UNIVERSITÄT BONN

Matthias Schott

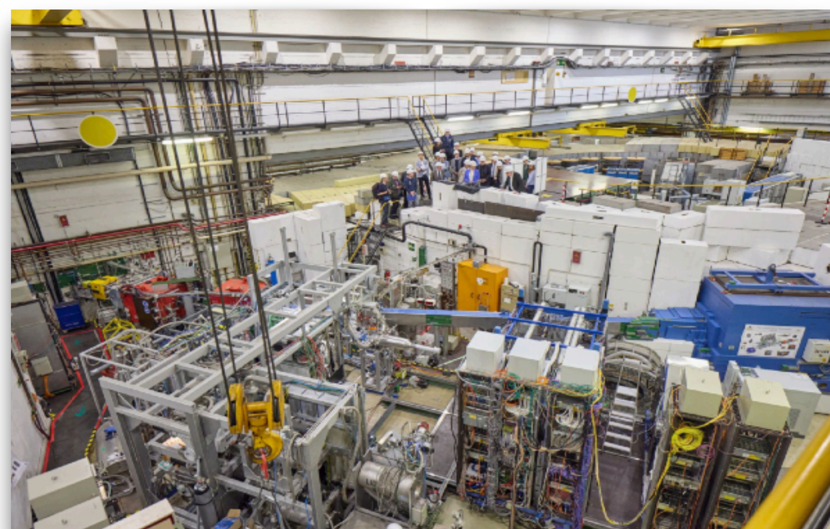
Bread and Butter Physics

City of Bonn

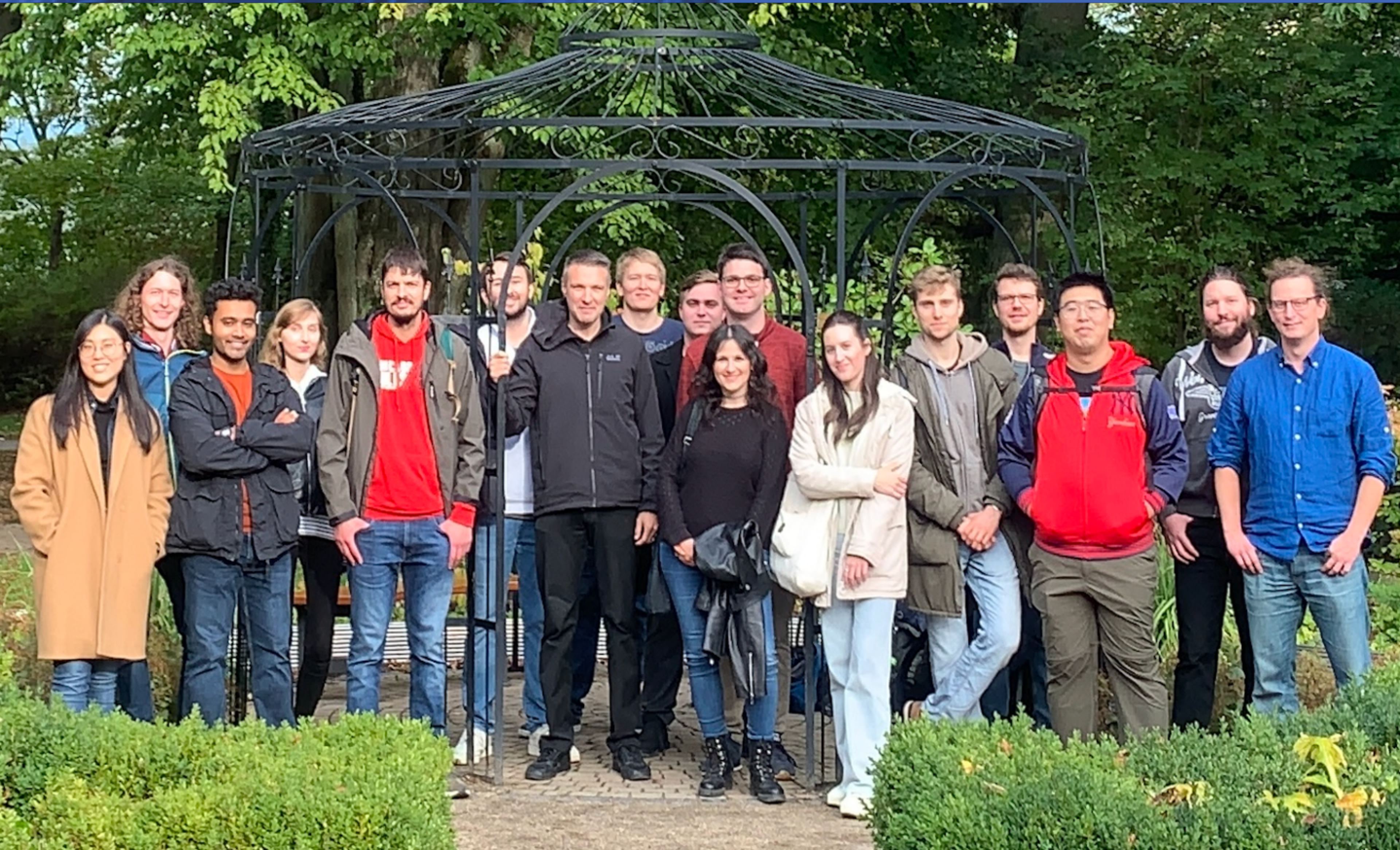
- Founded by the Romans 11 B.C. next to the Rhine River
- Middle ages: important religious centre
- Conquered by Napoleon in 1794
  - Afterwards part of Prussia
- Most famous citizen
  - Ludwig van Beethoven (1770*)
- Capital of Germany from 1949 to 1991

University of Bonn

- 1818: Foundation of the University
  - by King Friedrich Wilhelm III.
  - as partner university to Berlin
- Some Figures
  - 33.000 students
  - 6.000 PhD students
  - >600 professors in nearly all subjects
- Most successful excellence university within Germany
  - 5 Nobel-Prizes
  - 3 Field-medal winners
- … and our own accelerator



My Research Group in Bonn



What could we talk about today?

Searching for New Physics with Loops

- W-Boson Properties
- Light-by-Light Scattering
- Strong Coupling Constant
- (g-2) of the tau

Searches for Axions

- Axions at the LHC
- Light Through Wall
- Helioscope
- Cavity-based Searches

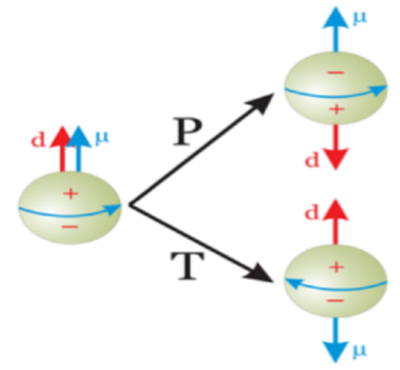
Matthias Schott

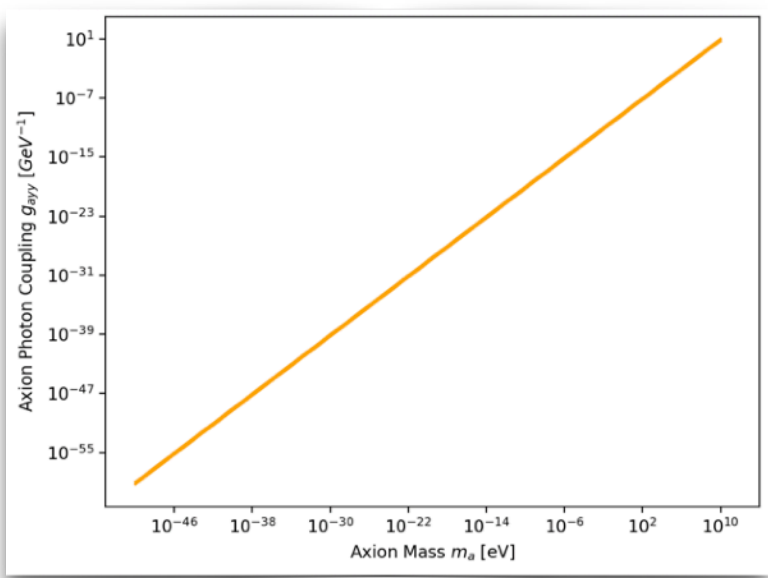
Search for Axion-Like Particles

Or why we did not find axion-like particles at the LHC, but might discover gravitational waves instead

Why Axions?

- Neutron Electric Dipole Moment
  - violates P and T symmetry
  - If CPT conserved, it violates CP
- Axions from Strong CP problem
  - Expected nEDM: ~10^{-18} e cm.
  - Exp. bound is a trillion times smaller
- Peccei-Quinn solution
  - global anomalous $U(1)_{PQ}$ symmetry
  - spontaneously broken
  - Axion is pseudo-Nambu-Goldstone boson
  - Predicted relation between mass and coupling





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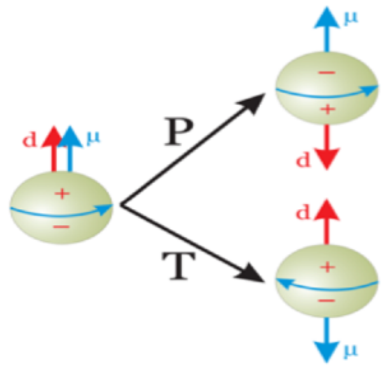

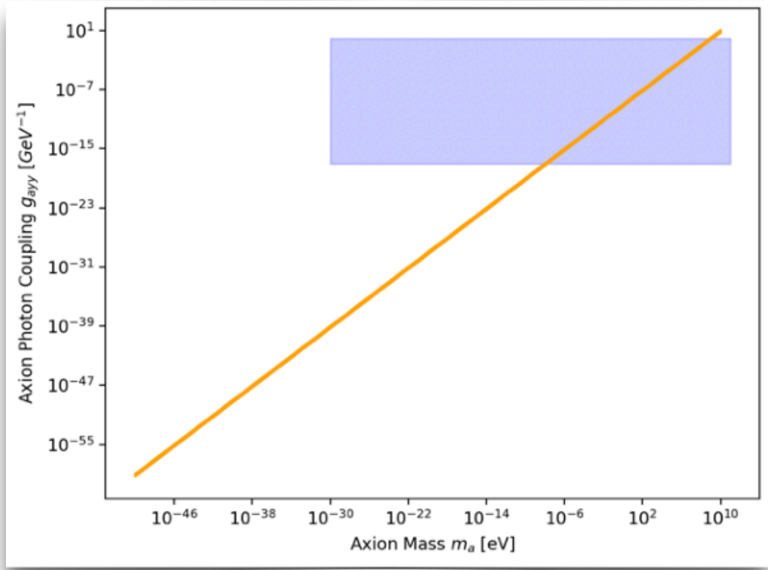

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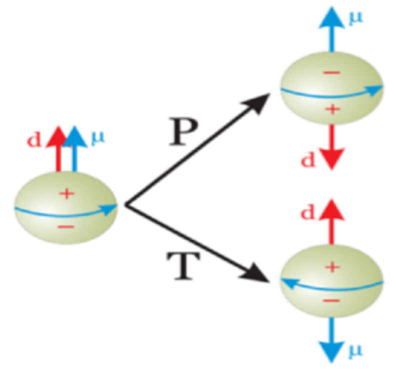
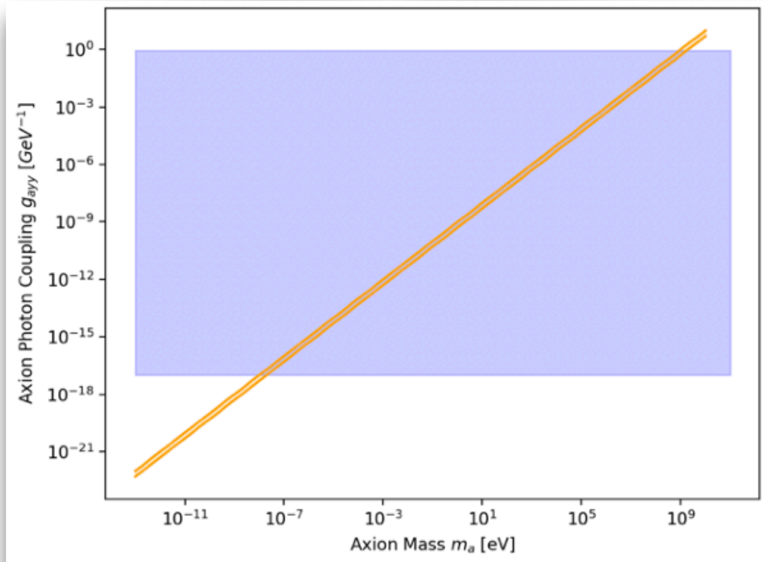
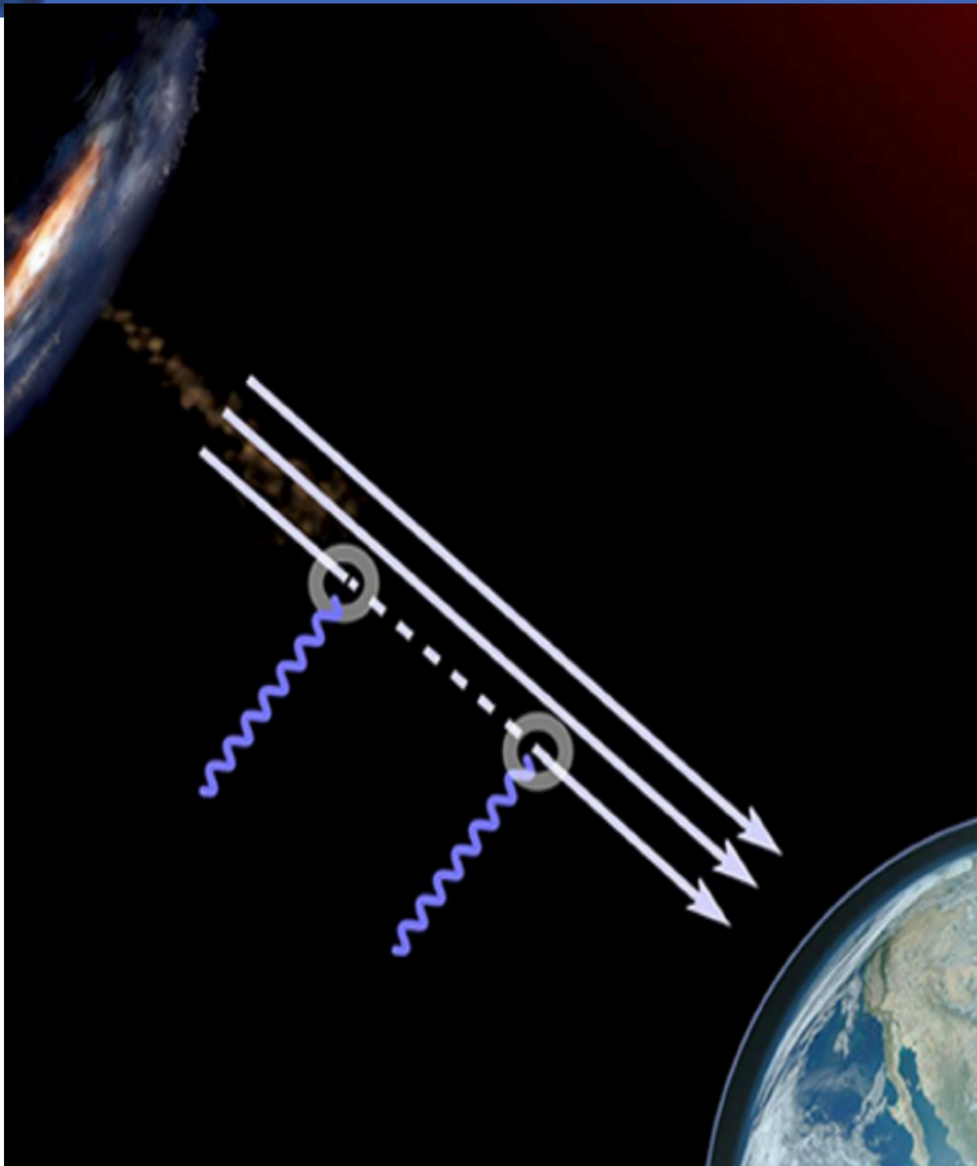




Why Axion-Like Particles?

- „No“ new physics model that does not have a gold-stone boson
  - e.g. pion in QCD
  - More general class of axion-like particles (ALPs)
    - coupling&mass are independent
- Many decay modes possible
  - This talk only covers photon decay modes
  - QCD Axion has two-photon vertex (Due to mixing with π^0)
- For large enough PQ symmetry breaking scale, the axion may be the main constituent of DM



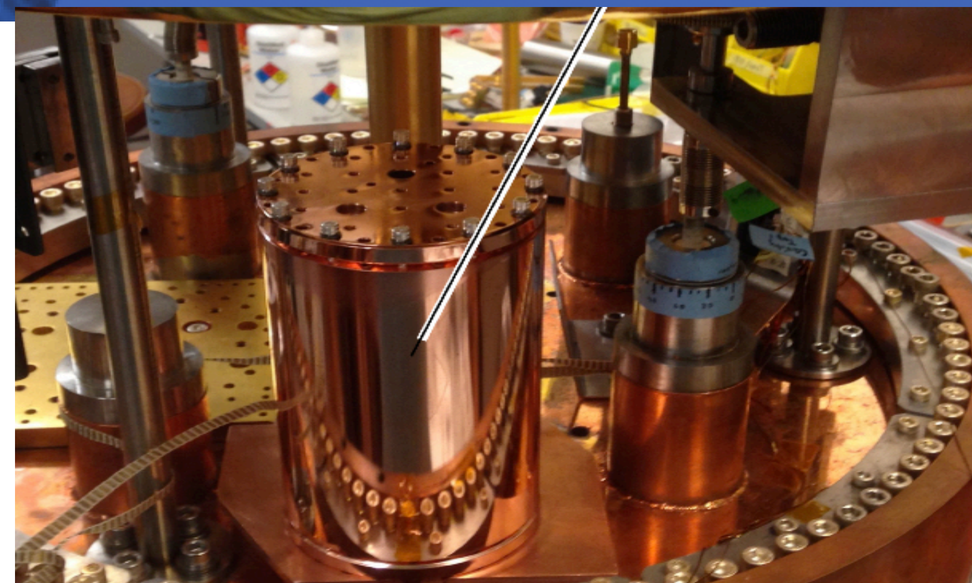
Overview of Searches for ALPS

- Light Through Wall (LWS) Type Experiments
  - Model-independence: yes
  - Couplings: yes
  - Mass: no / maybe
  - QCD-Axion: no

- Helioscopes: Look at the sun
  - Model-independence: a bit
  - Couplings: no
  - Mass: no / maybe
  - QCD-Axion: yes

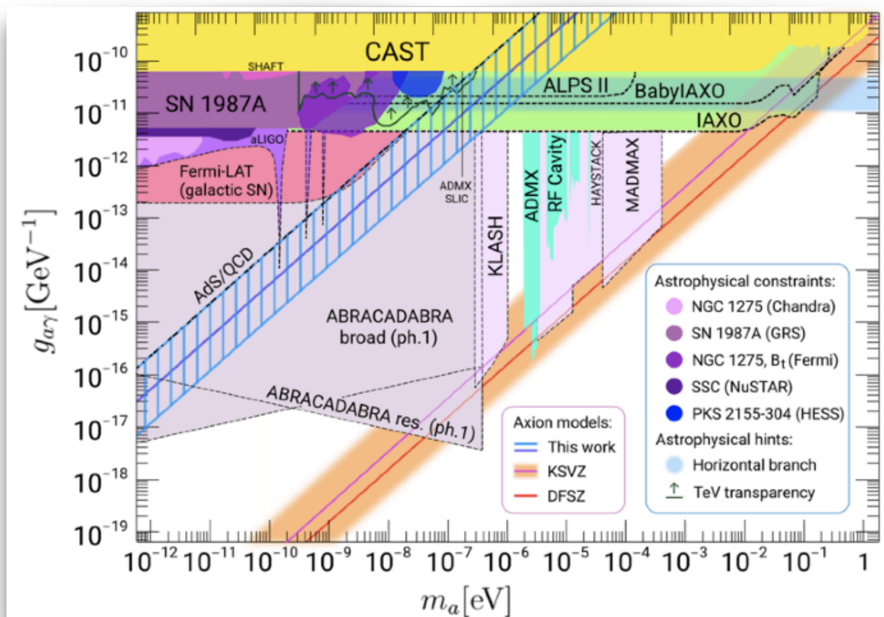
Overview of Searches for ALPS

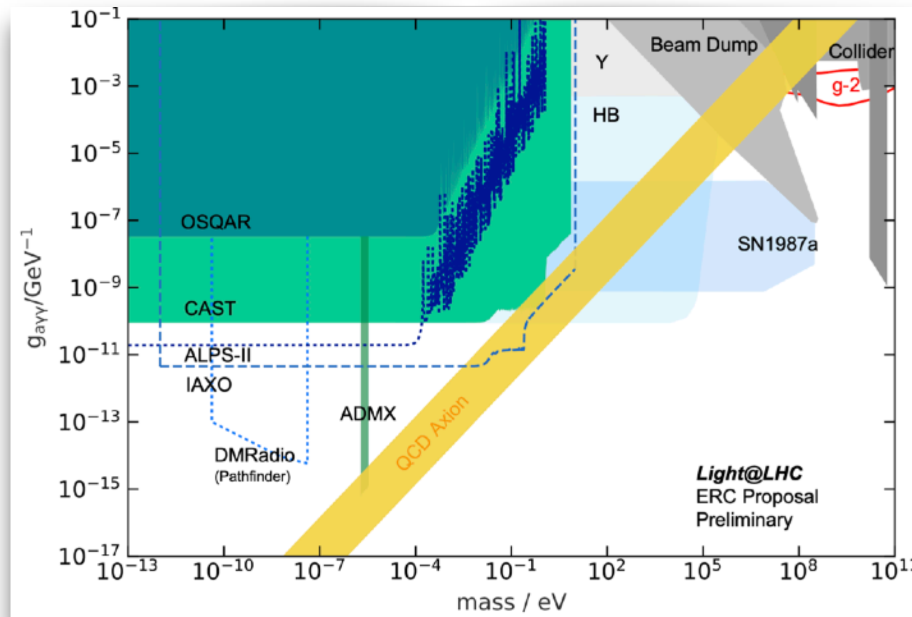
- Dark Matter Searches
  - Model-independence: no
  - Couplings: no
  - Mass: yes
  - QCD-Axion: yes



- Collider Based Searches
  - Model-independence: depends
  - Couplings: depends
  - Mass: yes
  - QCD-Axion: no

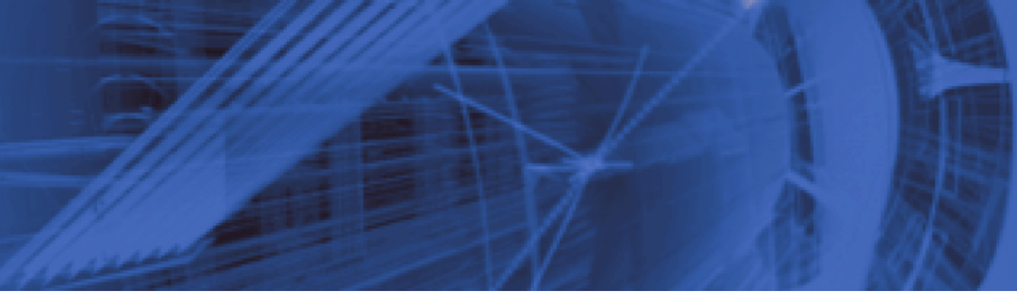
Be Careful when looking at Exclusion Plots!





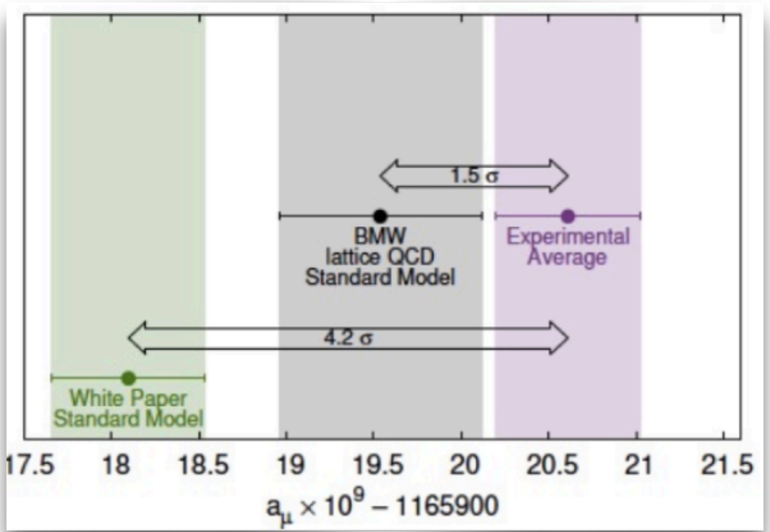
- Note: Not all future experiments are shown!
- Only few experiments can probe only some very small regions of the QCD-Axion
  - … and those are strongly model dependent.
  - … people only zoom into the regions where they are sensitive!
  - … it is a logarithmic plot!

Axions at Colliders



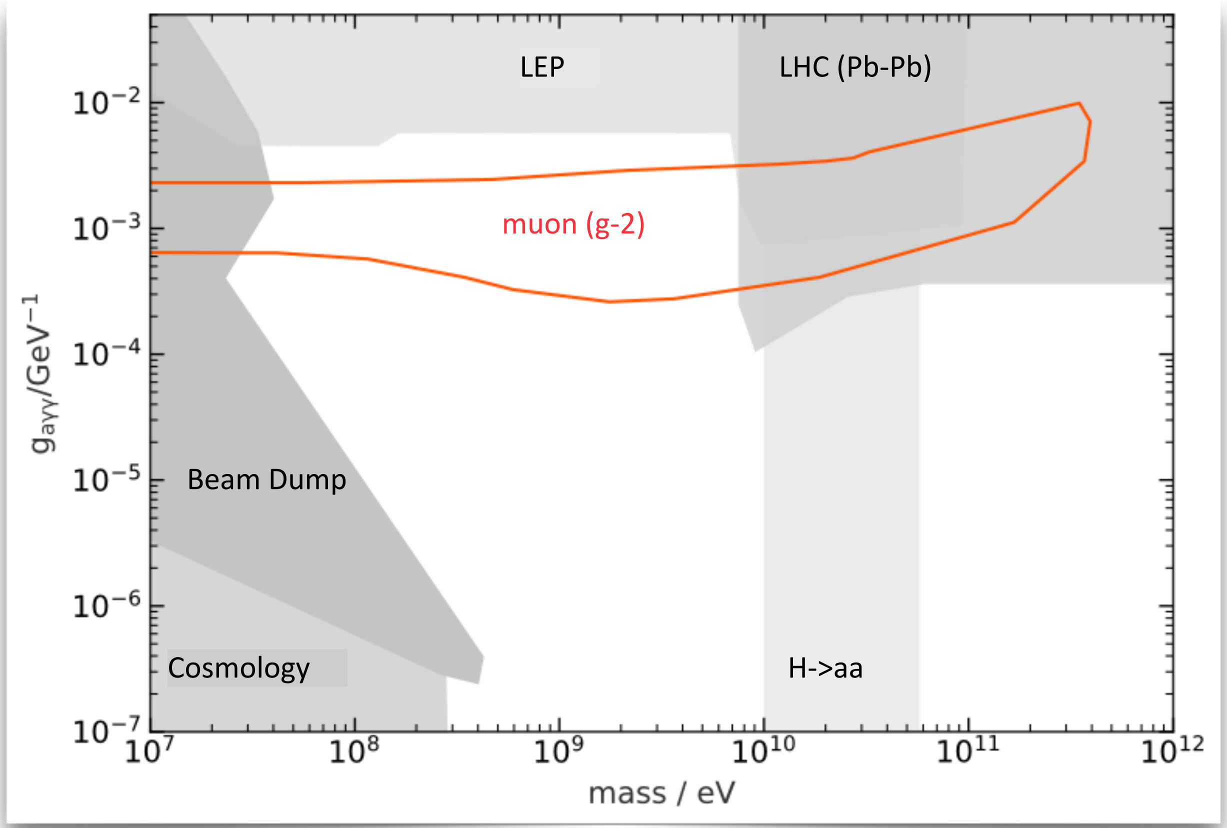
Axions at the LHC

- Many searches for light scalars ongoing, but most of them not really well motivated
- Higgs Portal could connect high energy physics with low energy phenomena
  - M. Bauer, M. Neubert, A. Thamm, Collider Probes of Axion-Like Particles
  - arXiv: 1708.00443v2
- Axion models that could explain the muonic (g-2) anomaly



Axions at the LHC

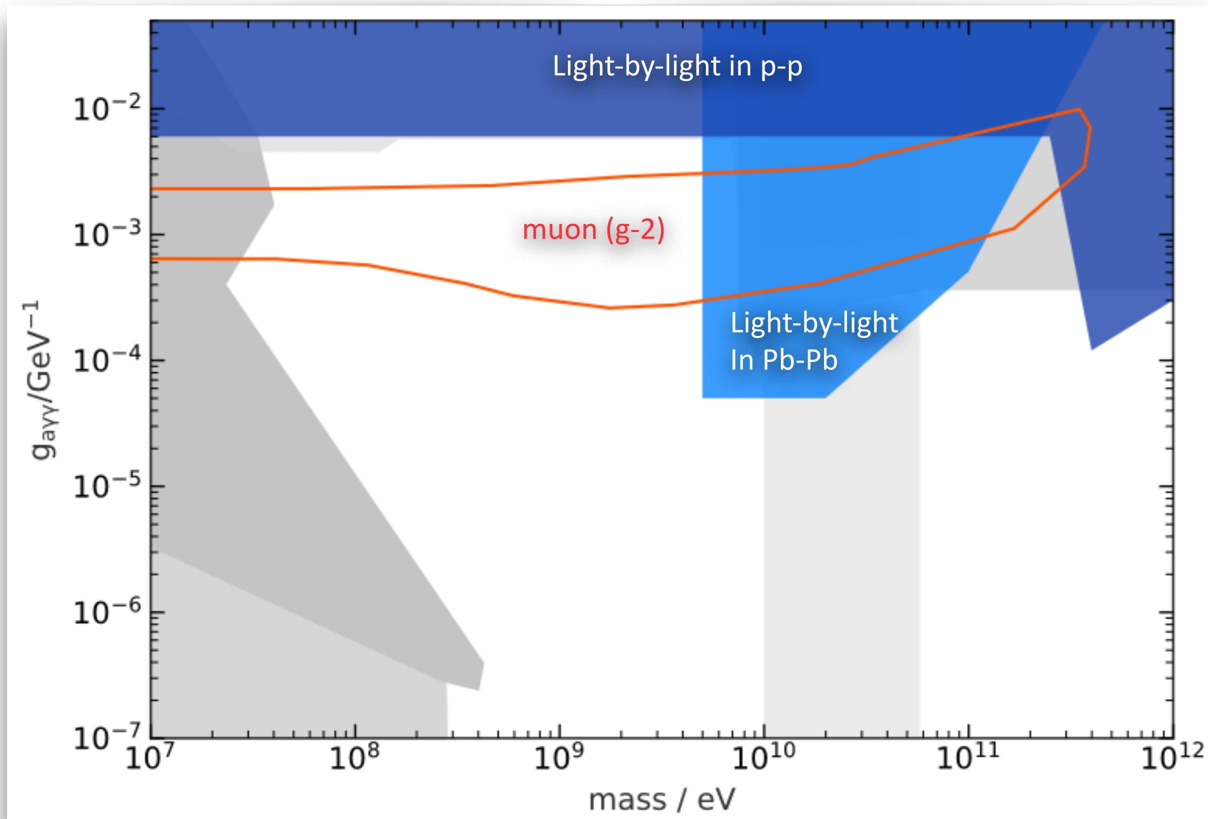
- Search for axion-like particles with masses from 10 MeV to 1 TeV using colliders



Axions at the LHC

- Search for axion-like particles with masses from 10 MeV to 1 TeV using colliders

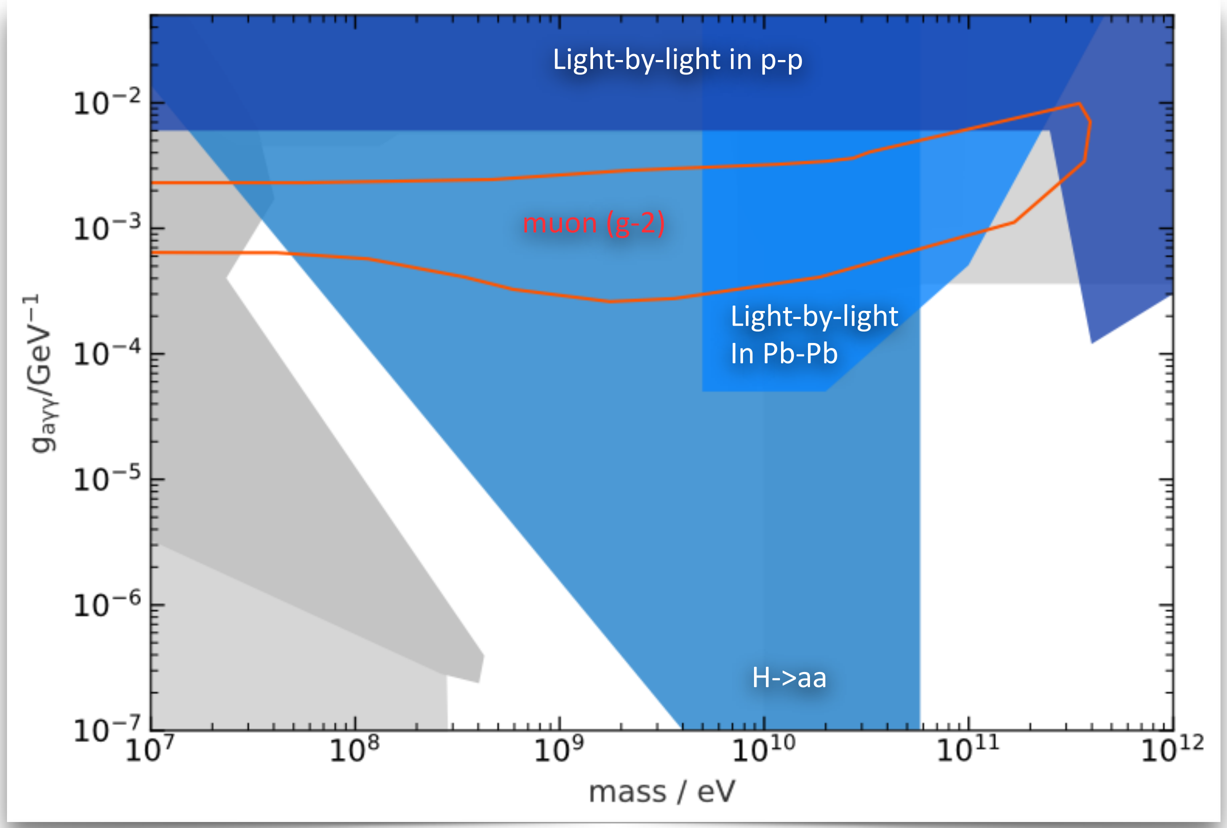
- 5 GeV < m_A < 1 TeV:
    - Light-by-light scattering



Axions at the LHC

- Search for axion-like particles with masses from 10 MeV to 1 TeV using colliders

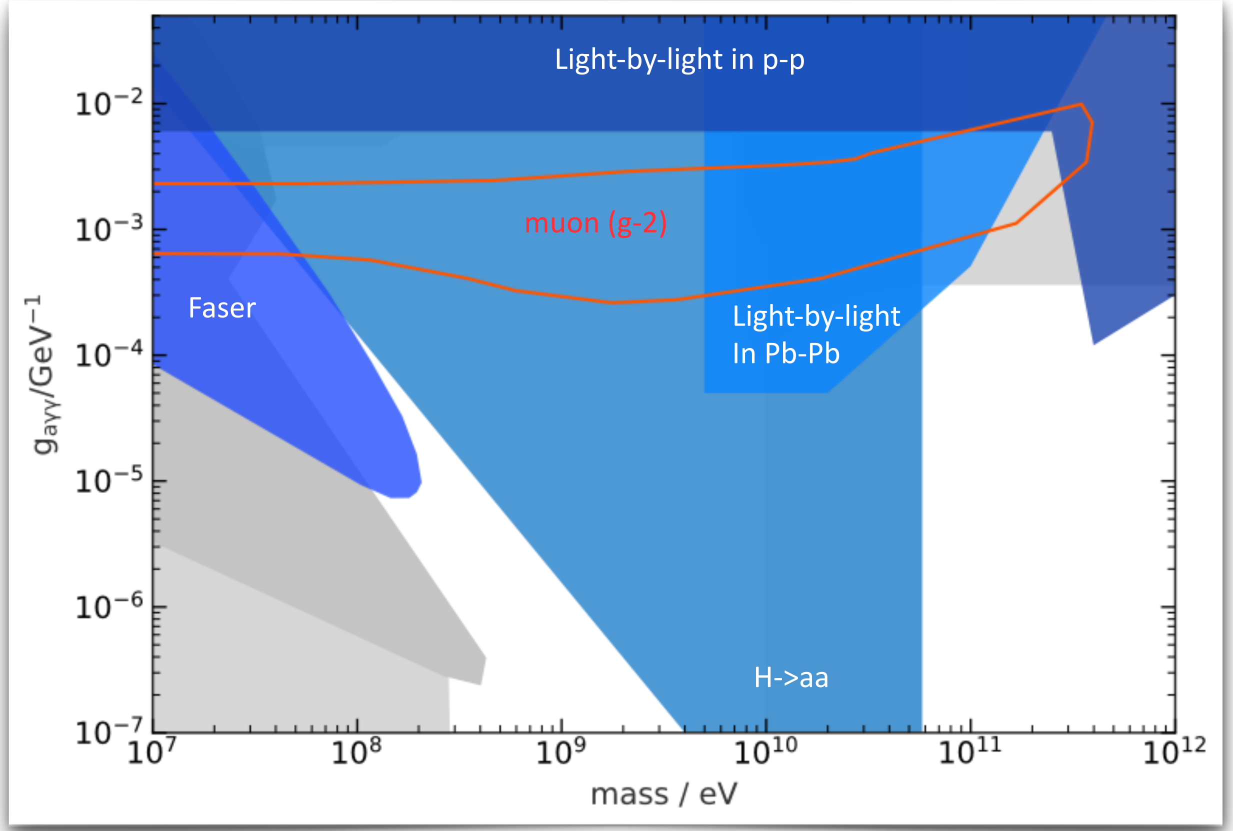
- 5 GeV < m_A < 1 TeV:
  - Light-by-light scattering
- 50 MeV < m_A < 62 GeV:
  - Anomalous Higgs boson decays into four photons



Axions at the LHC

- Search for axion-like particles with masses from 10 MeV to 1 TeV using colliders

- 5 GeV < m_A < 1 TeV:
  - Light-by-light scattering
- 50 MeV < m_A < 62 GeV:
  - Anomalous Higgs boson decays into four photons
- 10 MeV < m_A < 400 MeV
  - Search for ALPs at the FASER experiment



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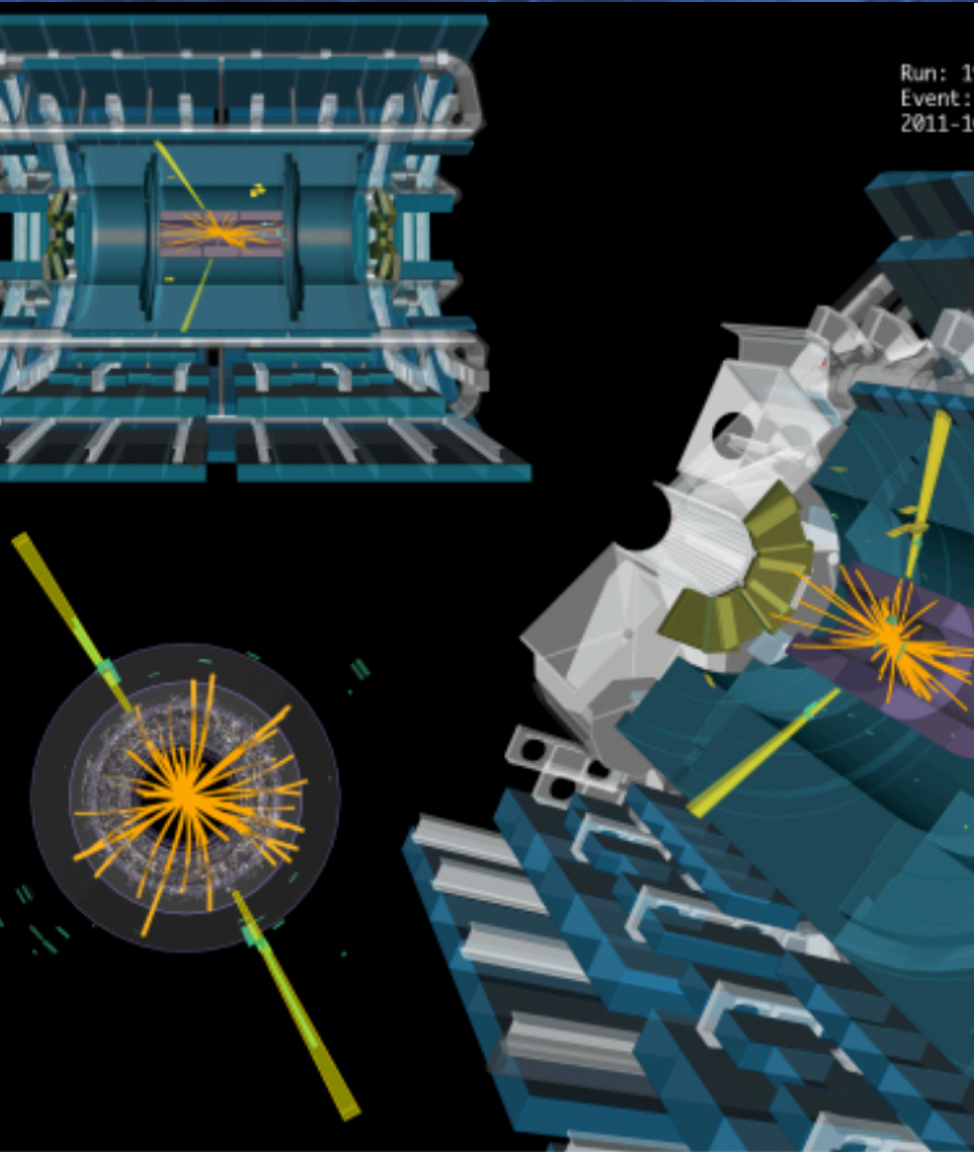
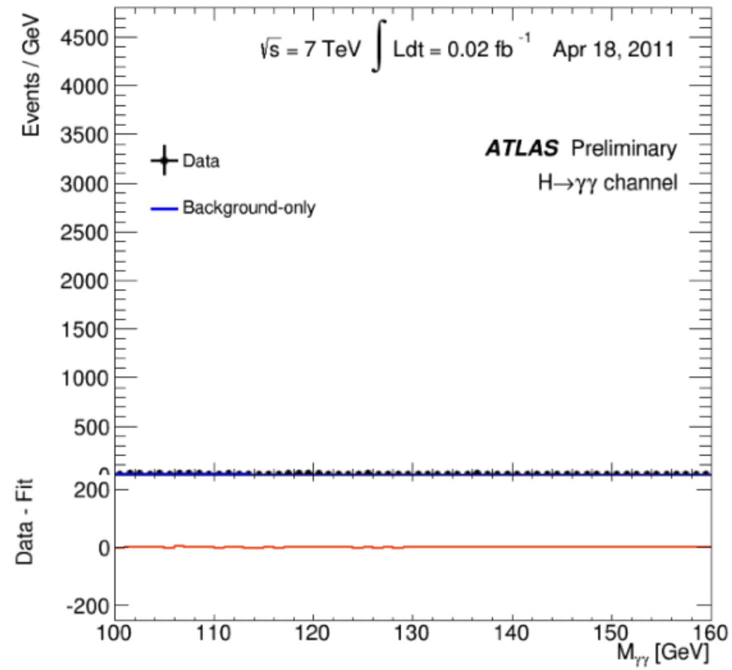
Light by Light Scattering

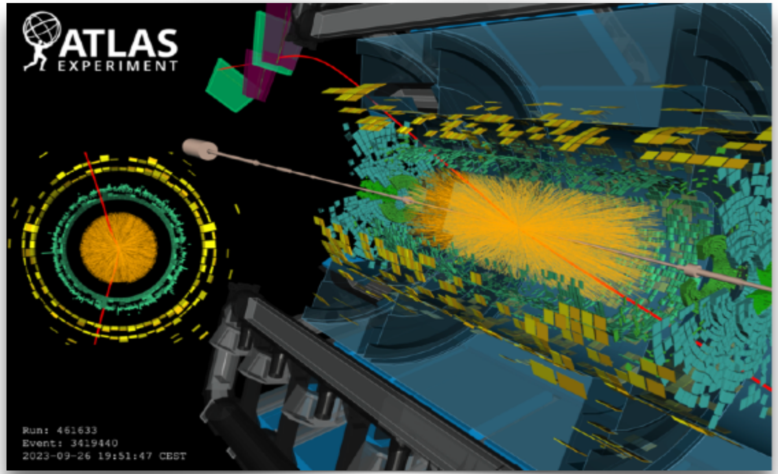
Typical Di-Photon Events in ATLAS and CMS

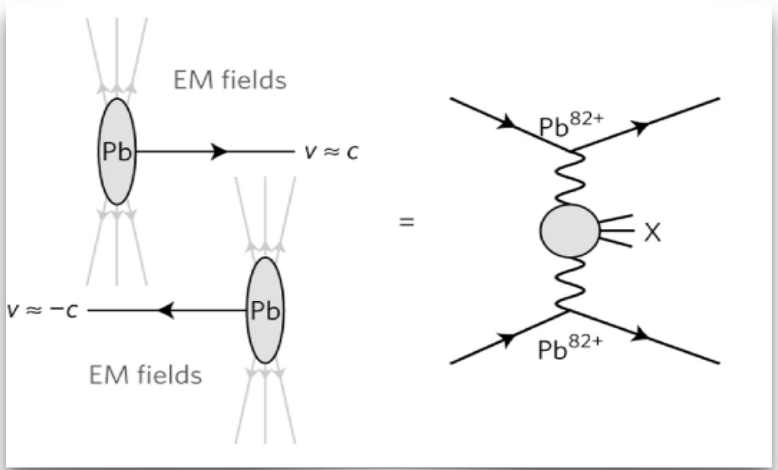




Electromagnetic interactions in p+p and Pb+Pb collisions

- Typical Heavy Ion Collisions are a huge mess
- Ion and proton beams with relativistic energies generate large EM-fields
- photon-induced reactions
  - Pb ions/protons escape into the beam pipe without remnants in the ATLAS detector
- in “ultra-peripheral collisions”: impact parameter is large
  - → suppress strong interactions

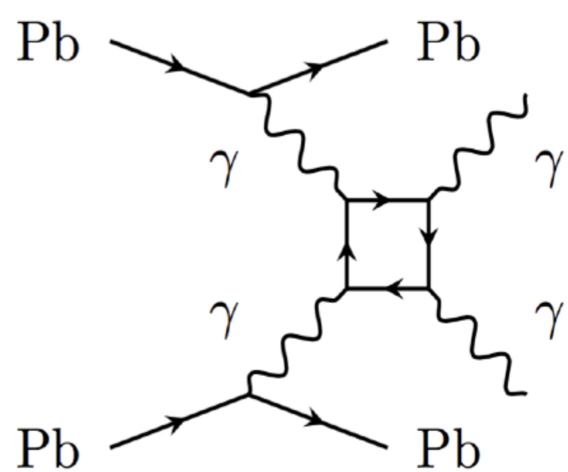




[Fermi, Nuovo Cim. 2 (1925) 143]
[Weizsacker, Z. Phys. 88 (1934) 612]
[Williams, Phys. Rev. 45 (10 1934) 729]

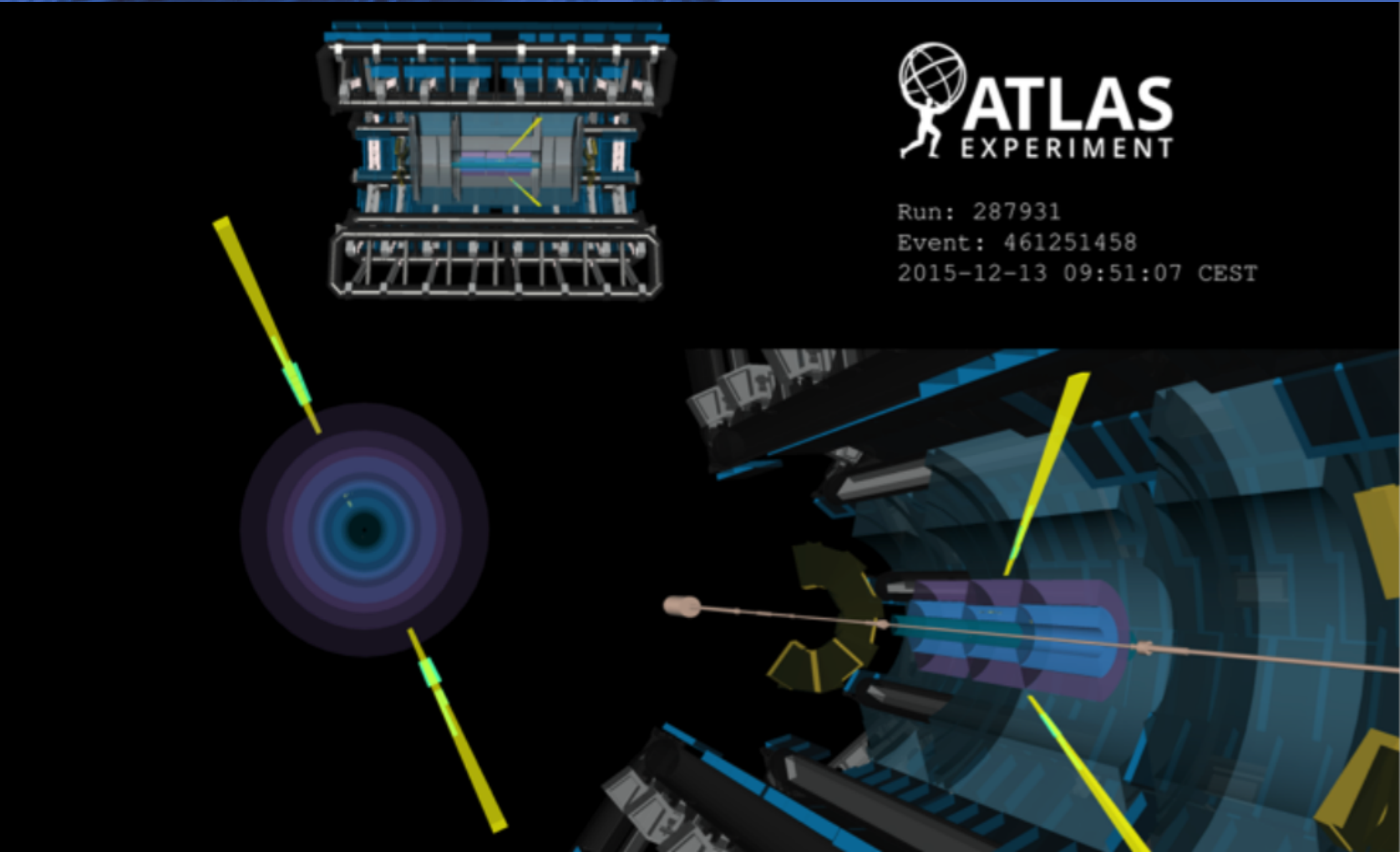
Search for Light-by-Light Scattering

- Light-by-light scattering in ultra-peripheral Pb+Pb collisions
  - arxiv:1702.01625
  - arxiv:2008.05355
- Idea based of this measurement based on [D. d'Enterria et al. PRL 111 (2013) 080405]
  - Follow up in [A. Szczurek et al. PRC 93 (2016) 4, 044907]
- What do we expect in the detector?



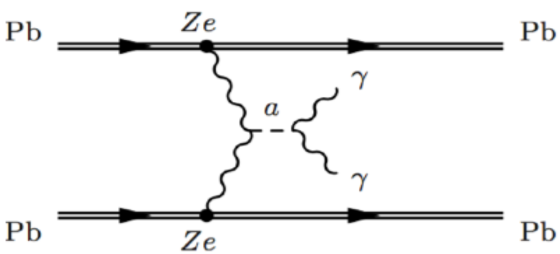
- Two photons and nothing else in the detector
  - $E_T > 3$ GeV and $|\eta| < 2.37$
  - $m_{\gamma\gamma} > 6$ GeV, $p_{T,\gamma\gamma} < 2$ GeV
- The Pb-ions would be scattered under a very small angle
- Veto event if it has charged tracks with hit in pixel
- Back-to-back photons
  - Acoplanarity = $1 - \Delta\phi / \pi < 0.01$ (reduces CEP background)

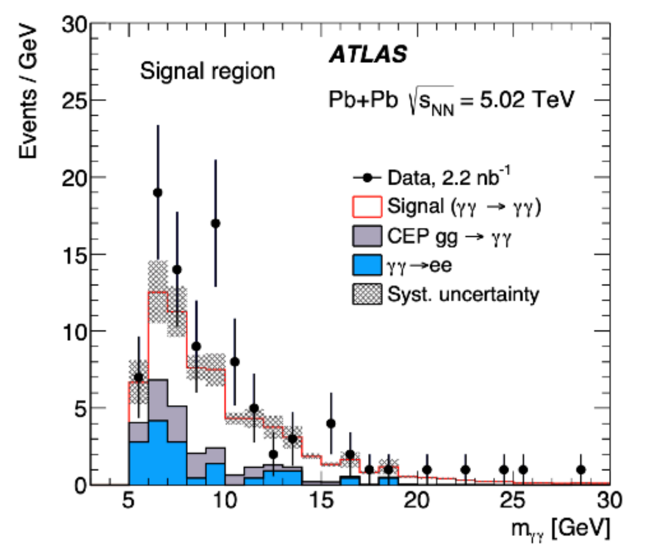
Light-by-Light Scattering Candidate

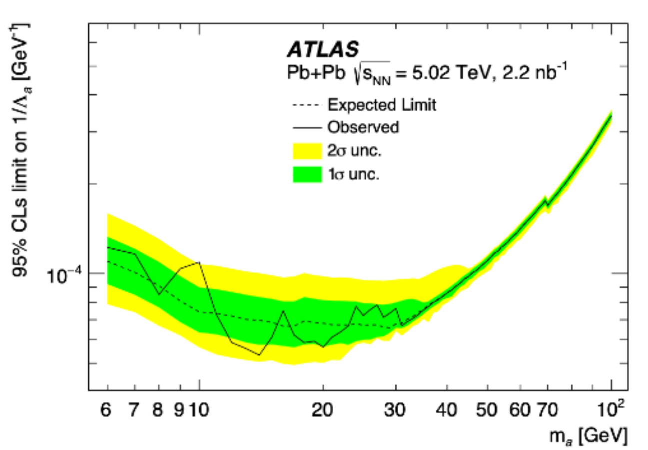


Observation of Light-by-Light Scattering

- 97 selected candidate events
  - a signal expectation of 45
  - a background expectation of 27 events
- x-sec measured in fiducial region
  - σ_{fid} = 120±17(stat.)±13(syst.)±4(lumi.)
  - σ_{SM} = 80 ± 8 nb
- Light-by-light scattering results at the LHC can be reinter-pretated in upper bounds for axion-models







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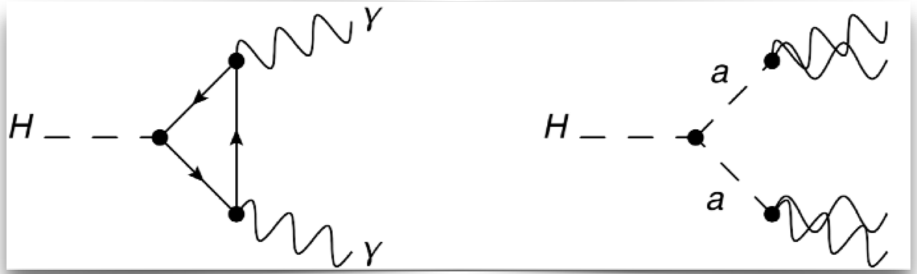
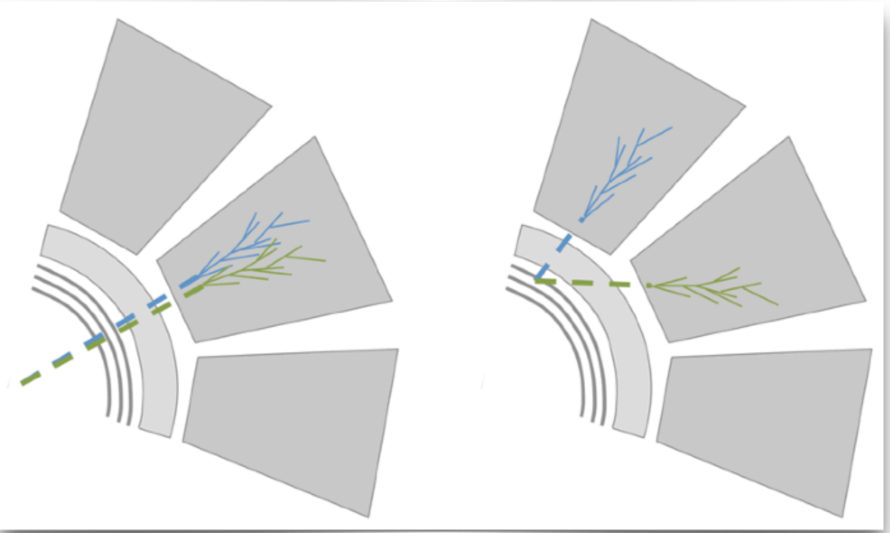
Higgs To Axion Decays

Higgs Boson Decays into Axions

- Convention: Axions couple to Higgs
  - Inhibits a certain model dependence



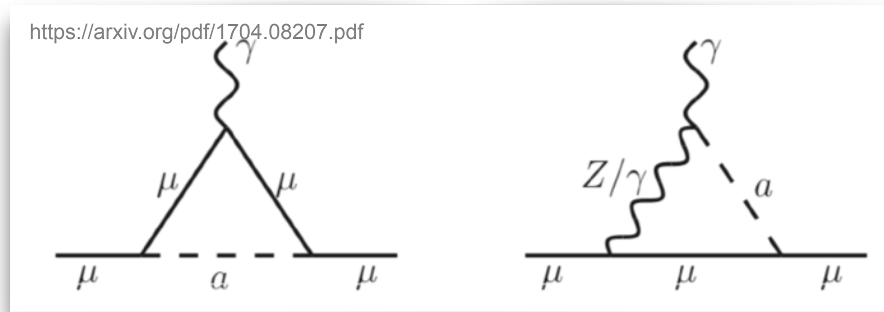
- State-of-the-art photon identification does not work for
  - Highly collimated photons
  - Axions decay close to the calorimeter

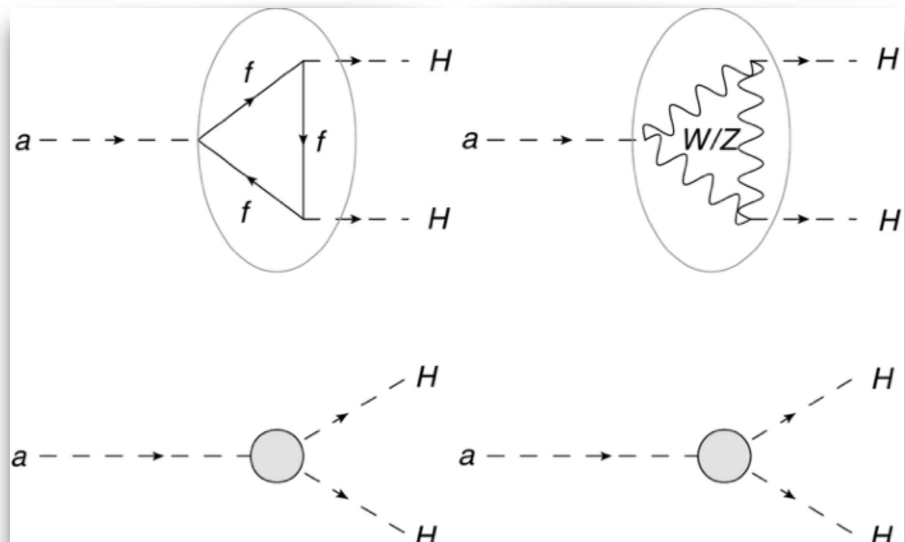


- Strategy:
  - High mass range (5-60 GeV)
    - Look for 3-4 Photon events
  - Low mass range (100 MeV - 5 GeV)
    - Try to separate close-by photons with neural network based classifiers

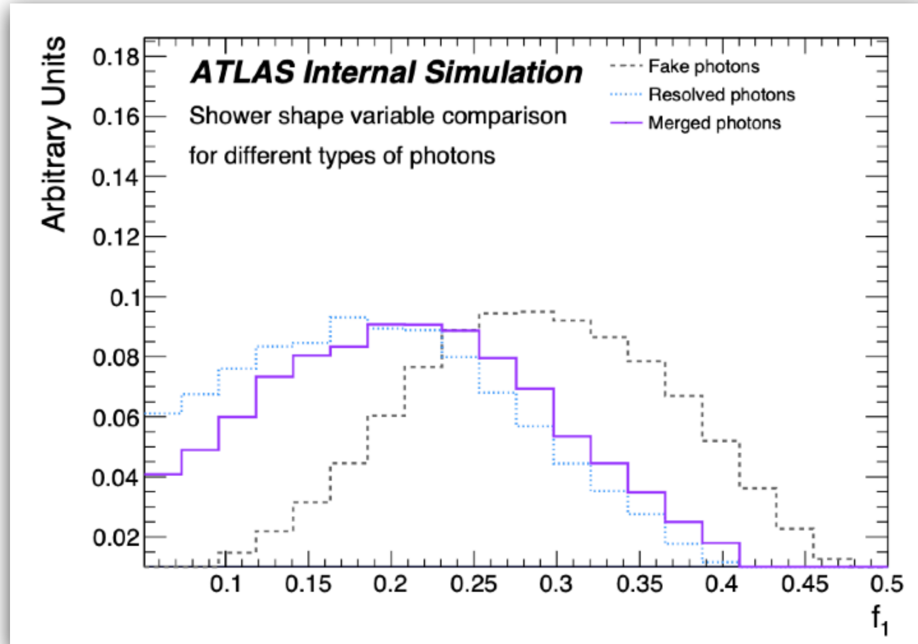
Some words on model-dependence

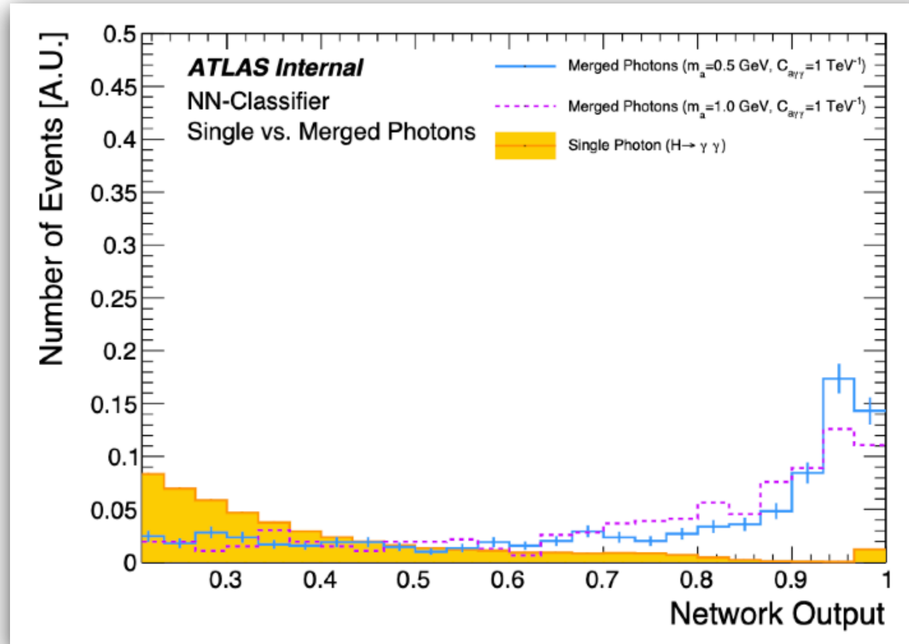
- Most axion models inhibit a certain model dependence
  - Exception: Axion-photon production
- Which axions could explain $(g-2)_\mu$
  - Requires coupling muons
    - Problem: wrong sign!
  - Solution: also coupling to photons
    - Wilson coefficient $C_{\gamma\gamma}$ needs to sufficiently large
- How about the assumption that axions couple to Higgs?
  - Trivially realized by loops: almost in every axion model the case
  - Sensitivity depends in the Wilson coefficient C_{ah}, that describes the axion-Higgs coupling.
  - I used $|C_{ah}|/\Lambda^2 = 0.01\ \mathrm{TeV}^{-2}$





Identify Merged Photons

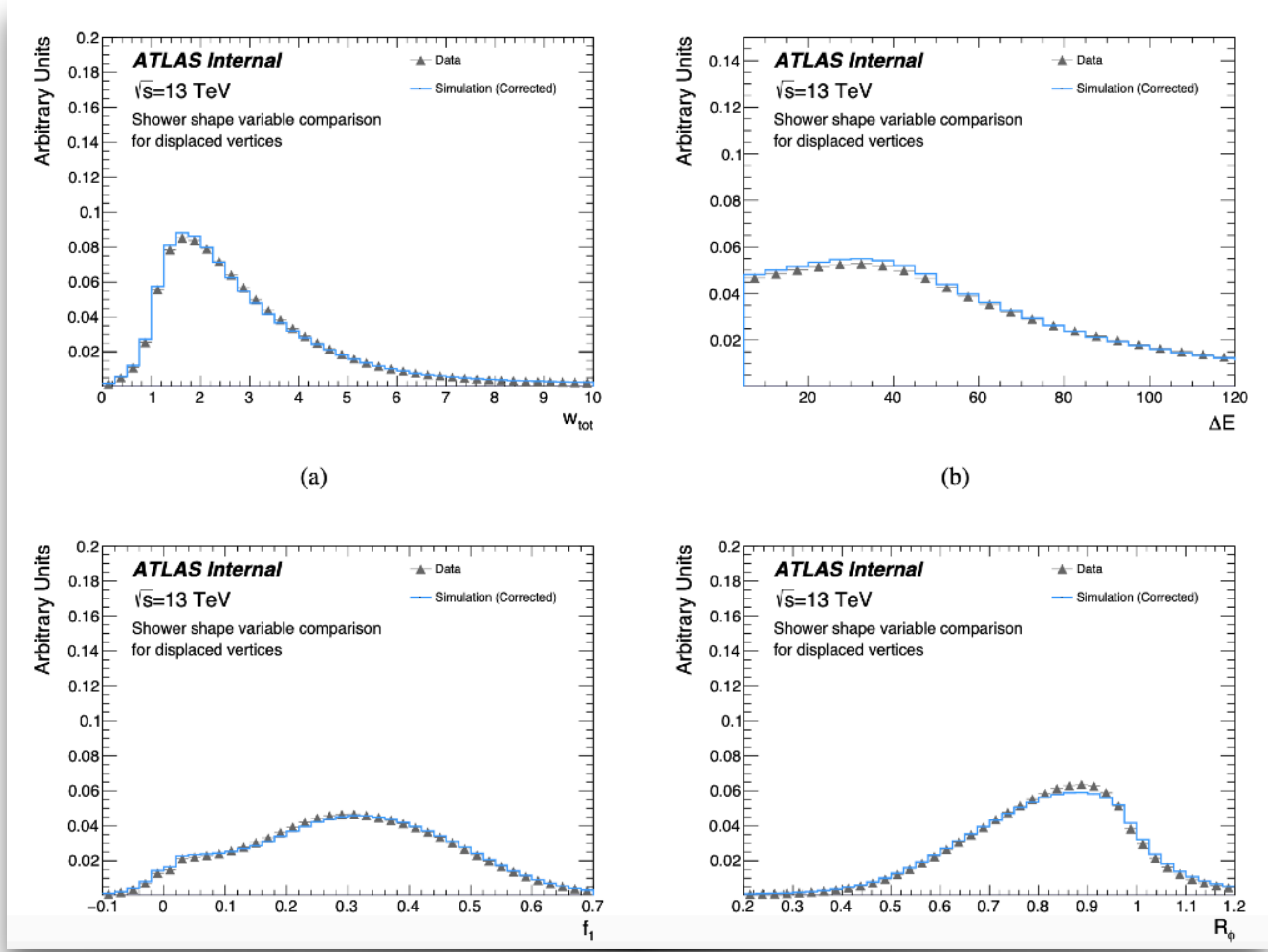




- NN based classifier using shower-shapes of the electromagnetic calorimeter
- Training data
  - Single Photons from Data and MC
  - Merged Photons only from MC
  - Systematics by varying shower shapes and $Z \rightarrow ee\gamma$ events

How about displaced photon signatures?

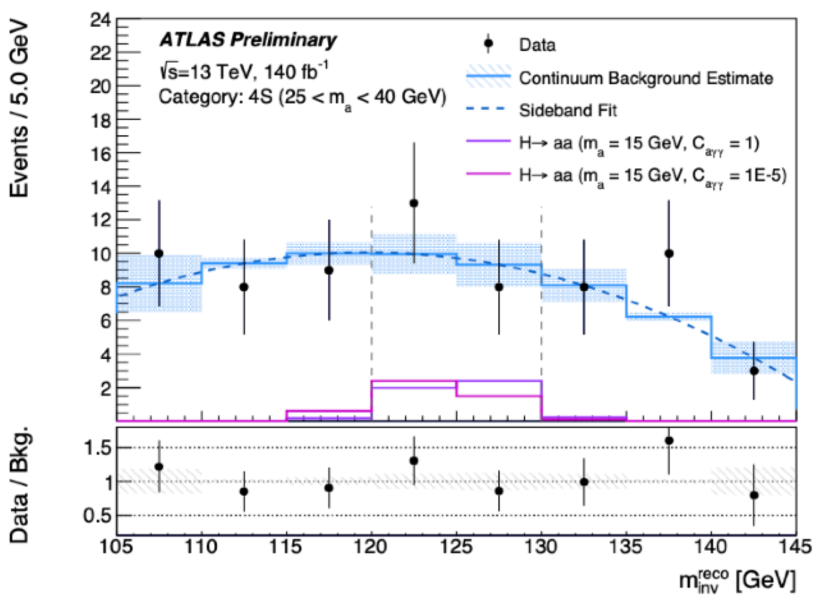
- Problem: how do we know the ATLAS detector response for displaced photons?
- Idea: Compare shower-shape variables of identified K-long decays
- Treat difference as systematic

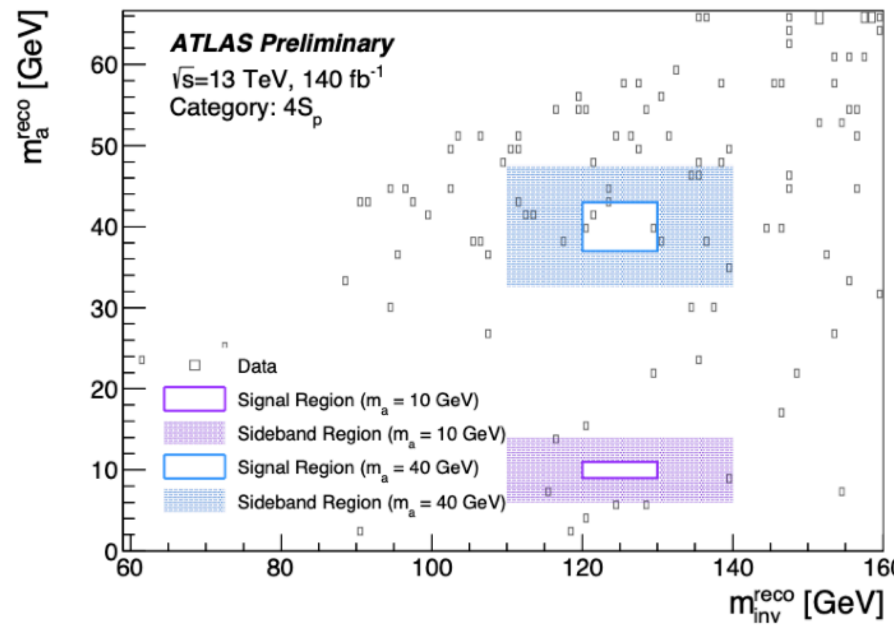


(a)

(b)

High Mass Region (5-62 GeV)



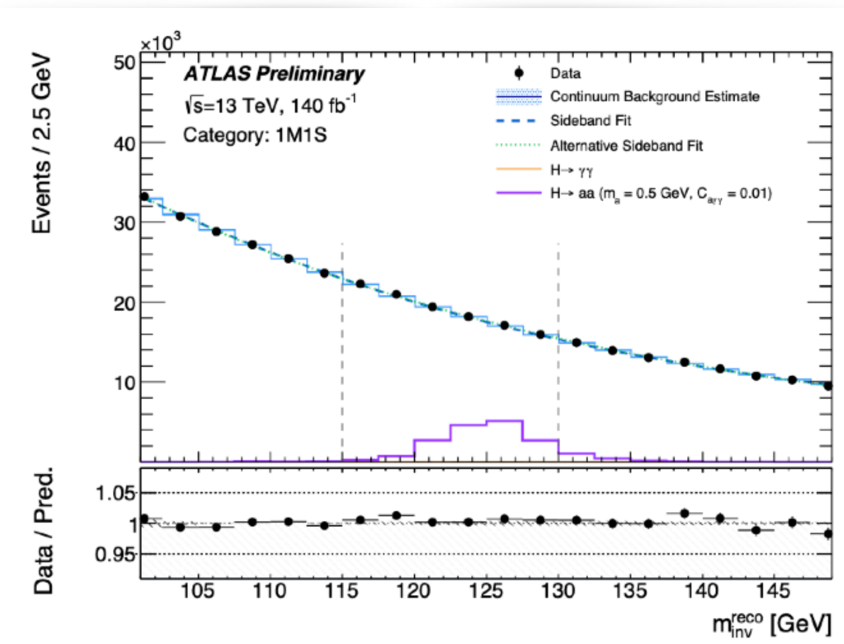


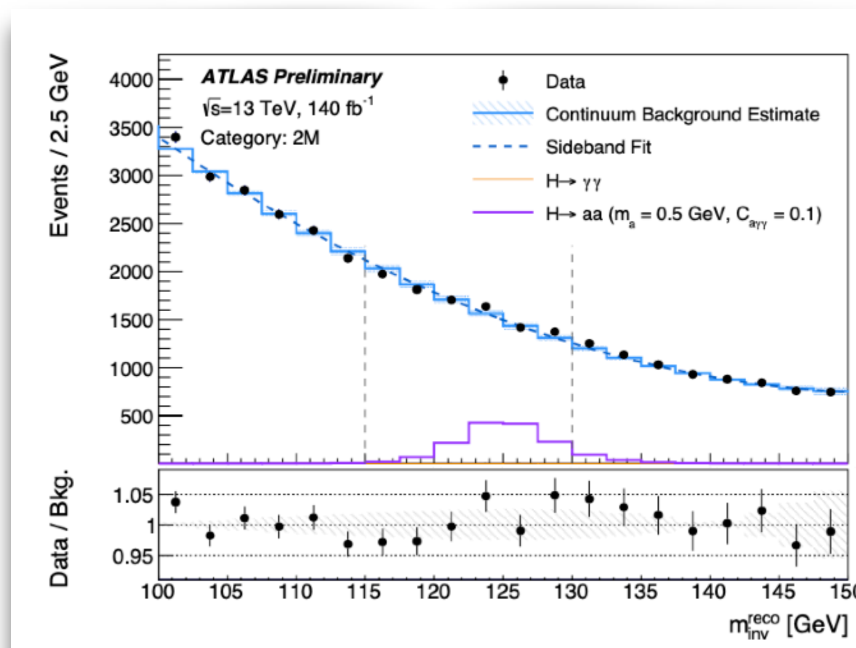
- Prompt decays (large $g_{a\gamma\gamma}$ couplings): 4 standard „tight" photons
  - Super small background
- Long-lived axions (smaller $g_{a\gamma\gamma}$ coupling): >1 standard „tight" photon, 3 loose/ displaced photons
  - Background estimated using simple sideband approach

Medium Mass Region (1-5 GeV)

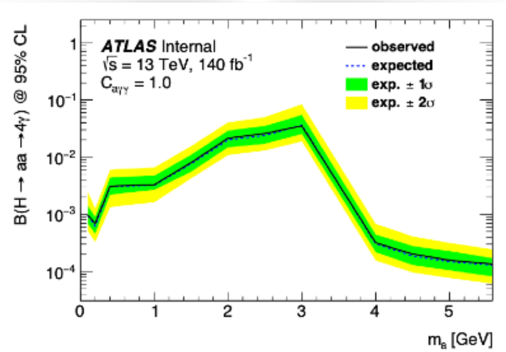
- Photons from axion decays start to appear merged
  - Simultaneously study two regions
  - 1 single photon + 1 merged photon
  - 2 merged photons

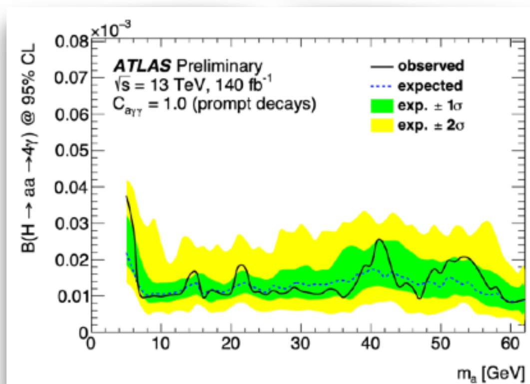
- Background estimation again with a simple sideband approach

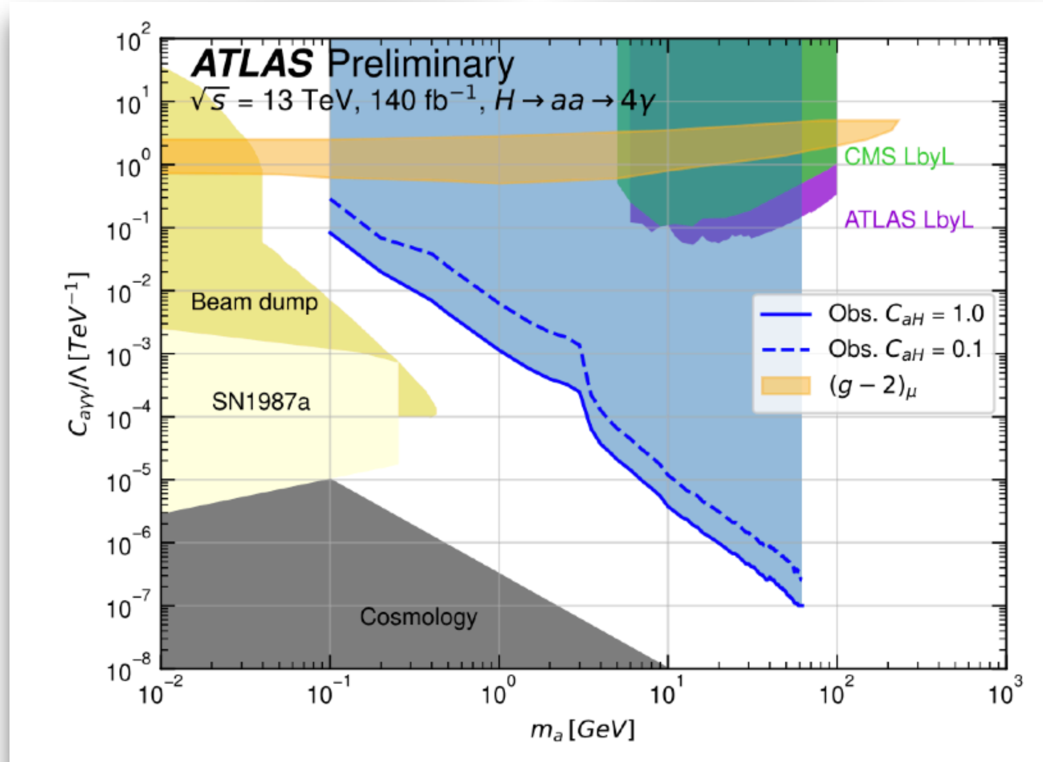




Do we see something?

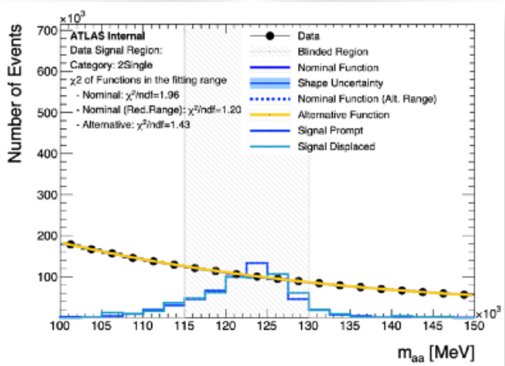


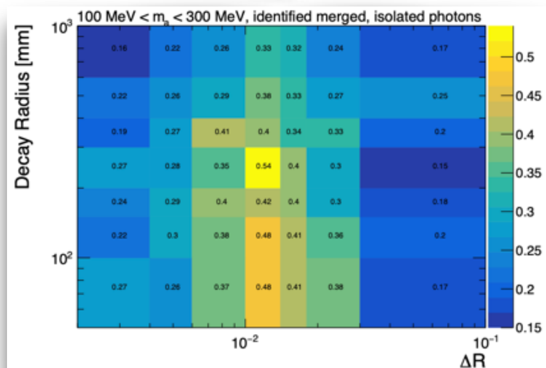


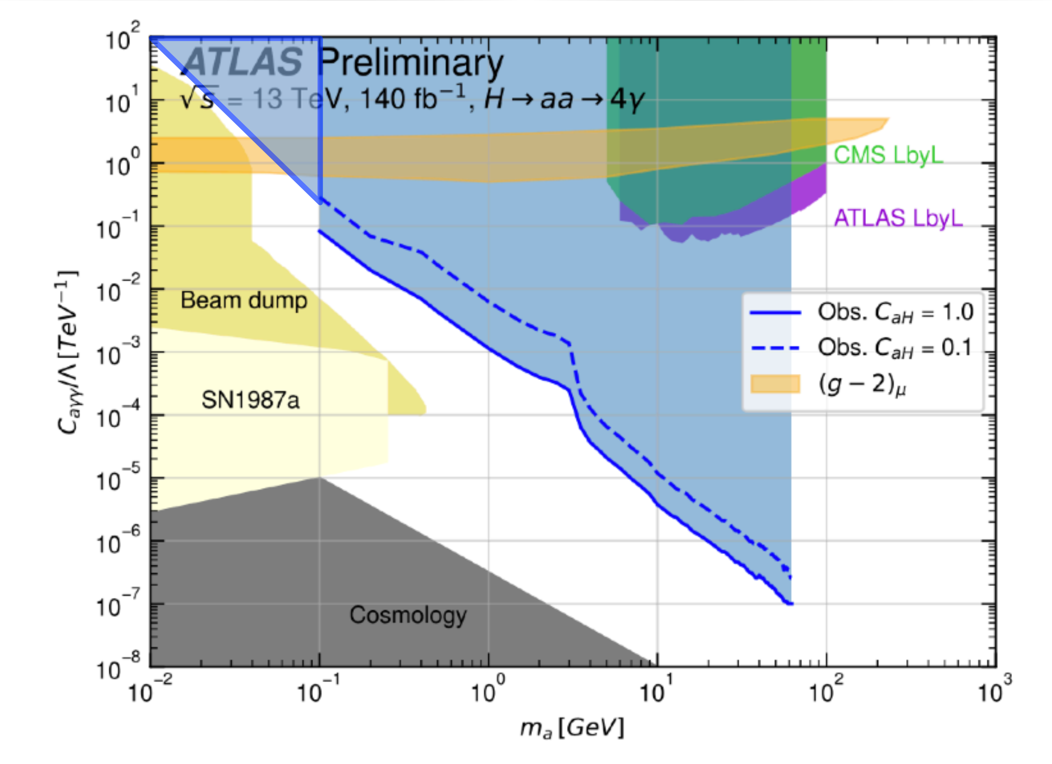


- Most stringent limits and first limits on ALP models with large lifetimes!
- Why didnt we find axions here? Well, because they are simply not there…
  - … but there is a small parameter region left unprobed

Low Mass Region (0.1-1 GeV) - I hereby apply for dinner!



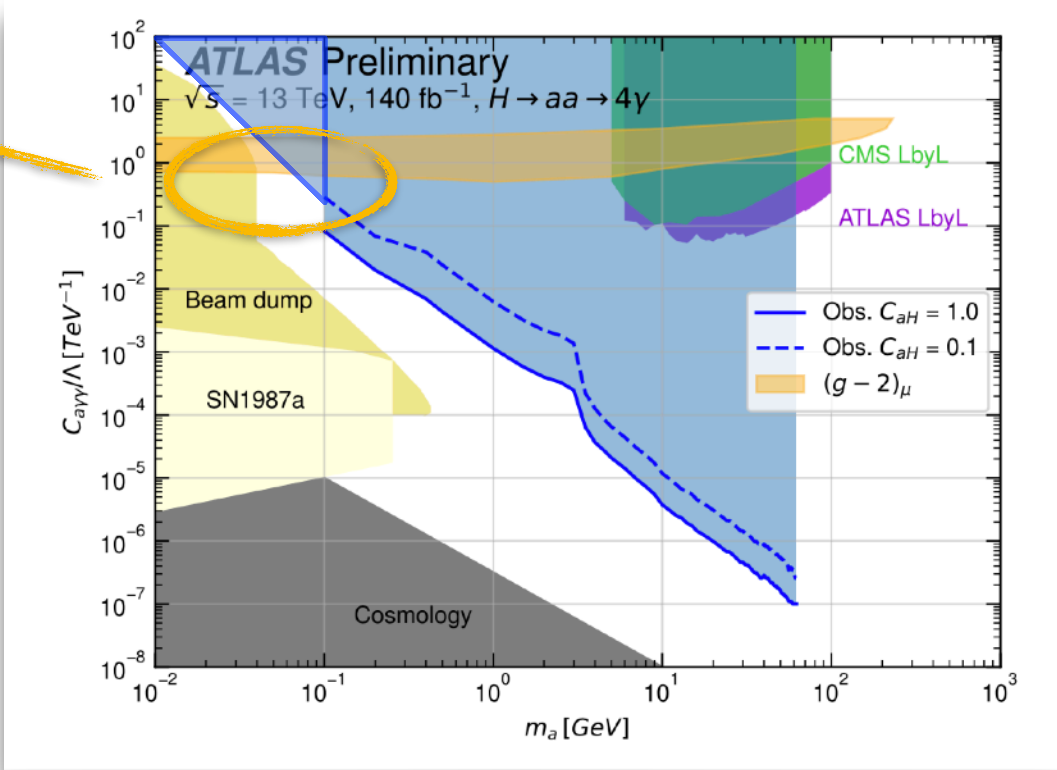




- My summer 2023 at SUNY: Select H→$\gamma\gamma$ events (2 Single) and reinterpret the those for very low axion masses
  - Highly collinear axions will pass the standard single photon selection
  - Based only on public results only, but I hope to bring this through ATLAS :)

We need to leave ATLAS for the Rest…

To probe this, we need FASER



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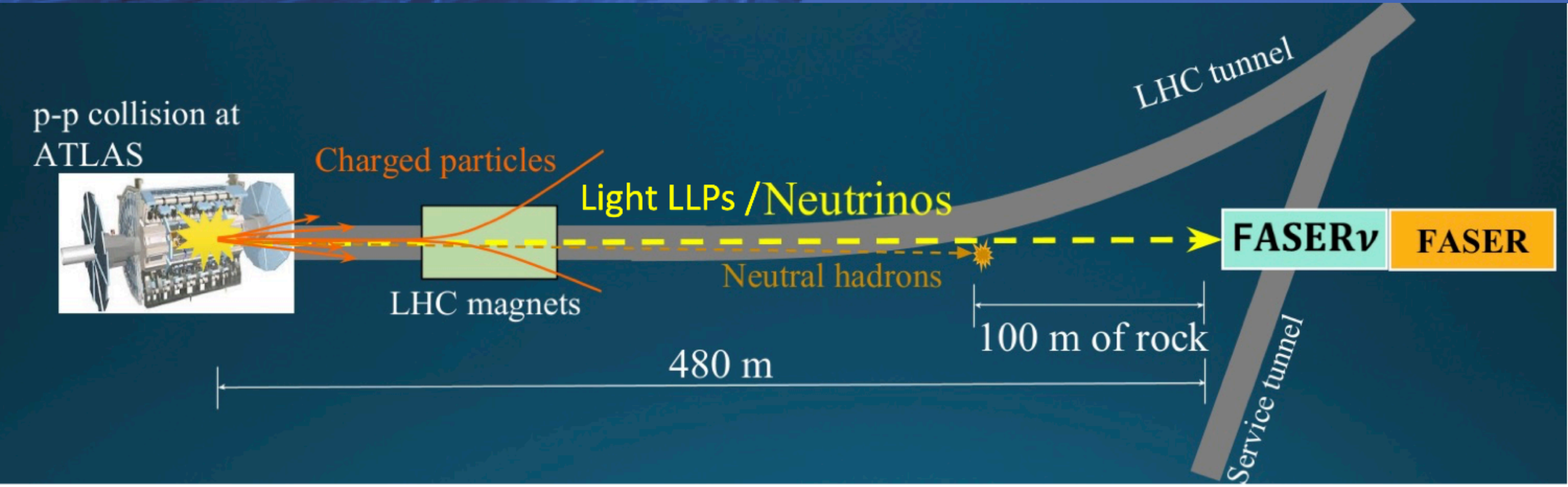
Future Axion Searches and finally Gravitational Waves

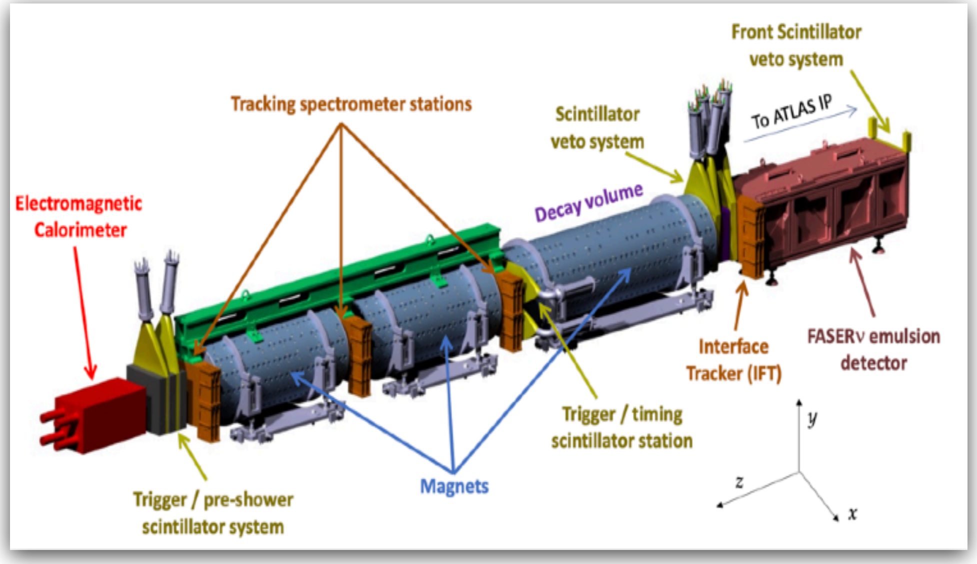
The FASER Experiment



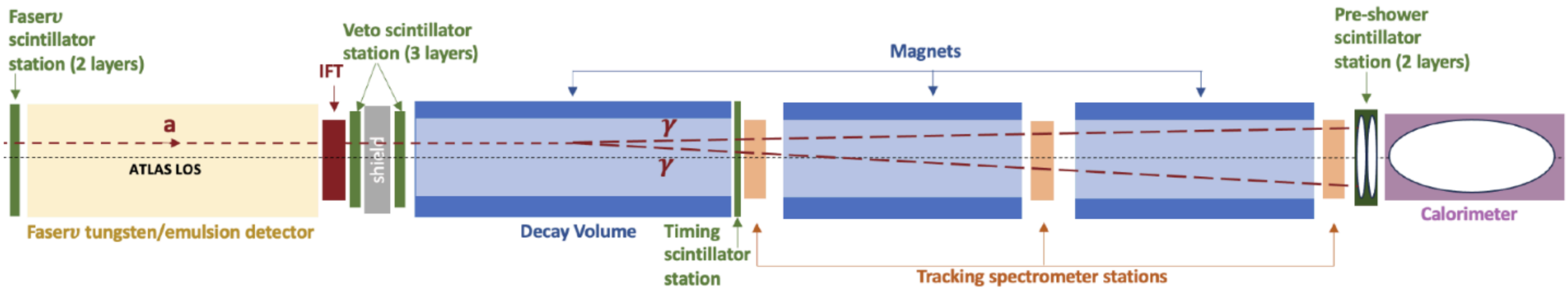
- LHC was designed to search (and study) for heavy strongly coupled particles
  - Existing experiments well suited for this, and performing well
- Huge number of light SM hadrons in the LHC collisions are produced in the forward direction
  - Weakly coupled, light new particles (dark sector)
  - Weak coupling means very rarely produced, and long-lived
  - Neutrinos produced in hadron decay
  - Weak coupling means rarely interacting

The FASER Detector System

- FASER is situated ~500m from the ATLAS collision point (η > 9.2)
  - on the beam collision axis
  - 0.6T permanent dipole magnets
  - 1.5m long decay volume
  - 2.5m long tracker (96 ATLAS SCTs)
  - Scintillators for veto, trigger, and preshower (particle ID)
  - 4 LHCb calorimeter modules
- Tungsten-emulsion FASERν detector for additional neutrino sensitivity

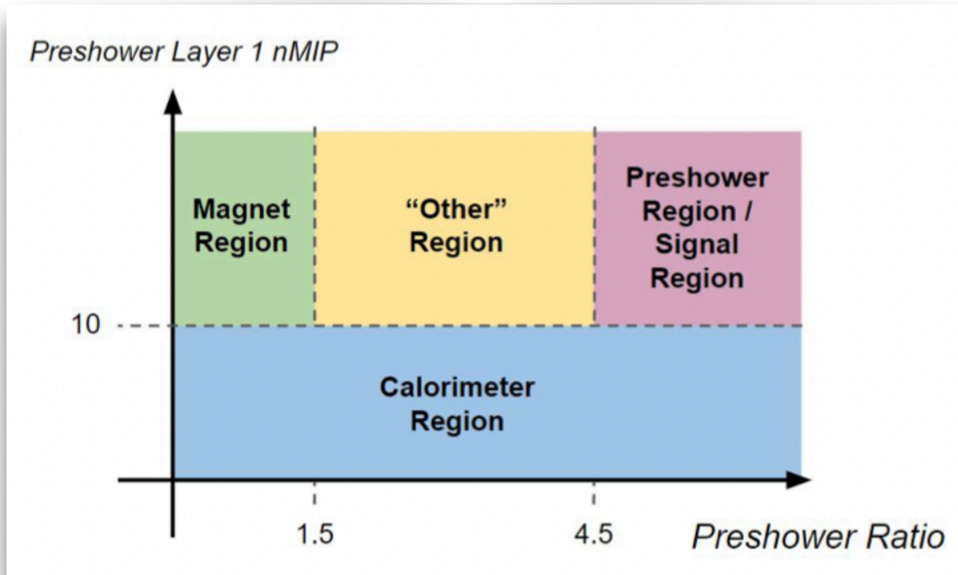


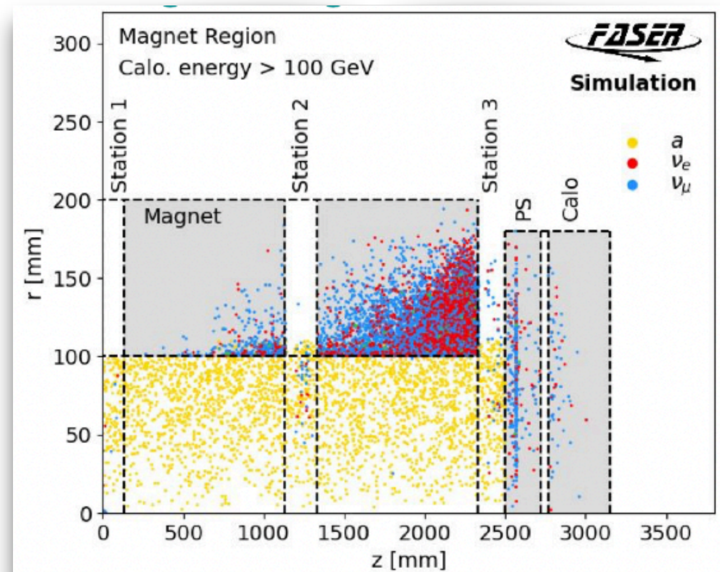
Search for Axion Like Particles: Signal and Selection



- Currently sensitive to axion-like particles (ALPs) coupling to $SU(2)_L$ gauge bosons
  - Mainly produced in B meson decays in our sensitivity range
  - Signature: as a → γγ appearing from ‘nothing’ with ~TeV of energy
  - Can decay anywhere in FASER

Search for Axion Like Particles: Signal and Selection



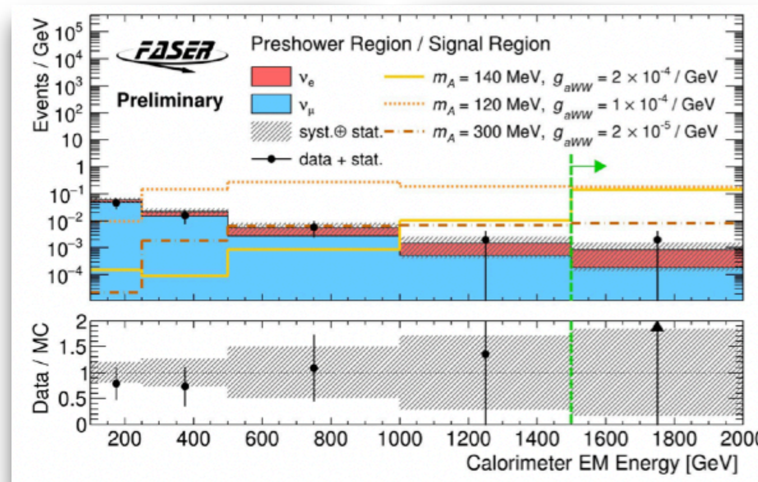


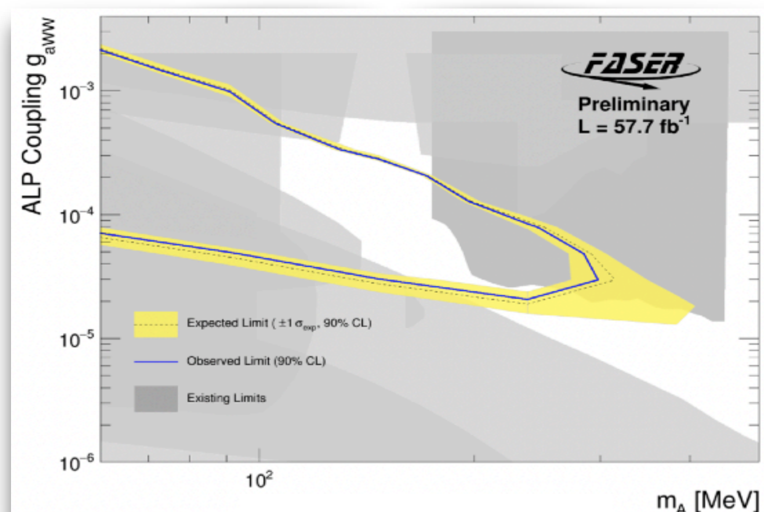
- Selection
  - Nothing in all 5 veto counters
  - Evidence of EM shower in preshower
  - > 1.5 TeV in calorimeter
  - In time with LHC collision
- Background
  - Neutrino interactions
  - Neutral hadrons
  - Large-angle muons
  - Non-collision / cosmics

- Data control regions and simulation used in blinded analysis to evaluate backgrounds

Search for Axion Like Particles: Signal Region and Limits

- New Results for Moriond 2024
- Observed 1 event in 58 fb^{-1} after unblinding
- Expecting 0.4 ± 0.4 from CC ν interactions in pre-shower
- Probing new parameter space of this ALPs Model
- Further Information:
  - https://faser.web.cern.ch/physics/publications

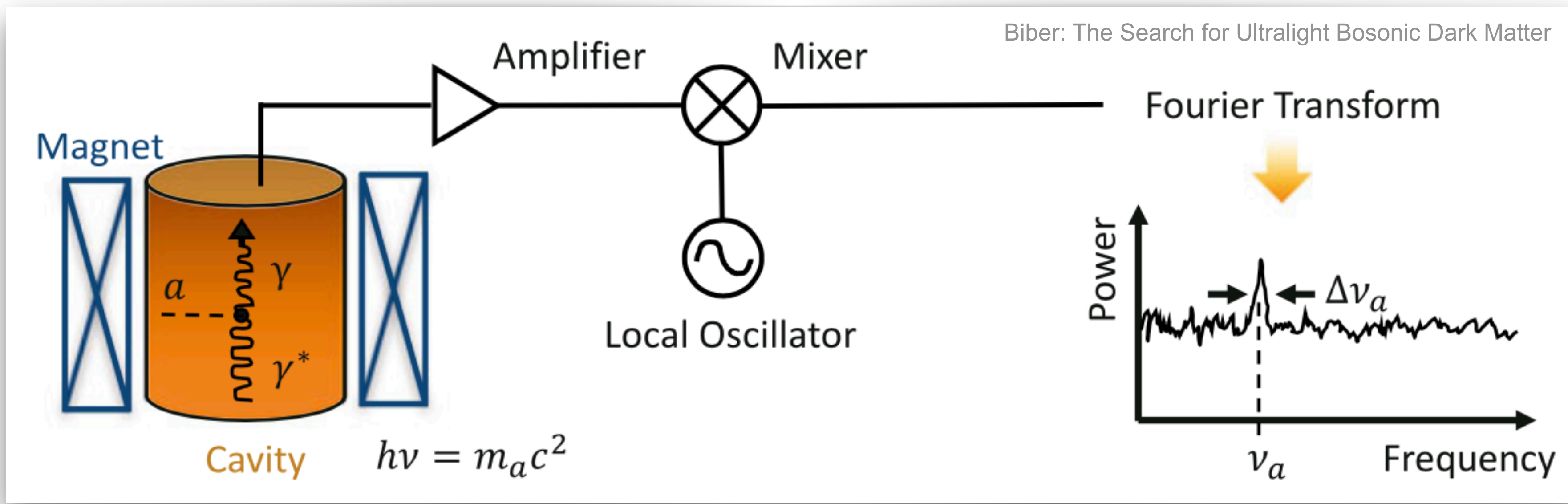




Future Axion Searches and finally Gravitational Waves

Recap of Searches with Cavities: ADMX, Haystack

- Galactic halo axion (or ALP DM)
  - photon conversion in cavity within a B-field if resonant
  - Galactic halo axions have speeds β=10^{-3}: 1.2 kHz spread in frequency
- Experimental Challenges
  - Signal Power: P=10^{-24}W
  - Only few kHz band-width can be observed at one time
  - Scanning required (tunable Cavity)

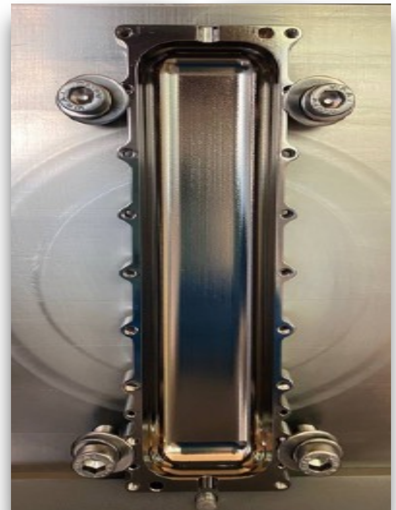
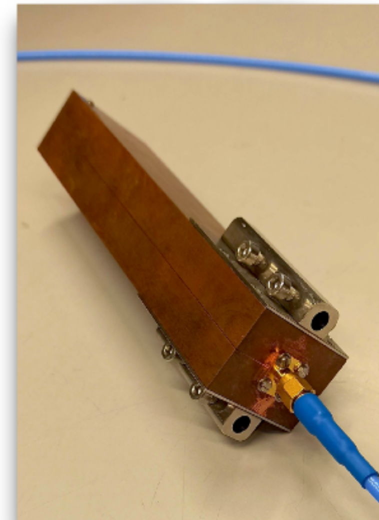



SUPAX: A New Experiment in Bonn

- Use new 14T Magnet with a bore diameter of 100 mm

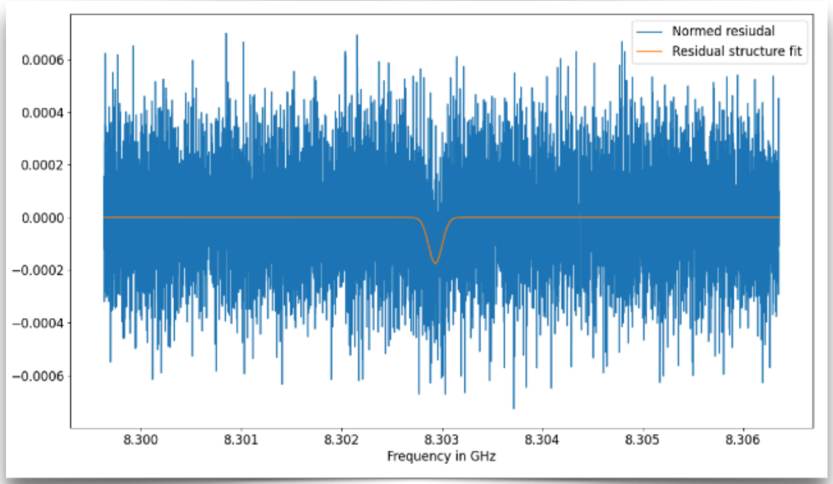
- Idea: To reduce noise, use a superconducting cavity
  - First time for an axion search experiment
  - Signal Power 10^{-24}W,
  - Q-Factor 10^{6}

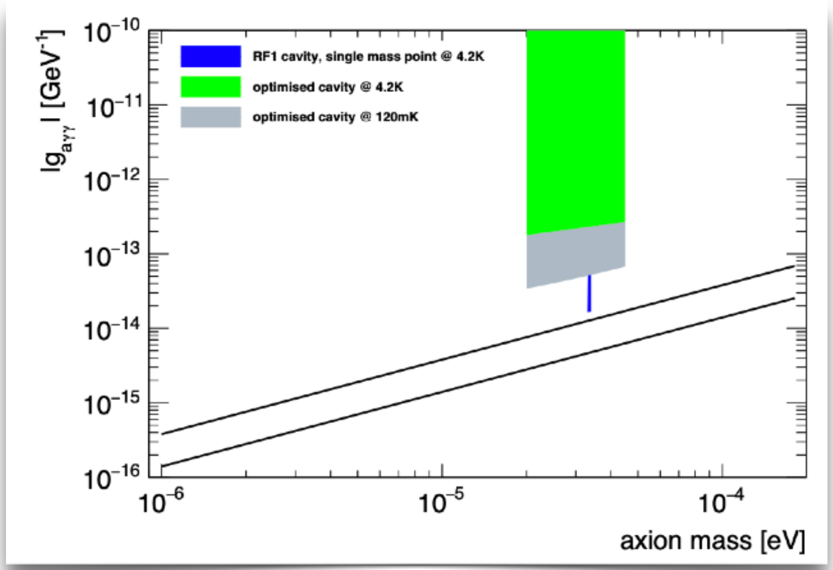
- Study only one frequency: 8-10 GHz 30 μeV to 40 μeV
  - Advantage: We do not need to tune the cavity and keep the Q-Factor high
  - Disadvantage: we need to be extremely lucky that we search at the right axion mass

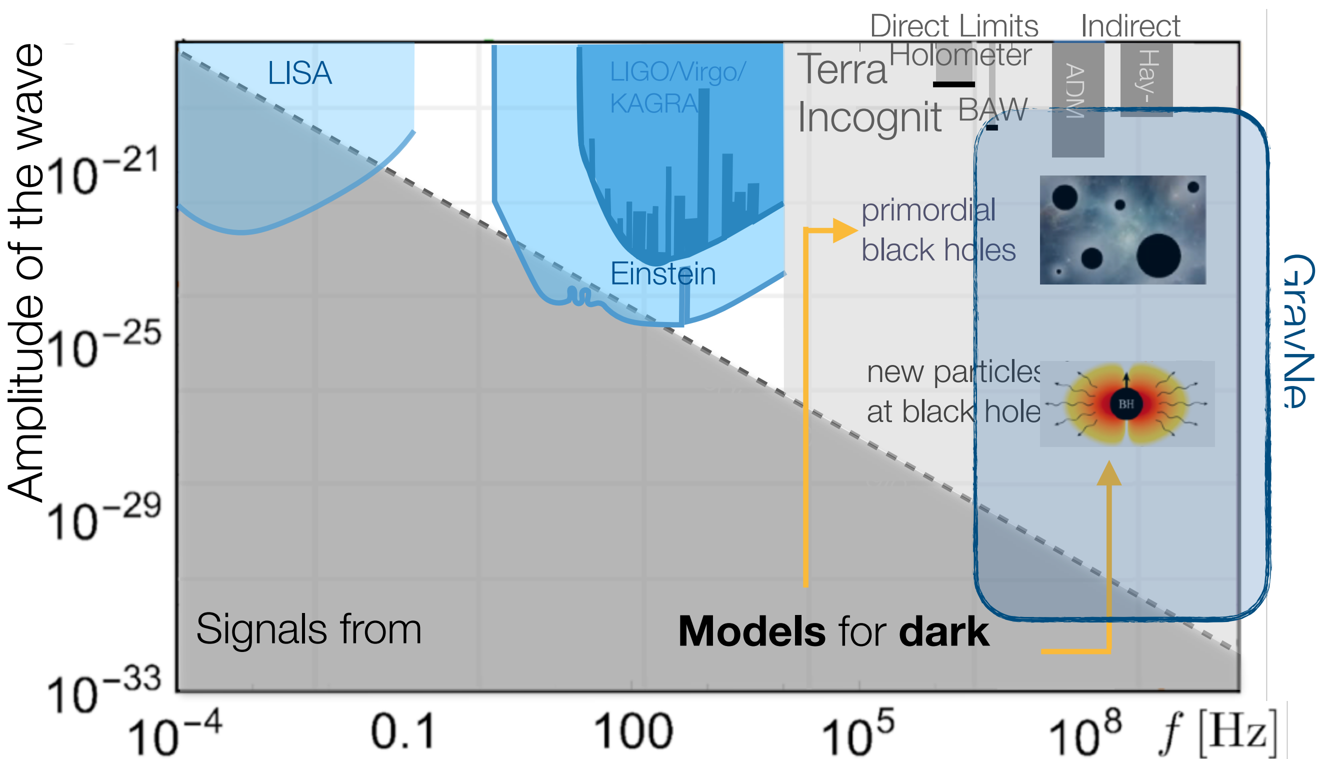

SUPAX: First Results and expected Sensitivity

- Dark Photons can convert within the SUPAX Cavity without B-field
  - First run sets limits on mixing parameter $\chi < 9.88 \cdot 10^{-14}$ for m_A=34.34 μeV
- Expect first data-taking with magnet in the coming months
  - Close to QCD Axion band when scanning one frequency
  - Developments for tunable cavity are ongoing
- … does this experiment makes sense?
  - Yes, because it is interesting R&D for superconducting cavities
  - … and clearly for HFGW





Gravitational Wave Landscape



- (HFGW) **sources**
  - could explain dark matter
  - could access the early universe
- Very mild limits for
  - f =1 MHz - 10 GHz

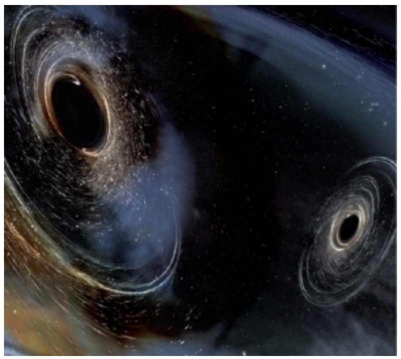
GravNet

A first dedicated effort probing high-frequency gravitational waves

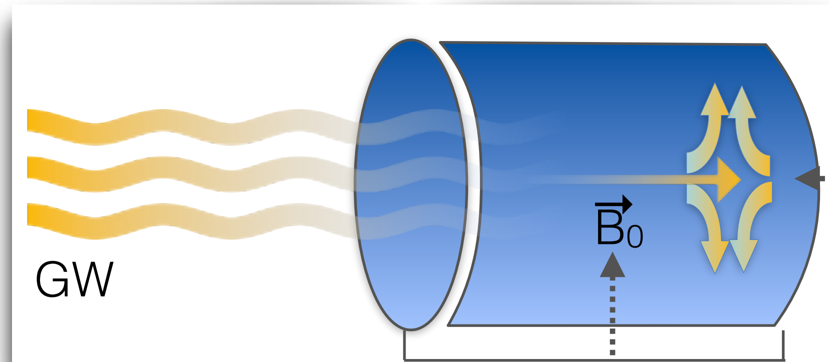
https://www.pi.uni-bonn.de/gravnet/

How to detect high frequency gravitational waves?

- Gravitational waves convert to photons in presence of magnetic fields



Source

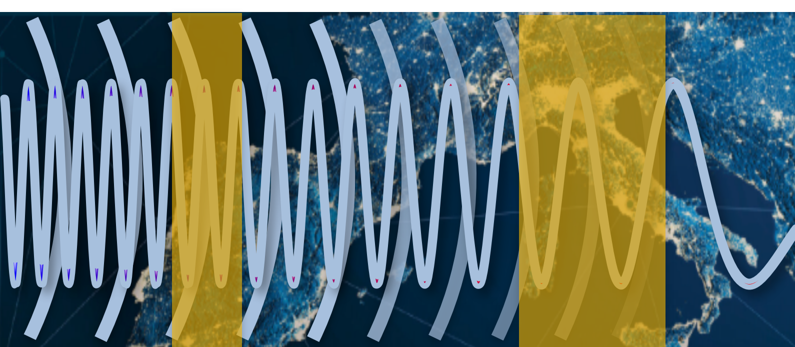


Strong static

Photons generated at $f = f_{GW}$

Expected signal power: $<10^{-24}$ W

- If photon matches **resonance** frequency of cavity, signal is enhanced and **detectable**

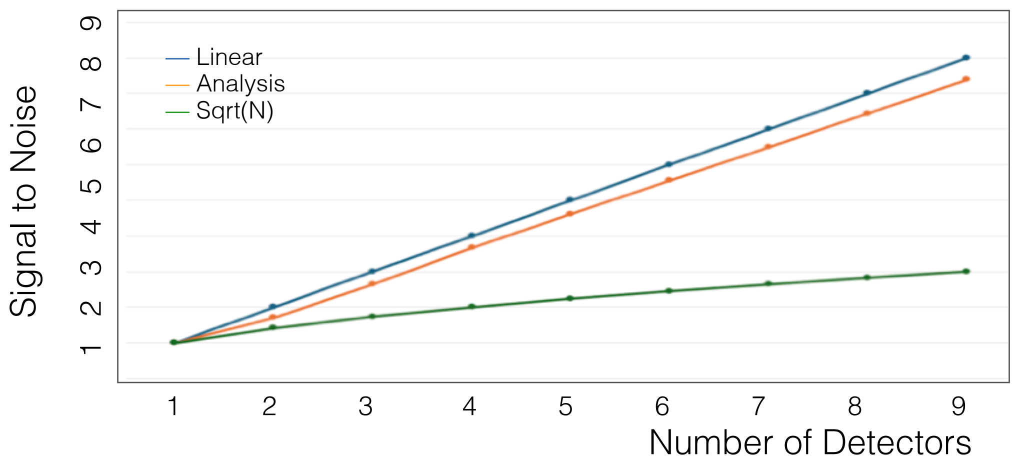


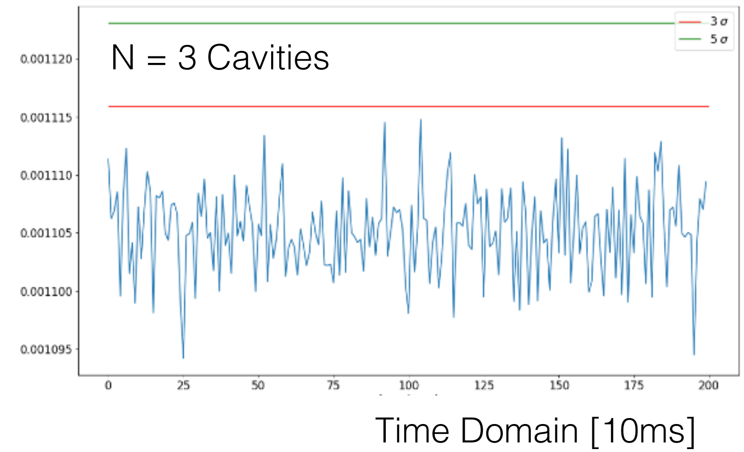
(1) HFGWs may sweep through frequency space
- Built the optimal detector for one frequency

(2) HFGWs yield coherent signals across Earth
- Built a network of optimal detectors

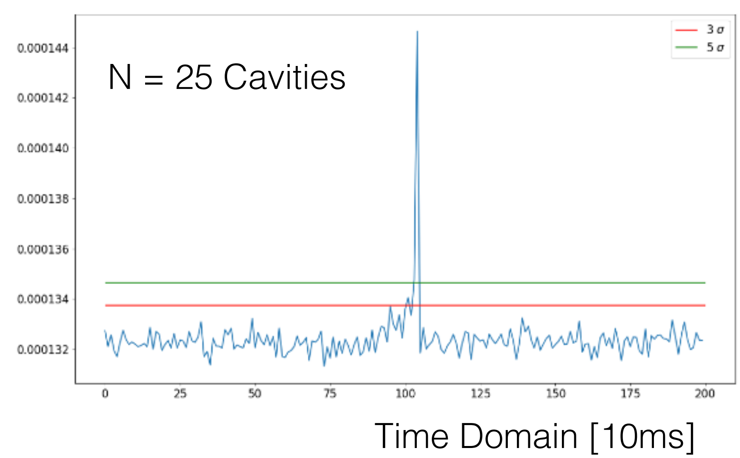
GravNet

How does a network of detectors help?





- Input data of one cavity
  - FFT of data in time-intervals
  - Signal power per time-interval as time-series
  - Future: Use directly recorded voltage as input
- Combine data of all cavities/experiments using an attention NN



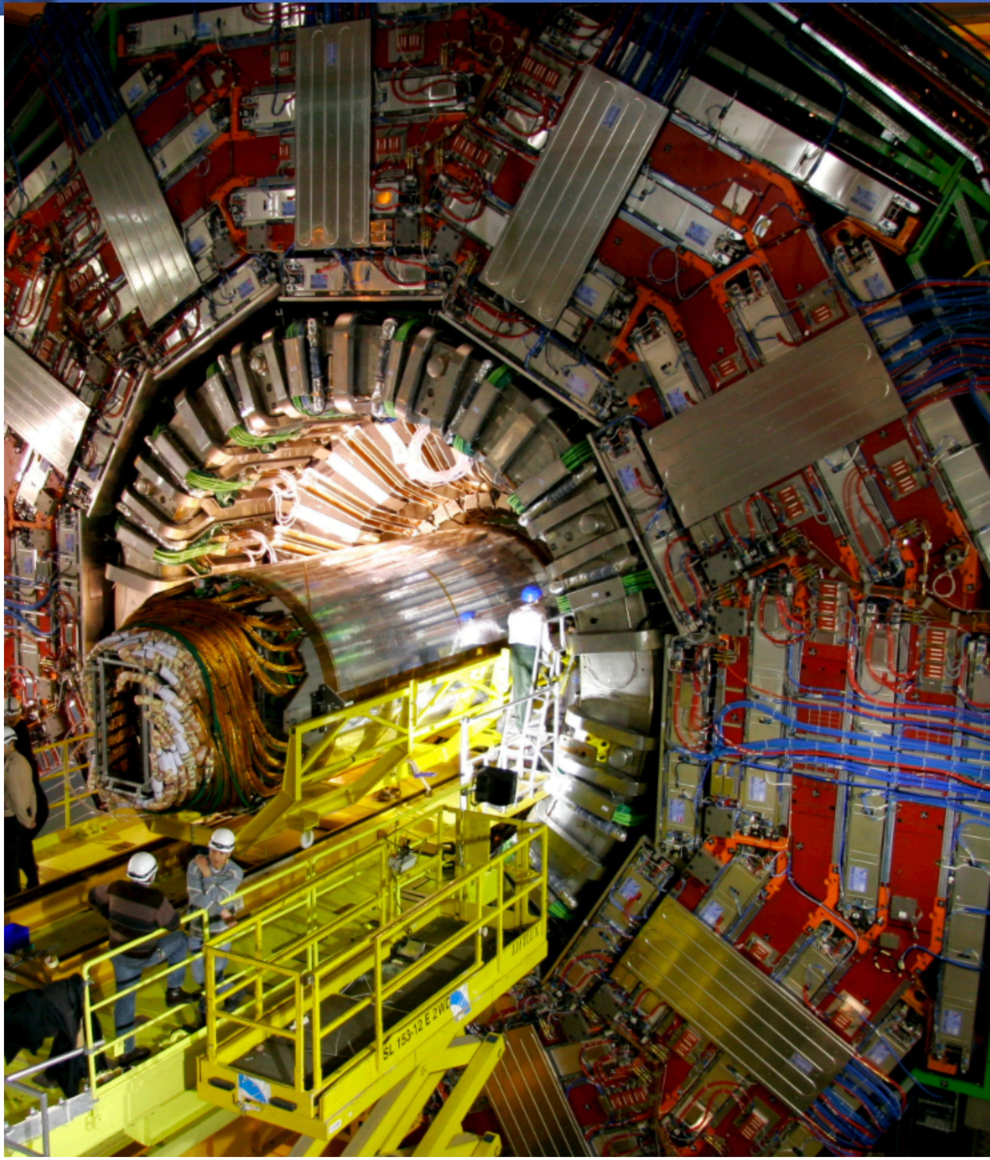
GravNet - If you have a magnet on site, we are happy to provide you with a GW detector

How to get even more sensitive?

- Signal Power of the axion in the cavity is given by

$$P_{\mathrm{sig}} = \left(\frac{g_\gamma \alpha_{em}}{\pi}\right)^2 \frac{(\hbar c)^3 \rho_a}{\Lambda^4} \frac{B^2 \beta}{(1+\beta)^2} Q_0 V C_{010} \frac{2\pi f_0}{\mu_0}$$

- Large volume and high magnetic fields drive the sensitivity
- Where can we find large volumes with high magnetic fields?
  - CMS!
- Problem: I guess CMS doesn't want to give up its LHC physics program
  - e-Print: 2209.12024 (in case you want to know what one could get)
  - … Let's see what the future brings



Funded by the European Union

European Research Council
Established by the European Commission

Summary

- ALPs are certainly a hot topic
  - Gravitational Waves might be even hotter
  - Lets combine efforts! GravNet is open to new members

- Some personal statements
  - Search for vaguely motivated new particles seems to me a waste of time during HLLHC
    - LHC searches will (have to) move to search for long-lived particles
  - IMHO: The future lies in precision physics!
    - Lepton collider

- Thanks to the ERC, which allowed this research

Light Through Wall

EDM in QCD

Characteristic scale of QCD when gets it strong

$\Lambda_{QCD} \sim 200$ MeV (m_π)

↳ $\Lambda_{QCD}^{-1} \sim 10^{-15}$ m

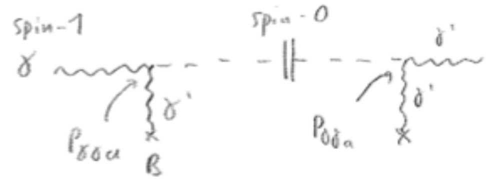
Quark charge $q = 1/3$

↳ $|d_n| \sim 10^{-15}\,\mathrm{m} \cdot \frac{1}{3} \cdot \theta$ "small"

more detailed

$|d_n| \sim 1 \ldots 10 \cdot 10^{-16}\,\theta$ ecm

LSW



$$P_{\gamma\to a} = \frac{g^2}{4} B^2 L^2 \underbrace{\frac{\sin\left(\frac{qL}{2}\right)}{(qL/2)}}_{\text{Form factor}}$$

$$q = \frac{m_a^2}{2E_\gamma}$$ lg. momentum difference btw. axion & photon

coherence (max. conv prob. when axion & photon field are in phase over L:

$qL < \pi$

↳ increasing m_a ⇒ momentum mismatch ⇒ suppression by

F

assume: $qL \ll \pi \Rightarrow F = 1$

$$N^\gamma_{Det} = N^\gamma_{Laser} \cdot \underset{\approx 1}{\eta} \cdot P^2 =$$

$$= N^\gamma_{Laser} \cdot \frac{1}{4^2} \cdot B^4 \cdot L^4 \cdot g^4_{A\gamma\gamma}$$

Limit von g gegeben durch

$$g_{A\gamma\gamma} < \left(\frac{N^\gamma_{Det}}{N^\gamma_{Laser}}\right) \cdot 2 \cdot B^{-1} L^{-1}$$

Beispiel:

$N^\gamma_{Laser} = 3 \cdot 10^{16}$ (Photons in 1 s)

$\lambda = 660$ nm

↳ $E = 2$ eV

$E_{Laser} = 1$ mJ

$B_{Helmholtz} = 0.001\,T = 0.2\,\mathrm{eV}^2$

$L_{Helmholtz} = 0.01\,\mathrm{m} = 5 \cdot 10^4$/eV

Sensitivität-Auge: 1000 γ

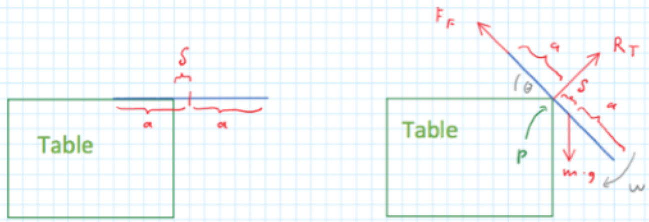
↳ $g_{A\gamma\gamma} < 100\,\mathrm{GeV}^{-1}$

↳ $m_a <$ 3 meV

Robert A J Matthews (Birmingham) Eur.J.Phys.16 (1995)

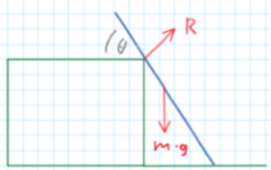
Toast: homogeneous rectangular lamina with mass m

Starting point: Overhanging toast with zero horizontal velocity



Frictional force F_F prevents sliding
- Consider turning lamina around fixed axis

Consider stable position



$$0 = R - m g \cdot \cos\theta$$

When toast is falling: additional acceleration in direction of R_T

$$m \cdot a = m \cdot \dot{v} = m S \dot{\omega}$$
($v = r \cdot \omega$)

$$\Rightarrow \quad (1) \quad m \cdot S \cdot \dot{\omega} = R_T - m \cdot g \cdot \cos\theta$$

Consider centrifugal force (opposite to FF)

$$F_Z = m \frac{v^2}{r} = m \frac{r^2 \omega^2}{r} = m S \omega^2$$

Not forget the component of the gravitational force

$$F_G = m \cdot g \cdot \sin\theta$$

$$F_Z + F_G = F_F$$

$$\Rightarrow \quad (2) \quad m S \omega^2 = F_F - m \cdot g \cdot \sin\theta$$

Last missing piece: Torque (Drehmoment)

$$M = J \cdot \dot{\omega} \qquad (\hat{=}\ F = m \cdot a)$$
(J: Moment of Inertia)

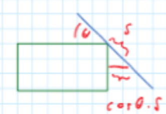
$$J_{Toast} = \int_V r_\perp^2 \rho(\vec{r})\, dV = \int r^2\, dm = \text{(for const. mass)} = \int_{-a}^{a} r^2 \frac{m}{2a}\, dr = \frac{1}{12} m (2a)^2 = \frac{m a^2}{3}$$

Steiners Theorem: Moment of inertia for parallel shifts of rotation axis

$$J = J_{old} + m d^2$$

$$\Rightarrow J = \frac{m a^2}{3} + m S^2 = m \left(a^2 \right.$$

What is the lever-force?



$$- m g S \cos\theta$$

Robert A J Matthews (Birmingham) Eur.J.Phys.16 (1995)

Torque and lever force

$$m\left(\frac{a^2}{3}+\delta^2\right)\dot{\omega} = -mg\delta\cos\theta \qquad (3)$$

We can derive a relation between ω and θ

$(3)\cdot 2\omega$

$$2\omega\left(\frac{a^2}{3}+\delta^2\right)\dot{\omega} + 2\omega g\delta\cos\theta = 0$$

$$2\dot{\theta}\left(\frac{a^2}{3}+\delta^2\right)\ddot{\theta} + 2\dot{\theta} g\delta\cos\theta = 0 \qquad \Big|\int \text{ with } \theta=\dot{\theta}=0$$

$$\left(\frac{a^2}{3}+\delta^2\right)\dot{\theta}^2 + 2g\delta\sin\theta = 0$$

$$\Rightarrow \omega^2 = \frac{2g\delta}{\frac{a^2}{3}+\delta^2}\sin\theta$$

Introduce overhanging parameter

$$\delta := \eta a \qquad (0<\eta\le 1)$$

Central Toast Formula

$$\omega^2 = \frac{6g}{a}\cdot\frac{\eta}{1+3\eta^2}\sin\theta \qquad (4)$$

(4) gives the angle velocity once the toast is detached. If the velocity is large enough, the toast will rotate more than 3π/2-ϕ, i.e. lands for sure on the jam-side up

↳ angle at detachment $\phi = \theta_0$

$$\Rightarrow \text{jam-up condition: } \omega_0\tau > \frac{3\pi}{2} - \phi \qquad (5)$$

$$\text{with } \tau = \sqrt{\frac{2(h-2a)}{g}} \qquad (6) \qquad \left(g=\ddot{x},\ \Rightarrow \frac{1}{2}gt^2 = x = h-2a\right)$$

What is the angle at which the sliding occurs?

- Force down must be larger than friction force

$$F = \mu R$$

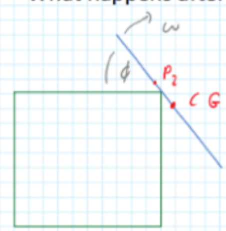
From (1), (2), (4) follows

$$\phi > \arctan\left[\frac{\mu}{1+9\eta^2}\right]$$

my own calculation $\phi > \arctan\left(\frac{\mu(1+3\eta^2)}{1+9\eta^2}\right)$

What is the free-falling angular rotation rate ω_0 ?

- What happens after sliding?



$P_2 \hat{=} CG - a(\eta+\varepsilon)$ slightly non overhanging

P_2 has rotationally-induced horizontal velocity component

$$a\cdot\varepsilon\cdot\omega\cdot\sin\phi$$

Sliding brings this point over the table => detachment

P_2 is essentially unchanged from initial conditions

- Free falling rotation rate is given by

$$\omega_0^2 = \frac{6g}{a}\left[\frac{\eta_0}{1+3\eta_0^2}\right]\sin\phi \qquad (8)$$

Calculate lower limit of η_0 to avoid jam side down - set ϕ=π/2 (highest rotation speed) use (5), (6) and (8)

$$\eta_0 > \frac{1-\sqrt{1-12\alpha^2}}{\alpha^2} \quad \text{with} \quad \alpha = \frac{\pi^2}{12\left(\frac{h}{a}-2\right)}$$