RHIC BES临界和非临界涨落

Shanjin Wu(吴善进)

兰州大学 LANZHOU UNIVERSITY Lanzhou University

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QCD phase diagram

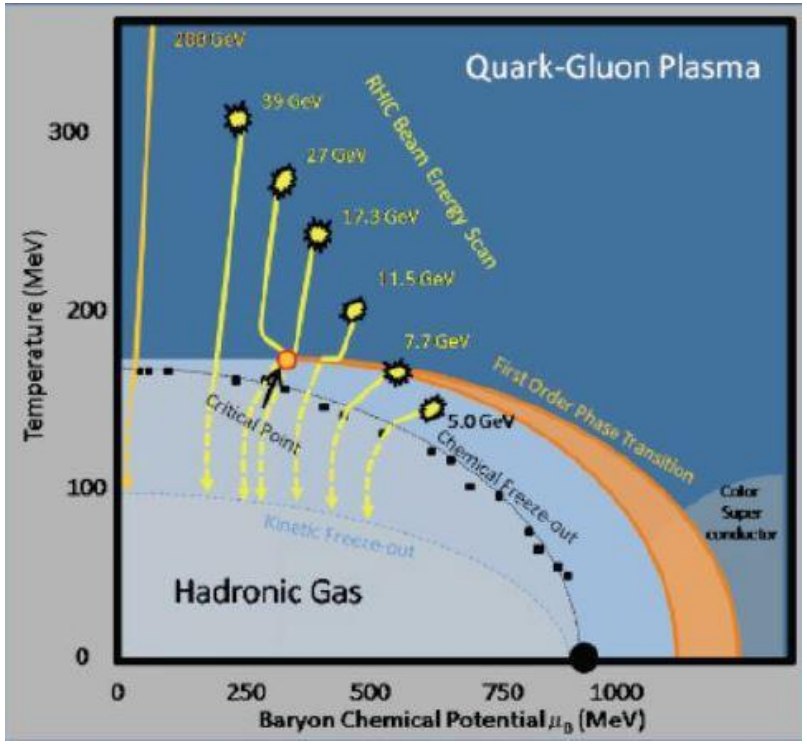
- **Lattice QCD** (small μ_B finite T):
  - Crossover Y. Aoki et al, Nature 443, 675 (2006).
- **Effective models** (large μ_B)
  - 1st order phase trans. W.J.Fu et al., PRD 77, 014006 S.X.Qin et al., PRL. 106, 172301 C. S. Fischer, PPNP. 105, 1……

→ Critical point

- Lattice QCD: sign problem at large μ_B
- Effective models: parameters dependent

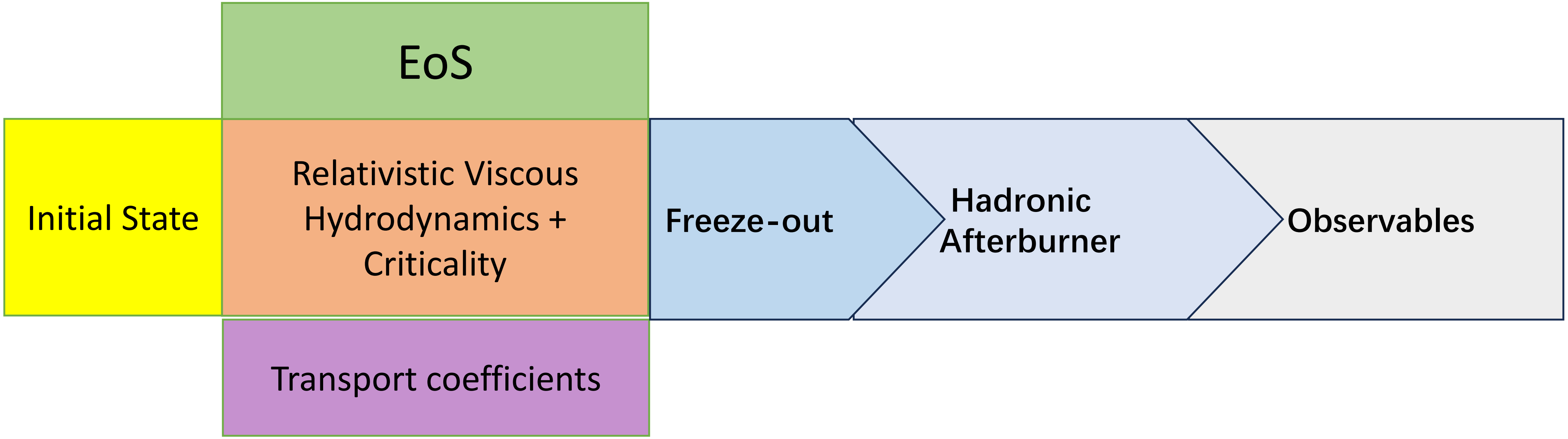
→ **Heavy-ion collisions :**

- tuning $\sqrt{s_{NN}}$,mapping $T-\mu$ phase diagram: RHIC(BES),NICA,FAIR,J_PARC….



Hybrid model for QCD critical point search

Jacquelyn Noronha-Hostler, CPOD2022



=>Bayesian analysis

Hybrid model for QCD critical point search

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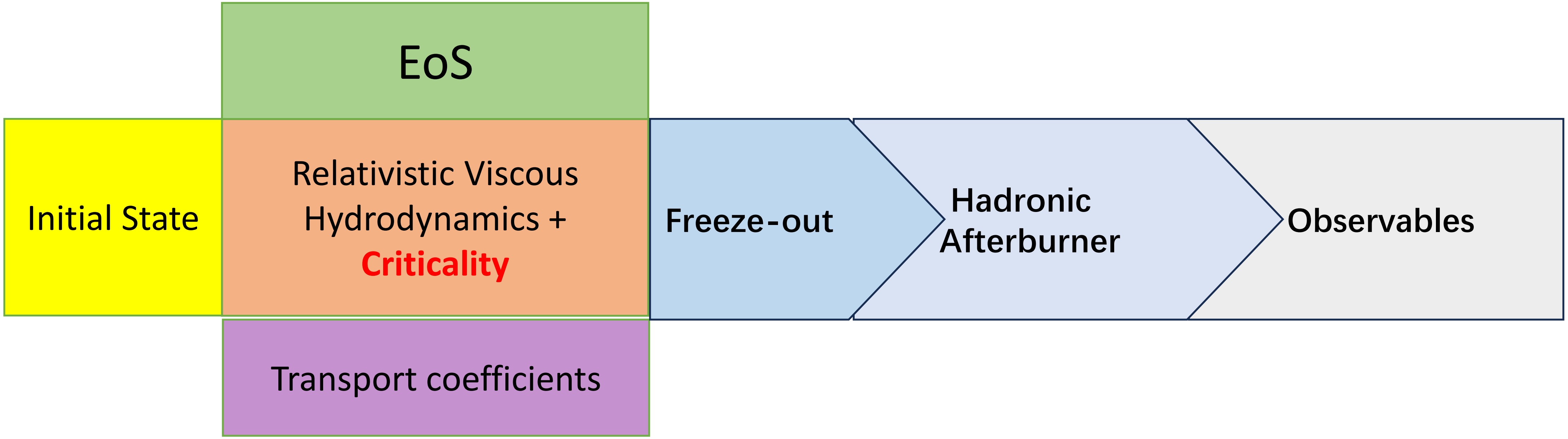
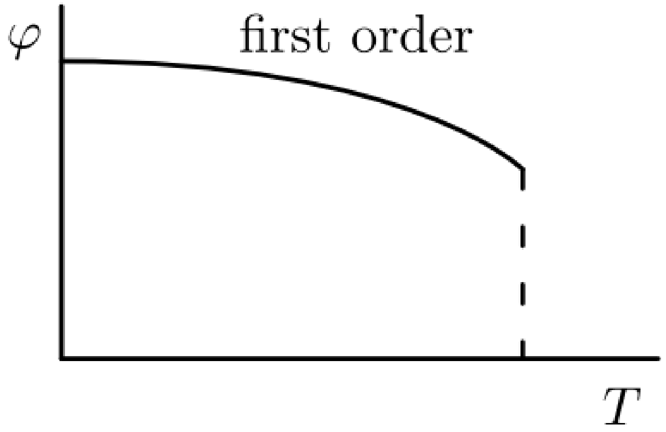
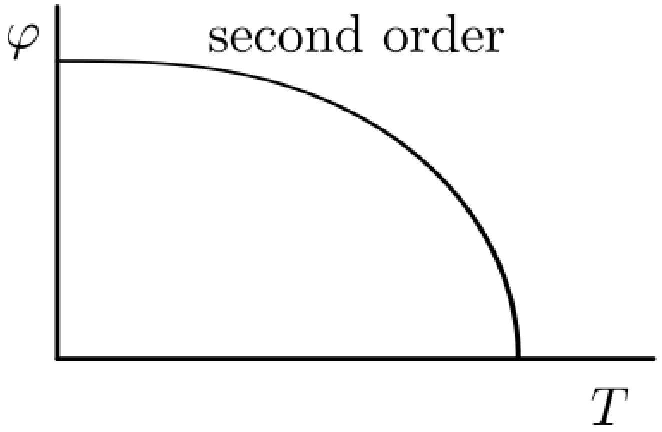


=>Bayesian analysis

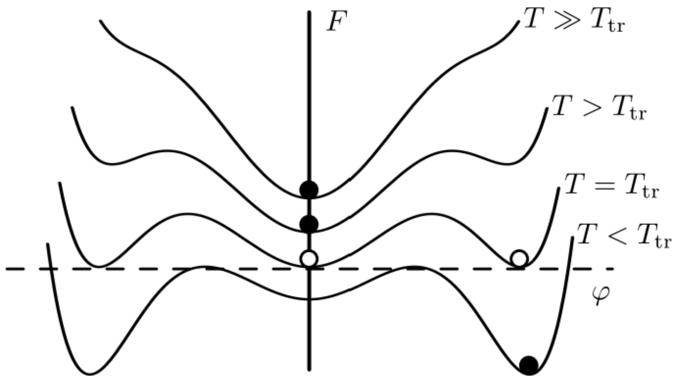
Theory of phase transition

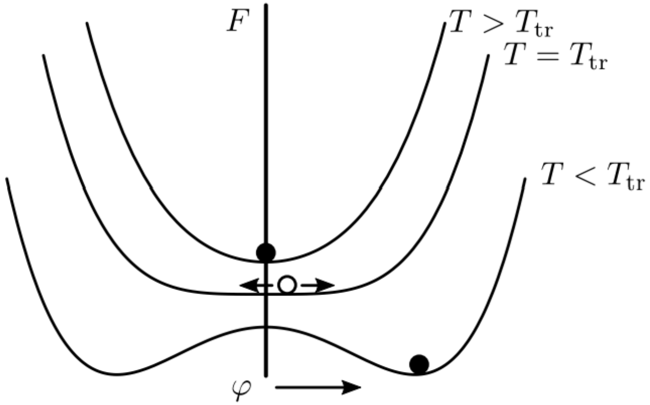




Lev Landau

Order parameter: identify symmetry and symmetry breaking





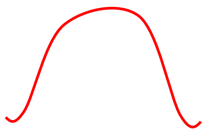
Higher order cumulants are sensitive to ξ

Stephanov PRL 102,032301(2009)

- Probability of order parameter σ :

$$P[\sigma] \sim \exp\{\Omega[\sigma]/T\},$$

$$\Omega = \int d^3x [\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \cdots]$$



- Coupling with hadrons via σNN (ξ=$1/m_\sigma$):

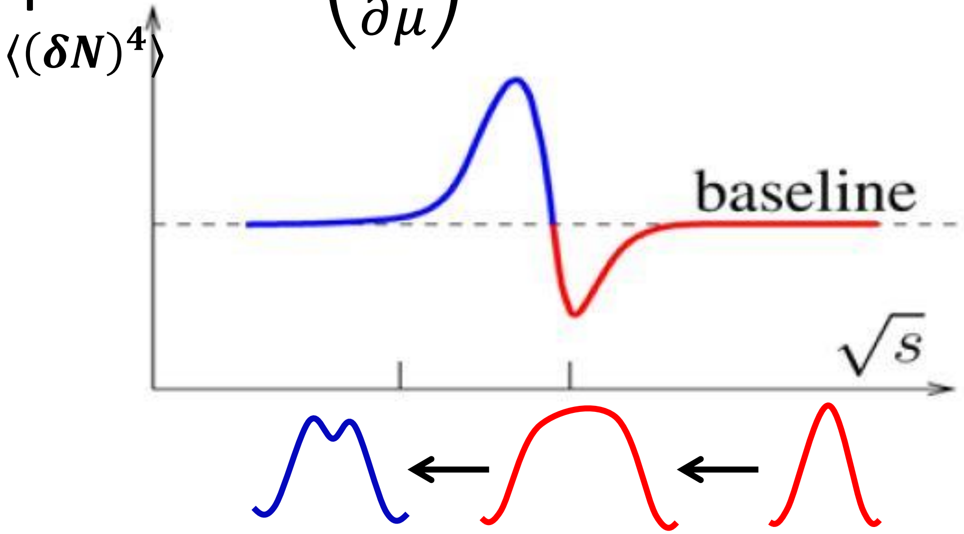
$$<(\delta N)^2> \sim \xi^2, \; <(\delta N)^3> \sim \xi^{9/2}, \; <(\delta N)^4> \sim \xi^7$$

Higher cumulants (ratios) are sensitive the shape of distribution

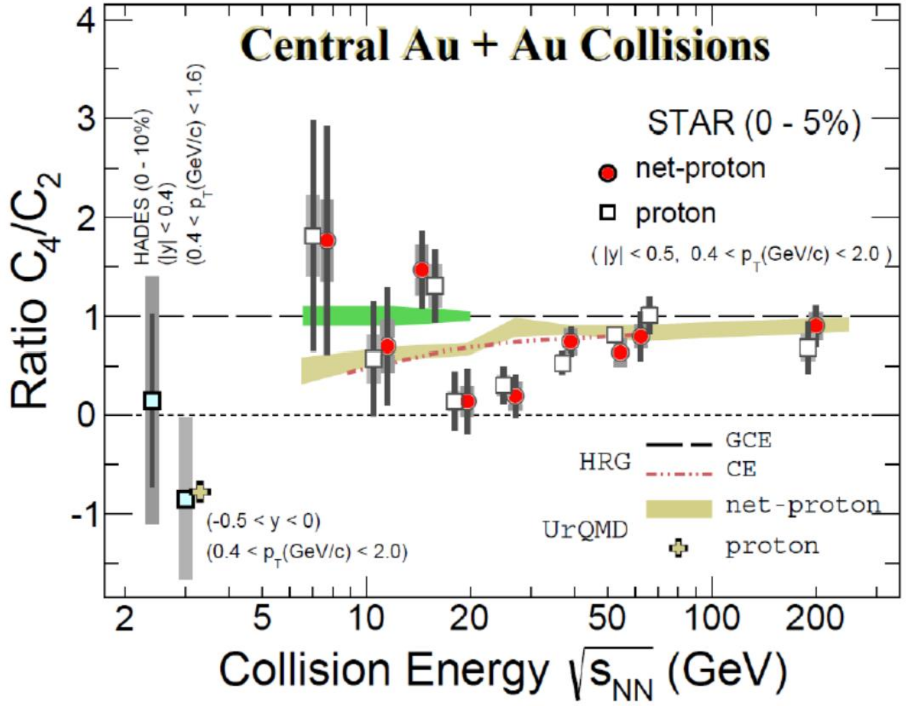
Net-proton fluctuations v.s. $\sqrt{s_{NN}}$

- Characteristic feature of critical point:
  - long range correlation
  - large fluctuations
- **Non-monotonicity** of Net-Proton Cumulant

$$\sim \text{susceptibilities} \left(\frac{\partial}{\partial\mu}\right)^n \ln Z$$



M.Stephanov, PRL 107,052301



STAR, PRL 126,092301
STAR,PRL 128,202303

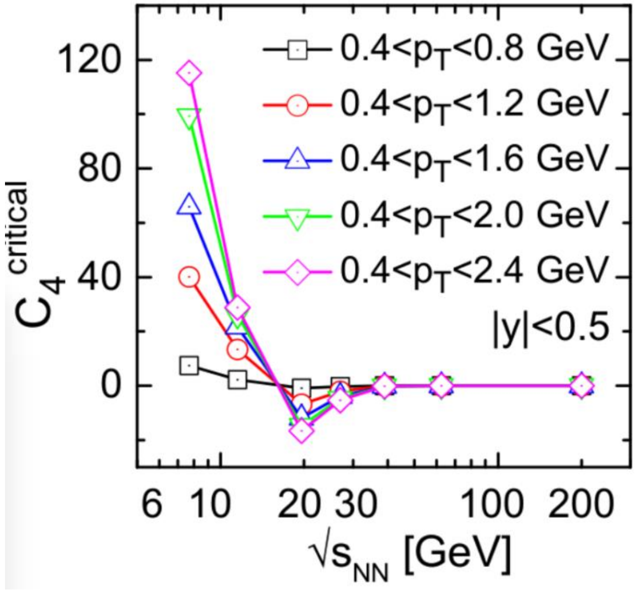
Long-range correlation => acceptance dependence

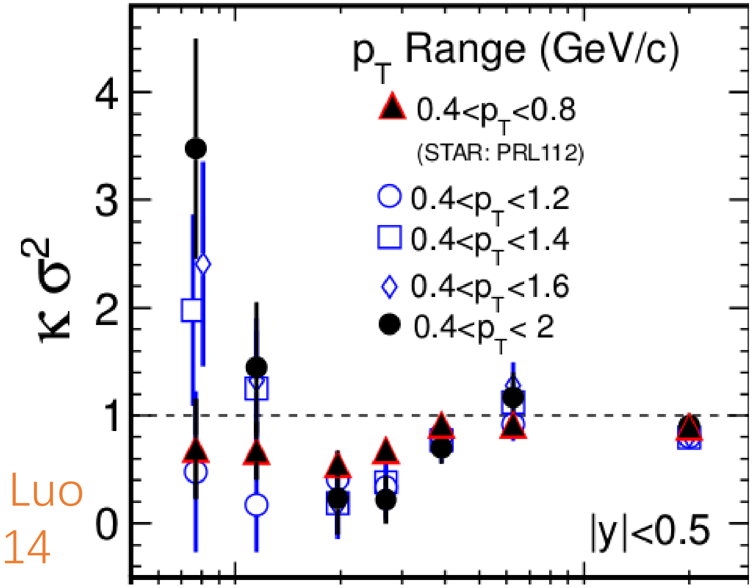
Jiang,Li,Song,PRC (2016)
Ling,Stephanov,PRC (2016)

- Multiplicity

$$N = \int_{p_{min}}^{p_{max}} \frac{d^3p}{E} \int_{\Sigma} \frac{p_\mu d\sigma^\mu}{2\pi^3} f(x,p)$$

Larger acceptance, **more particle correlated**



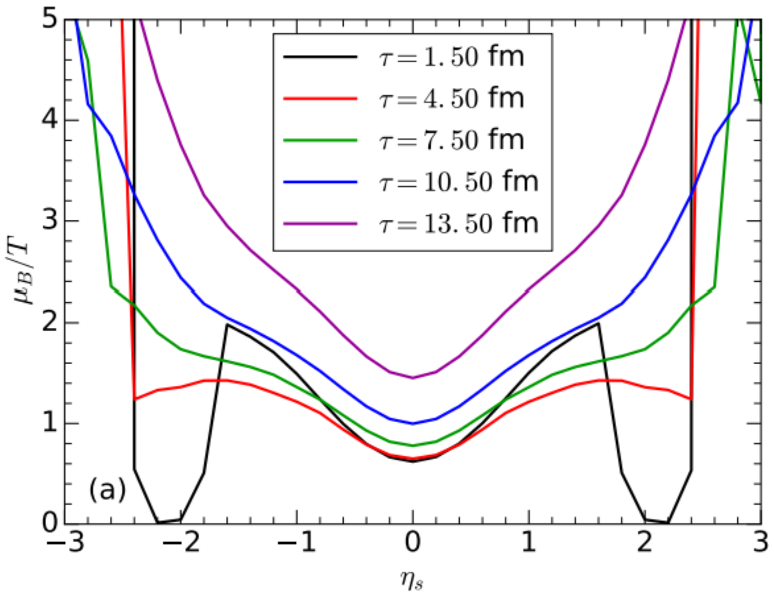


Xiaofeng Luo
CPOD 2014

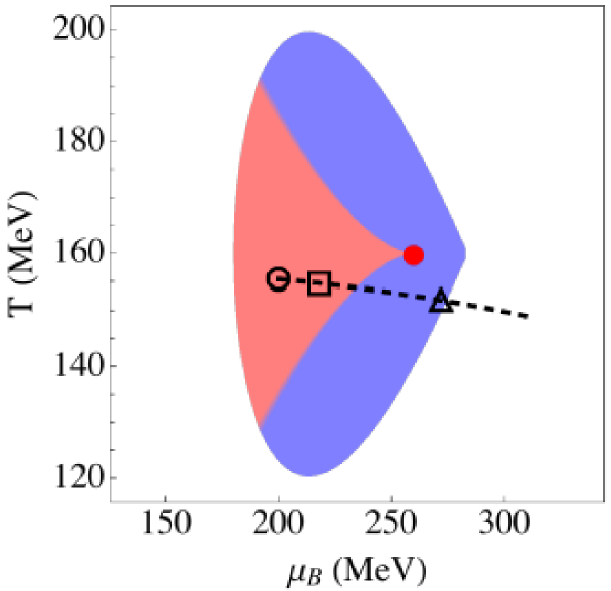
Acceptance dependence is non-trivial

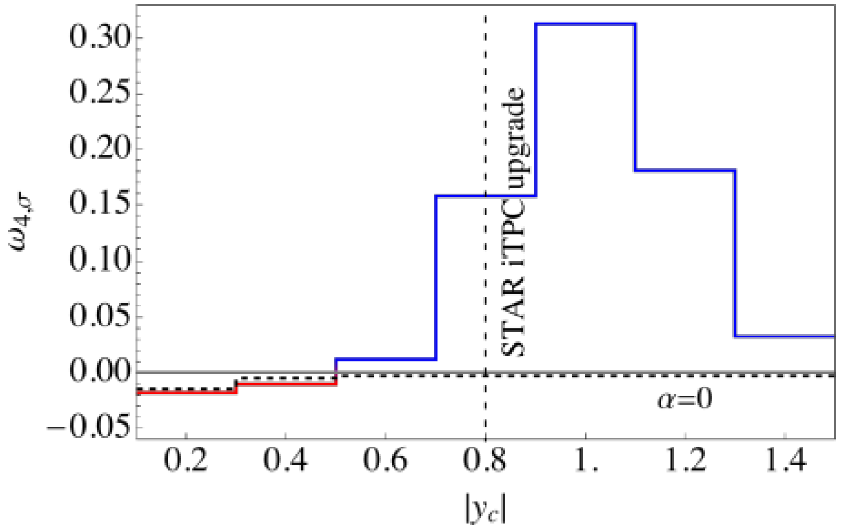
- Non-trivial
- Larger acceptance->**different critical region**

G. S. Denicol et al, PRC 98,034916



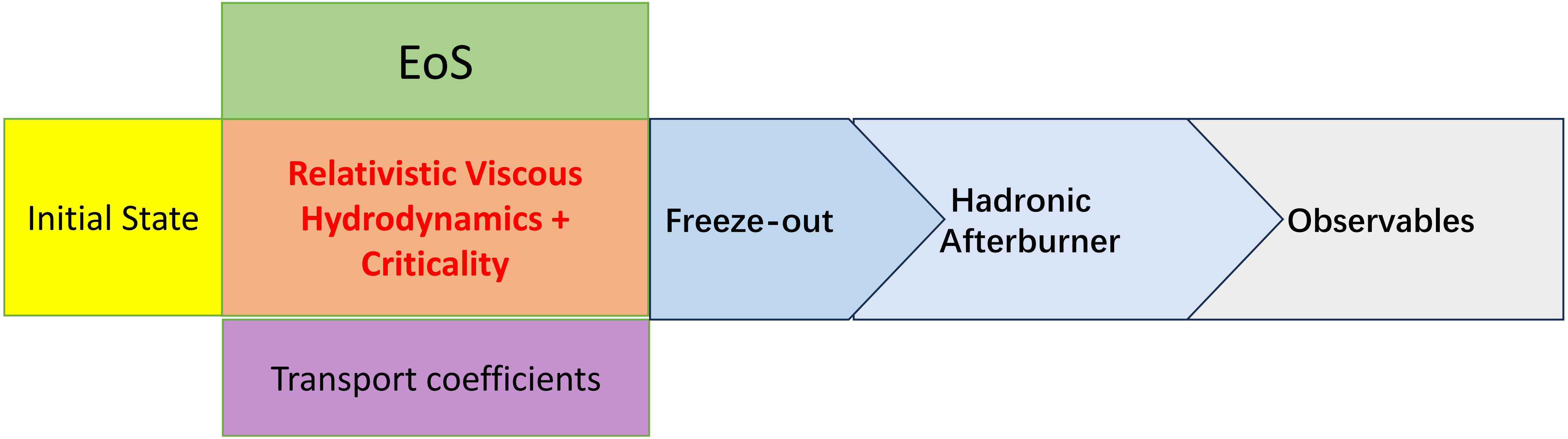
J. Brewer et al., 1804.10215





Non-equilibrium critical fluctuations

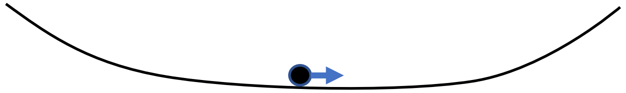
Jacquelyn Noronha-Hostler, CPOD2022



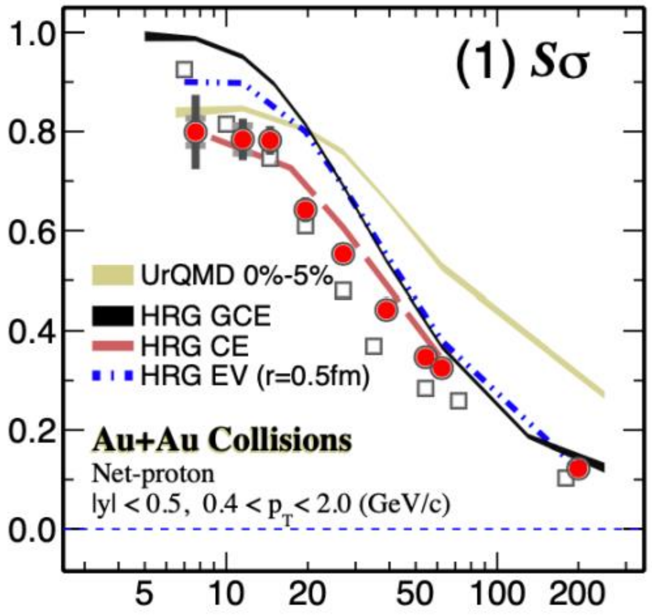
=>Bayesian analysis

Critical slowing down modifies cumulants

=>Long range correlation!
=>Critical slowing Down!



Slow, near critical point



Probability of order parameter $\sigma = \langle \bar{q} q \rangle$:

$$P[\sigma] \sim \exp\{F[\sigma]/T\},$$

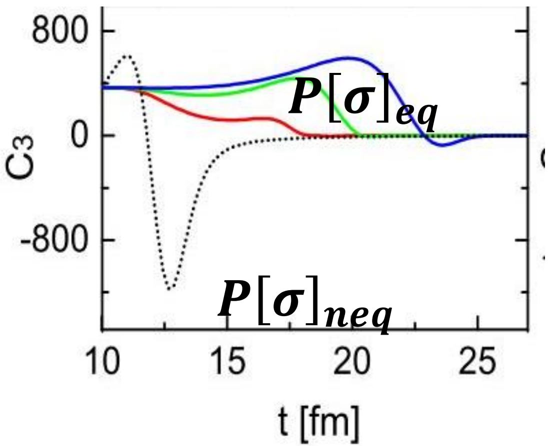
$$F = \int d^3x [\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \cdots]$$

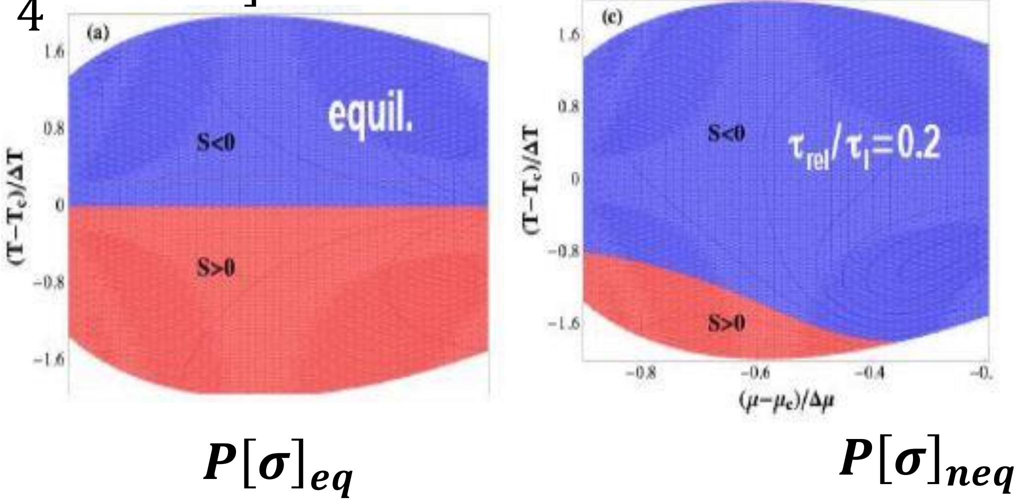
Fokker-Planck equation: $\partial_t P[\sigma(t)] = \cdots$

Mukherjee,Venugopalan, Yin, PRC 92.034912(2015)

Langevin equation: $\partial_t \sigma(t) = \cdot$

L.Jiang et al., 2112.04667

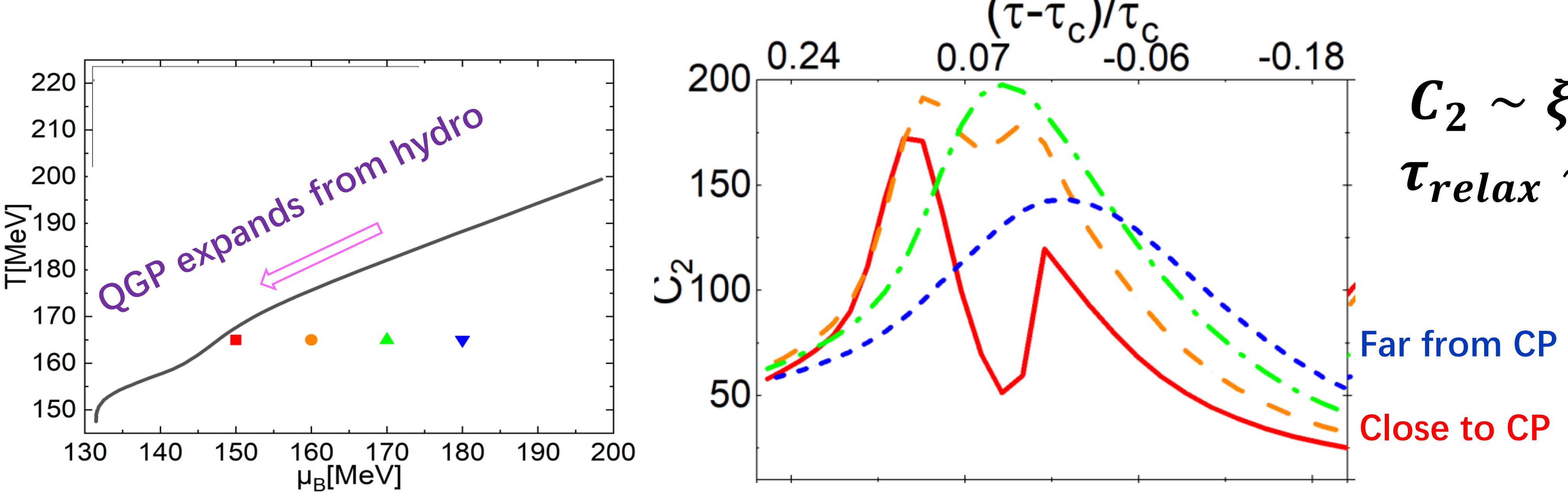




Largest fluctuations≠closest to critical point

S.Tang, SW, H.Song, 2303.15017

- Hydro background cools down => Critical Slowing Down.
- Critical slowing down effects suppress the fluctuations
- Fireball closer to critical point, Larger fluctuations but **larger suppression**



$C_2 \sim \xi^2$

$\tau_{relax} \sim \xi^z, z = 3$

Build the dynamical model near QCD critical point

- Near critical point, $\xi \to \infty$, only some macroscopic degree of freedoms are relevant.
- Only needs to build the dynamical model according to dynamical universality class.

P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. 49, 435 (1977)

- Near QCD critical point, dynamical system belongs to **Model H**.
- Relevant d.o.f for model H:
    - Conserved order parameter $(\sigma + n)$
    - Conserved transverse momentum
    - Poisson bracket between them

D. T. Son and M. A. Stephanov, Phys. Rev. D 70, 056001 (2004).

NχFD

- Non-equilibrium chiral fluid dynamics(NχFD). Langevin equation for sigma field: damping and noise from the interaction with the quarks:

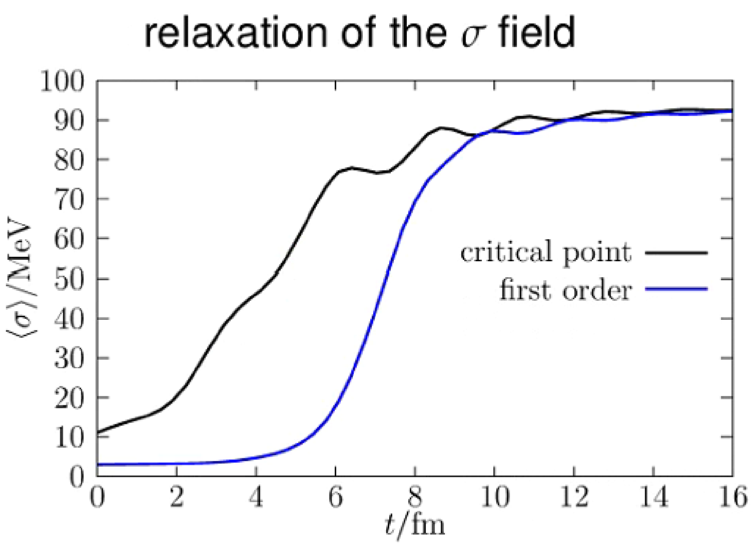
$$\eta \partial_\tau \sigma = \frac{\delta U}{\delta \sigma} + \text{noise}$$

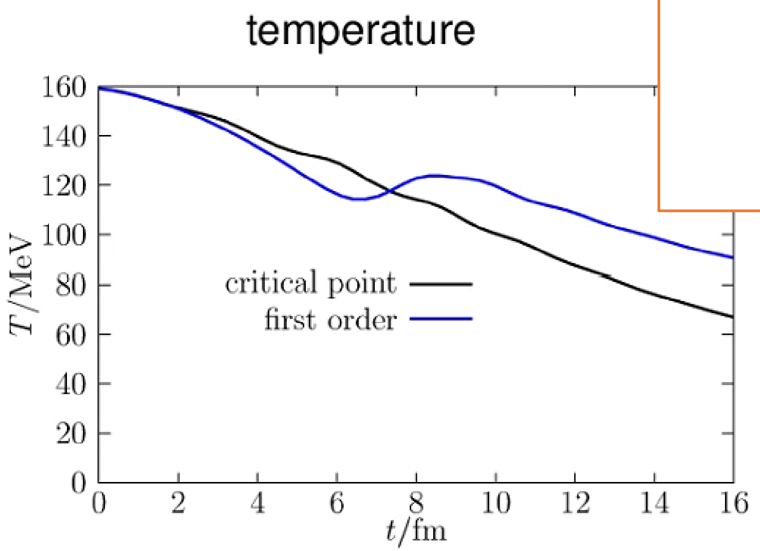
Marlene et al., arXiv:1105.1962

- Fluid dynamics expansion of the quark fluid=heat bath

$$\partial_\mu T_q^{\mu\nu} = S^\nu = -\partial_\mu T_\sigma^{\mu\nu}$$

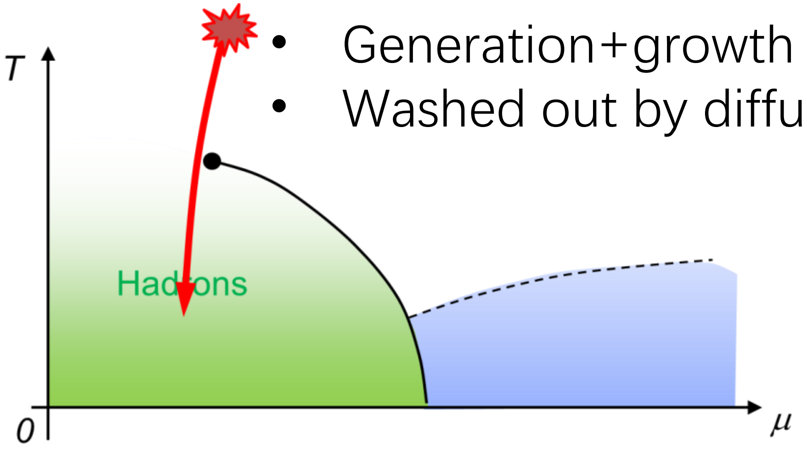
- Relevant d.o.f for model H:
  - ✓ Conserved order parameter ($\sigma + n$)
  - ✓ Conserved transverse momentum
  - ✓ Poisson bracket between them





Model B

- Relevant d.o.f for model H:
  - ✓ Conserved order parameter $(\sigma + n)$
  - X Conserved transverse momentum
  - X Poisson bracket between them



- Generation+growth
- Washed out by diffusion

- Dynamics of conserved charge:

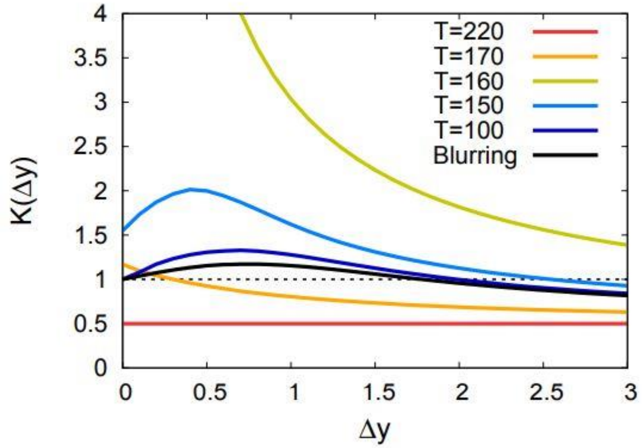
$$\partial_\tau n = \nabla^2 n + \text{noise}$$

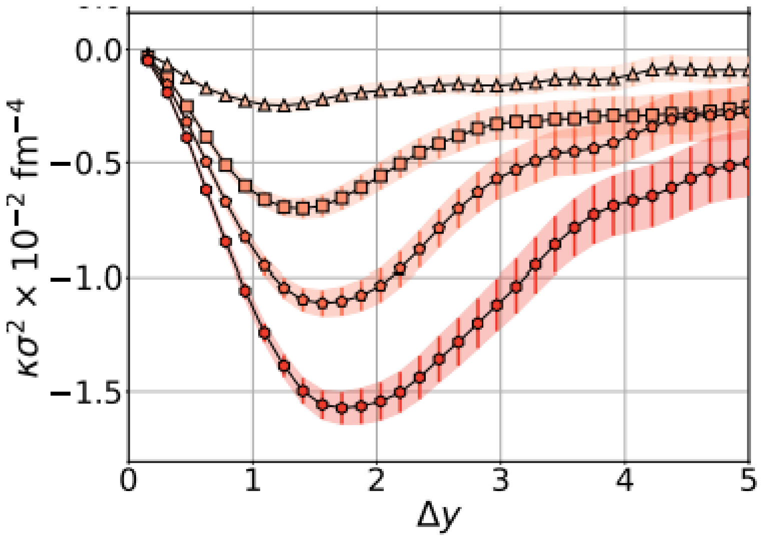
- Critical slowing down compete with diffusion

→non-monotonic

→Oscillation w.r.t accept.?

Sakaida et al, PRC.95.064905(2017)





- Dynamics of conserved charge with non-Gaussian terms:
- $\partial_\tau n = \nabla^2(n + n^2 + n^3) + \text{noise}$

G.Pihan et al., PRC.107.014908(2022), S.Wu et al., in progress

Hydro-Fluctuations

- Relevant d.o.f for model H:
  - X Conserved order parameter $(\sigma + n)$->noise
  - ✓ Conserved transverse momentum
  - ✓ Poisson bracket between them

- Thermal fluctuations in hydrodynamics equations:

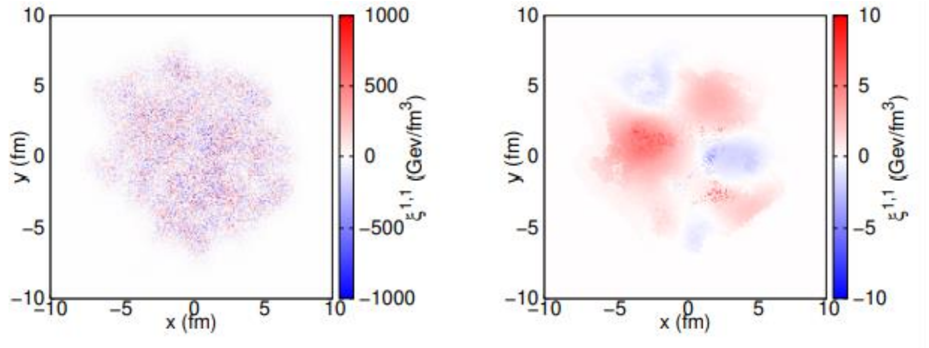
T.Hirano et al., arXiv:1809.04773
M.Singh et al., arXiv:1807.05451

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + T^{\mu\nu}_{\text{viscous}} + S^{\mu\nu}$$

- Thermal noise follows the fluctuation-dissipation theorem

$$\langle S^{\mu\nu}(x_1) S^{\mu\nu}(x_2)\rangle \sim 2T\eta\delta^4(x_1 - x_2) \sim 2T\eta/\Delta t \Delta x^3$$

- Depends the cut-off, requires renormalization



- Hydro-kinetic, determistic hydro-fluctuations:
- $\left[(u+v)\cdot\bar{\nabla} + f\cdot\partial_q\right]W_+ = -\gamma_L q^2\left(W_+ - W^0\right) + K''W_+$

Y.Akamatsu et al., arXiv:1606.07742
X.An et al., arXiv:1902.09517

Hydro+

- Relevant d.o.f for model H:
  - X Conserved order parameter ($\sigma + n$)->slow mode
  - ✓ Conserved transverse momentum
  - ✓ Poisson bracket between them

- Hydrodynamics equations:

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + T^{\mu\nu}_{\text{viscous}}$$

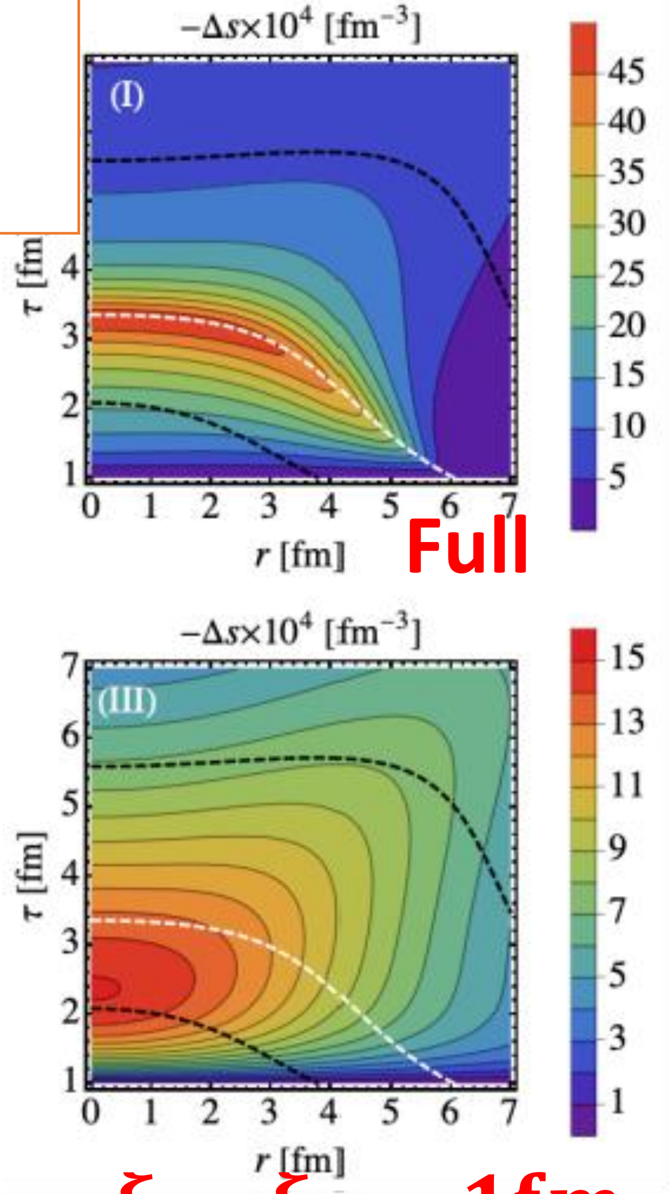
- Slow mode equation

$$(u^{\mu}\partial_{\mu})\phi(x^{\mu}; Q) = -\gamma(Q)[\delta s_{(+)}(\epsilon, n, \phi)/\delta\phi]$$

- $s_{(+)}(\epsilon, n, \phi)$is the renormalized entropy to include the slow mode effects

- Slow mode effects: $\frac{\Delta s}{s} \sim \mathcal{O}(10^{-4})$

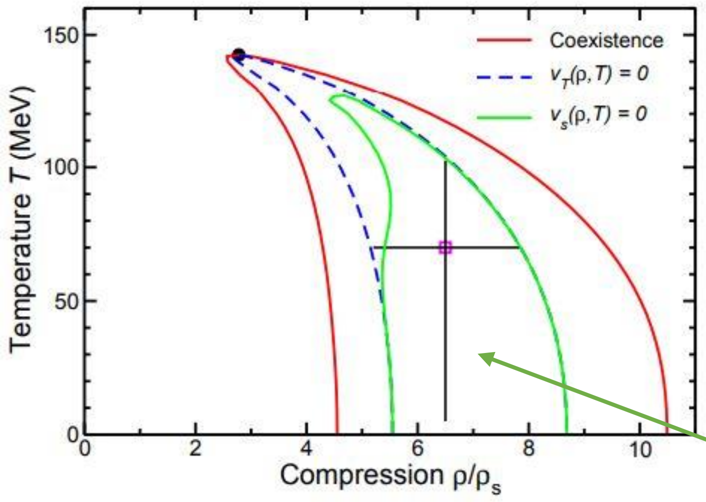
M.Stephanov et al., arXiv:1712.10305
K.Rajagopal et al., arXiv: 1908.08539
L.Du et al., arXiv:2004.02719



$\xi = \xi_0 = 1\text{fm}$

Spinodal decomposition at 1st order region

J.Randrup et al, PRC('09,'10,'13,'14),PRL('12)



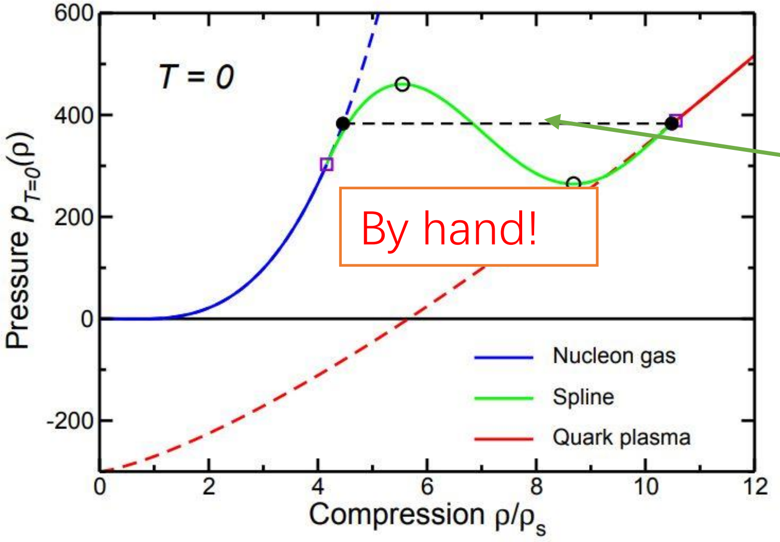
- Confined phase: ideal gas of pions nucleons+interaction

$$p^H = p_\pi + p_N + p_{\bar{N}} + p_w$$

- Deconfined phase: ideal gas of quarks and gluon:

$$p^Q = p_g + p_q + p_{\bar{q}} - B$$

- Interpolation: spline

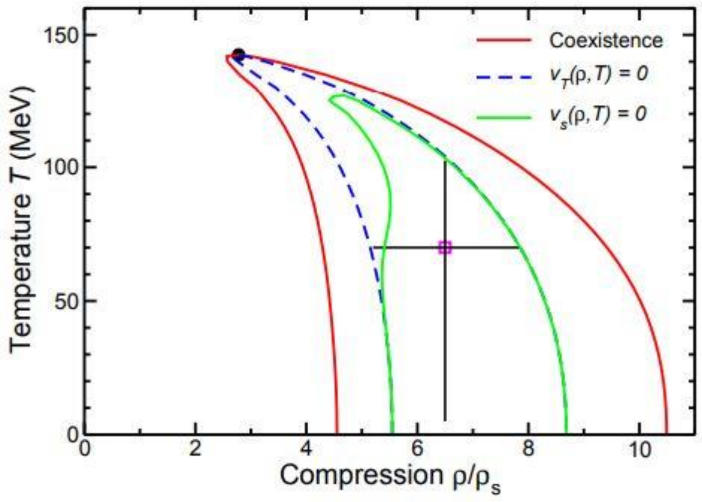


$c_s^2 < 0$

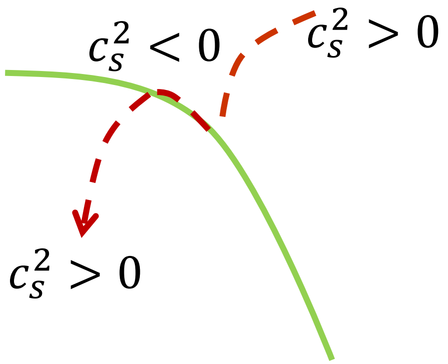
Unstable!

Spinodal decomposition at 1st order region

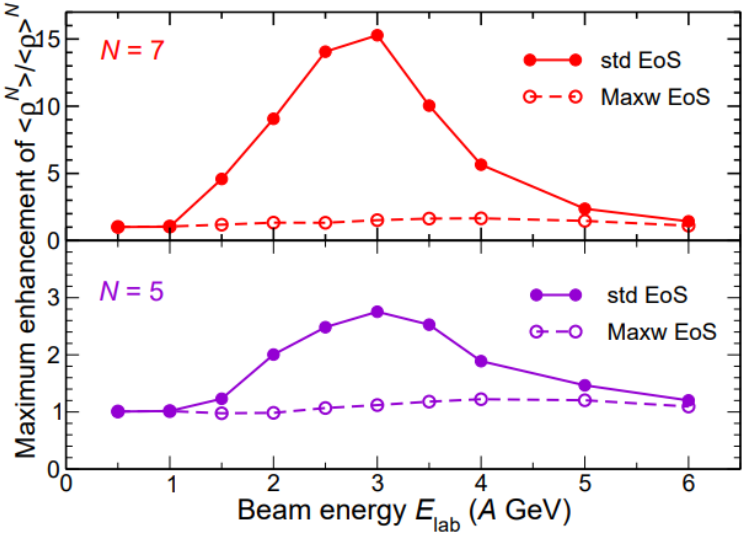
J.Randrup et al, PRC('09,'10,'13,'14),PRL('12)



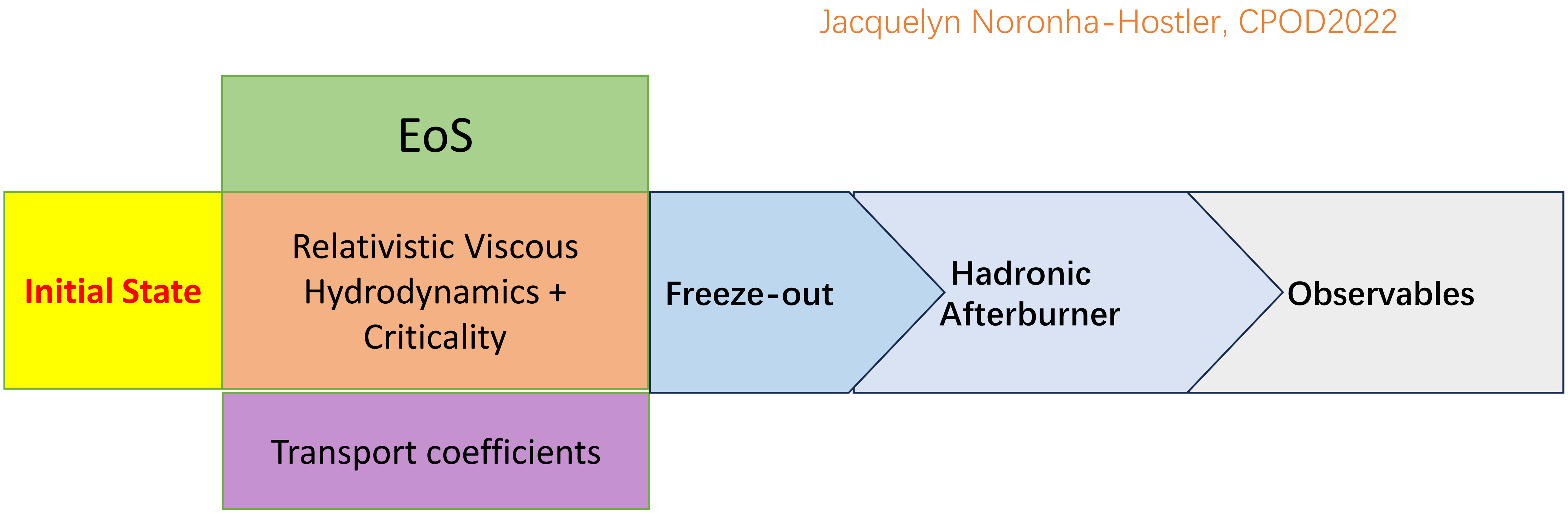
- Confined phase: ideal gas of pions nucleons+interaction
$$p^H = p_\pi + p_N + p_{\bar{N}} + p_w$$
- Deconfined phase: ideal gas of quarks and gluon:
$$p^Q = p_g + p_q + p_{\bar{q}} - B$$
- Interpolation: spline



$$e = e_0\left(\frac{\tau_0}{\tau}\right)^{1+c_s^2}$$



Initial condition

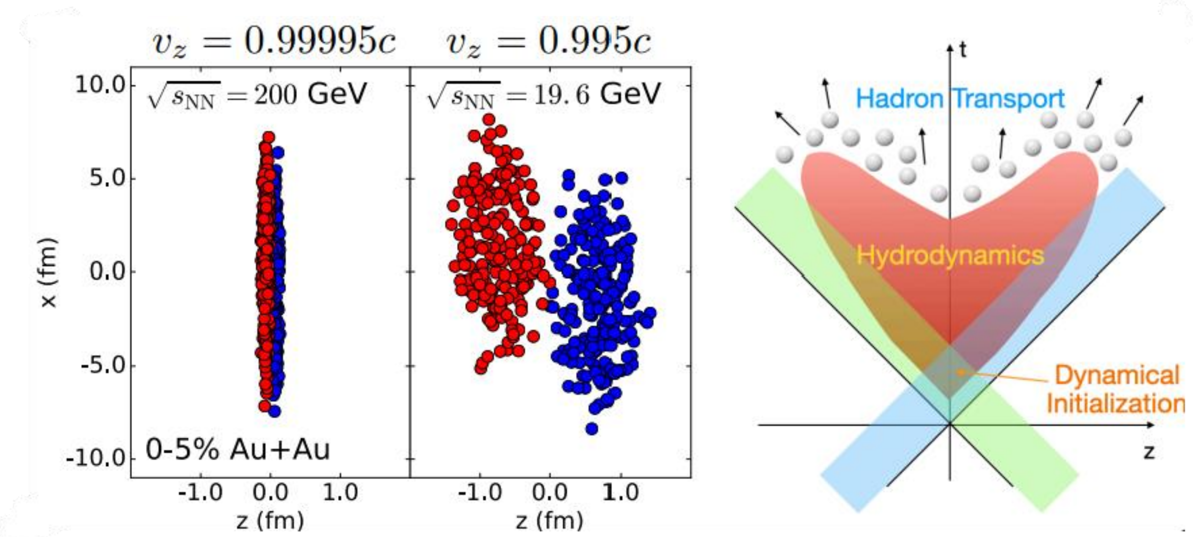


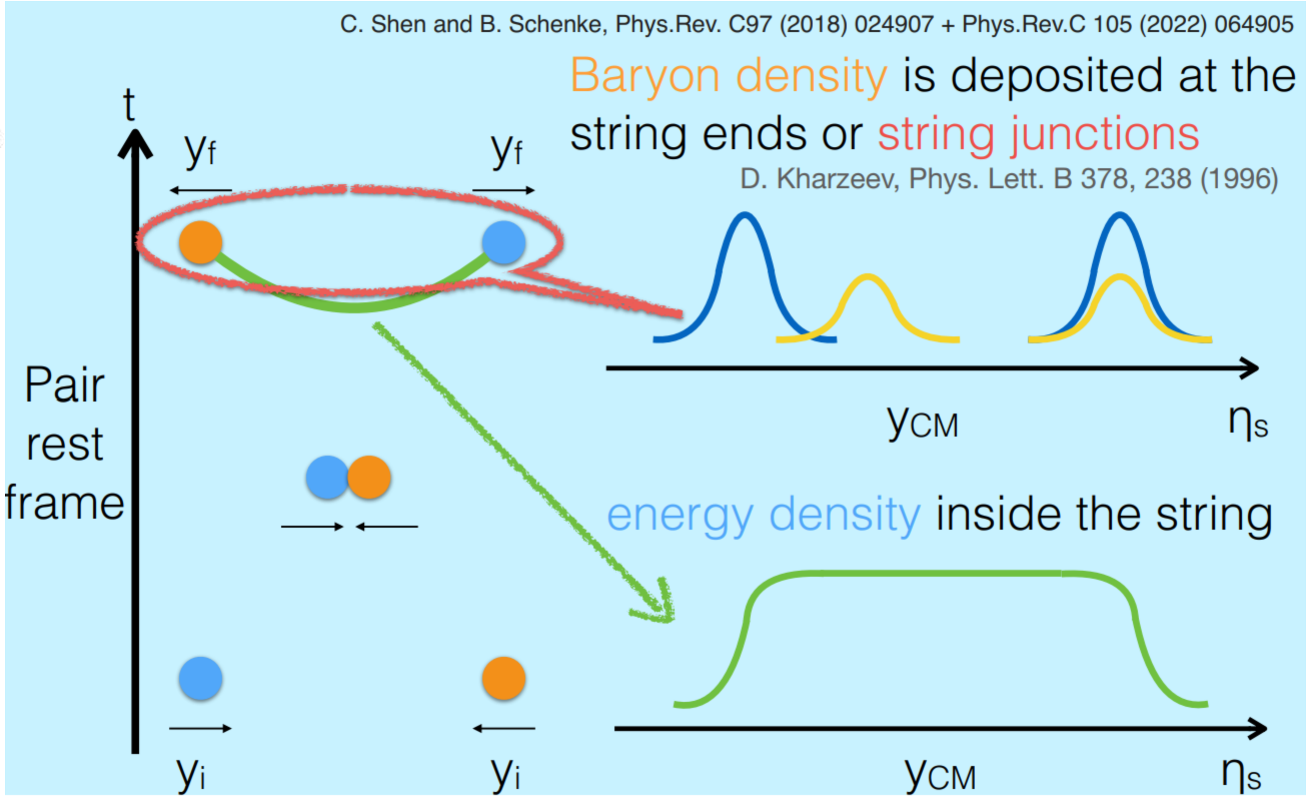
=>Bayesian analysis

Dynamical initial condition at lower energies

Chun Shen QM2023

$$\partial_\mu T^{\mu\nu} = J^{\nu}_{source}$$
$$\partial_\mu J^{\mu} = \rho_{source}$$

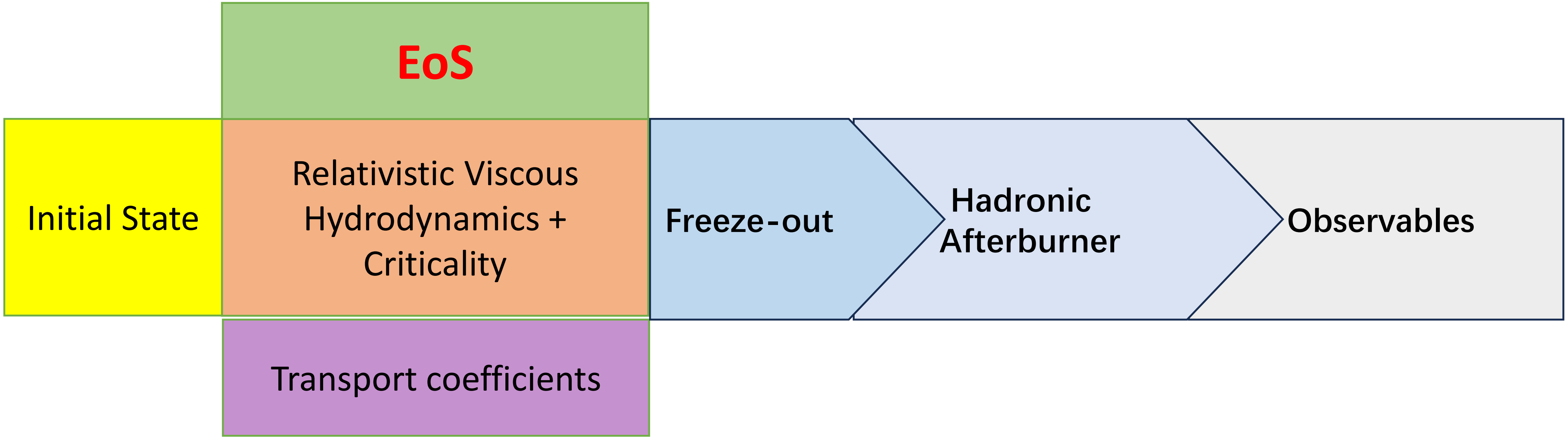




Requires initial condition for critical field

Equation of state

Jacquelyn Noronha-Hostler, CPOD2022



=>Bayesian analysis

Equation of state

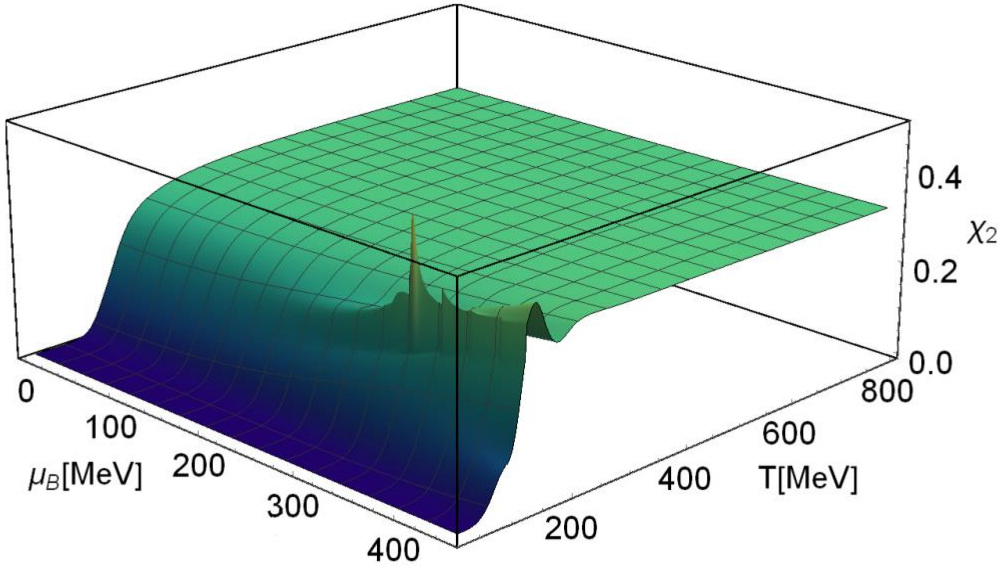
P. Parotto et al., 1805.05249

Lattice-based EoS(3-D Ising, same universality class):

$$P(T,\mu) = P_{\text{Lattice}} + P_{\text{Ising}}$$

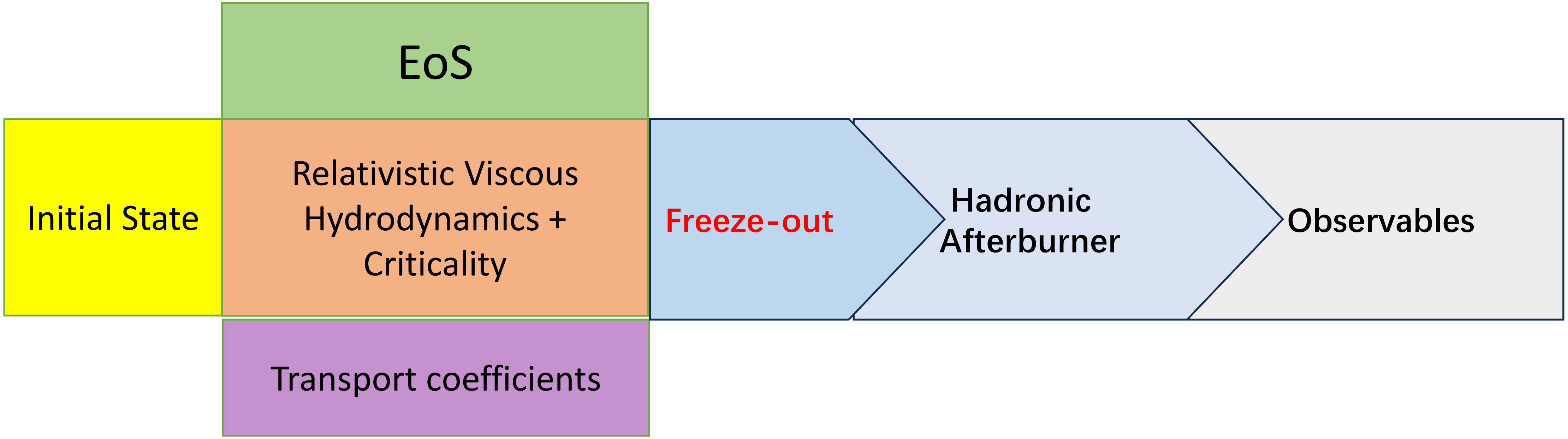
EoS also from QCD-based theories,

See Prof. Fu and Gao's talk



Freeze-out

Jacquelyn Noronha-Hostler, CPOD2022



=>Bayesian analysis

Freeze-out hydro+

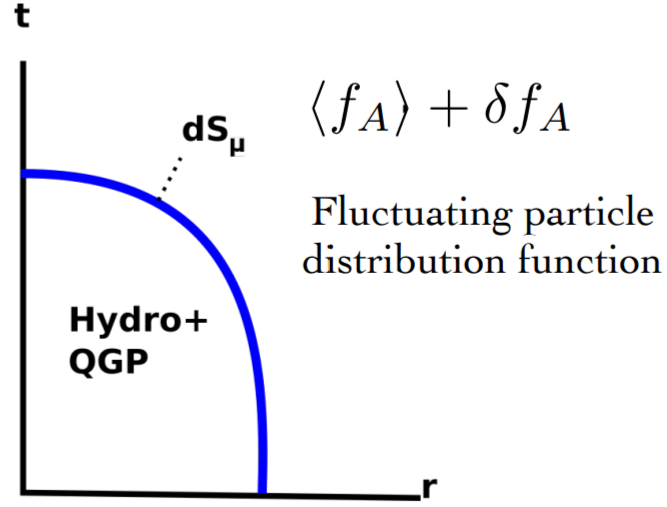
Maneesha Pradeep QM2023

Particle masses modification due to the interaction with critical sigma field

$$\delta m_A = g_A \sigma$$

Fluctuating particle distribution function

$$f_A = \langle f_A \rangle + g_A \frac{\partial \langle f_A \rangle}{\partial m_A} \sigma$$



Where the critical field:

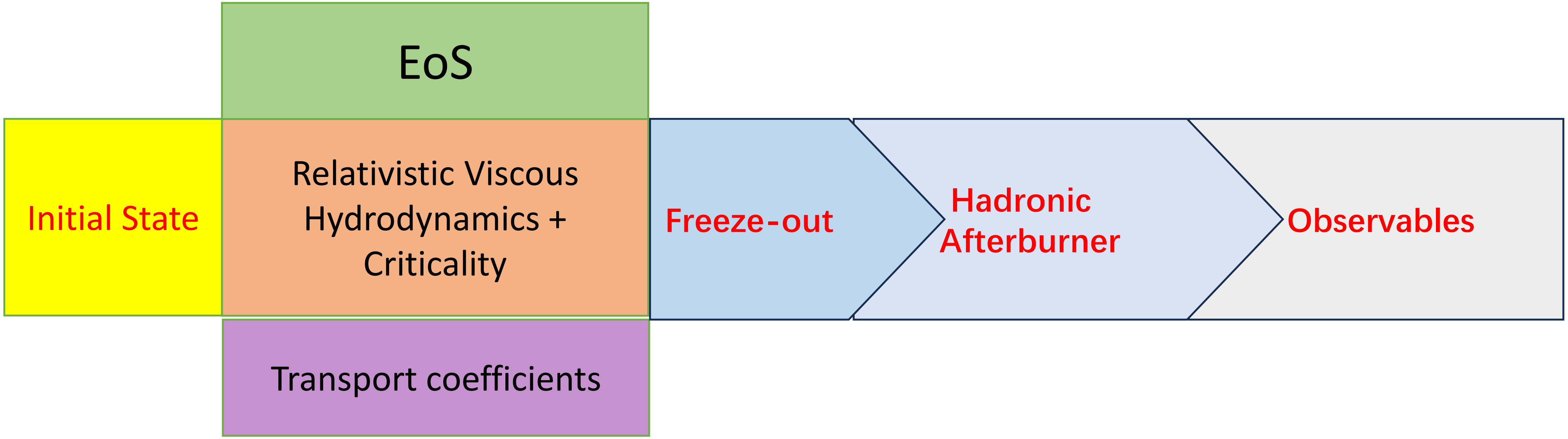
$$\langle \sigma \rangle = 0, \langle \sigma(x_+)\sigma(x_-) \rangle \sim \text{hydro}_+ \text{ slow mode}$$

And the proton fluctuations:

$$\langle \delta N_A \delta N_B \rangle = \langle N_A \rangle \delta_{AB} + \# \int_\Sigma \int_\Sigma \langle \sigma(x_+)\sigma(x_-) \rangle$$

Non-critical fluctuations

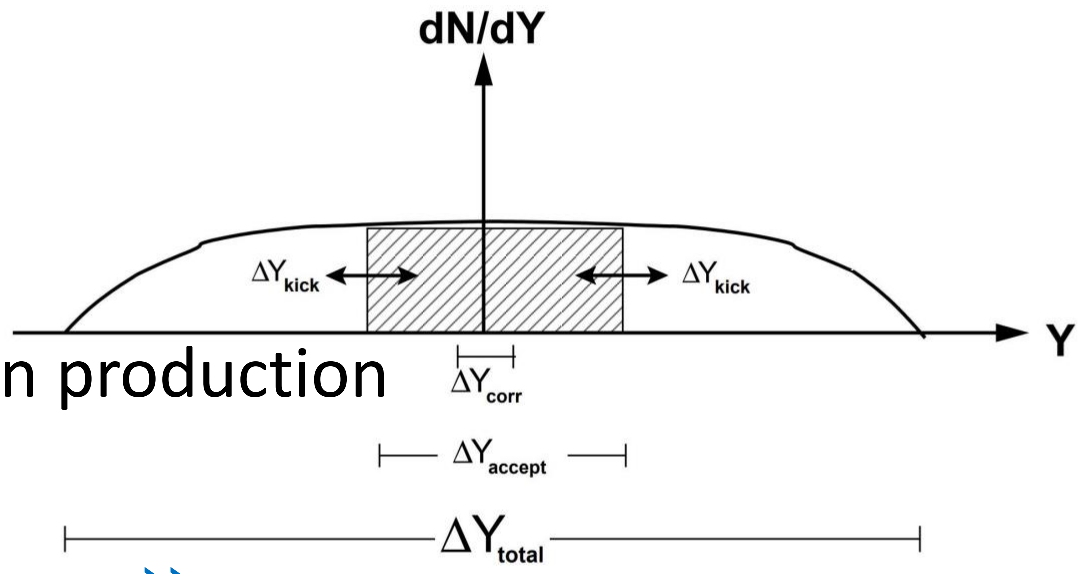
Jacquelyn Noronha-Hostler, CPOD2022



=>Bayesian analysis

Conservation

- Poisson distribution, uncorrelated proton production



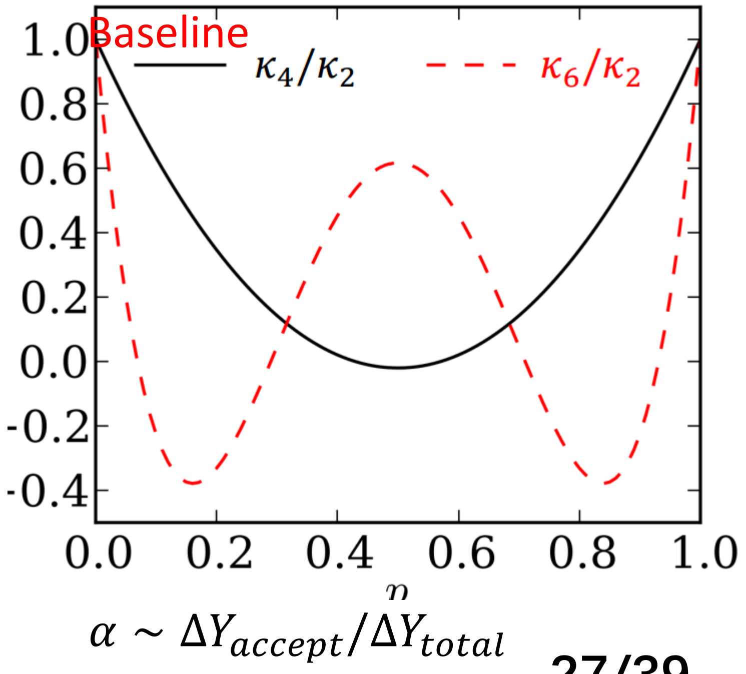
V. Koch, 0810.2520

- Grand-canonical ensemble applies if $\Delta Y_{total} \gg$ ΔY_accept. If $\Delta Y_{total} \gg$ ΔY_accept does not hold. corrections from conservation appear

scaled variance $\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1-\alpha)\,\frac{\chi_2^B}{\chi_1^B},$

skewness $\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1-2\alpha)\,\frac{\chi_3^B}{\chi_2^B},$

kurtosis $\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta\left(\frac{\chi_3^B}{\chi_2^B}\right)^2.$



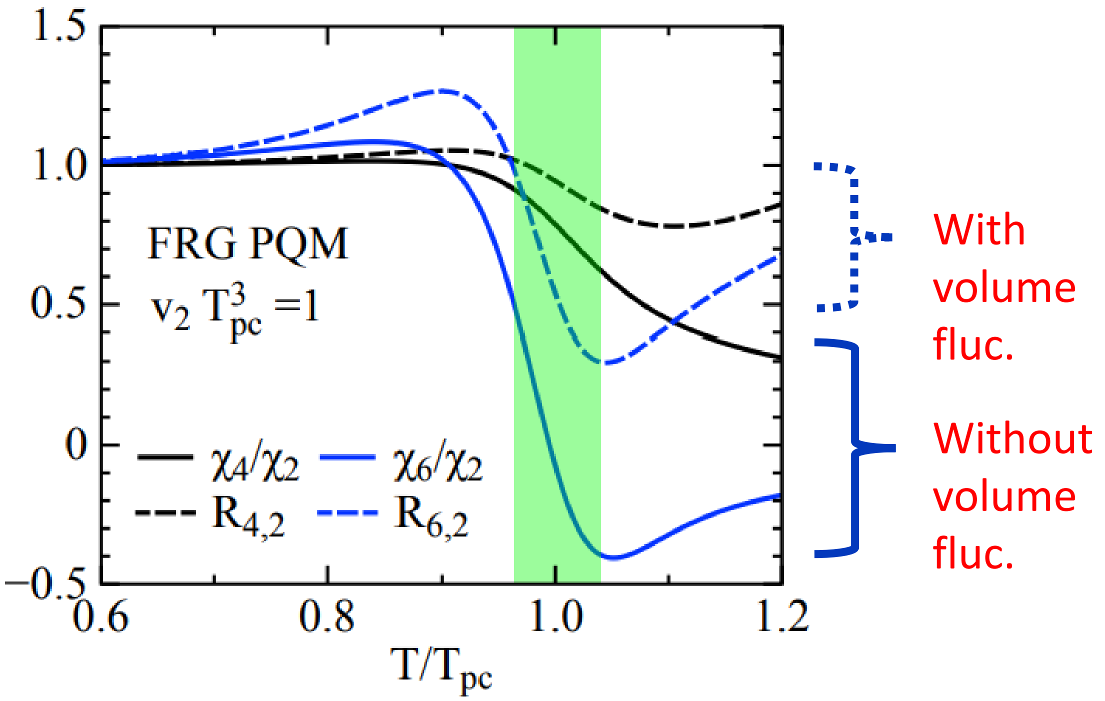
Bzdak et al., 1203.4529
V. Vovchenko et al., 2003.13905

Volume fluctuations

- Relates to the centrality selection, a source of initial fluctuations
- Overcome it by centrality bin width correction

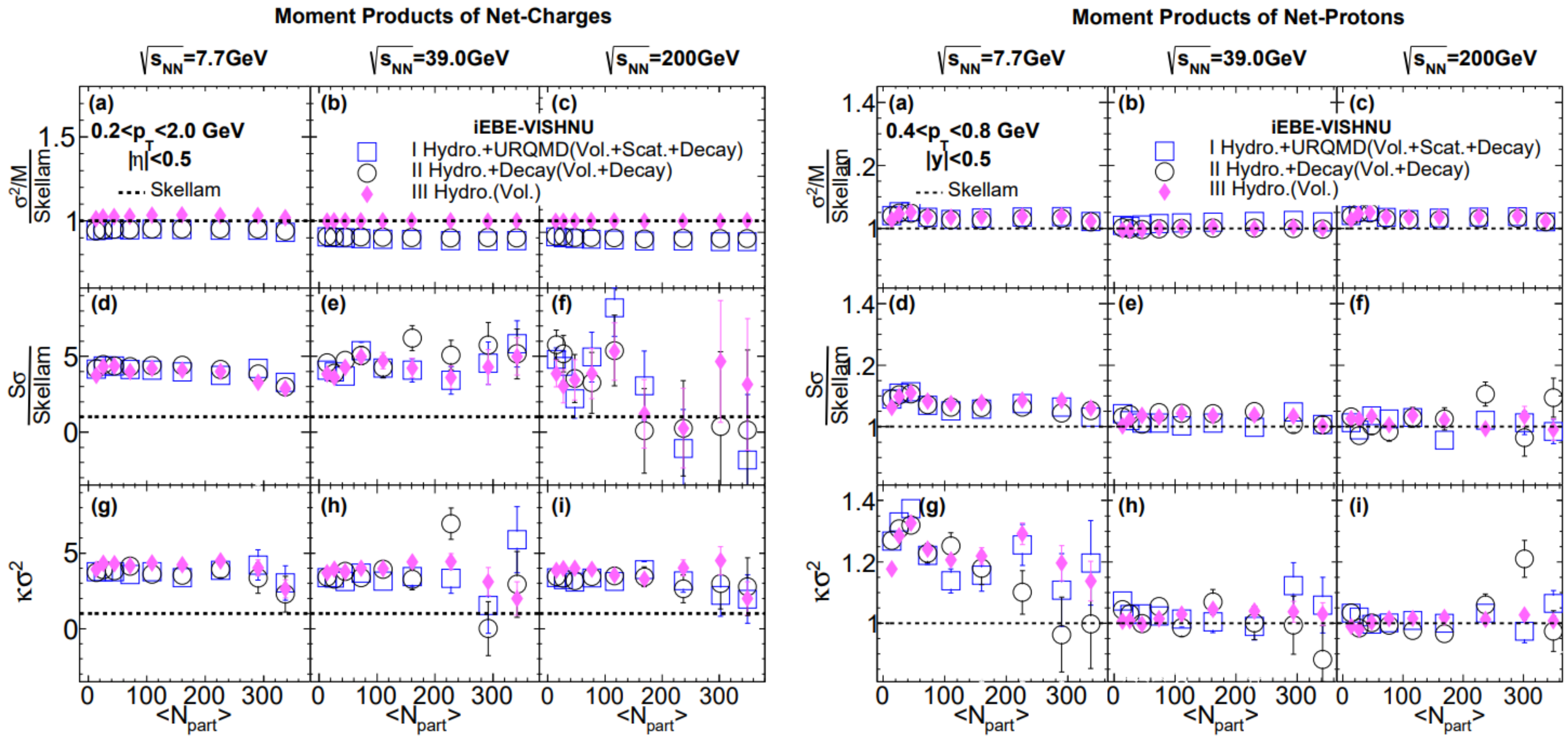
$$\sigma = \sum_r \omega_r \sigma_r \,, S = \sum_r \omega_r S_r, \kappa = \sum_r \omega_r \kappa_r$$

- $\omega_r = n_r / \sum_r n_r$, n_r is the number of events for r^{th} multiplicity



X.Luo et al., 1302.2332
V.Skokov et al., 1205.4756
M.I.Gorenstein et al., 1101.4865

Other sources of non-critical fluctuations



J.Li et al., 1707.09742

iEBE-VISHNU

	Volume fluc. for σ	Volume fluc. for $S\sigma$	Volume fluc. For $\kappa\sigma^2$	Scatt. and decay
Net-charge	small	Large	Large	Small
Net-proton	small	Small	Small	Small

Observables

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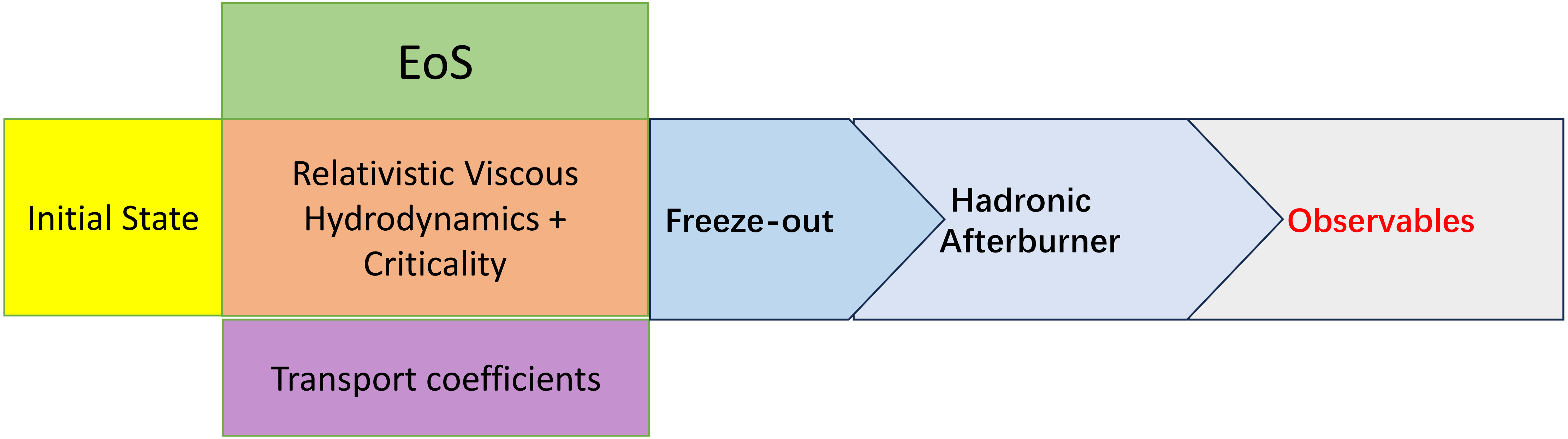
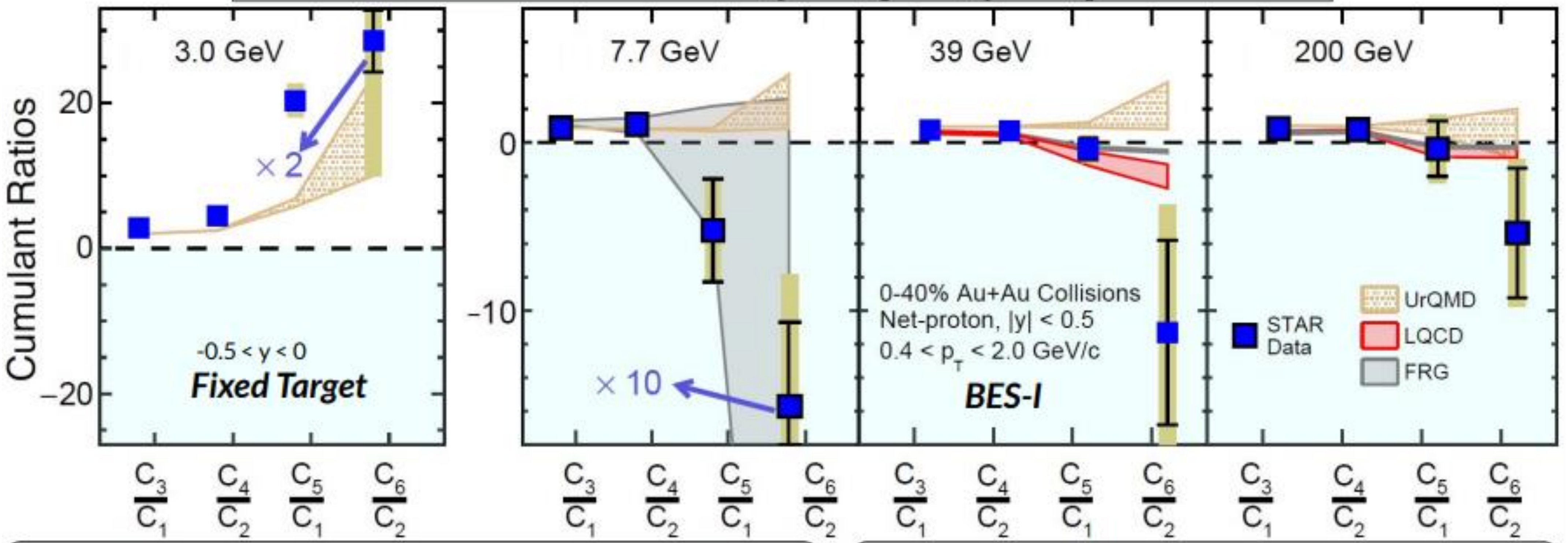


=>Bayesian analysis

Higher-order cumulants(C_5, C_6)

Ordering of ratios (Net-baryon): $\frac{C_3}{C_1} > \frac{C_4}{C_2} > \frac{C_5}{C_1} > \frac{C_6}{C_2}$ - LQCD, FRG

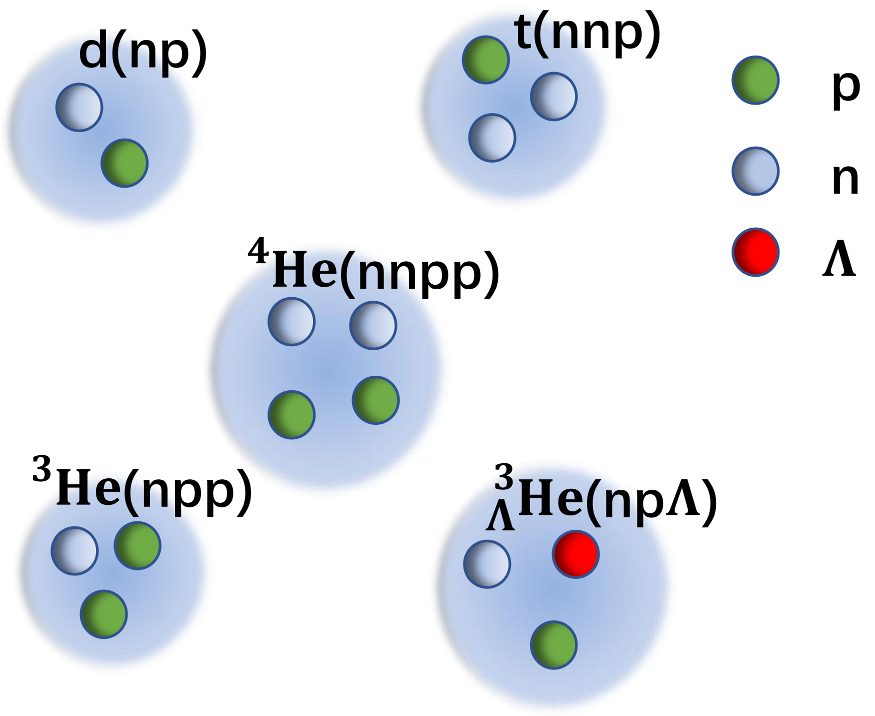


- Violation of ordering found at fixed target $\sqrt{s_{NN}}$ = 3 GeV
  - Trend reproduced by UrQMD
  - Suggests hadronic matter

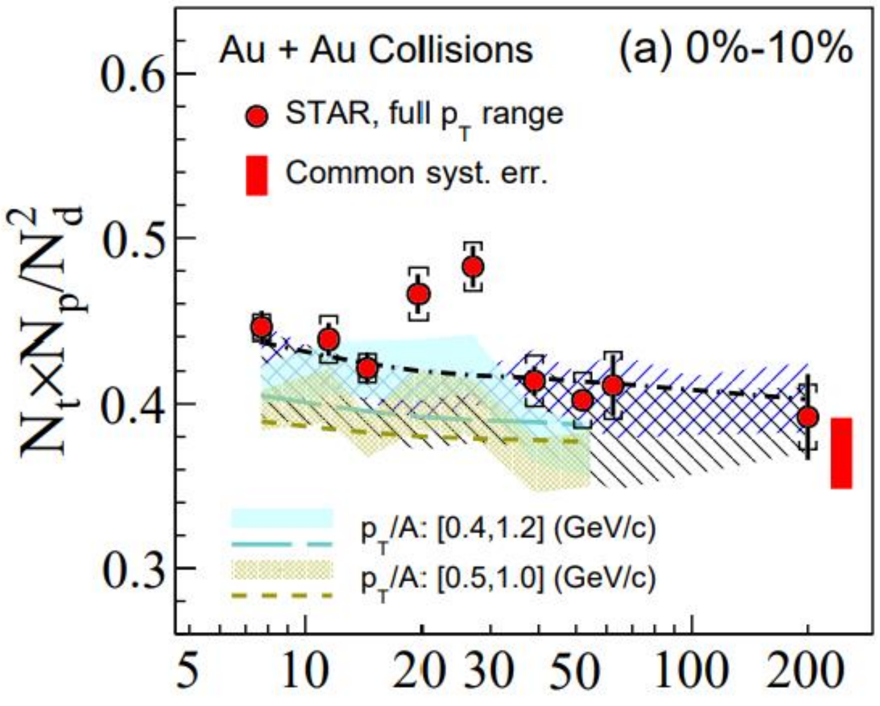
- Data trends appear consistent with predicted hierarchy between $\sqrt{s_{NN}}$ = 7.7 and 200 GeV

STAR: Phys Rev Lett 130, 082301 (2023)

Light Nuclei Production



STAR Collaboration, PRL 130.202301



- **Non-monotonic behavior** also been observed

  K-J.Sun et al., PRC (2021) …. K-J.Sun et al., Phys. Lett. B, 781:499–504(2018)
  K-J.Sun et al., Phys. Lett. B,774:103–107(2017)
  W.Zhao et al., PRC (2018), S.Wu et al., PRC(2022)…..

Light-Nuclei Yield with E-by-E Fluctuations

K.Murase, **SW,** In preparation

Single Event

- Non-trivial background distribution in a single event

$$f(\boldsymbol{r},\boldsymbol{p}) = f_0(\boldsymbol{r},\boldsymbol{p})$$

Assumption: No(Critical) fluctuations

Event-by-Event

$$f^{(1)}(\boldsymbol{r},\boldsymbol{p}), f^{(2)}(\boldsymbol{r},\boldsymbol{p}), f^{(3)}(\boldsymbol{r},\boldsymbol{p}) \cdots$$

$$N_A^{(1)}, \quad N_A^{(2)}, \quad N_A^{(3)} \cdots \to \langle N_A \rangle$$

Event average

- Initial fluctuations
- Hydro fluctuations
- Fluctuations induced by hard process
- etc..

Light-Nuclei Yield with E-by-E Fluctuations

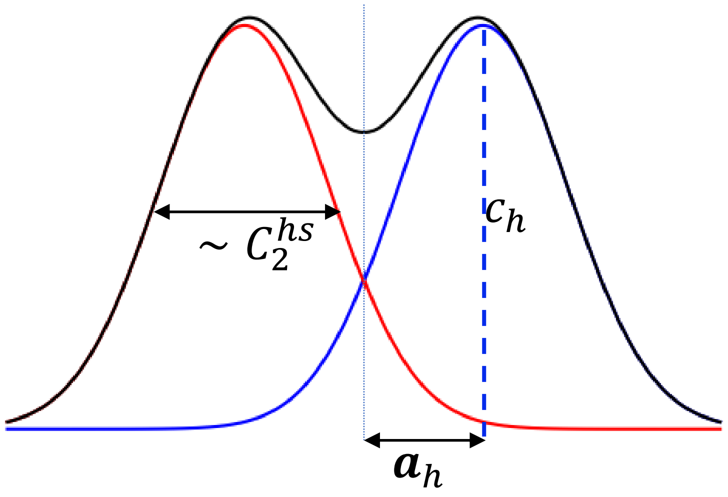
K.Murase, **SW**, In preparation

Light Nuclei in Single Event

$$N_A^f(\{c_h, \boldsymbol{a}_h\}) = g_A \int [\prod_i^A d^6 \boldsymbol{z}_i f(\boldsymbol{z}_i)_{\{c_{h_i}, \boldsymbol{a}_{h_i}\}}] W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^A)$$

Fluctuating Gaussian Profile

$$f(\boldsymbol{z})_{\{c_h, \boldsymbol{a}_h\}} = \frac{1}{n} \sum_{h=1}^{n} c_h \frac{1}{\sqrt{\det(2\pi \mathcal{C}_2^{hs})}} \exp[-\frac{1}{2}(\boldsymbol{z} - \boldsymbol{a}_h)^T (\mathcal{C}_2^{hs})^{-1} (\boldsymbol{z} - \boldsymbol{a}_h)]$$



Light Nuclei in Event-Averaged

$$N_A = \int [\prod_{h=1}^{n} dc_h d^6 \boldsymbol{a}_h] \mathrm{Pr}(\{c_h, \boldsymbol{a}_h\}) N_A^f(\{c_h, \boldsymbol{a}_h\})$$

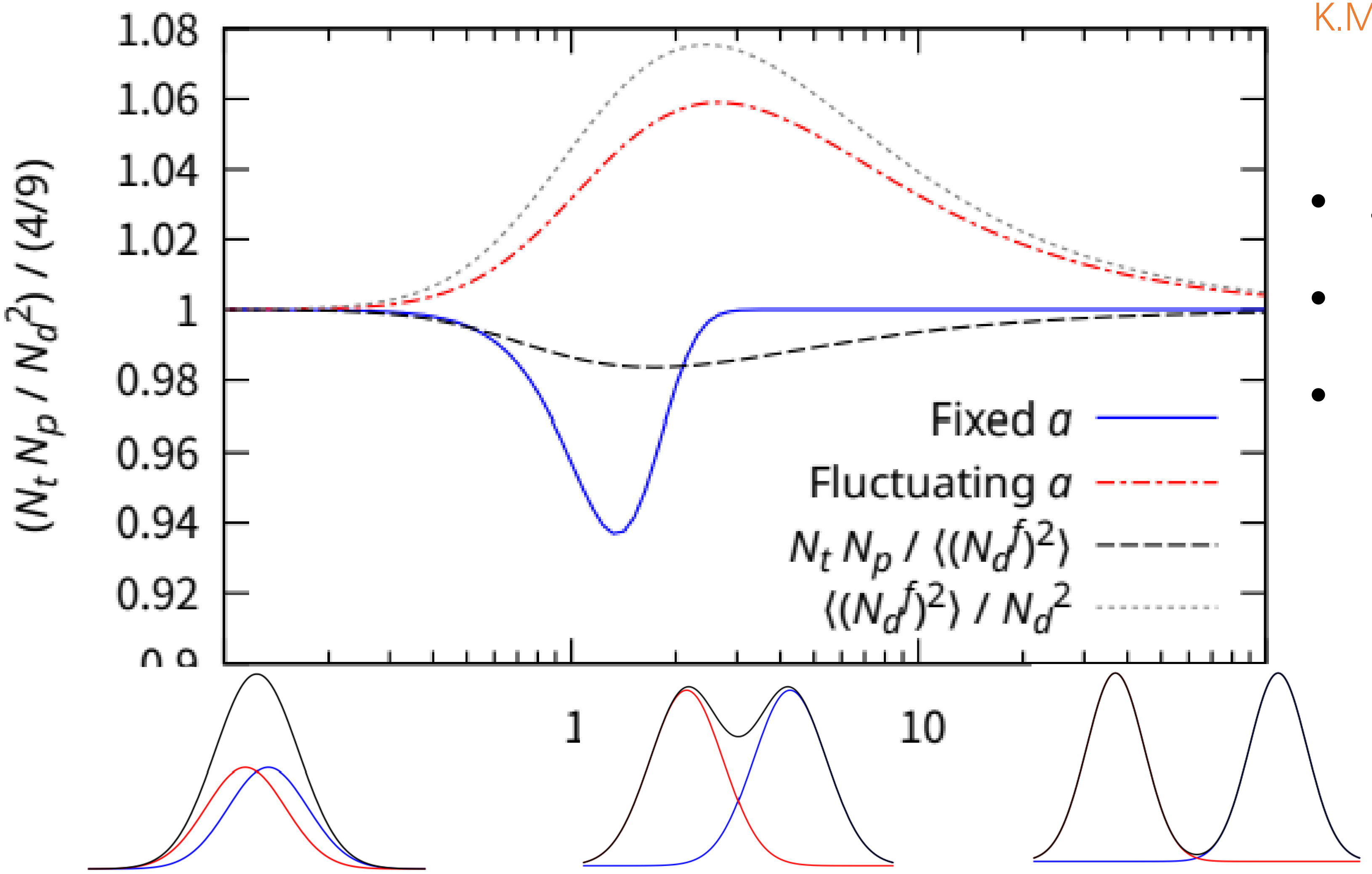
Distribution of Hot Spots

$$\mathrm{Pr}(\{c_h, \boldsymbol{a}_h\}) = \prod_{h=1}^{n} p(c_h) \frac{\exp[-\frac{1}{2}\boldsymbol{a}_h^T (\mathcal{C}_2^{hc})^{-1} \boldsymbol{a}_h]}{\sqrt{\det(2\pi \mathcal{C}_2^{hc})}}$$

$p(c_h)$: Distribution of hot spot magnitude c_h

C_2^{hc}: Covariance of hot-spot centers

Double-Gaussian(n=2) in 1d



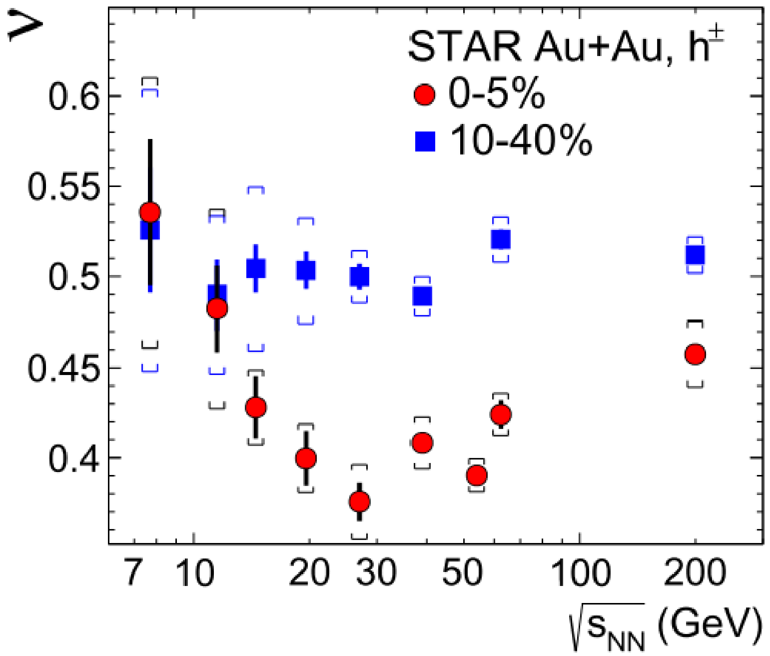
K.Murase, **SW,** In preparation

- Small a ~ 1 Gaussian
- Large a ~ sum of Gaussian
- Intermediate a ⇒ Non-Gaussian

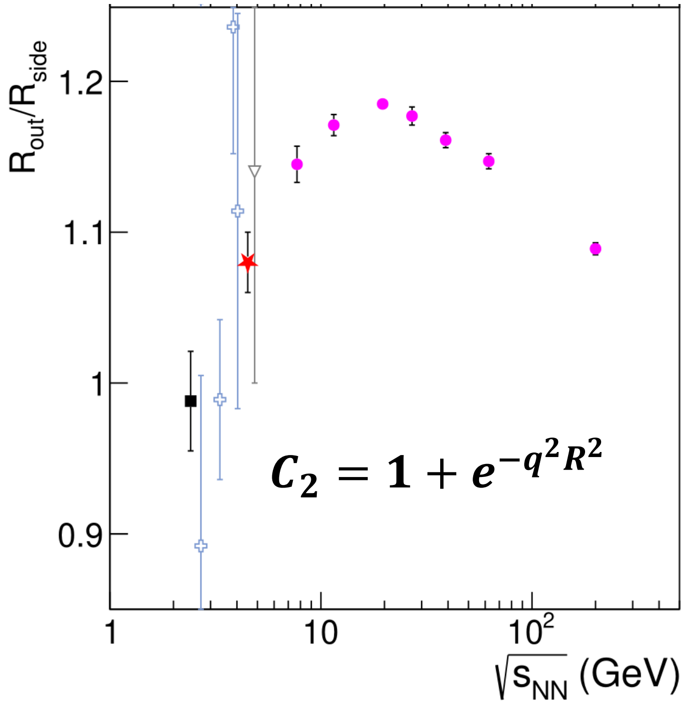
$$\frac{N_t N_p}{N_d^2} = \frac{N_t N_p}{\langle (N_d^f)^2 \rangle_f} \frac{\langle (N_d^f)^2 \rangle_f}{N_d^2}$$
$$= \frac{N_t N_p}{\langle (N_d^f)^2 \rangle_f} \left[1 + \frac{\langle (N_d^f - N_d)^2 \rangle_f}{N_d^2} \right]$$

Position fluctuations increase the ratio

Other possible observables



STAR PRC 103, 034908 (2021)



$$C_2 = 1 + e^{-q^2 R^2}$$

- v ~strength of intermittency,
- Intermittency~moments

STAR, 2301.11062

- HBT-radius

Bayesian analysis

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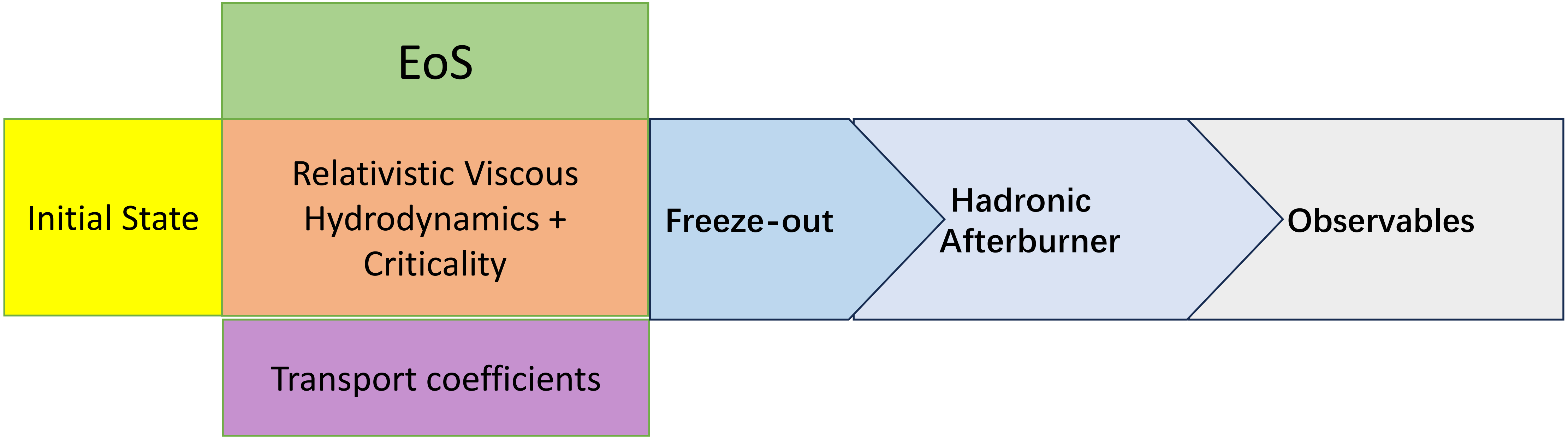
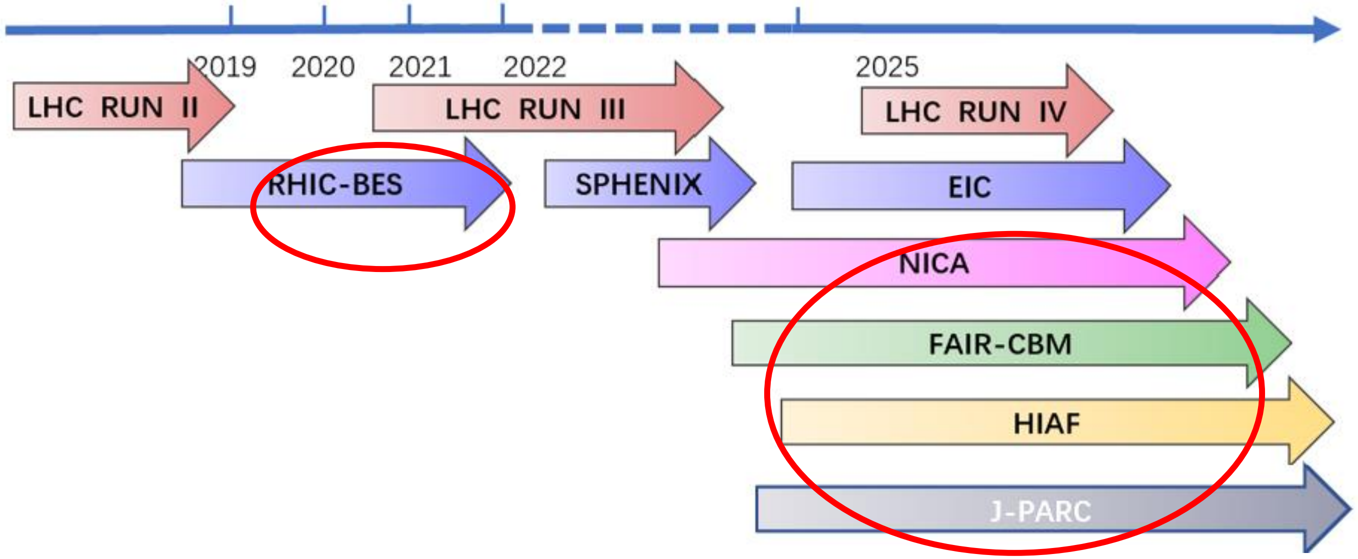
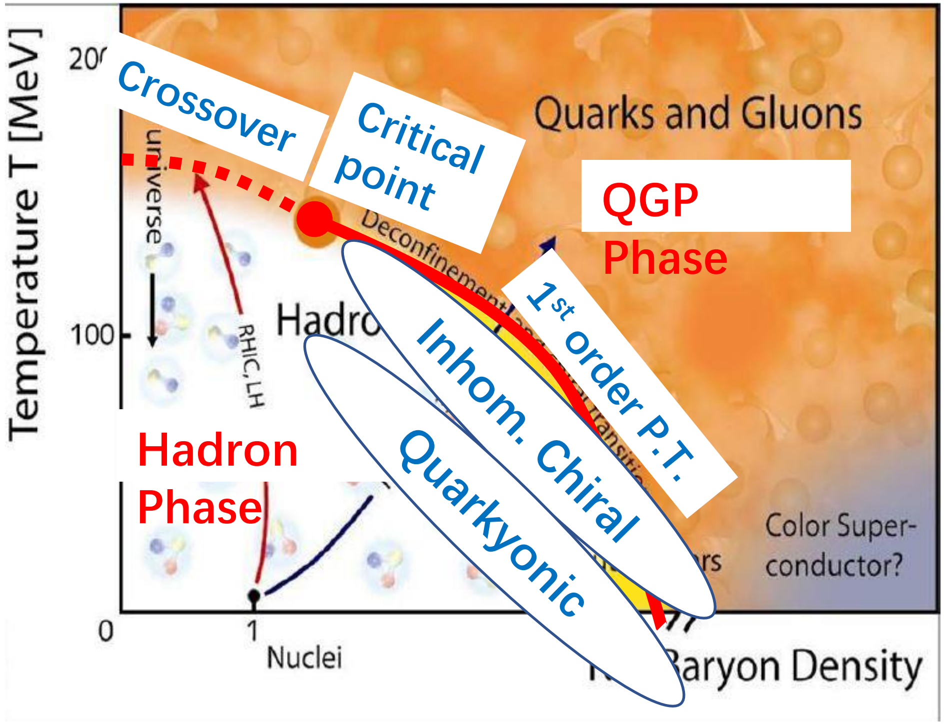


=>Bayesian analysis

Other topics and opportunities on QCD phase diagram





Larry McLerran
Fabian Rennecke, W.J.Fu, Fukushima
….

Conclusion and Outlook

- Exploring the QCD structure is one of the most important goals in Beam Energy Scan program at RHIC. Preliminary theoretical and experimental researches indicate the possible existence of QCD critical point.
- Theoretically, great efforts have been made approaching the hybrid model near the QCD critical point in heavy-ion collisions, including the studies on non-equilibrium critical fluctuations and non-critical fluctuations.
- Looking forward for the coming higher statistic data from BES-II and more exciting theoretical progress.

Thank You!