Measurement of $\Sigma^+\bar{\Sigma}^-$ electromagnetic form factors using initial-state-radiation technique

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Electromagnetic Form Factors

- The cross section for the process $e^+e^- \to B\overline{B}$ via one-photon exchange, where B is a spin 1/2 baryon, can be expressed in terms of the electric and magnetic FFs G_E and G_M by following formula:

$$\sigma_{B\bar{B}}(s) = \frac{4\pi\alpha^2 C\beta}{3s}\left[|G_M(s)|^2 + \frac{1}{2\tau}|G_E(s)|^2\right].$$

- s is the invariant mass of the hadronic system
- α =1/137.036 is the fine-structure constant
- $\beta = \sqrt{1 - 4M_B^2/s}$ is the velocity
- $\tau = s/4M_B^2$, M_B is the mass of the baryon.

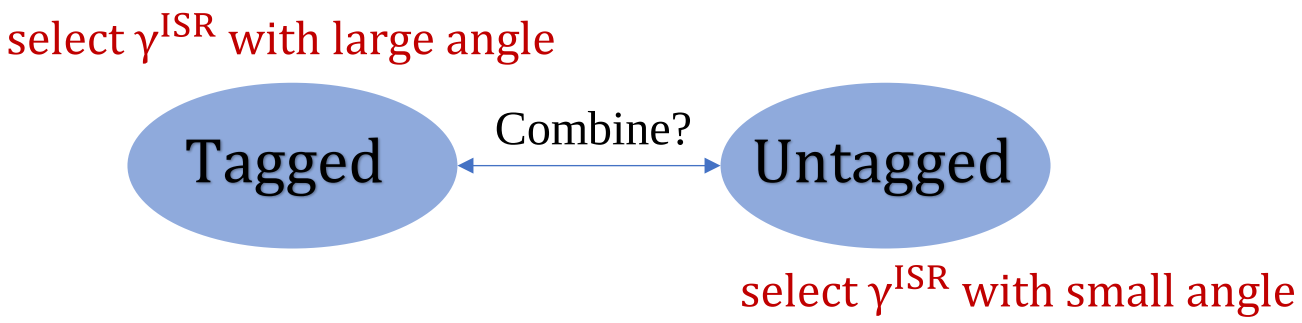
$$\boxed{|G_{\text{eff}}(s)| = \sqrt{\frac{2\tau|G_M(s)|^2 + |G_E(s)|^2}{2\tau + 1}}.}$$

- Coulomb correction factor $C = \begin{cases} 1, for\ pairs\ of\ neutral\ baryons \\ y/(1-e^{-y}), y = \pi\alpha(1+\beta^2)/\beta, for\ pairs\ of\ charged\ baryons \end{cases}$

Analysis Strategy

$$e^+e^- \longrightarrow \gamma^{ISR}\Sigma^+\overline{\Sigma}^- \longrightarrow \gamma^{ISR}p\pi^0\overline{p}\pi^0$$

- We use two method to select the signal events:
- 1、**Tagged method:** Selecting all final particles including $\gamma^{ISR}p\pi^0\overline{p}\pi^0$. However, this method can only detect γ^{ISR} entering the EMC, and small-angle γ^{ISR} cannot be detected.
- 2、**Untagged method:** We select $p\pi^0\overline{p}\pi^0$ and miss γ^{ISR}. Since most γ^{ISR} are emitted at small angles, we can exclude a large amount of background by restricting their angles.


select γ^{ISR} with large angle

select γ^{ISR} with small angle

Data Sets

- 1、Data :

$\sqrt{s}\ [GeV]$	Sample Type	Run number	Luminosity [pb^{-1}]	Total Luminosity
3.773	Round03 (2010)	11414-13988,14395-14604	2931.8±0.2±13.8	20247.8 pb^{-1}
	Round04 (2011)	20488-23454		
	Round15 (2022)	70522-73929	4995±19	
	Round16 (2023)	74031-78536	8157±31	
	Round17 (2024)	78615-81094	4191±16	

- 2、Monte Carlo simulations(MC) ($\sqrt{s} = 3.773$ GeV):

The signal MC:

The generator software package **ConExc** is used to simulate the signal MC samples.

Event Selection

Good charged tracks：

- $|\cos\theta| \leq 0.93$, $|V_{xy}| \leq 2cm$, $|V_z| \leq 10cm$

PID：

- p:prob(p)>prob(π)&prob(p)>prob(K)

Photon：

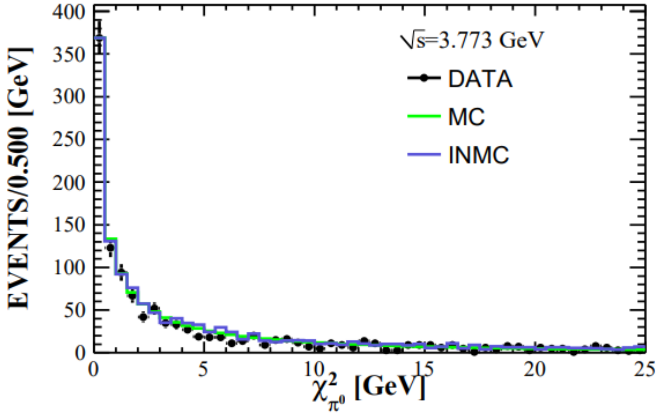
- $E_\gamma > 25MeV(|\cos\theta| \leq 0.86)$
- $E_\gamma > 50MeV(0.86 \leq |\cos\theta| \leq 0.92)$
- Time cut: $0 \leq T \leq 700ns$
- The angle of photon and proton $> 10^\circ$
- The angle of photon and anti-proton $> 20^\circ$

Other selections:

- Number of $p = 1\ and\ \bar{p} = 1$
- For untagged method: $\gamma \geq 4\ and\ \pi^0 \geq 2$
- For Tagged method: $\gamma \geq 5\ and\ \pi^0 \geq 2$

Reconstruct π^0:

- $|M(\gamma\gamma) - M(\pi^0)| \in [-60,40]MeV$
- A kinematic fit be used, $\chi^2 < 25$



Event Selection

For Untagged Method：

- Select minimum $\Delta_m = \sqrt{\left(M_{p\pi^0} - M_{\Sigma^+}\right)^2 + \left(M_{\bar{p}\pi^0} - M_{\bar{\Sigma}^-}\right)^2}$

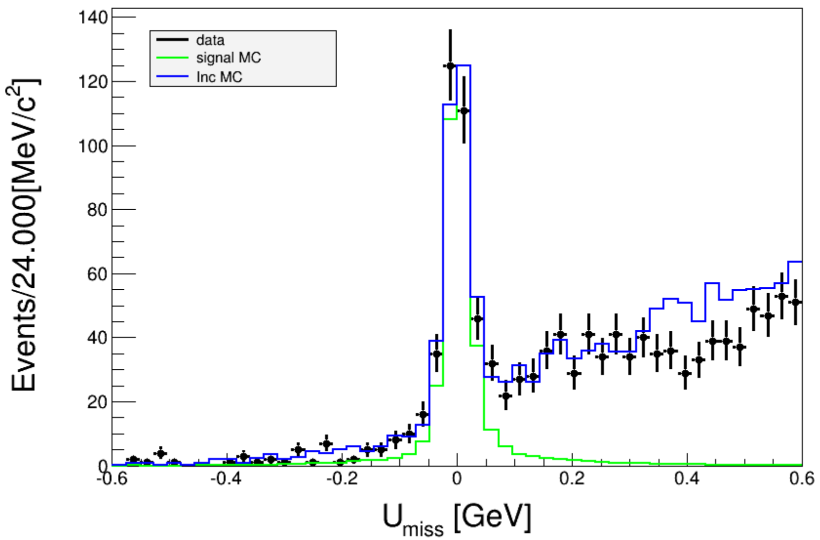
- Mass cut: $M_{\Sigma^+(\bar{\Sigma}^-)} \in [1.16, 1.21]\ [GeV]$

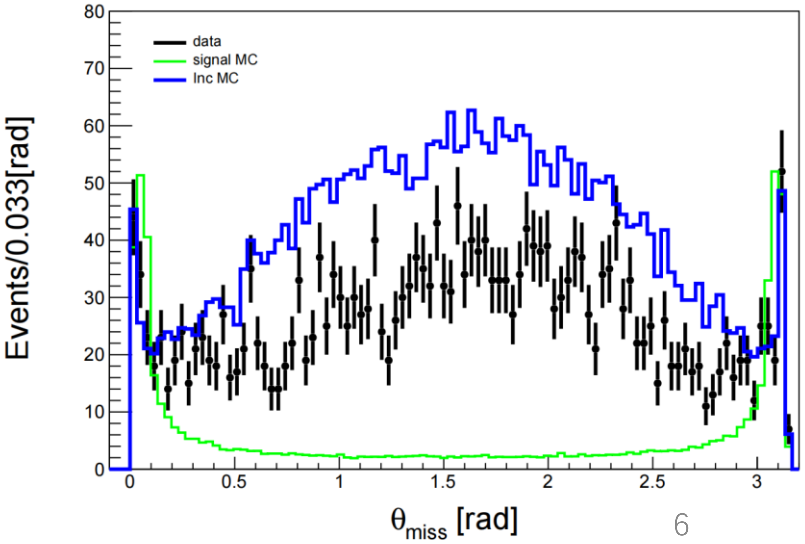
- U_{miss} cut: $-0.14 < U_{miss} < 0.06\ [GeV]$

$$U_{miss} = E^{rec}_{\Sigma^+\bar{\Sigma}^-} - P^{rec}_{\Sigma^+\bar{\Sigma}^-}$$

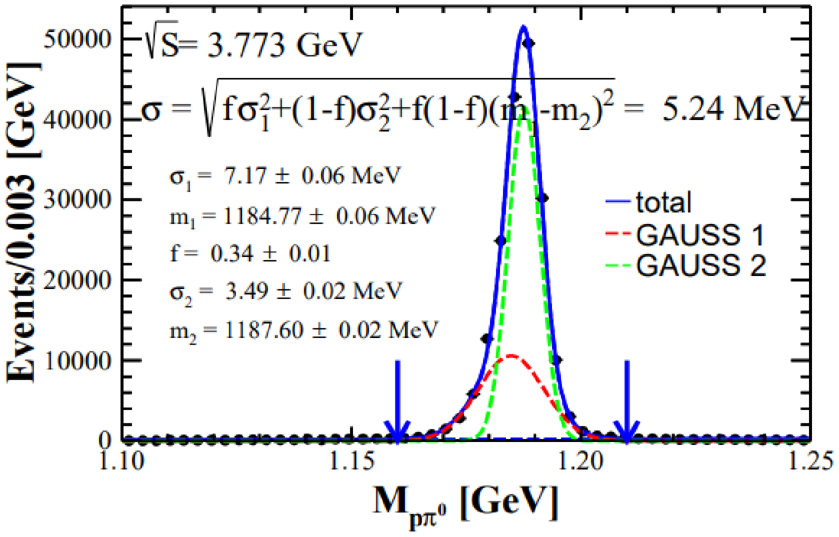
- θ_{miss} cut: $\theta_{miss} < 0.25\ or\ \theta_{miss} > 2.90\ [rads]$

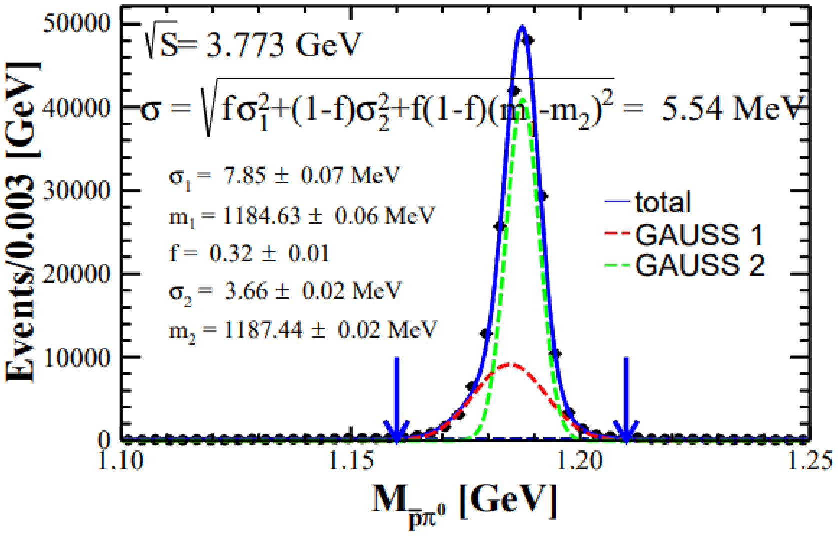
  θ_{miss} means the angle between the momentum of the recoiling against the $\Sigma^+\bar{\Sigma}^-$ system and beam direction

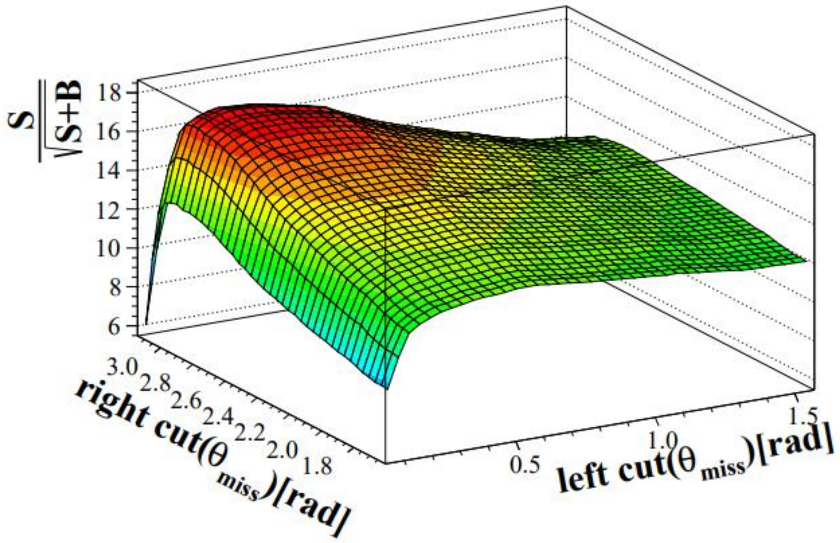


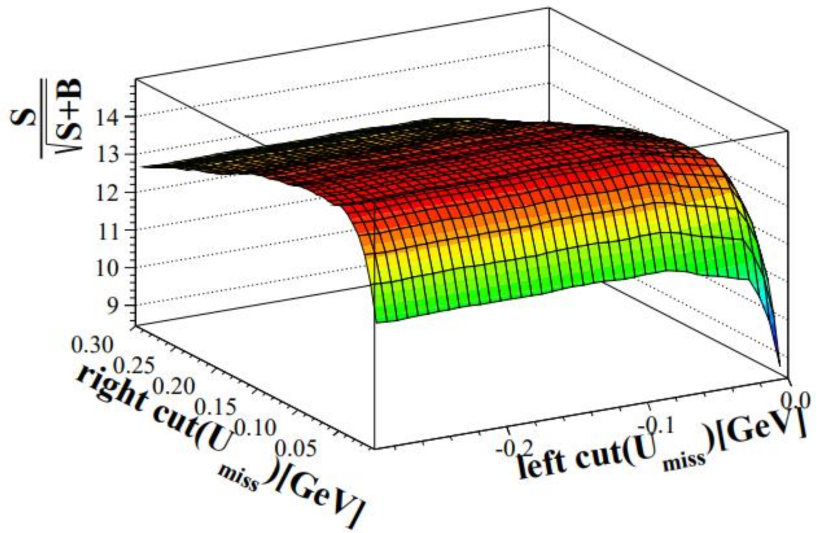


Study of Event Selection





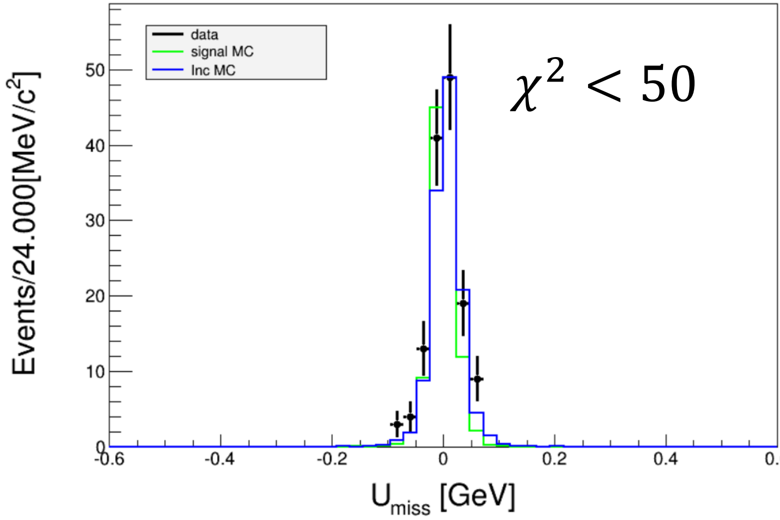


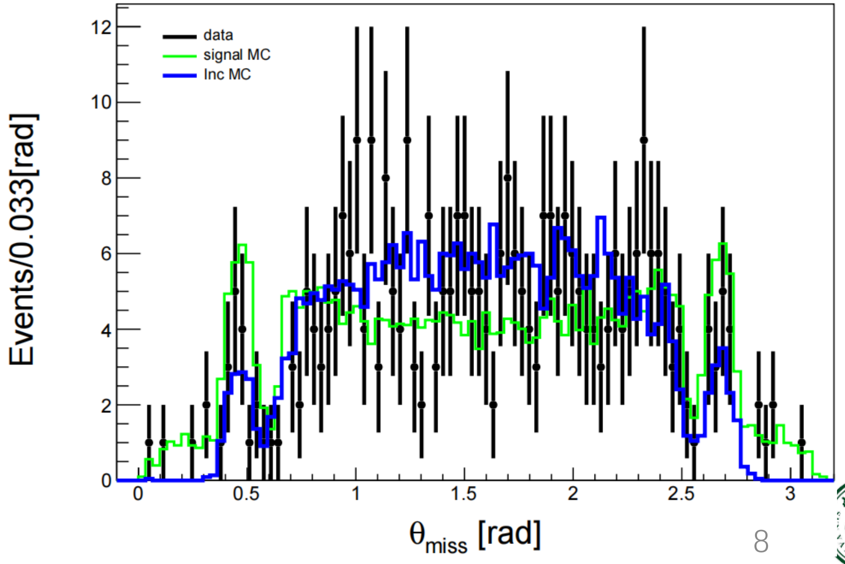


Event Selection

For Tagged Method：

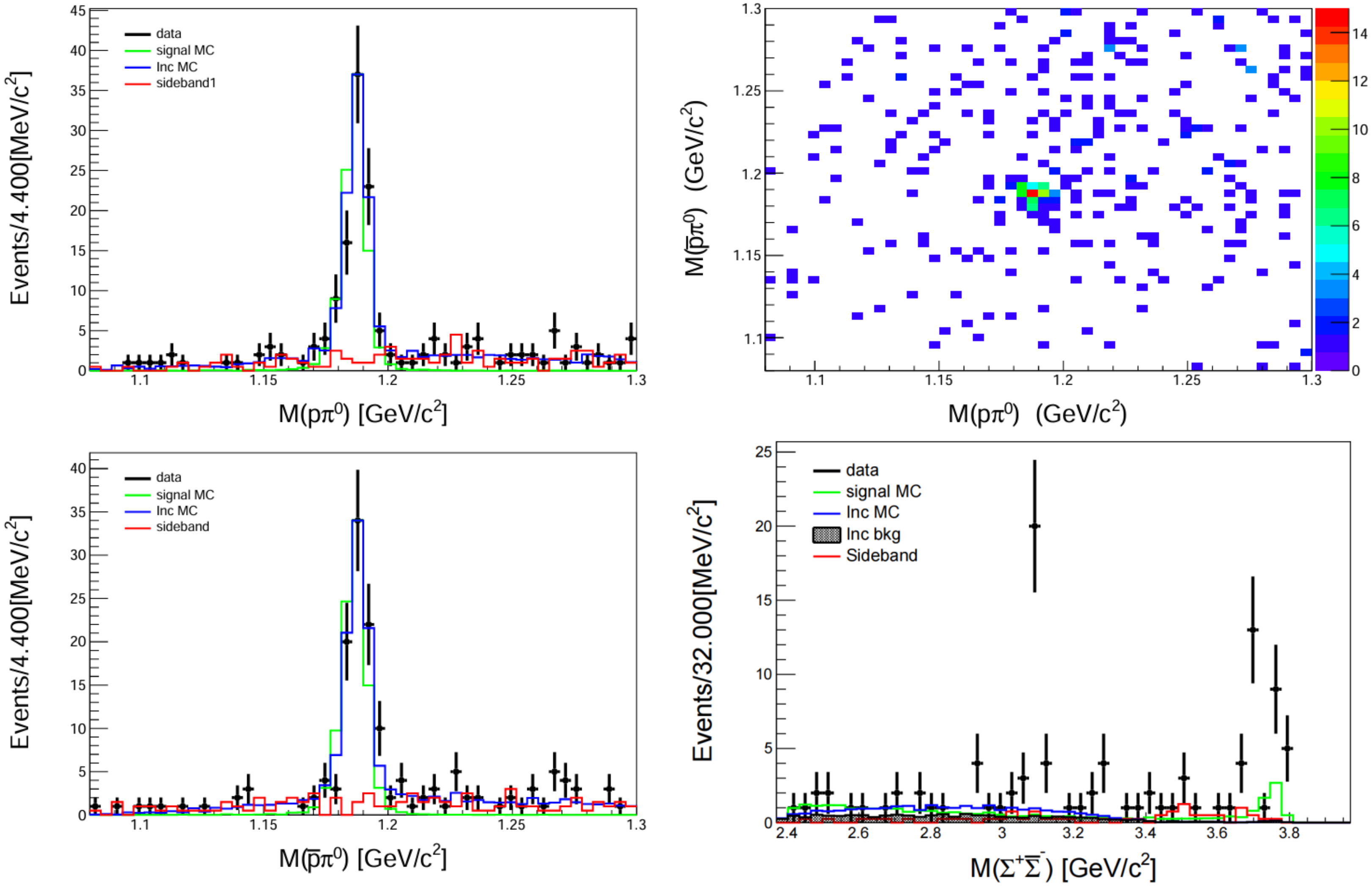
- Select minimum $\Delta_m = \sqrt{\left(M_{p\pi^0} - M_{\Sigma^+}\right)^2 + \left(M_{\bar{p}\pi^0} - M_{\bar{\Sigma}^-}\right)^2}$
- A kinematic fit is used, there are $5\gamma,\ 1p\ and\ 1\ \bar{p}$. And we require $\chi^2 < 50$.
- A kinematic fit of background is used $(6\gamma, 1p, 1\bar{p})$. If $\chi^2_{BG} < \chi^2_{sig}$, then we exclude this event. (exclude $\pi^0\Sigma^+\bar{\Sigma}^-$)
- θ_{miss} cut: $0.25 < \theta_{miss} < 2.90\ [rads]$
- U_{miss} cut: $-0.06 < U_{miss} < 0.06\ [GeV]$
- Mass cut: $M_{\Sigma^+(\bar{\Sigma}^-)} \in [1.16, 1.21]\ [GeV]$





Tagged Result

2010-11 3773 data



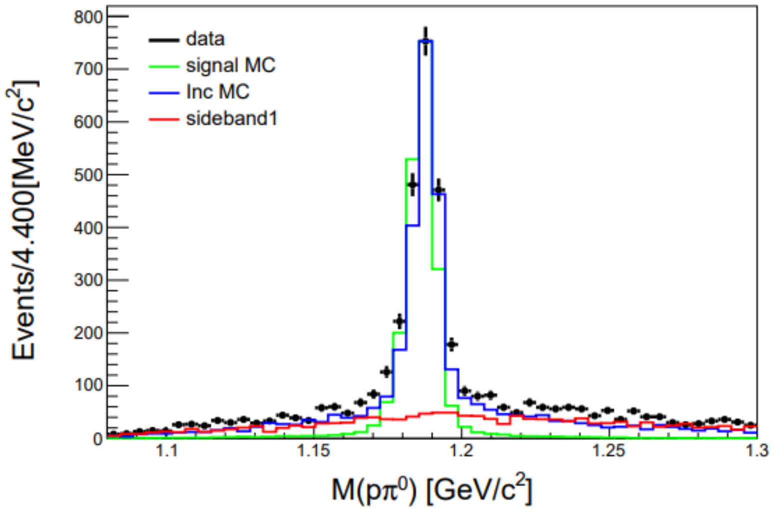
Sideband Region:

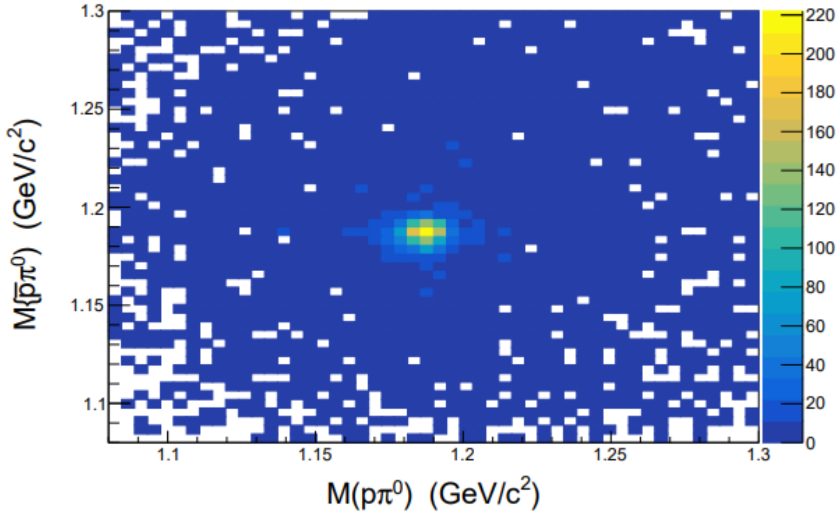
- BKG1: $1.10 \le M_{\Sigma^+} \le 1.15$ GeV and $1.22 \le M_{\bar{\Sigma}^-} \le 1.27$ GeV,
- BKG2: $1.22 \le M_{\Sigma^+} \le 1.27$ GeV and $1.22 \le M_{\bar{\Sigma}^-} \le 1.27$ GeV,
- BKG3: $1.10 \le M_{\Sigma^+} \le 1.15$ GeV and $1.10 \le M_{\bar{\Sigma}^-} \le 1.15$ GeV,
- BKG4: $1.22 \le M_{\Sigma^+} \le 1.27$ GeV and $1.10 \le M_{\bar{\Sigma}^-} \le 1.15$ GeV.

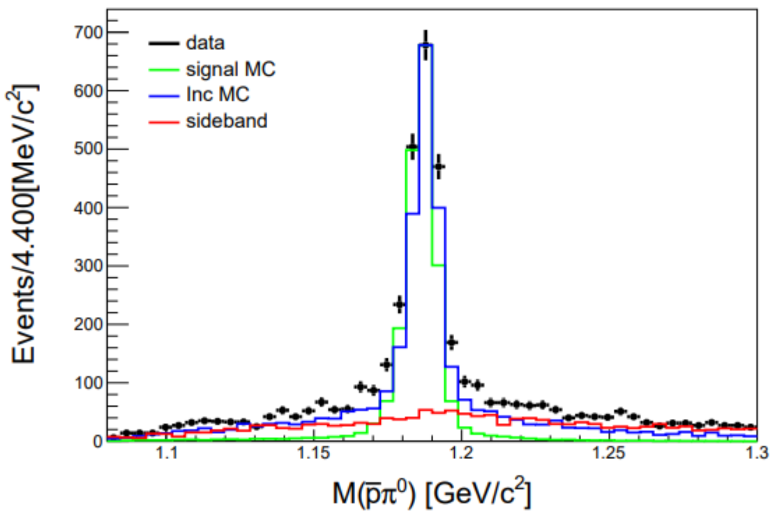
For the **tagged method**, the main background are $\pi^0\ \Sigma\Sigma$ from IncMC topology. The sideband method is used to estimate other background. Considering the low efficiency and high background, we have abandoned this method.

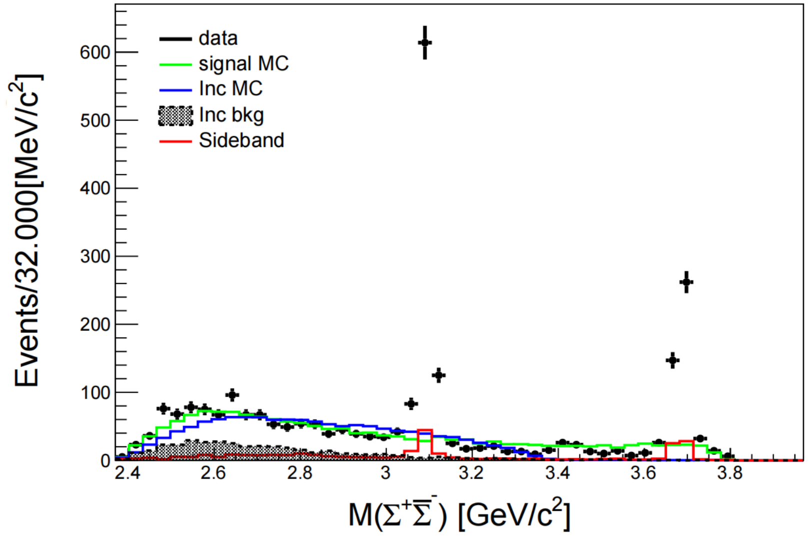
Untagged Results

Whole 3773 data









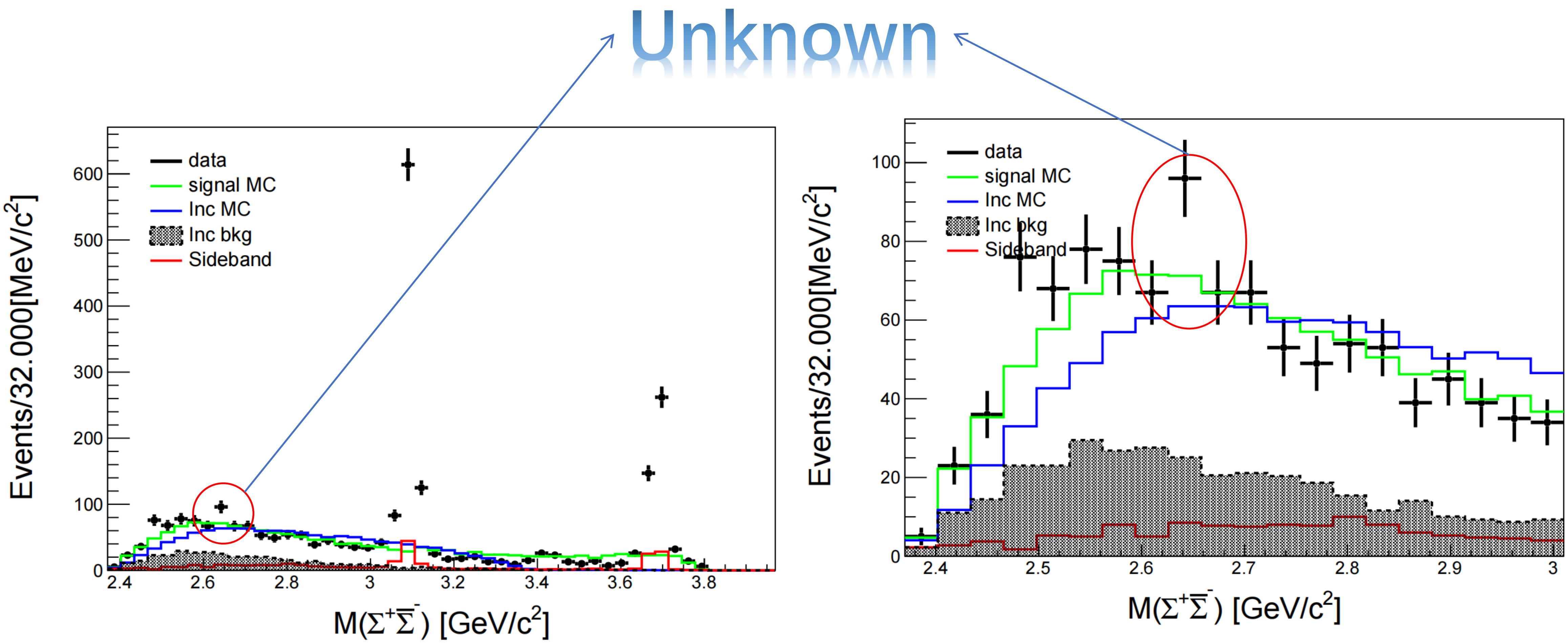
Sideband Region:

- BKG1: $1.10 \le M_{\Sigma^+} \le 1.15$ GeV and $1.22 \le M_{\bar{\Sigma}^-} \le 1.27$ GeV,
- BKG2: $1.22 \le M_{\Sigma^+} \le 1.27$ GeV and $1.22 \le M_{\bar{\Sigma}^-} \le 1.27$ GeV,
- BKG3: $1.10 \le M_{\Sigma^+} \le 1.15$ GeV and $1.10 \le M_{\bar{\Sigma}^-} \le 1.15$ GeV,
- BKG4: $1.22 \le M_{\Sigma^+} \le 1.27$ GeV and $1.10 \le M_{\bar{\Sigma}^-} \le 1.15$ GeV.

For the **untagged method**, the background is cleaner. We should estimate its background. There are two methods:

1. Estimate using IncMC;
2. Estimate using sideband + π^0 $\Sigma\Sigma$

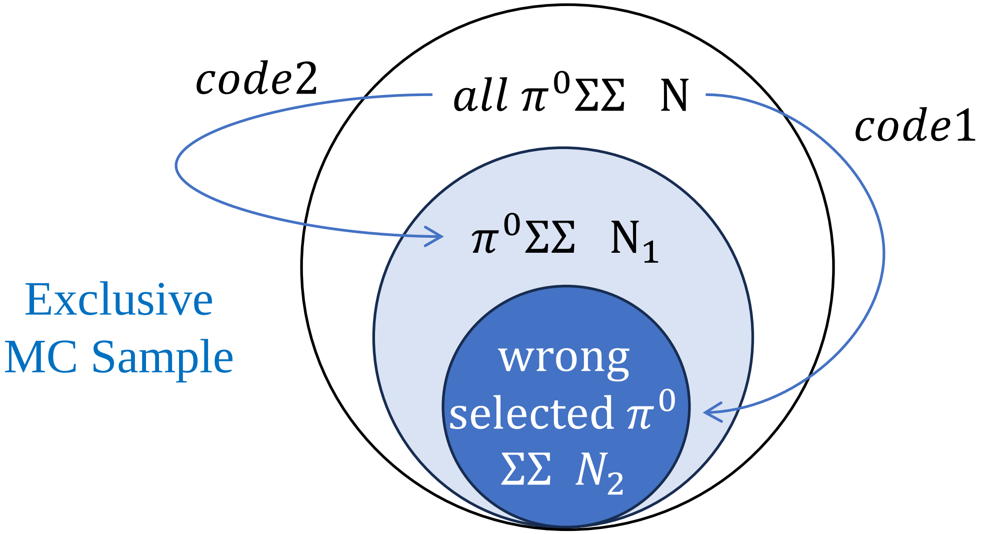
Untagged Result

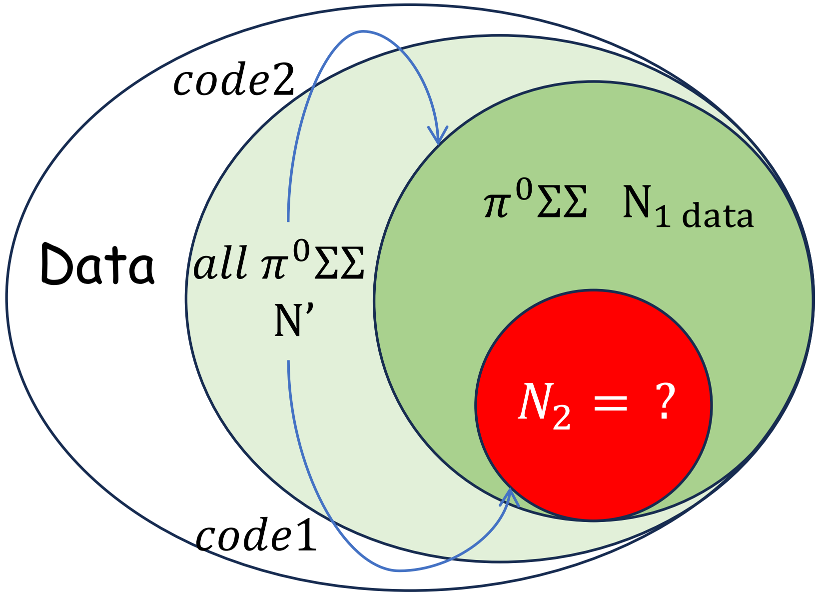


Unknown

$\pi^0\Sigma\Sigma$ BG Estimate

We use another code to select $\boldsymbol{\pi^0\Sigma\Sigma}$ code1: select $\boldsymbol{\gamma\Sigma\Sigma}$(signal). code2: select $\boldsymbol{\pi^0\Sigma\Sigma}$(bkg)


$code2$
$all\ \pi^0\Sigma\Sigma$ N
$code1$
$\pi^0\Sigma\Sigma$ N_1
Exclusive MC Sample
wrong selected π^0 $\Sigma\Sigma$ N_2


$code2$
Data
$all\ \pi^0\Sigma\Sigma$ N'
$\pi^0\Sigma\Sigma$ $\mathrm{N}_{1\ \mathrm{data}}$
$N_2 = ?$
$code1$

Code2 Event Selection:

a The same as untagged method, select $p\,\pi^0\,\bar{p}\,\pi^0$ tracks first.

b Limit $U_{miss} < -0.14\ or\ U_{miss} > 0.06$ to exclude $\boldsymbol{\gamma\Sigma\Sigma}$ signal.

c Loop all π^0 and make kmfit, select the minimum χ^2 event.

$\pi^0\Sigma\Sigma$ BG Estimate

code1: select $\gamma\Sigma\Sigma$(signal). code2: select $\pi^0\Sigma\Sigma$(bkg)

In Exclusive MC:

1、ε^i_{sig} means the number of bkg($\pi^0\Sigma\Sigma$) events N_i misidentified as signal($\gamma\Sigma\Sigma$) ratio --> Code1

$$\varepsilon^i_{sig} = N^i_2/N^i$$

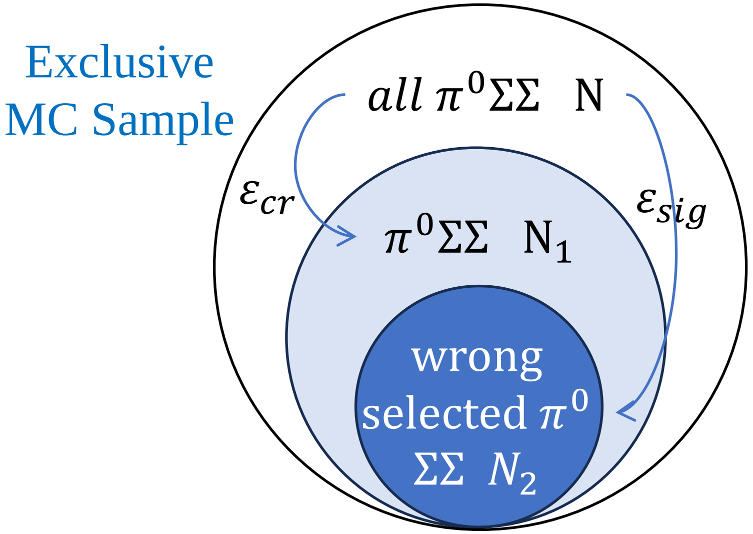
2、ε^i_{cr} means the efficiency of bkg($\pi^0\Sigma\Sigma$) selection --> Code2

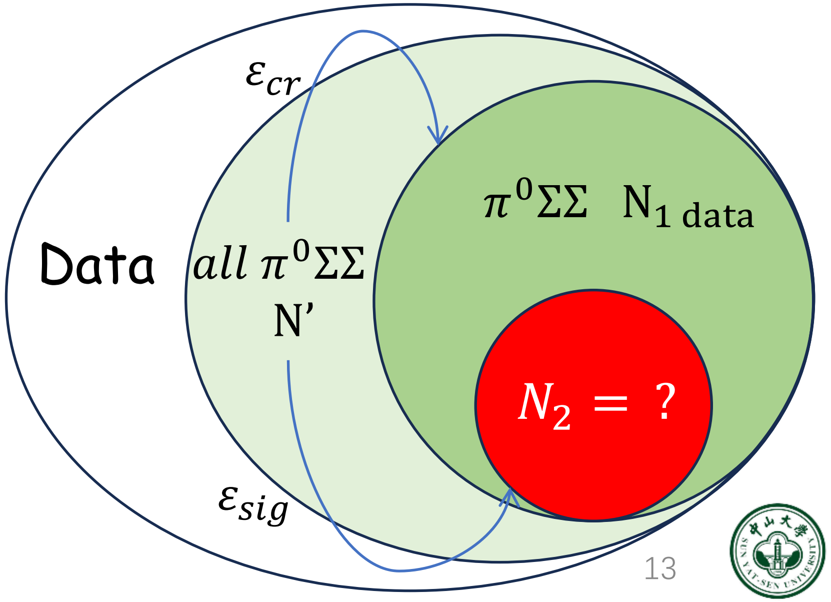
$$\varepsilon^i_{cr} = N^i_1/N^i$$

In Data:

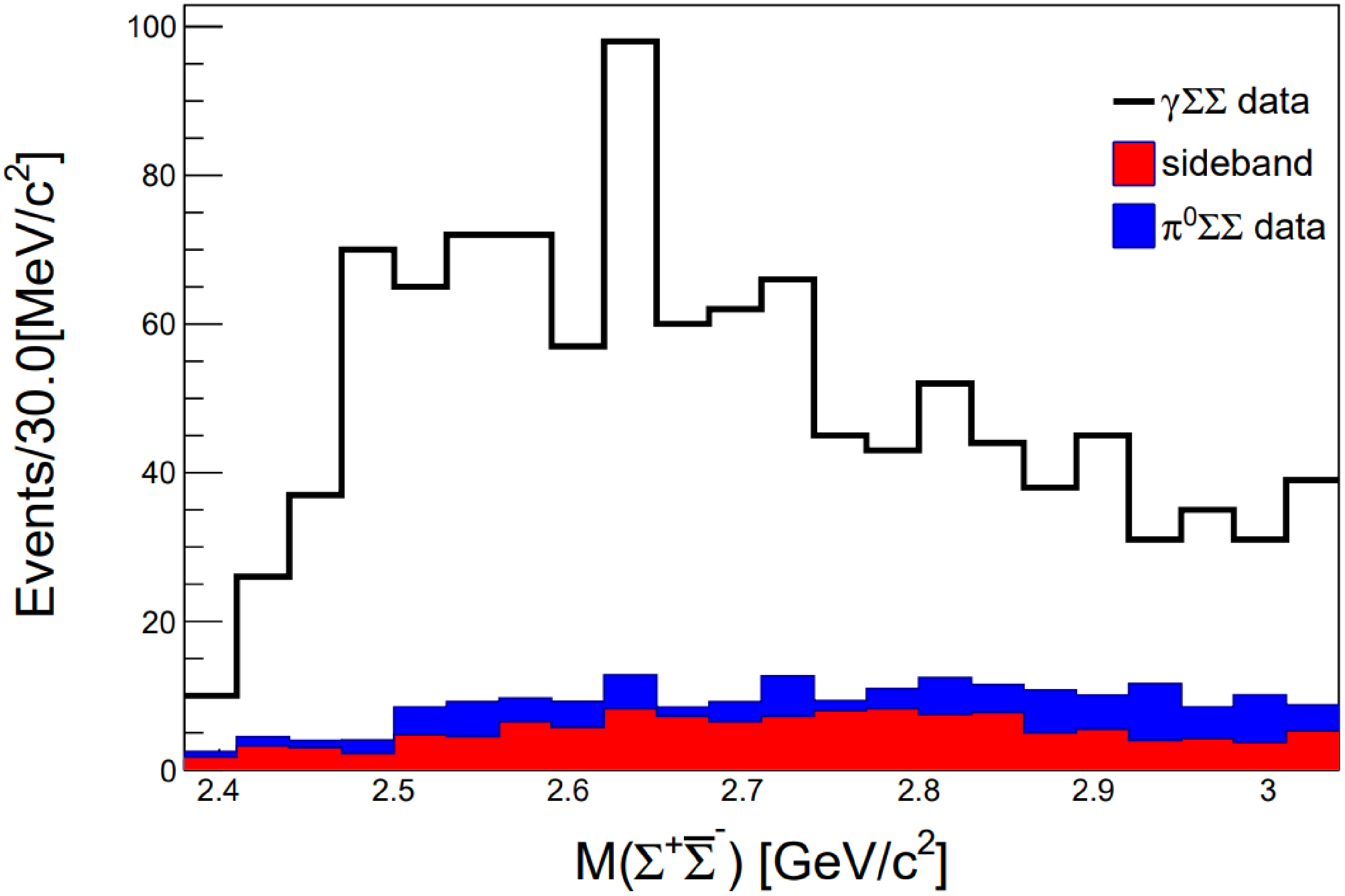
3、We can estimate the number of $\pi^0\Sigma\Sigma$ misidentified as signal($\gamma\Sigma\Sigma$) number:

$$N_{2\,data} = N' \cdot \varepsilon^i_{sig} = N^i_{1\,data}/\varepsilon^i_{cr} \cdot \varepsilon^i_{sig} = RN^i_{1\,data} \qquad \left(R = \frac{\varepsilon^i_{sig}}{\varepsilon^i_{cr}}\right)$$





BG estimate



Events/30.0[MeV/c^2]
0
20
40
60
80
100
2.4
2.5
2.6
2.7
2.8
2.9
3
M($\Sigma^+\bar{\Sigma}^-$) [GeV/c^2]
$\gamma\Sigma\Sigma$ data
sideband
$\pi^0\Sigma\Sigma$ data

Calculation the Cross Section

Cross Section： $$\sigma_{\Sigma^+\bar{\Sigma}^-}(M_{\Sigma^+\bar{\Sigma}^-}) = \frac{(dN_{sig}/dM_{\Sigma^+\bar{\Sigma}^-})}{\varepsilon\cdot\mathcal{B}(\Sigma^+\to p\pi^0)\mathcal{B}(\bar{\Sigma}^-\to\bar{p}\pi^0)\mathcal{B}^2(\pi^0\to\gamma\gamma)\cdot(d\mathcal{L}_{int}/dM_{\Sigma^+\bar{\Sigma}^-})}.$$

Here, $d\mathcal{L}_{int}/dM_{\Sigma^+\bar{\Sigma}^-} = W(s,x)\cdot\mathcal{L}_{int}$ $$W(s,x) = \frac{\alpha}{\pi x}\left[\ln\left(\frac{s}{M_e^2}\right) - 1\right]\cdot(2-2x+x^2)\,, x = 1 - \frac{M_{\Sigma^+\bar{\Sigma}^-}^2}{s}$$

If higher-order terms are considered (to more accurately describe the ISR process), then $W(s,x)$:

$$W(s,x) = kx^{k-1}\left[1+\frac{\alpha}{\pi}\left(\frac{\pi^2}{3}-\frac{1}{2}\right)+\frac{3}{4}k+k^2\left(\frac{37}{96}-\frac{\pi^2}{12}-\frac{1}{72}\ln\frac{s}{m_e^2}\right)\right]-k\left(1-\frac{1}{2}x\right)$$
$$+\frac{1}{8}k^2\left[4(2-x)\ln\frac{1}{x}-\frac{1+3(1-x)^2}{x}\ln(1-x)-6+x\right], k=\frac{2\alpha}{\pi}\left[\ln\frac{s}{m_e^2}-1\right],$$

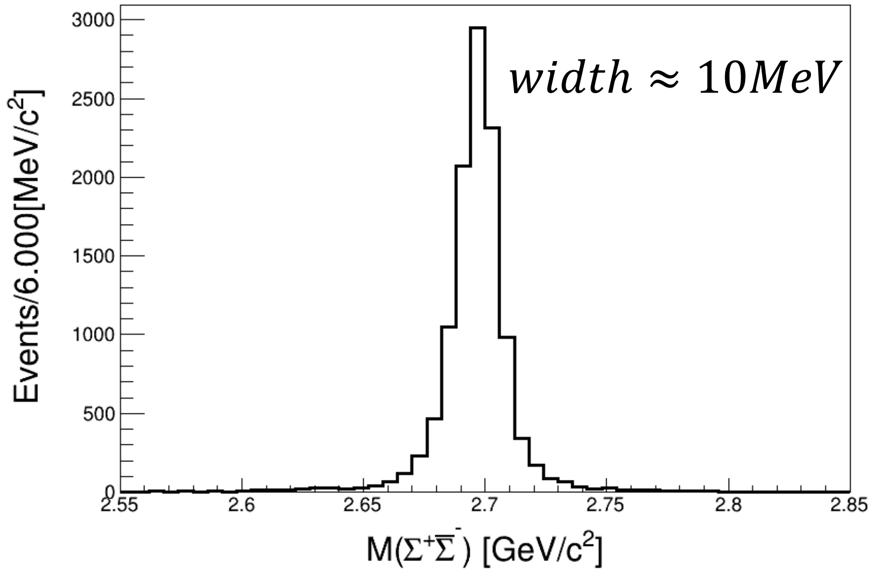
From this, we can calculate the production cross section of ΣΣ in each bin.

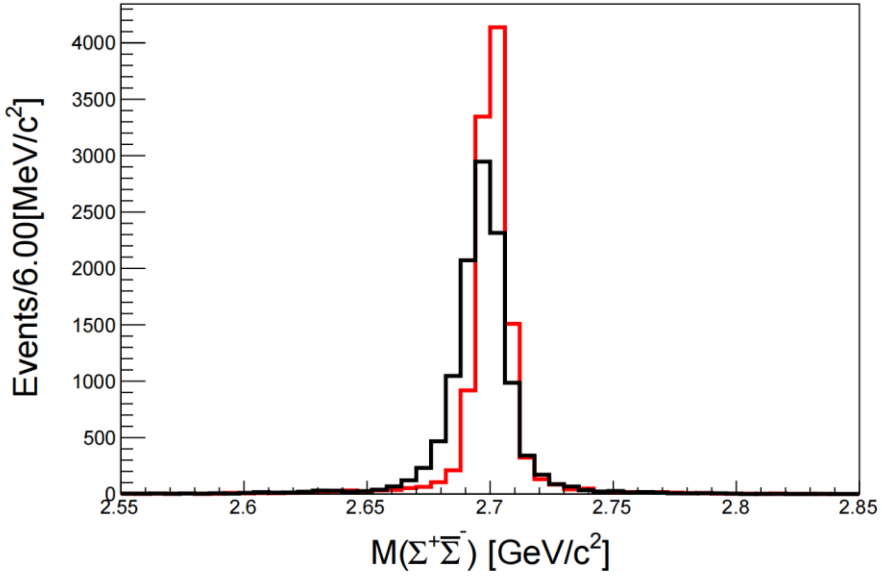
Resolution Study

To improve the $M_{\Sigma\Sigma}$ mass resolution, we correct $M_{\Sigma\Sigma}$ to

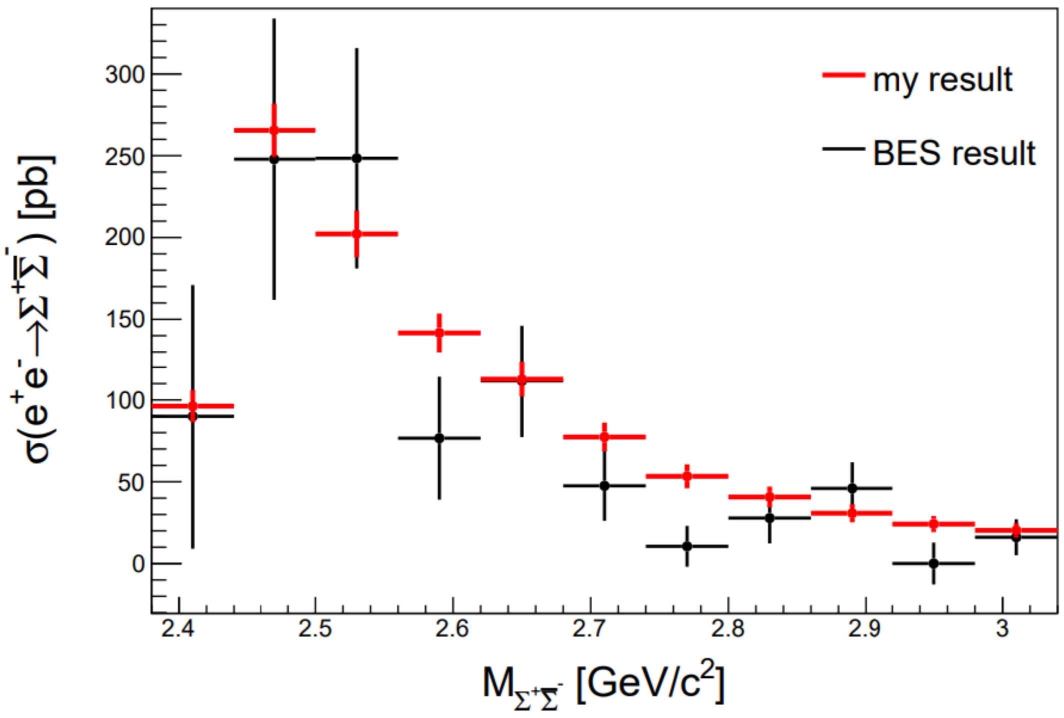
$$(M_{\Sigma\Sigma})_{correct} = M_{\Sigma\Sigma} - M_{p\pi^0} - M_{\bar{p}\pi^0} + 2M_{\Sigma}(pdg)$$

We generate mass = 2.7GeV, width = 0 particle MC Sample





MSigmaSigma [GeV/c^2]	N1(Origin)	N2(sideband)	Nbkg(pi0SigmaSigma)	Nsig	ε	Leff	σ[pb]	Geff [10-2]
2.38-2.44	25	5	2.14	17.86	2.19%	32.61	96.22	13.04
2.44-2.50	104	5.25	2.70	96.05	3.98%	35.04	265.49	21.66
2.50-2.56	141	9.25	8.45	123.30	6.22%	37.74	202.17	18.90
2.56-2.62	142	12.25	6.63	123.12	8.21%	40.75	141.64	15.82
2.62-2.68	143	15.5	5.81	121.69	9.42%	44.13	112.70	14.11
2.68-2.74	123	13.75	7.83	101.42	10.53%	47.93	77.35	11.69
2.74-2.80	100	16.25	3.85	79.90	11.04%	52.24	53.33	9.71
2.80-2.86	93	15.25	8.56	69.19	11.46%	57.15	40.67	8.48
2.86-2.92	82	10.5	10.75	60.75	12.11%	62.78	30.77	7.37
2.92-2.98	73	8.25	11.73	53.02	12.21%	69.30	24.14	6.53
2.98-3.04	72	9	10.34	52.66	12.99%	76.89	20.29	5.99



Uncertainty

$$\sigma_{\Sigma^+\bar{\Sigma}^-}(M_{\Sigma^+\bar{\Sigma}^-}) = \frac{(dN_{sig}/dM_{\Sigma^+\bar{\Sigma}^-})}{\varepsilon \cdot \mathcal{B}(\Sigma^+ \to p\pi^0)\mathcal{B}(\bar{\Sigma}^- \to \bar{p}\pi^0)\mathcal{B}^2(\pi^0 \to \gamma\gamma) \cdot (d\mathcal{L}_{int}/dM_{\Sigma^+\bar{\Sigma}^-})}.$$

Source	Uncertainty [%]										
Tracking and PID (p and $\bar{p}$)	1.6										
π^0 reconstruction	3.26?										
Σ mass window	0.98										
U_{miss} requirement	1.11										
θ_{miss} requirement	2.76										
Background estimation	4.11	1.01	1.61	0.10	2.74	2.30	5.15	4.16	1.48	1.11	2.37
Angular distribution	4.66	9.71	9.06	7.41	8.08	3.92	4.87	4.73	3.79	4.01	4.33
Luminosity	0.5										
Related branching fraction	1.18										

1、Umiss Uncertainty

$E_\gamma \in [0.20, 0.35]$ GeV.

The $\Sigma^+/\bar{\Sigma}^-$ mass window cut is set to be [1.175,1.195] GeV for $\Sigma^+/\bar{\Sigma}^-$ signal region.

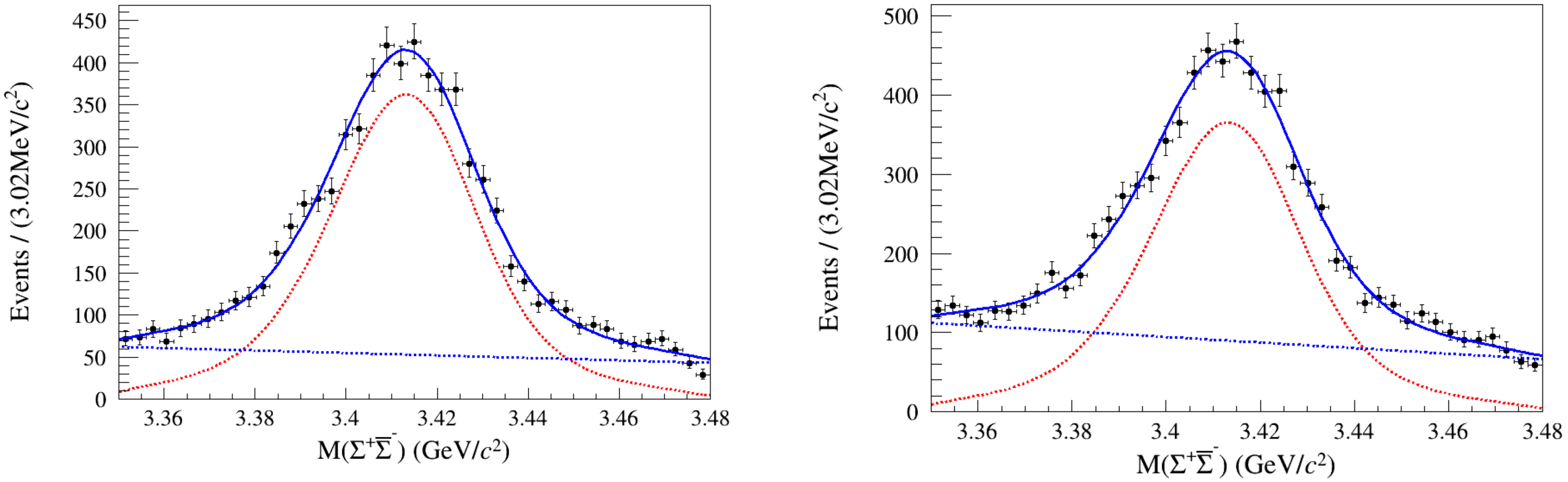
$U_{miss} \in [-0.14, 0.06]$

The uncertainty due to the U_{miss} requirement is estimated via studied $\psi(3686) \rightarrow \gamma\chi_{c0}, \chi_{c0} \rightarrow \Sigma^+\bar{\Sigma}^-$ decay.

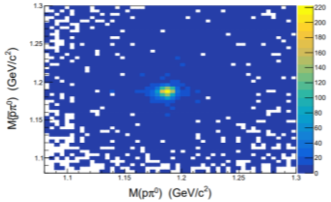
$$\epsilon_{DATA/MC} = \frac{n^{with}_{DATA/MC}}{n^{without}_{DATA/MC}}$$

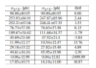
Source	ε_{data}	ε_{MC}	Uncertainty
Umiss requirement	99.60%	98.49%	1.11%

$$\delta = \varepsilon_{MC}/\varepsilon_{DATA} - 1$$



2、sideband uncertainty





To estimate the uncertainties of background events, we changed the 2D sideband regions for background descriptions, by changing the $M_{p\pi^0(\bar{p}\pi^0)}$ sideband regions from [1.10,1.15] and [1.22,1.27] GeV to [1.095,1.145] and [1.225,1.275] GeV. The differences are taken as the systematic uncertainties com-

BES result

	Nominal Value	
$\sigma_{\Sigma^+\bar{\Sigma}^-}$ [pb]	$\sigma_{\Sigma^+\bar{\Sigma}^-}$ [pb]	Diff
90.00±80.95	90.00±80.95	0.00
253.93±84.91	247.87±85.96	2.44
252.21±67.04	248.41±67.35	1.53
76.73±37.58	76.63±37.58	0.13
109.67±34.62	111.68±34.37	-1.79
45.69±21.68	47.52±21.4	-3.84
11.99±12.17	10.54±12.47	13.76
29.18±15.22	27.82±15.48	4.89
44.61±16.16	45.95±15.98	-2.91
-0.96±12.99	0.04±12.82	-2499.99
17.85±10.71	16.10±11.00	10.87

My result

	σ[pb]		
	sideband region 1	sideband region 2	diff [%]
Bin0	155.968	162.373	4.11%
Bin1	277.694	280.498	1.01%
Bin2	196.426	199.596	1.61%
Bin3	126.024	126.153	0.10%
Bin4	126.716	130.193	2.74%
Bin5	81.586	83.462	2.30%
Bin6	45.4936	47.8387	5.15%
Bin7	42.0678	43.816	4.16%
Bin8	31.1578	30.6968	-1.48%
Bin9	21.0988	21.3333	1.11%
Bin10	19.5274	19.991	2.37%

3、MC efficiency correction for π^0

Based on the study of control samples of $\psi(3686) \to \pi^0\pi^0 J/\psi$ and $e^+e^- \to \omega\pi^0$ reconstruction efficiency is studied [48]. The relative difference of the π^0 reconstruction efficiencies $\Delta\varepsilon_{\pi^0}(p)$ obtained on the two data sets are consistent with each other. A momentum dependency of $\Delta\varepsilon_{\pi^0}(p)$ is observed. The combined results is $\Delta\varepsilon_{\pi^0}(p) = (0.06 - 2.41p)\%$, and the error of this correction is 0.87% for the slope and 0.24% for the offset. Considering the error propagation, the systematic uncertainty is $(0.06-2.41p-\sqrt{0.76p^2+1.15+0.39p})\%$ [48]. The resulting uncertainty is obtained by reweighting according to the number of events in each energy bin, see below:

$$\frac{n_1}{N}\Delta\varepsilon_{\pi^0}(p_1) + \frac{n_2}{N}\Delta\varepsilon_{\pi^0}(p_2) + \frac{n_3}{N}\Delta\varepsilon_{\pi^0}(p_3) + \dots \tag{7}$$

where n is the number of π^0 events in MC corresponding to a single bin,N is the total number of π^0 events in MC. The numerical results is listed in Table 5. The systematic uncertainties from the reconstruction of π^0 in the $\Sigma^+ \to p\pi^0$ and $\bar{\Sigma}^- \to \bar{p}\pi^0$ processes are 1.64% and 1.62%, respectively, for the $e^+e^- \to \Sigma^+\bar{\Sigma}^-$ decay mode. Therefore, the total uncertainty due to the π^0 is about 3.26%.

Tab. 5: The results of the n and $\Delta\varepsilon_{\pi^0}(p)$ in various momentum ranges of π^0 at $\sqrt{}= 3.773$ GeV.

P_{π^0}[GeV]	π^0 from Σ^+		π^0 from $\bar{\Sigma}^-$		$\Delta\varepsilon_{\pi^0}(p)(\%)$
	n	$n/N(\%)$	n	$n/N(\%)$	
(0.0,0.2]	47998	38.54	51613	41.45	1.28
(0.2,0.4]	68936	55.36	66382	53.30	1.82
(0.4,0.6]	7601	6.10	6540	5.25	2.38
(0.6,0.8]	0	0.00	0	0.00	2.97
Uncertainty (%)	1.64		1.62		-

Thank you!

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