QCD: Few to Many

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(秦思学)

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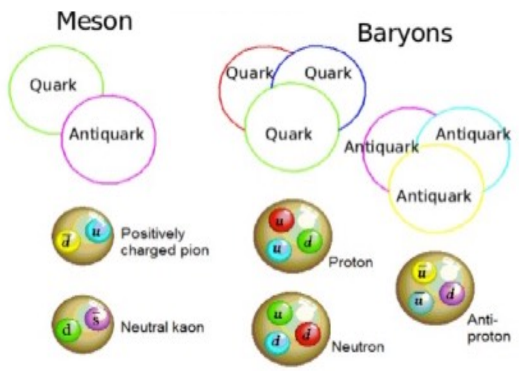
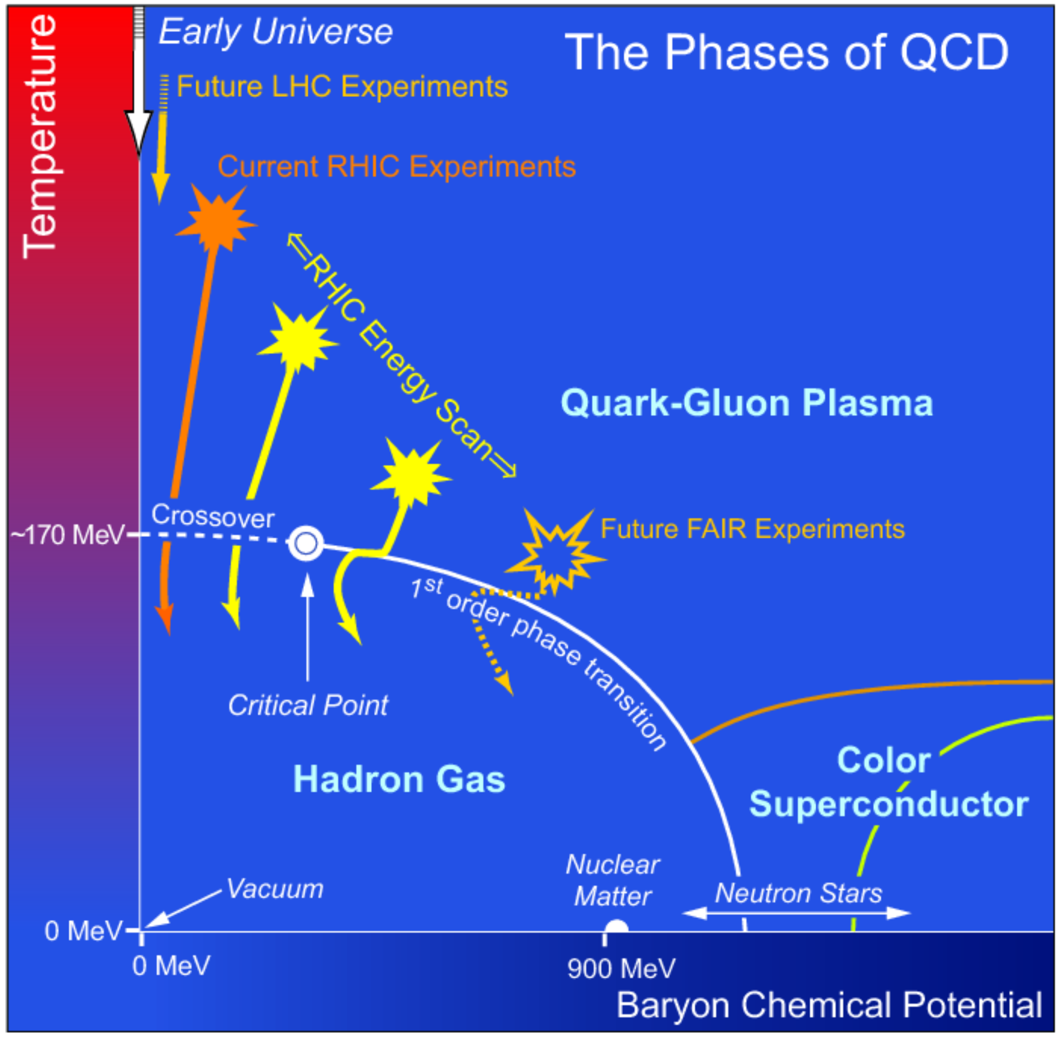
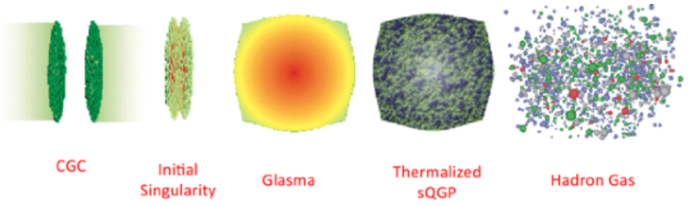
Background: QCD frontiers

Hadron

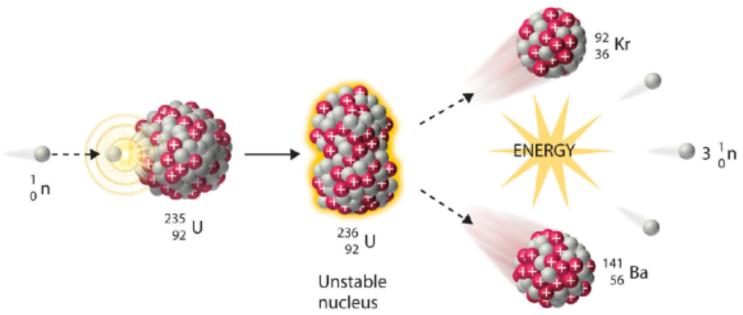




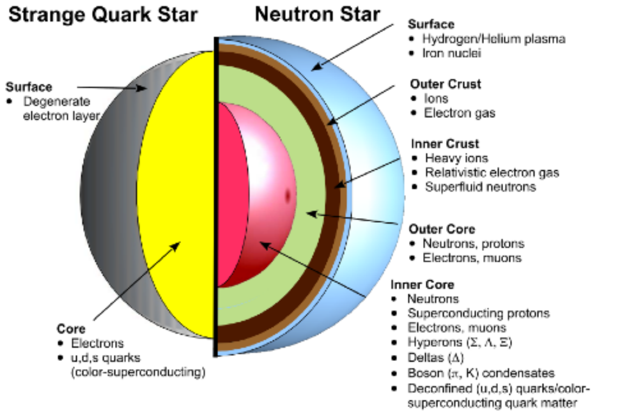
QGP



Nucleus



Compact Star



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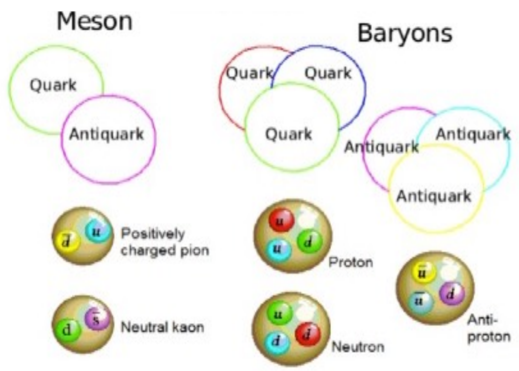

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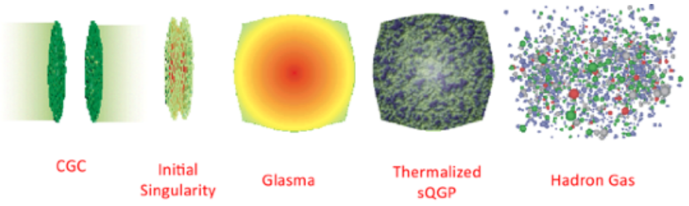

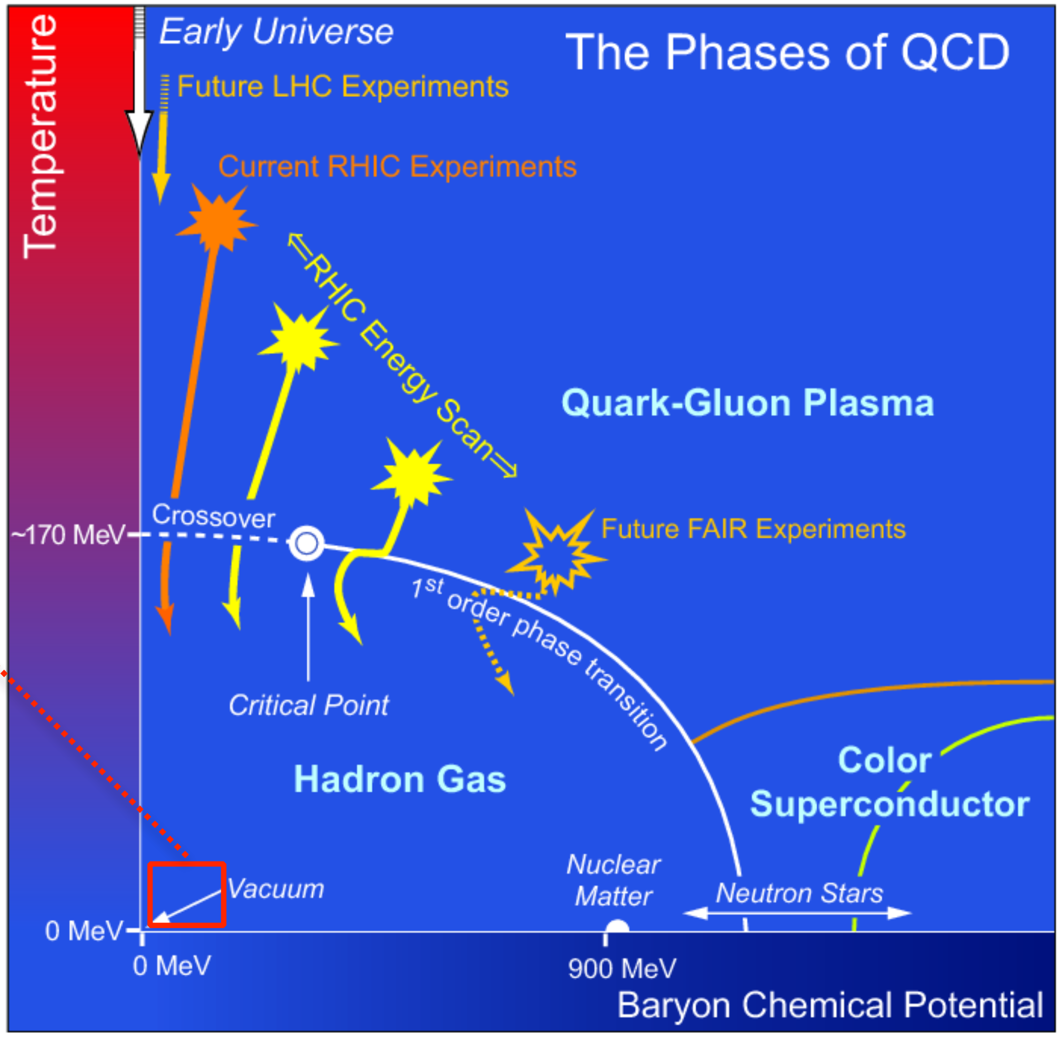

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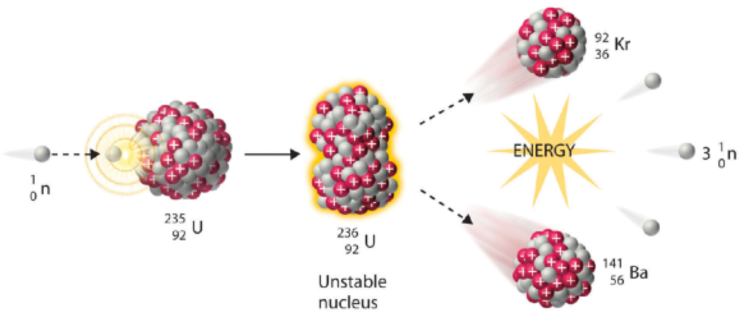

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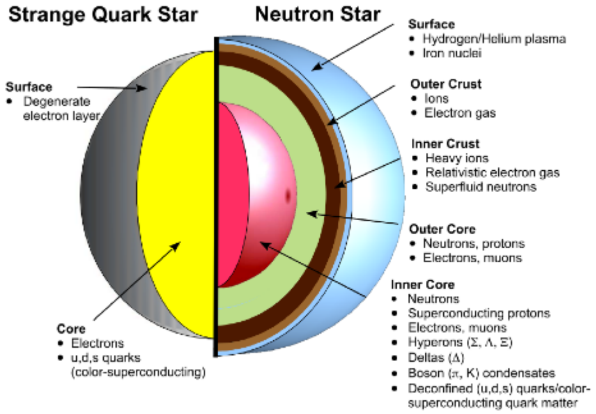

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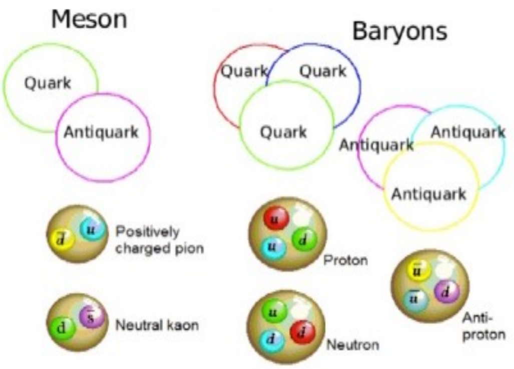

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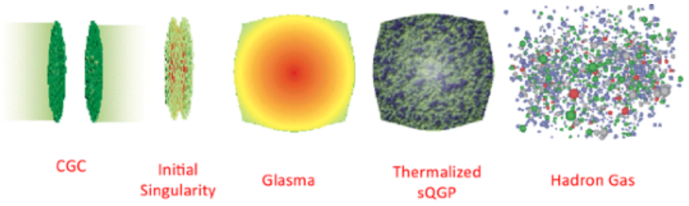

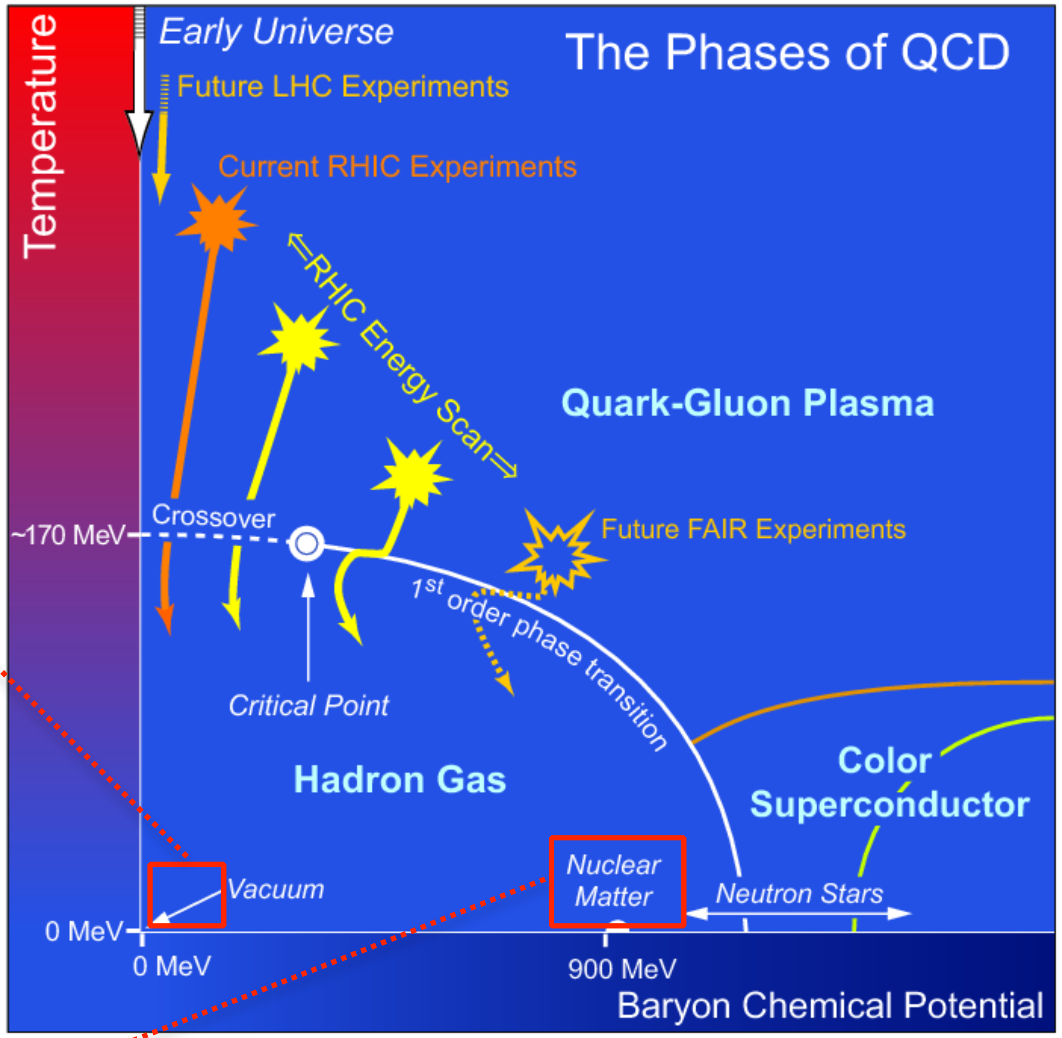

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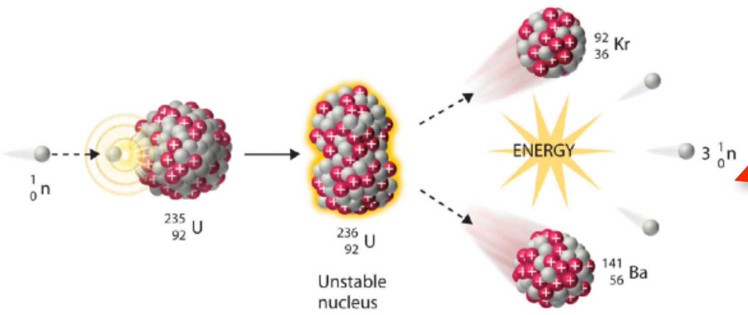

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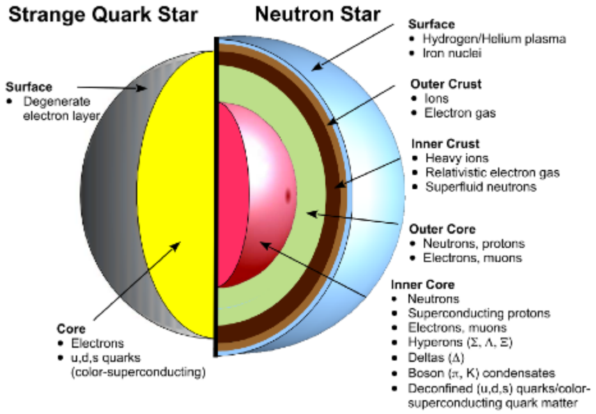

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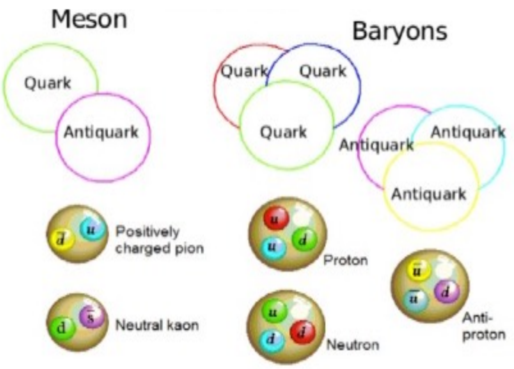

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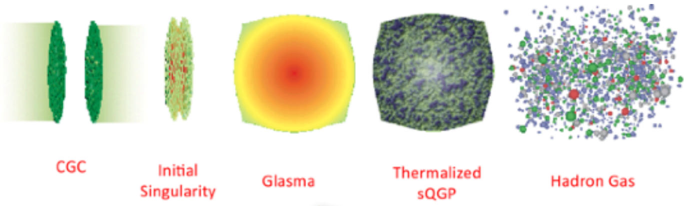

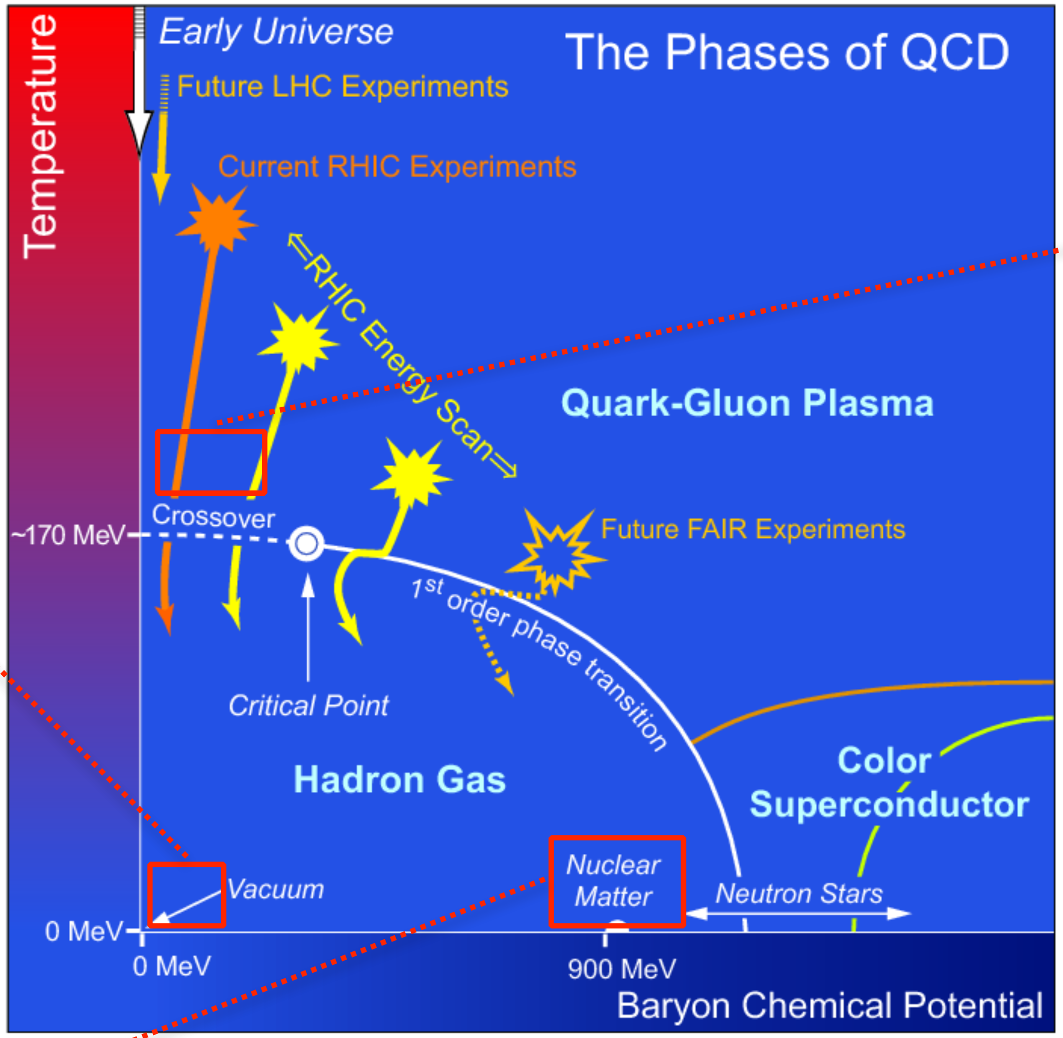

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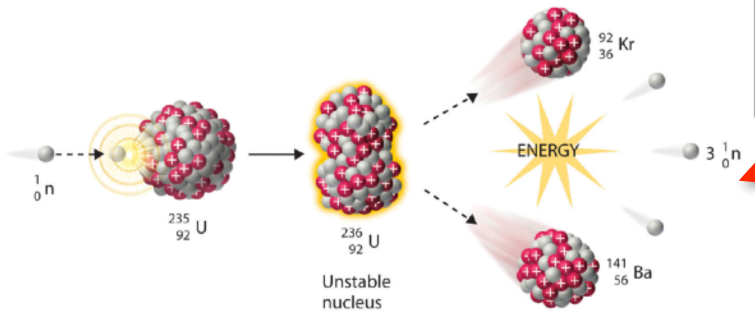

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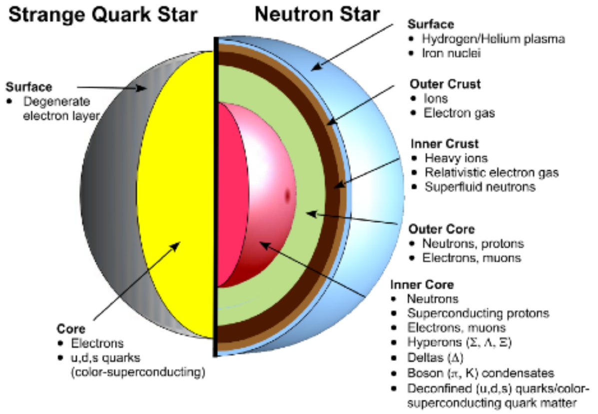

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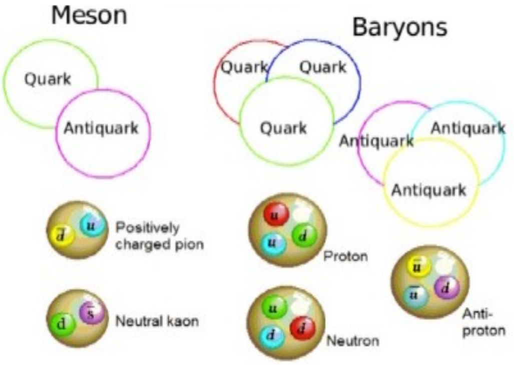

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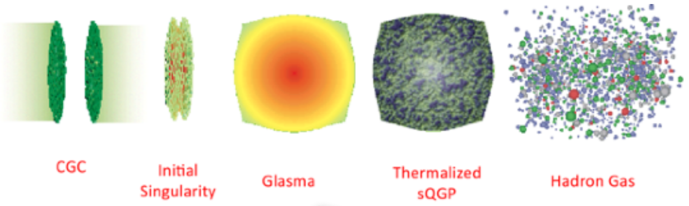

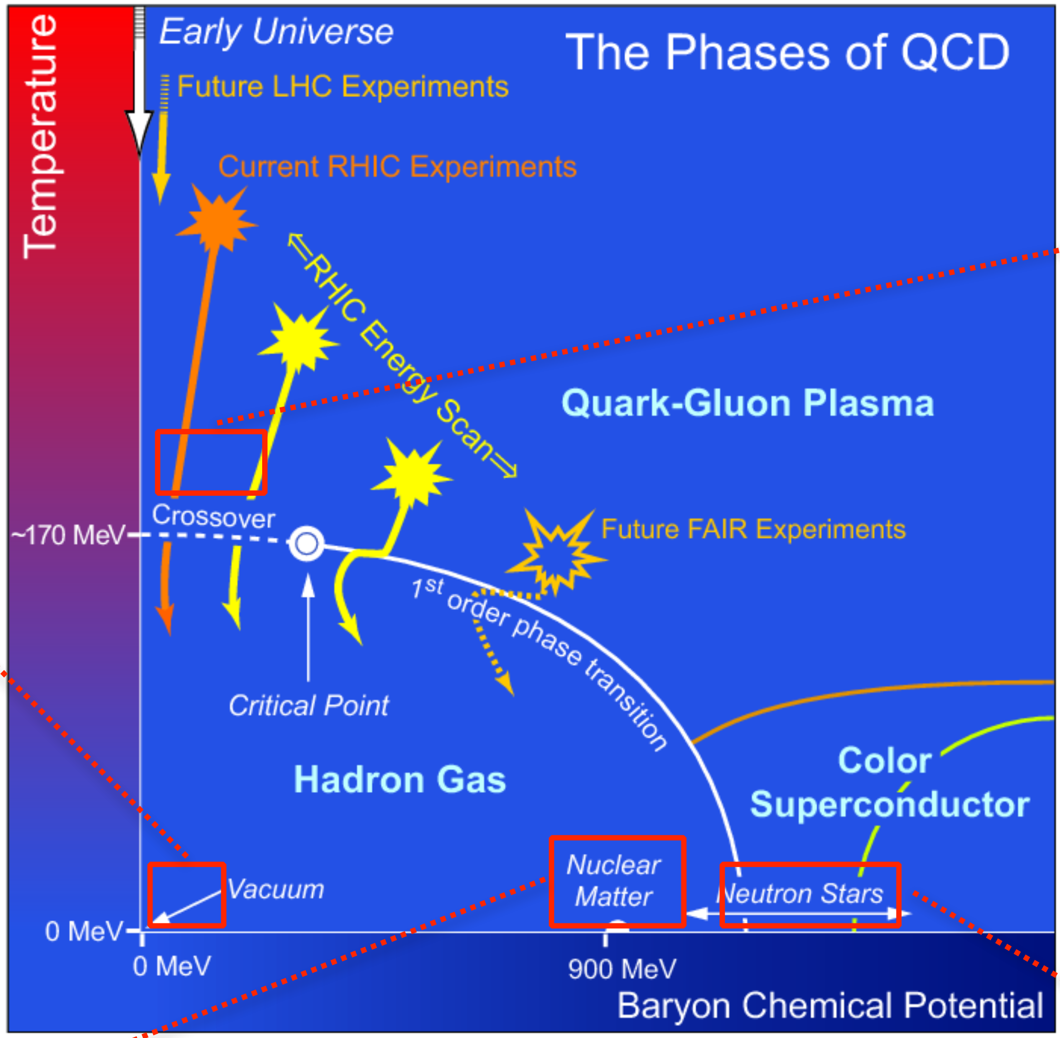

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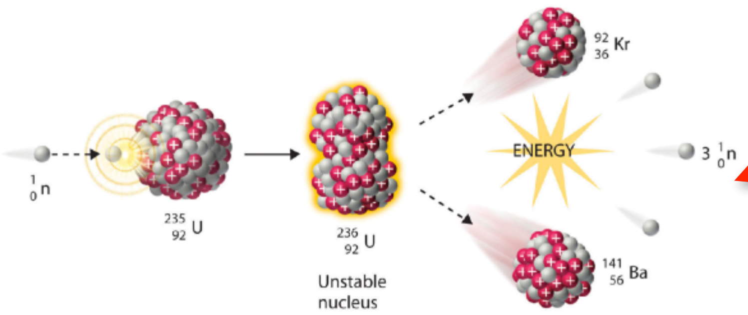

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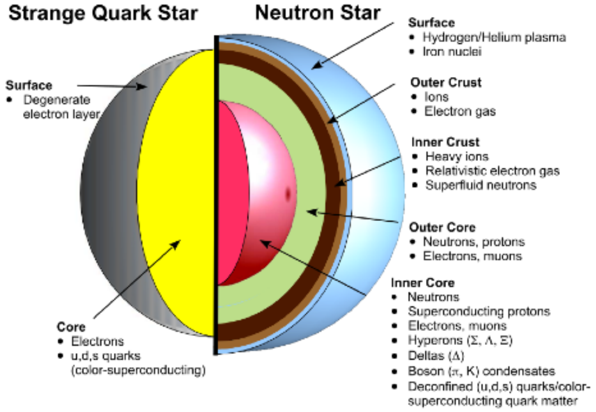

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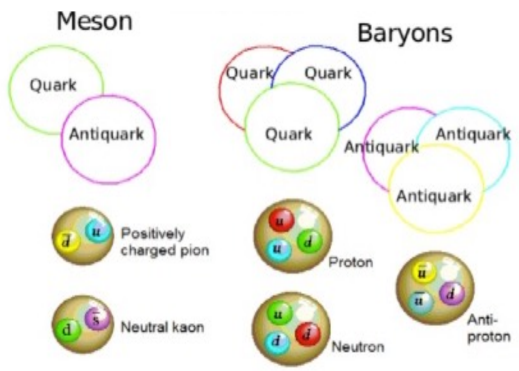

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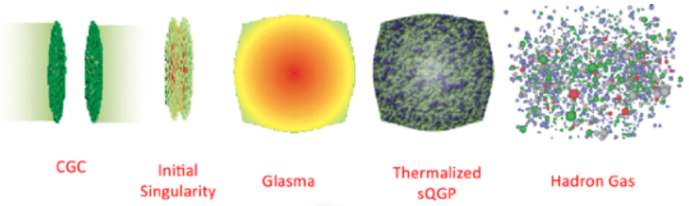

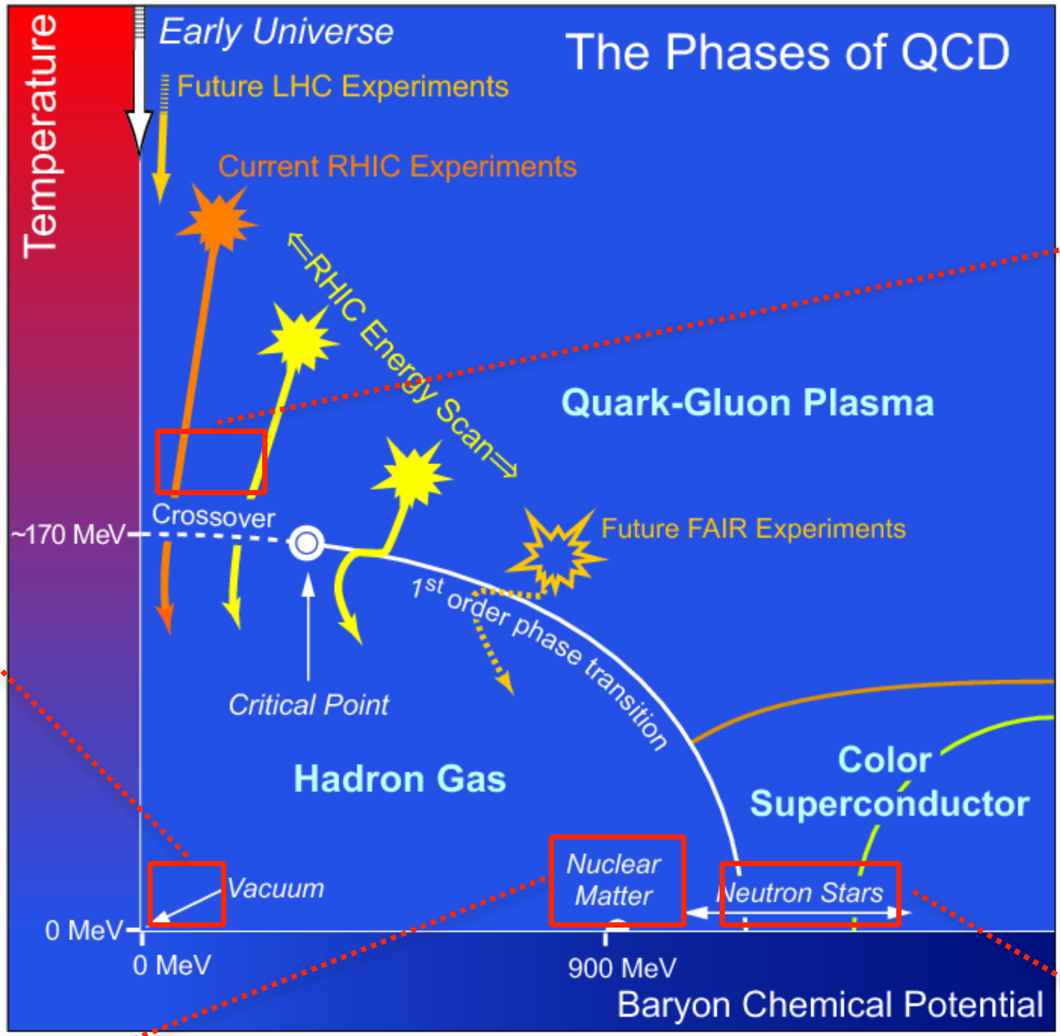

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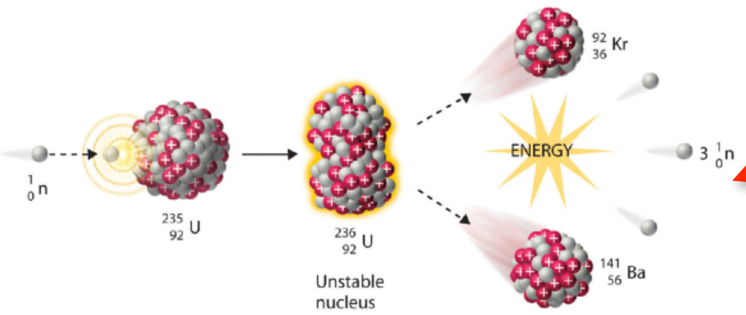

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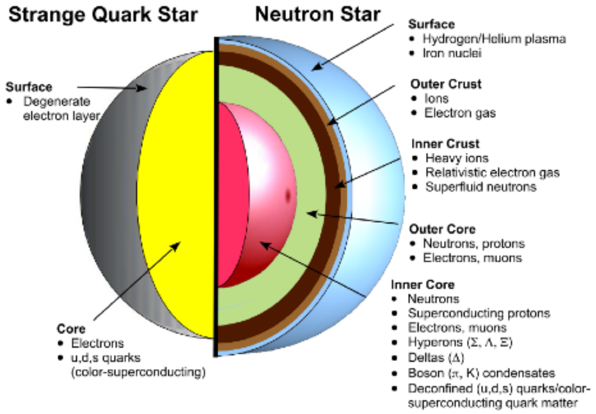
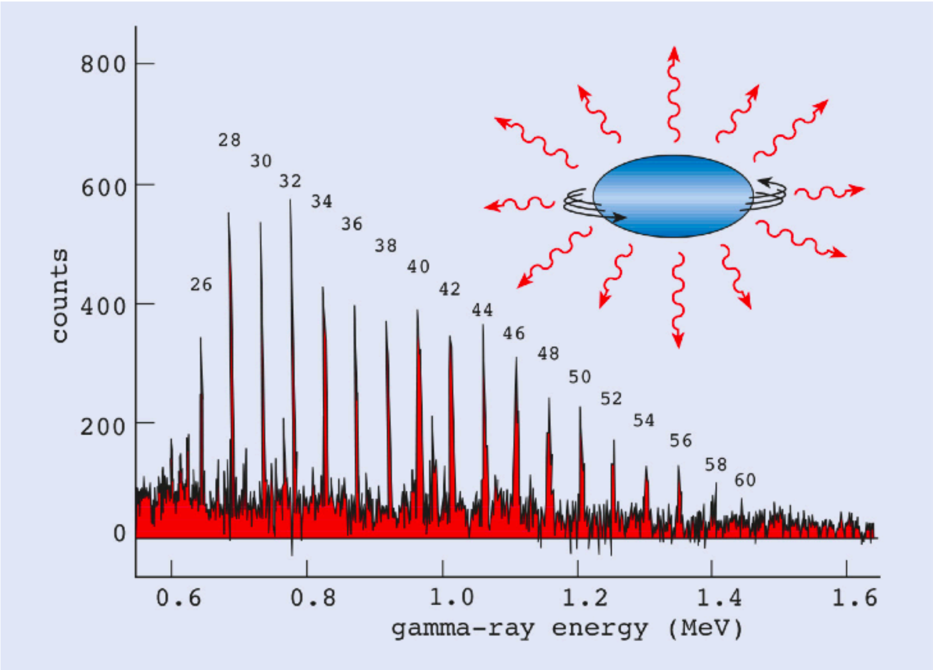


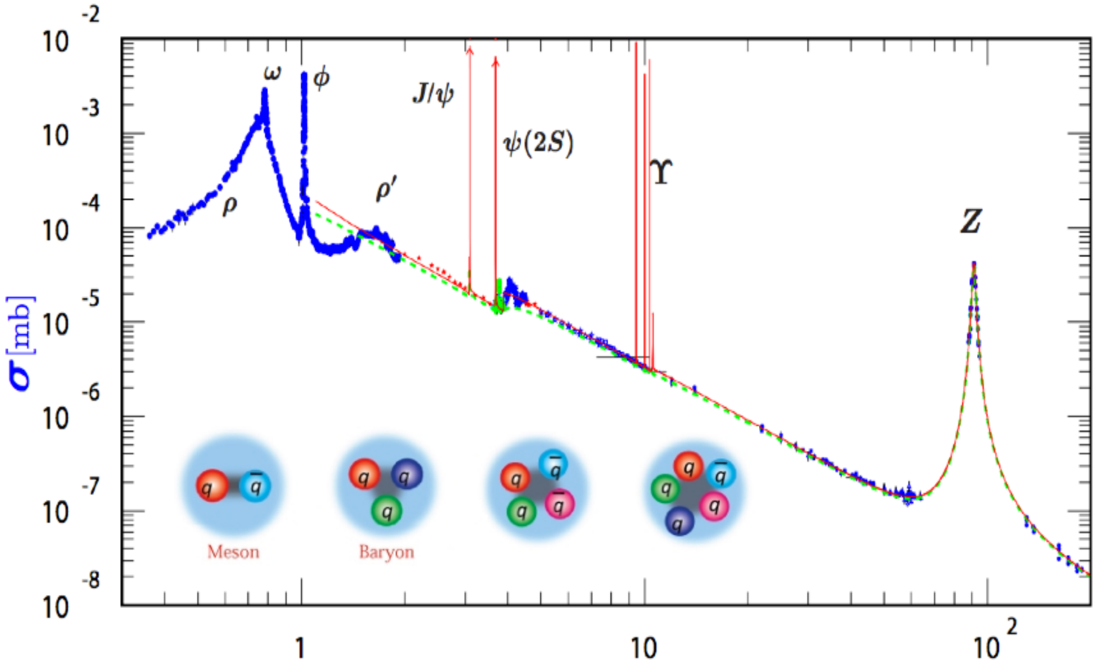
Few-body

Many-body

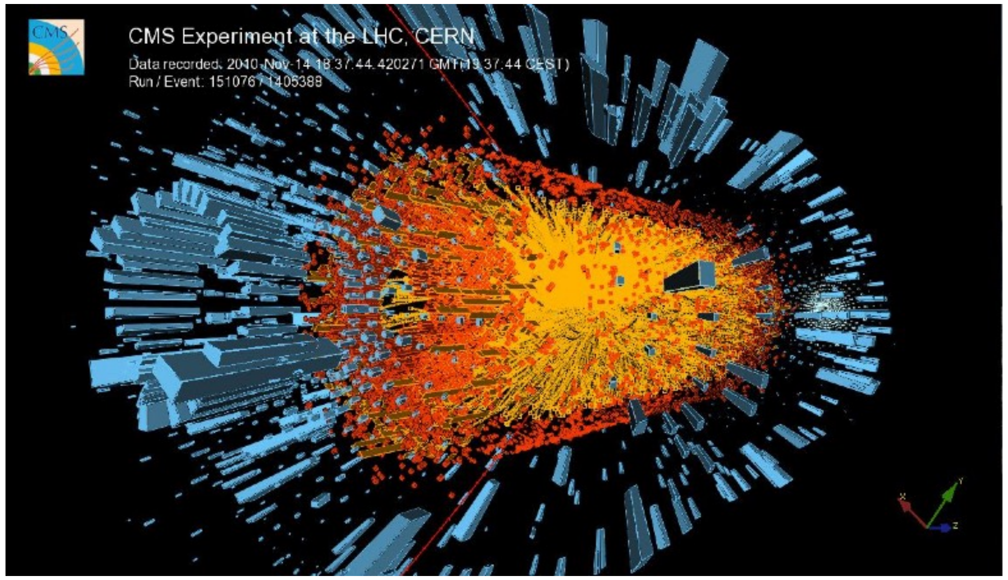
Background: How to study QCD



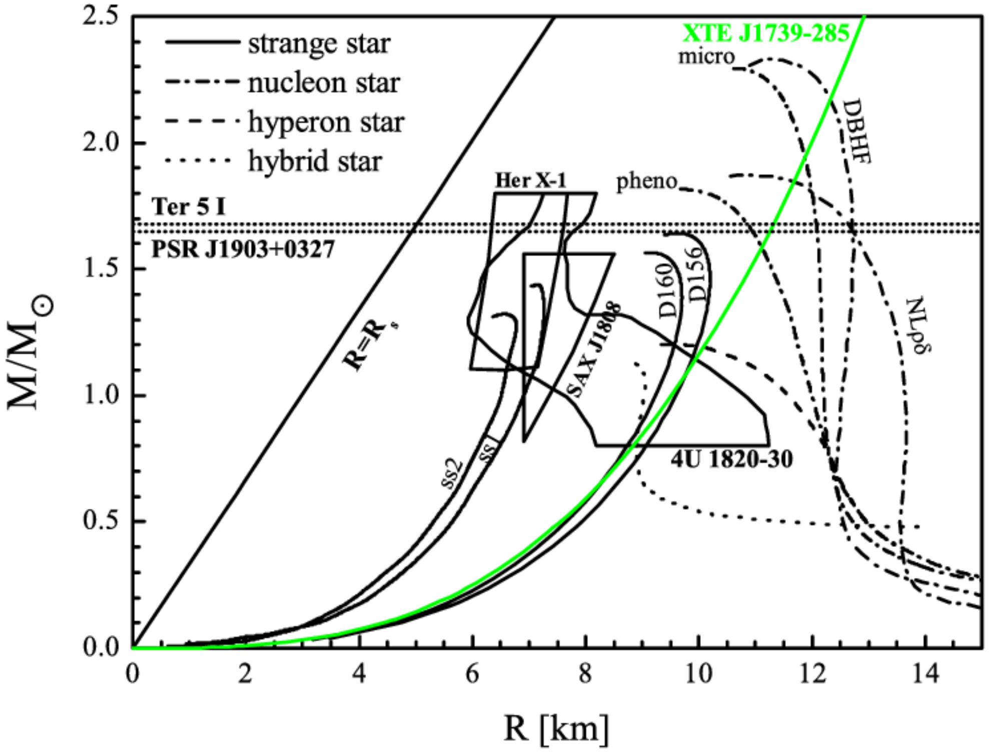
Novel states of nuclei



e⁺e⁻ hadronic annihilation

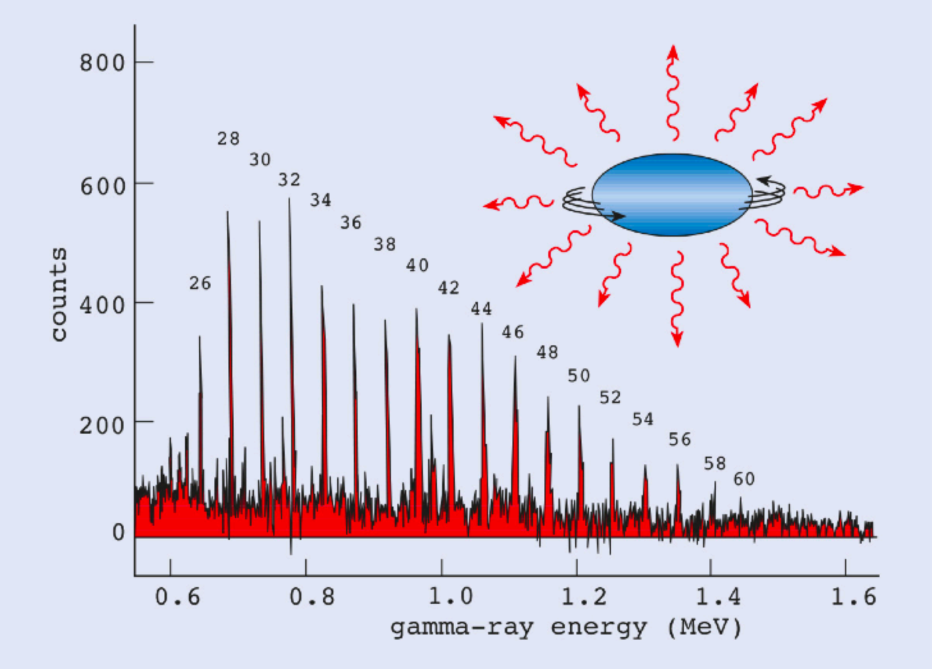


relativistic heavy-ion collision



mass-radius relation of compact stars

Background: How to study QCD



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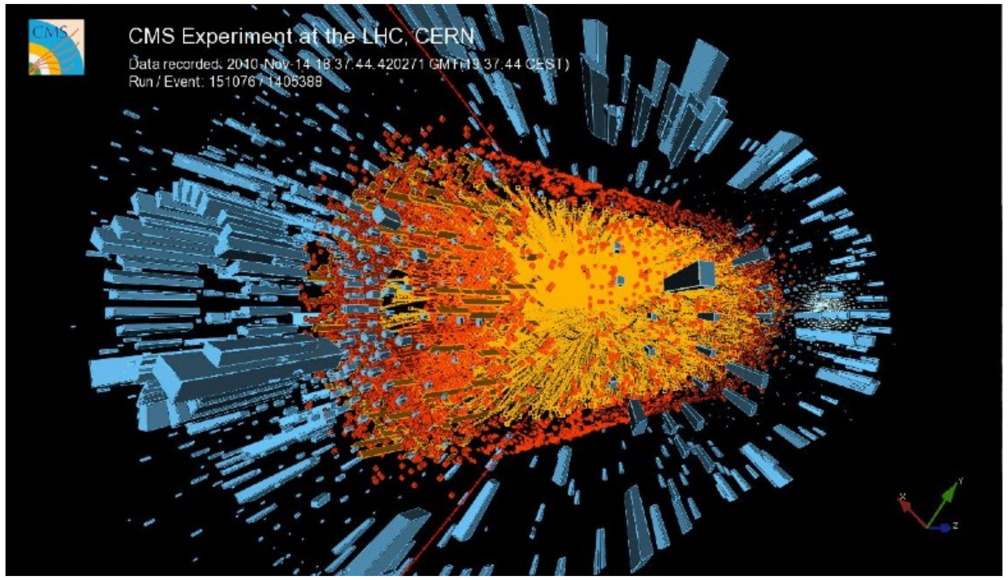
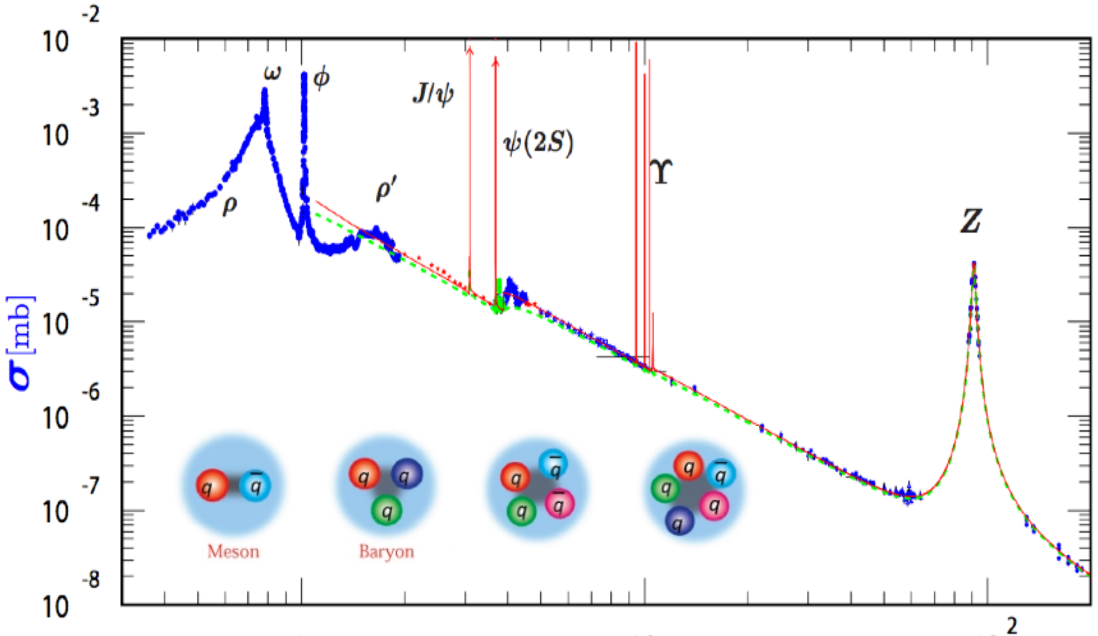
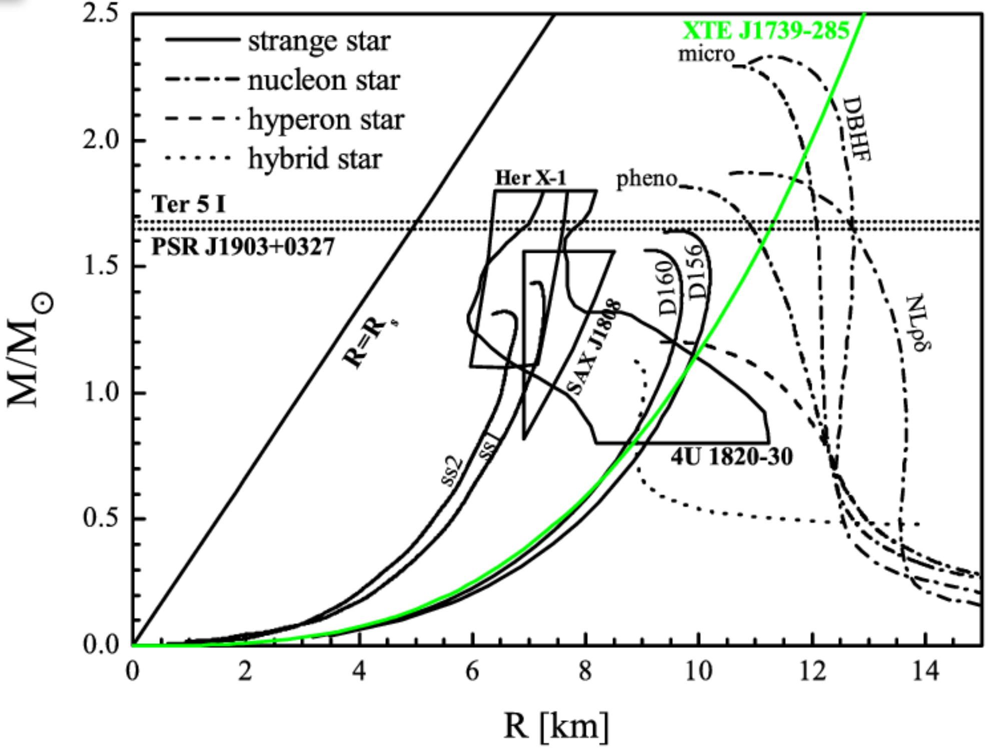


relativistic heavy-ion collision

Solve QCD



e^+e^- hadronic annihilation



mass-radius relation of compact stars

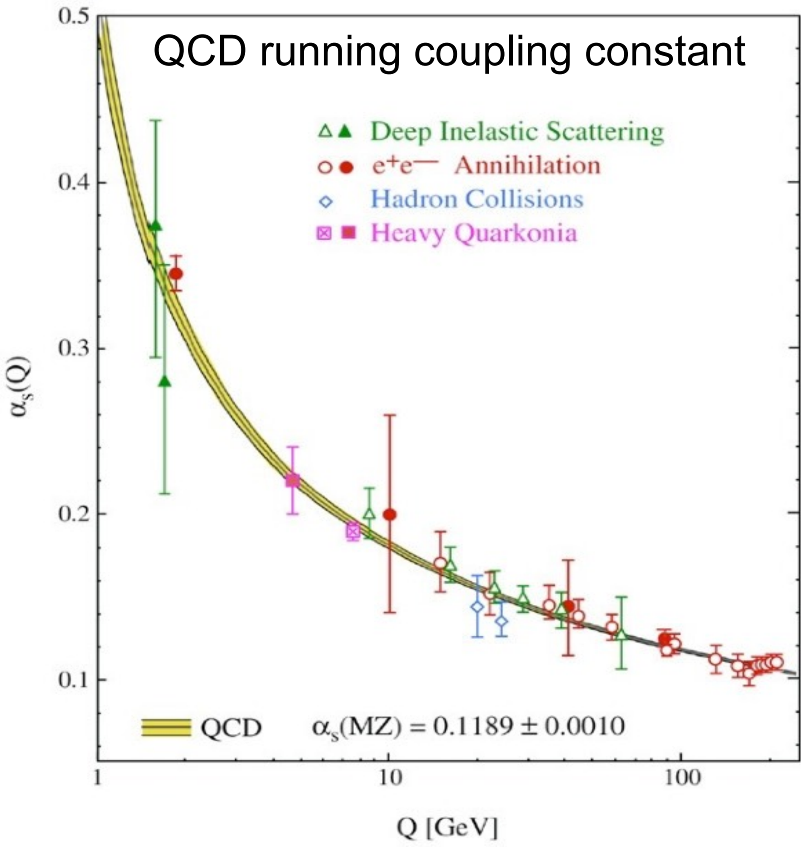
Background: Hadron — The difficult problem

• Relativistic bound states

> *“These problems are those involving bound states […] such problems necessarily involve a breakdown of ordinary perturbation theory. […] The pole therefore can only arise from a divergence of the sum of all diagrams […]”*

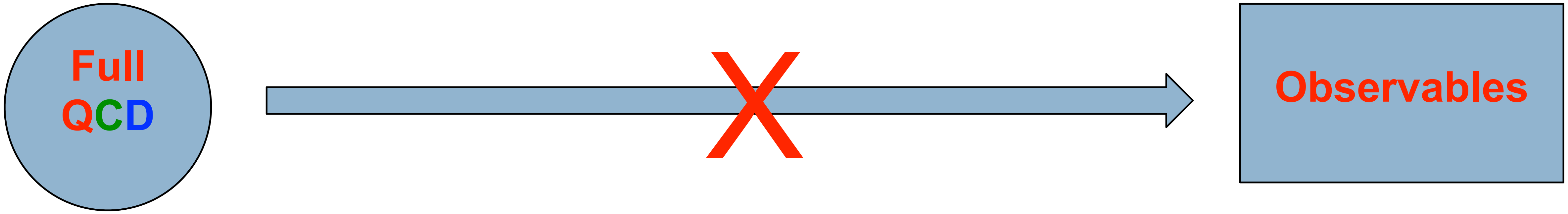
The QFT book vol1 p564 Weinberg

• Strongly coupled systems

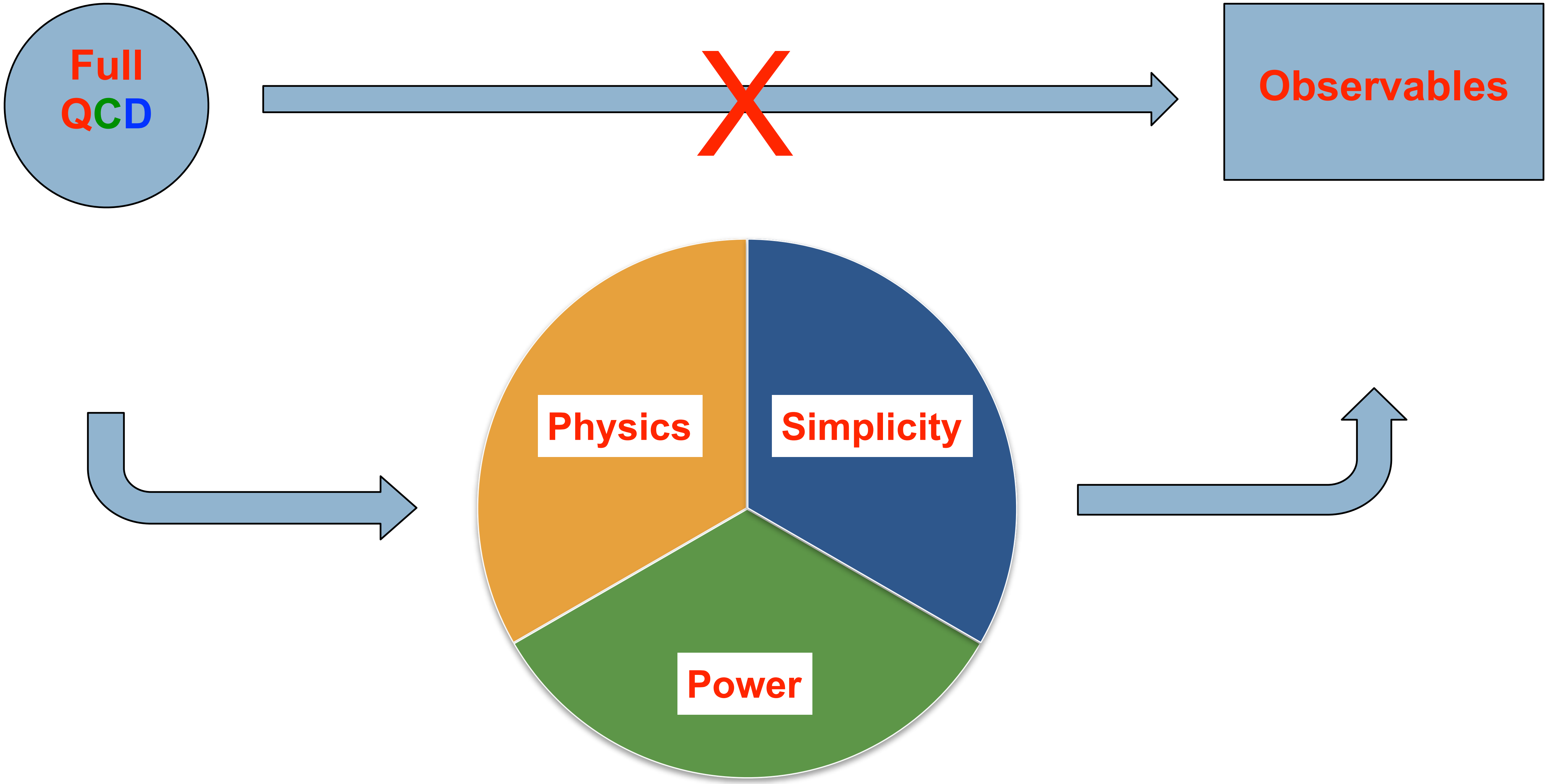


- **Color Confinement**: No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon (Millennium Problems).
- **Dynamical Chiral Symmetry Breaking**: Mystery of bound state masses, e.g., current quark mass (Higgs) is small, and no degeneracy between *parity partners.*

Background: Non-perturbative approaches of **QCD**



Background: Non-perturbative approaches of QCD



Full QCD

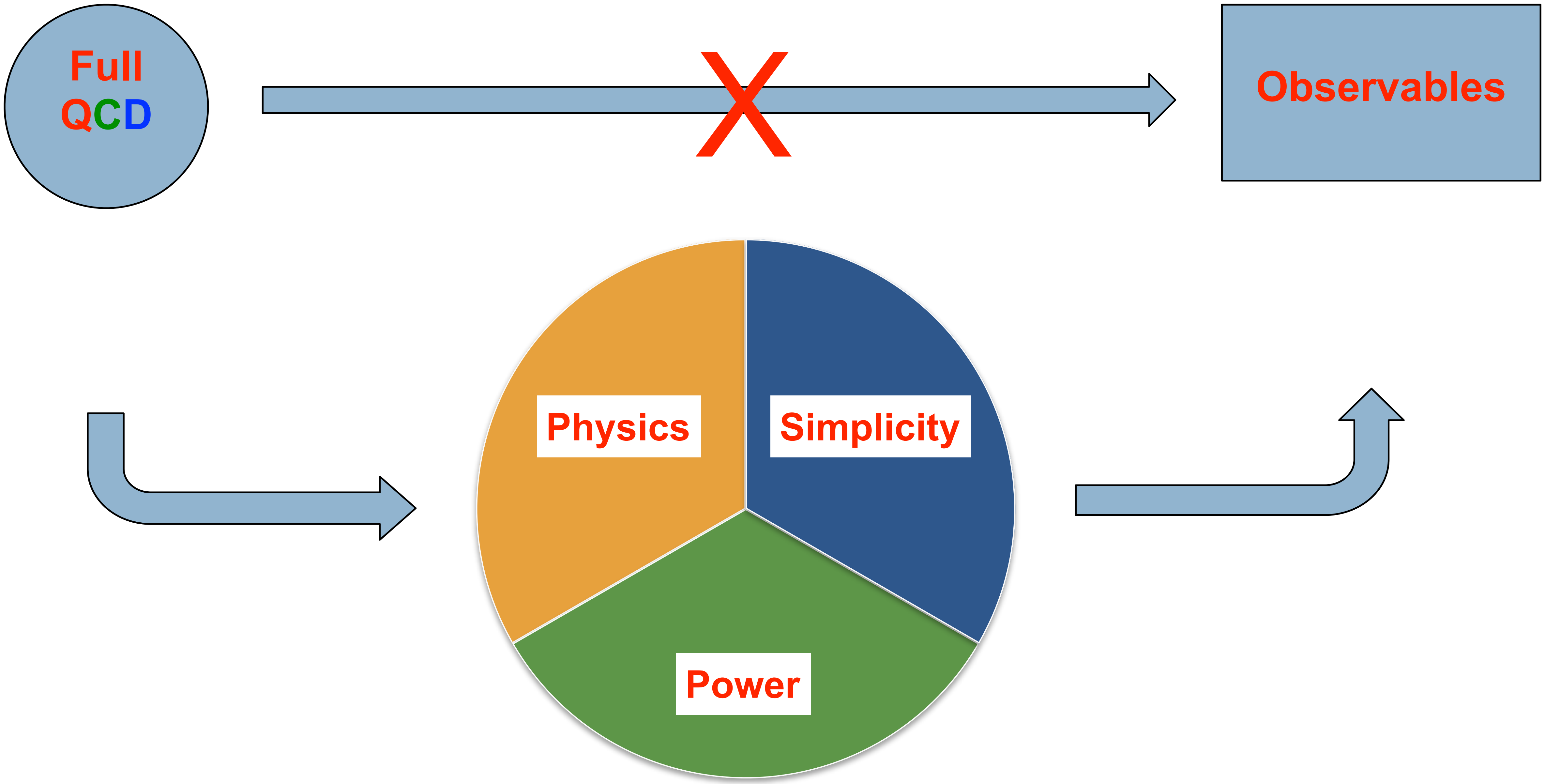
Observables

Physics

Simplicity

Power

Background: Non-perturbative approaches of QCD



Full QCD

Observables

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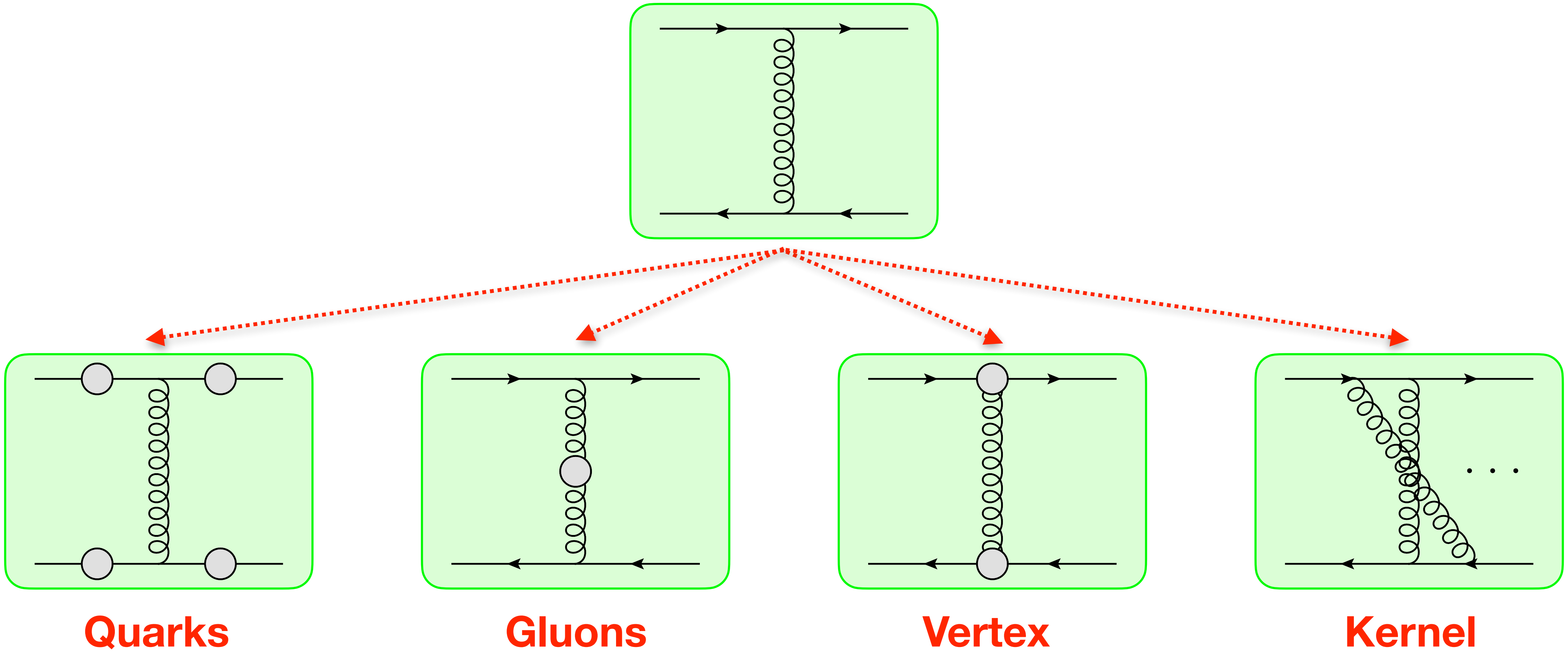
Lattice QCD, Dyson-Schwinger equations, AdS/QCD, NJL model, Effective theories…

Chapter I: Theory

Physics of quark, gluon, vertex, and kernel

Continuum QCD: Interaction between quarks



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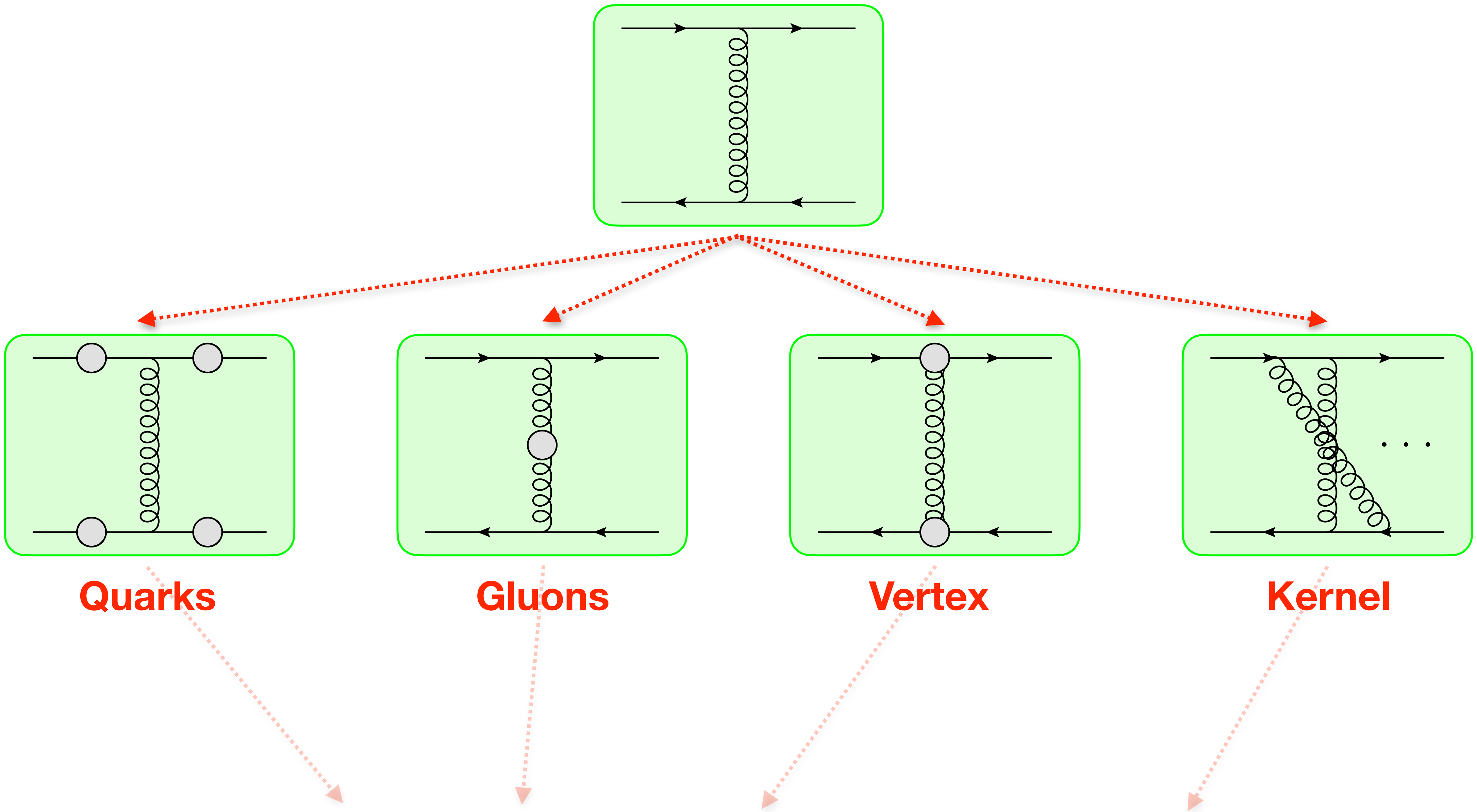


Quarks

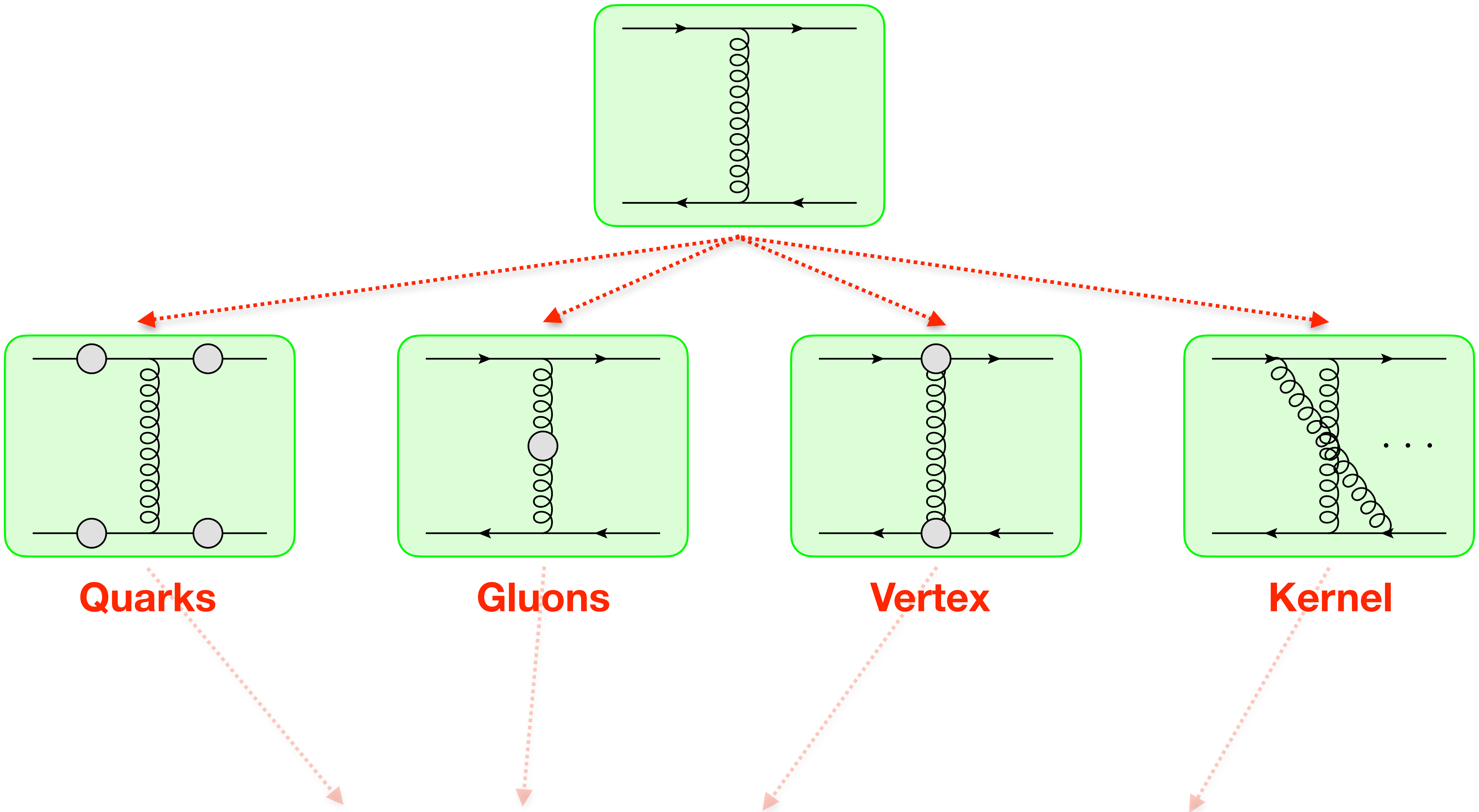
Gluons

Vertex

Kernel

$$H = H_{\text{kinetic}} + H_{\text{Coulomb}} + H_{\text{spin-orbit}} + H_{\text{relativistic}} + H_{\text{QED}}$$

Continuum QCD: Interaction between quarks



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Ground states — **Excited states** → M

Continuum QCD: Equations of motion

Principle of Least Action

$$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$$

Dyson-Schwinger Equations

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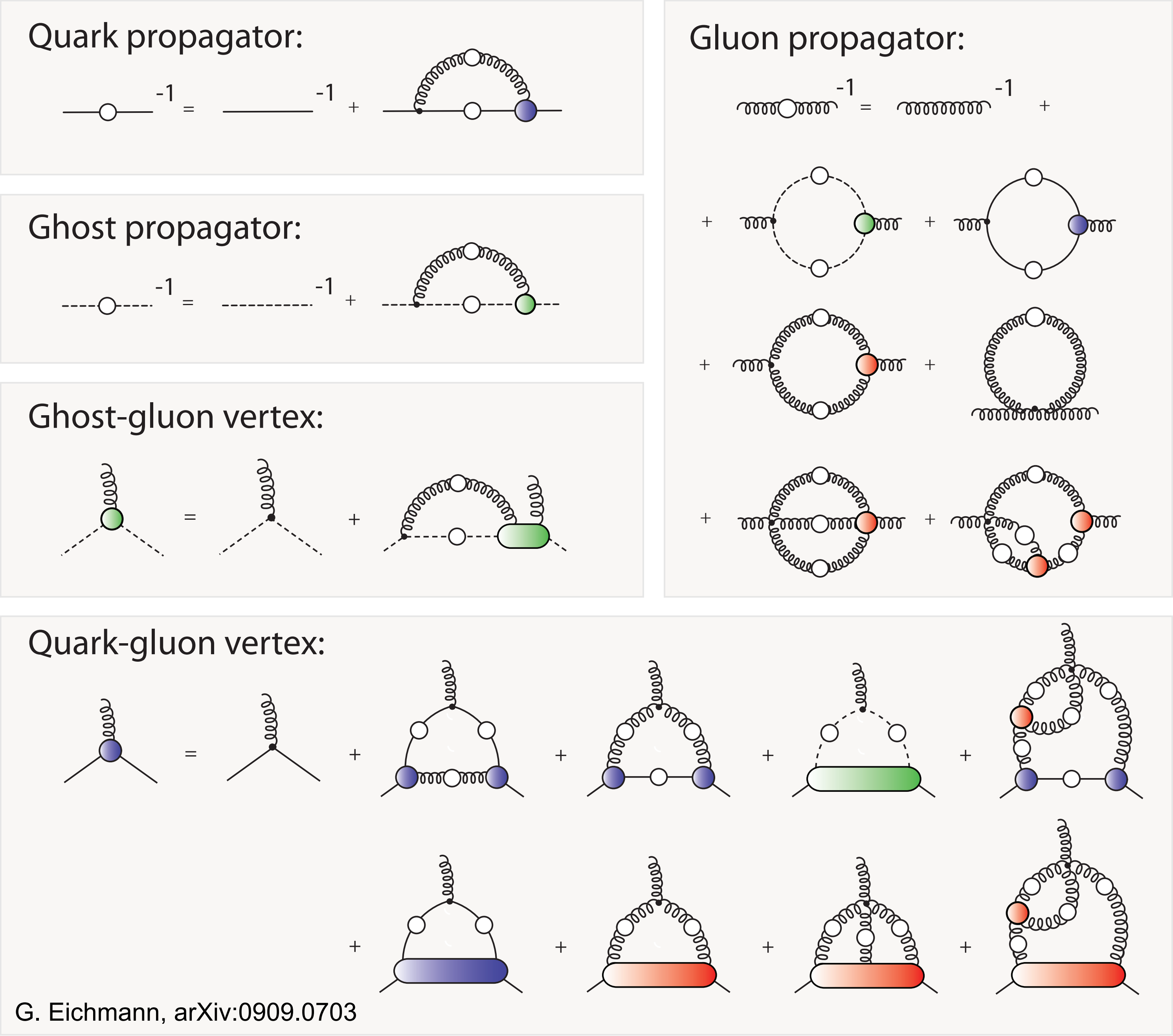
Quark propagator:

Gluon propagator:

Ghost propagator:

Ghost-gluon vertex:

Quark-gluon vertex:

G. Eichmann, arXiv:0909.0703

Continuum QCD: Equations of motion

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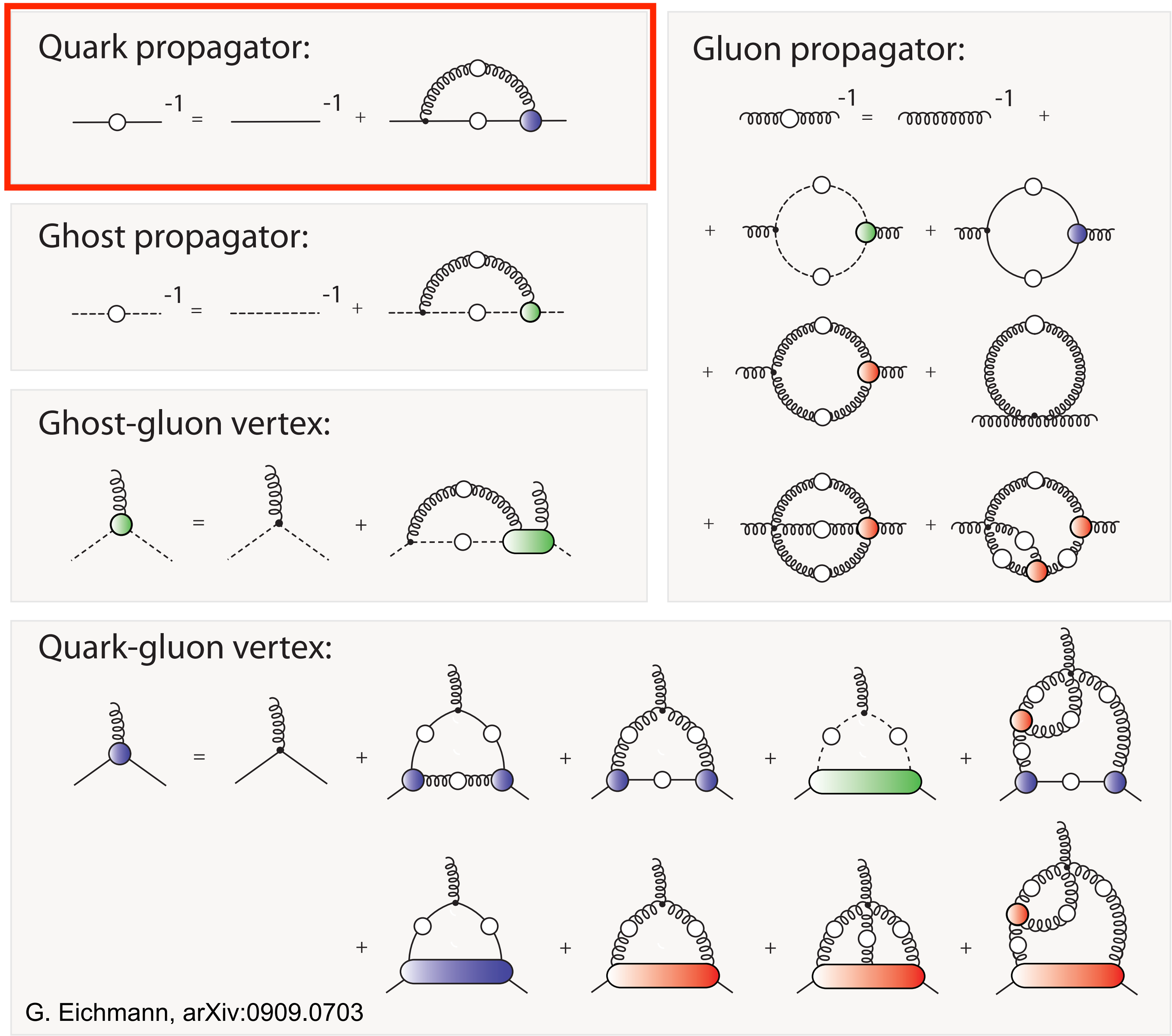
Dyson-Schwinger Equations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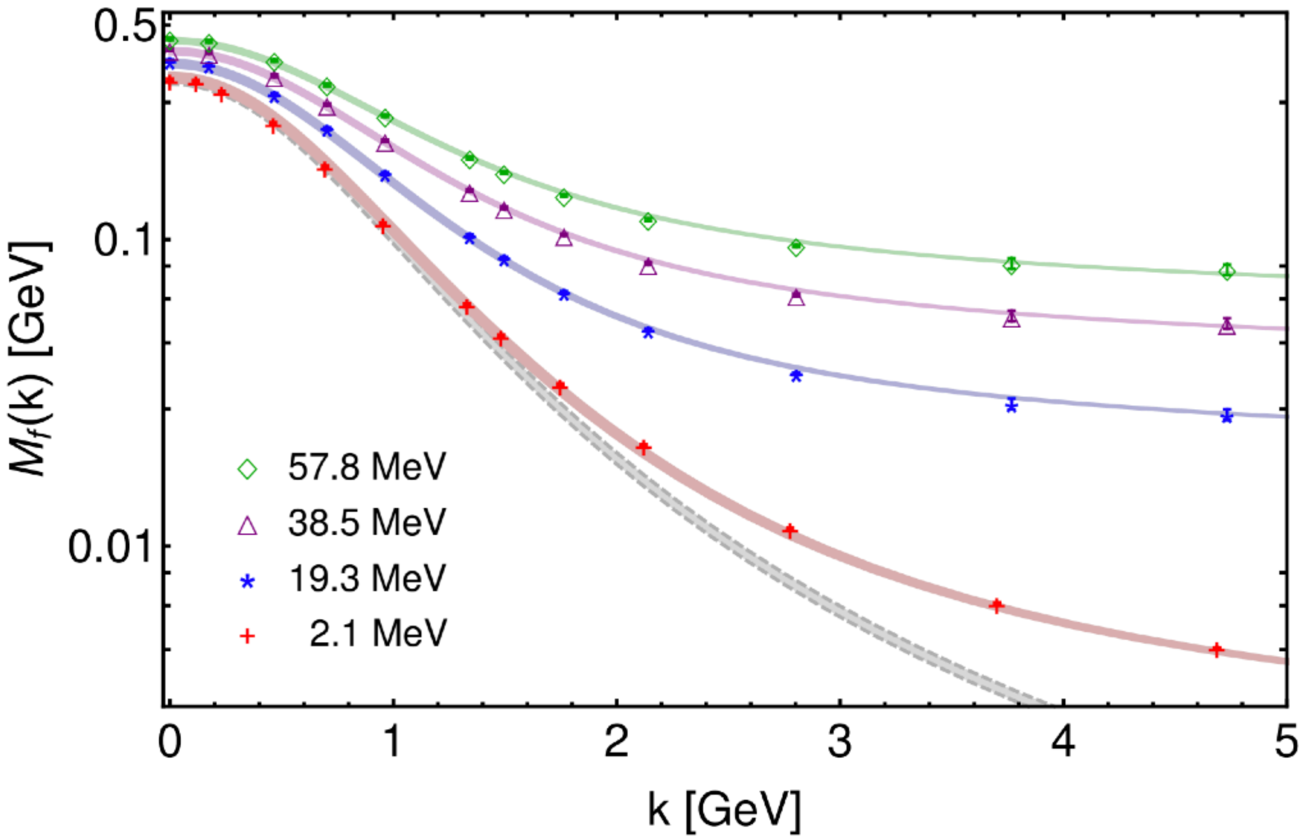
Quark: Running mass function

$$S(p) = \frac{1}{i\gamma\cdot pA(p^2)+B(p^2)} = \frac{Z(p^2)}{i\gamma\cdot p+M(p^2)}$$

Chang et. al., PRD 104, 094509 (2021)



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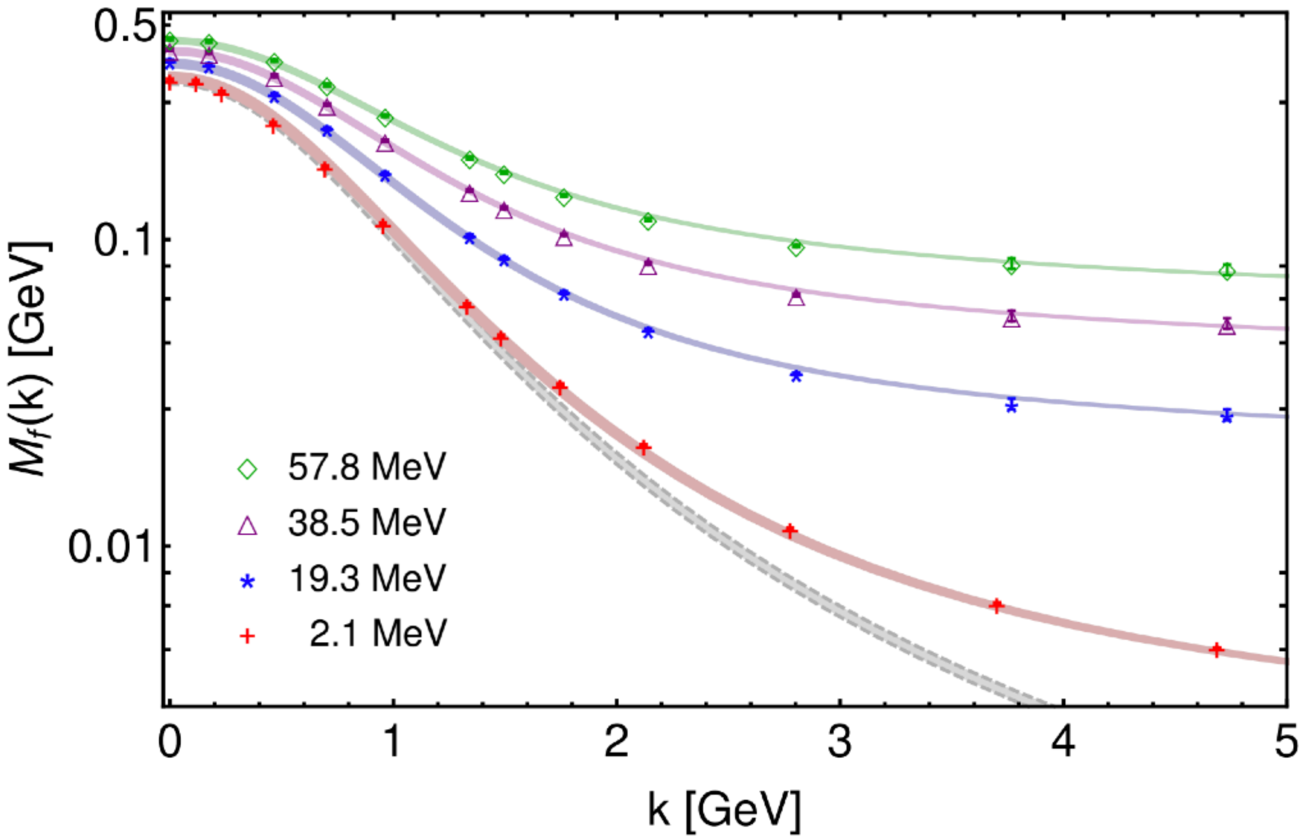

✦ **Now:**

1. The quark's **effective mass** runs with its momentum.
2. The most **constituent mass** of a light quark comes from a cloud of gluons.

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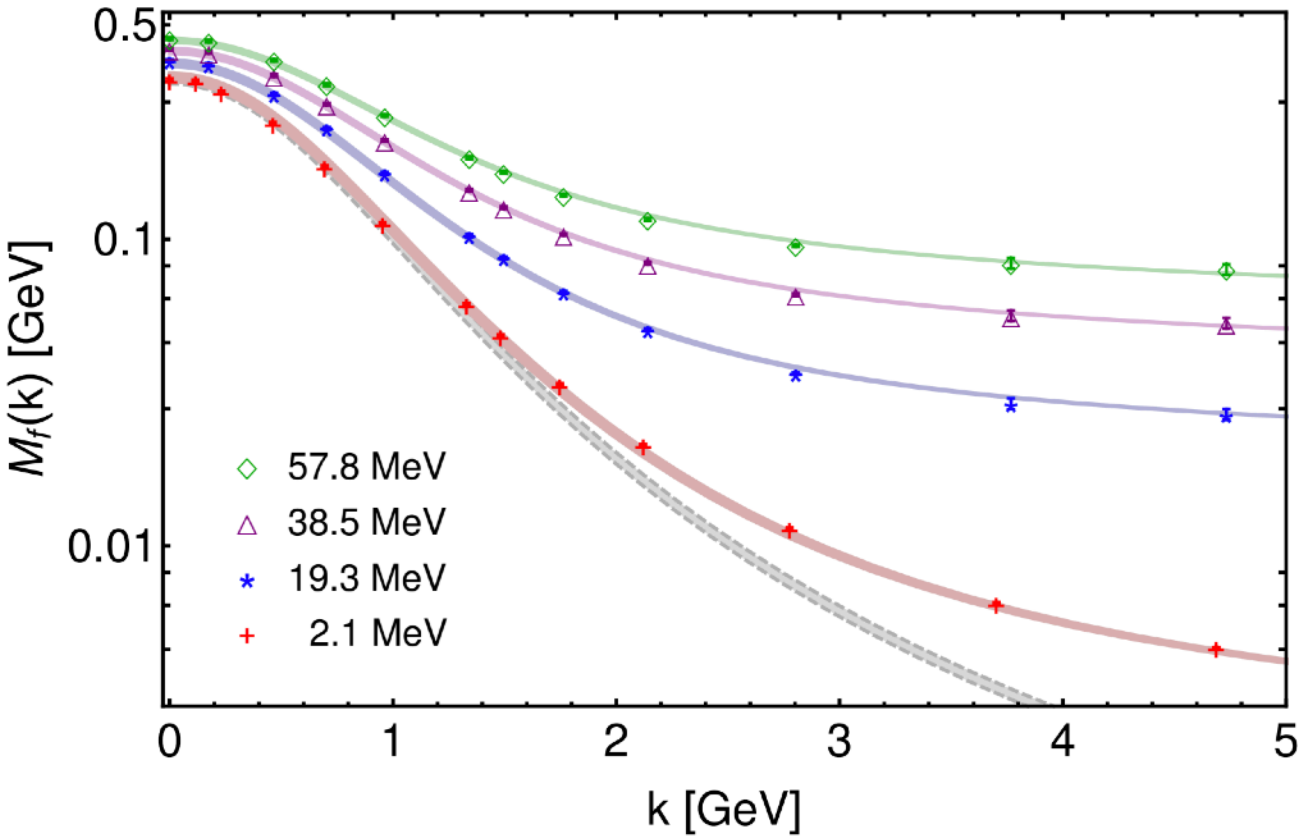

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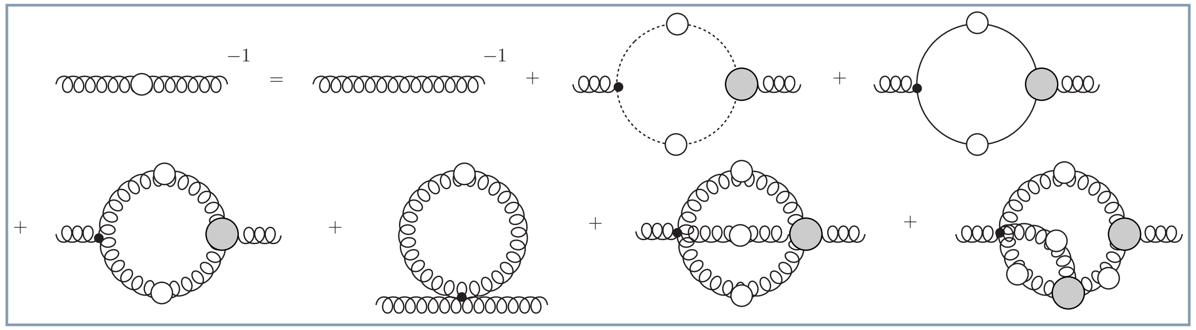
✦ **Next:**

1. What is the **infrared scale** of quark mass function?
2. How does the **transition** connect the non-perturbative and perturbative regions?

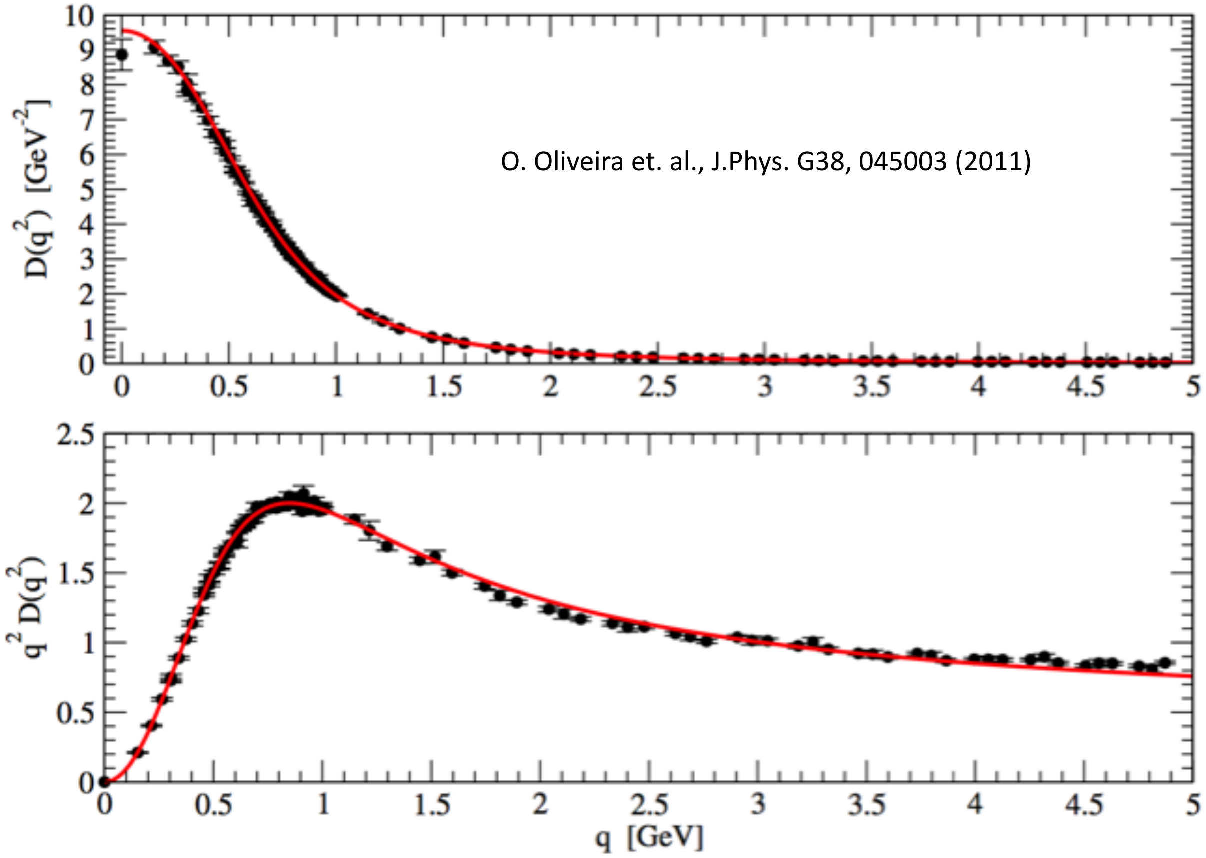
Gluon: Dynamical mass scale

Gluon gap equation:

Aguilar, Binosi, Papavassiliou and Rodriguez-Quintero



Lattice QCD simulations:



O. Oliveira et. al., J.Phys. G38, 045003 (2011)

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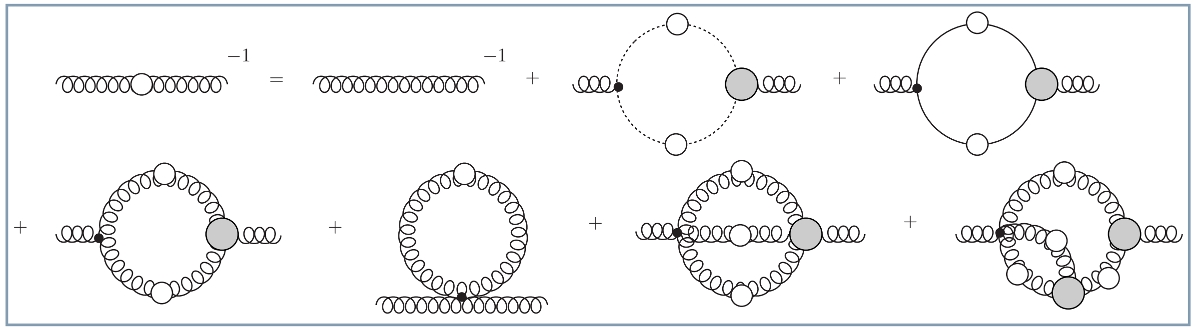

- The interaction can be decomposed: ***gluon running mass*** + ***effective running coupling***

$$g^2 D_{\mu\nu}(k) = \mathcal{G}(k^2)\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)$$

$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_{RL}(k^2)}{k^2 + m_g^2(k^2)}$$

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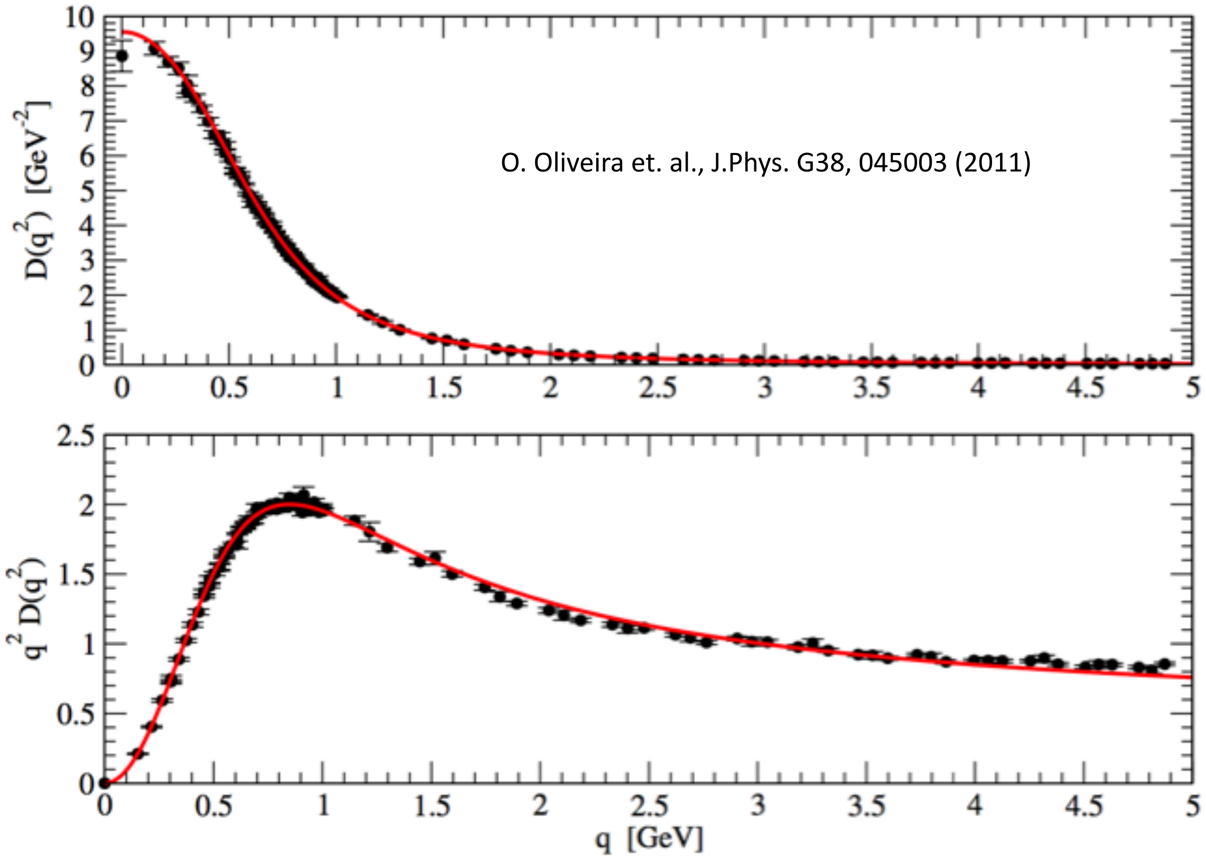

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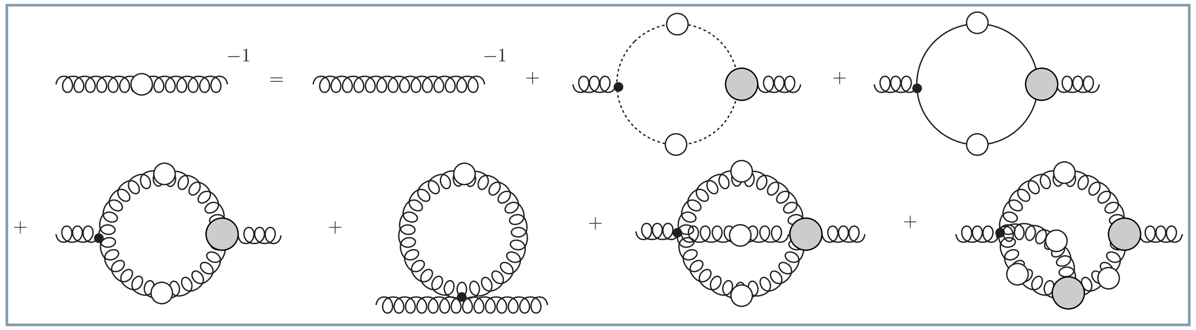
Gluon gap equation: Aguilar, Binosi, Papavassiliou and Rodriguez-Quintero



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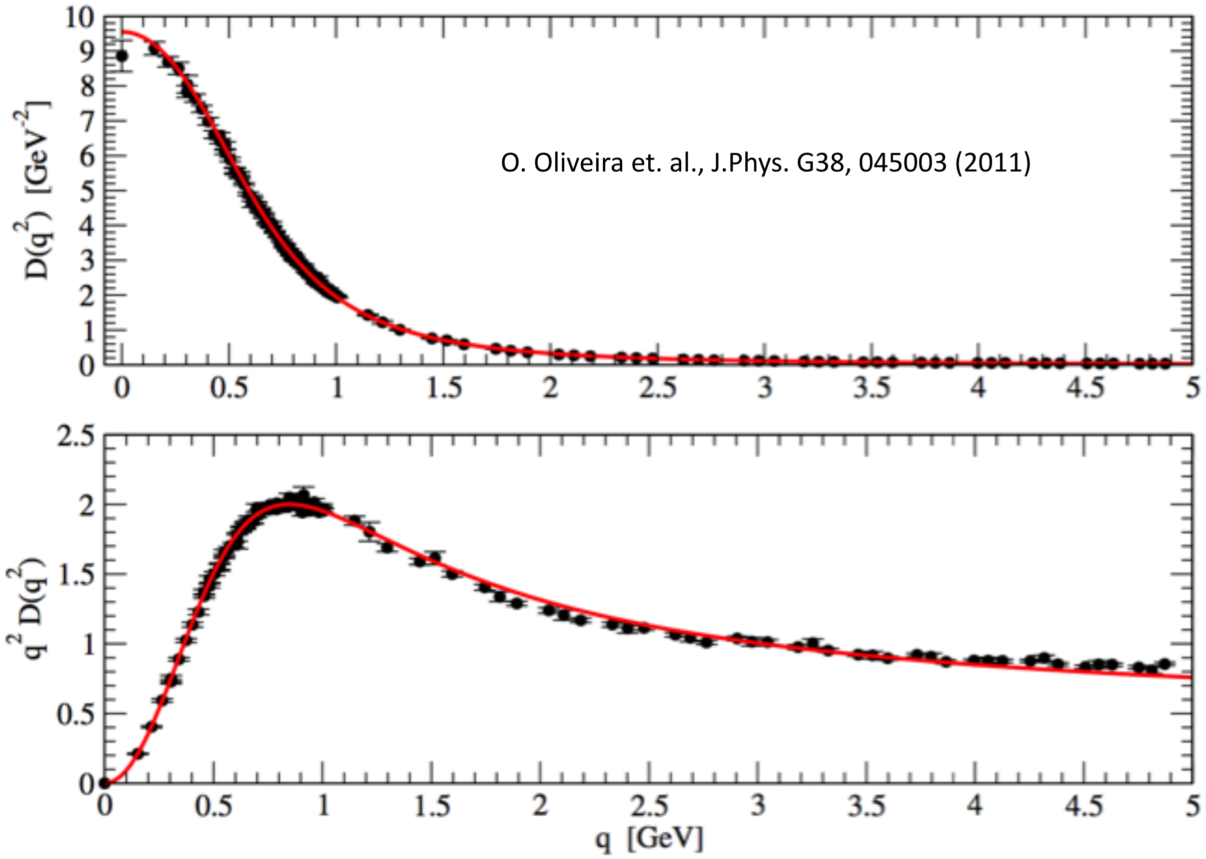

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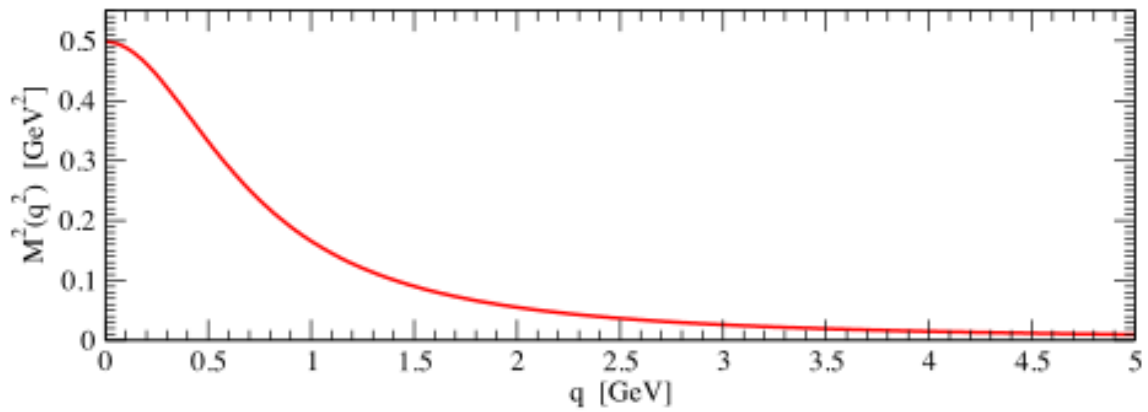
- In QCD: Gluons are ***cannibals*** – a particle species whose members become **massive** by eating each other!

Gluon: Dynamical mass scale

Gluon mass function:

O. Oliveira et. al., J.Phys. G38, 045003 (2011)



Running coupling:

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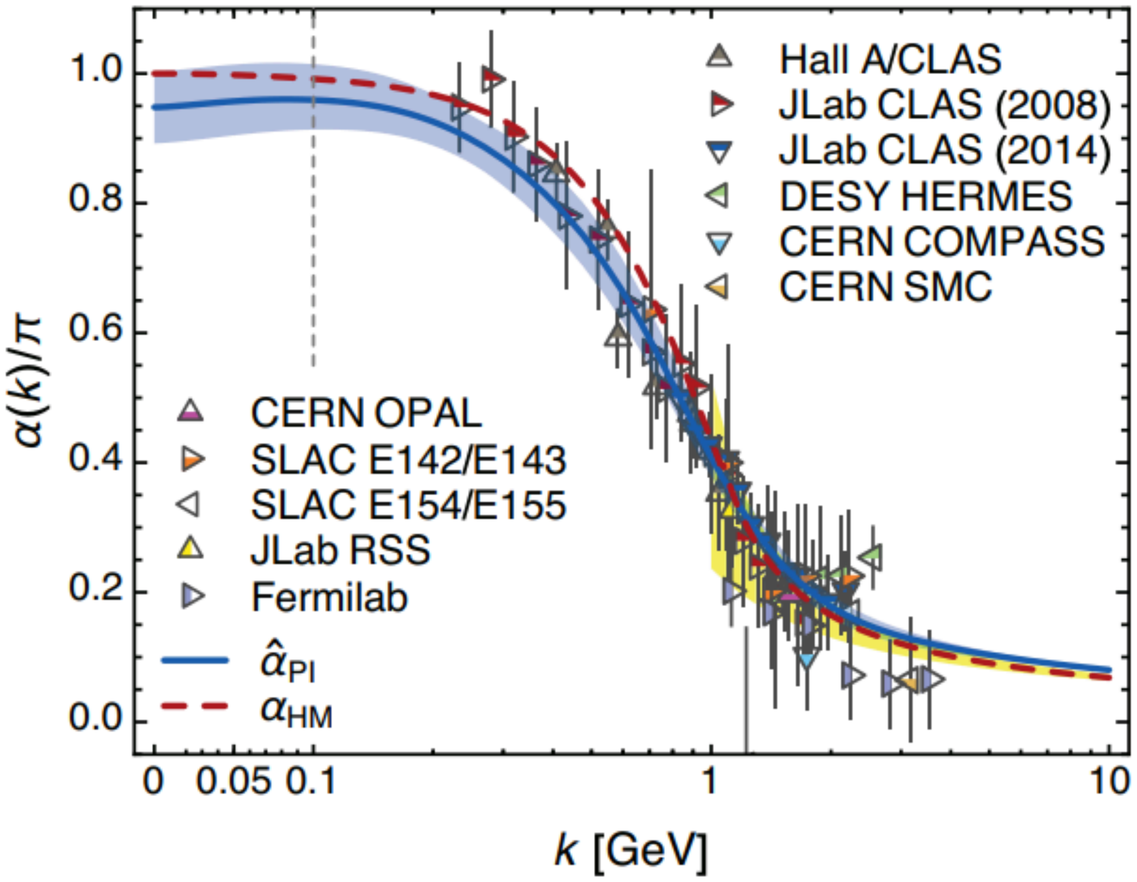

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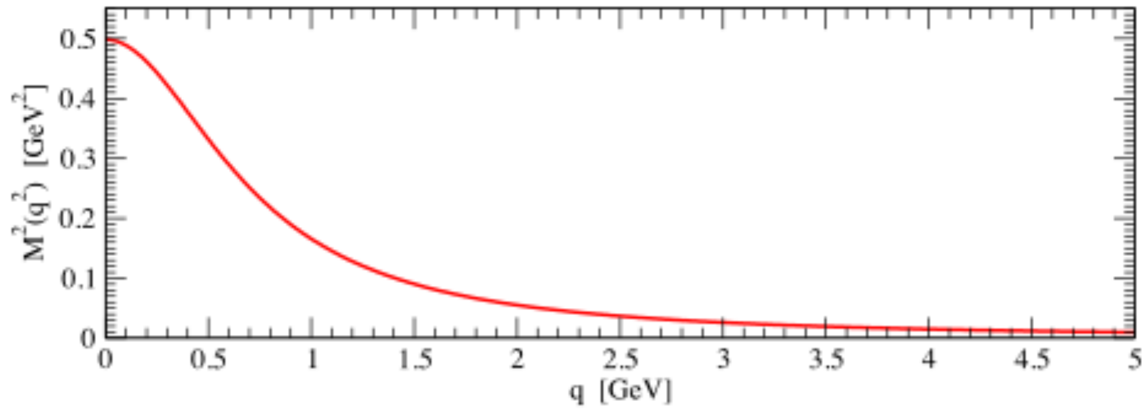
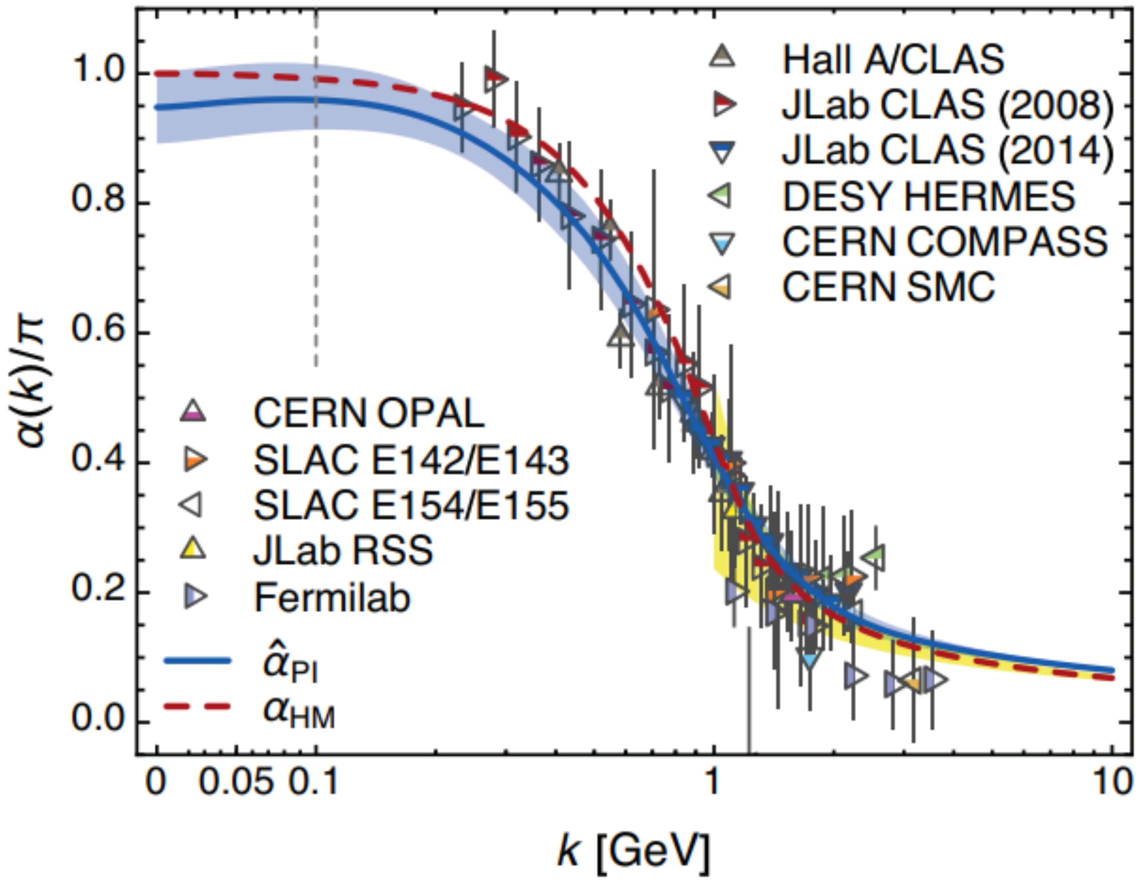


Running coupling: Binosi, Mezrag, Papavassiliou, Roberts and Rodriguez-Quintero



✦ **Now:**

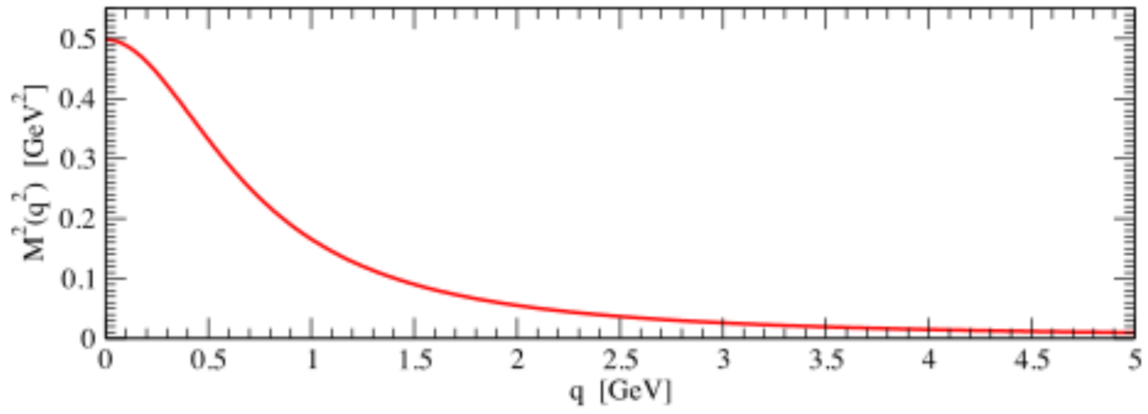
1. The dressed gluon can be well parameterized by a **mass scale**

$$m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2}$$

2. The effective running coupling **saturates** in the infrared limit.

Gluon: Dynamical mass scale

Gluon mass function: O. Oliveira et. al., J.Phys. G38, 045003 (2011)



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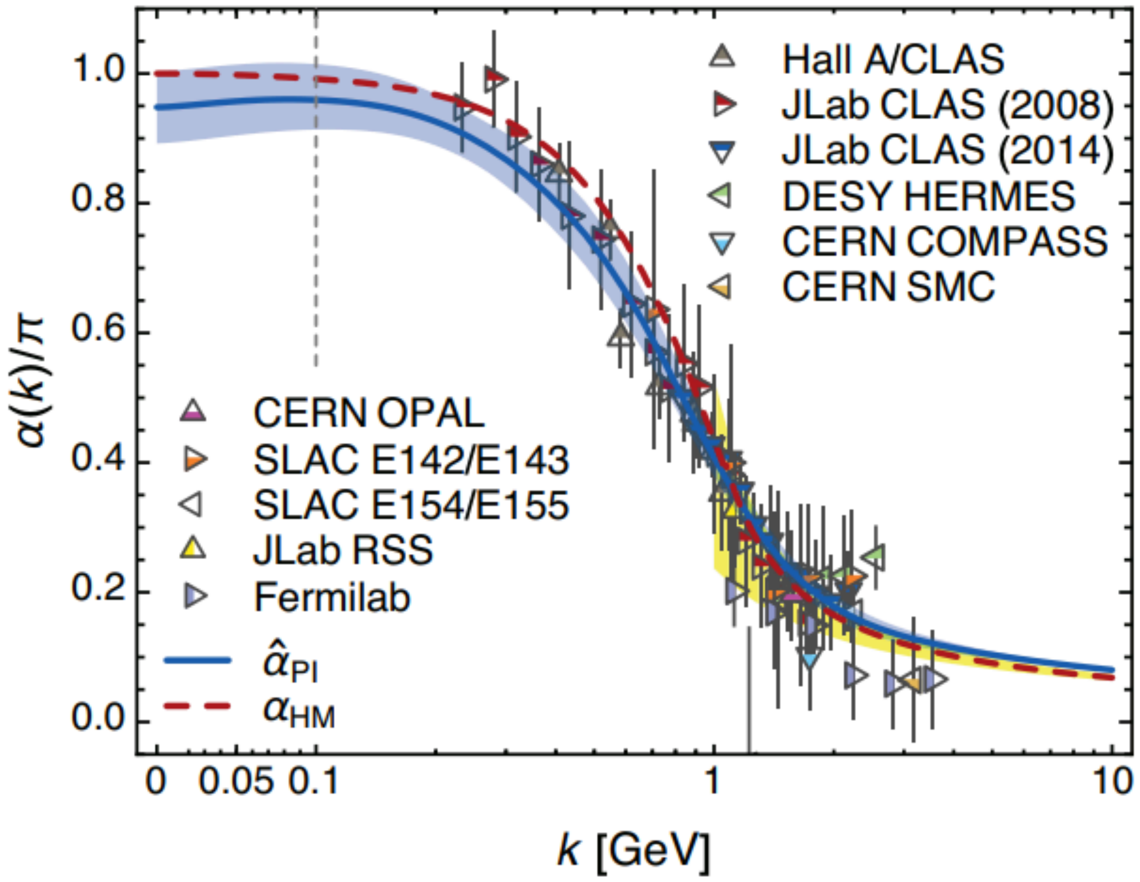

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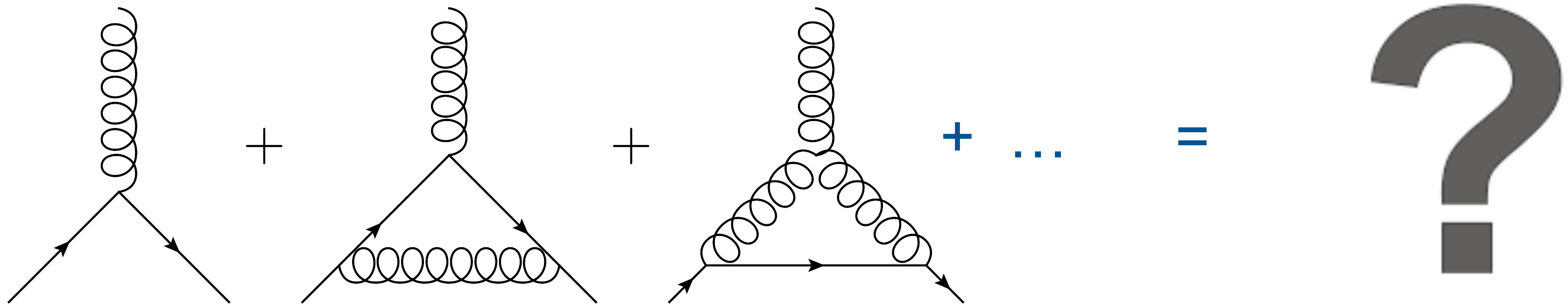
2. The effective running coupling **saturates** in the infrared limit.

✦ **Next:**

1. What is the **mass scale** of gluon?
2. What is the **infrared magnitude** of running coupling?

Vertex: DCSB feedback

Quark-gluon vertex:



See, e.g., PLB722, 384 (2013)

Vertex: DCSB feedback

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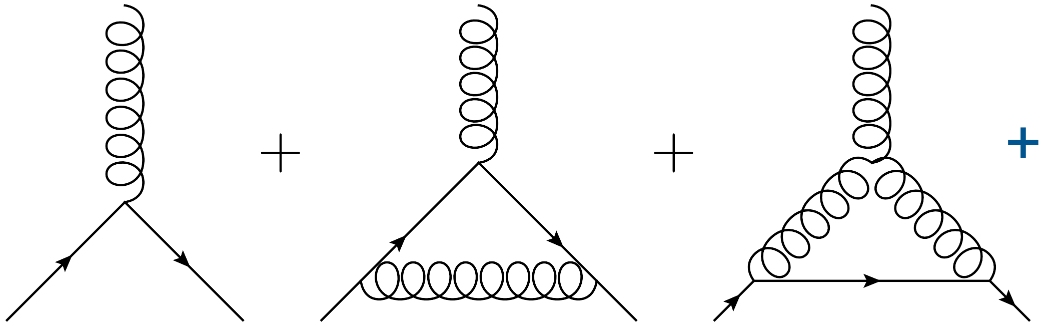
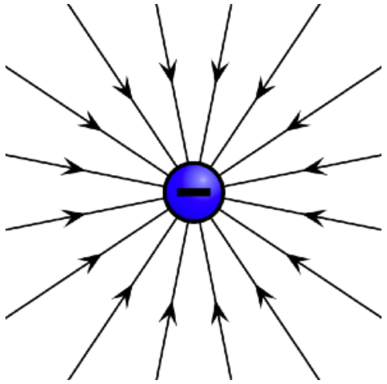
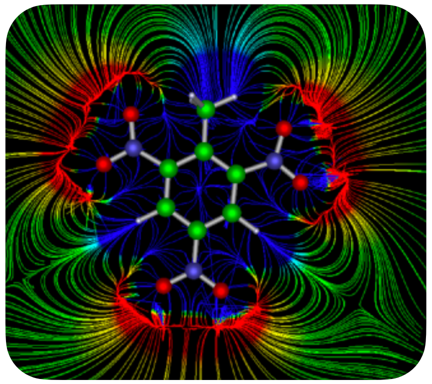


+ … = ?

point charge





distributed charge

See, e.g., PLB722, 384 (2013)

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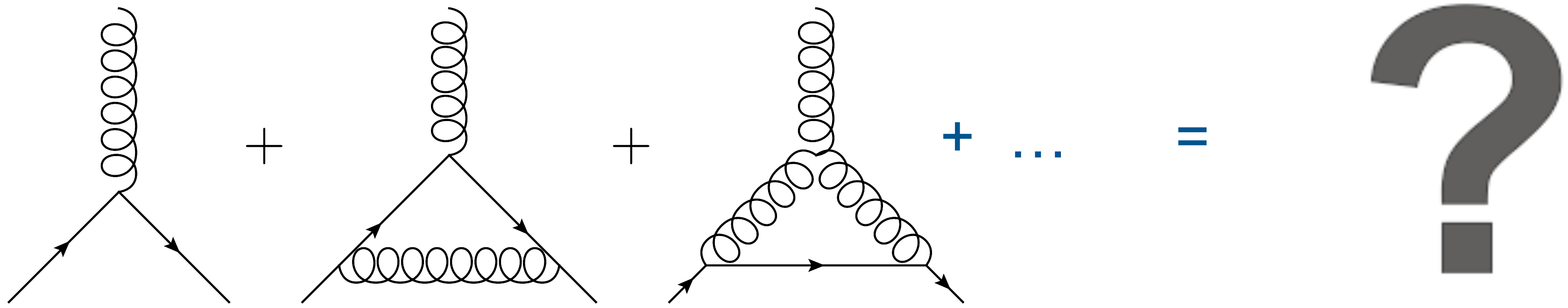

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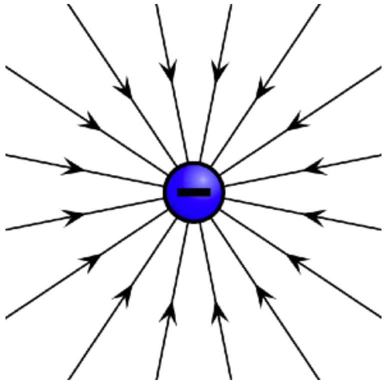

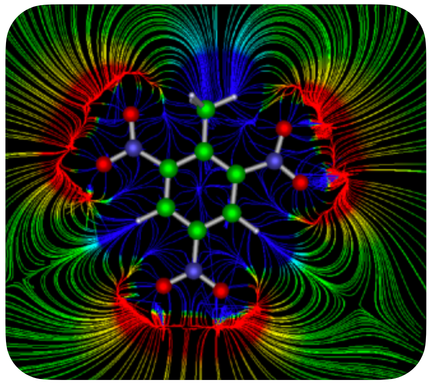

distributed charge

✦ The **Dirac** and **Pauli** terms: for an on-shell fermion, the vertex can be decomposed by two form factors:

$$\Gamma^{\mu}(P', P) = \gamma^{\mu} F_1(Q^2) + \frac{i\sigma_{\mu\nu}}{2M_f} Q^{\nu} F_2(Q^2)$$

✦ The form factors express (color-)**charge** and (color-)**magnetization** densities. And the so-called **anomalous magnetic moment** is proportional to the Pauli term.

See, e.g., PLB722, 384 (2013)

Vertex: DCSB feedback

Spacetime

- ☐ **Poincaré symmetry**

Fields

- ☐ **Gauge symmetry**
- ☐ **Chiral symmetry**

“Symmetry dictates interaction.” — CN Yang

See, e.g., PLB722, 384 (2013)

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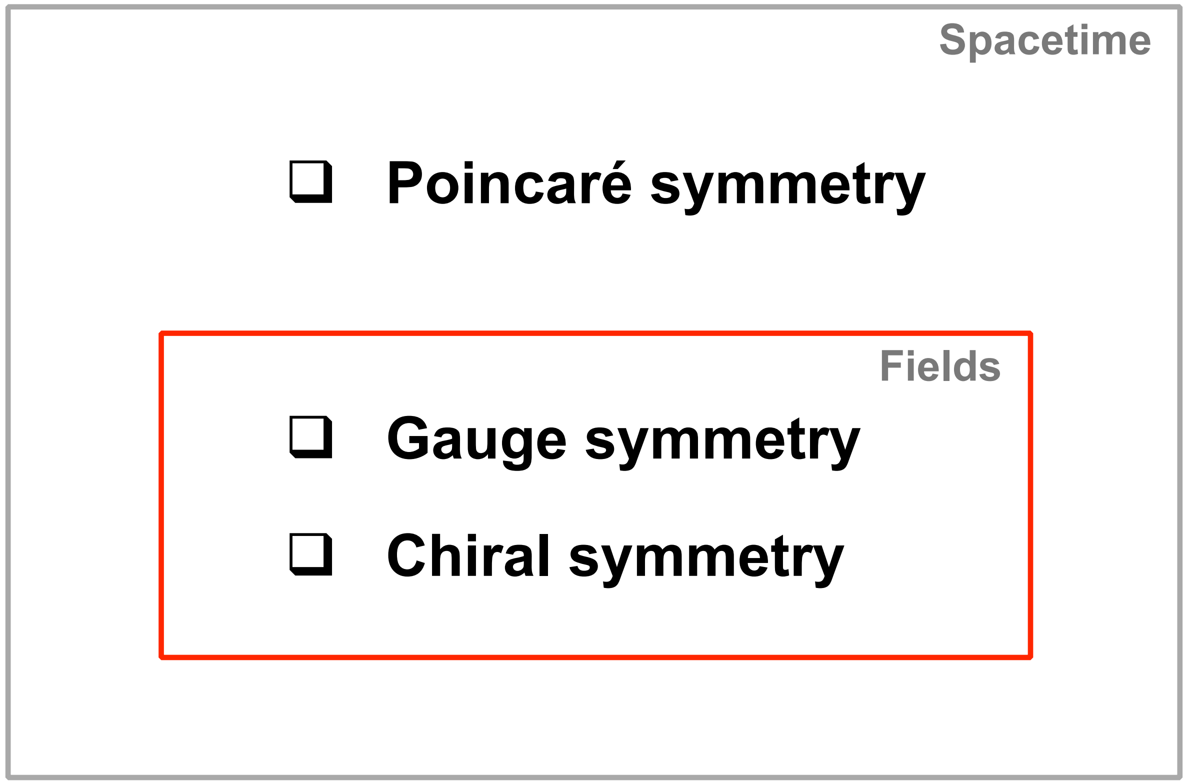

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☐ **Gauge symmetry:** Longitudinal WGTI

$$\boxed{iq_\mu \Gamma_\mu(k,q) = S^{-1}(k) - S^{-1}(p)}$$

☐ **Lorentz symmetry + :** Transverse WGTIs

$$\begin{aligned} q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) &= S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) \\ &\quad + 2im\Gamma_{\mu\nu}(k,p) + t_\lambda \epsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) \\ &\quad + A^V_{\mu\nu}(k,p)\,, \\ q_\mu \Gamma^A_\nu(k,p) - q_\nu \Gamma^A_\mu(k,p) &= S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu} S^{-1}(k) \\ &\quad + t_\lambda \epsilon_{\lambda\mu\nu\rho}\Gamma_\rho(k,p) \\ &\quad + V^A_{\mu\nu}(k,p)\,, \qquad \sigma^5_{\mu\nu} = \sigma_{\mu\nu}\gamma_5 \end{aligned}$$

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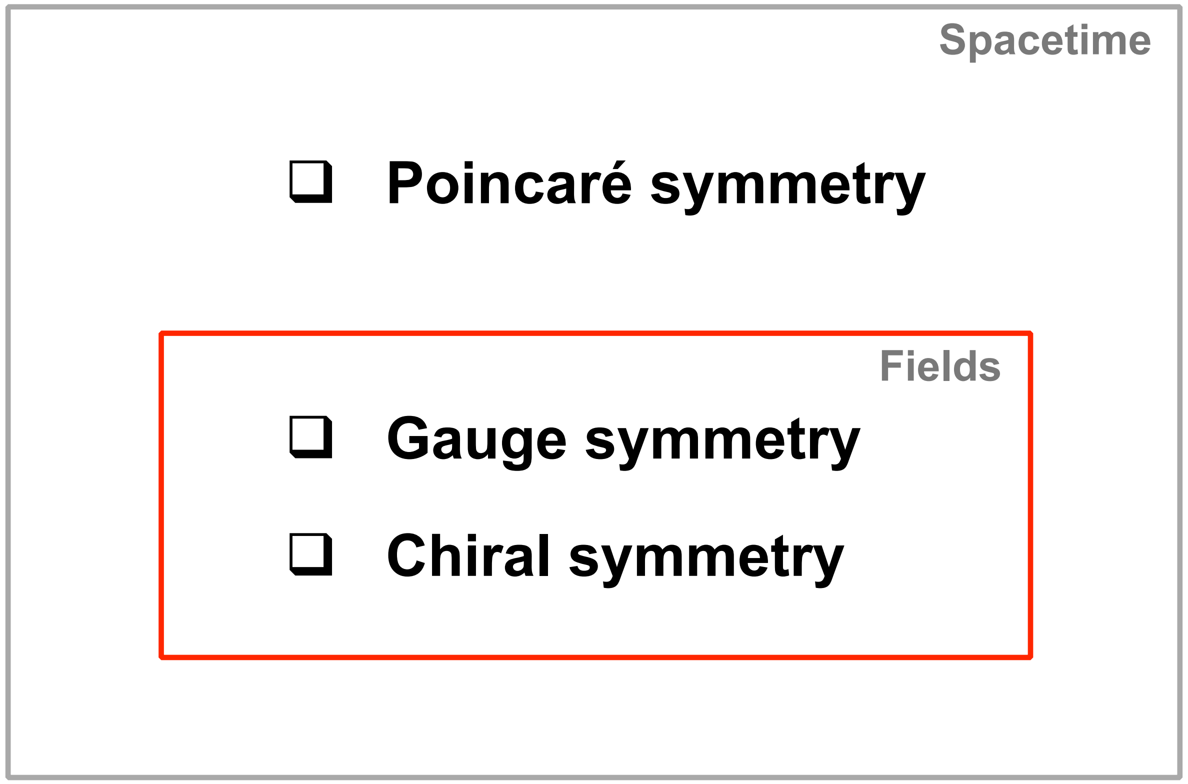

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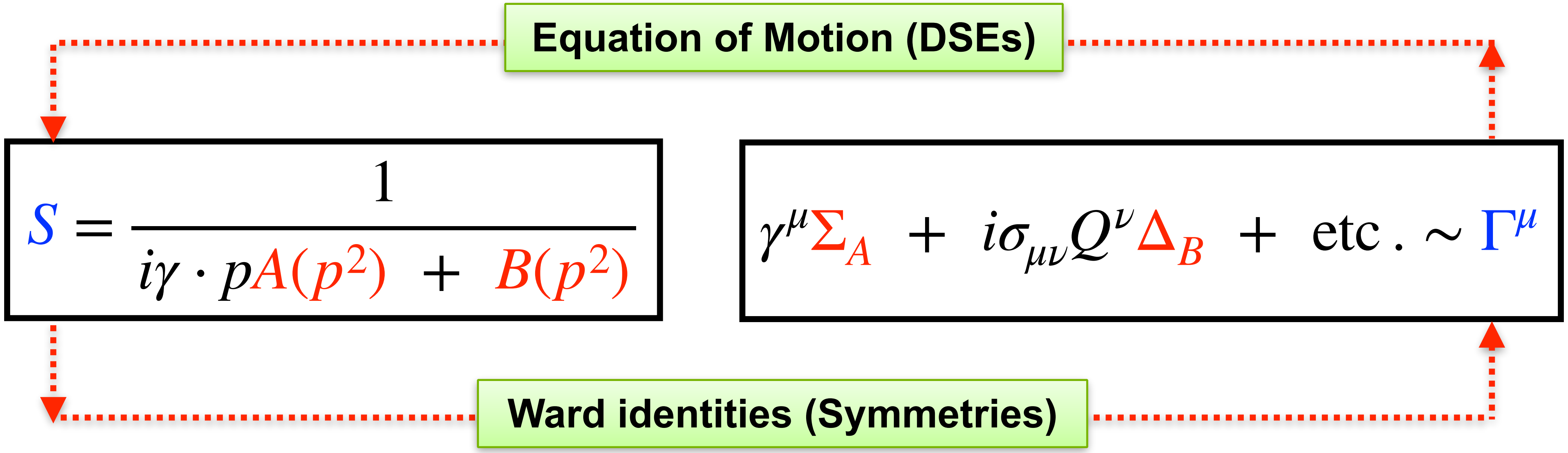
The WGTIs of the vertices can be decoupled and (partially) solved.

See, e.g., PLB722, 384 (2013)

Vertex: DCSB feedback



Equation of Motion (DSEs)

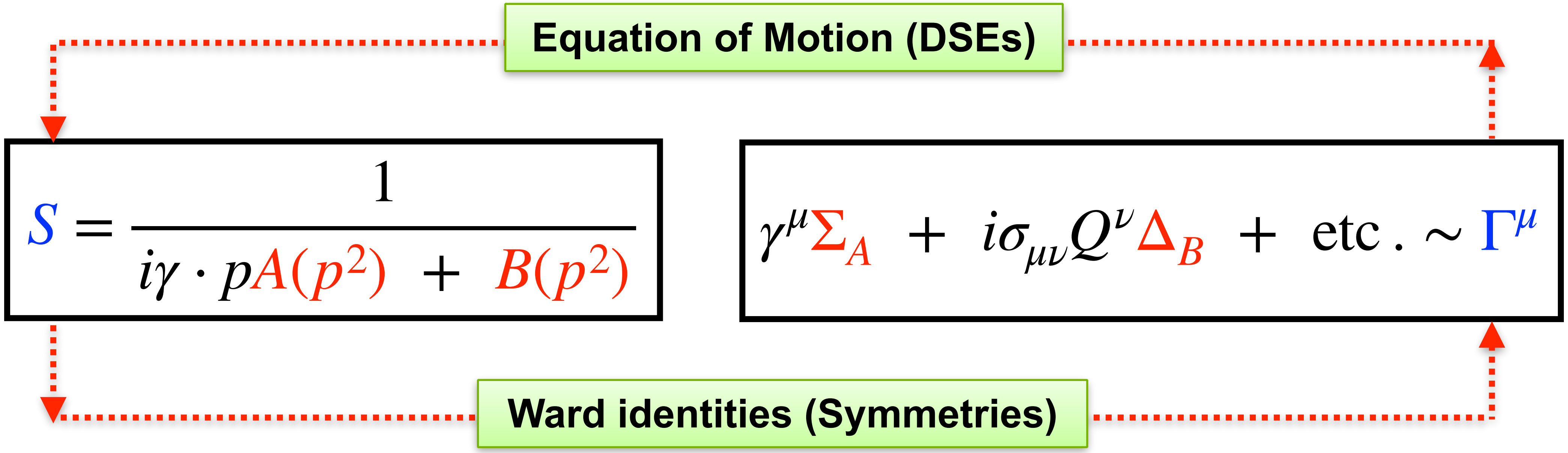
$$S = \frac{1}{i\gamma\cdot p A(p^2) \; + \; B(p^2)}$$

$$\gamma^\mu \Sigma_A \; + \; i\sigma_{\mu\nu} Q^\nu \Delta_B \; + \; \text{etc.} \sim \Gamma^\mu$$

Ward identities (Symmetries)

See, e.g., PLB722, 384 (2013)

Vertex: DCSB feedback



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✦ **Now:**

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See, e.g., PLB722, 384 (2013)

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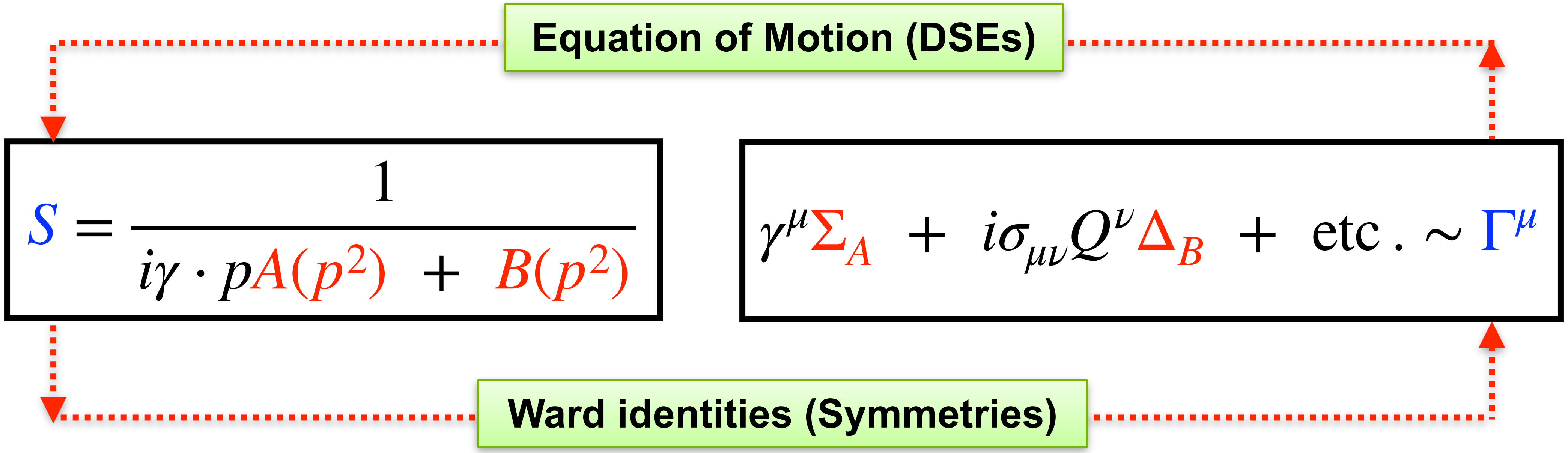

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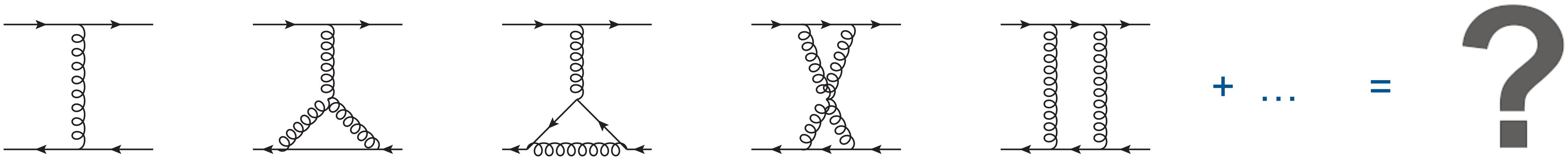
✦ **Next:**

1. What are the exact **strengths** of the terms in the vertex?

2. What the exact **behaviors** of the form factors in the vertex?

See, e.g., PLB722, 384 (2013)

Kernel: Twofold role of pion



+ … = ?

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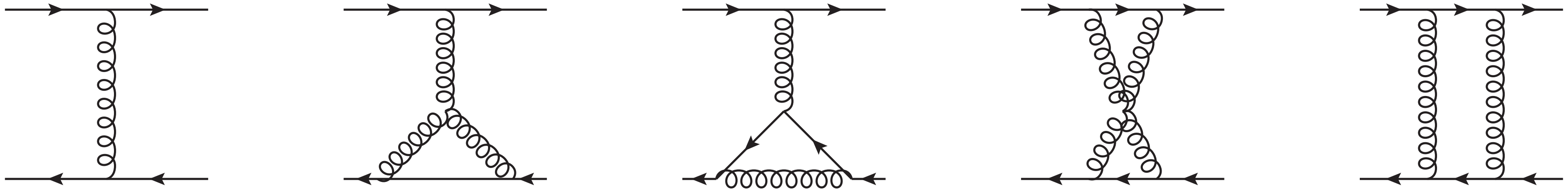

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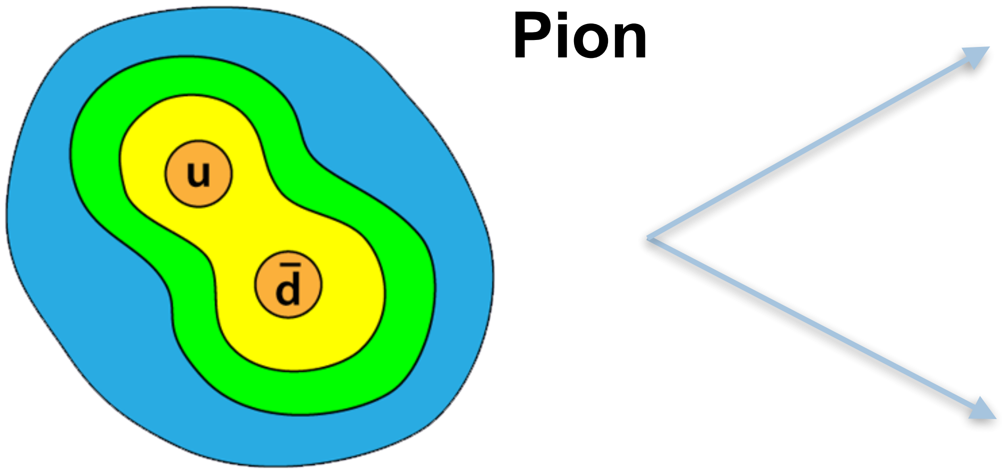

- **Bound state** of quark and anti-quark, but abnormally light:

$$M_\pi \ll M_u + M_{\bar{d}}$$

- **Goldstone's theorem:** If a generic continuous symmetry is spontaneously broken, then new massless scalar particles appear in the spectrum of possible excitations.

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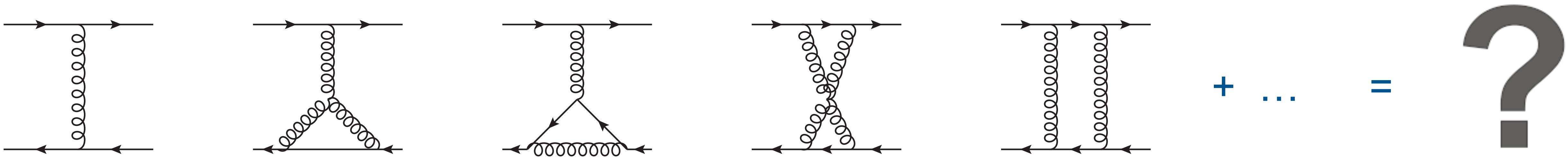

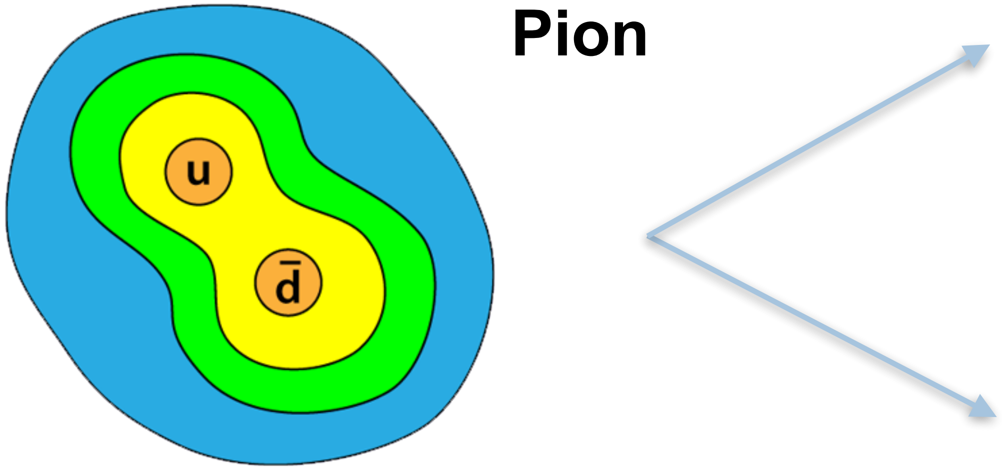

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- ✦ The **discrete** and **continuous** **symmetries** strongly constrain the kernel:

Poincaré symmetry

C-, P-, T-symmetry

Gauge symmetry

Chiral symmetry

Kernel: Twofold role of pion

✦ **Now:**

1. A deep connection between one-body and two-body problem:

$$\boxed{f_\pi E_\pi(k^2) = B(k^2)} \left\{ \right.$$

Pion exists if, and only if, the **quark mass** is dynamically generated.

Two-body problem solved, almost completely, once solution of **one-body** problem is known.

See, e.g., CPL 38 (2021) 7, 071201

Kernel: Twofold role of pion

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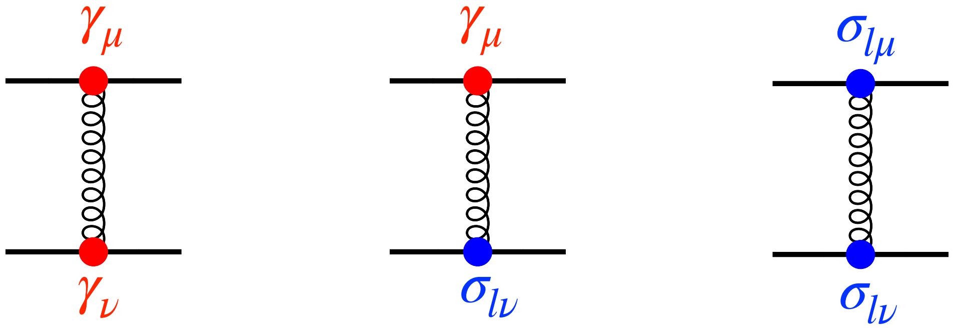

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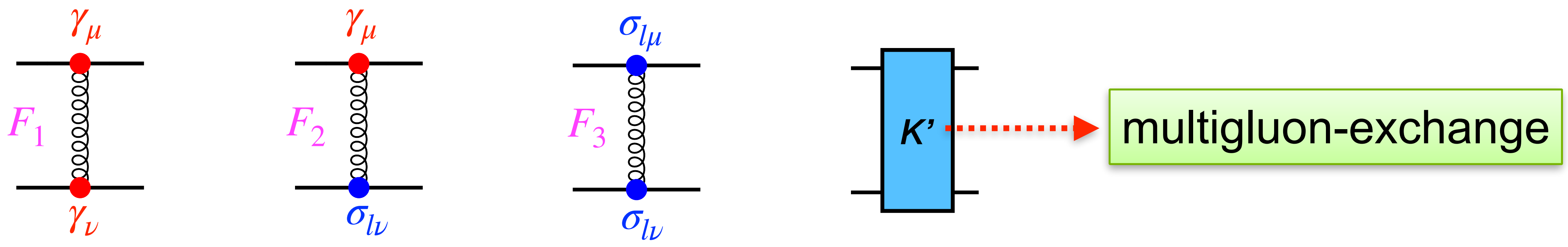

multigluon-exchange

See, e.g., CPL 38 (2021) 7, 071201

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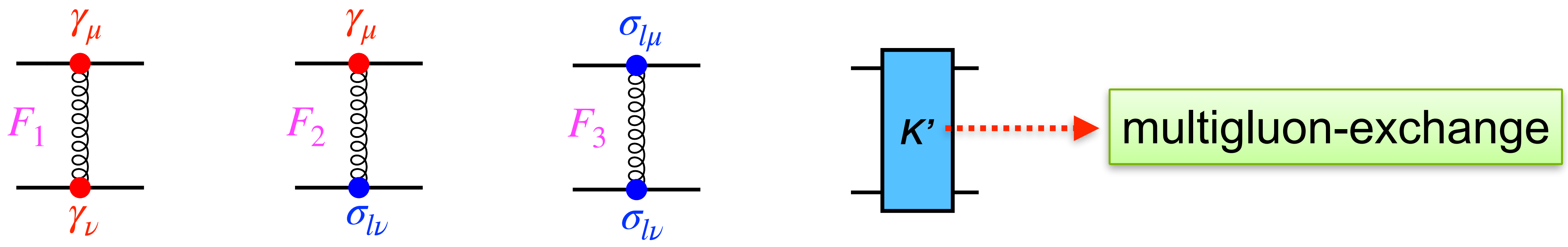

✦ **Next:**

1. How to further **pin down structures** of the kernel?

2. How to **simplify the kernel** for more practical applications?

See, e.g., CPL 38 (2021) 7, 071201

Chapter II: Applications

Properties of few-body and many-body systems

Few: Partial-wave structures of mesons

✦ Structure of wave function, e.g. ***ρ*** meson:

$$
\begin{aligned}
\tau^{1}_{1^-} &= i\gamma^{T}_{\mu}, \\
\tau^{2}_{1^-} &= i\big[3k^{T}_{\mu}\gamma\cdot k^{T} - \gamma^{T}_{\mu}k^{T}\cdot k^{T}\big], \\
\tau^{3}_{1^-} &= ik^{T}_{\mu}\,k\cdot P\,\gamma\cdot P, \\
\tau^{4}_{1^-} &= i\big[\gamma^{T}_{\mu}\gamma\cdot P\,\gamma\cdot k^{T} + k^{T}_{\mu}\gamma\cdot P\big], \\
\tau^{5}_{1^-} &= k^{T}_{\mu}, \\
\tau^{6}_{1^-} &= k\cdot P\big[\gamma^{T}_{\mu}\gamma^{T}\cdot k - \gamma\cdot k^{T}\gamma^{T}_{\mu}\big], \\
\tau^{7}_{1^-} &= (k^{T})^{2}\left(\gamma^{T}_{\mu}\gamma\cdot P - \gamma\cdot P\gamma^{T}_{\mu}\right) - 2k^{T}_{\mu}\gamma\cdot k^{T}\gamma\cdot P, \\
\tau^{8}_{1^-} &= k^{T}_{\mu}\gamma\cdot k^{T}\gamma\cdot P.
\end{aligned}
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See, e.g., PRC 85, 035202 (2012)

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\end{aligned}
$$

$$
\boxed{\mathbf{J = 1}\ \begin{cases} \mathbf{S = 0,\ L = 1} \\ \mathbf{S = 1,\ L = 0} \\ \mathbf{S = 1,\ L = 2} \end{cases}}
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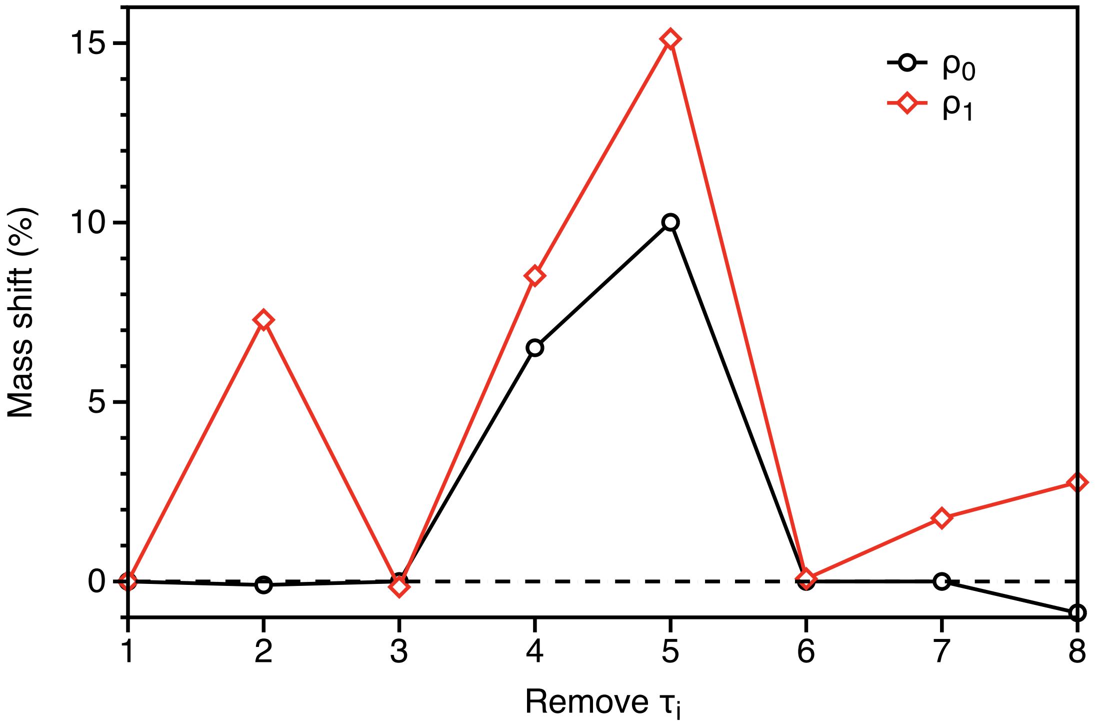
\tau_{1^-}^{5} &= k_\mu^T, \\
\tau_{1^-}^{6} &= k\cdot P\left[\gamma_\mu^T\gamma^T\cdot k-\gamma\cdot k^T\gamma_\mu^T\right], \\
\tau_{1^-}^{7} &= (k^T)^2\left(\gamma_\mu^T\gamma\cdot P-\gamma\cdot P\gamma_\mu^T\right)-2k_\mu^T\gamma\cdot k^T\gamma\cdot P, \\
\tau_{1^-}^{8} &= k_\mu^T\gamma\cdot k^T\gamma\cdot P.
\end{aligned}
$$



$$
\boxed{\mathbf{J=1}\ \begin{cases}\mathbf{S=0,\ L=1}\\ \mathbf{S=1,\ L=0}\\ \mathbf{S=1,\ L=2}\end{cases}}
$$

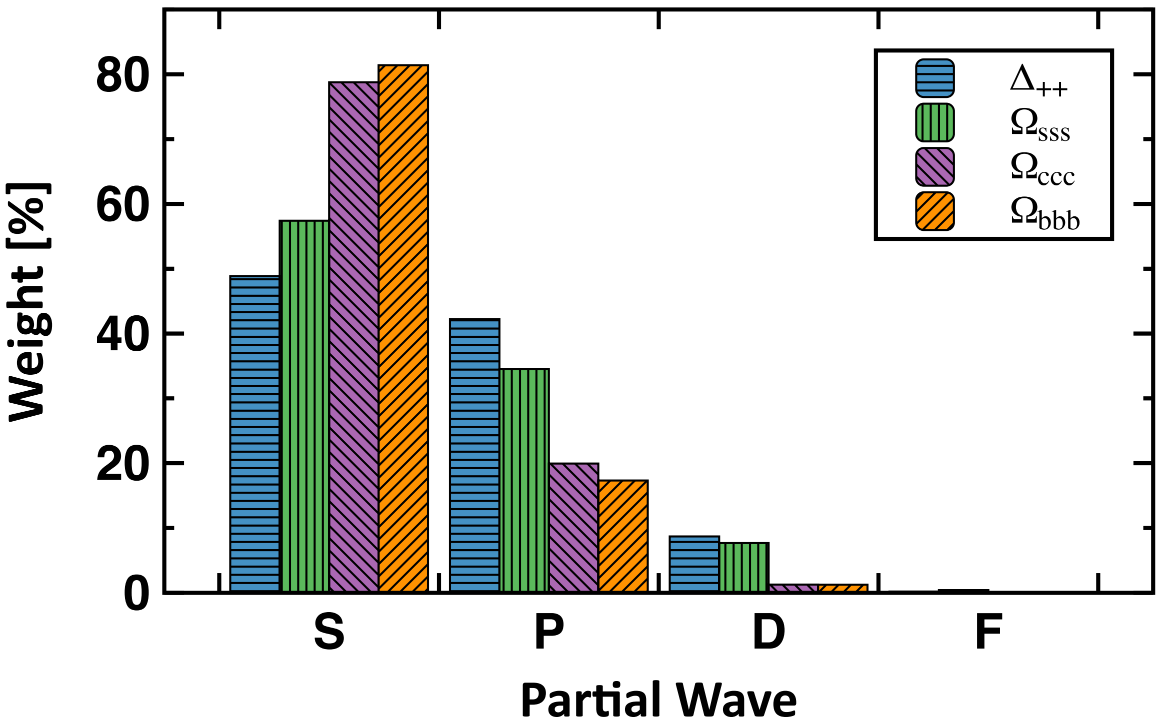
✦ Total angular momentum **J** is a good quantum number, but **S** and **L** are not. The partial waves mix together.

$$P \neq (-1)^{L+1}$$

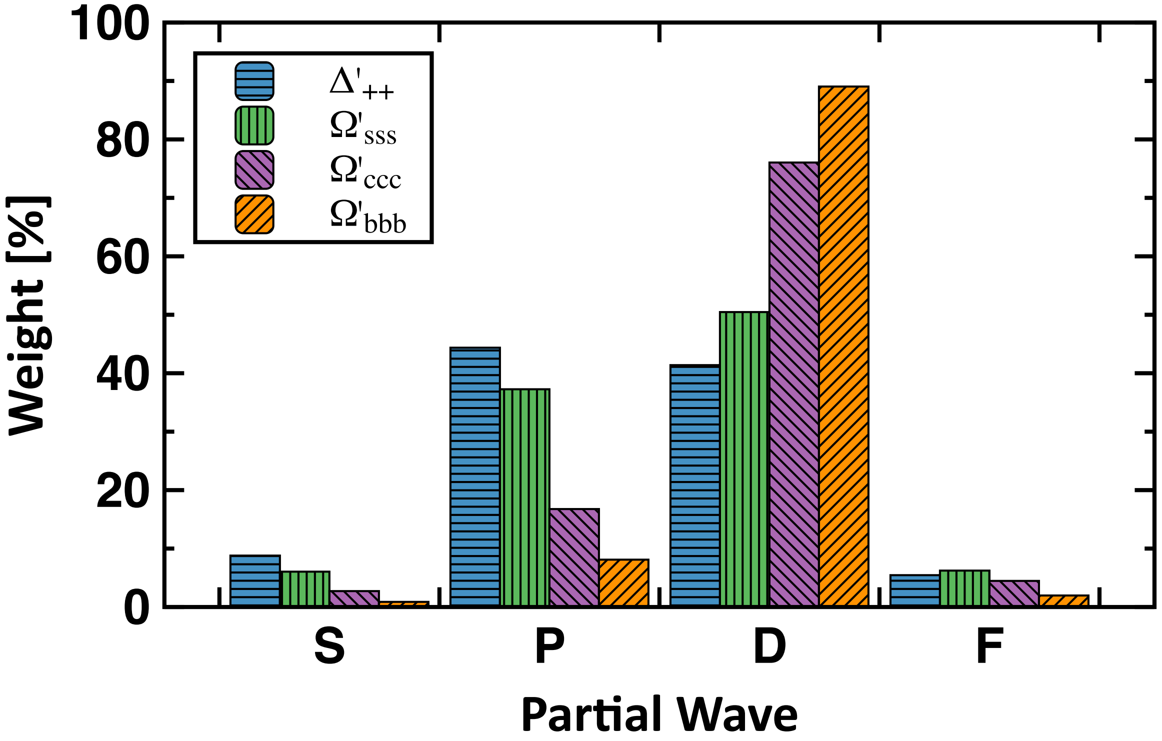
✦ Missing some partial waves could remarkably affect the mass, especially of radial excitation states.

See, e.g., PRC 85, 035202 (2012)

Few: Partial-wave structures of baryons



✓ S-waves dominate for ground states, but p-waves grow for light baryons.



✓ D-waves dominate for excited states, but p-waves grow for light baryons.

See, e.g., PRD 97, 114017 (2018)

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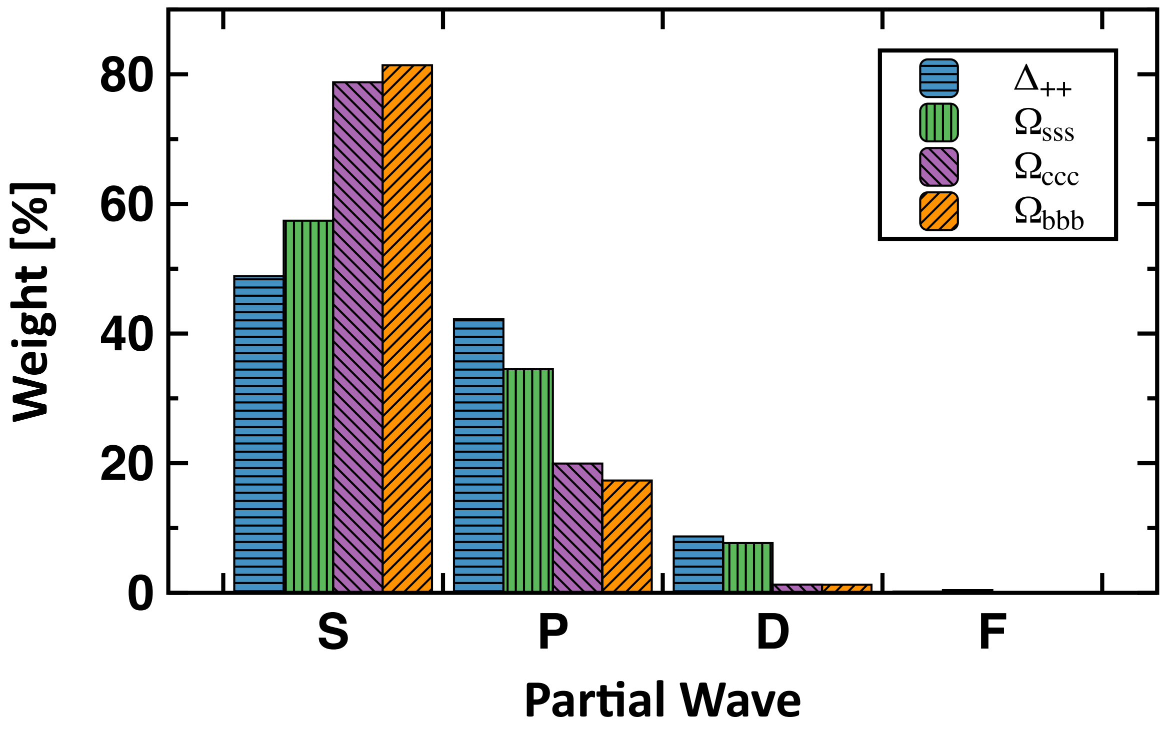

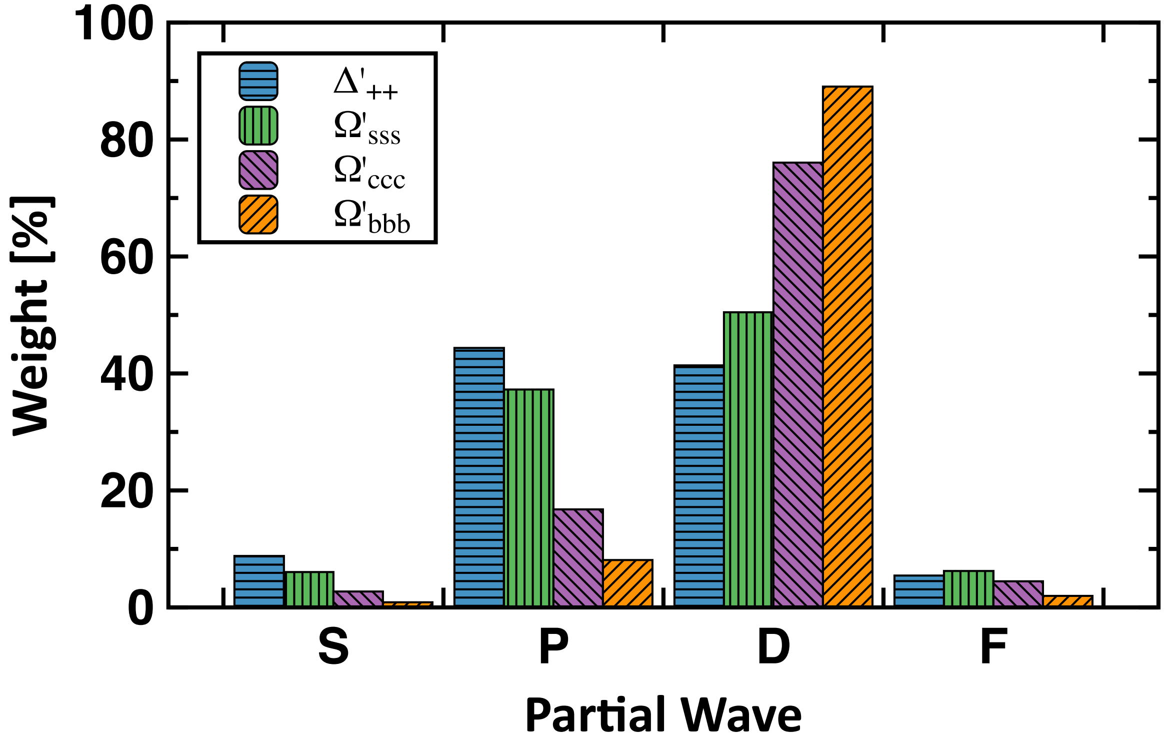

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How NR quark model works

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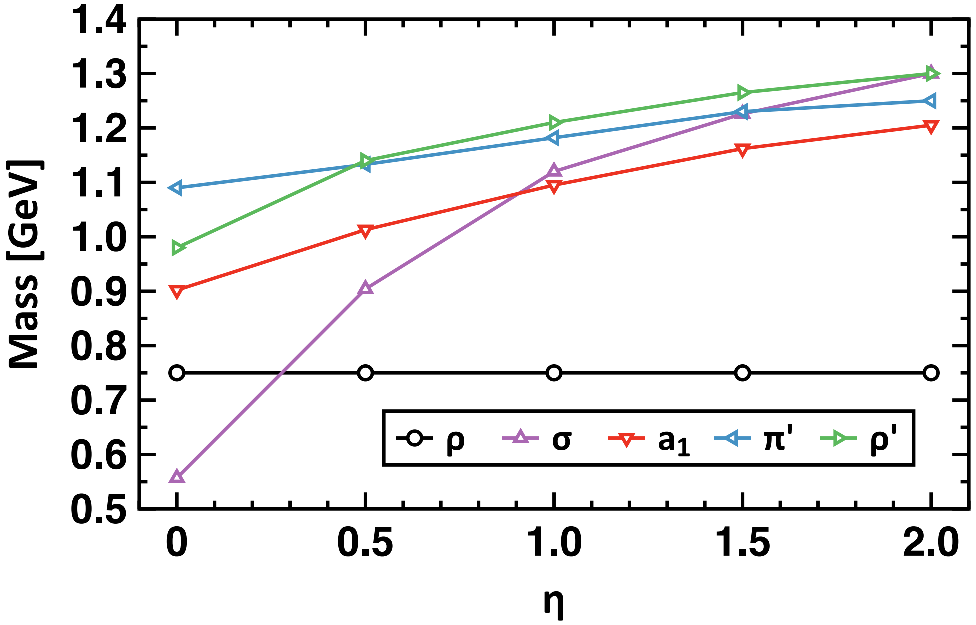
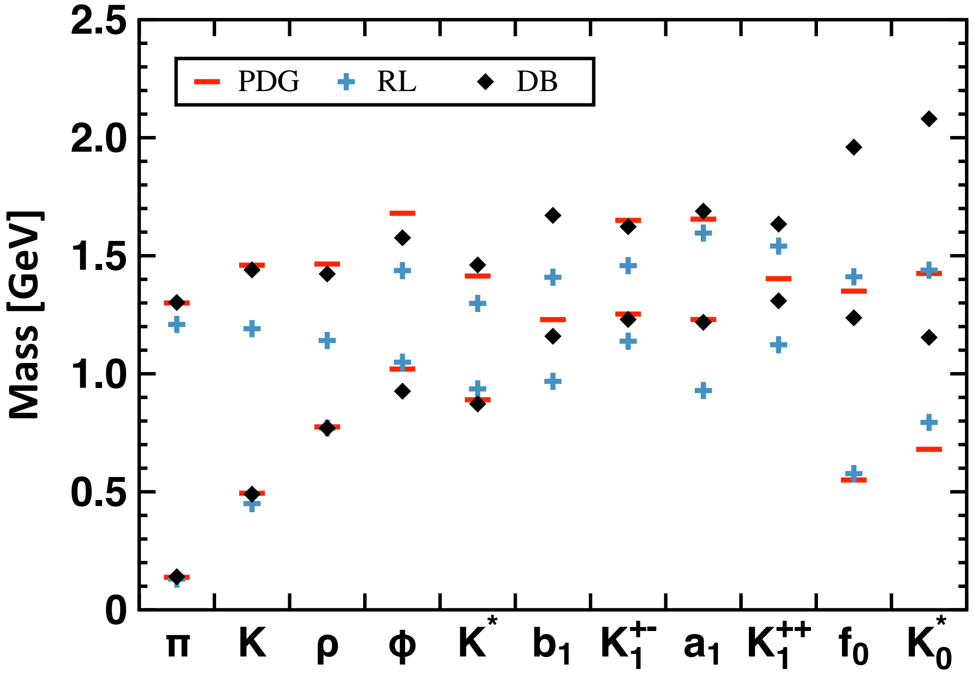
Few: Spectra of mesons

- Impact of the Pauli term (AM):



- Light-flavor meson spectrum:



See, e.g., CPL 38 (2021) 7, 071201

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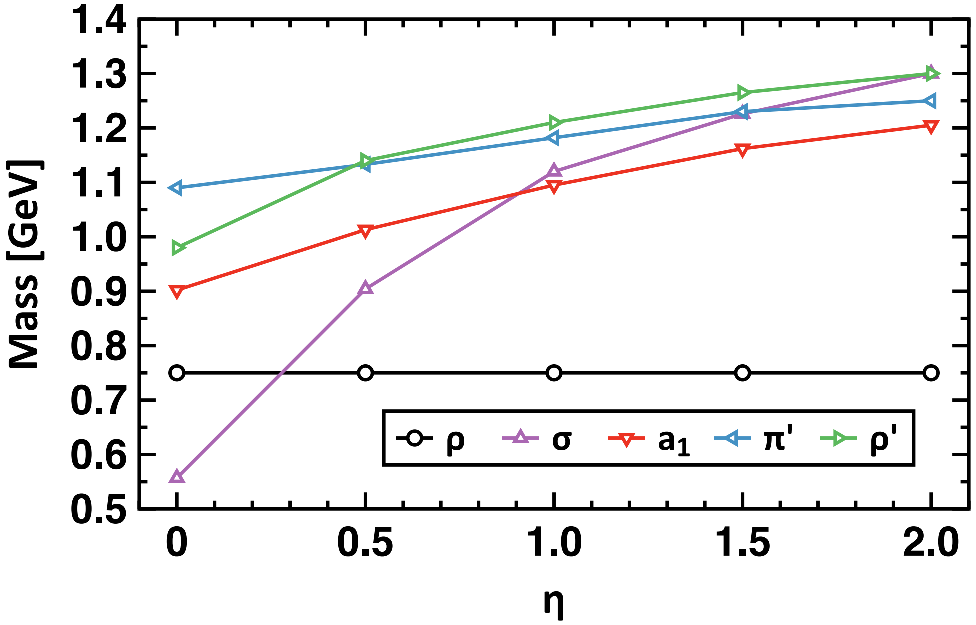

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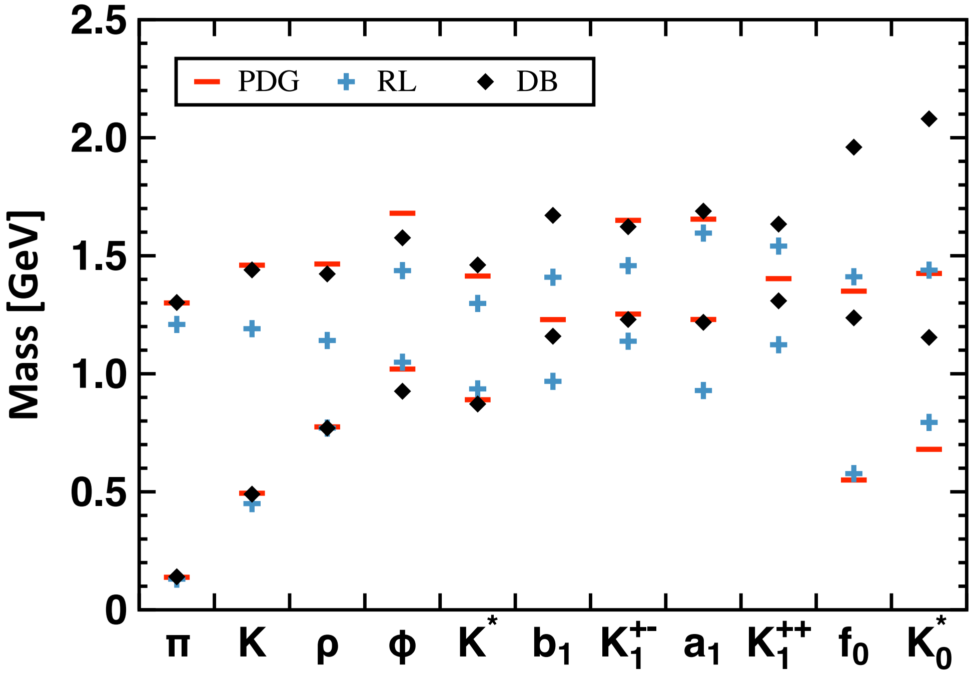

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See, e.g., CPL 38 (2021) 7, 071201

Few: Spectra of mesons

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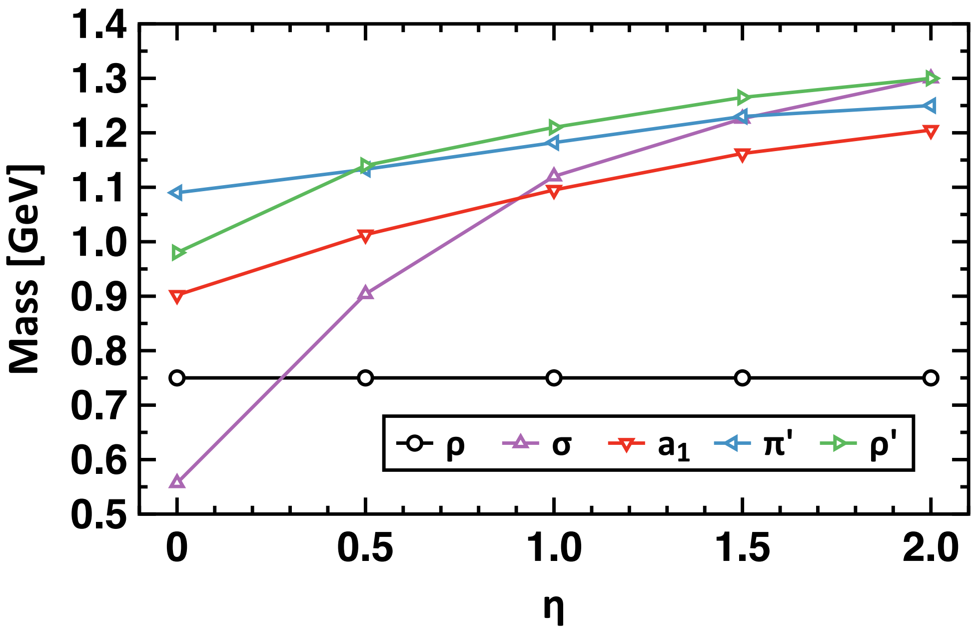

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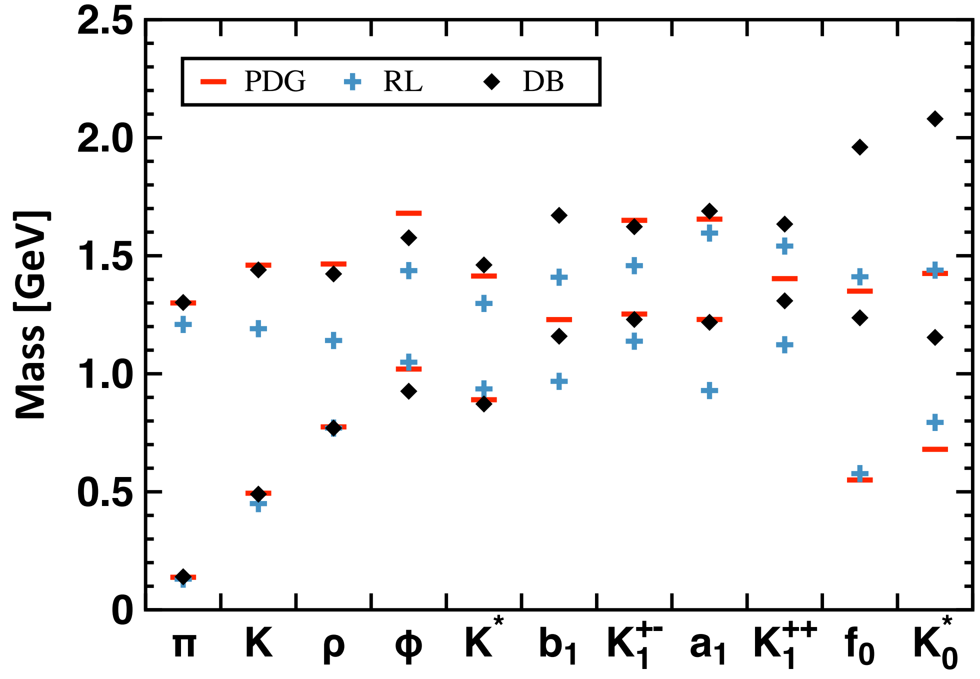

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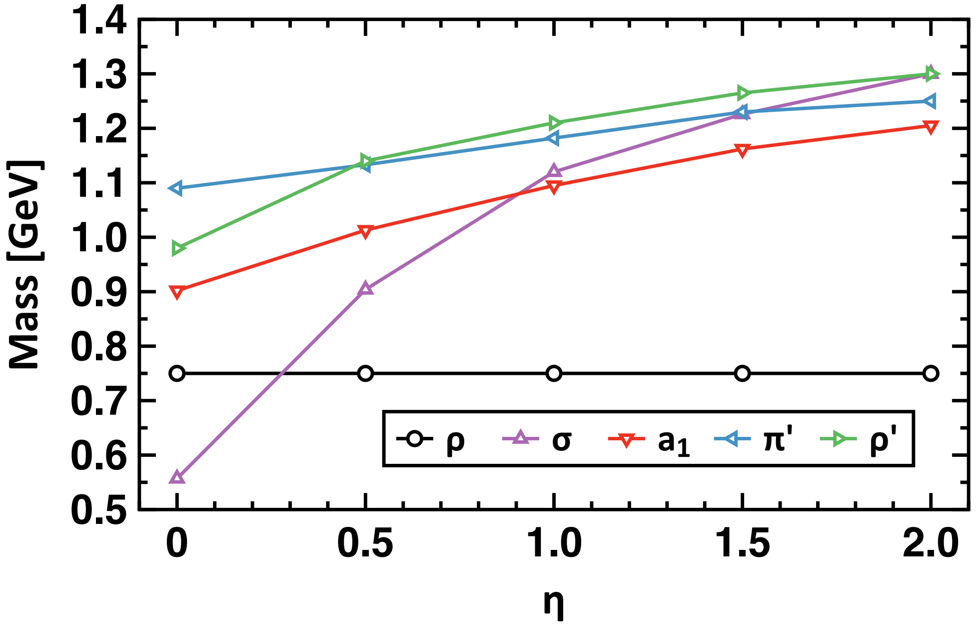

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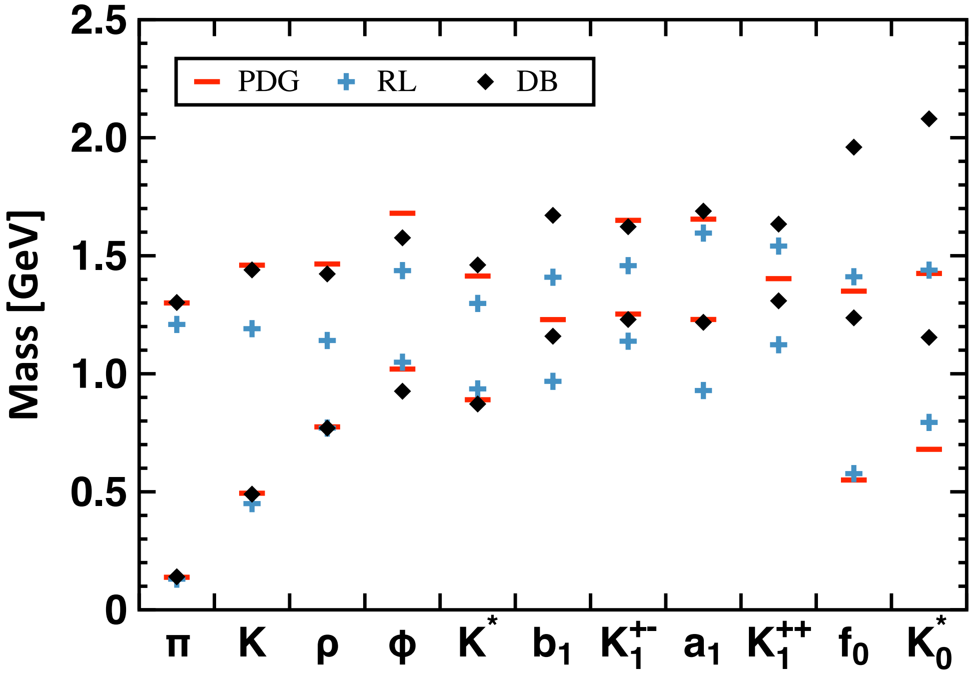

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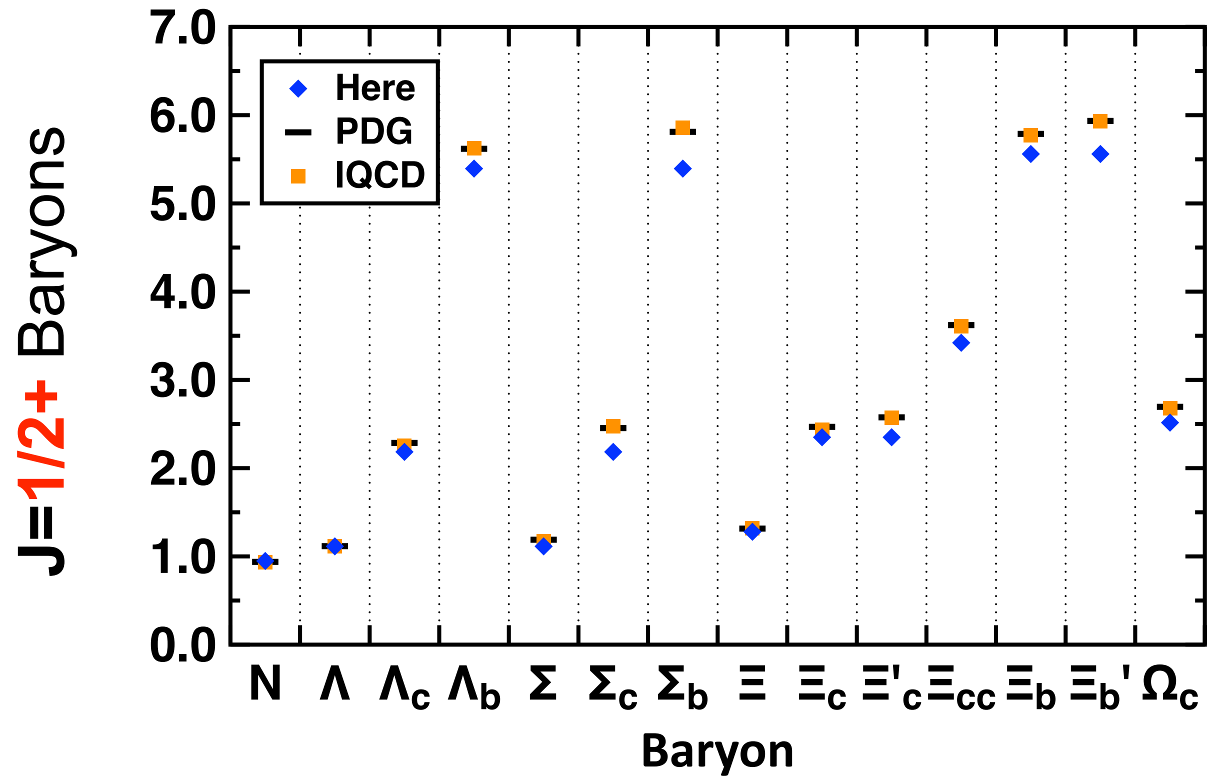
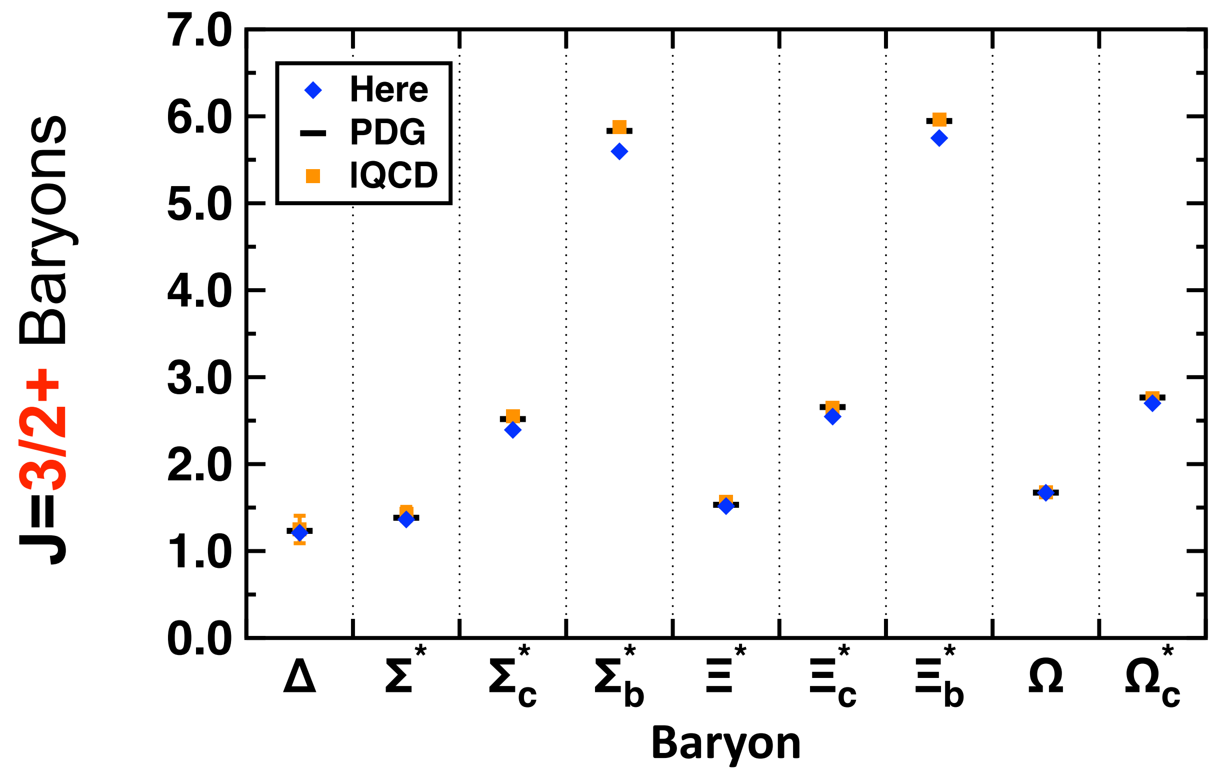
- The magnitude and ordering of radial excitation states can be fixed with the DCSB-enhanced kernel.

See, e.g., CPL 38 (2021) 7, 071201

Few: Spectra of baryons





See, e.g., Few-Body Syst 60, 26 (2019)

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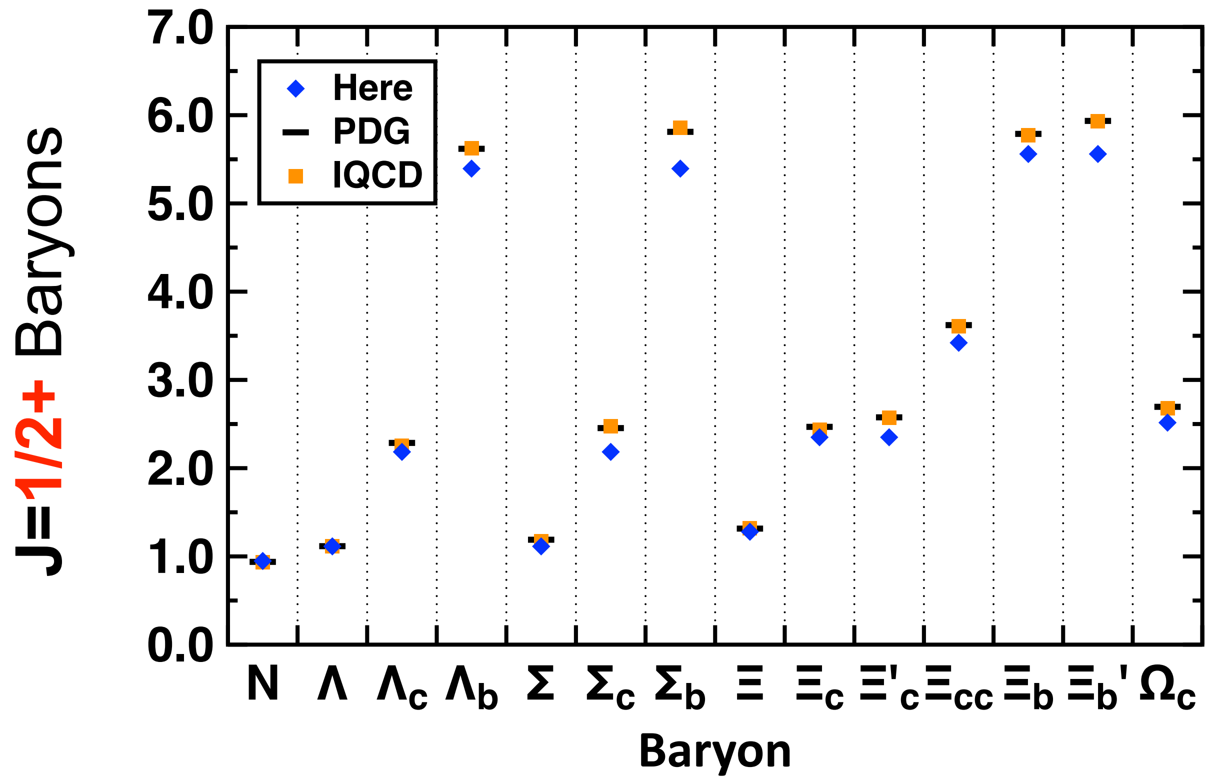

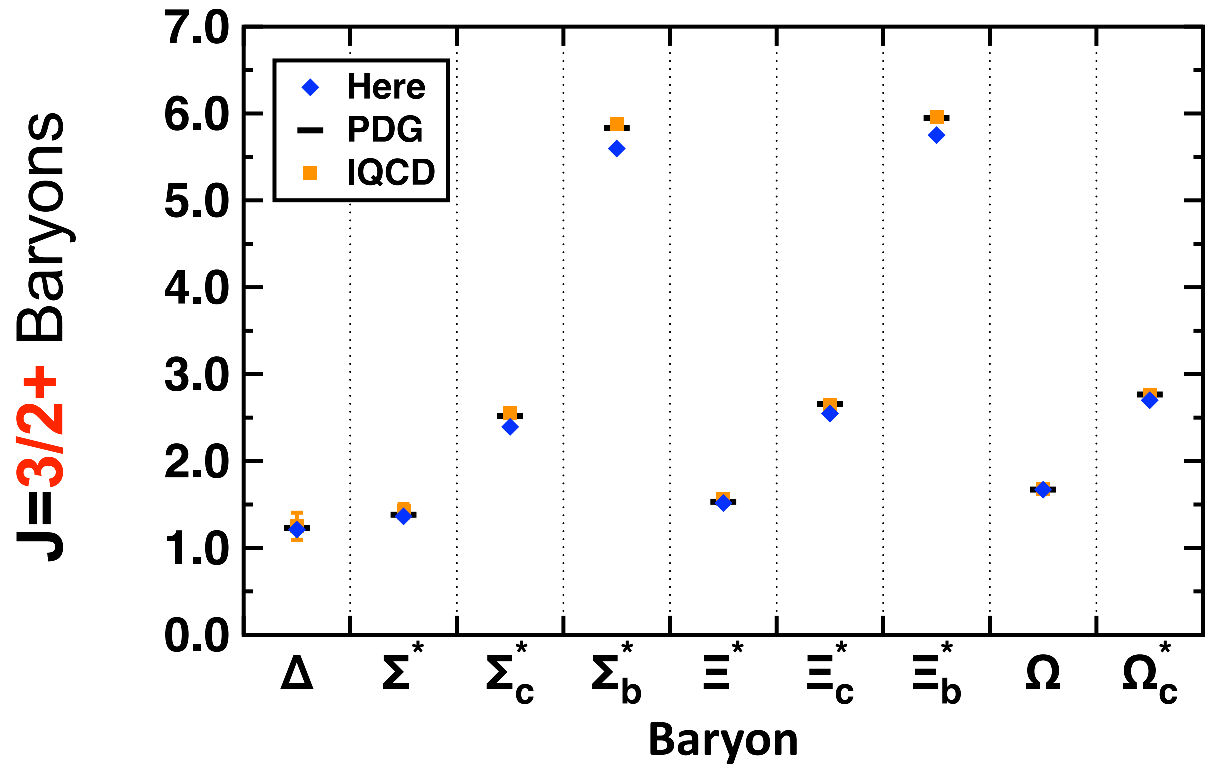

- ✦ The ground states of Nucleon and Delta families can be described by a simple kernel.

See, e.g., Few-Body Syst 60, 26 (2019)

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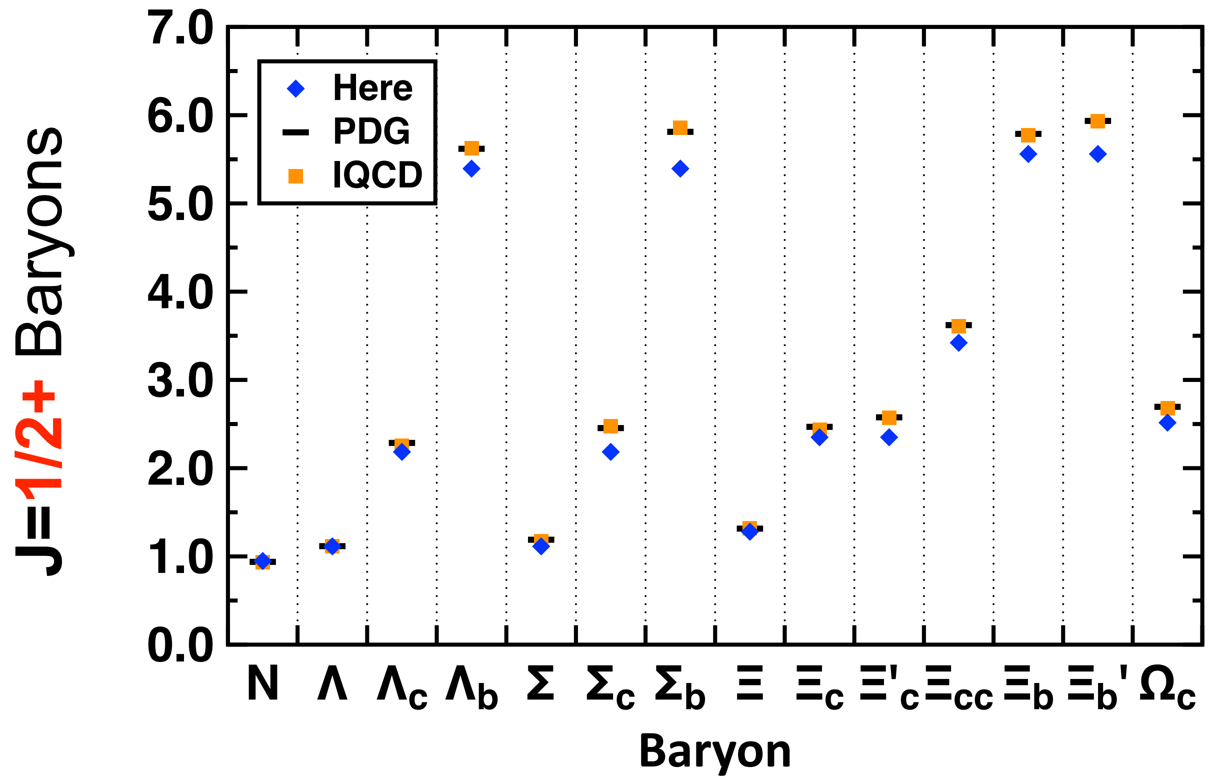

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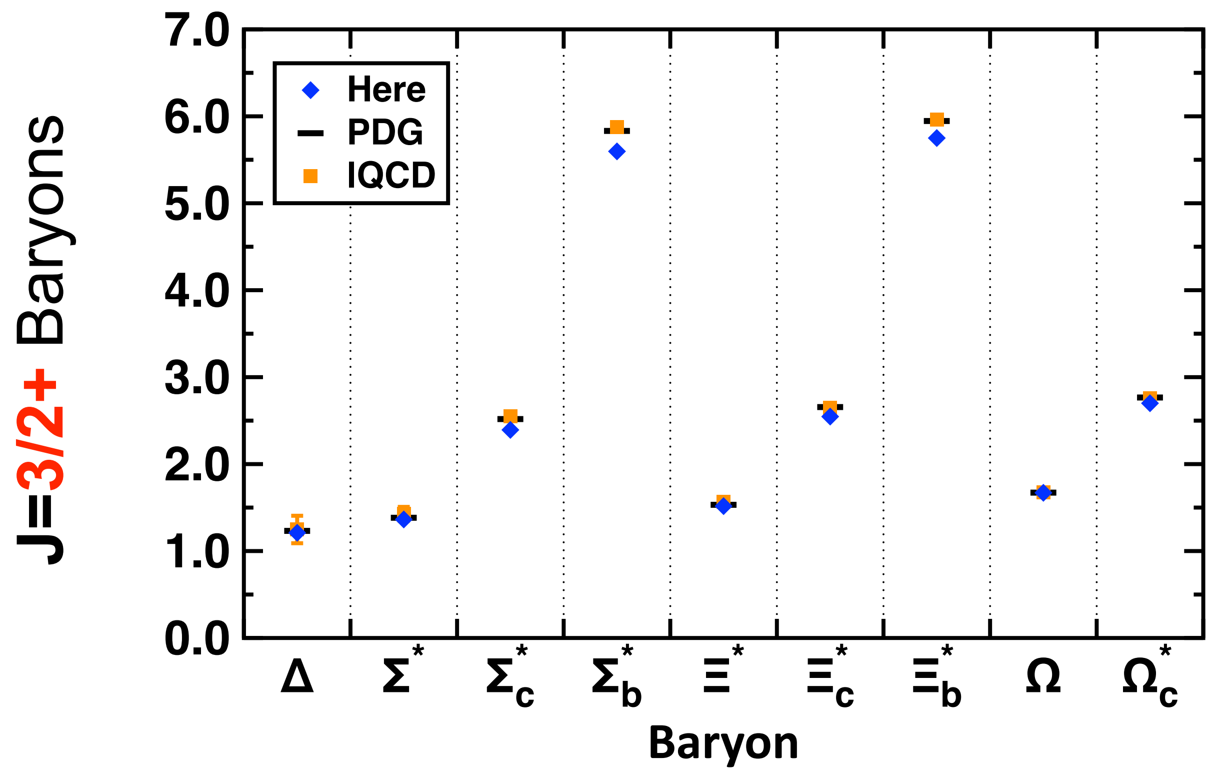
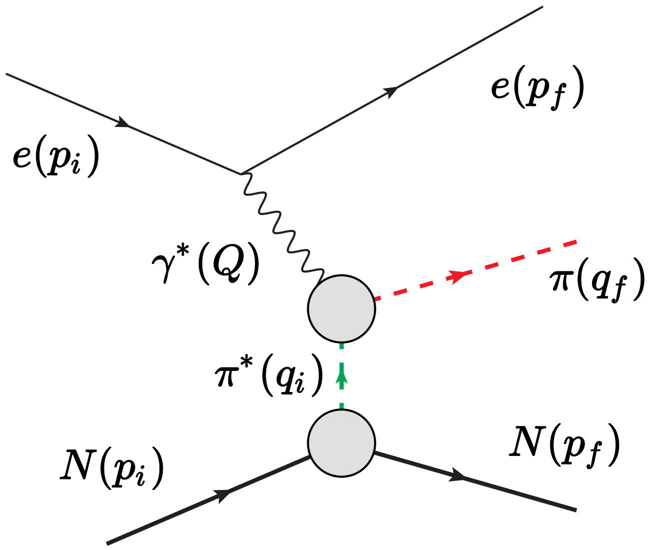
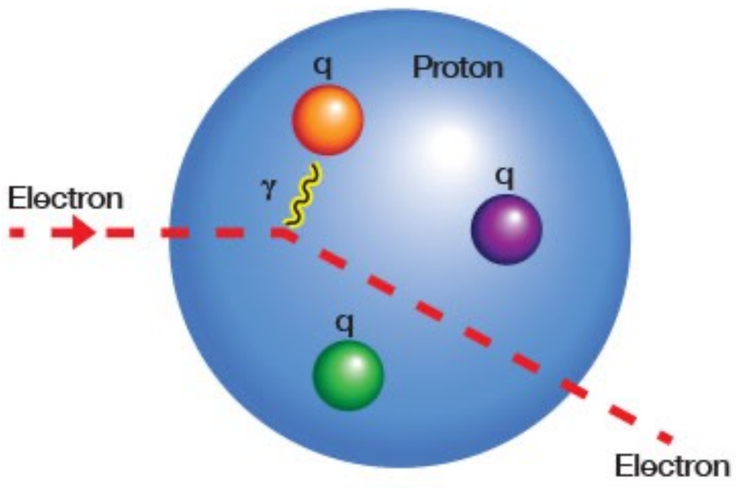


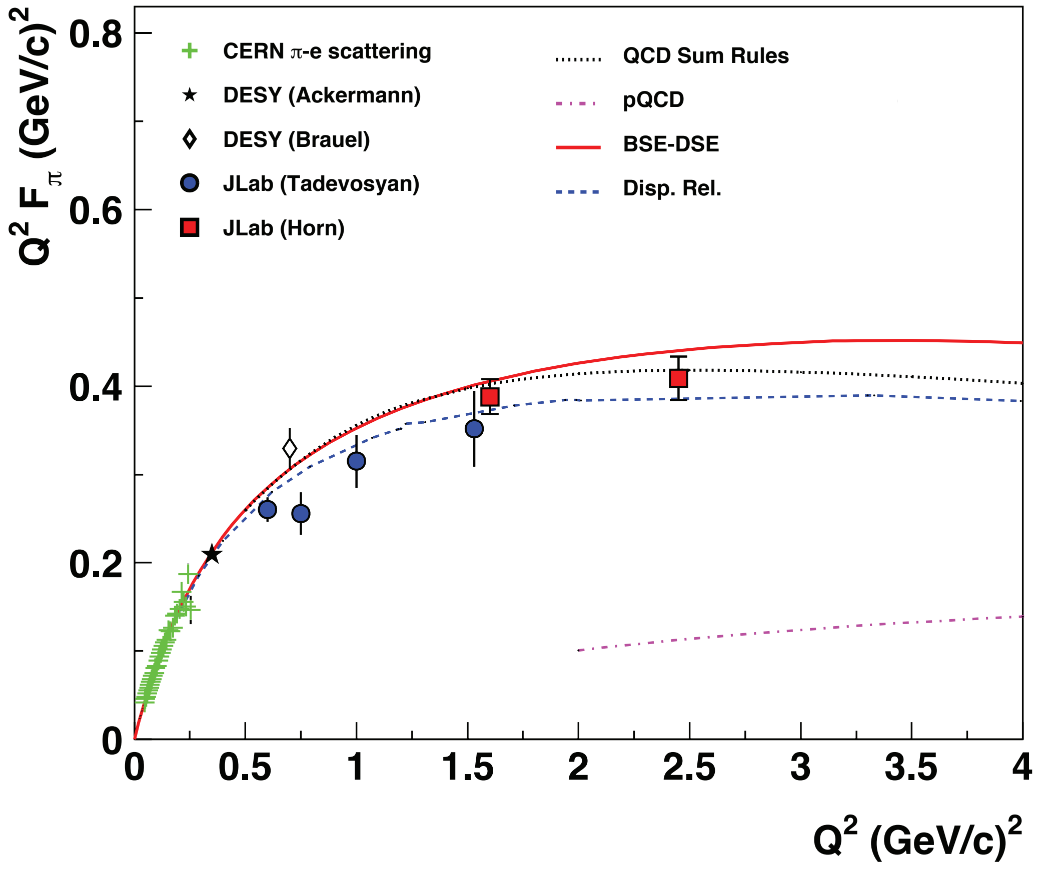
- ✦ The excited states and the parity partners require a DCSB-enhanced kernel (to be published).

See, e.g., Few-Body Syst 60, 26 (2019)

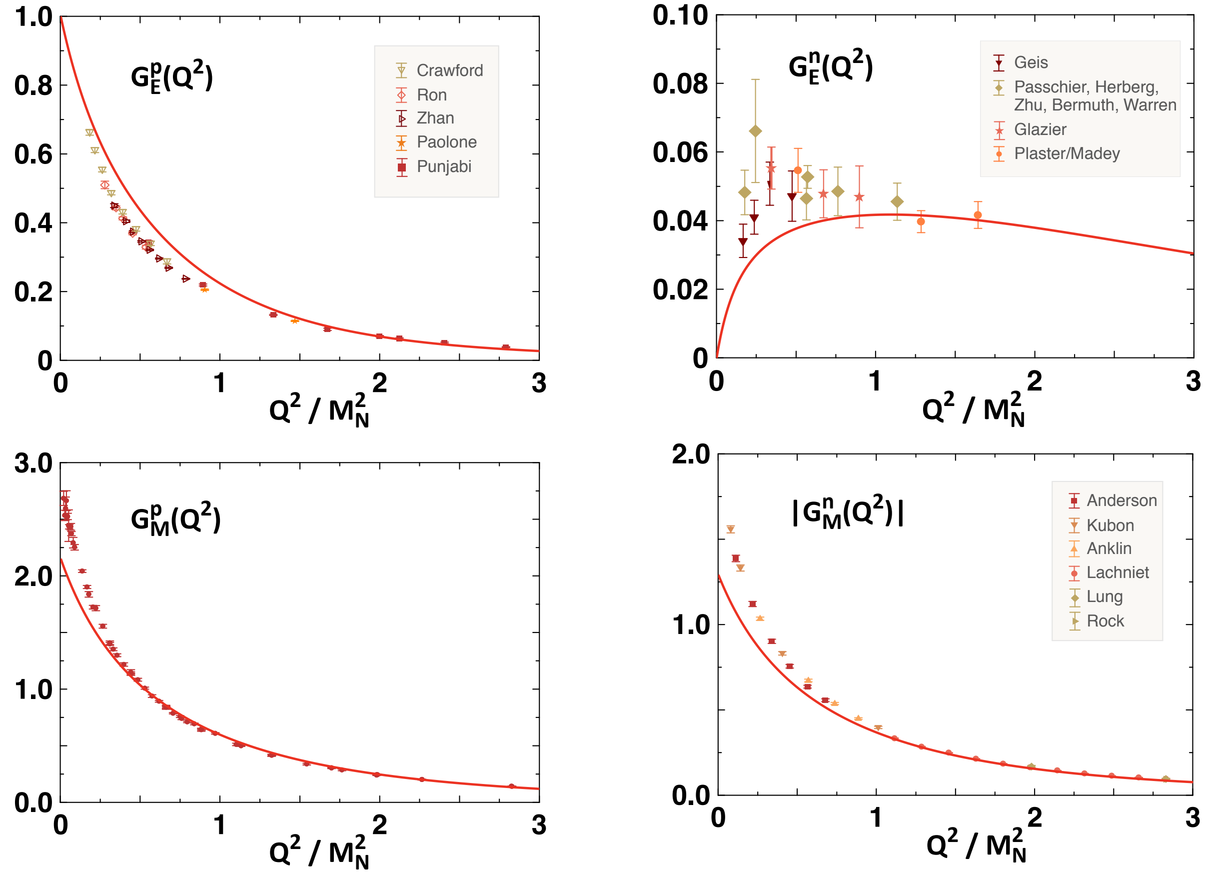
Few: Structures of hadrons





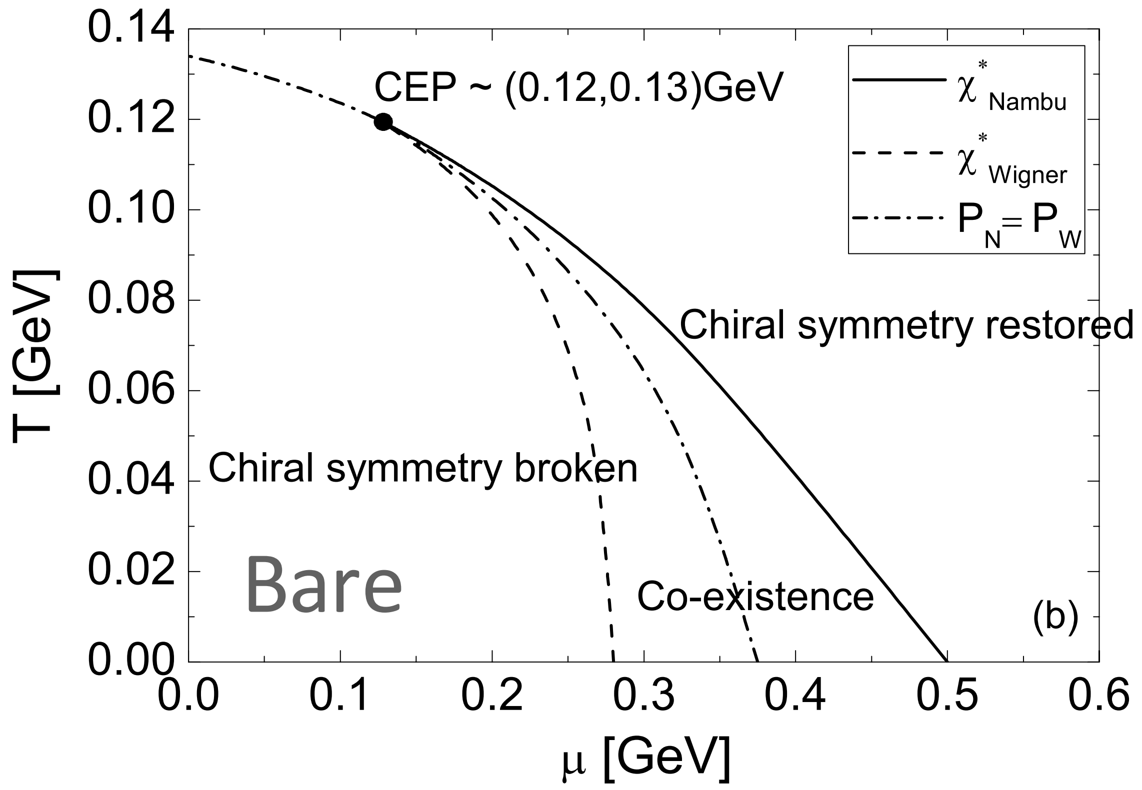


See, e.g., PRL 97, 192001 (2006)



See, e.g., PRD 84, 014014 (2011)

Many: Phase diagram of nuclear mater



Ferromagnet	**QCD**
Order parameter: ***M***	Order parameter: $-\langle \bar{q}q \rangle$
External source: ***B***	External source: ***m***
Susceptibility: $\chi = \frac{\partial M}{\partial B}$	Susceptibility: $\chi_m = -\frac{\partial \langle \bar{q}q \rangle}{\partial m}$

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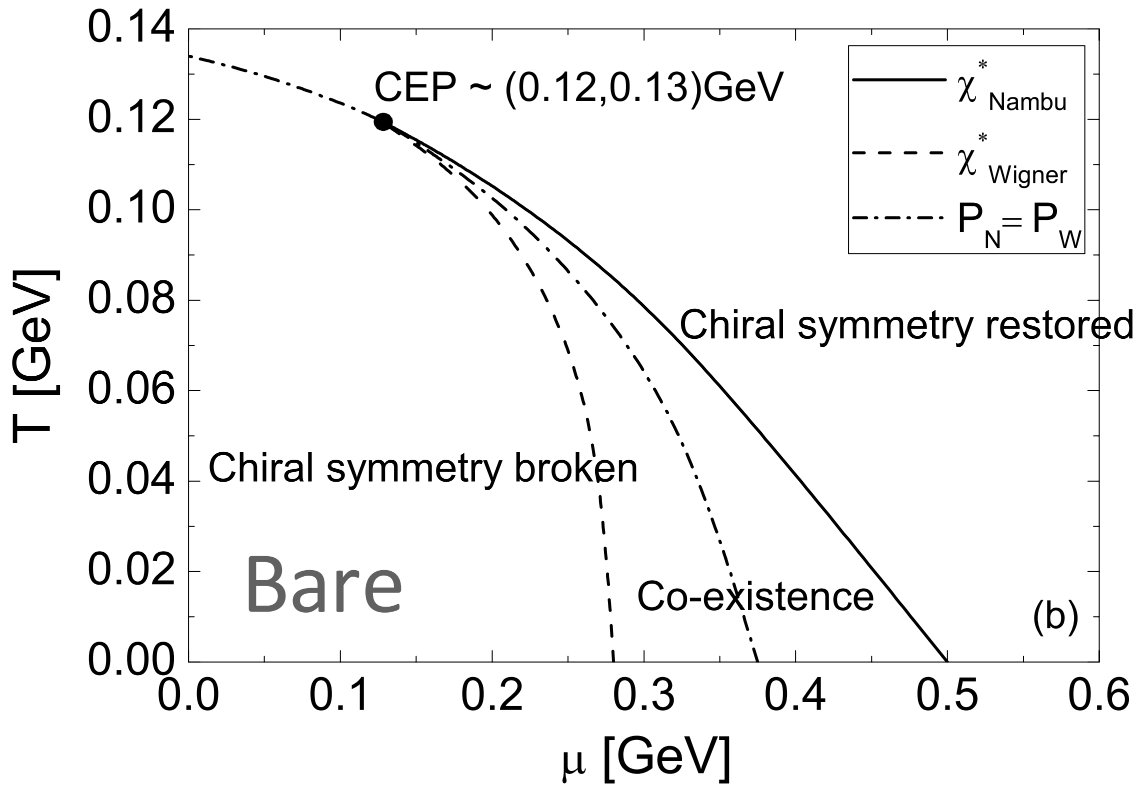

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1. The CEP rotates toward the *T*-axis.
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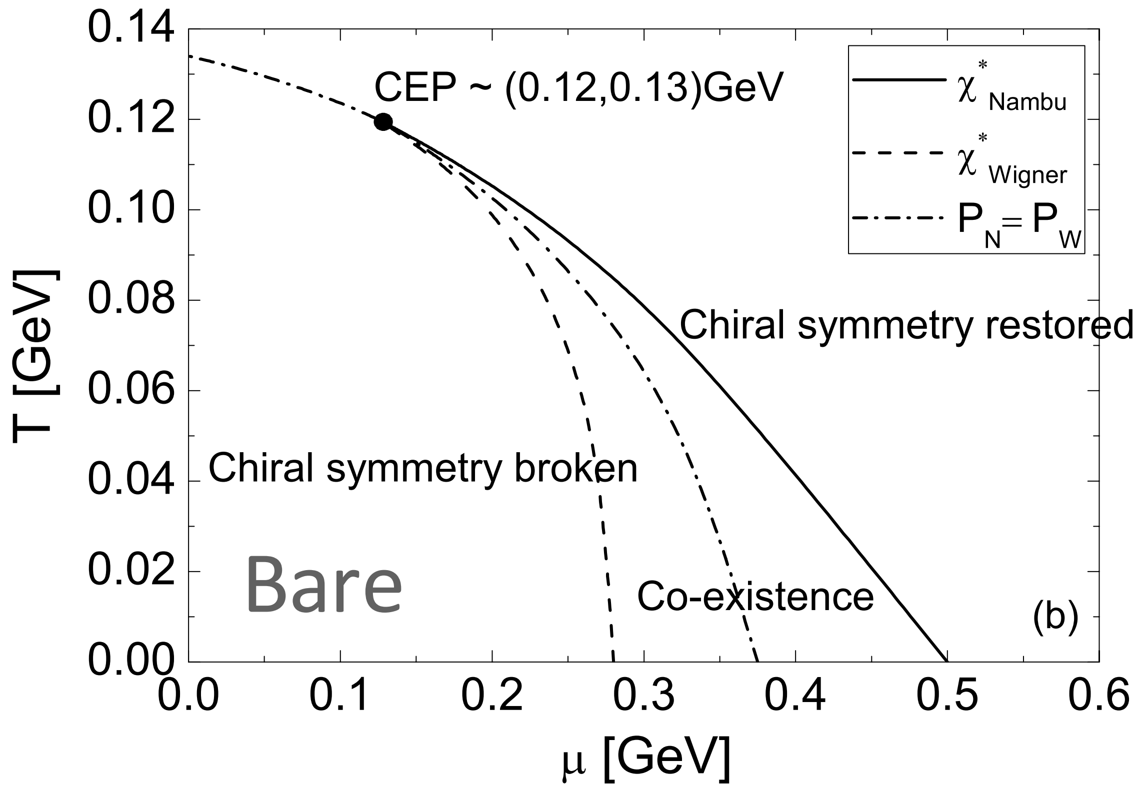

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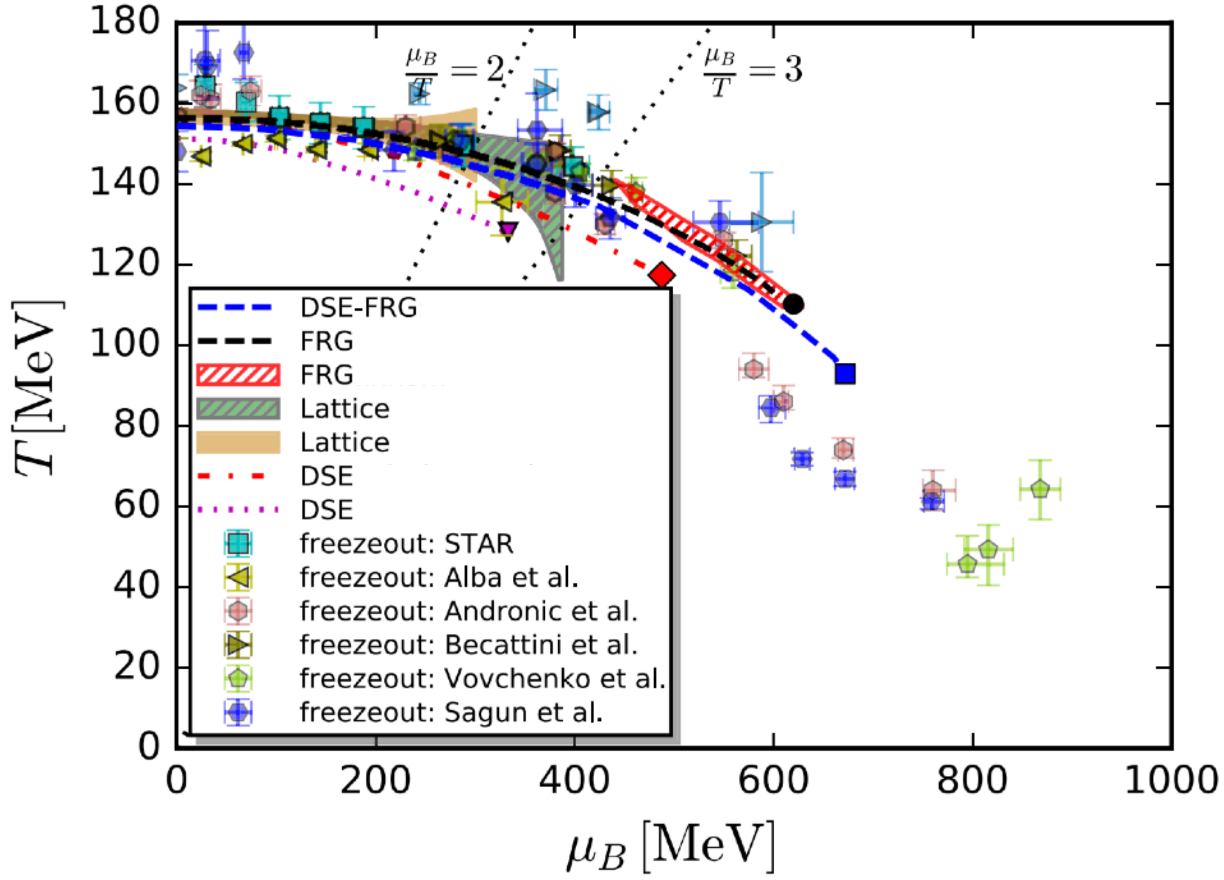

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Many: Emergent states of hot matter

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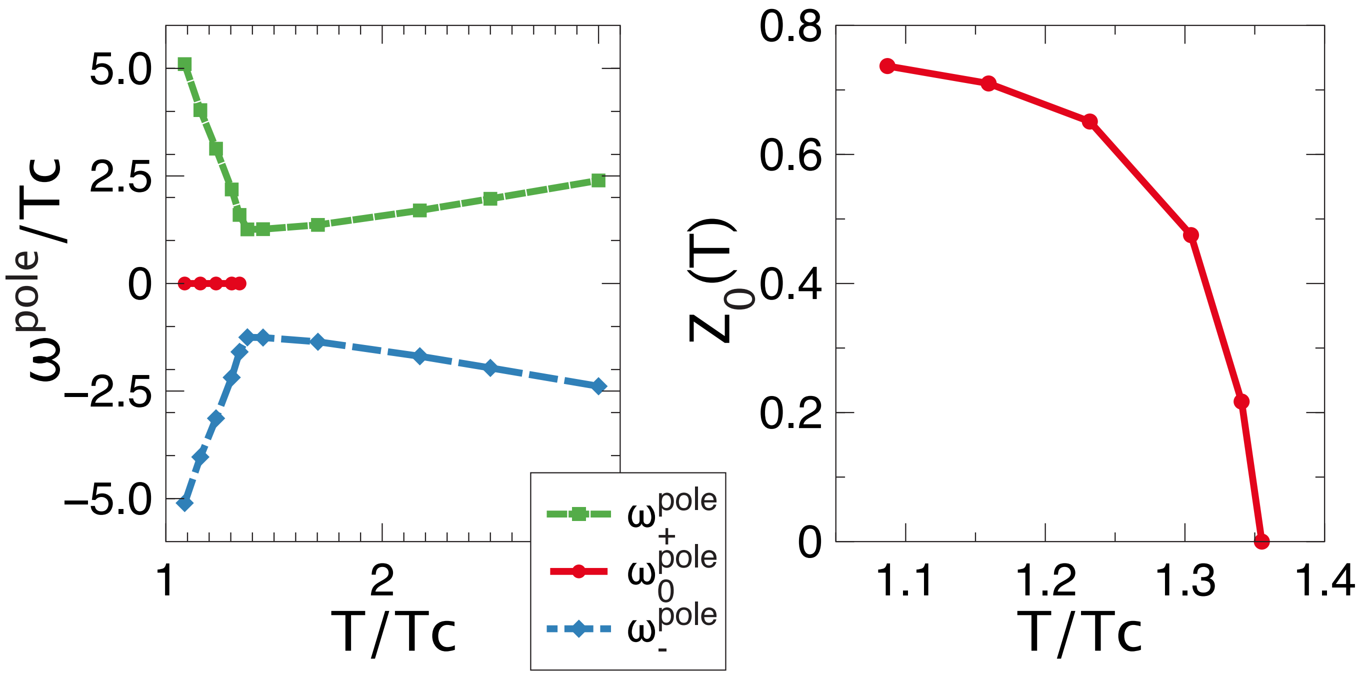

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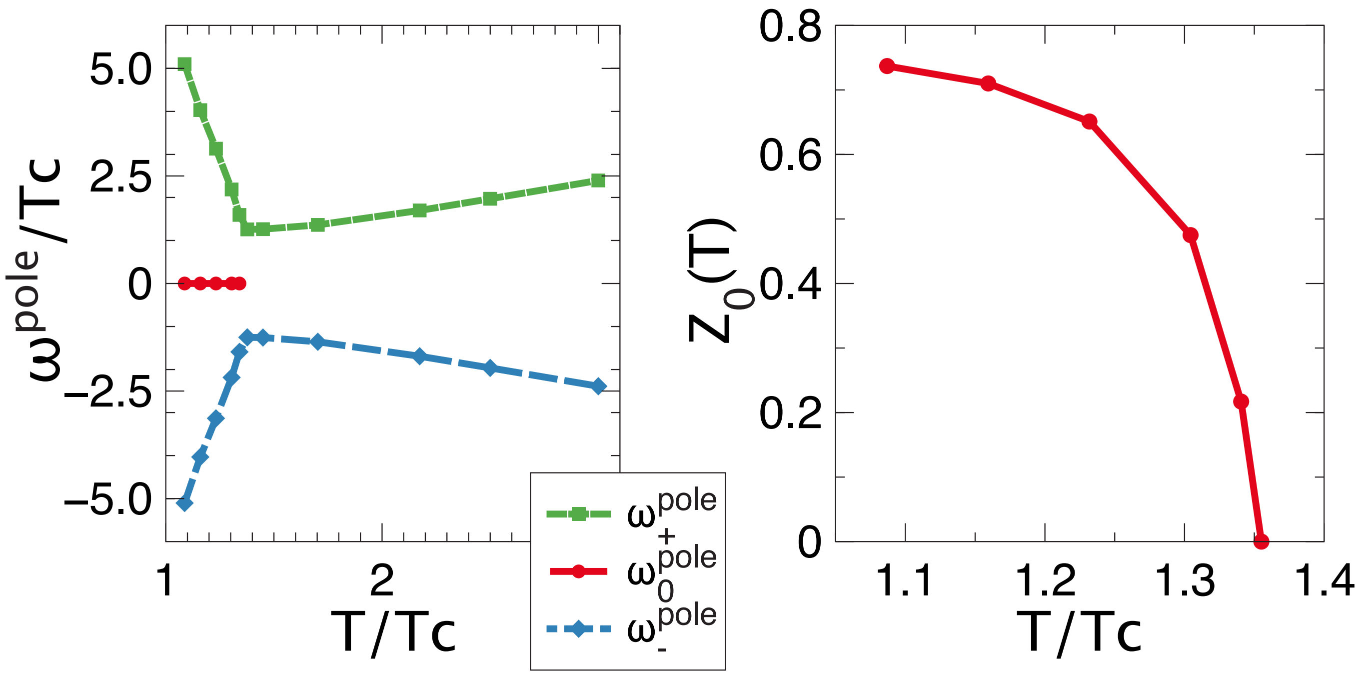
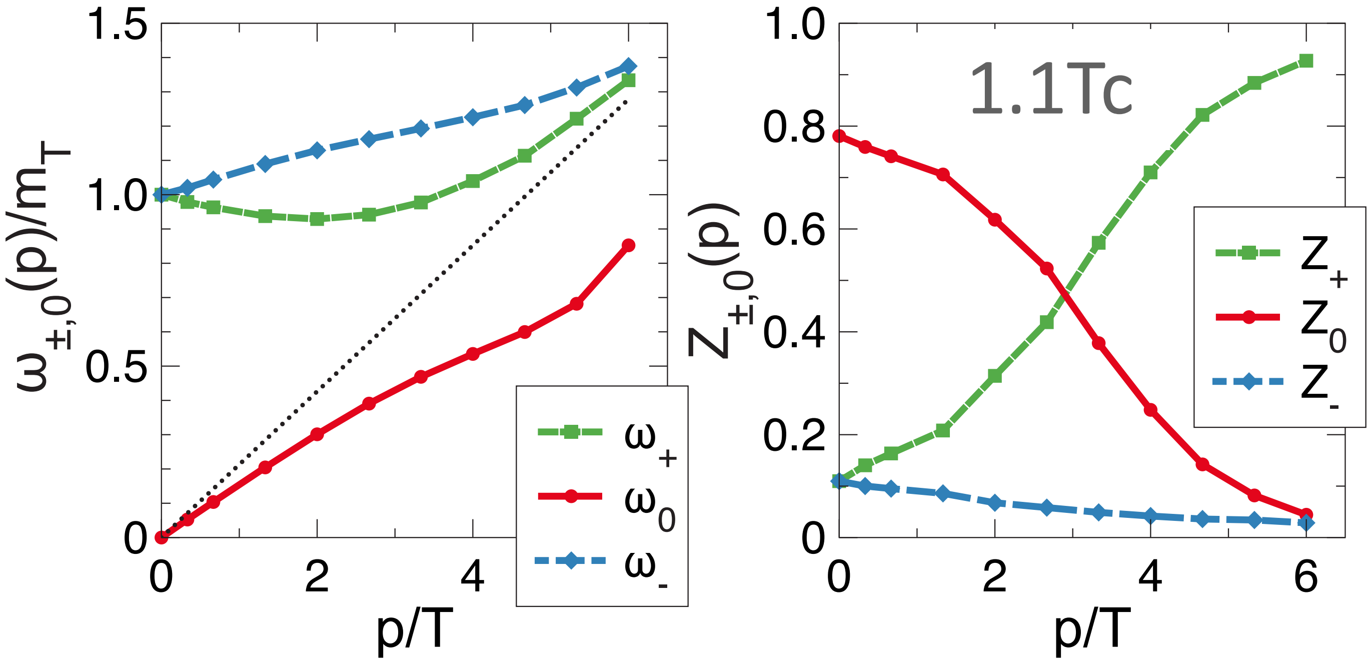


- Atypical dispersion relations of excitations:



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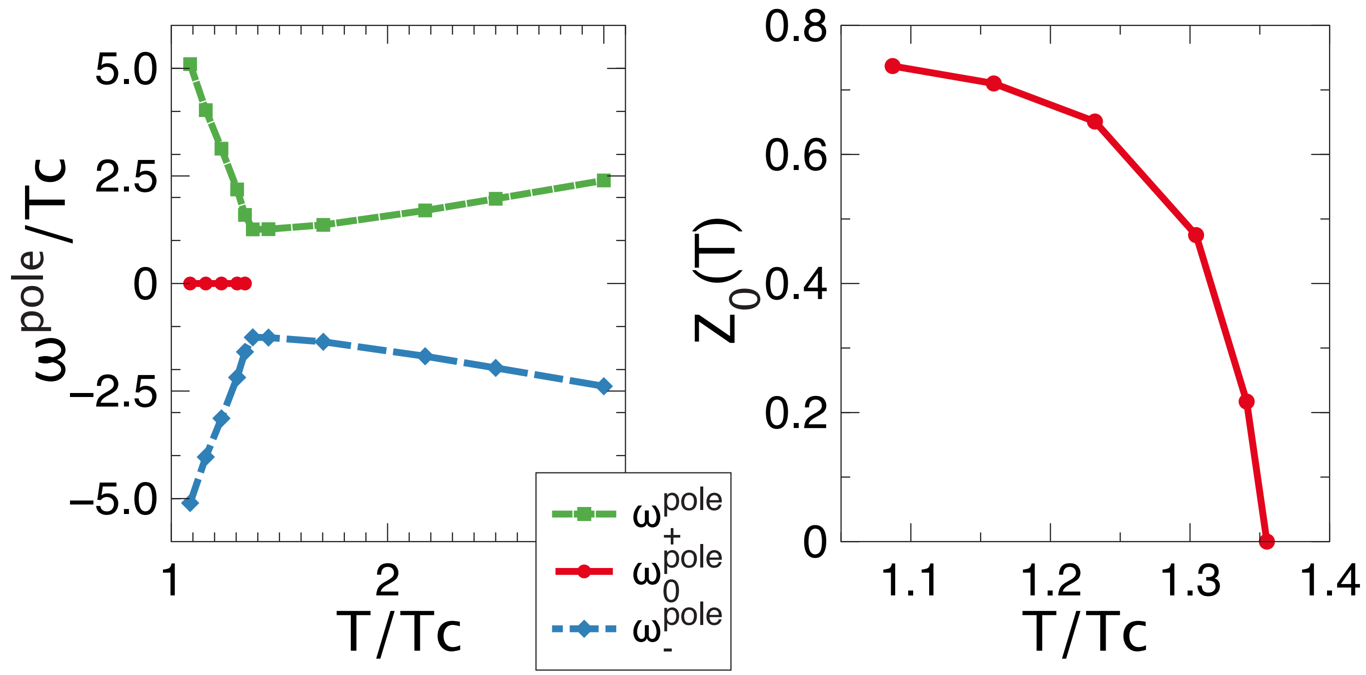

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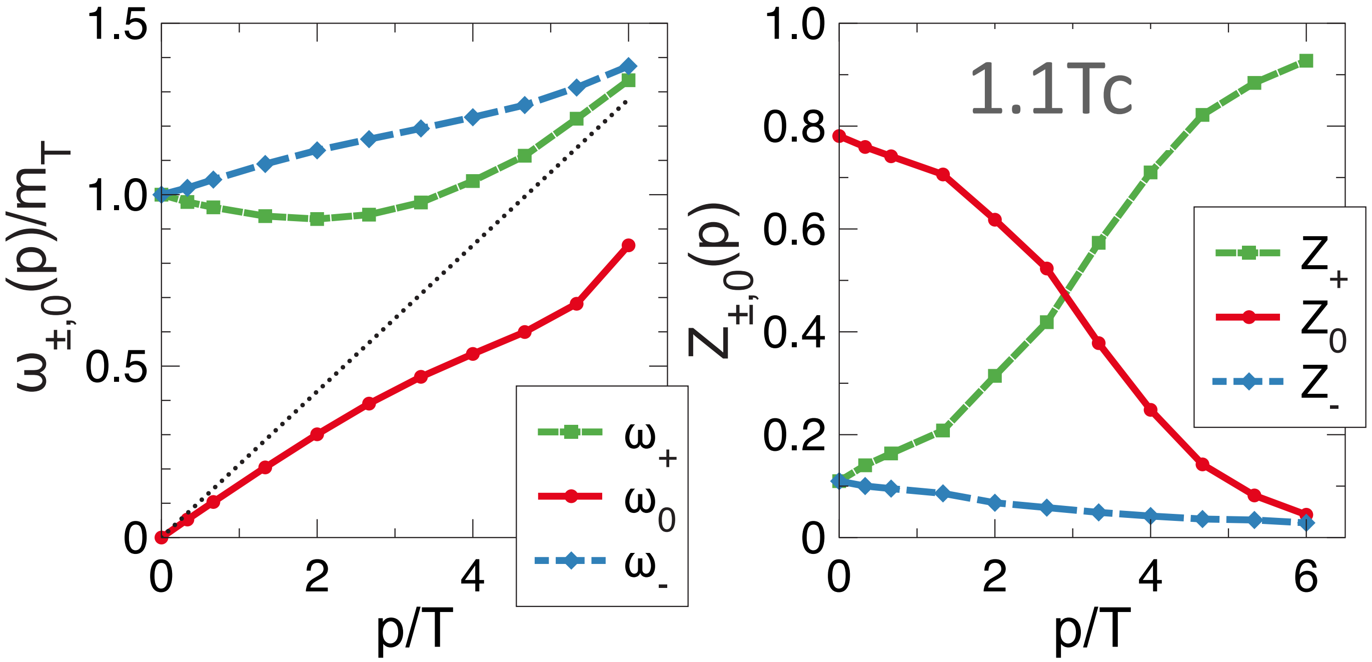

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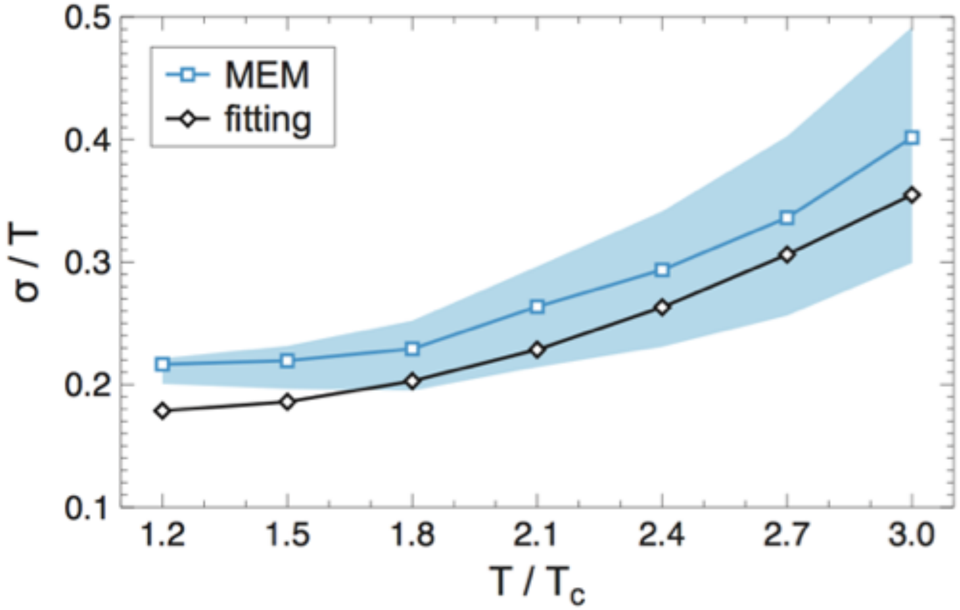

$\sigma/T \sim 0.4$	Aarts et. al., PRL 99 (2007)
$0.2 < \sigma/T < 0.3$ ($1\sim2T_c$)	Amato et. al., PRL 111 (2013)
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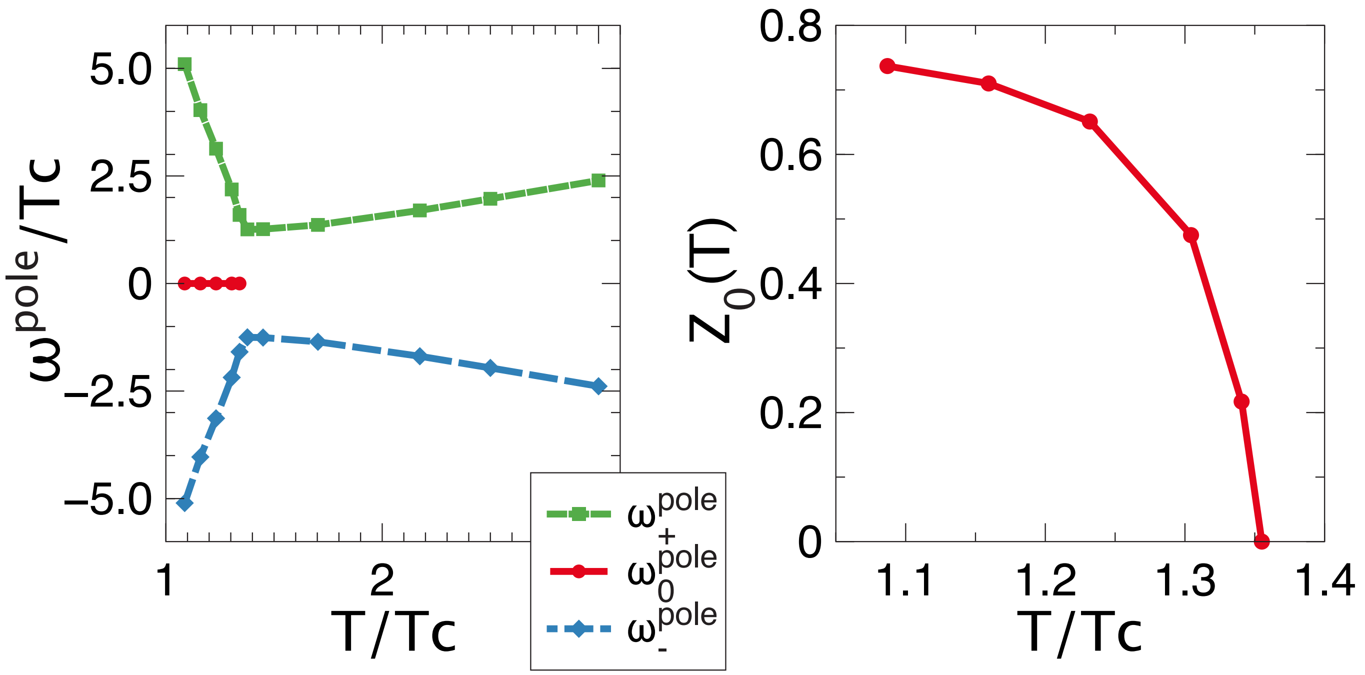

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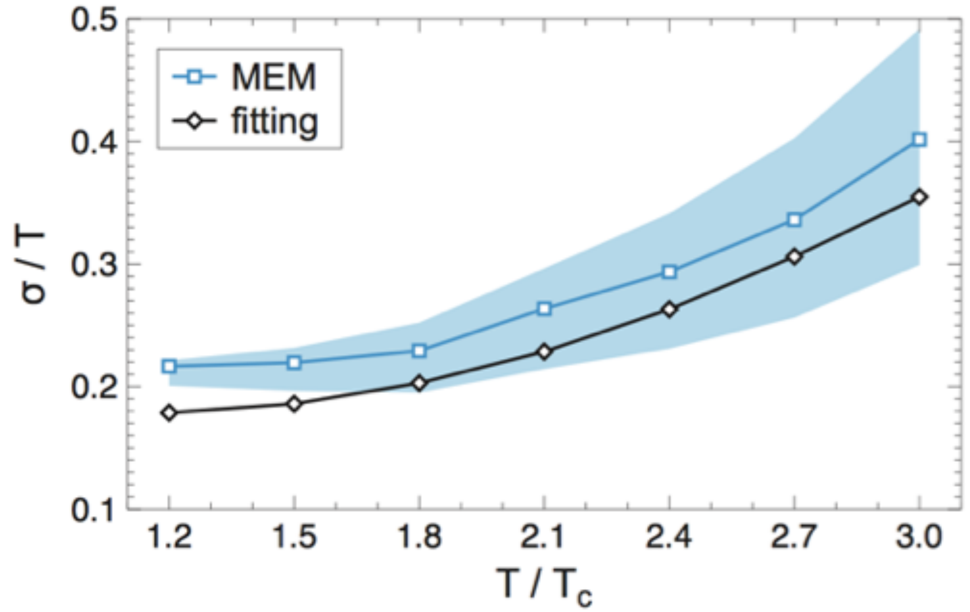

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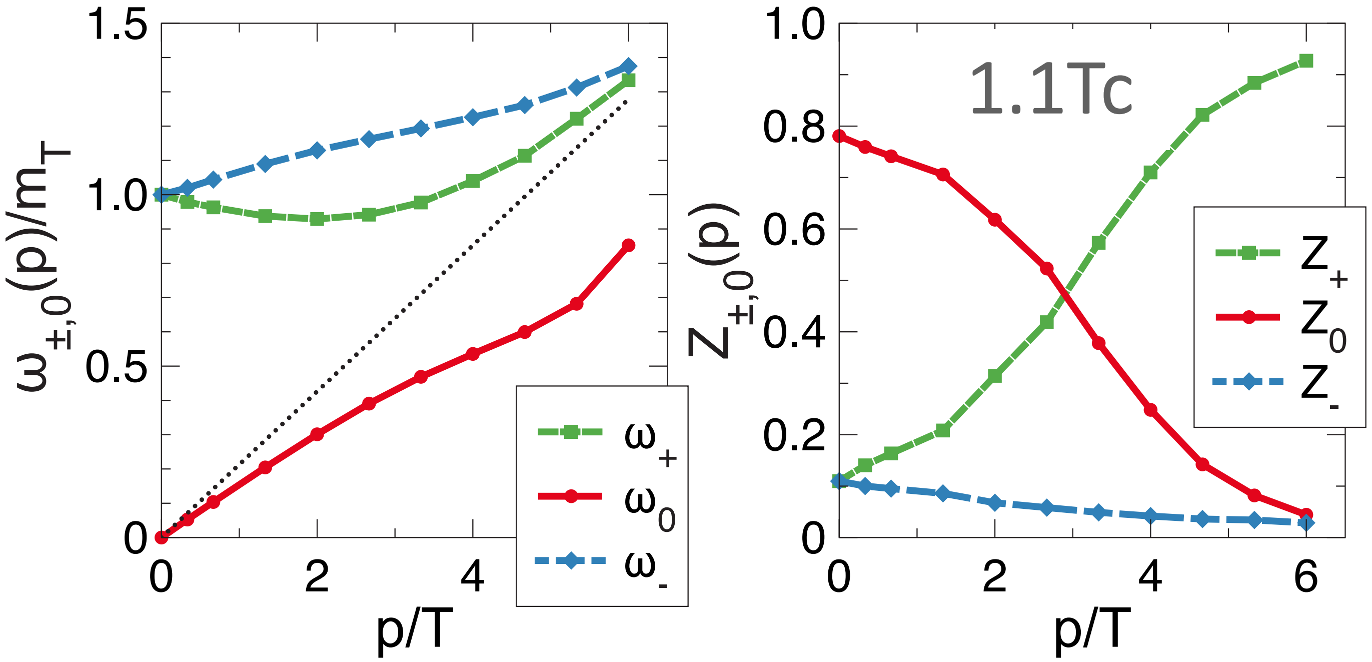

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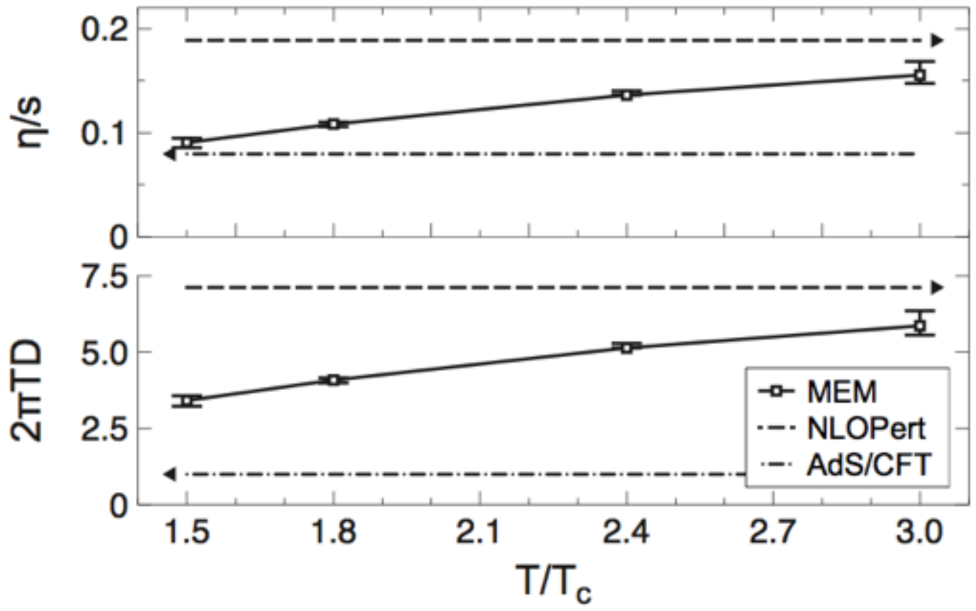

LQCD: $2\pi TD \sim 2$, Ding et. al, PRD, 86, (2012)

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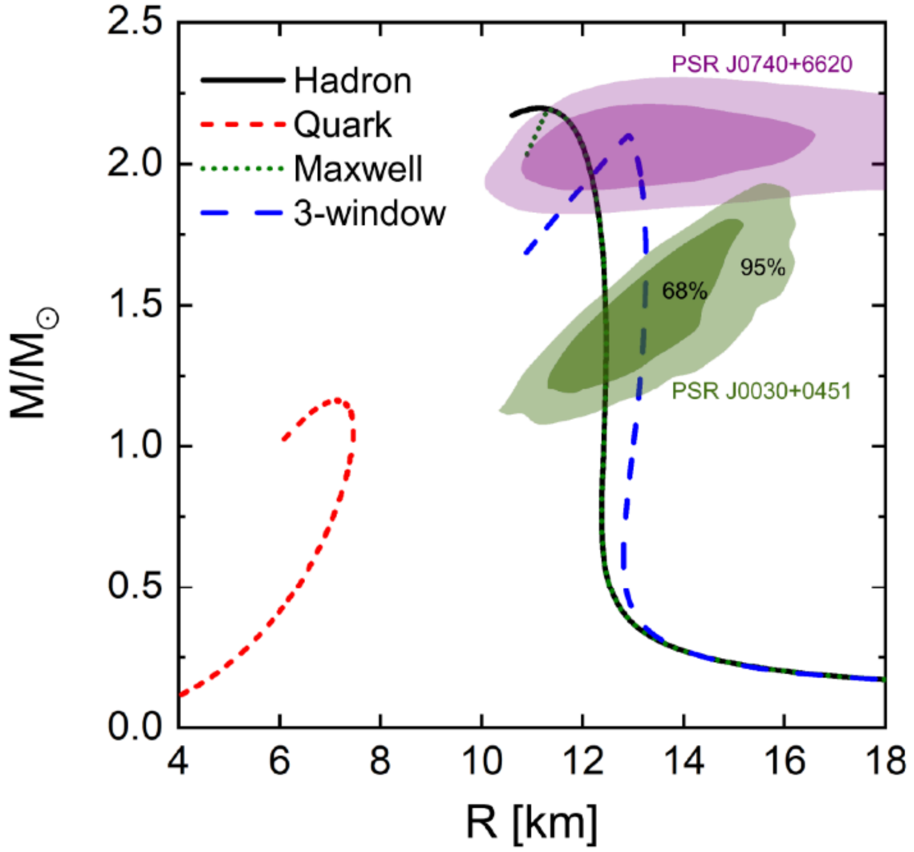
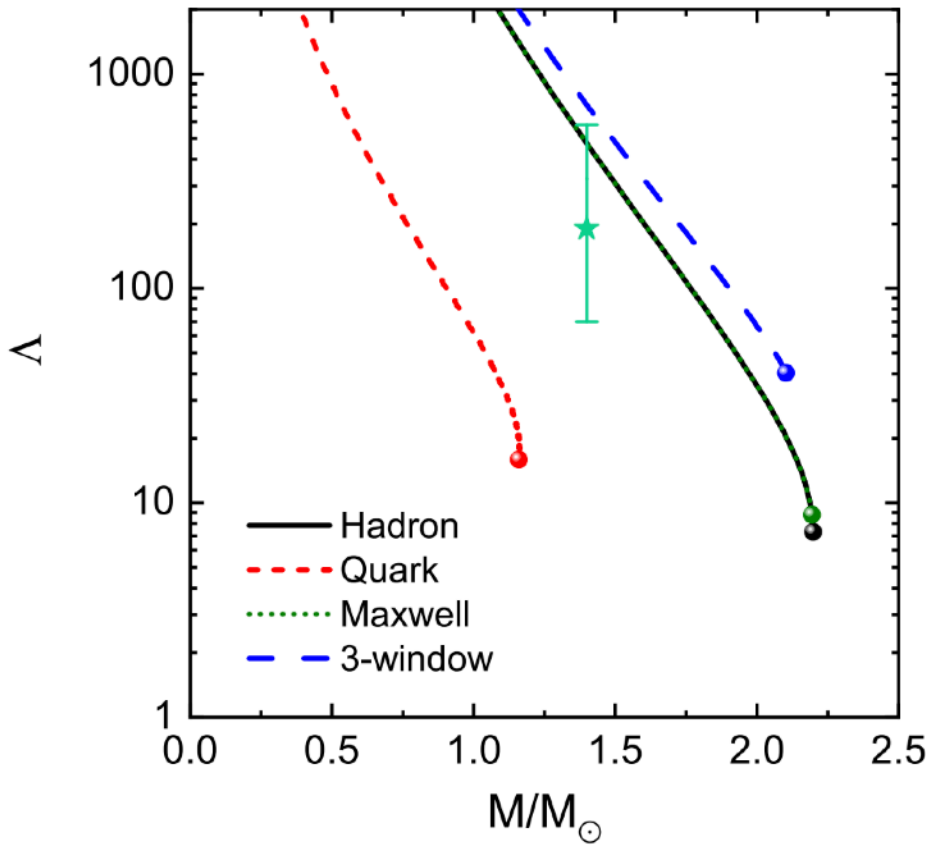
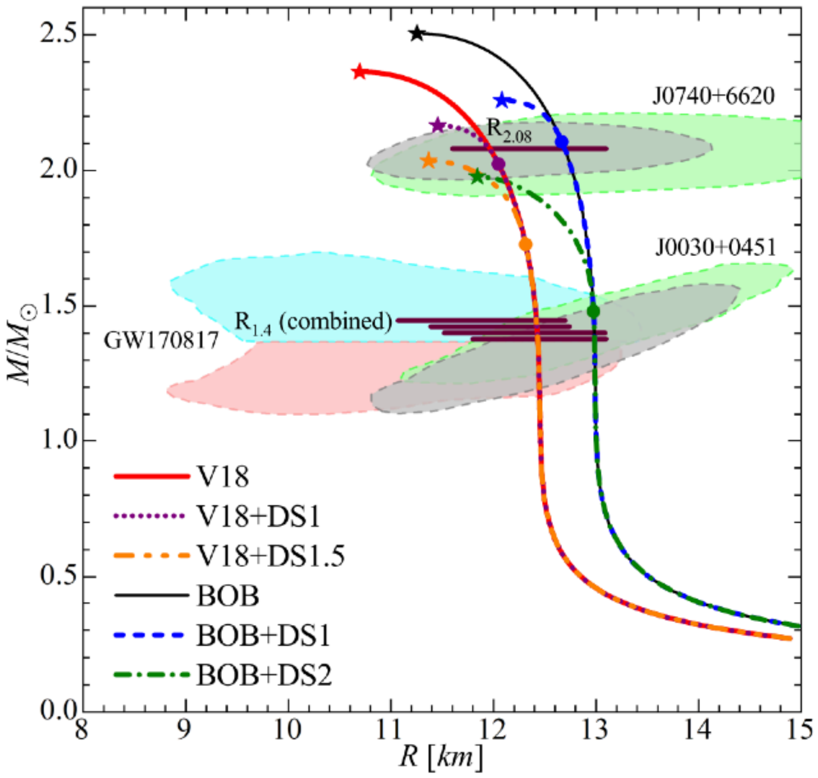
v_2: $4 < 2\pi TD < 6$, PHENIX, PRC, 84, (2011)

Many: Emergent states of dense matter







See, e.g., PRD 107, 103009 (2023), 107, 103048 (2023), 108, 014018 (2023)

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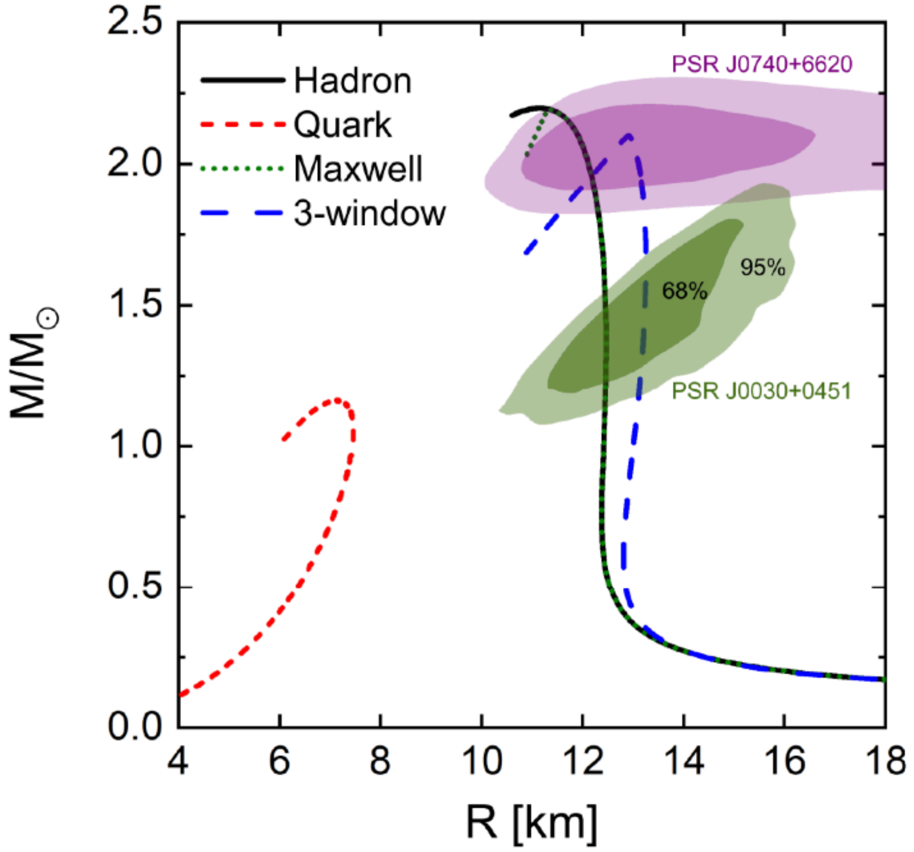

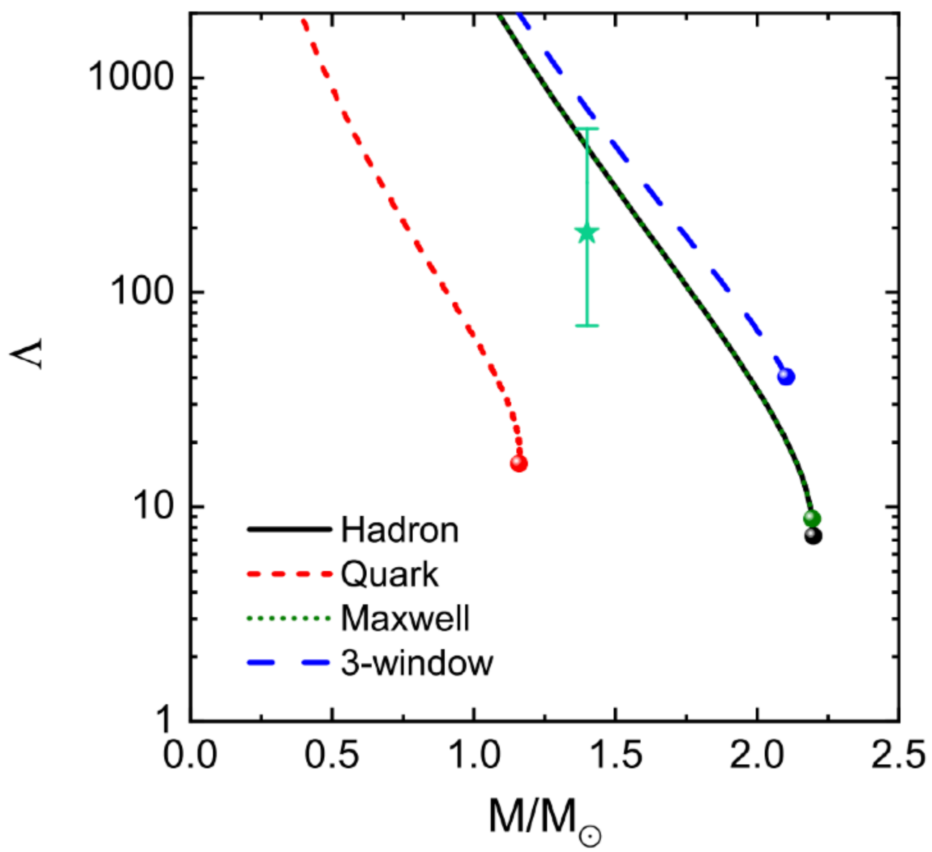

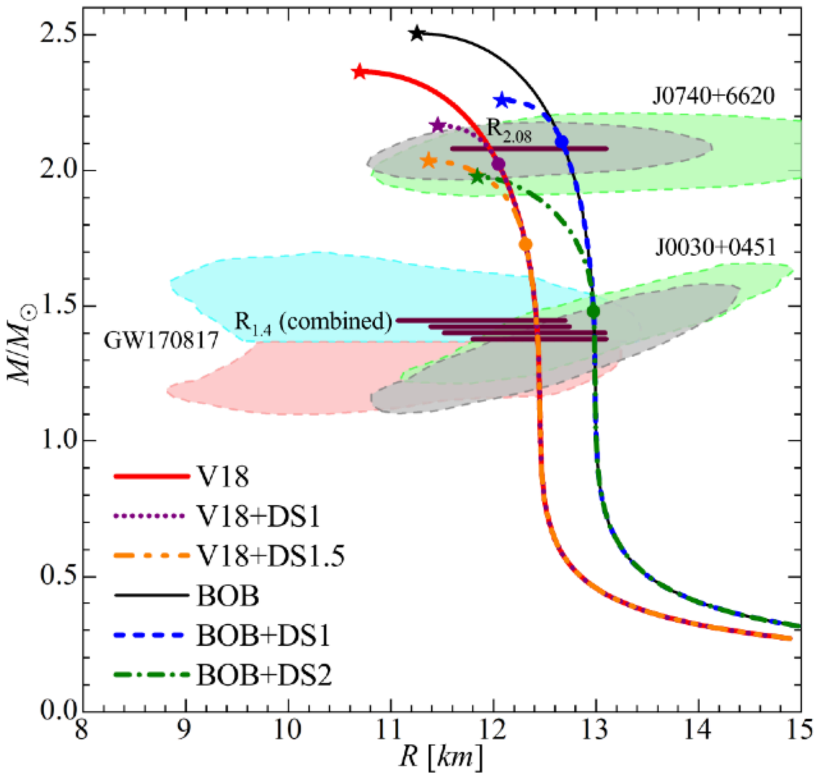

✦ Hadron phase: RMF, BHF, DBHF, RBHF

✦ Quark phase: DSE with effective model

✦ Construction: thermodynamic constrains

See, e.g., PRD 107, 103009 (2023), 107, 103048 (2023), 108, 014018 (2023)

Summary

✦ The **DSE** is a **balanced** tool to solve nonperturbative QCD in the continuum QFT framework.

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Outlook

- **Unify** the DSE elements at $T = 0$ and $\mu = 0$, produce properties of in-vacuum hadrons.

- **Extend** the DSE framework to $T \neq 0$ and $\mu \neq 0$, predict properties of novel nuclear matter.