The equation of state of QCD matter in functional QCD approach

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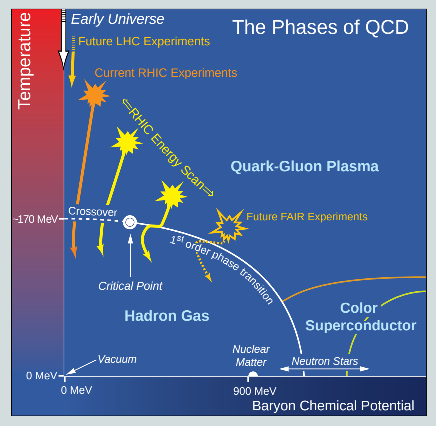
based on arXiv:2310.16345 and 2310.xxxxx

QCD is running from perturbative to non purturbative region as the energy scale changes

- The main character of QCD phase transition is chiral phase transition
- Fine structures to classify phases like inhomogeneous phase, chiral spin symmetric phase, color superconductivity phase
- The chiral phase transition driven by different combination of temperature and chemical potential, and connected with crossover at low density by critical end point (CEP)



[1] The frontiers of nuclear science, A long range plan[J]. 2008.

Framework of functional QCD method

Dyson-Schwinger equations (DSEs) and functional renormailization group (fRG) approach are the nonperturbative approach in continuum QCD which can capture the running behaivor of QCD

DSEs are the equations of motions in quantum field theory:

$$\frac{\partial S}{\partial \phi} = J$$

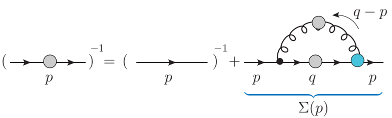
fRG is based on the idea of homotopy with one more dimension for the renormalization scale:

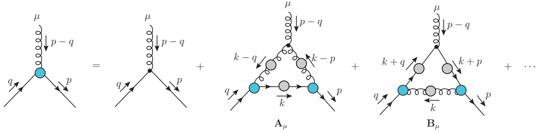
$$f(\lambda) = \int_0^\infty dx e^{-\lambda x^2} \rightarrow \frac{\partial f(\lambda)}{\partial \lambda} = -\int_0^\infty dx x^2 e^{-\lambda x^2} = -\frac{f(\lambda)}{2\lambda}$$

The functional QCD methods

The truncation is required in functional QCD methods as the equations are not closed.

- How to generally evaluate the truncation?
- How to reduce the higher order correction and make the truncation controllable?

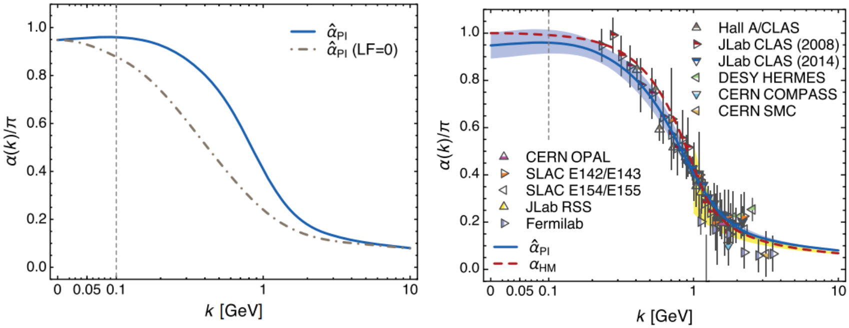




Effective charge

The hints from effective charge:

D. Binosi et al, PRD96, 054026 (2017); A. Aguilar et al, PRD80, 085018 (2009). A. Deur et al, Prog. Part.Nucl. Phys. 90, 1 (2016);

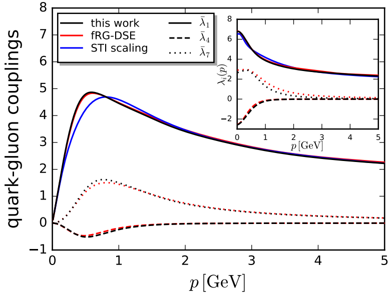


- The efforts beyond props are "perturbative", and can be captured by the inclusion of vertex.
- Might be a fixed point: defines an "perturbative" expansion in infrared and thus Only the running of propagator and vertex is singular.

quark gluon vertex

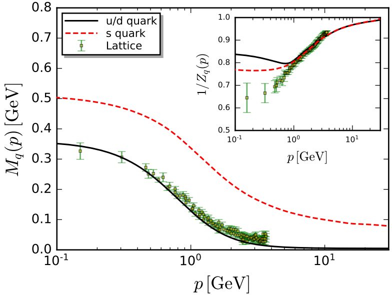
The quark gluon vertex is also complete with propagator and vertex DSEs.

The running coupling coincides with 2 loop running till 3 GeV.



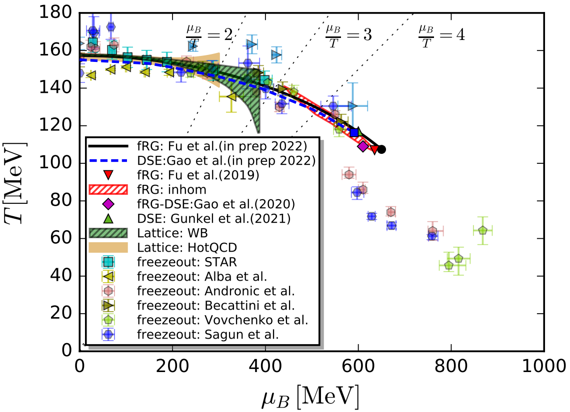
DSE:
FG, J. Papavassiliou, J. Pawlowski, PRD103, 094013(2021).

Running mass of quark:



lattice:
P. O. Bowman et al, PRD71, 054507 (2005)
fRG:
W.-j. Fu et al, PRD 101, 054032 (2020)
fRG-DSE:
FG et al, PRD 102, 034027 (2020)

Phase diagram in temperature-chemical potential region for 2+1 flavour QCD



The fQCD computations of chiral phase transition are converging:

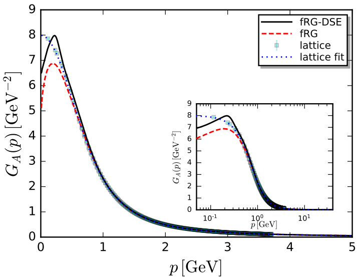
- $T_c = 155\,\mathrm{MeV}$ and $\kappa \sim 0.16$
- Estimated range of CEP: $T \in (100, 110)$ MeV $\mu_B \in (600, 700)$ MeV

W.-j. Fu et al, PRD 101, 054032 (2020)
FG and Jan M. Pawlowski, PLB 820, 136584(2021)
P.J. Gunkel, C. S. Fischer, PRD 104, 054022 (2021).

- A full computation with propagator and vertex DSEs.

The truncation that describes both the vacuum and the phase transition region requires:

- *Describe the running mass of quark and gluon*
- *Describe the running of the coupling*



The Yang-Mills sector is relatively separable. One can apply the data in vacuum:

Lattice:
A. G. Duarte et al, PRD 94, 074502 (2016),
P. Boucaud et al, PRD 98, 114515 (2018),
S. Zafeiropoulos et al, PRL122, 162002 (2019)

fRG:
W.-j. Fu et al, PRD 101, 054032 (2020)
Cyrol, Fister, Mitter, Pawlowski, Strodthoff, PRD 94 (2016) 5, 054005

Compute the difference between finite T/μ and vacuum:

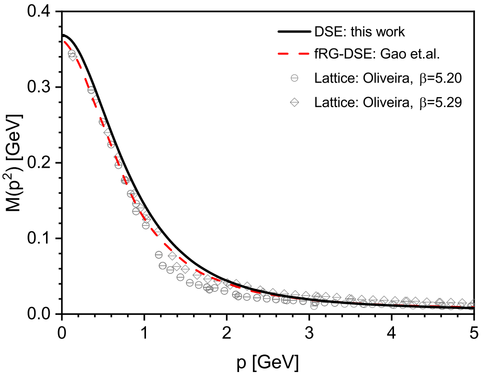
$$D^{-1}_{\mu\nu}(k)|_{T,\mu} = D^{-1}_{\mu\nu}(k)|_{0,\mathbf{0}} + \Delta\Pi^{\rm gauge}_{\mu\nu}(k) + \Delta\Pi^{\rm qrk}_{\mu\nu}(k)$$

The minimal truncation:

In Landau gauge:

$$\Gamma^{\mu}(q,-p)=\sum_{i=1}^{8}\lambda_i(q,-p)P^{\mu\nu}(q-p)\mathcal{T}_i^{\nu}(q,-p)\,,$$

With all quantities are expressed by the running of two point functions, The Quark Mass function:



The optimised truncation:

$$\mathcal{T}_1(p,q)=-i\gamma^{\mu}\,,\mathcal{T}_4^{\mu}(p,q)=(\not p+\not q)\gamma^{\mu}\,,$$

$$\lambda_1(p,q)=F(k^2)\frac{A(p^2)+A(q^2)}{2}$$

$$\lambda_4(p,q)=\left[Z(k^2)\right]^{-1/2}\frac{B(p^2)-B(q^2)}{p^2-q^2}$$

L. Chang, YX Liu, and C. D. Roberts, PRL 106, 072001(2011)
SX Qin, L. Chang, YX Liu, C. D. Roberts, S. M. Schmidt, PLB 722, 384(2013)

The Cornwall–Jackiw–Tomboulis (CJT) effective potential:

$$\Gamma(S) = -Tr[\ln(S_0^{-1}S) - S_0^{-1}S + 1] + \Gamma_2(S), \quad (1)$$

where S_0 and S stands for the bare and full quark propagator,Γ_2 is the 2PI contribution. Calculating the variation respective to quark propagator, we have:

$$\frac{\partial^2\Gamma}{\partial S^2} = S^{-2} + \frac{\partial\Gamma_2(S)}{\partial S}. \quad (2)$$

combing with the derivative on the quark propagator DSE as:

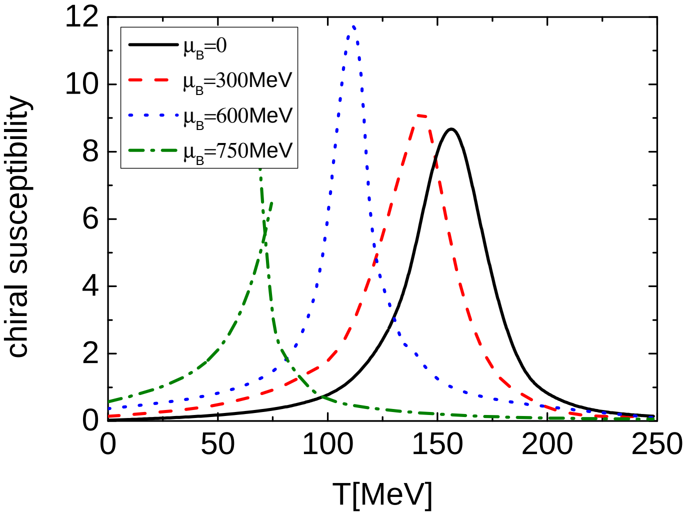
$$-S^{-2}\frac{\partial S}{\partial T} = 1 + \frac{\partial\Gamma_2(S)}{\partial S}\frac{\partial S}{\partial T}, \quad (3)$$

The criterion is then given by[1]:

$$\frac{\partial S}{\partial T} = -\frac{1}{\partial^2\Gamma/\partial S^2}. \quad (4)$$

[1] **Fei Gao**, Yu-xin Liu. Phys. Rev. D 94 (2016) 7, 076009.

Scanning the susceptibility in the whole temperature-chemical potential plane:



- low chemical potential, crossover
- large chemical potential, first order phase transition

A first estimation of QCD phase transition line:

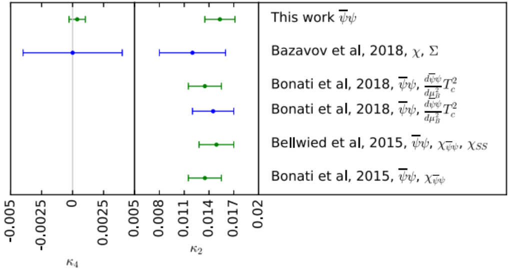
$$\frac{d(P_N - P_W)}{dT} = \left(\frac{\partial P_N}{\partial \mu} - \frac{\partial P_W}{\partial \mu}\right)\frac{\partial \mu}{\partial T} + \left(\frac{\partial P_N}{\partial T} - \frac{\partial P_W}{\partial T}\right) = 0\,.$$

Therefore, the phase transition line should bend down typically:

$$\frac{\partial \mu}{\partial T} = -\frac{s_N - s_W}{n_N - n_W} < 0\,.$$

The line can be parametrized as:

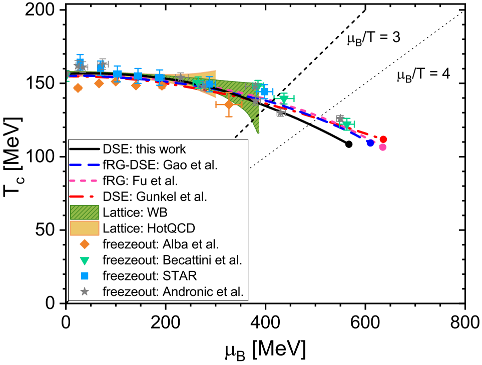
$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 + \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4 + \cdots,$$



Here : $\kappa_2 = 0.0169(6)$, $\kappa_4 \approx 5 \times 10^{-4}$

QCD phase diagram at high chemical potential

Phase diagram in temperature-chemical potential region for 2+1 flavour QCD



The fQCD computations of chiral phase transition are converging:

- The present approximation to the minimal DSE scheme is reliable up to $\mu_B/T \lesssim 3$
- CEP at (108.5,567) MeV.
- The full computation estimated range of CEP: $T \in (100, 110)$ MeV $\mu_B \in (600, 650)$ MeV
- A benchmark for experiments and also the model construction.

W.-j. Fu et al, PRD 101, 054032 (2020)
FG and Jan M. Pawlowski, PLB 820, 136584(2021)
P.J. Gunkel, C. S. Fischer, PRD 104, 054022 (2021).

QCD thermodynamic properties

Currently, the functional QCD approaches can only calculate the quark potential directly, while the gluon sector still awaits further investigations.

One may incorporate the lattice QCD simulation at $\mu = 0$ here to combine the advantages of the two methods. One can calculate the quark number densities $\{n_q\}$ at finite chemical potential and obtain the pressure by:

$$P(T, \boldsymbol{\mu}) = P_{Latt.}(T, \mathbf{0}) + \sum_q \int_0^{\mu_q} n_q(T, \mu)\, d\mu$$

$$n_q^f(T, \mu_B) \simeq -N_c Z_2^f\, T \sum_n \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \mathrm{tr}_D \left[\gamma_4 S^f(p)\right]$$

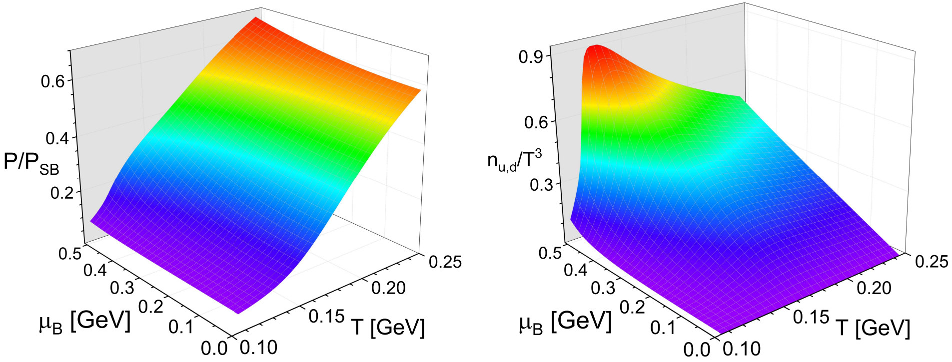
[1]private comm. with N. Wink and J. M. Pawlowski

[2]P. Isserstedt, C.S. Fischer and T. Steinert, PRD103 (2021) 054012

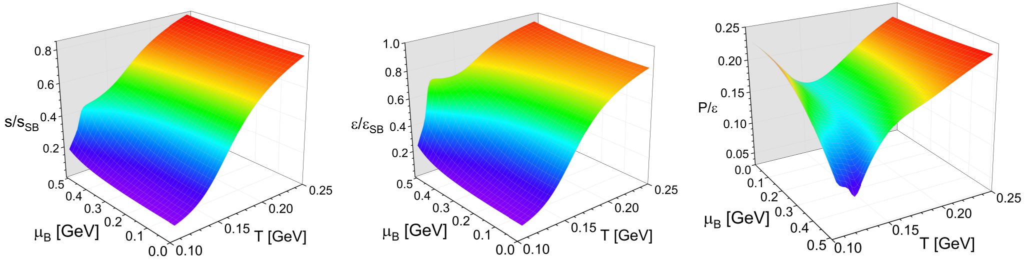
[3]**FG**, Yuxin Liu, PRD 94 (2016) 9, 094030

[4]H. Chen, M. Baldo, G. F. Burgio, and H.-J. Schulze, PRD86(2012)045006

The obtained number density and the integrated pressure:

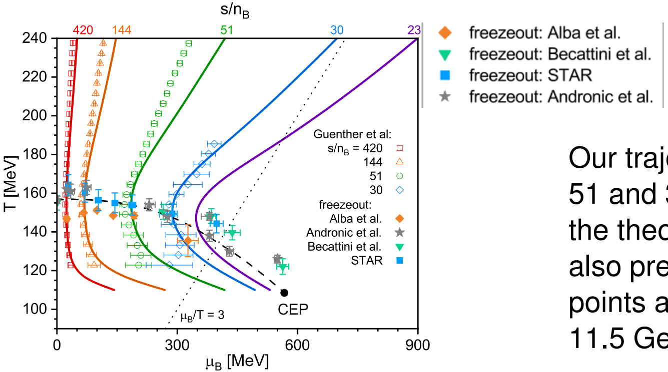


The calculated number density, entropy and energy density in the plane of temperature and chemical potential:



As getting closer to CEP, the slopes of the thermodynamics quantities become sharper.

isentropic trajectories in the up to date scheme:



Our trajectories for $s/n_B = 420$, 144, 51 and 30 which values are chosen in the theoretical studies,
also precisely meet with the freezeout points at $\sqrt{s_{\rm NN}} = 200$, 62.4, 19.6 and 11.5 GeV, respectively.

The Polyakov loop potential is missing and the high T dependence is incorrect.

functional QCD based approximation(arXiv:2310.16345)

Quark propagator with chiral and deconfinement phase transition:

$$S_q^{-1}(p) \simeq \mathrm{i}(\omega_n + \mathrm{i}\mu_q + gA_0)\gamma_4 + \mathrm{i}\gamma \cdot \boldsymbol{p} + M_q$$

with the dynamical quark mass M_q and the gluon condensate $gA_0 = 2\pi T \phi \tau^3$. A_0 is also related to the Polyakov loop $\mathcal{L}$:

$$\mathcal{L} = [1 + 2\cos(\pi\varphi)]/3, \quad \text{with} \quad \phi_{\text{fund}} = \{\pm\varphi/2, 0\}.$$

If one neglects the momentum dependence of M_q and A_0, then the number density can be expressed analytically:

$$\rho_q(T, \mu_q) = 2N_f N_c \int \frac{d^3\boldsymbol{k}}{(2\pi)^3}[f(\epsilon, \mathcal{L}, \mu_q) - f(\epsilon, \mathcal{L}, -\mu_q)],$$

$$f(\epsilon, \mathcal{L}, \mu) = \frac{\mathcal{L}e^{2(\epsilon-\mu)/T} + 2\mathcal{L}e^{(\epsilon-\mu)/T} + 1}{e^{3(\epsilon-\mu)/T} + 3\mathcal{L}e^{2(\epsilon-\mu)/T} + 3\mathcal{L}e^{(\epsilon-\mu)/T} + 1},$$

$$\epsilon(\boldsymbol{k}, M_q) = \sqrt{\boldsymbol{k}^2 + M_q^2}.$$

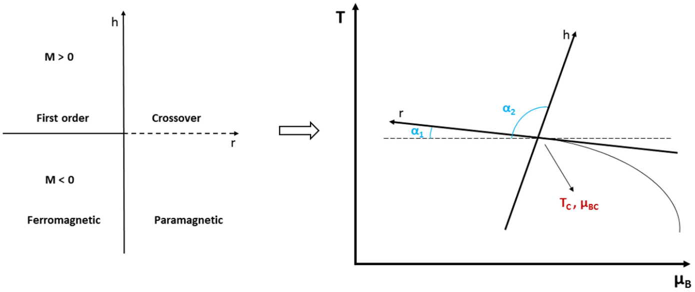
Chiral phase transition:

The quark mass is parameterized from the Ising order parameter:

Parotto, Bluhm, Mroczek et.al. PRC 101, 034901 (2020);

$$\mathcal{M}_{\mathrm{Ising}} = \mathcal{M}_0 R^{\beta}\theta, \quad h = h_0 R^{\beta\delta}\tilde{h}(\theta), \quad r = R(1-\theta^2),$$

Map between the Ising parameters and the QCD phase diagram: $(T, \mu_B) \leftrightarrow (r, h)$.



However, a global map of the QCD phase transition line to the Ising variables is required:

$$\frac{\mu_B}{\mu_B^E} - 1 = -r\omega\rho\cos\alpha_1 - h\omega\cos\alpha_2, \quad \frac{T}{T^E} - 1 = f_{\mathrm{PT}}(r) + h\omega\sin\alpha_2.$$

Phase transition happens at $h = 0$, which gives the constraint on the map function f_{PT}:

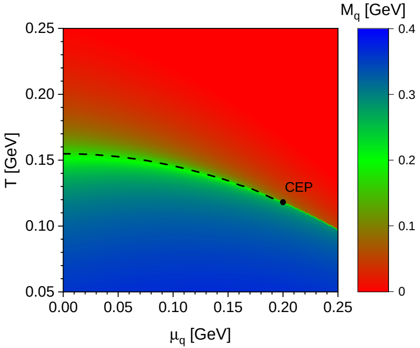
$$f_{\mathrm{PT}}(r) = \frac{T_c(\mu_B)}{T^E} - 1, \quad \mu_B = \mu_B^E\,(1 - r\omega\rho\cos\alpha_1)\,.$$

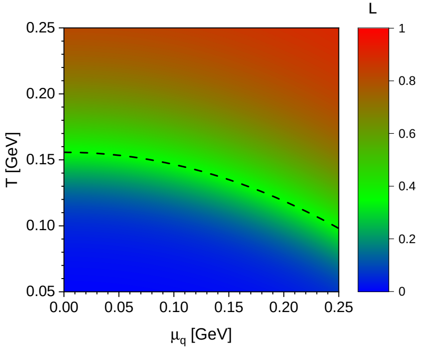
Finally, the Ising mapping starts from a critical point. Its position is set from the predictions of functional QCD studies:

$$\mu_B^E = 3\,\mu_q^E = 600\,\mathrm{MeV},\, T^E = T_c(\mu_B^E) = 118\,\mathrm{MeV}.$$

Deconfinement transition:
The Polyakov loop data at zero μ_B is taken from the fQCD result; at finite μ_B, the temperature scaling is suggested in Refs. :
(Fu and Pawlowski, PRD 92, 116006 (2015); S. Borsanyi et.al. (WB-collaboration), PRL. 126, 232001 (2021))

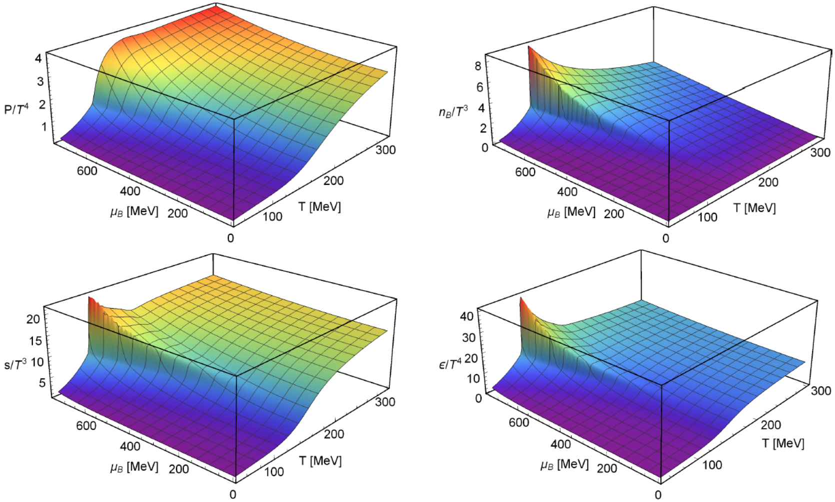
$$\mathcal{L}(T,\mu_q) = \mathcal{L}_{\rm fRG}(T',0), \quad \frac{T'}{T_c(0)} = \frac{T}{T_c(0)} + \kappa\left(\frac{3\,\mu_q}{T_c(0)}\right)^2 .$$





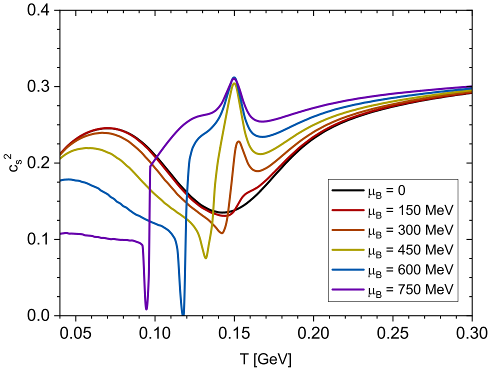
EoS from phase diagram mapping

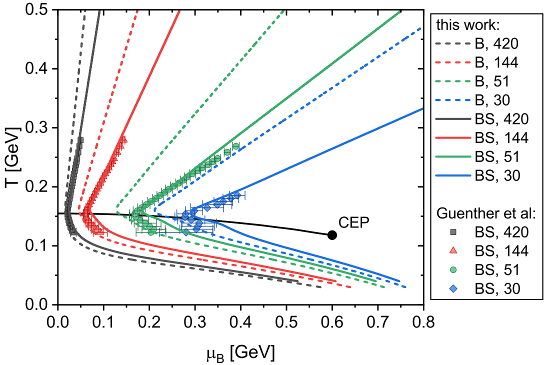
$s = \partial P/\partial T, \quad \epsilon = Ts - P + \mu_B n_B, \quad c_s^2 = \partial P/\partial \epsilon, \ldots$



EoS from phase diagram mapping

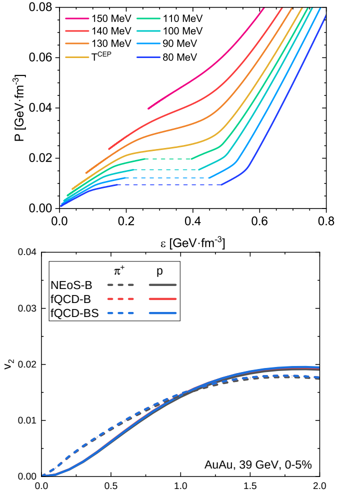
The speed of sound for different μ_B

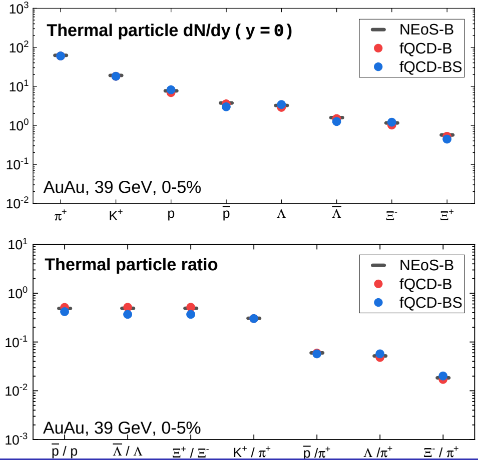




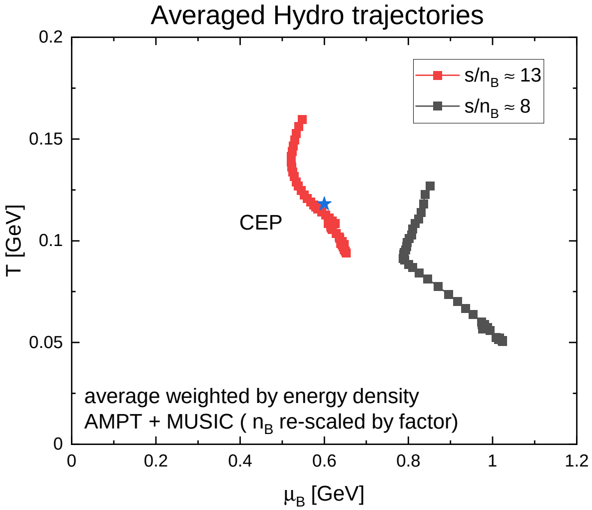
Combine with hydrodynamic simulation

The EoS can be applied in hydrodynamic simulations after mapping the data into (ϵ, n_B) plane together with Maxwell construction.





The hydrodynamic simulation can be extended to first order region within the current EoS:



In the future:

- Incorporating the EoS of QCD into hydrodynamics simulations and Universe evolution;
- Studying the global properties of QCD matter generated in HIC, for instance, the transport coefficients and the polarization structure.
- Investigating the spectral function of QCD states at finite T and μ.
- Investigating the possible new phases of QCD at finite T and μ.

Thank you!