Parameterization

Formalism

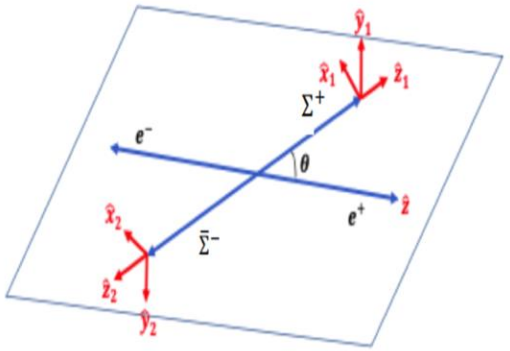
- Reaction described by $\xi = (\theta, \theta_1, \theta_2, \phi_1, \phi_2)$
- Decay distribution given by

$$\mathcal{W}(\xi) = \mathcal{F}_0(\xi) + \alpha \mathcal{F}_5(\xi)$$ SPIN CORRELATIONS

$$+\alpha_1\alpha_2\left(\mathcal{F}_1(\xi) + \sqrt{1-\alpha^2}\cos(\Delta\Phi)\,\mathcal{F}_2(\xi) - \alpha\mathcal{F}_6(\xi)\right)$$

$$+\sqrt{1-\alpha^2}\sin(\Delta\Phi)(-\alpha_1\mathcal{F}_3(\xi) + \alpha_2\mathcal{F}_4(\xi))$$ POLARIZATIONS

- $R = \sqrt{\tau}\sqrt{\frac{1-\alpha}{1+\alpha}}, \tau = \frac{q^2}{4m^2}$
- Both the values of R and ΔΦ could be extracted by MLL fit.



$\mathcal{F}_0 = 1,$

$\mathcal{F}_1 = \sin^2\theta\,\sin\theta_1\sin\theta_2\cos\phi_1\cos\phi_2 - \cos^2\theta\,\cos\theta_1\cos\theta_2,$

$\mathcal{F}_2 = \sin\theta\,\cos\theta(\sin\theta_1\cos\theta_2\cos\phi_1 - \cos\theta_1\sin\theta_2\cos\phi_2),$

$\mathcal{F}_3 = \sin\theta\,\cos\theta\,\sin\theta_1\,\sin\phi_1,$

$\mathcal{F}_4 = \sin\theta\,\cos\theta\,\sin\theta_2\,\sin\phi_2,$

$\mathcal{F}_5 = \cos^2\theta,$

$\mathcal{F}_6 = \cos\theta_1\cos\theta_2 - \sin^2\theta\,\sin\theta_1\sin\theta_2\sin\phi_1\sin\phi_2$

$$\mathcal{W}(\boldsymbol{\xi}) = \mathcal{T}_0 + \eta\mathcal{T}_5 - \alpha_\Lambda^2\left[\mathcal{T}_1 + \sqrt{1-\eta^2}\cos(\Delta\Phi)\mathcal{T}_2 + \eta\mathcal{T}_6\right]$$
$$+\alpha_\Lambda\sqrt{1-\eta^2}\sin(\Delta\Phi)(\mathcal{T}_3 - \mathcal{T}_4), \qquad (3)$$

where α_Λ denotes the decay asymmetry of the $\Lambda \to p\pi^-$ decay. The seven functions $\mathcal{T}_k(\boldsymbol{\xi})$ do not depend on the parameters η and $\Delta\Phi$, but only on the measured angles:

$\mathcal{T}_0(\boldsymbol{\xi}) = 1,$

$\mathcal{T}_1(\boldsymbol{\xi}) = \sin^2\theta\sin\theta_1\sin\theta_2\cos\phi_1\cos\phi_2 + \cos^2\theta\cos\theta_1\cos\theta_2,$

$\mathcal{T}_2(\boldsymbol{\xi}) = \sin\theta\cos\theta(\sin\theta_1\cos\theta_2\cos\phi_1 + \cos\theta_1\sin\theta_2\cos\phi_2),$

$\mathcal{T}_3(\boldsymbol{\xi}) = \sin\theta\cos\theta\sin\theta_1\sin\phi_1,$

$\mathcal{T}_4(\boldsymbol{\xi}) = \sin\theta\cos\theta\sin\theta_2\sin\phi_2,$

$\mathcal{T}_5(\boldsymbol{\xi}) = \cos^2\theta,$

$\mathcal{T}_6(\boldsymbol{\xi}) = \cos\theta_1\cos\theta_2 - \sin^2\theta\sin\theta_1\sin\theta_2\sin\phi_1\sin\phi_2.$

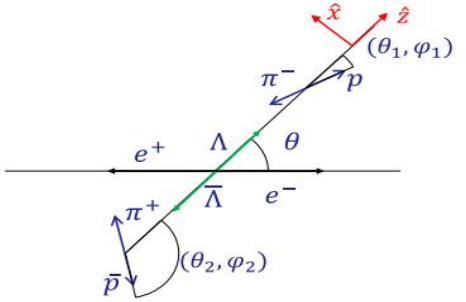


FIG. 1. Definition of the coordinate system used to describe the $e^+e^- \to \Lambda\bar{\Lambda}$ ($\Lambda \to p\pi^-$, $\bar{\Lambda} \to \bar{p}\pi^+$) process.

ML fit

$$\mathcal{L} = \prod_{i=1}^{N} \mathcal{P}(\xi_i; \alpha, \Delta\Phi) = \prod_{i=1}^{N} \mathcal{C}\mathcal{W}(\xi_i; \alpha, \Delta\Phi)\epsilon(\xi_i),$$

- i is the event index
- $P(\xi_i; \alpha, \Delta\Phi)$ is the probability density function of ξ_i
- $\epsilon(\xi_i)$ is the efficiency of each event
- $\mathcal{C}^{-1} = \int \mathcal{W}(\xi; \alpha, \Delta\Phi)\epsilon(\xi)d\xi$ is the normalization factor $\mathcal{C}$

$$S = -\ln \mathcal{L}_{\mathrm{Data}} + \ln \mathcal{L}_{\mathrm{Bkg}},$$

Decay asymmetry — Maximum likelihood fit

$$\mathscr{L} = \prod_{i=1}^{N} \frac{\mathscr{W}(\xi, \alpha_\gamma)_i \varepsilon(\xi)_i}{C}$$

$$C = \frac{1}{N} \sum_{i=1}^{N} \frac{\mathscr{W}(\xi, \alpha_\gamma)_i}{\mathscr{W}(\xi_0, \alpha_\gamma)_i}$$

$$-\ln \mathscr{L}_{sig} = -\ln \mathscr{L}_{data} + \ln \mathscr{L}_{bkg}$$
$$= -\sum_{data} \ln \mathscr{W} + \sum_{bkg} \ln \mathscr{W} + (N_{data} - N_{bkg}) * \ln C$$

14.4.2 期望值估计法

任何一个积分都可以表示为某个随机变量的数学期望，因此，可以用该随机变量的子样平均值作为积分的近似值.

设欲求的积分为

$$I = \int_{V_s} g(\boldsymbol{x})\mathrm{d}\boldsymbol{x}, \tag{14.4.16}$$

其中 $\boldsymbol{x} = \{x_1, x_2, \cdots, x_s\}$ 表示 S 维空间的点，V_s 表示积分区域. 令 $f(\boldsymbol{x})$ 为 V_s 上的任一随机变量 $\boldsymbol{\xi}$ 的概率密度函数

$$\int_{V_s} f(\boldsymbol{x})\mathrm{d}\boldsymbol{x} = 1, \tag{14.4.17}$$

则积分 I 可表示为随机变量 $h(\boldsymbol{x}) = g(\boldsymbol{x})/f(\boldsymbol{x})$ 的数学期望

$$I = \int_{V_s} g(\boldsymbol{x})\mathrm{d}\boldsymbol{x} = \int_{V_s} \frac{g(\boldsymbol{x})}{f(\boldsymbol{x})} f(\boldsymbol{x})\mathrm{d}\boldsymbol{x} = E\left[\frac{g(\boldsymbol{x})}{f(\boldsymbol{x})}\right] = E[h(\boldsymbol{x})]. \tag{14.4.18}$$

当从随机变量 $\boldsymbol{\xi}$ 抽取容量为 n 的随机子样 $\boldsymbol{\xi}_1, \boldsymbol{\xi}_2, \cdots, \boldsymbol{\xi}_n$（即服从分布 $f(\boldsymbol{x})$ 的随机数），可求得随机变量 $h(\boldsymbol{x})$ 的子样 $h_1, h_2, \cdots, h_n$，

$$h_i = h(\boldsymbol{\xi}_i) = g(\boldsymbol{\xi}_i)/f(\boldsymbol{\xi}_i). \tag{14.4.19}$$

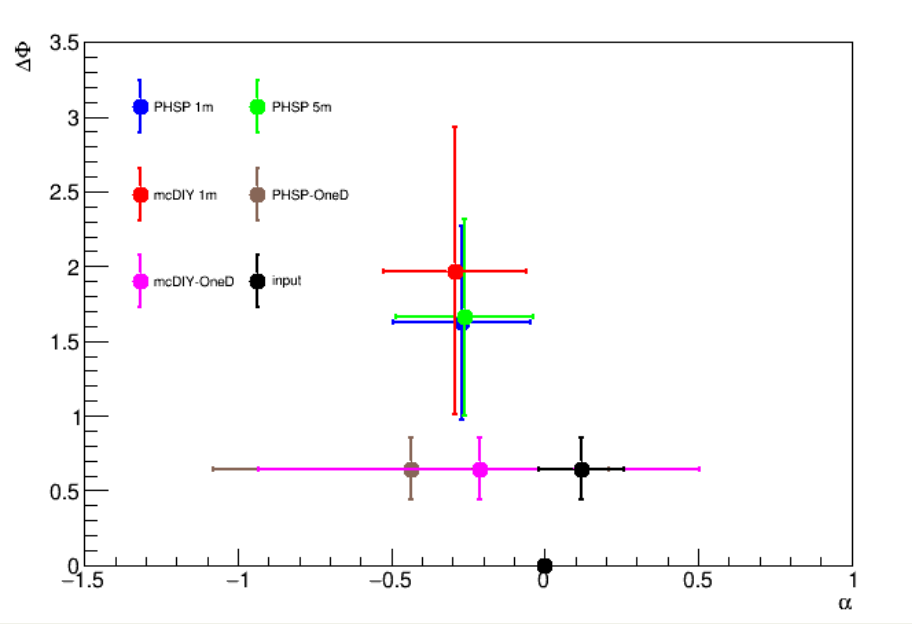
而根据大数定律，当 $n \to \infty$ 时，子样平均

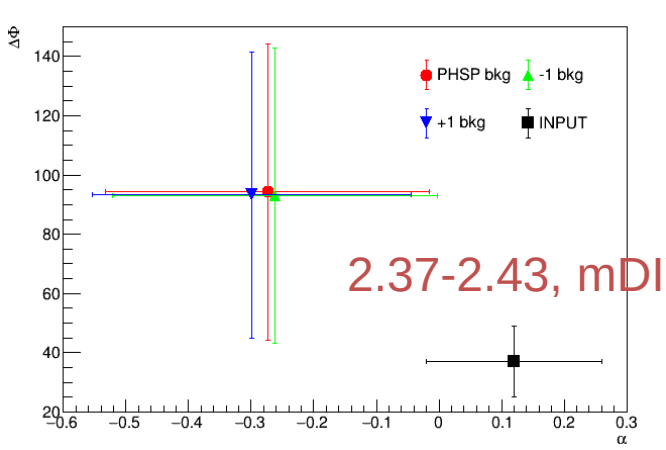
$$\hat{h} = \frac{1}{n}\sum_{i=1}^{n} h_i \tag{14.4.20}$$

的期望值 $E[\hat{h}]$ 与总体 h 的数学期望 $E[h]$ 相等，所以当 n 充分大时，有

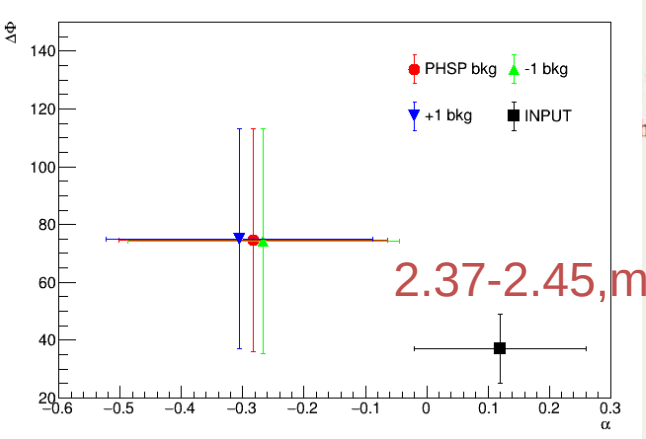
$$I = E[h] = E[\hat{h}] \approx \frac{1}{n}\sum_{i=1}^{n} h_i \equiv I_n. \tag{14.4.21}$$

		3773		4128-4258	
M_l	M_r	Ndata	Nsig	Ndata	Nsig
2.23	2.31	22	19.92961	1	0.87886
2.31	2.33	20	17.27213	7	6.71859
2.33	2.35	36	31.37735	2	1.36996
2.35	2.37	40	33.60154	13	11.989
2.37	2.39	61	54.1591	10	8.5271
2.39	2.41	53	44.92773	13	11.1579
2.41	2.43	51	42.69043	11	9.06577
2.43	2.45	51	42.63131	15	12.8528
2.45	2.47	44	35.83603	7	4.69727
2.47	2.49	49	40.72529	13	10.7822
2.49	2.51	32	23.62878	7	4.71956
2.51	2.53	32	23.97409	17	14.7041
2.53	2.55	38	30.34397	8	5.86071
2.55	2.57	28	20.82146	9	6.76202
2.57	2.59	29	21.41971	3	0.85978
2.59	2.61	30	22.76648	9	7.01061
2.61	2.63	25	18.10658	13	11.0077

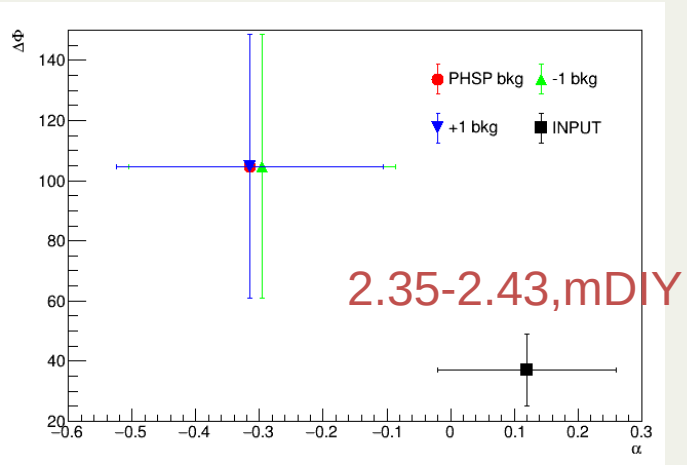




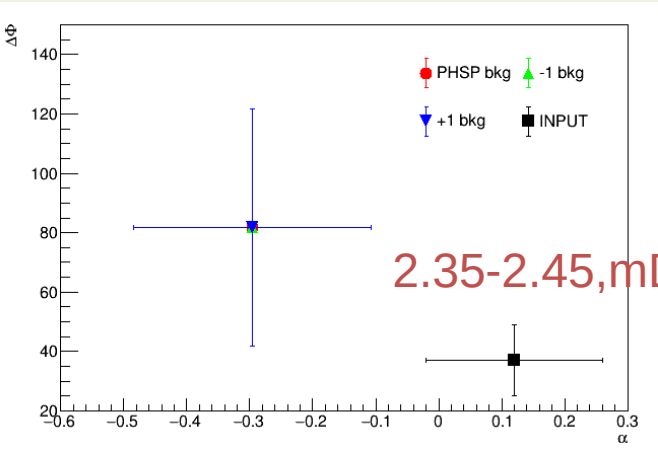
2.37-2.43, mDIY



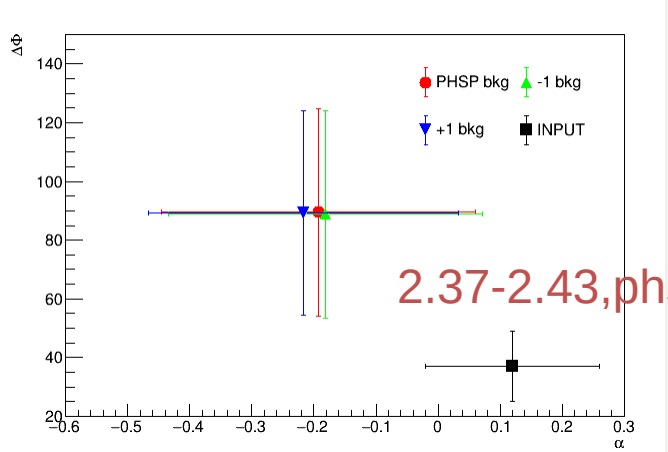
2.37-2.45,mDIY



2.35-2.43,mDIY



2.35-2.45,mDIY



2.37-2.43,phsp

MCtruth

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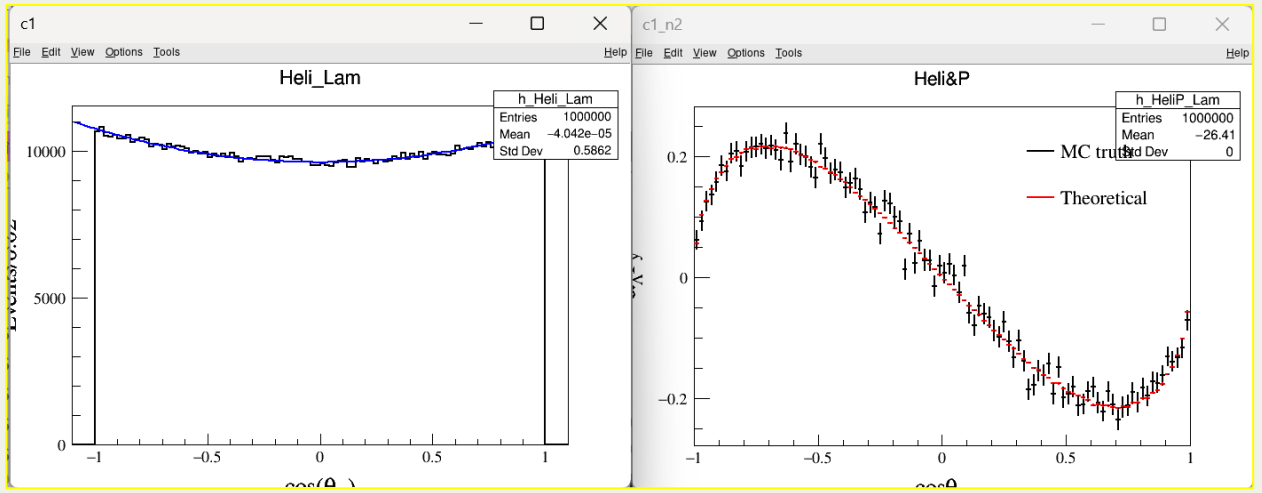
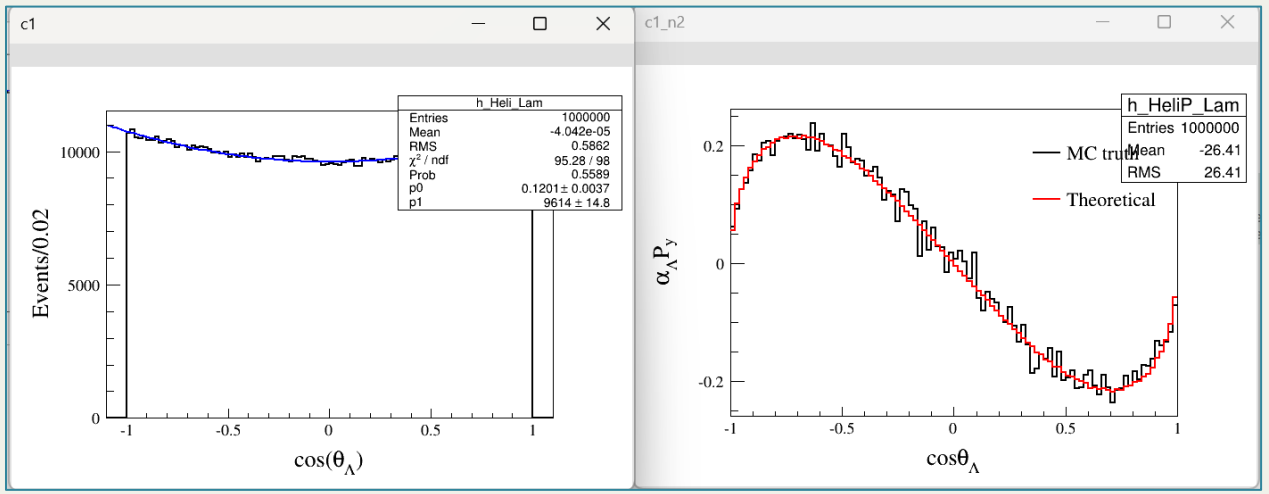
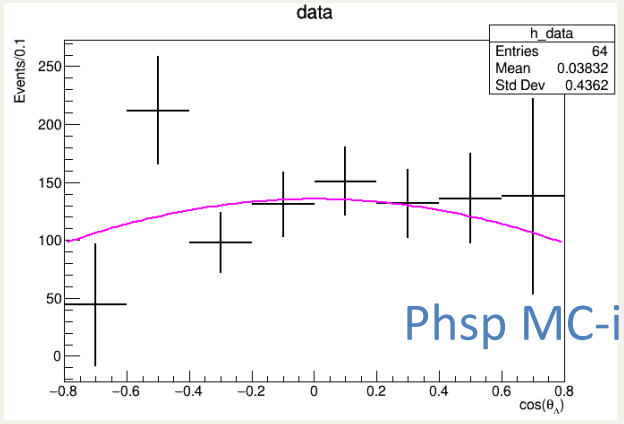




NO.	NAME	VALUE	ERROR
1	p0	1.20123e-01	3.66497e-03
2	p1	9.61413e+03	1.48280e+01

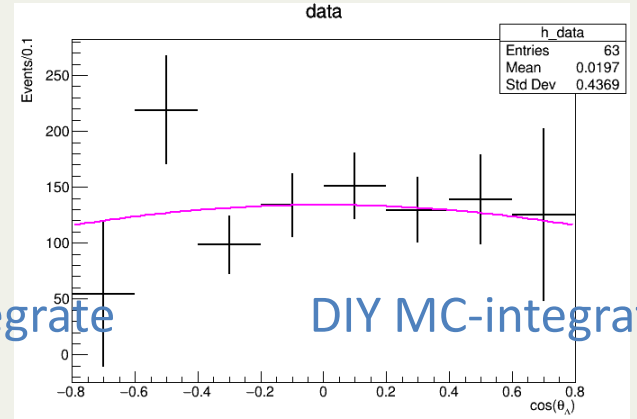
1D fit

Check the distribution of $cos\theta_{\Lambda}$ in the $\Lambda\bar{\Lambda}$-CMS system (production level): (Data-bkg)/eff



Phsp MC-integrate

NO.	NAME	VALUE	ERROR
1	p0	-4.36793e-01	6.44871e-01
2	p1	1.35521e+02	1.63747e+01



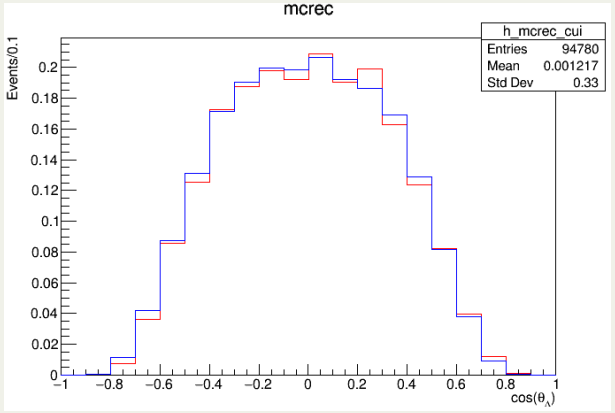
DIY MC-integrate

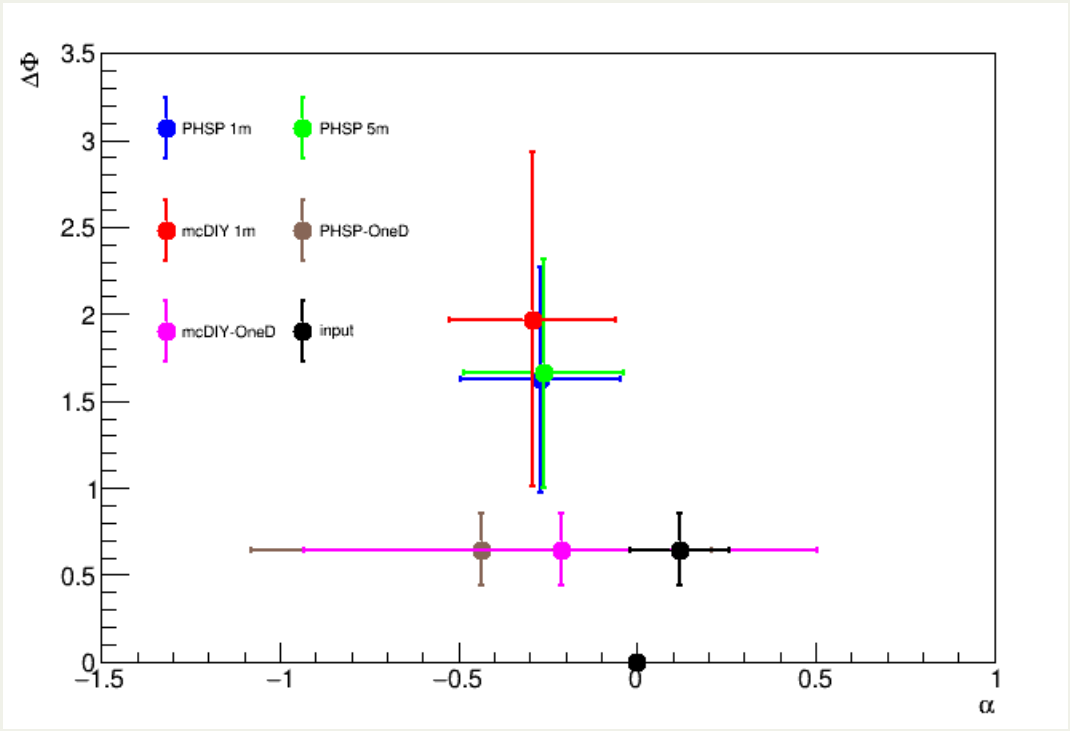
NO.	NAME	VALUE	ERROR
1	p0	-2.12145e-01	7.18564e-01
2	p1	1.33960e+02	1.66731e+01

eff:

Phsp

mDIY





Pull distribution (pure signal)

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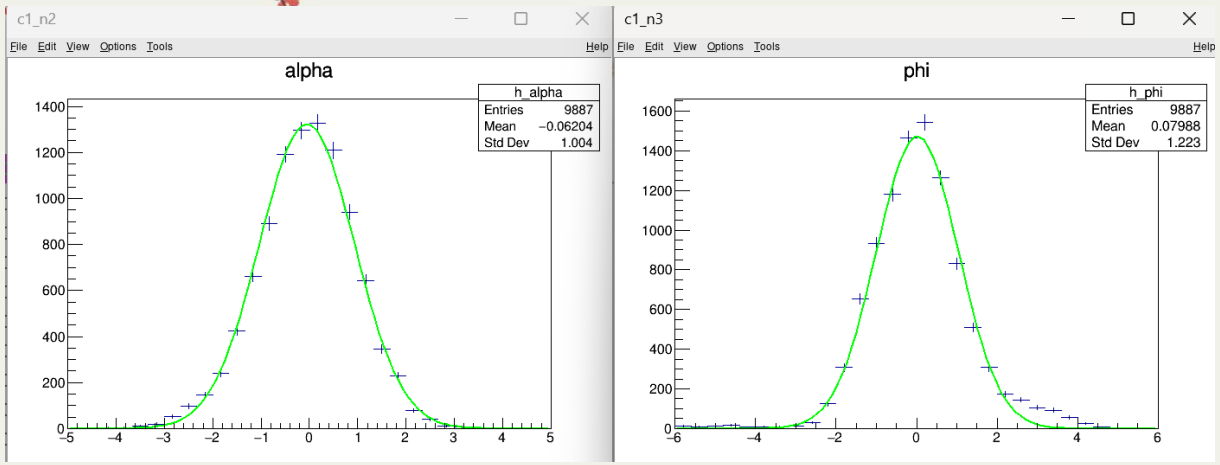
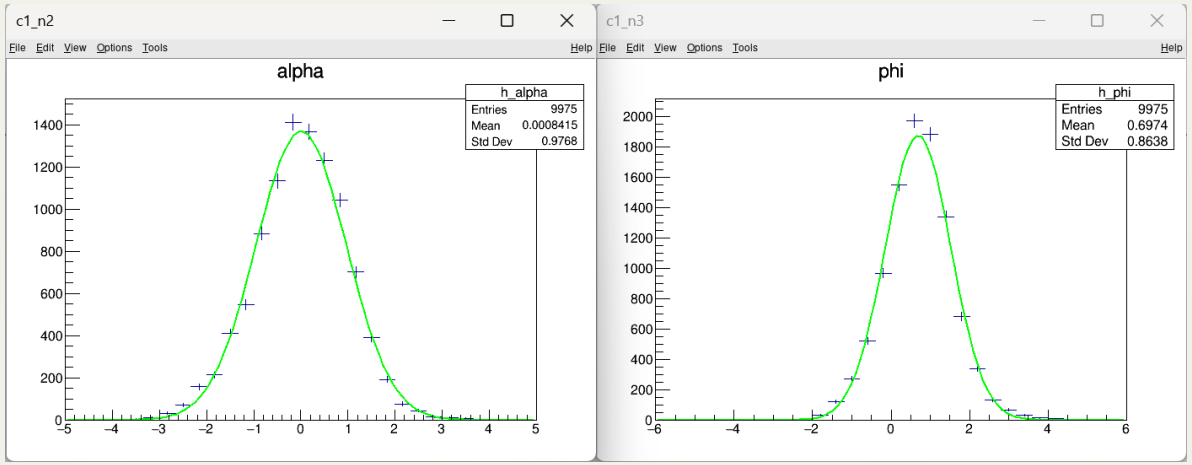
DIY MC-integrate



10000 times fit with 207 events randomly selected in DIY MC (no bkg) , 9810 of them give good fit.

```
FCN=91.0484 FROM MIGRAD    STATUS=CONVERGED      61 CALLS          62 TOTAL
                    EDM=2.02553e-07    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                   STEP         FIRST
 NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
  1  Constant     1.32379e+03   1.69342e+01   6.26573e-02  -4.04824e-05
  2  Mean        -5.10196e-02   1.01911e-02   4.66036e-05  -5.34806e-03
  3  Sigma        9.84108e-01   7.72683e-03   9.39926e-06  -2.95688e-01
FCN=432.252 FROM MIGRAD    STATUS=CONVERGED      80 CALLS          81 TOTAL
                    EDM=2.10133e-08    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                   STEP         FIRST
 NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
  1  Constant     1.47155e+03   1.94407e+01   1.54189e-01   2.66221e-06
  2  Mean         2.08370e-02   1.10481e-02   1.06905e-04  -1.64374e-02
  3  Sigma        1.02030e+00   8.42603e-03   1.91119e-05  -5.86133e-03
```

Phsp MC-integrate



10000 times fit with 207 events randomly selected in DIY MC (no bkg) , 9975 of them give good fit.

```
FCN=111.128 FROM MIGRAD    STATUS=CONVERGED      62 CALLS          63 TOTAL
                    EDM=2.2297e-08    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                   STEP         FIRST
 NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
  1  Constant     1.36895e+03   1.76523e+01   7.12671e-02  -1.54437e-06
  2  Mean         6.30233e-03   9.90158e-03   4.98924e-05   9.99424e-03
  3  Sigma        9.58198e-01   7.72001e-03   1.05322e-05  -6.84750e-02
FCN=123.057 FROM MIGRAD    STATUS=CONVERGED      62 CALLS          63 TOTAL
                    EDM=5.17046e-07    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                   STEP         FIRST
 NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
  1  Constant     1.87313e+03   2.46820e+01   1.02767e-01  -9.00425e-06
  2  Mean         6.89587e-01   8.45455e-03   4.59315e-05  -1.43940e-02
  3  Sigma        8.37098e-01   7.09508e-03   1.14794e-05  -4.14684e-01
```

Pull distribution (signal + bkg)

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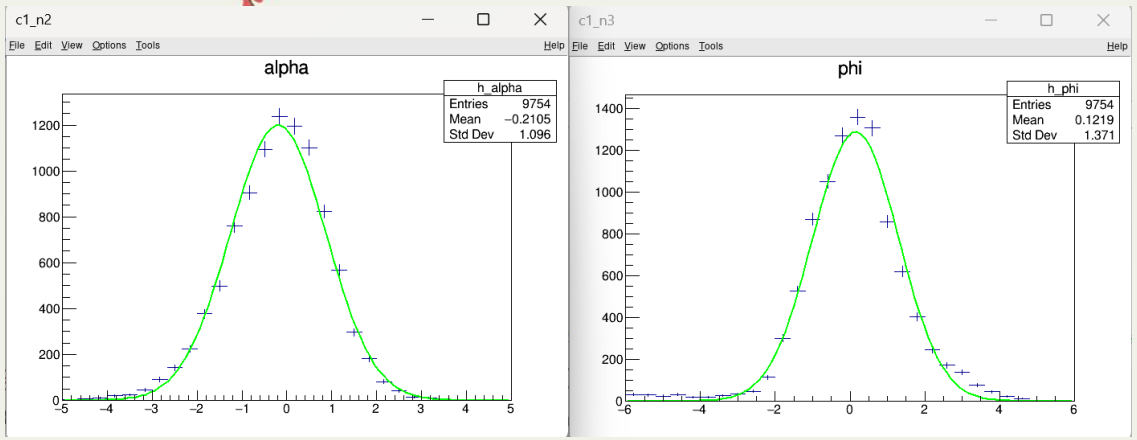
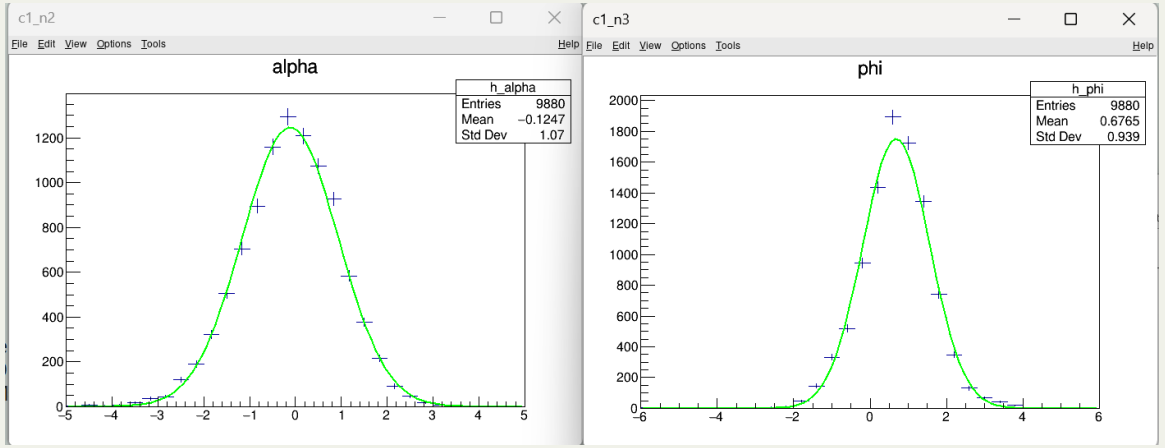
DIY MC-integrate



10000 times fit, 181 signal events, 26 bkg.
9754 of them give good fit.

```
FCN=135.549 FROM MIGRAD    STATUS=CONVERGED      62 CALLS          63 TOTAL
                     EDM=2.95805e-07    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                   STEP         FIRST
 NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
  1  Constant     1.19848e+03   1.55595e+01   6.97322e-02  -1.83378e-05
  2  Mean        -1.95368e-01   1.13616e-02   6.20938e-05   2.71081e-02
  3  Sigma        1.06703e+00   8.57352e-03   1.14844e-05  -2.89480e-01
FCN=408.028 FROM MIGRAD    STATUS=CONVERGED      77 CALLS          78 TOTAL
                     EDM=1.35763e-08    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                   STEP         FIRST
 NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
  1  Constant     1.28998e+03   1.79389e+01   1.32365e-01  -2.25017e-06
  2  Mean         1.51220e-01   1.24652e-02   1.17576e-04  -8.33676e-03
  3  Sigma        1.14587e+00   1.05880e-02   2.00112e-05  -4.45720e-02
```

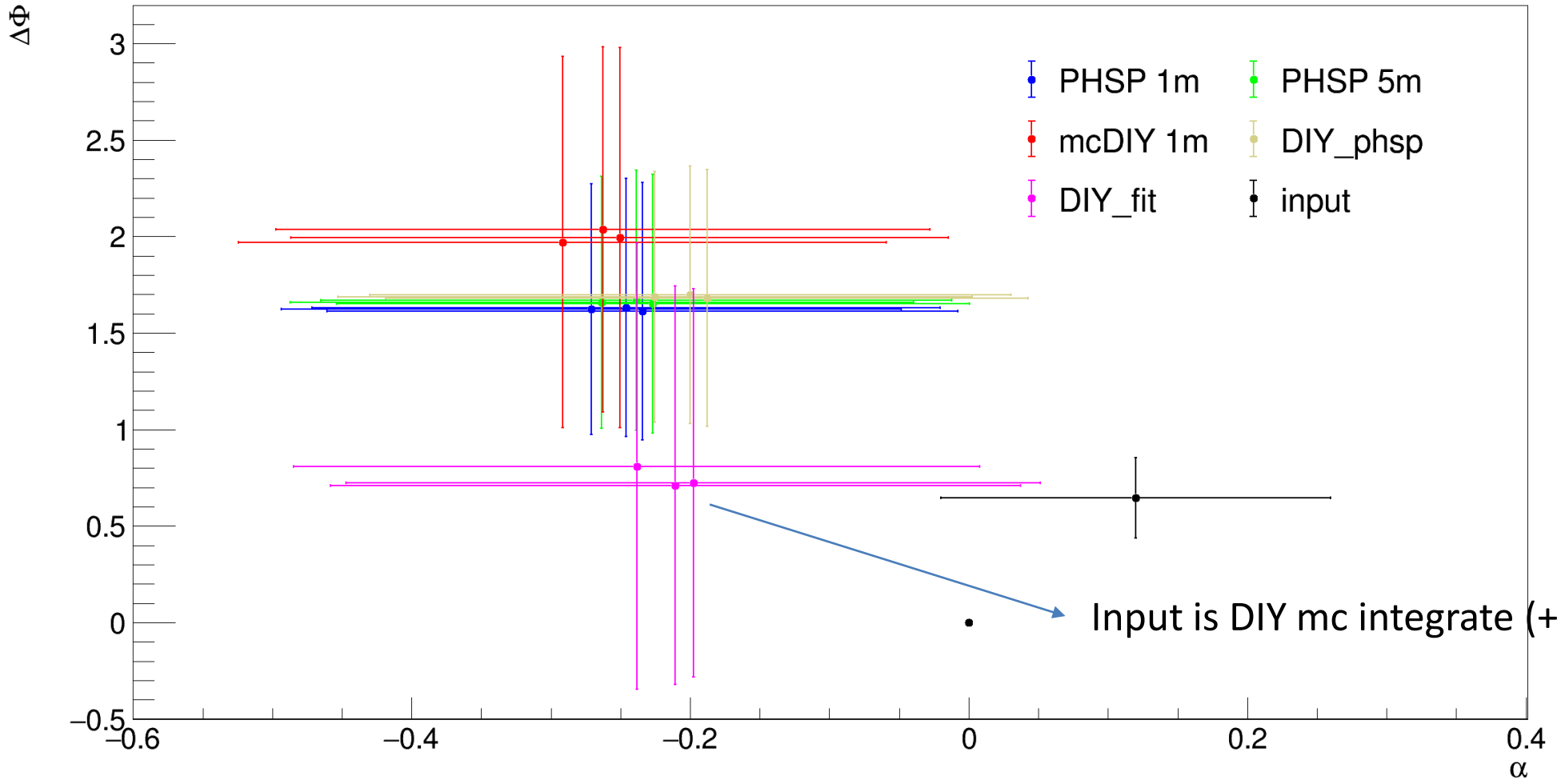
Phsp MC-integrate



10000 times fit, 181 signal events, 26 bkg.
9880 of them give good fit.

```
FCN=79.973 FROM MIGRAD    STATUS=CONVERGED      62 CALLS          63 TOTAL
                     EDM=3.10088e-08    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                   STEP         FIRST
 NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
  1  Constant     1.24656e+03   1.57687e+01   5.53411e-02  -7.87055e-07
  2  Mean        -1.15753e-01   1.07642e-02   4.63934e-05   6.50065e-03
  3  Sigma        1.04492e+00   7.95257e-03   8.66852e-06  -9.31871e-02
FCN=159.39 FROM MIGRAD    STATUS=CONVERGED      70 CALLS          71 TOTAL
                     EDM=3.77343e-13    STRATEGY= 1      ERROR MATRIX ACCURATE
 EXT PARAMETER                                   STEP         FIRST
 NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
  1  Constant     1.75149e+03   2.34510e+01   1.10400e-01   1.79952e-09
  2  Mean         6.87408e-01   9.02634e-03   5.52765e-05   9.70489e-05
  3  Sigma        8.76896e-01   7.61227e-03   1.29998e-05   6.57810e-05
```

Fit result


ΔΦ
3
2.5
2
1.5
1
0.5
0
−0.5
−0.6
−0.4
−0.2
0
0.2
0.4
α
PHSP 1m
PHSP 5m
mcDIY 1m
DIY_phsp
DIY_fit
input
Input is DIY mc integrate (+1 bkg)