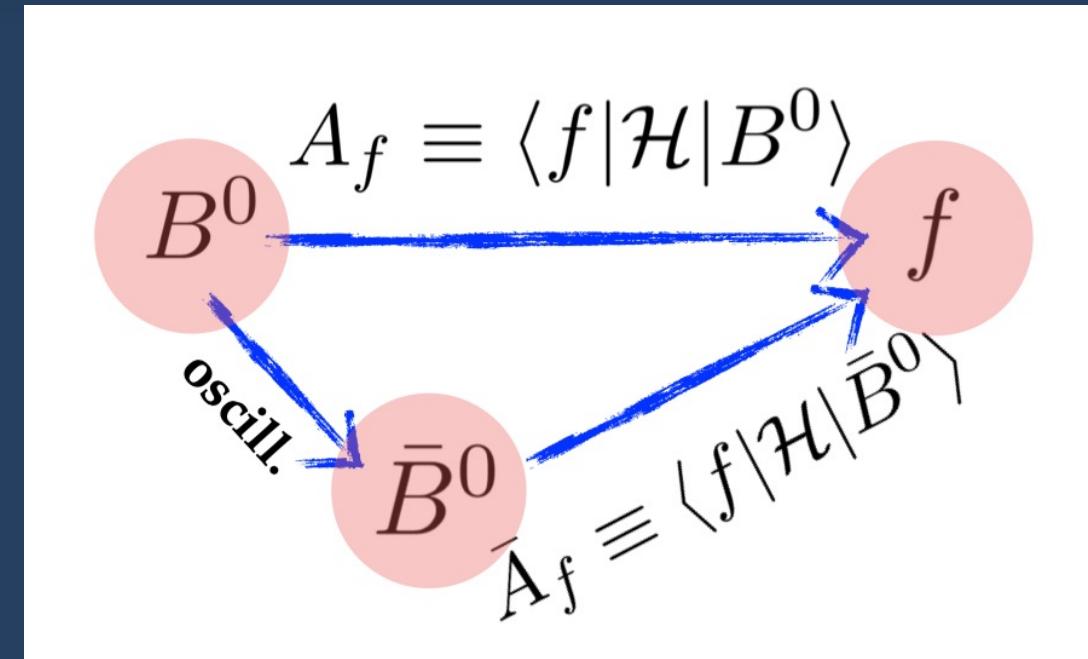
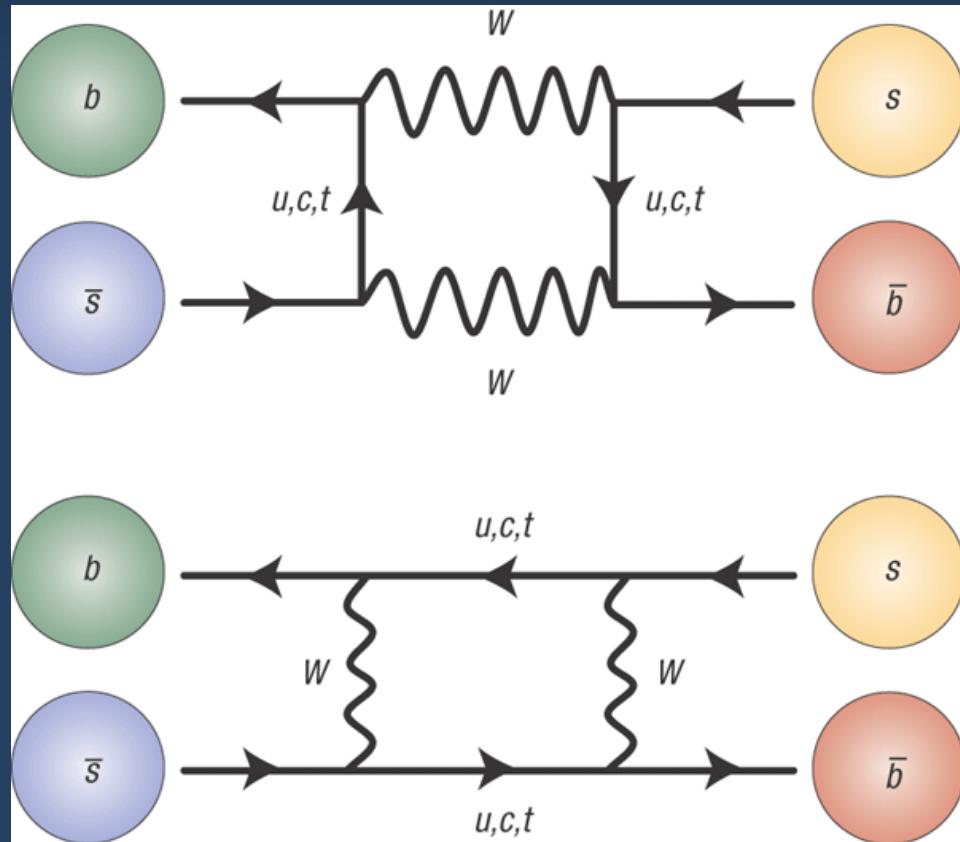
$$\mathcal{A}_{f_{CP}}(t) = S_f \sin(\Delta m t) - C_f \cos(\Delta m t)$$

$$S_f \equiv \frac{2 \operatorname{Im} \lambda_f}{1 + |\lambda_f|^2}, \quad C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}.$$

$$\lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f}.$$



CPV in interference between decays with and without mixing



CPV in interference between decays with and without mixing

# Theoretical foundations

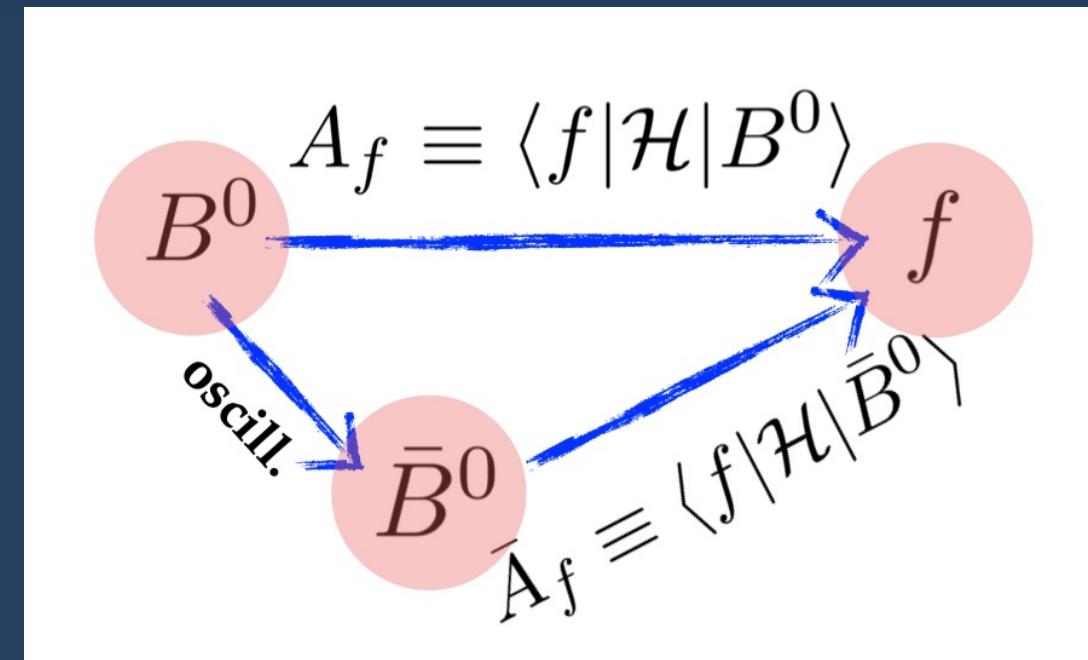
13

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{\frac{d}{dt}\Gamma[\bar{B}^0(t) \rightarrow f_{CP}] - \frac{d}{dt}\Gamma[B^0(t) \rightarrow f_{CP}]}{\frac{d}{dt}\Gamma[\bar{B}^0(t) \rightarrow f_{CP}] + \frac{d}{dt}\Gamma[B^0(t) \rightarrow f_{CP}]}$$

$$\mathcal{A}_{f_{CP}}(t) = S_f \sin(\Delta m t) - C_f \cos(\Delta m t)$$

$$S_f \equiv \frac{2 \operatorname{Im} \lambda_f}{1 + |\lambda_f|^2}, \quad C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}$$

$$\frac{q}{p} = e^{-i\phi_B} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*},$$



CPV in interference between decays with and without mixing

$$\mathcal{A}_{f_{CP}}(t) = S_f \sin(\Delta m t) - C_f \cos(\Delta m t)$$

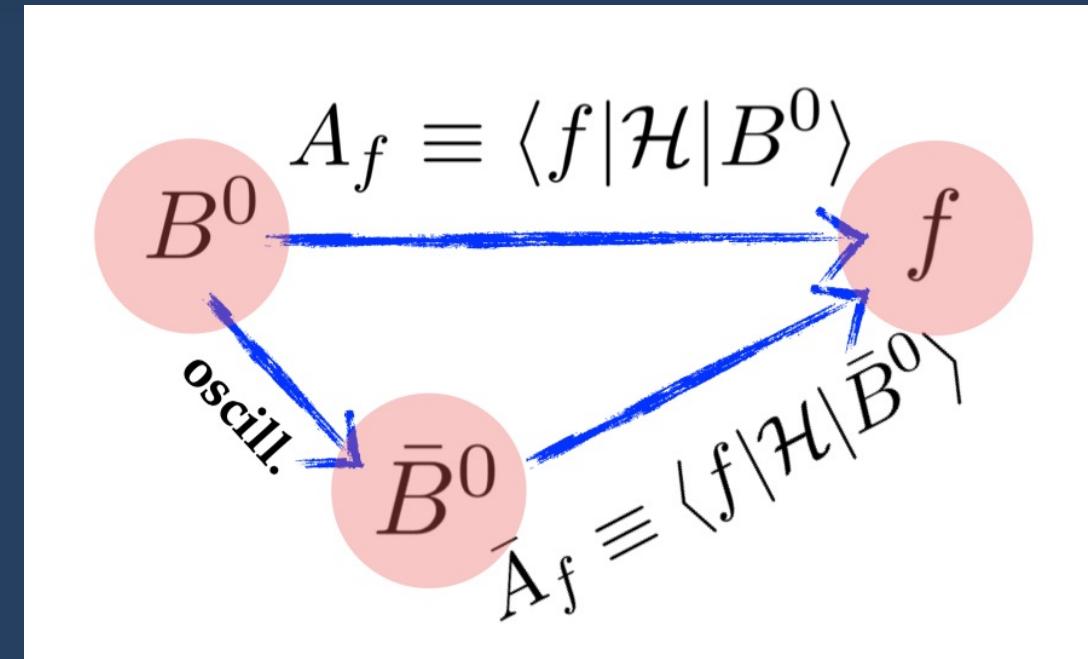
$$S_f \equiv \frac{2 \operatorname{Im} \lambda_f}{1 + |\lambda_f|^2}, \quad C_f \equiv \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}$$

$$\frac{\bar{A}_{J/\psi K_S}}{A_{J/\psi K_S}} = \eta_f \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} + \dots$$

$$\lambda_{J/\psi K_S} = \eta_f \frac{V_{tb}^* V_{td} V_{cb} V_{cs}^*}{V_{tb} V_{td}^* V_{cb}^* V_{cs}} = \eta_f e^{-i2\beta}$$

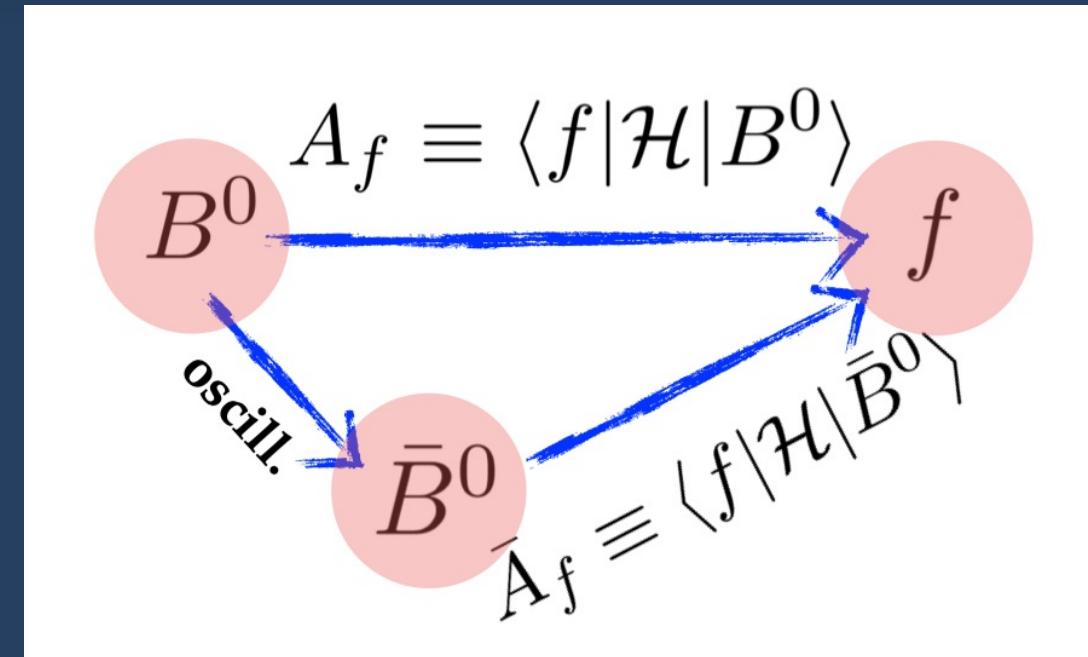
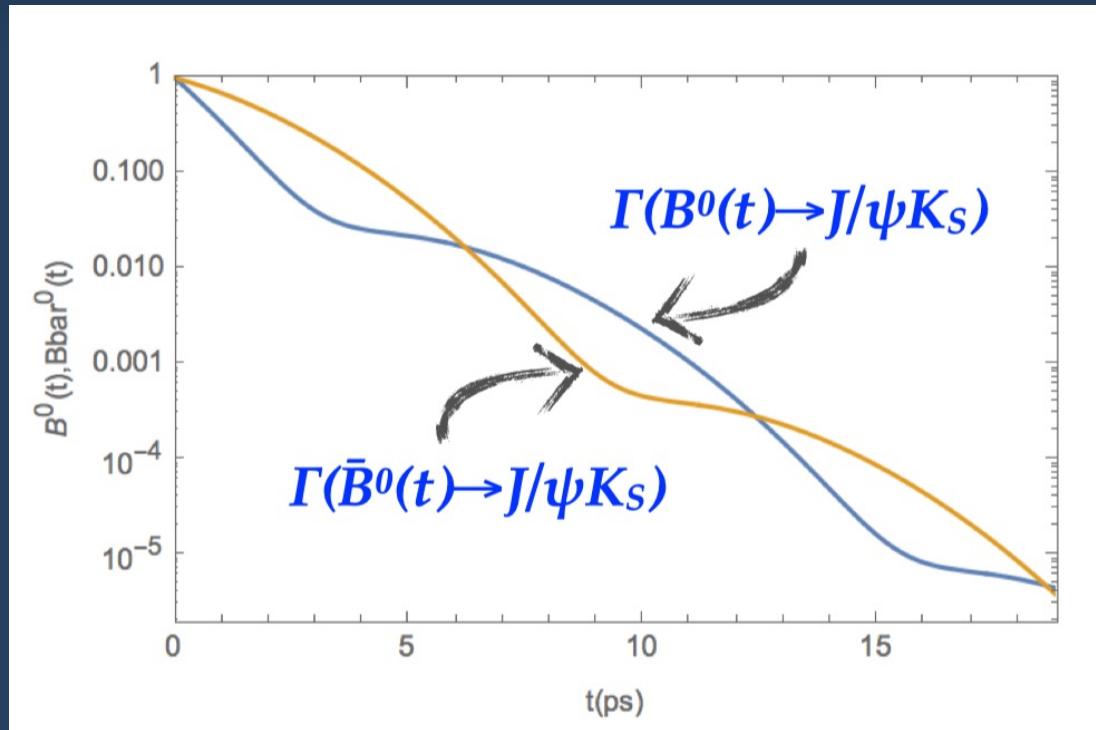
$$\operatorname{Im} \lambda_{J/\psi K_S} = \sin 2\beta$$

CPV in interference between decays with and without mixing

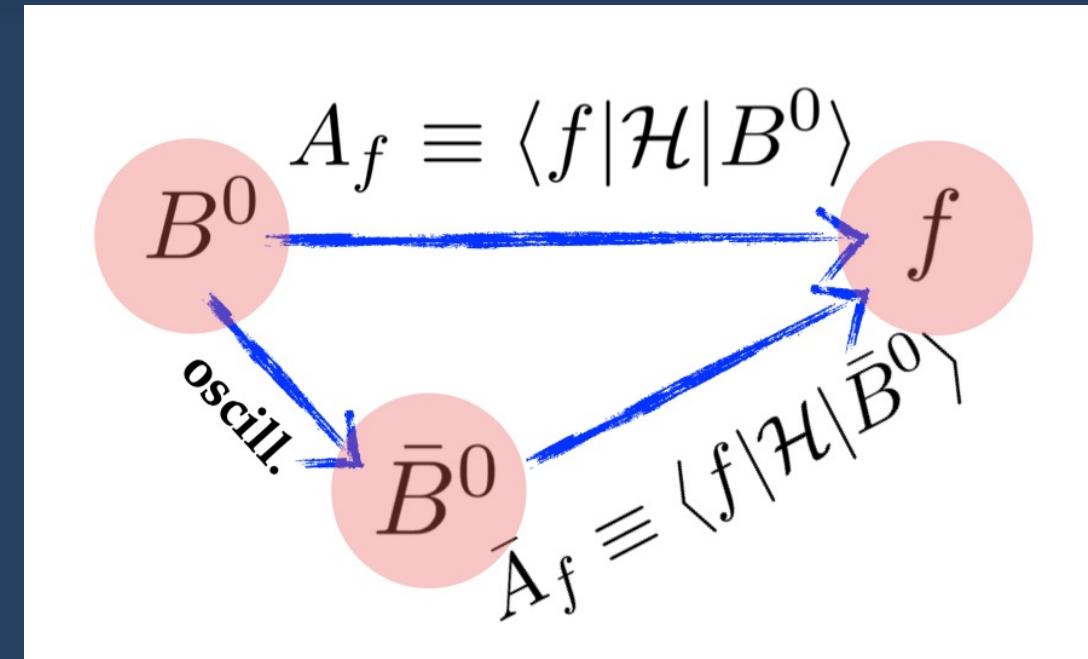
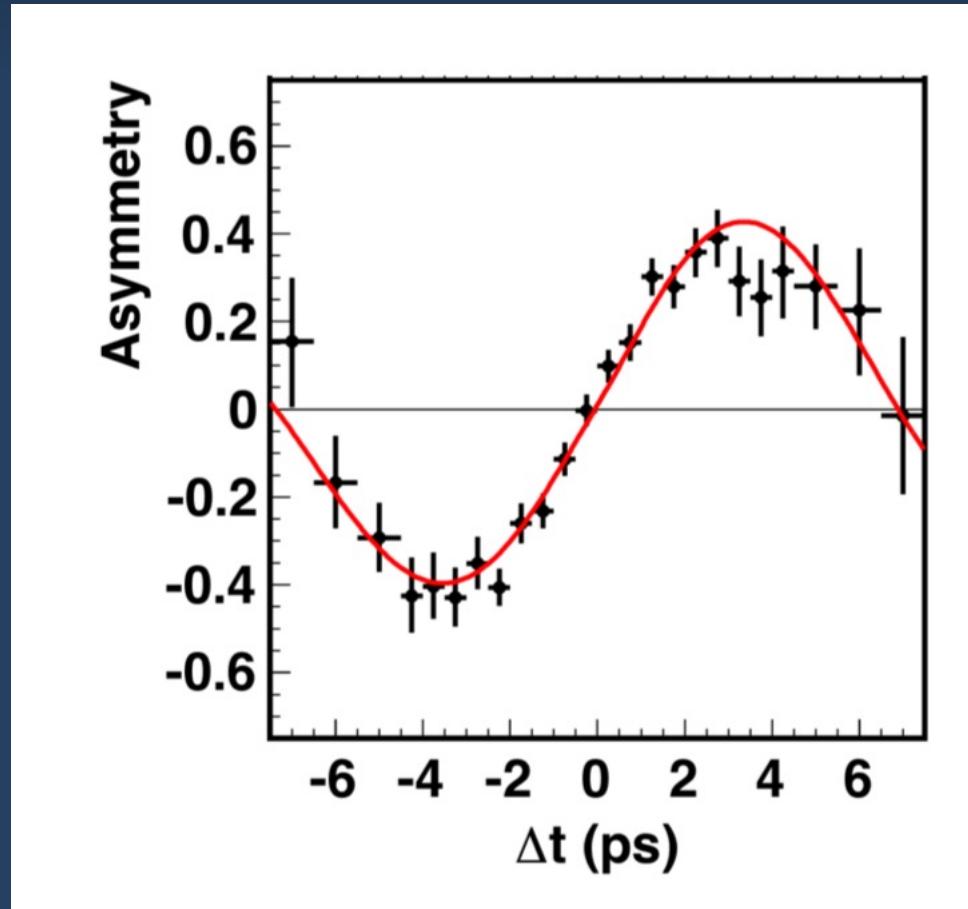


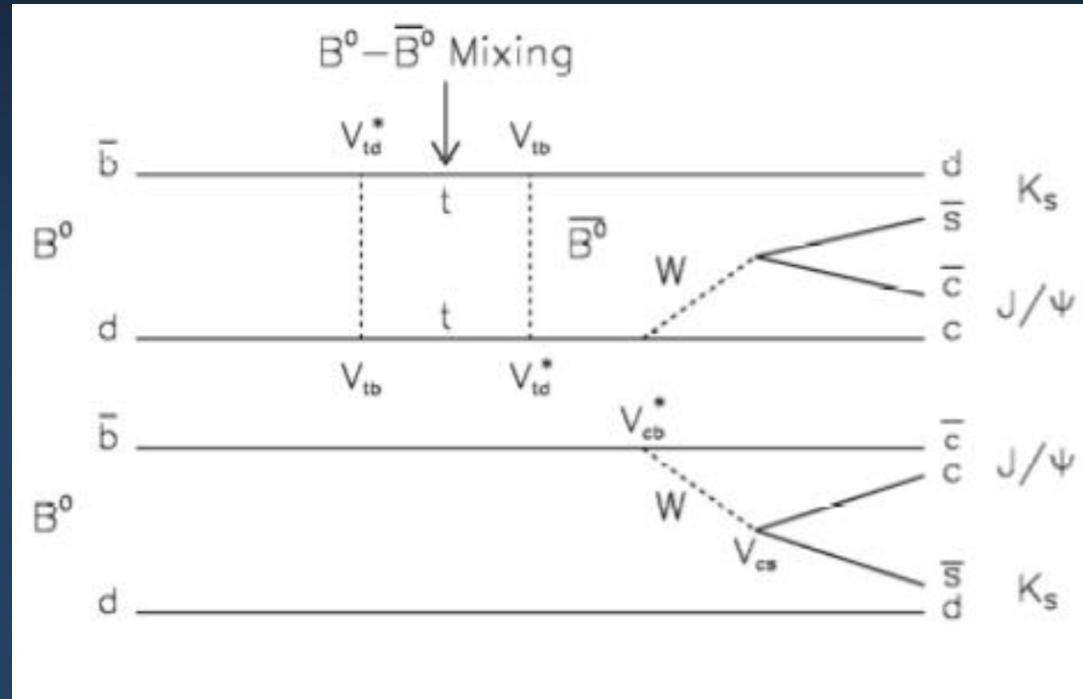
# Theoretical foundations

15



CPV in interference between decays with and without mixing





I. Bigi, A. Sanda

$$\mathcal{A}_{f_{CP}}(t) = S_f \sin(\Delta m t) - C_f \cos(\Delta m t)$$

Cabibbo  
1963

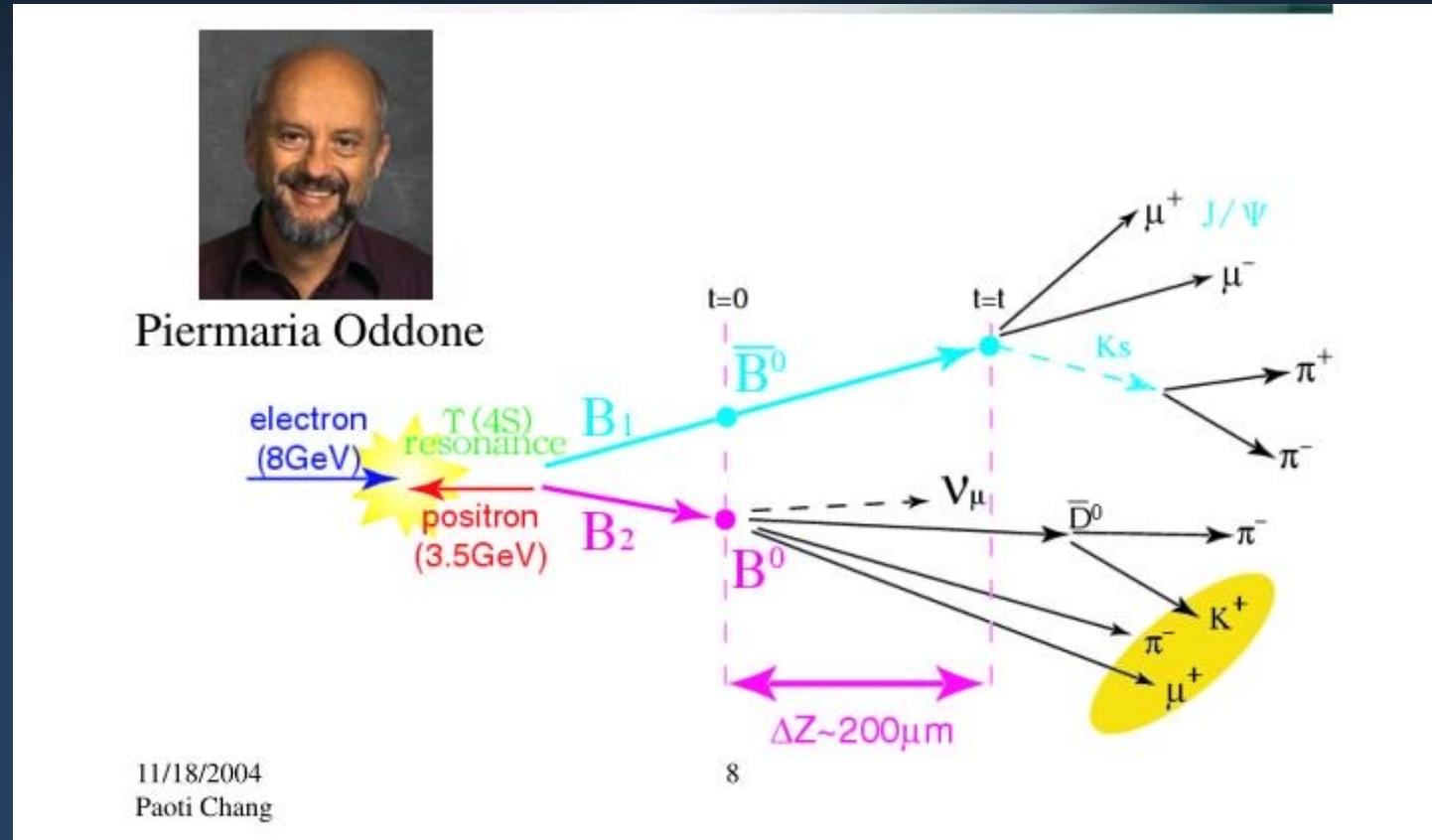
Sakharov  
1967

B → J/ψK  
1981



K CPV  
1967

KM  
1973



Cabibbo  
1963

Sakharov  
1967

$B \rightarrow J/\Psi K$   
1981



K CPV  
1967

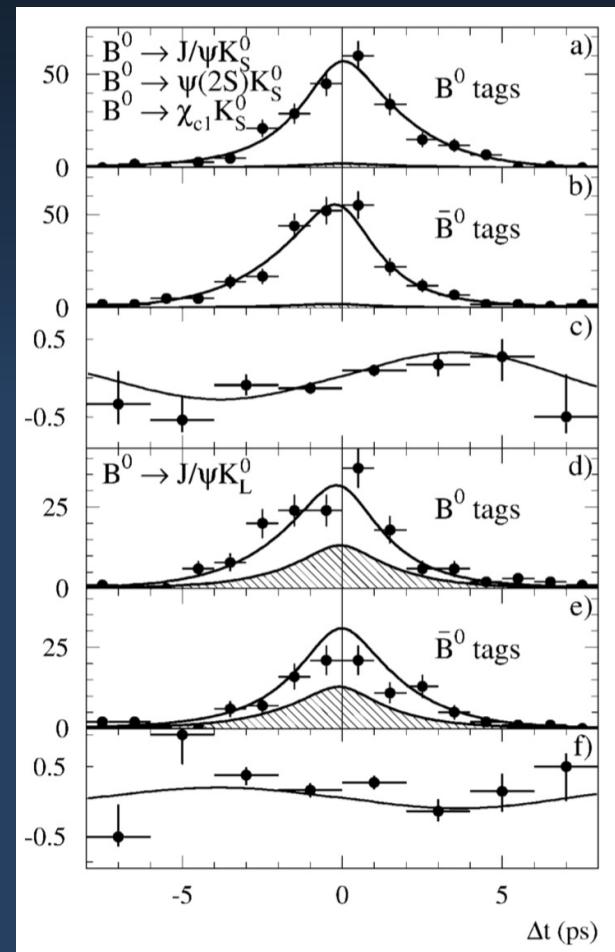


KM  
1973



Asymmetric  
1987





Cabibbo  
1963

Sakharov  
1967

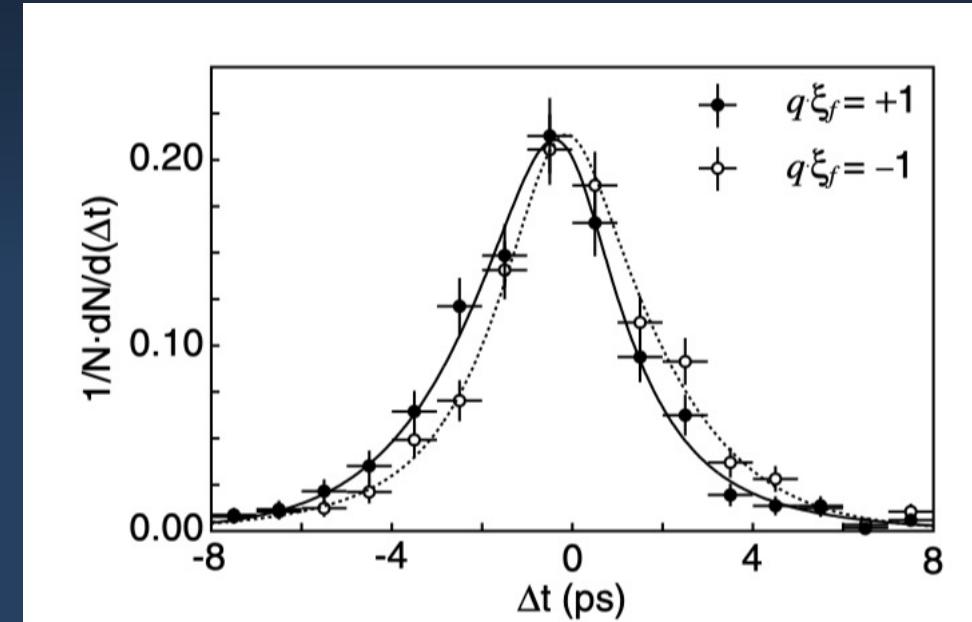
$B \rightarrow J/\Psi K$   
1981

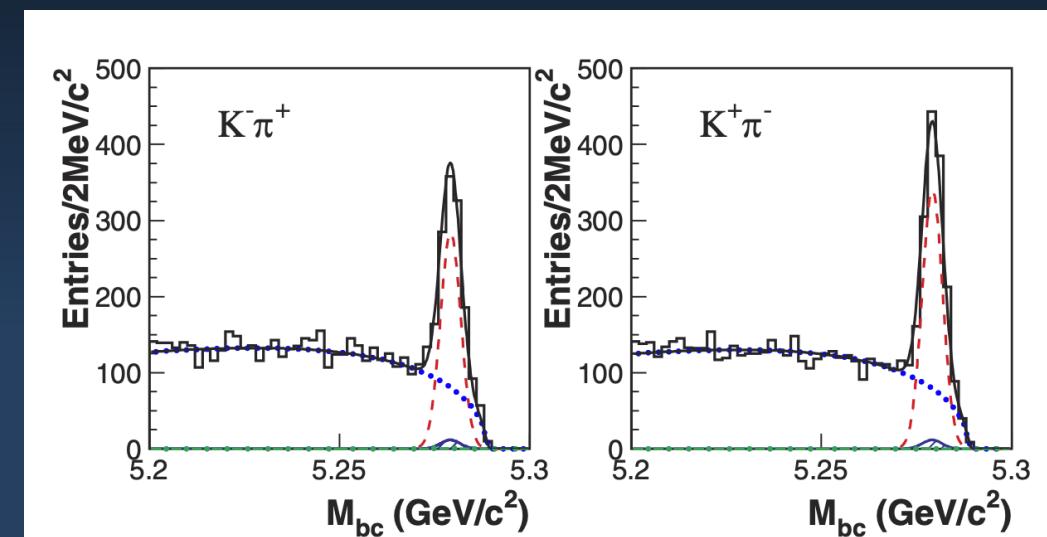
B CPV(Large)  
2001

K CPV  
1967

KM  
1973

Asymmetric  
1987





Sample	$N_{B\bar{B}}$	$n_{K\pi}$	$\mathcal{A}_{K\pi}$
1999–2001	21.1	$142 \pm 15$	$-0.240 \pm 0.102$
2002	66.4	$479 \pm 27$	$-0.102 \pm 0.055$
2003	34.1	$241 \pm 19$	$-0.109 \pm 0.079$
2004	104.9	$743 \pm 33$	$-0.142 \pm 0.044$

Babar, PRL93.131801(2004)

Mode	Signal Yield	$\mathcal{A}_{CP}$	Bkg	$\mathcal{A}_{CP}$
$K^\mp \pi^\pm$	$2140 \pm 53$	$-0.101 \pm 0.025 \pm 0.005$	$-0.001 \pm 0.005$	
$K^\mp \pi^0$	$728 \pm 34$	$0.04 \pm 0.05 \pm 0.02$	$-0.02 \pm 0.01$	
$\pi^\mp \pi^0$	$315 \pm 29$	$-0.02 \pm 0.10 \pm 0.01$	$-0.01 \pm 0.01$	

Belle, PRL93.191802(2004)

Cabibbo  
1963

Sakharov  
1967

$B \rightarrow J/\Psi K$   
1981

B CPV(Large)  
2001



K CPV  
1967



KM  
1973

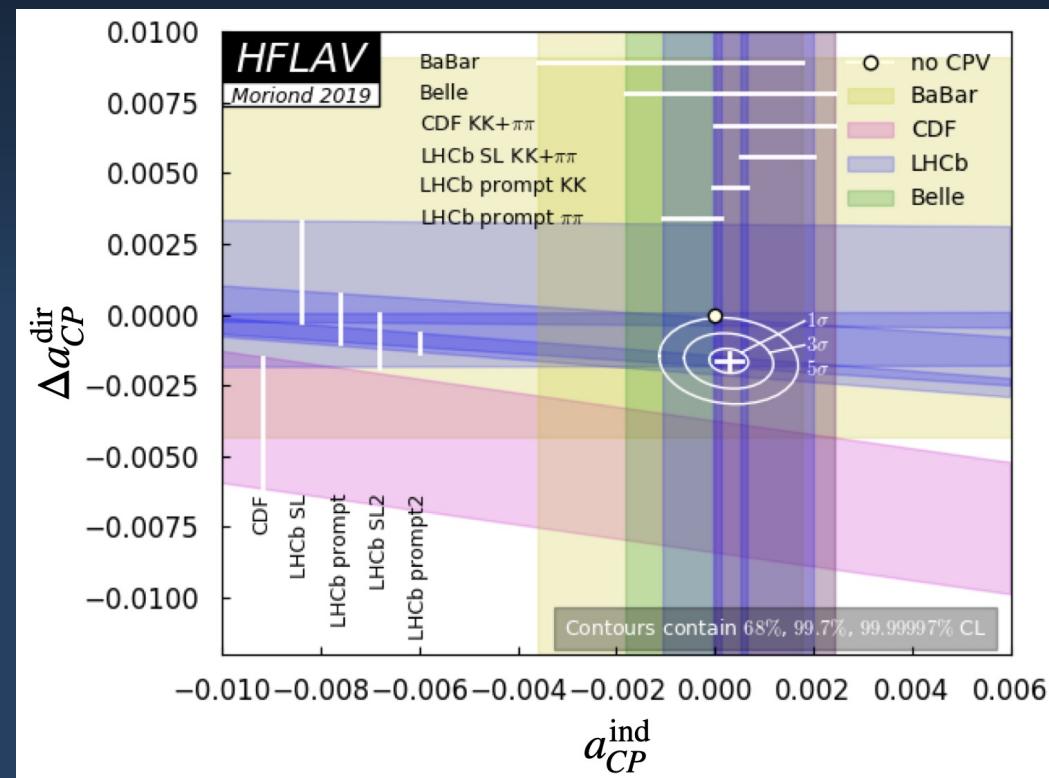


Asymmetric  
1987



Large Dir CPV  
2004





Cabibbo  
1963

Sakharov  
1967

$B \rightarrow J/\Psi K$   
1981

B CPV(Large)  
2001

Charm Dir CPV  
2019



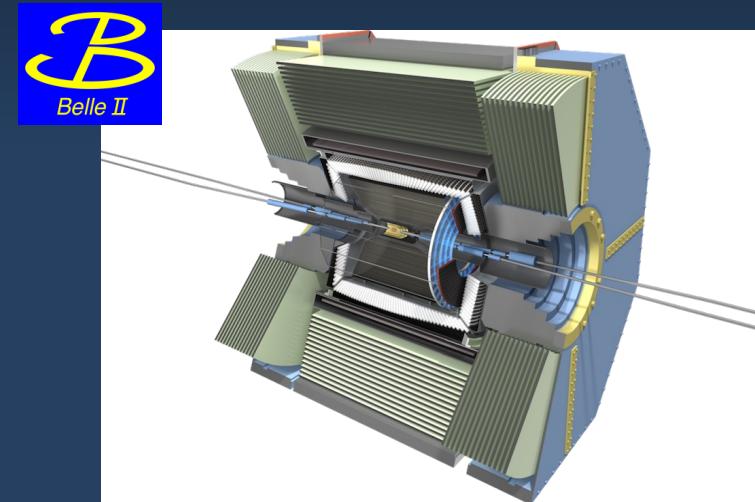
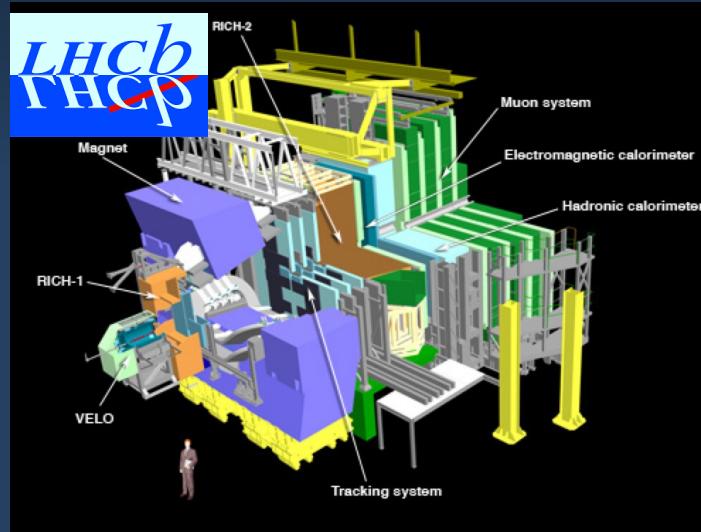
K CPV  
1967

KM  
1973

Asymmetric  
1987

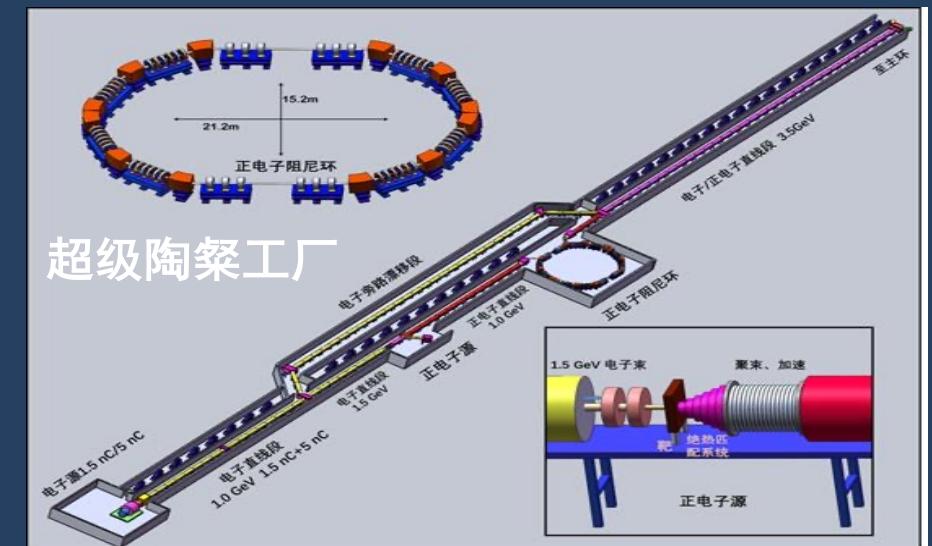
Large Dir CPV  
2004





高精度的实验数据对理论计算精度提出了相应的需求

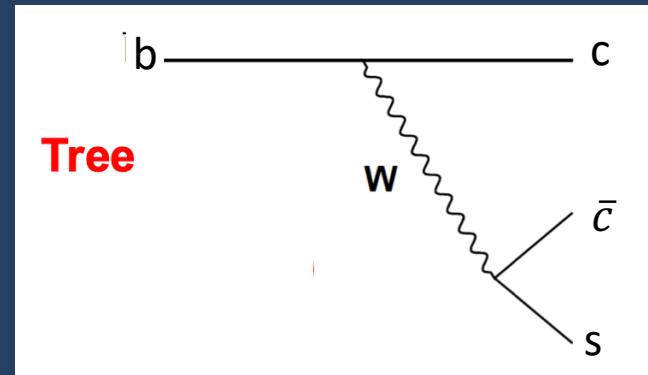
See Wen-bin, Pei-Lian, and Xiaodong's talks.



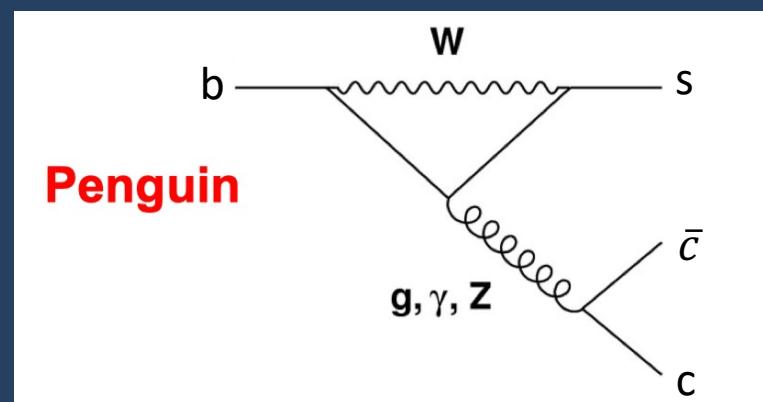
# Challenges

23

$$\sin(2\beta) = 0.699 \pm 0.017$$
$$\beta = (22.2 \pm 0.7)^\circ$$

Golden Channel:  $B \rightarrow J/\psi K_s$ 

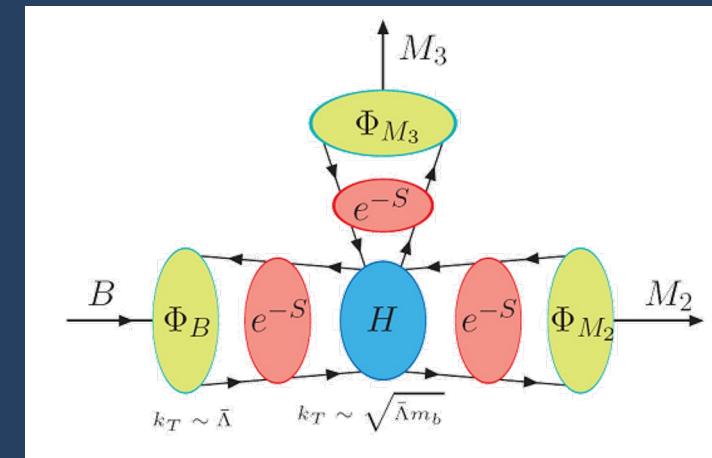
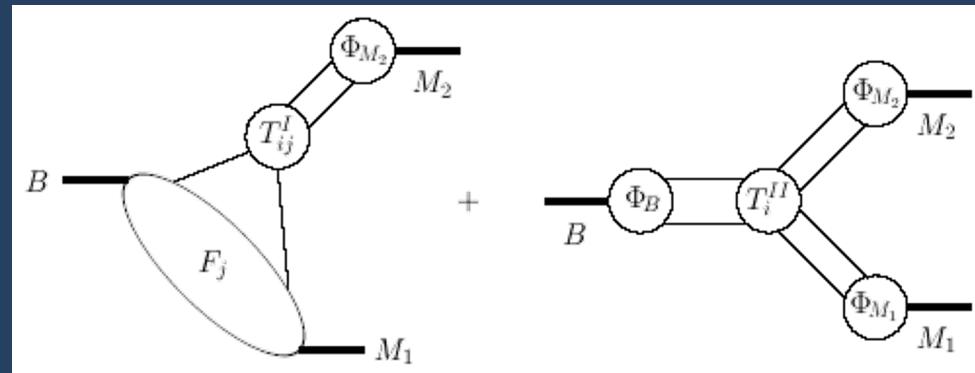
Tree Amplitudes:  
 $V_{cb} V_{cs} \sim 40 \times 10^{-3}$   
 $a_2 \sim 0.1 - 1$



Penguin Amplitudes:  
 $V_{ub} V_{us} \sim 0.8 \times 10^{-3}$   
 $a_4 \sim 0.04$

# Challenges

24



$$A(B \rightarrow \pi\pi) \propto \phi_\pi \otimes T_I \otimes F^{B\pi} + \phi_\pi \otimes T_{II} \otimes \phi_B \otimes \phi_\pi$$

$$A \sim \int d^4k_1 d^4k_2 d^4k_3 \text{Tr} [ C(t) \Phi_B(k_1) \Phi_1(k_2) \Phi_2(k_3) H(k_1, k_2, k_3, t) ] \exp \{-S(t)\}$$

A incomplete list of recent theoretical progress:

High Precision:

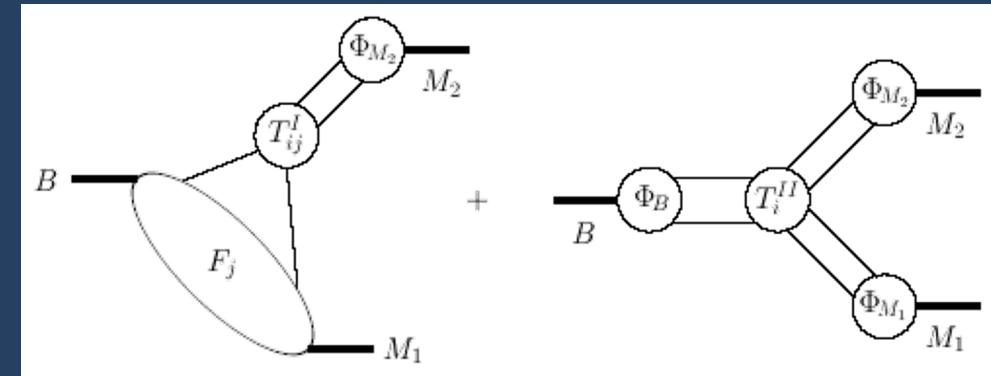
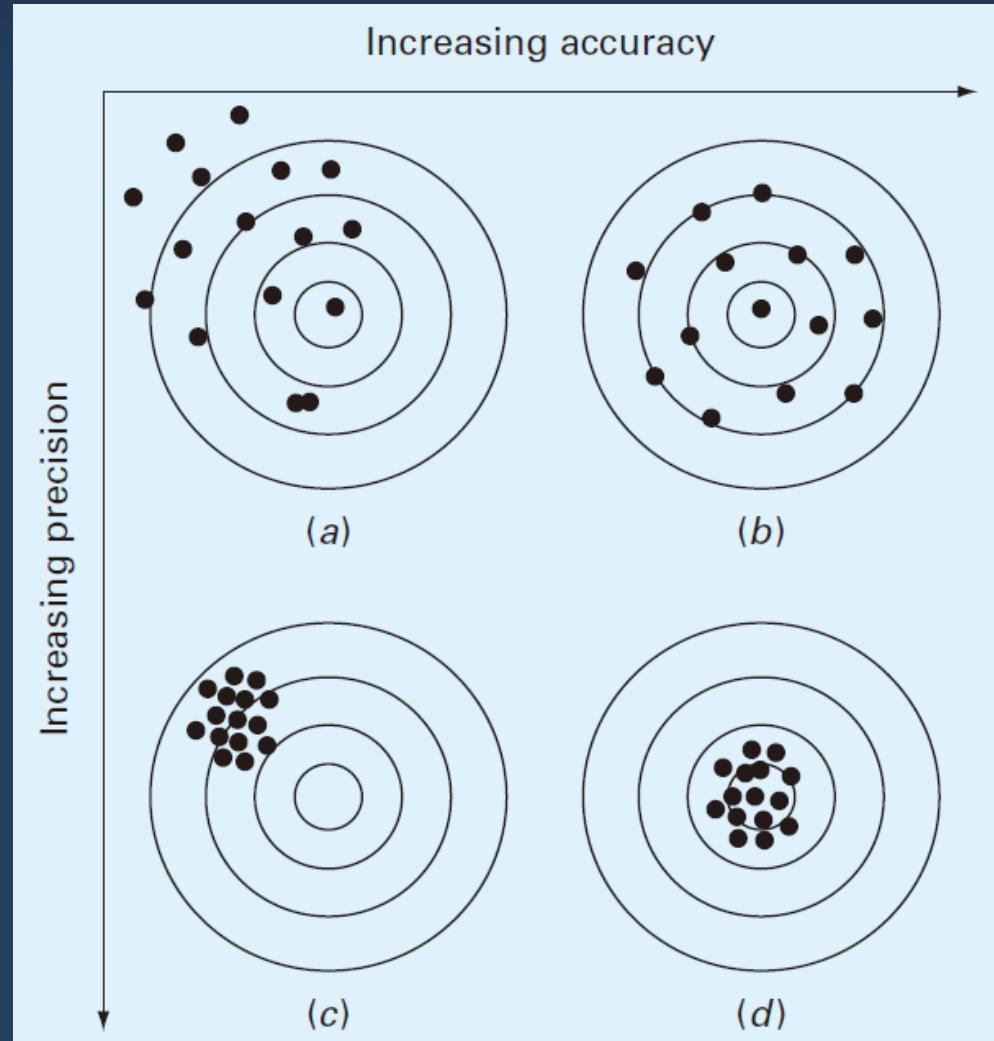
- ✓ QCD corrections
- ✓ Power corrections vs Factorization
- ✓ Low energy Inputs
- ✓ . . .

New Observables:

- ✓ Baryon CPV:
- ✓ Multi-body CPV:



# Prospect:

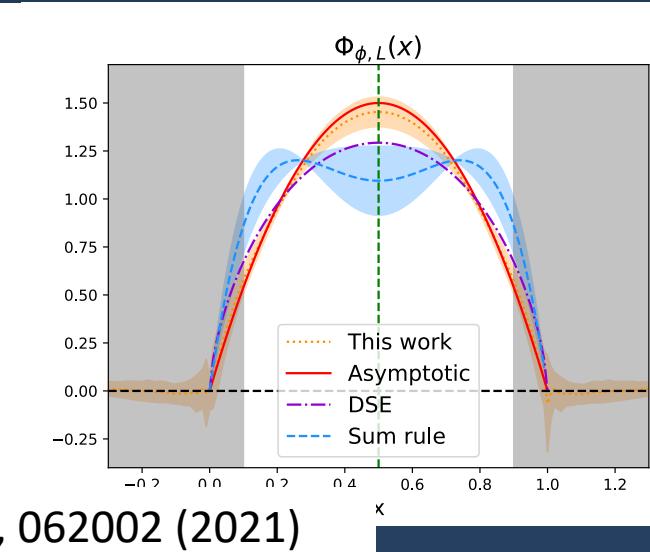
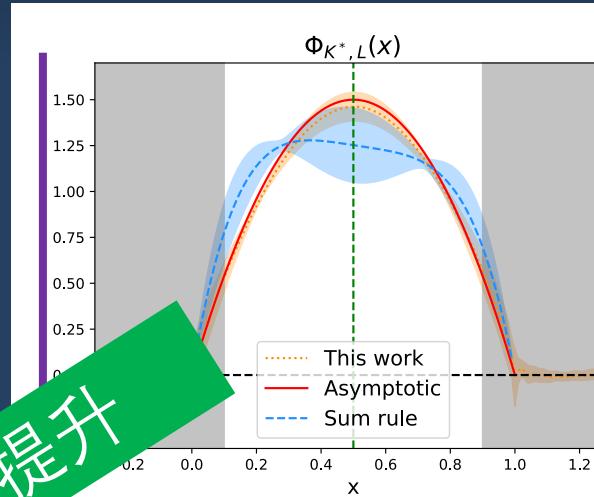
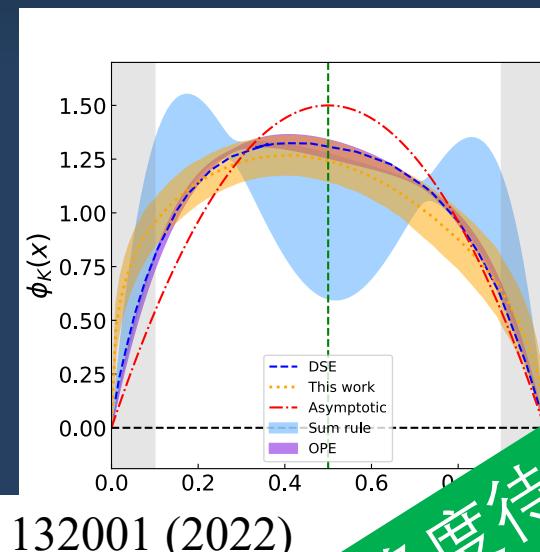
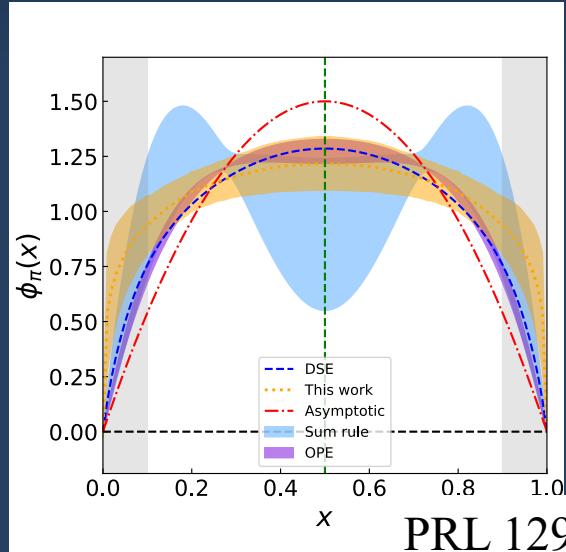


$$A(B \rightarrow \pi\pi) \propto \phi_\pi \otimes T_I \otimes F^{B\pi} + \phi_\pi \otimes T_{II} \otimes \phi_B \otimes \phi_\pi$$

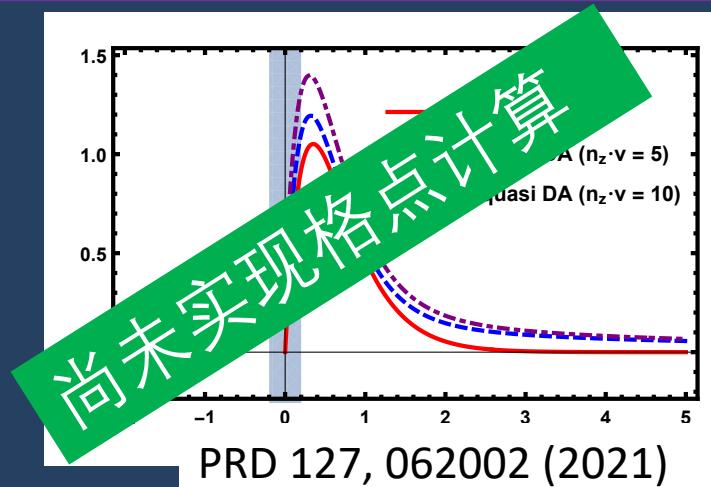
高精度 = 正确 + 精确

# Prospect:

27



精确度待提升



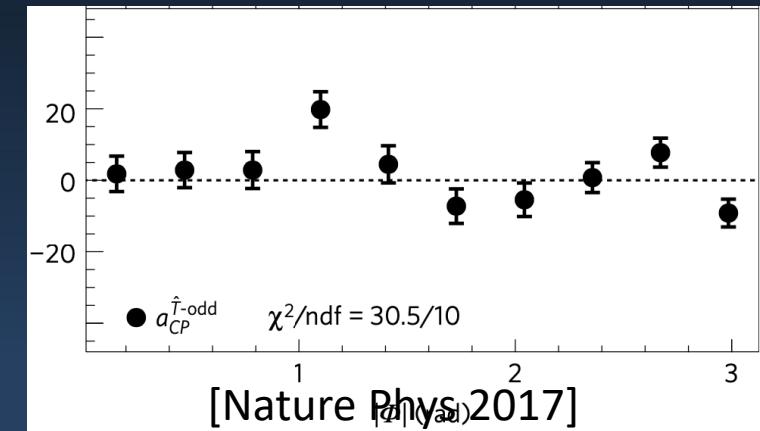
尚未实现格点计算

# Prospect: Baryon CPV

28

- LHCb实验在重味底夸克重子过程  $\Lambda_b^0 \rightarrow p\pi^-\pi^+\pi^-$  中发现了 $3\sigma$ 的CP破坏迹象
- 我国BESIII实验上也测量到目前关于超子CP破坏最严格的上限
- 虽未发现重子CP破坏，但实验精度已经达到了1%量级

重子CP破坏的实验研究已取得重要进展



BESIII:  $\Lambda \rightarrow p\pi$   
 $A_{CP} = (-0.6 \pm 1.4)\%$   
[Nature Phys 2019]

BESIII:  $\Xi \rightarrow \Lambda\pi$   
 $A_{CP} = (0.6 \pm 1.4)\%$   
[Nature 2022]

LHCb:  $\Lambda_b \rightarrow p\pi$   
 $A_{CP} = (-3.5 \pm 2.6)\%$   
[PLB 2018]

- 以往寻找底重子CP破坏主要通过两类观测量：
  - 通过衰变宽度定义的直接CP破坏
  - 由动量定义的三重积诱导CP破坏
- 但都未发现重子CP破坏

直接CP破坏

$$A_{CP} = \frac{\Gamma(\Lambda_b) - \Gamma(\bar{\Lambda}_b)}{\Gamma(\Lambda_b) + \Gamma(\bar{\Lambda}_b)}$$

三重积诱导CP破坏

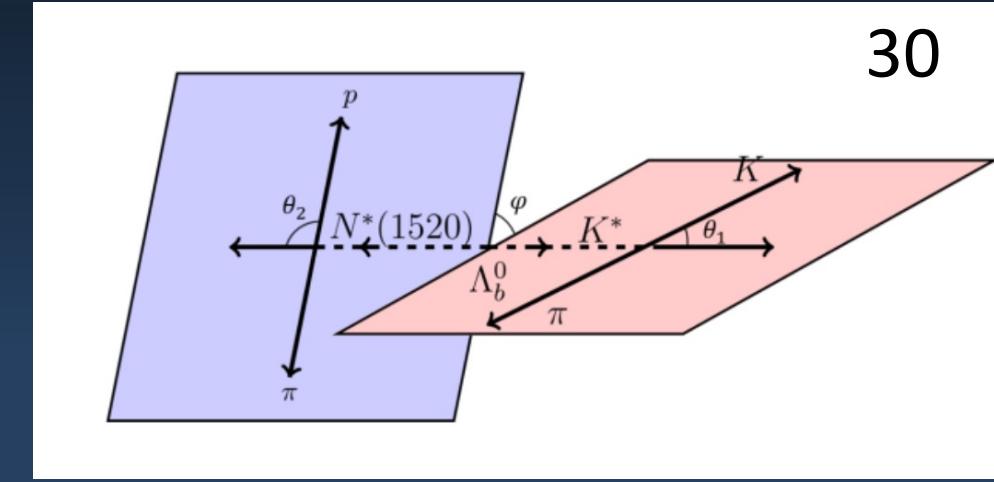
$$C_{\hat{T}} = \vec{p}_1 \cdot (\vec{p}_2 \times \vec{p}_3)$$

$$A_{\hat{T}}(C_{\hat{T}}) = \frac{N(C_{\hat{T}} > 0) - N(C_{\hat{T}} < 0)}{N(C_{\hat{T}} > 0) + N(C_{\hat{T}} < 0)}$$

$$A_{CP} = A_{\hat{T}} - \bar{A}_{\hat{T}}$$

- ① 用螺旋度振幅方法，推导底重子衰变各类过程的角分布公式，探讨角分布和分波等各类观测量
- ② 结合PQCD计算振幅，指出寻找重子CP破坏的黄金衰变道和最佳观测量

J.P. Wang, et.al, 2211.07332



$$\frac{d\Gamma}{dc_1 \, dc_2 \, d\varphi} \propto \begin{array}{l} \text{直接CP破坏} \\ \text{三重积诱导CP破坏} \end{array}$$

$$\begin{aligned}
 & -\frac{s_1^2 s_2^2}{\sqrt{3}} \text{Im} \left( \mathcal{H}_{+1,+\frac{3}{2}} \mathcal{H}_{-1,-\frac{1}{2}}^* + \mathcal{H}_{+1,+\frac{1}{2}} \mathcal{H}_{-1,-\frac{3}{2}}^* \right) \sin 2\varphi \\
 & + \frac{s_1^2 s_2^2}{\sqrt{3}} \text{Re} \left( \mathcal{H}_{+1,+\frac{3}{2}} \mathcal{H}_{-1,-\frac{1}{2}}^* + \mathcal{H}_{+1,+\frac{1}{2}} \mathcal{H}_{-1,-\frac{3}{2}}^* \right) \cos 2\varphi \\
 & - \frac{4s_1 c_1 s_2 c_2}{\sqrt{6}} \text{Im} \left( \mathcal{H}_{+1,+\frac{3}{2}} \mathcal{H}_{0,+\frac{1}{2}}^* + \mathcal{H}_{0,-\frac{1}{2}} \mathcal{H}_{-1,-\frac{3}{2}}^* \right) \sin \varphi \\
 & + \frac{4s_1 c_1 s_2 c_2}{\sqrt{6}} \text{Re} \left( \mathcal{H}_{+1,+\frac{3}{2}} \mathcal{H}_{0,+\frac{1}{2}}^* + \mathcal{H}_{0,-\frac{1}{2}} \mathcal{H}_{-1,-\frac{3}{2}}^* \right) \cos \varphi
 \end{aligned}$$

存在更多观测量

# Summary

CPV in (Heavy) Quarks has played an important role in SM  
Great Progresses have been made and we are entering a precision era.

High Precision Prediction vs New Observables

Warning: Small CPV BSM vs SM Uncertainties

Thank you very much!

# Backup

# Unitarity constraints: the triangles

$$ds \quad \begin{array}{c} V_{cd}V_{cs}^* \\ \diagdown \quad \diagup \\ V_{ud}V_{us}^* \end{array} \quad V_{td}V_{ts}^*$$

$$uc \quad \begin{array}{c} V_{ud}V_{cd}^* \\ \diagup \quad \diagdown \\ V_{ub}V_{cb}^* \end{array} \quad V_{us}V_{cs}^*$$

$$sb \quad \begin{array}{c} V_{ts}V_{tb}^* \\ \diagup \quad \diagdown \\ \phi_s \end{array} \quad V_{us}V_{ub}^*$$

$$ct \quad \begin{array}{c} V_{cs}V_{ts}^* \\ \diagup \quad \diagdown \\ V_{cd}V_{td}^* \end{array} \quad V_{cb}V_{tb}^*$$

$$bd \quad \begin{array}{c} V_{cb}V_{cd}^* \\ \gamma \quad \beta \\ \diagup \quad \diagdown \\ V_{ub}V_{ud}^* \end{array} \quad V_{tb}V_{td}^*$$

$$tu \quad \begin{array}{c} V_{ts}V_{us}^* \\ \diagup \quad \diagdown \\ V_{td}V_{ud}^* \end{array} \quad V_{tb}V_{ub}^*$$