Spin polarization in strongly coupled QGP

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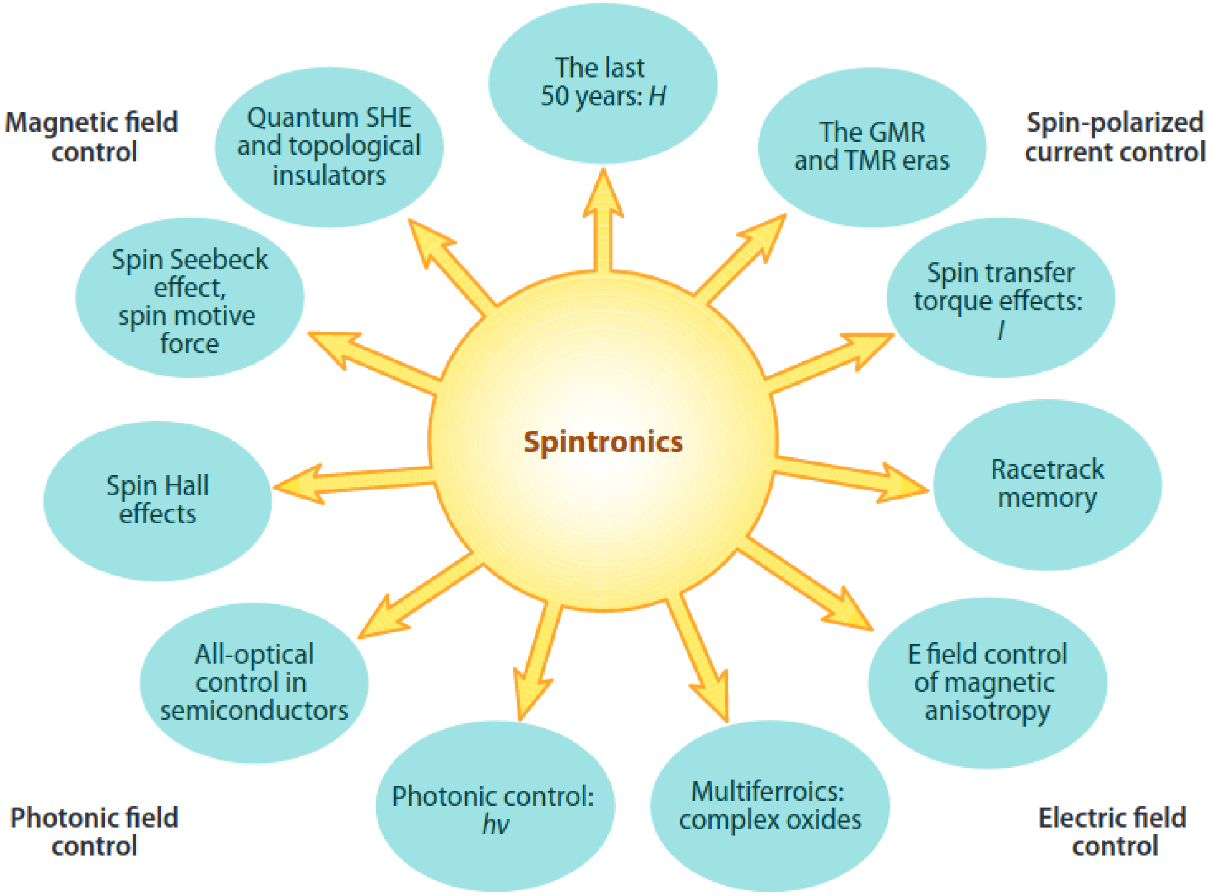
S.-W. Li, SL, to appear
SL, Tian, 2410.22935

Outline

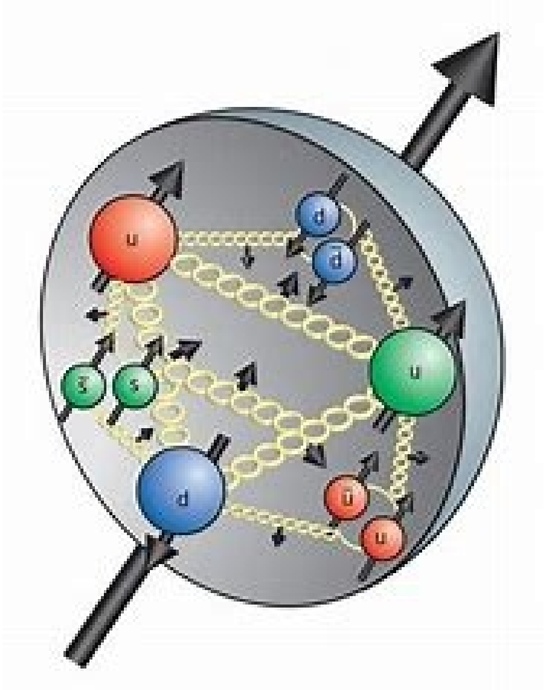
- Spin physics in different areas
- Spin in heavy ion collisions: polarization of Λ
- Uncertaity in local polarization of Λ
- Holographic model for spin polarization
- Fermionic spectral function in strongly coupled QGP
- Steady state effect on local polarization
- Conclusion and outlook

Spintronics in condensed matter physics



Bader+Parkin
ARCMP 2010

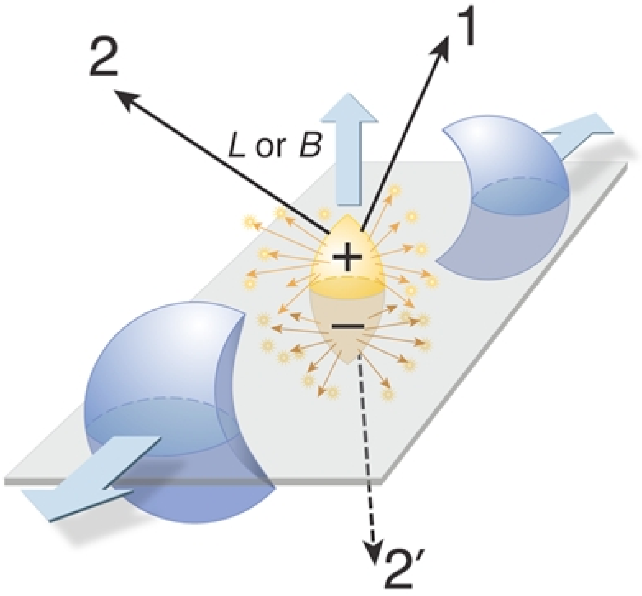
Spin in particle physics



Proton spin puzzle (1988-now)

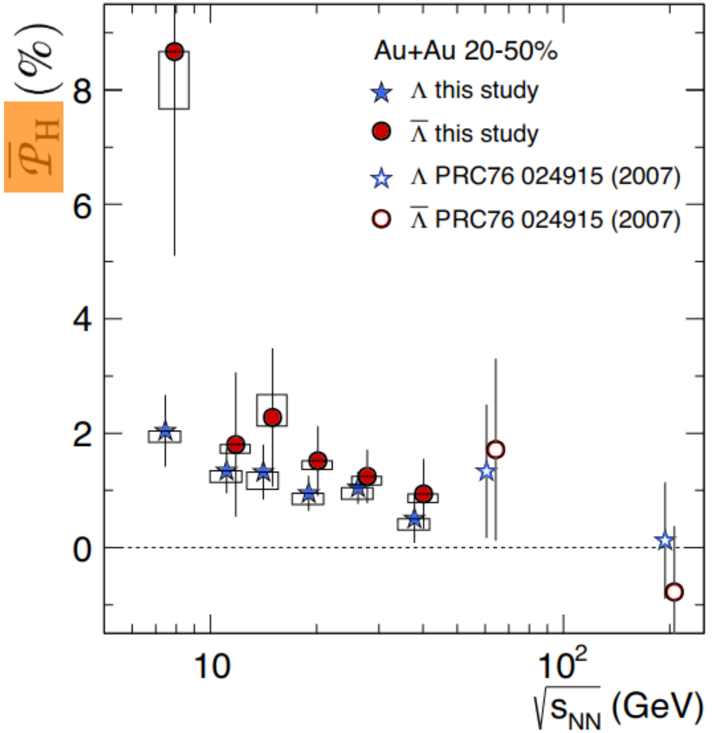
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

HIC: global polarization from vorticity



$$L_{ini} \sim 10^5 \hbar \rightarrow S_{final}$$

Liang, Wang, PRL 2005, PLB 2005

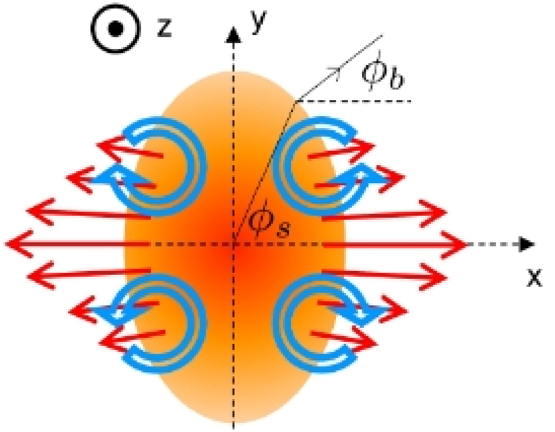


X. Sun's talk

STAR collaboration, Nature 2017

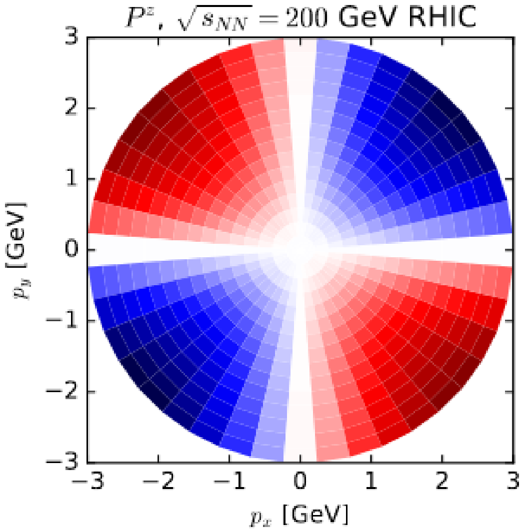
$$e^{-\beta(H_0 - \mathbf{S}\cdot\omega)}$$

HIC: local polarization from vorticity



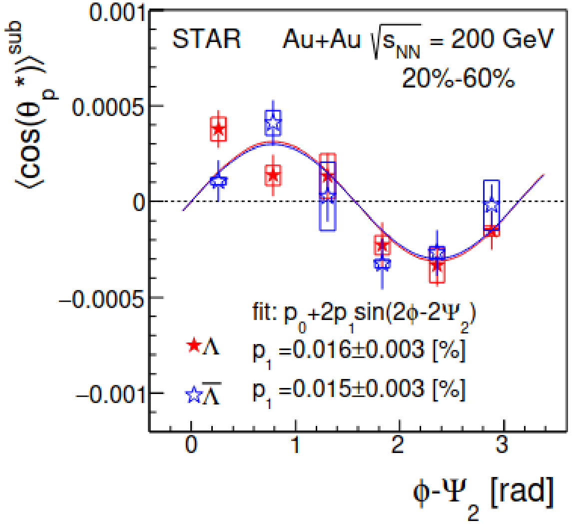
$$S^i \sim \omega^i$$

X. Sun's, Z. Chen's talks



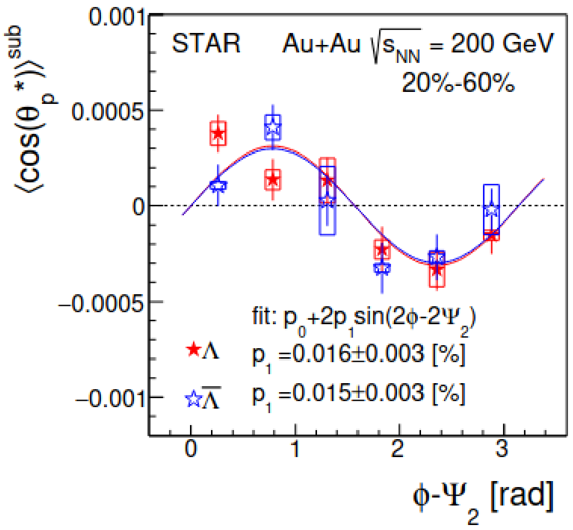
Becattini, Karpenko, PRL 2018
Wei, Deng, Huang, PRC 2019
Wu, Pang, Huang, Wang, PRR 2019
Fu, Xu, Huang, Song, PRC 2021

wrong sign



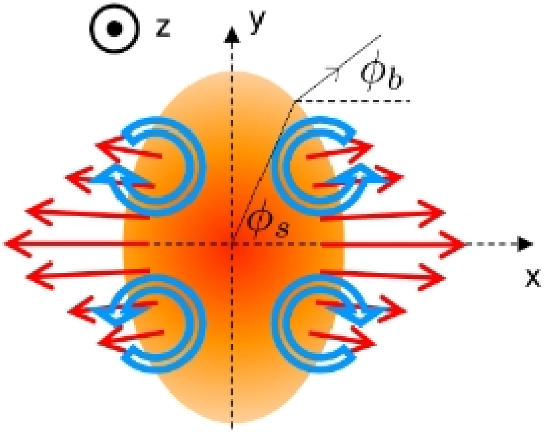
STAR collaboration, PRL 2019

HIC: local polarization from vorticity + shear



STAR collaboration, PRL 2019

vorticity



$$S^i \sim \omega^i$$

wrong sign

shear

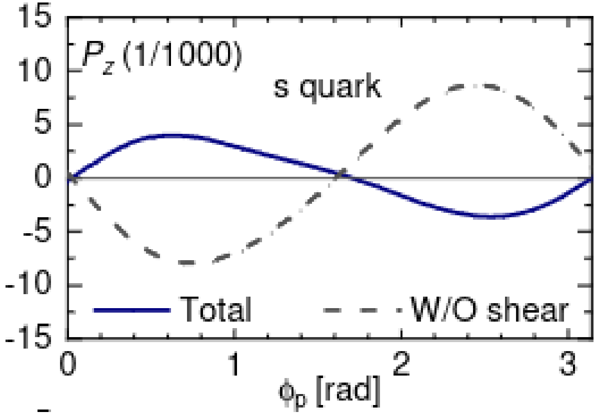
$$S^i \sim \epsilon^{ijk}\hat{p}_k\hat{p}_l\sigma_{jl}$$

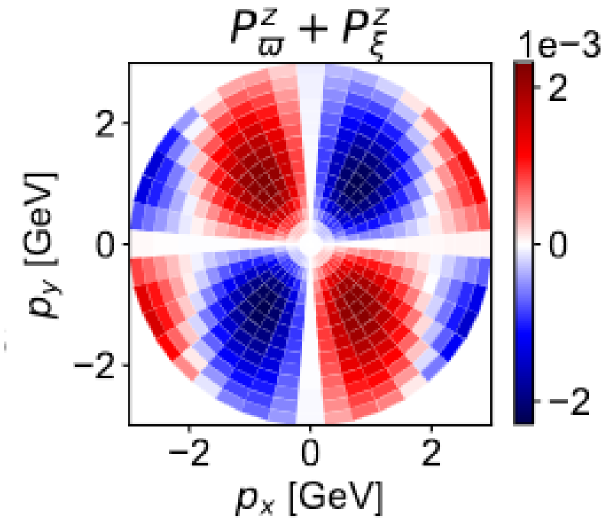
$$S^z \sim \left(\langle p_y^2\rangle - \langle p_x^2\rangle\right)\partial_y u_x$$

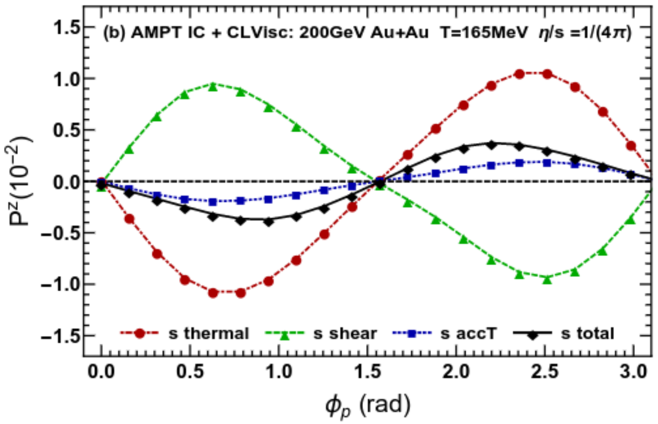
right sign

Hidaka, Pu, Yang, PRD 2018
Liu, Yin, JHEP 2021
Becattini, et al, PLB 2021

Uncertainty in spin response to shear







Fu, Liu, Pang, Song, Yin, PRL 2021
Becattini, et al, PRL 2021
Yi, Pu, Yang, PRC 2021

two scenarios
Λ: point particle
Λ: quark model

Z. T. Liang's, B.-Q. Ma's talks

$$S^\mu(p^\alpha) = \frac{1}{4m} \frac{\int d\Sigma \cdot p\, n_0(1-n_0)\mathcal{A}^\mu}{\int d\Sigma \cdot p\, n_0}$$

Lesson from quantum kinetic theory

free theory
point particle

$$S^i \sim \left(\omega^i + \epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{kl} + \frac{\partial_i T}{T}\right)\delta(P^2 - m^2)$$

- shear correction from collision (perturbative O(1))

SL, Wang, JHEP 2022, PRD 2025
Fang, Pu, Yang, PRD 2024
Fang, Pu, PRD 2025

- shear/vorticity correction to spectral function (perturbative $O(g^2)$, usually ignored)

SL, Tian, 2410.22935
Fang, Pu, Yang, 2503.13320

Y. Yin's talk

Complication for Λ polarization

- shear correction from collision (perturbative O(1))

SL, Wang, JHEP 2022, PRD 2025
Fang, Pu, Yang, PRD 2024
Fang, Pu, PRD 2025

- shear/vorticity correction to spectral function (perturbative O(g^2))

SL, Tian, 2410.22935
Fang, Pu, Yang, 2503.13320

correction to Λ structure model (non-perturbative)

Alternative approach? Holographic model

Holographic model for baryon

$$S = i\int d^{D+1}x\sqrt{-g}\bar{\psi}\left(\Gamma^M\nabla_M - m\right)\psi,$$

Iqbal, Liu 2009

5D Dirac fermion ⟷ 4D Weyl fermion

$$\psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$$

"lump of quark/gluon"

treat all interaction effects uniformly

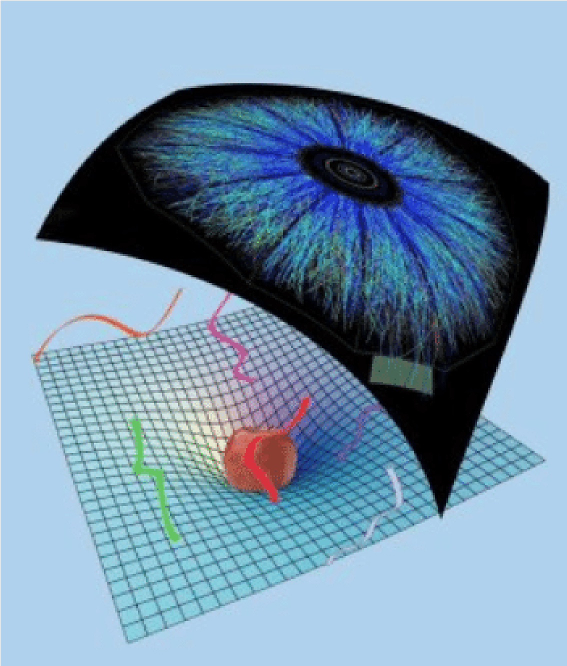
- Collisional correction (steady state effect)
- Spectral correction (structure effect)

focus of the talk

Holographic model for QGP

$$ds^2 = -2u_\mu(x)dx^\mu dr - r^2 f(b(x)r)u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu}dx^\mu dx^\nu$$ local equilibrium

$$+ 2r^2 bF(br)\sigma_{\mu\nu}dx^\mu dx^\nu + \frac{2}{3}ru_\mu u_\nu \partial_\lambda u^\lambda dx^\mu dx^\nu - ru^\lambda \partial_\lambda(u_\mu u_\nu)dx^\mu dx^\nu$$ steady state



Bhattacharyya et al, JHEP 2008

$$\qquad O(\partial^0) \qquad\qquad\qquad O(\partial)$$

$$T^{\mu\nu} = (\pi T)^4 (\eta^{\mu\nu} + 4u^\mu u^\nu) - 2(\pi T)^3 \sigma^{\mu\nu}$$

local equilibrium

η

steady state

σ_{ij} ~~$\partial_0 b = \frac{1}{3}\partial_i u_i$~~ $\partial_i b = \partial_0 u_i$

Spectral function generalities

$$G^R_{\alpha\beta}(\omega,\vec{p}) = i\int d^4x e^{ip\cdot x}\theta(x^0)\left\langle \Psi_\alpha(x)\Psi^\dagger_\beta(0) + \Psi^\dagger_\beta(0)\Psi_\alpha(x)\right\rangle$$

$$\rho_{\alpha\beta}(\omega,\vec{p}) = \int d^4x e^{ip\cdot x}\left\langle \Psi_\alpha(x)\Psi^\dagger_\beta(0) + \Psi^\dagger_\beta(0)\Psi_\alpha(x)\right\rangle$$

$$\langle\cdots\rangle = \mathrm{Tr}[D\cdots]$$

D: density matrix

D hermitian

$$\rho = 2\mathrm{Im}G^R$$

assume D T-even

$$\rho_{\alpha\beta}(\omega,\vec{p}) = (\sigma_2\,\rho(-\omega,\vec{p})\,\sigma_2)^*_{\alpha\beta}$$

Application: equilibrium spectral function

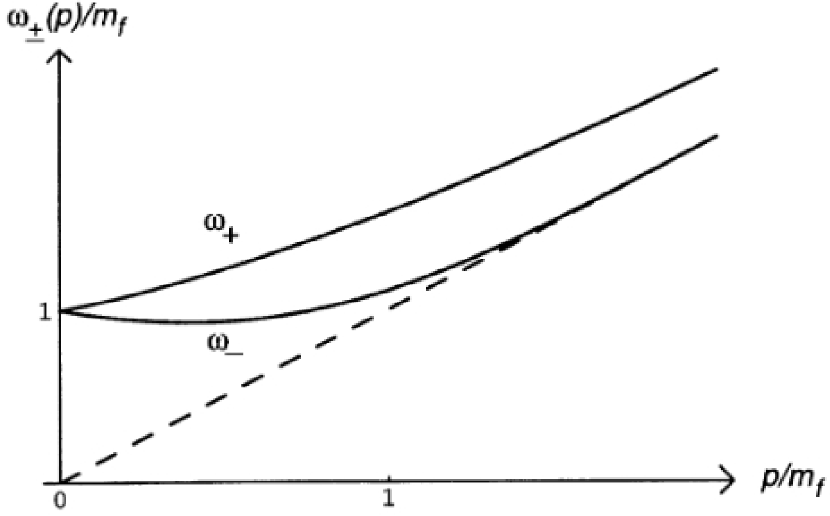
$$G_R(\omega, \vec{p}) = A(\omega, p) + B(\omega, p)\hat{p} \cdot \vec{\sigma}$$

$$\rho_{\alpha\beta}(\omega, \vec{p}) = (\sigma_2\, \rho(-\omega, \vec{p})\, \sigma_2)^*_{\alpha\beta} \quad \Longrightarrow \quad \begin{aligned} \mathrm{Im} A(\omega, \vec{p}) &= \mathrm{Im} A(-\omega, \vec{p}) \\ \mathrm{Im} B(\omega, \vec{p}) &= -\mathrm{Im} B(-\omega, \vec{p}) \end{aligned}$$

example: spectral function in HTL

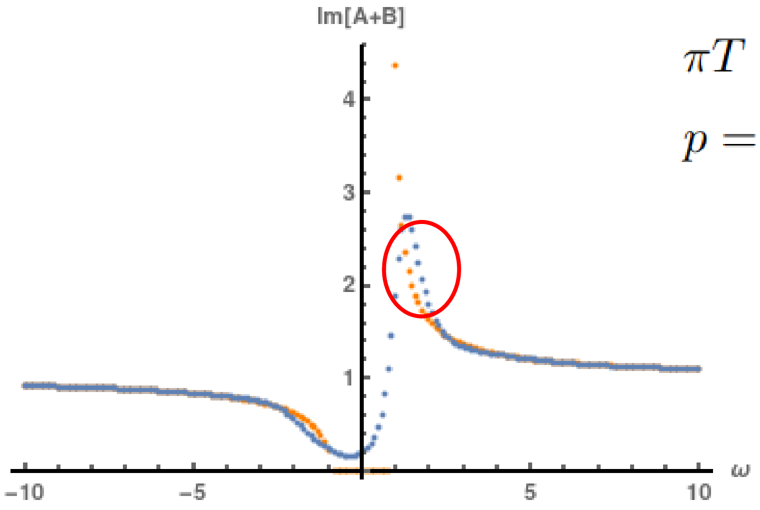
$$\rho = \rho_+ \frac{1 - \hat{p} \cdot \vec{\sigma}}{2} + \rho_- \frac{1 + \hat{p} \cdot \vec{\sigma}}{2}$$

$$\rho_\pm \sim Z_\pm \delta(\omega - \omega_\pm(p)) + Z_\mp \delta(\omega + \omega_\mp)$$



Holography: equilibrium spectral function

$$G_R(\omega,\vec{p}) = A(\omega,p) + B(\omega,p)\hat{p}\cdot\vec{\sigma}$$



$\pi T = 1$

$p = 0.9$

orange: vacuum

$\mathrm{Im}[A+B] \sim (\omega - p)^{-1/2}$ Iqbal, Liu 2009

no spacelike spectral

blue: equilibrium QGP
soften the singularity
develop spacelike spectral

focus on timelike spectral for baryon

Off-equilibrium: gradient correction

baryon as a probe to QGP

$\omega, p \gg \partial_i T, \partial_i u_j$ Wigner transform

$$G^R_{\alpha\beta}(\omega, \vec{p}) = i \int d^4x e^{ip\cdot x} \theta\left(x^0\right) \left\langle \Psi_\alpha(x) \Psi^\dagger_\beta(0) + \Psi^\dagger_\beta(0) \Psi_\alpha(x) \right\rangle$$

solve bulk Dirac equation in gradient expansion

$$\left(\Gamma^M \nabla_M - m\right) \psi = 0$$

Gradient corrections

$$ds^2 = -2u_\mu(x)dx^\mu dr - r^2 f(b(x)r)u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu}dx^\mu dx^\nu$$ local equilibrium

$$+ 2r^2 bF(br)\sigma_{\mu\nu}dx^\mu dx^\nu + \frac{2}{3}\cancel{r u_\mu u_\nu \partial_\lambda u^\lambda dx^\mu dx^\nu} - r u^\lambda \partial_\lambda(u_\mu u_\nu)dx^\mu dx^\nu$$ steady state

$$\left(\Gamma^M \nabla_M - m\right)\psi = 0$$

steady state of QGP → polarization from shear, T-grad, no vorticity

$$\left(\Gamma^M \nabla_M - m\right)\psi = 0$$

local equilibrium QGP → polarization from vorticity, shear, T-grad

strong vs weak coupling

$$\delta G^R = D_1 \partial_i b\, \sigma_i + iD_2 \epsilon^{ijk} \hat{p}_i \partial_j b\, \sigma_k$$

$$\delta G^R = E_1 \omega_i \sigma_i + iE_2 \epsilon^{ijk} \hat{p}_i \omega_j \sigma_k \qquad b = \frac{1}{\pi T}$$

$$\delta G^R = F_1 \hat{p}_j \sigma_{ij} \sigma_i + iF_2 \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{kl} \sigma_i$$

strong coupling $D_{1,2},\ E_{1,2},\ F_{1,2} \sim O(\lambda^0)$

weak coupling $D_2,\ E_1\ F_2 \sim O(g^2)$ no $D_1,\ E_2,\ F_1$

$$\frac{g^2 C_F}{2(2\pi)^2} \frac{\pi}{2p} \gamma^5 \gamma_i \left[0.95\omega_{\parallel}^i + 1.48\omega_{\perp}^i - 0.52\epsilon^{ijk}\hat{p}_j \hat{p}_l \sigma_{kl} - 0.02\epsilon^{ijk}\hat{p}_j \frac{\partial_k \beta}{\beta}\right]$$

SL, Tian, 2410.22935

radiative correction can't access steady state

T-symmetry

$$\delta G^R = D_1 \partial_i b\, \sigma_i + i D_2 \epsilon^{ijk} \hat{p}_i \partial_j b\, \sigma_k$$

$$\delta G^R = E_1 \omega_i \sigma_i + i E_2 \epsilon^{ijk} \hat{p}_i \omega_j \sigma_k$$

$$b = \frac{1}{\pi T}$$

$$\delta G^R = F_1 \hat{p}_j \sigma_{ij} \sigma_i + i F_2 \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{kl} \sigma_i$$

$$\rho_{\alpha\beta}(\omega, \vec{p}) = (\sigma_2\, \rho(-\omega, \vec{p})\, \sigma_2)^*_{\alpha\beta}$$

$$\delta\langle \cdots \rangle = \mathrm{Tr}[\delta D \cdots]$$

green: consistent with T-symmetry
purple: inconsistent with T-symmetry

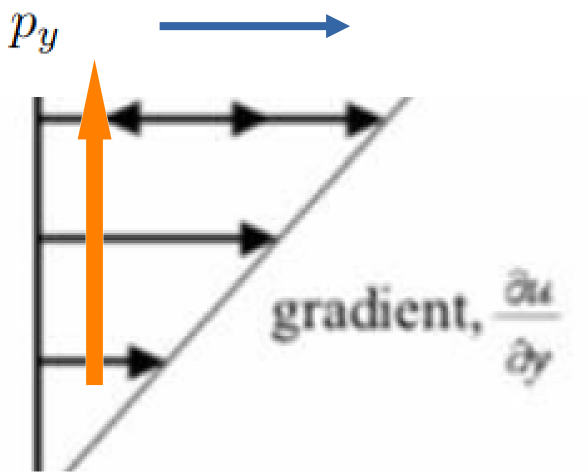
T-even part of δD
T-odd part of δD

Steady state effect: shear example

$$\delta G^R = F_1 \hat{p}_j \sigma_{ij} \sigma_i + i F_2 \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{kl} \sigma_i$$

steady state effect of baryon, not contribute to polarization

acceleration balanced by collision



$$\Delta p_x \to S_x$$

contribute to polarization of Weyl fermion, cancels in polarization of baryon (axial)

baryon = R-Weyls + L-Weyl

do modify vector component of spectral function!

Expected contribution at $O(\partial^2)$

Expect mixed contributions to polarization

$$S^i \sim \quad \epsilon^{ijk}\partial_j b\hat{p}_l\sigma_{kl} \qquad \Delta p_k$$

steady state effect

$$\begin{aligned}\mathrm{tr}\left[\gamma^\mu\gamma^5 W_{\mathrm{quad}}(x,p)\right] \; = \; & \frac{\delta(p^2-m^2)\mathrm{sgn}(p^0)}{(2\pi)^3(p^0)^2} n_F(p)[1-n_F(p)][1-2n_F(p)](y^0_\Sigma - x^0) \\ & \times p^{\lambda'}\partial_{\lambda'}\left[p^{\tau'}\beta_{\tau'}(x)-\zeta(x)\right] \\ & \times\left\{2\epsilon^{\mu\nu\rho\lambda}p_\nu\hat{t}_\rho\partial_\lambda\left[p^\tau\beta_\tau(x)-\zeta(x)\right]+(p^\mu p_\tau - g^\mu_\tau m^2)\hat{t}_\rho\epsilon^{\rho\nu\lambda\tau}\Omega_{\nu\lambda}(x)\right\}\end{aligned}$$

to be added to

$$\xi_{ij} \sim \sigma_{ij} \qquad \epsilon^{ijk}\varpi_{jk} \sim \omega_i$$

Sheng, Becattini, Huang, Zhang, PRC 2024

Conclusion

- Uncertainty in local polarization related to Λ structure
- Holographic model for probe baryon in strongly coupled QGP
- Gardient corrections to baryon spectral function: local equilibrium + steady state contribution
- Expect mixed contribution to polarization at second order

Outlook

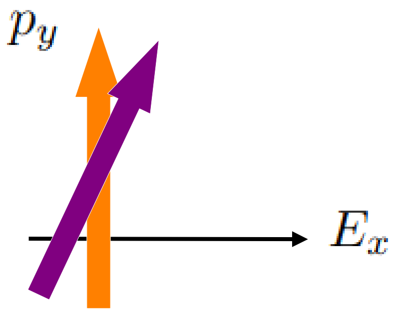
- Schwinger-Keldysh extended holographic model for complete gradient correction to polarization

Thank you!

Spin shear coupling & Spin Hall effect

$$\mathcal{P}^i \sim q_f \epsilon^{ijk} \hat{p}_j E_l$$

steady state $\quad j_x = \sigma E_x$



$$\mathcal{P}^i \sim \epsilon^{ijk} \hat{p}_j \hat{p}_l \sigma_{kl}$$

steady state $\quad T_{xy} = \eta \sigma_{xy}$