Accessing 3-D and spin dependent fragmentation functions in e^+e^- annihilation

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- Introduction
- Collinear expansion and 3-D FFs
- Accessing tensor polarized 3-D FFs in $e^+e^- \to V\pi X$
- Energy dependence of hadron polarizations
- Summary

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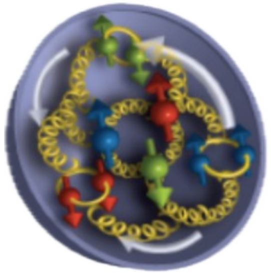
■ Parton distribution functions and fragmentation functions

Parton distribution functions (**PDFs**):

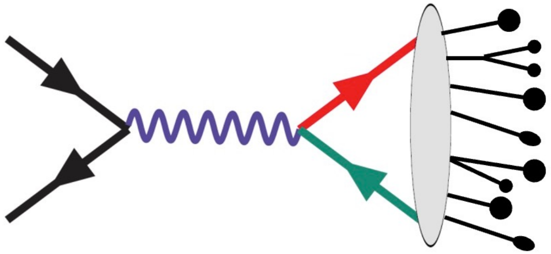
Parton momentum distribution inside a hadron



Hadron structure

Fragmentation functions (**FFs**):

Hadron momentum distribution inside a parton jet

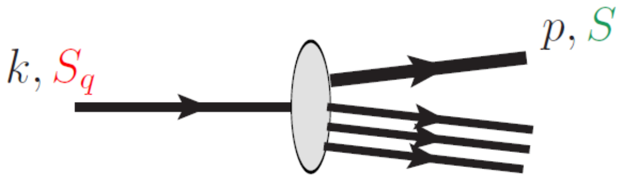


Hadronization mechanism

Why PDFs and FFs important ?

- Insights into the properties of strong interaction
- Understanding hadron structure and hadronization mechanism
- Prerequisite and inputs for new physics researches
- … …

■ Intuitive definition of FFs



$z = p^+/k^+$

One-dimensional (1-D)

Unpolarized: $D_1^{q\to h}(k;p) = D_1^{q\to h}(z)$

Number density of hadron h carrying momentum fraction z of the fragmenting quark q

Polarized: $D\left(k, S_q; p, S\right) = D_1(z) + \lambda_q \lambda G_{1L}(z) + \left(\vec{S}_{\perp q}\cdot\vec{S}_T\right)H_{1T}(z)$

Spin-1/2

Longitudinal spin transfer — Transverse spin transfer

Three-dimensional (3-D)

Transverse momentum dependent (TMD)

$$D\left(k, S_q; p, S\right) = D_1(z, p_T) + \lambda_q \lambda G_{1L}(z, p_T) + \left(\vec{S}_{\perp q}\cdot\vec{S}_T\right)H_{1T}(z, p_T) + \frac{1}{M}\vec{S}_T\cdot\left(\hat{k}\times\vec{p}_T\right)D_{1T}^{\perp}(z, p_T)$$

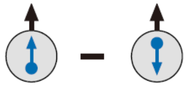
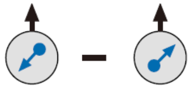
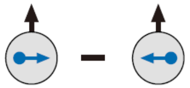
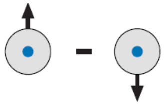
$$+\frac{1}{M}\lambda_q\left(\vec{S}_T\cdot\vec{p}_T\right)G_{1T}^{\perp}(z, p_T) + \frac{1}{M}\vec{S}_{\perp q}\cdot\left(\hat{k}\times\vec{p}_T\right)H_1^{\perp}(z, p_T)$$

$$+\frac{1}{M^2}\left(\vec{S}_{\perp q}\cdot\vec{p}_T\right)\left(\vec{S}_T\cdot\vec{p}_T\right)H_{1T}^{\perp}(z, p_T) + \frac{1}{M}\lambda\left(\vec{S}_{\perp q}\cdot\vec{p}_T\right)H_{1L}^{\perp}(z, p_T)$$

Sivers-type FF

Collins function

Introduction

Illustration	TMD FFs	Integrate over p_T	Naming/interpretation
	$D_1(z, p_T)$	$D_1(z)$	Number density
	$G_{1L}(z, p_T)$	$G_{1L}(z)$	Longitudinal spin transfer
	$H_{1T}(z, p_T)$	$H_{1T}(z)$	Transverse spin transfer
	$H_{1T}^{\perp}(z, p_T)$		
	$G_{1T}^{\perp}(z, p_T)$	×	Longitudinal to transvers spin transfer
	$H_{1L}^{\perp}(z, p_T)$	×	Transvers to longitudinal spin transfer
	$D_{1T}^{\perp}(z, p_T)$	×	Sivers-type FF
	$H_1^{\perp}(z, p_T)$	×	Collins FF

- What's the quantum field theoretical definition of these FFs?
- How to probe them?

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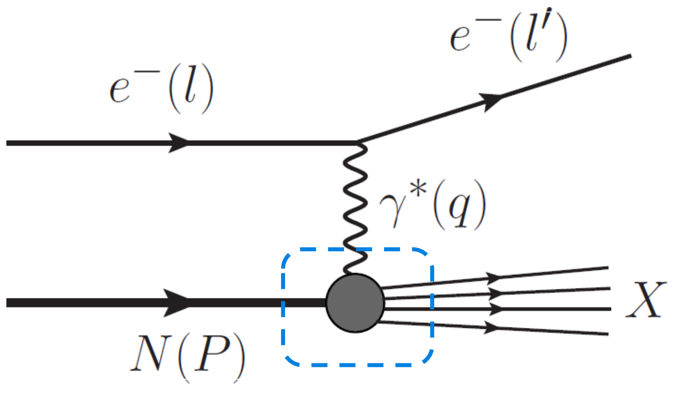
Collinear expansion and 3-D FFs

■ Study PDFs: deeply inelastic scattering (DIS)

$$d\sigma = \frac{2\alpha^2}{sQ^4} L^{\mu\nu}(l, l', \lambda_l) W_{\mu\nu}(q, P) \, \frac{d^3 l'}{2E'}$$

$$L_{\mu\nu}(l, l', \lambda_l) = 2\left(l_\mu l'_\nu + l_\nu l'_\mu - g_{\mu\nu} l \cdot l'\right) + 2i\lambda_l \varepsilon_{\mu\nu\rho\sigma} l^\rho l'^\sigma$$

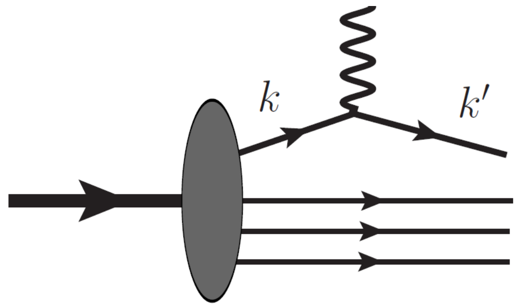
$$W_{\mu\nu}(q, P) = \frac{1}{2\pi} \sum_X \langle P | J_\mu(0) | X \rangle \langle X | J_\nu(0) | P \rangle (2\pi)^4 \delta^4(P + q - P_X)$$



Naïve parton model

$$|\mathcal{M}(eN \to eX)|^2 = \sum_q \int dx \, f_1^q(x) \, \left|\widehat{\mathcal{M}}(eq \to eq)\right|^2$$

$f_1^q(x)$: parton distribution functions (PDFs).
Parton number density of flavor q in the nucleon.
(momentum fraction: $x = k/P$)



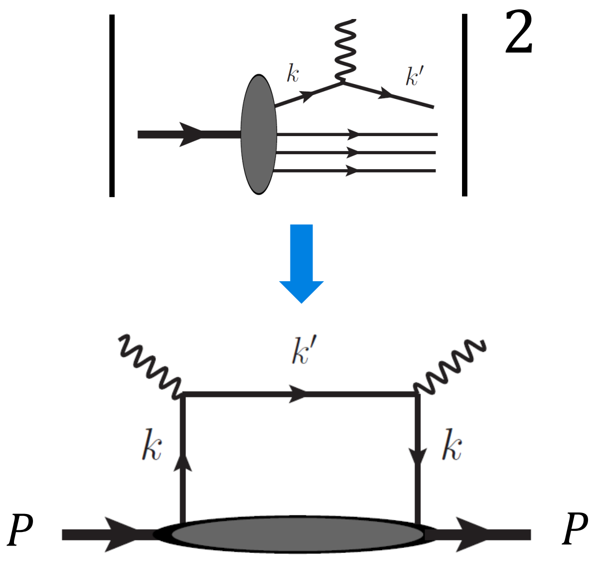
Collinear expansion and 3-D FFs

■ PDFs in terms of QFT operators

$$W_{\mu\nu}(q,P) = \frac{1}{2\pi}\int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}^{(0)}_{\mu\nu}(k,q)\hat{\phi}^{(0)}(k,P)\right]$$

$$\hat{H}^{(0)}_{\mu\nu}(k,q) = \gamma_\mu(\not{k}+\not{q})\gamma_\nu\,(2\pi)\delta^+((k+q)^2)$$

$$\hat{\phi}^{(0)}(k,P) = \int d^4y\, e^{ik\cdot y}\langle P|\bar{\psi}(0)\psi(y)|P\rangle$$



Collinear approximation: $k \approx xP$

$$\hat{H}^{(0)}_{\mu\nu}(k,q) \approx \hat{H}^{(0)}_{\mu\nu}(x)$$

$$W_{\mu\nu} = \frac{1}{2\pi}\int dx\ \mathrm{Tr}\left[\hat{H}^{(0)}_{\mu\nu}(x)\hat{\phi}^{(0)}(x,P)\right]$$

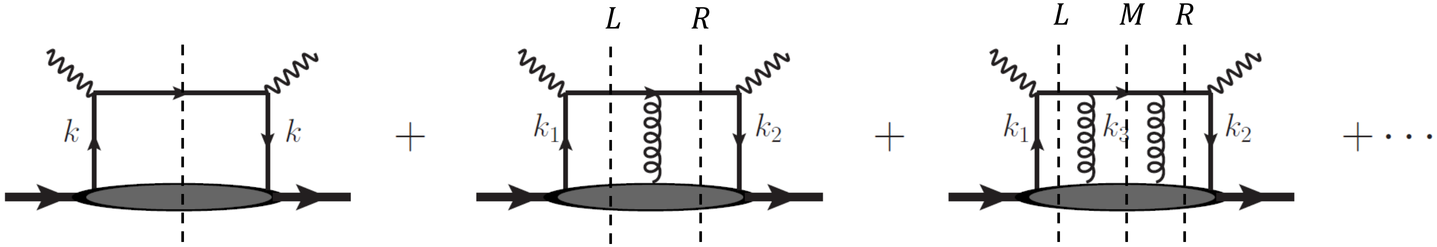
$$\hat{\phi}^{(0)}(x,P) = \frac{1}{2} f_1(x)\, P_\alpha\gamma^\alpha + \cdots$$

Operator definition of one dimensional PDF:

$$f_1(x) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \left\langle P\left|\bar{\psi}(0)\frac{\gamma^+}{2}\psi(y^-)\right|P\right\rangle$$

However, not gauge invariant!

■ Final state interactions



$$W_{\mu\nu} = W_{\mu\nu}^{(0)} + W_{\mu\nu}^{(1,c)} + W_{\mu\nu}^{(2,c)} + \cdots$$

$$W_{\mu\nu}^{(1,L)}(q,P) = \frac{1}{2\pi}\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(1,L)\rho}(k_1,k_2,q)\hat{\phi}_\rho^{(1,L)}(k_1,k_2,P)\right]$$

$$\hat{H}_{\mu\nu}^{(1,L)\rho} = \frac{\gamma_\mu(\not{k}_2+\not{q})\gamma^\rho}{(k_2+q)^2-i\epsilon}(\not{k}_1+\not{q})\gamma_\nu\,(2\pi)\delta^+\left((k_1+q)^2\right),\quad \hat{\phi}_\rho^{(1,L)} = \int d^4y\,d^4z e^{ik_1\cdot y+i(k_2-k_1)\cdot z}\langle P|\bar{\psi}(0)gA_\rho(z)\psi(y)|P\rangle$$

Collinear approximation

$k_i \approx x_i P,\ A_\rho \approx A^+\bar{n}_\rho$

- no power suppression for A^+ gluon exchange
- lead to the same hard part $P_\rho \hat{H}_{\mu\nu}^{(1,L)\rho}(x_1,x_2) = \frac{\hat{H}_{\mu\nu}^{(0)}(x_1)}{x_2-x_1-i\epsilon}$, … …

$\Rightarrow$ Gauge invariant quark-quark correlator: $\hat{\Phi}^{(0)}(x,P) = \int \frac{p^+dy^-}{2\pi} e^{ixp^+y^-}\langle P|\bar{\psi}(0)\mathcal{L}(0,y^-)\psi(y^-)|P\rangle$,

Gauge link: $\mathcal{L}(0,y^-) = \mathcal{P}\exp[ig\int_0^{y^-} d\eta^- A^+(\eta^-)]$

Higher twist contribution ? → Collinear expansion

Collinear expansion and 3-D FFs

■ Collinear expansion for inclusive DIS $eN \to eX$

General steps:

R.K. Ellis, W. Furmanski and R. Petronzio (1982,1983); J.W. Qiu and G. Sterman (1991)

- Expand the hard parts at $k_i = x_i P$, such as: $\hat{H}^{(0)}_{\mu\nu}(k) = \hat{H}^{(0)}_{\mu\nu}(x) + \frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k_\rho} \omega_\rho{}^{\rho'} k_{\rho'} + \cdots$

$\omega_\rho^{\rho'} = g_\rho^{\rho'} - \bar{n}_\rho n^{\rho'}$

$\omega_\rho^{\rho'} k_{\rho'} = (k - xp)_\rho$

- Decompose the gluon field: $A_\rho = A^+ \bar{n}_\rho + \omega_\rho{}^{\rho'} A_{\rho'}$
- Apply Ward identities, such as: $P_\rho \hat{H}^{(1,L)\rho}_{\mu\nu}(x_1, x_2) = \frac{\hat{H}^{(0)}_{\mu\nu}(x_1)}{x_2 - x_1 - i\epsilon}, \quad \frac{\partial \hat{H}^{(0)}_{\mu\nu}(x)}{\partial k_\rho} = -\sum_{c=L,R} \hat{H}^{(1,c)\rho}_{\mu\nu}(x, x), \cdots$
- Add same hard parts together: $W_{\mu\nu} = \hat{H}^{(0)}_{\mu\nu}(x) \otimes \hat{\Phi}^{(0)}(x) + \hat{H}^{(1,L)\rho}_{\mu\nu}(x_1, x_2) \omega_\rho{}^{\rho'} \otimes \hat{\varphi}^{(1,L)}_{\rho'}(x_1, x_2) + \cdots$

quark-quark correlator: $\hat{\Phi}^{(0)} = \int \frac{p^+ dy^-}{2\pi} e^{ixp^+y^-} \langle P | \bar{\psi}(0) \mathcal{L}(0, y^-) \psi(y^-) | P \rangle \Rightarrow$ twist-2,3,4

quark-gluon-quark correlator: $\hat{\varphi}^{(1,L)}_\rho = \int \frac{p^+ dy^-}{2\pi} e^{ixp^+y^-} \langle P | \bar{\psi}(0) D_\rho(0) \mathcal{L}(0, y^-) \psi(y^-) | P \rangle \Rightarrow$ twist-3,4,5

$D_\rho \equiv -i\partial_\rho + gA_\rho$

Collinear expansion: to obtain gauge invariant definition of parton correlators and PDFs;
to calculate leading and higher twist contributions systematically.

Collinear expansion and 3-D FFs

■ Collinear expansion for semi-inclusive DIS $eN \to eq(\text{jet})X$

The only difference is the kinematical factor in hard parts, i.e.,

$$W_{\mu\nu}^{(j,c,si)} = \hat{H}_{\mu\nu}^{(j,c,si)} \otimes \hat{\phi}^{(j,c)} \qquad \hat{H}_{\mu\nu}^{(j,c,si)}(k_i,k',q) = \hat{H}_{\mu\nu}^{(j,c)}(k_i,q)\times K(k_c,k',q)$$

Such as: $\hat{H}_{\mu\nu}^{(0,si)}(k,k',q) = \hat{H}_{\mu\nu}^{(0)}(k,q)\times 2E_{k'}(2\pi)^3\delta^3(\vec{k}'-\vec{k}-\vec{q})$

$$\hat{H}_{\mu\nu}^{(1,L,si)\rho}(k_1,k_2,k',q) = \hat{H}_{\mu\nu}^{(1,L)\rho}(k_1,k_2,q)\times 2E_{k'}(2\pi)^3\delta^3(\vec{k}'-\vec{k}_1-\vec{q})$$

Collinear expansion procedures are not affected!

Z.T. Liang and X.N. Wang (2007)

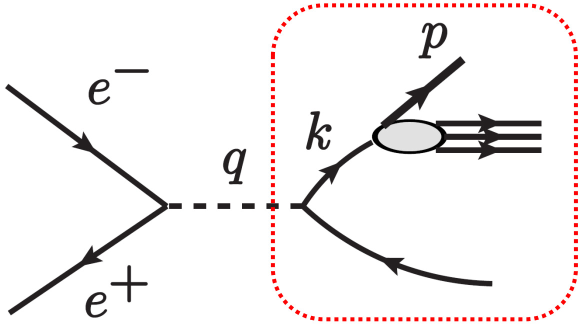
Collinear expansion can be naturally extended to $eN \to eq(\text{jet})X$

- A way to access 3-D hadron structure, the gauge invariant PDFs are transverse momentum dependent, e.g.,

$$f_1(x,k_\perp) = \int \frac{dy^- d^2y_\perp}{2(2\pi)^3} e^{ixp^+y^- - i\vec{k}_\perp\cdot\vec{y}_\perp} \langle P|\bar{\psi}(0)\gamma^+\mathcal{L}(0,y)\psi(y)|P\rangle$$

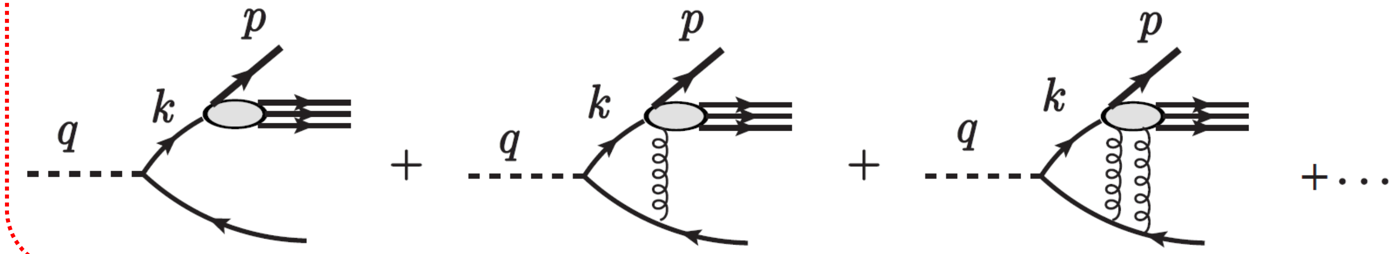
- Novel observables involved: azimuthal asymmetries, transverse spin and momentum correlations.

■ Study FFs through e^+e^- annihilation

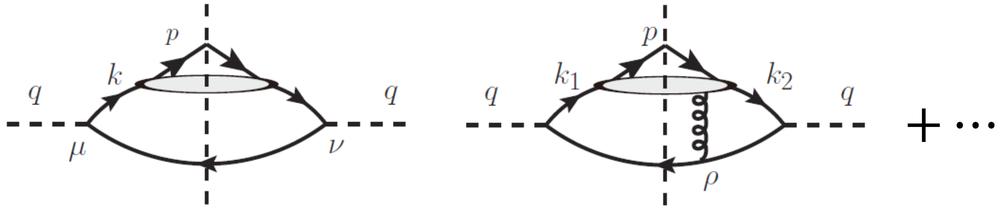


$e^+e^- \to h + \bar{q} + X$

Final state interactions, multi-gluon rescattering



$W_{\mu\nu} =$



Collinear expansion

$$W_{\mu\nu}(q,p,S) = \widetilde{W}^{(0)}_{\mu\nu}(q,p,S) + \widetilde{W}^{(1,L)}_{\mu\nu}(q,p,S) + \cdots$$

$$\widetilde{W}^{(0)}_{\mu\nu}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}[\hat{H}^{(0)}_{\mu\nu}(z)\hat{\Xi}^{(0)}(k;p,S)]$$

$$\widetilde{W}^{(1,L)}_{\mu\nu}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} \mathrm{Tr}[\hat{H}^{(1,L)\rho}_{\mu\nu}(z_1,z_2)\omega_\rho{}^{\rho'}\hat{\Xi}^{(1,L)}_{\rho'}(k_1,k_2;p,S)]$$

S.Y. Wei, Y.K. Song and Z.T. Liang, PRD (2014)
S.Y. Wei, K.B. Chen, Y.K. Song and Z.T. Liang, PRD (2015)

Collinear expansion and 3-D FFs

■ Quark-quark correlator

Gauge link: $\mathcal{L}(\xi,\infty) = \mathcal{P}Pe^{ig\int_{\xi^-}^{\infty} d\eta^- n\cdot A(\eta^-;\xi^+,\vec{\xi}_\perp)}$

$$\hat{\Xi}^{(0)}(z,k_\perp;p,S) = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-i(p^+\xi^-/z+k_\perp\cdot\xi_\perp)} \langle 0|\mathcal{L}^\dagger(0;\infty)\psi(0)|p,S;X\rangle\langle p,S;X|\bar{\psi}(\xi)\mathcal{L}(\xi;\infty)|0\rangle$$

4×4 matrix in Dirac space. Expand under Dirac-Γ matrices

$= I\cdot\Xi^{(0)}(z,k_\perp;p,S)$ → Scalar

$+i\gamma_5\tilde{\Xi}^{(0)}(z,k_\perp;p,S)$ → Pseudo-scalar

$+\gamma^\alpha\Xi_\alpha^{(0)}(z,k_\perp;p,S)$ → Vector

$+\gamma_5\gamma^\alpha\tilde{\Xi}_\alpha^{(0)}(z,k_\perp;p,S)$ → Pseudo-vector

$+i\sigma^{\alpha\beta}\gamma_5\Xi_{\alpha\beta}^{(0)}(z,k_\perp;p,S)$ → Tensor

Lorentz decomposition of the coefficients → **3-D FFs**

Hermiticity: $\hat{\Xi}^{\dagger(0)}(k;p,S) = \gamma^0\,\hat{\Xi}^{(0)}(k;p,S)\,\gamma^0$

Parity: $\hat{\Xi}^{(0)}(k;p,S) = \gamma^0\,\hat{\Xi}^{(0)}\left(k^{\mathcal{P}};p^{\mathcal{P}},S^{\mathcal{P}}\right)\gamma^0$

■ FFs defined via quark-quark correlator

Unpolarized part

Decomposition of the correlation functions using kinematic variables:

$p_\alpha, \ k_{\perp\alpha}, \ n_\alpha$

- Scalar: p^2 or M — Pseudo-scalar: none
- Vector: $p_\alpha, \ k_{\perp\alpha}, \ n_\alpha$ — Pseudo-vector: $\varepsilon_{\perp\alpha\beta}k_\perp^\beta$
- Tensor: $p_{[\rho}\varepsilon_{\perp\alpha]\beta}k_\perp^\beta, \ \varepsilon_{\perp\rho\alpha}, \ n_{[\rho}\varepsilon_{\perp\alpha]\beta}k_\perp^\beta$

$$p_\alpha = p^+\bar{n}_\alpha + \frac{M^2}{2p^+}n_\alpha$$

$$\varepsilon_{\perp\alpha\beta} = \varepsilon_{\mu\nu\alpha\beta}\bar{n}^\mu n^\nu$$

$$z\Xi^{U(0)}(z,k_\perp;p) = ME(z,k_\perp),$$

$$z\Xi_\alpha^{U(0)}(z,k_\perp;p) = p^+\bar{n}_\alpha D_1(z,k_\perp) + k_{\perp\alpha}D^\perp(z,k_\perp) + \frac{M^2}{p^+}n_\alpha D_3(z,k_\perp),$$

$$z\tilde{\Xi}_\alpha^{U(0)}(z,k_\perp;p) = \varepsilon_{\perp\alpha\beta}k_\perp^\beta G^\perp(z,k_\perp),$$

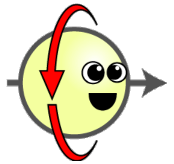
$$z\Xi_{\rho\alpha}^{U(0)}(z,k_\perp;p) = \frac{p^+\bar{n}_{[\rho}\varepsilon_{\perp\alpha]\beta}k_\perp^\beta}{M}H_1^\perp(z,k_\perp) + M\varepsilon_{\perp\rho\alpha}H(z,k_\perp) + \frac{Mn_{[\rho}\varepsilon_{\perp\alpha]\beta}k_\perp^\beta}{p^+}H_3^\perp(z,k_\perp).$$

8 unpolarized TMD FFs

Operator definition: $D_1(z,k_\perp) = \sum_X \int \frac{z\, d\xi^- d^2\xi_\perp}{8\pi} e^{-ip^+\xi^-/z - ik_\perp\cdot\xi_\perp} \mathrm{Tr}\left[\gamma^+\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|p,X\rangle\langle p,X|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle\right]$

Collinear expansion and 3-D FFs

Spin dependence



$S = 1/2$ 2×2 spin density matrix $\rho = \frac{1}{2}(1 + \vec{S}\cdot\vec{\sigma})$

Polarization vector: $S^\mu = (0, \vec{S}_T, \lambda)$, $\vec{S}_T = (S_T^x, S_T^y)$

Three parameters for vector polarized part

$S = 1$ 3×3 spin density matrix $\rho = \frac{1}{3}\left(1 + \frac{3}{2}\vec{S}\cdot\vec{\Sigma} + 3T^{ij}\Sigma^{ij}\right)$, $\Sigma^{ij} = \frac{1}{2}(\Sigma^i\Sigma^j + \Sigma^j\Sigma^i) - \frac{2}{3}\delta^{ij}\mathbf{I}$.

Polarization vector: $S^\mu = (0, \vec{S}_T, \lambda)$

Five parameters for tensor polarized part

Polarization tensor: $$\mathbf{T} = \frac{1}{2}\begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^{x} \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^{y} \\ S_{LT}^{x} & S_{LT}^{y} & \frac{4}{3}S_{LL} \end{pmatrix}.$$

S_{LL}

$S_{LT}^{\mu} = (0, S_{LT}^x, S_{LT}^y, 0)$

$S_{TT}^{x\mu} = (0, S_{TT}^{xx}, S_{TT}^{xy}, 0)$

Bacchetta and Mulders, PRD 62, 114004 (2000)

Vector polarized part

24 vector polarized 3-D FFs

$$\tilde{k}_{\perp\alpha} \equiv \varepsilon_{\perp\rho\alpha} k_\perp^\rho$$

$$z\Xi^{V(0)}(z,k_\perp;p,S) = (\tilde{k}_\perp \cdot S_T) E_T^\perp(z,k_\perp),$$

$$z\tilde{\Xi}^{V(0)}(z,k_\perp;p,S) = M\left[\lambda E_L(z,k_\perp) + \frac{k_\perp \cdot S_T}{M} E_T'^\perp(z,k_\perp)\right],$$

$$z\Xi_\alpha^{V(0)}(z,k_\perp;p,S) = p^+ \bar{n}_\alpha \frac{\tilde{k}_\perp \cdot S_T}{M} D_{1T}^\perp(z,k_\perp) - M\tilde{S}_{T\alpha} D_T(z,k_\perp) - \tilde{k}_{\perp\alpha}\left[\lambda D_L^\perp(z,k_\perp) + \frac{k_\perp \cdot S_T}{M} D_T^\perp(z,k_\perp)\right]$$
$$+ \frac{M}{p^+} n_\alpha (\tilde{k}_\perp \cdot S_T) D_{3T}^\perp(z,k_\perp),$$

$$z\tilde{\Xi}_\alpha^{V(0)}(z,k_\perp;p,S) = p^+ \bar{n}_\alpha \left[\lambda G_{1L}(z,k_\perp) + \frac{k_\perp \cdot S_T}{M} G_{1T}^\perp(z,k_\perp)\right] - MS_{T\alpha} G_T(z,k_\perp) - k_{\perp\alpha}\left[\lambda G_L^\perp(z,k_\perp) + \frac{k_\perp \cdot S_T}{M} G_T^\perp(z,k_\perp)\right]$$
$$+ \frac{M^2}{p^+} n_\alpha \left[\lambda G_{3L}(z,k_\perp) + \frac{k_\perp \cdot S_T}{M} G_{3T}^\perp(z,k_\perp)\right]$$

$$z\Xi_{\rho\alpha}^{V(0)}(z,k_\perp;p,S) = p^+ \bar{n}_{[\rho} S_{T\alpha]} H_{1T}(z,k_\perp) + \frac{p^+}{M} \bar{n}_{[\rho} k_{\perp\alpha]}\left[\lambda H_{1L}^\perp(z,k_\perp) + \frac{k_\perp \cdot S_T}{M} H_{1T}^\perp(z,k_\perp)\right] + k_{\perp[\rho} S_{T\alpha]} H_T^\perp(z,k_\perp)$$
$$+ M\bar{n}_{[\rho} n_{\alpha]}\left[\lambda H_L(z,k_\perp) + \frac{k_\perp \cdot S_T}{M} H_T'^\perp(z,k_\perp)\right] + \frac{M^2}{p^+} n_{[\rho} S_{T\alpha]} H_{3T}(z,k_\perp) + \frac{M}{p^+} n_{[\rho} k_{\perp\alpha]}\left[\lambda H_{3L}^\perp(z,k_\perp) + \frac{k_\perp \cdot S_T}{M} H_{3T}^\perp(z,k_\perp)\right]$$

Tensor polarized part

$S_{LL} \leftrightarrow$ unpolarized, $\quad S_{LT}^{\alpha} \leftrightarrow \varepsilon_{\perp}^{\alpha\beta} S_{T\beta}, \quad S_{TT}^{\alpha\beta} k_{\perp\beta} \leftrightarrow S_{LT}^{\alpha}$

$$z\Xi^{T(0)}(z,k_\perp;p,S) = M\left[S_{LL}E_{LL}(z,k_\perp) + \frac{k_\perp\cdot S_{LT}}{M}E_{LT}^{\perp}(z,k_\perp) + \frac{S_{TT}^{kk}}{M^2}E_{TT}^{\perp}(z,k_\perp)\right],$$

$$z\tilde{\Xi}^{T(0)}(z,k_\perp;p,S) = M\left[\frac{\tilde{k}\cdot S_{LT}}{M}E_{LT}^{\prime\perp}(z,k_\perp) + \frac{S_{TT}^{\tilde{k}k}}{M^2}E_{TT}^{\perp}(z,k_\perp)\right],$$

$$z\Xi_{\alpha}^{T(0)}(z,k_\perp;p,S) = p^+\bar{n}_\alpha\left[S_{LL}D_{1LL}(z,k_\perp) + \frac{k_\perp\cdot S_{LT}}{M}D_{1LT}^{\perp}(z,k_\perp) + \frac{S_{TT}^{kk}}{M^2}D_{1TT}^{\perp}(z,k_\perp)\right] + MS_{LT\alpha}D_{LT}(z,k_\perp)$$
$$+k_{\perp\alpha}\left[S_{LL}D_{LL}^{\perp}(z,k_\perp) + \frac{k_\perp\cdot S_{LT}}{M}D_{LT}^{\perp}(z,k_\perp) + \frac{S_{TT}^{kk}}{M^2}D_{TT}^{\perp}(z,k_\perp)\right] + S_{TT\alpha}^{k}D_{TT}^{\prime\perp}(z,k_\perp)$$
$$+\frac{M^2}{p^+}n_\alpha\left[S_{LL}D_{3LL}(z,k_\perp) + \frac{k_\perp\cdot S_{LT}}{M}D_{3LT}^{\perp}(z,k_\perp) + \frac{S_{TT}^{kk}}{M^2}D_{3TT}^{\perp}(z,k_\perp)\right]$$

Tensor polarized part

$$z\tilde{\Xi}_{\alpha}^{T(0)}(z,k_\perp;p,S) = p^+\bar{n}_\alpha\left[\frac{\tilde{k}_\perp\cdot S_{LT}}{M}G_{1LT}^\perp(z,k_\perp) + \frac{S_{TT}^{\tilde{k}k}}{M^2}G_{1TT}^\perp(z,k_\perp)\right] - M\tilde{S}_{LT\alpha}G_{LT}(z,k_\perp) - \tilde{S}_{TT\alpha}^{k}G_{TT}'^\perp(z,k_\perp)$$

$$-\tilde{k}_{\perp\alpha}\left[S_{LL}G_{LL}^\perp(z,k_\perp) + \frac{k_\perp\cdot S_{LT}}{M}G_{LT}^\perp(z,k_\perp) + \frac{S_{TT}^{kk}}{M^2}G_{TT}^\perp(z,k_\perp)\right] + \frac{M^2}{p^+}n_\alpha\left[\frac{\tilde{k}\cdot S_{LT}}{M}G_{3LT}^\perp(z,k_\perp) + \frac{S_{TT}^{\tilde{k}k}}{M^2}G_{3TT}^\perp(z,k_\perp)\right],$$

$$z\Xi_{\rho\alpha}^{T(0)}(z,k_\perp;p,S) = -p^+\bar{n}_{[\rho}\tilde{S}_{LT\alpha]}H_{1LT}(z,k_\perp) - \frac{p^+}{M}\bar{n}_{[\rho}\tilde{S}_{TT\alpha]}^{k}H_{1TT}'^\perp(z,k_\perp)$$

$$-\frac{p^+}{M}\bar{n}_{[\rho}\tilde{k}_{\perp\alpha]}\left[S_{LL}H_{1LL}^\perp(z,k_\perp) + \frac{k_\perp\cdot S_{LT}}{M}H_{1LT}^\perp(z,k_\perp) + \frac{S_{TT}^{kk}}{M^2}H_{1TT}^\perp(z,k_\perp)\right]$$

$$+M\varepsilon_{\perp\rho\alpha}\left[S_{LL}H_{LL}(z,k_\perp) + \frac{k_\perp\cdot S_{LT}}{M}H_{LT}^\perp(z,k_\perp) + \frac{S_{TT}^{kk}}{M^2}H_{TT}^\perp(z,k_\perp)\right] + \bar{n}_{[\rho}n_{\alpha]}\left[(\tilde{k}_\perp\cdot S_{LT})H_{LT}'^\perp(z,k_\perp) + \frac{S_{TT}^{\tilde{k}k}}{M^2}H_{TT}'^\perp(z,k_\perp)\right]$$

$$-\frac{M}{p^+}n_{[\rho}\tilde{k}_{\perp\alpha]}\left[S_{LL}H_{3LL}^\perp(z,k_\perp) + \frac{k_\perp\cdot S_{LT}}{M}H_{3LT}^\perp(z,k_\perp) + \frac{S_{TT}^{kk}}{M^2}H_{3TT}^\perp(z,k_\perp)\right]$$

$$-\frac{M}{p^+}\left[Mn_{[\rho}\tilde{S}_{LT\alpha]}H_{3LT}(z,k_\perp) + n_{[\rho}\tilde{S}_{TT\alpha]}^{k}H_{3TT}'^\perp(z,k_\perp)\right].$$

40 tensor polarized TMD FFs

Collinear expansion and 3-D FFs

Quark pol.	Hadron pol.	Chiral-even		Chiral-odd	
		T-even	T-odd	T-even	T-odd
U	U	D_1 $D^{\perp}$ D_3		E	
	L		$D_L^{\perp}$		
	T		$D_{1T}^{\perp}$ D_T $D_T^{\perp}$ $D_{3T}^{\perp}$		$E_T^{\perp}$
	LL	D_{1LL} $D_{LL}^{\perp}$ D_{3LL}		E_{LL}	
	LT	$D_{1LT}^{\perp}$ D_{LT} $D_{LT}^{\perp}$ $D_{3LT}^{\perp}$		$E_{LT}^{\perp}$	
	TT	$D_{1TT}^{\perp}$ $D_{TT}^{\perp}$ $D_{TT}^{\prime\perp}$ $D_{3TT}^{\perp}$		$E_{TT}^{\perp}$	
L	U		$G^{\perp}$		
	L	G_{1L} $G_L^{\perp}$ G_{3L}			E_L
	T	$G_{1T}^{\perp}$ G_T $G_T^{\perp}$ $G_{3T}^{\perp}$			$E_T^{\prime\perp}$
	LL		$G_{LL}^{\perp}$		
	LT		$G_{1LT}^{\perp}$ G_{LT} $G_{LT}^{\perp}$ $G_{3LT}^{\perp}$	$E_{LT}^{\prime\perp}$	
	TT		$G_{1TT}^{\perp}$ $G_{TT}^{\perp}$ $G_{TT}^{\prime\perp}$ $G_{3TT}^{\perp}$	$E_{TT}^{\prime\perp}$	
T	U				$H_1^{\perp}$ H $H_3^{\perp}$
	L			$H_{1L}^{\perp}$ H_L $H_{3L}^{\perp}$	
	T(∥)			H_{1T} $H_T^{\perp}$ H_{3T}	
	T(⊥)			$H_{1T}^{\perp}$ $H_T^{\prime\perp}$ $H_{3T}^{\perp}$	
	LL				$H_{1LL}^{\perp}$ H_{LL} $H_{3LL}^{\perp}$
	LT				H_{1LT} $H_{1LT}^{\perp}$ $H_{LT}^{\perp}$ $H_{LT}^{\prime\perp}$ H_{3LT} $H_{3LT}^{\perp}$
	TT				$H_{1TT}^{\perp}$ $H_{1TT}^{\prime\perp}$ $H_{TT}^{\perp}$ $H_{TT}^{\prime\perp}$ $H_{3TT}^{\perp}$ $H_{3TT}^{\prime\perp}$

72 TMD FFs from quark-quark correlator

■ FFs defined via quark-gluon-quark correlator

A complete results up to twist-3 need the contribution from quark-gluon-quark correlator

$$\hat{\Xi}^{(1)}_{\rho}(k;p,S)=\frac{1}{2\pi}\sum_X\int d^4\xi\, e^{-ik\xi}\langle 0|\mathcal{L}^{\dagger}(0;\infty)D_\rho(0)\psi(0)|p,S;X\rangle\langle p,S;X|\bar{\psi}(\xi)\mathcal{L}(\xi;\infty)|0\rangle$$

$$\hat{\Xi}^{(1)}_{\rho}=\Xi^{(1)}_{\rho}+i\gamma_5\tilde{\Xi}^{(1)}_{\rho}+\gamma^{\alpha}\Xi^{(1)}_{\rho\alpha}+\gamma_5\gamma^{\alpha}\tilde{\Xi}^{(1)}_{\rho\alpha}+i\sigma^{\alpha\beta}\gamma_5\Xi^{(1)}_{\rho\alpha\beta}$$

e.g., unpolarized part:

$$z\Xi^{U(1)}_{\rho\alpha}(z,k_\perp;p)=-p^+\bar{n}_\alpha k_{\perp\rho}D^{\perp}_{d}(z,k_\perp)+\cdots,$$
$$z\tilde{\Xi}^{U(1)}_{\rho\alpha}(z,k_\perp;p)=ip^+\bar{n}_\alpha\varepsilon_{\perp\rho\sigma}k^{\sigma}_{\perp}G^{\perp}_{d}(z,k_\perp)+\cdots,$$
$$z\Xi^{U(1)}_{\rho\alpha\beta}(z,k_\perp;p)=-p^+[M\varepsilon_{\perp\rho[\alpha}\bar{n}_{\beta]}H_d(z,k_\perp)+\frac{1}{M}\varepsilon_{\perp\rho\sigma}k^{\sigma}_{\perp}k_{F\perp[\alpha}\bar{n}_{\beta]}H^{\perp}_{d}(z,k_\perp)]+\cdots.$$

Collinear expansion and 3-D FFs

■ Relations between twist-3 FFs

QCD equation of motion： $\gamma^\mu D_\mu(y)\psi(y)=0$

Chiral even:

$$D_{dS}^K(z,k_\perp)+G_{dS}^K(z,k_\perp)=\frac{1}{z}[D_S^K(z,k_\perp)+iG_S^K(z,k_\perp)]$$

K	S
null	T LT
$\perp$	null L T LL LT TT
$'\perp$	TT

Chiral odd:

$$H_{dS}^K(z,k_\perp)+\frac{k_\perp^2}{2M^2}H_{dS}^{K'}(z,k_\perp)=\frac{1}{2z}[H_S^K(z,k_\perp)-iE_S^K(z,k_\perp)]$$

K	K'	S
null	$\perp$	null L LL
$\perp$	$\perp'$	T LT TT
$'\perp$	$'\perp'$	T LT TT

Twist-3 FFs defined from quark-gluon-quark correlator can be replaced by those from quark-quark correlator.

Contents

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- Collinear expansion and 3-D FFs
- Accessing tensor polarized 3-D FFs in $e^+e^- \to V\pi X$
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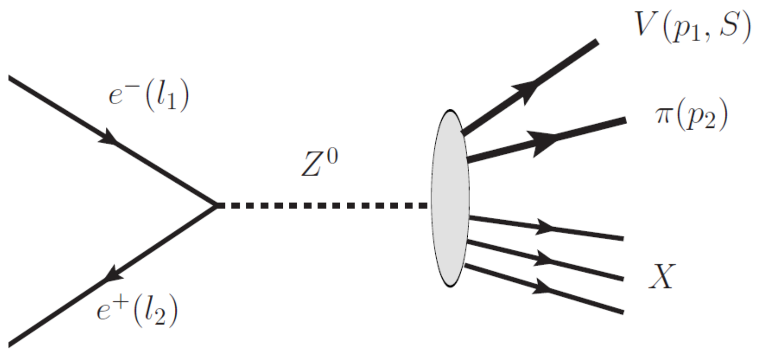
Accessing tensor polarized 3-D FFs in $e^+e^- \to V\pi X$

$e^+e^- \to V\pi X$：The best process for accessing tensor polarized 3-D FFs.

■ General kinematics

$$\frac{E_1 E_2 d\sigma}{d^3p_1 d^3p_2} = \frac{\alpha^2\chi}{2sQ^4} L_{\mu\nu}(l_1, l_2) W^{\mu\nu}(q, p_1, S, p_2)$$



$$L^{\mu\nu}(l_1, l_2) = c_1^e\left[l_1^\mu l_2^\nu + l_1^\nu l_2^\mu - (l_1 \cdot l_2) g^{\mu\nu}\right] + i c_3^e \varepsilon^{\mu\nu\rho\sigma} l_{1\rho} l_{2\sigma}$$

$$W^{\mu\nu}(q, p_1, S, p_2) = \frac{1}{(2\pi)^4} \sum_X \langle 0|J^\nu(0)|p_1, S, p_2, X\rangle\langle p_1, S, p_2, X|J^\mu(0)|0\rangle\, (2\pi)^4 \delta^4(q - p_1 - p_2 - p_X)$$

General kinematic analysis: to construct the general form of the hadronic tensor.

Available kinematic variables: q^μ, p_1^μ, p_2^μ, S^μ, S_{LL}, S_{LT}^μ, $S_{TT}^{\mu\nu}$, $g^{\mu\nu}$, $\varepsilon^{\mu\nu\rho\sigma}$

Accessing tensor polarized 3-D FFs in $e^+e^- \to V\pi X$

$$W^{\mu\nu}(q,p_1,S,p_2) = W^{S\mu\nu}(\text{symmetric part}) + iW^{A\mu\nu}(\text{anti}-\text{symmetric part})$$

$$= \sum_{\sigma,i} W^S_{\sigma i}(s,\xi_1,\xi_2,\xi_{12})\ h^{S\mu\nu}_{\sigma i} + i\sum_{\sigma,j} W^A_{\sigma j}(s,\xi_1,\xi_2,\xi_{12})\ h^{A\mu\nu}_{\sigma j}$$ Parity even

$$+\sum_{\sigma,k} \widetilde{W}^S_{\sigma k}(s,\xi_1,\xi_2,\xi_{12})\ \tilde{h}^{S\mu\nu}_{\sigma k} + i\sum_{\sigma,l} \widetilde{W}^A_{\sigma l}(s,\xi_1,\xi_2,\xi_{12})\ \tilde{h}^{A\mu\nu}_{\sigma l}$$ Parity odd

Scalar coefficients

Basic Lorentz tensors (BLTs)

$(\sigma = U,V,LL,LT,TT)$

$s = q^2$

$\xi_1 = 2q\cdot p_1/q^2$

$\xi_2 = 2q\cdot p_2/q^2$

$\xi_{12} = (p_1+p_2)^2/q^2$

Hermiticity: $W^{*\mu\nu}(q,p_1,S,p_2) = W^{\nu\mu}(q,p_1,S,p_2)$

Current conservation: $q_\mu W^{\mu\nu}(q,p_1,S,p_2) = q_\nu W^{\mu\nu}(q,p_1,S,p_2) = 0$

Accessing tensor polarized 3-D FFs in $e^+e^- \to V\pi X$

Unpolarized part

$$h_{Ui}^{S\mu\nu} = \left\{g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2},\quad p_{1q}^\mu p_{1q}^\nu,\quad p_{1q}^{\{\mu} p_{2q}^{\nu\}},\quad p_{2q}^\mu p_{2q}^\nu\right\}$$

$$\tilde{h}_{Ui}^{S\mu\nu} = \left\{\varepsilon^{\{\mu q p_1 p_2} p_{1q}^{\nu\}},\quad \varepsilon^{\{\mu q p_1 p_2} p_{2q}^{\nu\}}\right\}$$

$$h_U^{A\mu\nu} = p_{1q}^{[\mu} p_{2q}^{\nu]}$$

$$\tilde{h}_{Ui}^{A\mu\nu} = \{\varepsilon^{\mu\nu q p_1},\quad \varepsilon^{\mu\nu q p_2}\}$$

9 basic Lorentz tensors

5(P-even) + 4(P-odd)

$$\left(p_q^\mu = p^\mu - \frac{p\cdot q}{q^2} q^\mu,\quad q \cdot p_q = 0 \right)$$

Vector polarized part

$$h_{Vi}^{S\mu\nu} = \{[(q\cdot S),(p_2\cdot S)]\times\tilde{h}_{Ui}^{S\mu\nu},\quad \varepsilon^{Sqp_1p_2}\times h_{Uj}^{S\mu\nu}\}$$

$$\tilde{h}_{Vi}^{S\mu\nu} = \{[(q\cdot S),(p_2\cdot S)]\times h_{Ui}^{S\mu\nu},\quad \varepsilon^{Sqp_1p_2}\times\tilde{h}_{Uj}^{S\mu\nu}\}$$

$$h_{Vi}^{A\mu\nu} = \{[(q\cdot S),(p_2\cdot S)]\times\tilde{h}_{Ui}^{A\mu\nu},\quad \varepsilon^{Sqp_1p_2}\times h_{U}^{A\mu\nu}\}$$

$$\tilde{h}_{Vi}^{A\mu\nu} = \{[(q\cdot S),(p_2\cdot S)]\times h_{U}^{A\mu\nu},\quad \varepsilon^{Sqp_1p_2}\times\tilde{h}_{Uj}^{A\mu\nu}\}$$

Rules

Polarized BLTs = Polarization dependent (pseudo-)scalars

× Unpolarized BLTs

3 × 9 = 27 BLTs

Accessing tensor polarized 3-D FFs in $e^+e^- \to V\pi X$

■ Tensor polarized part

$$\begin{Bmatrix} h_{LLi}^{S\mu\nu} \\ \tilde{h}_{LLi}^{S\mu\nu} \\ h_{LL}^{A\mu\nu} \\ \tilde{h}_{LLi}^{A\mu\nu} \end{Bmatrix} = S_{LL} \begin{Bmatrix} h_{Ui}^{S\mu\nu} \\ \tilde{h}_{Ui}^{S\mu\nu} \\ h_{U}^{A\mu\nu} \\ \tilde{h}_{Ui}^{A\mu\nu} \end{Bmatrix}$$

$$h_{LTi}^{S\mu\nu} = \{(p_2 \cdot S_{LT})\times h_{Ui}^{S\mu\nu}, \quad \varepsilon^{S_{LT}qp_1p_2}\times \tilde{h}_{Uj}^{S\mu\nu}\}$$

$$\tilde{h}_{LTi}^{S\mu\nu} = \{(p_2 \cdot S_{LT})\times \tilde{h}_{Ui}^{S\mu\nu}, \quad \varepsilon^{S_{LT}qp_1p_2}\times h_{Uj}^{S\mu\nu}\}$$

$$h_{LTi}^{A\mu\nu} = \{(p_2 \cdot S_{LT})\times h_{U}^{A\mu\nu}, \quad \varepsilon^{S_{LT}qp_1p_2}\times \tilde{h}_{Uj}^{A\mu\nu}\}$$

$$\tilde{h}_{LTi}^{A\mu\nu} = \{(p_2 \cdot S_{LT})\times \tilde{h}_{Ui}^{A\mu\nu}, \quad \varepsilon^{S_{LT}qp_1p_2}\times h_{U}^{A\mu\nu}\}$$

$$h_{TTi}^{S\mu\nu} = \{S_{TT}^{p_2p_2}\times h_{Ui}^{S\mu\nu}, \quad \varepsilon^{S_{LT}^{p_2}qp_1p_2}\times \tilde{h}_{Uj}^{S\mu\nu}\}$$

$$\tilde{h}_{TTi}^{S\mu\nu} = \{S_{TT}^{p_2p_2}\times \tilde{h}_{Ui}^{S\mu\nu}, \quad \varepsilon^{S_{TT}^{p_2}qp_1p_2}\times h_{Uj}^{S\mu\nu}\}$$

$$h_{TTi}^{A\mu\nu} = \{S_{TT}^{p_2p_2}\times h_{U}^{A\mu\nu}, \quad \varepsilon^{S_{TT}^{p_2}qp_1p_2}\times \tilde{h}_{Uj}^{A\mu\nu}\}$$

$$\tilde{h}_{TTi}^{A\mu\nu} = \{S_{TT}^{p_2p_2}\times \tilde{h}_{Ui}^{A\mu\nu}, \quad \varepsilon^{S_{TT}^{p_2}qp_1p_2}\times h_{U}^{A\mu\nu}\}$$

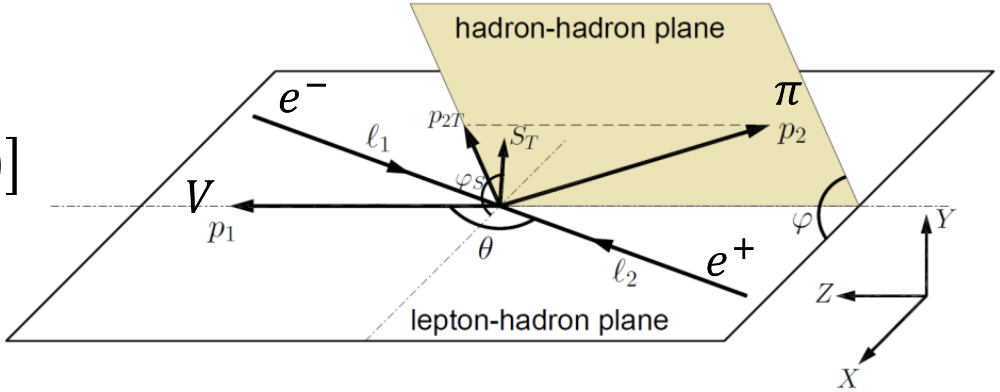
1 × 9   2 × 9   2 × 9

In total: 9 × 9 = 81 BLTs for unpolarized + vector polarized + tensor polarized

K.B. Chen, W.H. Yang, S.Y. Wei and Z.T. Liang, PRD 94, 034003 (2016)

■ Cross section in terms of structure functions

$$\frac{E_1 E_2 d\sigma}{d^3p_1 d^3p_2} = \frac{\alpha^2\chi}{2s^2}\Big[\big(\mathcal{F}_U + \tilde{\mathcal{F}}_U\big) + \lambda\big(\mathcal{F}_L + \tilde{\mathcal{F}}_L\big) + |S_T|\big(\mathcal{F}_T + \tilde{\mathcal{F}}_T\big) + S_{LL}\big(\mathcal{F}_{LL} + \tilde{\mathcal{F}}_{LL}\big) + |S_{LT}|\big(\mathcal{F}_{LT} + \tilde{\mathcal{F}}_{LT}\big) + |S_{TT}|\big(\mathcal{F}_{TT} + \tilde{\mathcal{F}}_{TT}\big)\Big]$$



$$\mathcal{F}_U = (1+\cos^2\theta)F_{1U} + \sin^2\theta\, F_{2U} + \cos\theta\, F_{3U}$$
$$+ \cos\varphi\left[\sin\theta\, F_{1U}^{\cos\varphi} + \sin 2\theta\, F_{2U}^{\cos\varphi}\right] \quad \Longleftrightarrow \quad \{\tilde{\mathcal{F}}_L,\ \mathcal{F}_{LL}\}$$
$$+ \cos 2\varphi \sin^2\theta\, F_U^{\cos 2\varphi}$$

$$\tilde{\mathcal{F}}_U = \sin\varphi\left[\sin\theta\, \tilde{F}_{1U}^{\sin\varphi} + \sin 2\theta\, \tilde{F}_{2U}^{\sin\varphi}\right] \quad \Longleftrightarrow \quad \{\mathcal{F}_L,\ \tilde{\mathcal{F}}_{LL}\}$$
$$+ \sin 2\varphi \sin^2\theta\, \tilde{F}_U^{\sin 2\varphi}$$

$$p_1 = (E_1, 0,0, p_{1z})$$
$$p_2 = (E_2, |\vec{p}_{2T}|\cos\varphi, |\vec{p}_{2T}|\sin\varphi, p_{2z})$$
$$l_1 = \frac{Q}{2}(1, \sin\theta, 0, \cos\theta)$$
$$l_2 = \frac{Q}{2}(1, -\sin\theta, 0, -\cos\theta)$$
$$q = l_1 + l_2 = (Q, 0,0,0)$$

Accessing tensor polarized 3-D FFs in $e^+e^- \to V\pi X$

$$
\begin{aligned}
\mathcal{F}_T = {} & \sin\varphi_S\,[\sin\theta\, F_{1T}^{\sin\varphi_S} + \sin 2\theta\, F_{2T}^{\sin\varphi_S}] \\
& + \sin(\varphi_S+\varphi)\sin^2\theta\, F_T^{\sin(\varphi_S+\varphi)} \\
& + \sin(\varphi_S-\varphi)\,[(1+\cos^2\theta) F_{1T}^{\sin(\varphi_S-\varphi)} \\
& \qquad + \sin^2\theta\, F_{2T}^{\sin(\varphi_S-\varphi)} + \cos\theta\, F_{3T}^{\sin(\varphi_S-\varphi)}] \\
& + \sin(\varphi_S-2\varphi)\,[\sin\theta\, F_{1T}^{\sin(\varphi_S-2\varphi)} + \sin 2\theta\, F_{2T}^{\sin(\varphi_S-2\varphi)}] \\
& + \sin(\varphi_S-3\varphi)\sin^2\theta\, F_T^{\sin(\varphi_S-3\varphi)}
\end{aligned}
$$

$(\varphi_S \to \varphi_{LT}, F_{iT} \to \tilde{F}_{iLT})$

$\tilde{\mathcal{F}}_{LT}$

$(\varphi_{LT} \to 2\varphi_{TT} - \varphi,\ \tilde{F}_{iLT} \to \tilde{F}_{iTT})$

$$
\begin{aligned}
\tilde{\mathcal{F}}_{TT} = {} & \sin 2\varphi_{TT}\sin^2\theta\, \tilde{F}_{TT}^{\sin 2\varphi_{TT}} \\
& + \sin(2\varphi_{TT}-\varphi)\,[\sin\theta\, \tilde{F}_{1TT}^{\sin(2\varphi_{TT}-\varphi)} + \sin 2\theta\, \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-\varphi)}] \\
& + \sin(2\varphi_{TT}-2\varphi)\,[(1+\cos^2\theta) \tilde{F}_{1TT}^{\sin(2\varphi_{TT}-2\varphi)} \\
& \qquad + \sin^2\theta\, \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-2\varphi)} + \cos\theta\, \tilde{F}_{3TT}^{\sin(2\varphi_{TT}-2\varphi)}] \\
& + \sin(2\varphi_{TT}-3\varphi)\,[\sin\theta\, \tilde{F}_{1TT}^{\sin(2\varphi_{TT}-3\varphi)} + \sin 2\theta\, \tilde{F}_{2TT}^{\sin(2\varphi_{TT}-3\varphi)}] \\
& + \sin(2\varphi_{TT}-4\varphi)\sin^2\theta\, \tilde{F}_{TT}^{\sin(2\varphi_{TT}-4\varphi)}
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\mathcal{F}}_T = {} & \cos\varphi_S\,[\sin\theta\, \tilde{F}_{1T}^{\cos\varphi_S} + \sin 2\theta\, \tilde{F}_{2T}^{\cos\varphi_S}] \\
& + \cos(\varphi_S+\varphi)\sin^2\theta\, \tilde{F}_T^{\cos(\varphi_S+\varphi)} \\
& + \cos(\varphi_S-\varphi)\,[(1+\cos^2\theta) \tilde{F}_{1T}^{\cos(\varphi_S-\varphi)} \\
& \qquad + \sin^2\theta\, \tilde{F}_{2T}^{\cos(\varphi_S-\varphi)} + \cos\theta\, \tilde{F}_{3T}^{\cos(\varphi_S-\varphi)}] \\
& + \cos(\varphi_S-2\varphi)\,[\sin\theta\, \tilde{F}_{1T}^{\cos(\varphi_S-2\varphi)} + \sin 2\theta\, \tilde{F}_{2T}^{\cos(\varphi_S-2\varphi)}] \\
& + \cos(\varphi_S-3\varphi)\sin^2\theta\, \tilde{F}_T^{\cos(\varphi_S-3\varphi)}
\end{aligned}
$$

$(\varphi_S \to \varphi_{LT}, \tilde{F}_{iT} \to F_{iLT})$

$\mathcal{F}_{LT}$

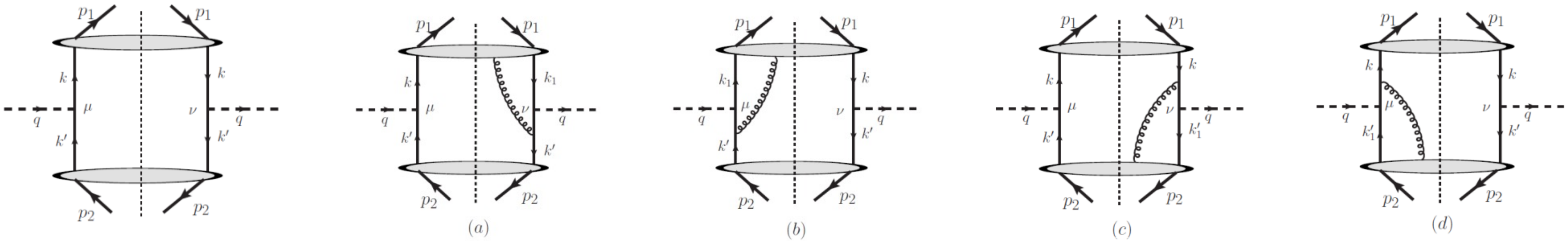
$(\varphi_{LT} \to 2\varphi_{TT} - \varphi,\ F_{iLT} \to F_{iTT})$

$$
\begin{aligned}
\mathcal{F}_{TT} = {} & \cos 2\varphi_{TT}\sin^2\theta\, F_{TT}^{\cos 2\varphi_{TT}} \\
& + \cos(2\varphi_{TT}-\varphi)\,[\sin\theta\, F_{1TT}^{\cos(2\varphi_{TT}-\varphi)} + \sin 2\theta\, F_{2TT}^{\cos(2\varphi_{TT}-\varphi)}] \\
& + \cos(2\varphi_{TT}-2\varphi)\,[(1+\cos^2\theta) F_{1TT}^{\cos(2\varphi_{TT}-2\varphi)} \\
& \qquad + \sin^2\theta\, F_{2TT}^{\cos(2\varphi_{TT}-2\varphi)} + \cos\theta\, F_{3TT}^{\cos(2\varphi_{TT}-2\varphi)}] \\
& + \cos(2\varphi_{TT}-3\varphi)\,[\sin\theta\, F_{1TT}^{\cos(2\varphi_{TT}-3\varphi)} + \sin 2\theta\, F_{2TT}^{\cos(2\varphi_{TT}-3\varphi)}] \\
& + \cos(2\varphi_S-4\varphi)\sin^2\theta\, F_{TT}^{\cos(2\varphi_{TT}-4\varphi)}
\end{aligned}
$$

Accessing tensor polarized 3-D FFs in $e^+e^- \to V\pi X$

■ Hadronic tensor in pQCD parton model

Leading order in pQCD and up to twist-3



$$W_{\mu\nu} = \widetilde{W}_{\mu\nu}^{(0)} + \widetilde{W}_{\mu\nu}^{(1a)} + \widetilde{W}_{\mu\nu}^{(1b)} + \cdots$$

$$\widetilde{W}_{\mu\nu}^{(0)} = \frac{1}{p_1^+ p_2^-} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(\vec{k}_\perp + \vec{k}'_\perp - \vec{q}_\perp) \mathrm{Tr}[\hat{\Xi}^{(0)}(z_1, k_\perp, p_1, S) \Gamma_\mu \hat{\bar{\Xi}}^{(0)}(z_2, k'_\perp, p_2) \Gamma_\nu]$$

$$\widetilde{W}_{\mu\nu}^{(1a)} = \frac{-1}{\sqrt{2} Q p_1^+ p_2^-} \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{d^2 k'_\perp}{(2\pi)^2} \delta^2(\vec{k}_\perp + \vec{k}'_\perp - \vec{q}_\perp) \mathrm{Tr}[\hat{\Xi}_\rho^{(1)}(z_1, k_\perp, p_1, S) \Gamma_\mu \hat{\bar{\Xi}}^{(0)}(z_2, k'_\perp, p_2) \gamma^\rho \not{n} \Gamma_\nu]$$

……

Accessing tensor polarized 3-D FFs in $e^+e^- \to V\pi X$

Structure function results in terms of TMD FFs

27 nonzero at twist-2

$$F_{1U1} = 2c_1^e c_1^q \mathcal{C}[D_1 \bar{D}_1]$$

$$F_{3U1} = 4c_3^e c_3^q \mathcal{C}[D_1 \bar{D}_1]$$

$$F_{U1}^{\cos 2\varphi} = -8c_1^e c_2^q \mathcal{C}[w_{hh} H_1^\perp \bar{H}_1^\perp]$$

$$F_{1T1}^{\sin(\varphi_S-\varphi)} = 2c_1^e c_1^q \mathcal{C}[w_1 D_{1T}^\perp \bar{D}_1]$$

$$F_{3T1}^{\sin(\varphi_S-\varphi)} = 4c_3^e c_3^q \mathcal{C}[w_1 D_{1T}^\perp \bar{D}_1]$$

$$\tilde{F}_{1T1}^{\cos(\varphi_S-\varphi)} = 2c_1^e c_3^q \mathcal{C}[w_1 G_{1T}^\perp \bar{D}_1]$$

$$\tilde{F}_{3T1}^{\cos(\varphi_S-\varphi)} = 4c_3^e c_1^q \mathcal{C}[w_1 G_{1T}^\perp \bar{D}_1]$$

$$F_{T1}^{\sin(\varphi_S+\varphi)} = -8c_1^e c_2^q \mathcal{C}[\bar{w}_1 \mathcal{H}_{1T}^\perp \bar{H}_1^\perp]$$

$$F_{T1}^{\sin(\varphi_S-3\varphi)} = -8c_1^e c_2^q \mathcal{C}[w_{hh}^t H_{1T}^\perp \bar{H}_1^\perp]$$

$$\tilde{F}_{1L1} = -2c_1^e c_3^q \mathcal{C}[G_{1L} \bar{D}_1]$$

$$\tilde{F}_{3L1} = -4c_3^e c_1^q \mathcal{C}[G_{1L} \bar{D}_1]$$

$$F_{L1}^{\sin 2\varphi} = -8c_1^e c_2^q \mathcal{C}[w_{hh} H_{1L}^\perp \bar{H}_{1L}^\perp]$$

$$F_{1LT1}^{\cos(\varphi_{LT}-\varphi)} = -2c_1^e c_1^q \mathcal{C}[w_1 D_{1LT}^\perp \bar{D}_1]$$

$$F_{3LT1}^{\cos(\varphi_{LT}-\varphi)} = -4c_3^e c_3^q \mathcal{C}[w_1 D_{1LT}^\perp \bar{D}_1]$$

$$\tilde{F}_{1LT1}^{\sin(\varphi_{LT}-\varphi)} = -2c_1^e c_3^q \mathcal{C}[w_1 G_{1LT}^\perp \bar{D}_1]$$

$$\tilde{F}_{3LT1}^{\sin(\varphi_{LT}-\varphi)} = -4c_3^e c_1^q \mathcal{C}[w_1 G_{1LT}^\perp \bar{D}_1]$$

$$F_{LT1}^{\cos(\varphi_{LT}+\varphi)} = -8c_1^e c_2^q \mathcal{C}[\bar{w}_1 \mathcal{H}_{1LT}^\perp \bar{H}_1^\perp]$$

$$F_{LT1}^{\cos(\varphi_{LT}-3\varphi)} = 8c_1^e c_2^q \mathcal{C}[w_{hh}^t H_{1LT}^\perp \bar{H}_1^\perp]$$

$$F_{1LL1} = 2c_1^e c_1^q \mathcal{C}[D_{1LL} \bar{D}_1]$$

$$F_{3LL1} = 4c_3^e c_3^q \mathcal{C}[D_{1LL} \bar{D}_1]$$

$$F_{LL1}^{\cos 2\varphi} = -8c_1^e c_2^q \mathcal{C}[w_{hh} H_{1LL}^\perp \bar{H}_1^\perp]$$

$$F_{1TT1}^{\cos(2\varphi_{TT}-2\varphi)} = 2c_1^e c_1^q \mathcal{C}[w_{dd}^{tt} D_{1TT}^\perp \bar{D}_1]$$

$$F_{3TT1}^{\cos(2\varphi_{TT}-2\varphi)} = 4c_3^e c_3^q \mathcal{C}[w_{dd}^{tt} D_{1TT}^\perp \bar{D}_1]$$

$$\tilde{F}_{1TT1}^{\sin(2\varphi_{TT}-2\varphi)} = 2c_1^e c_3^q \mathcal{C}[w_{dd}^{tt} G_{1TT}^\perp \bar{D}_1]$$

$$\tilde{F}_{3TT1}^{\sin(2\varphi_{TT}-2\varphi)} = 4c_3^e c_1^q \mathcal{C}[w_{dd}^{tt} G_{1TT}^\perp \bar{D}_1]$$

$$F_{TT1}^{\cos(2\varphi_{TT}-4\varphi)} = -4c_1^e c_2^q \mathcal{C}[w_{dd}^{tt} H_{1TT}^\perp \bar{H}_1^\perp]$$

$$F_{TT1}^{\cos 2\varphi_{TT}} = 8c_1^e c_2^q \mathcal{C}[w_2 H_{1TT}^{\perp\prime} \bar{H}_1^\perp]$$

$$\mathcal{C}[wD\bar{D}] = \frac{1}{z_1 z_2} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{d^2k'_\perp}{(2\pi)^2} \delta^2(\vec{k}_\perp + \vec{k}'_\perp - \vec{q}_\perp) w(k_\perp, k'_\perp) D(z_1, k_\perp) \bar{D}(z_2, k'_\perp)$$

Accessing tensor polarized 3-D FFs in $e^+e^- \to V\pi X$

- Structure function results in terms of TMD FFs

36 nonzero at twist-3

$$F_{1U2}^{\cos\varphi} = \frac{8c_3^e c_3^q}{z_1 z_2 Q} C[M_1 w_1 D^\perp z_2 \bar{D}_1 + M_2 \bar{w}_1 z_1 D_1 \bar{D}^{\perp\prime}],$$

$$F_{2U2}^{\cos\varphi} = \frac{4c_1^e}{z_1 z_2 Q}\Big\{c_1^q C[M_1 w_1 D^\perp z_2 \bar{D}_1 + M_2 \bar{w}_1 z_1 D_1 \bar{D}^{\perp\prime}] + 4c_2^q C[M_1 \bar{w}_1 H z_2 \bar{H}_1^\perp - M_2 w_1 z_1 H_1^\perp \bar{H}^{\perp\prime}]\Big\},$$

$$\tilde{F}_{1U2}^{\sin\varphi} = \frac{8c_3^e}{z_1 z_2 Q}\Big\{c_1^q C[(M_1 w_1 G^\perp z_2 \bar{D}_1 - M_2 \bar{w}_1 z_1 D_1 \bar{G}^\perp)] + 2c_2^q C[(M_1 \bar{w}_1 E z_2 \bar{H}_1^\perp - M_2 w_1 z_1 H_1^\perp \bar{E})]\Big\},$$

$$\tilde{F}_{2U2}^{\sin\varphi} = \frac{4c_1^e c_3^q}{z_1 z_2 Q} C[M_1 w_1 G^\perp z_2 \bar{D}_1 - M_2 \bar{w}_1 z_1 D_1 \bar{G}^\perp],$$

$$\tilde{F}_{1L2}^{\cos\varphi} = \frac{8c_3^e}{z_1 z_2 Q}\Big\{c_1^q C[M_1 w_1 G_L^\perp z_2 \bar{D}_1 - M_2 \bar{w}_1 z_1 G_{1L} \bar{D}^{\perp\prime}] + 2c_2^q C[-M_1 \bar{w}_1 E_L z_2 \bar{H}_1^\perp + M_2 w_1 z_1 H_{1L}^\perp \bar{E}]\Big\},$$

$$\tilde{F}_{2L2}^{\cos\varphi} = \frac{4c_1^e c_3^q}{z_1 z_2 Q} C[M_1 w_1 G_L^\perp z_2 \bar{D}_1 - M_2 \bar{w}_1 z_1 G_{1L} \bar{D}^{\perp\prime}],$$

$$F_{1L2}^{\sin\varphi} = \frac{8c_3^e c_3^q}{z_1 z_2 Q} C[-M_1 w_1 D_L^\perp z_2 \bar{D}_1 + M_2 \bar{w}_1 z_1 G_{1L} \bar{G}^\perp],$$

$$F_{2L2}^{\sin\varphi} = \frac{4c_1^e}{z_1 z_2 Q}\Big\{c_1^q C[-M_1 w_1 D_L^\perp z_2 \bar{D}_1 + M_2 \bar{w}_1 z_1 G_{1L} \bar{G}^\perp] + 4c_2^q C[M_1 \bar{w}_1 H_L z_2 \bar{H}_1^\perp - M_2 w_1 z_1 H_{1L}^\perp \bar{H}^{\perp\prime}]\Big\}$$

…… ……

Accessing tensor polarized 3-D FFs in $e^+e^- \to V\pi X$

■ Azimuthal asymmetries

Twist-2:

$$\langle \cos 2\varphi \rangle_U^{(0)} = -\frac{C(y)\sum_q c_1^e c_2^q \mathcal{C}[w_{hh} H_1^\perp \bar{H}_1^\perp]}{\sum_q T_0^q(y)\mathcal{C}[D_1 \bar{D}_1]}$$

Collins effect

$$\langle \sin 2\varphi \rangle_L^{(0)} = -\frac{\lambda C(y)\sum_q c_1^e c_2^q \mathcal{C}[w_{hh} H_{1L}^\perp \bar{H}_1^\perp]}{\sum_q T_0^q(y)\mathcal{C}[(D_1 - \lambda G_{1L})\bar{D}_1]}$$

$$\langle \cos 2\varphi \rangle_{LL}^{(0)} = -\frac{C(y)\sum_q c_1^e c_2^q \mathcal{C}[w_{hh}(H_1^\perp + S_{LL} H_{1LL}^\perp)\bar{H}_1^\perp]}{\sum_q T_0^q(y)\mathcal{C}[(D_1 + S_{LL} D_{1LL})\bar{D}_1]}$$

Polarization dependent

Twist-3:

$$\langle \cos \varphi \rangle_U^{(1)} = -\frac{2D(y)}{z_1 z_2 Q \sum_q T_0^q(y)\mathcal{C}[D_1 \bar{D}_1]} \times \sum_q \{T_2^q(y)[M_1 \mathcal{C}(w_1 D^\perp z_2 \bar{D}_1) + M_2 \mathcal{C}(\bar{w}_1 z_1 D_1 \bar{D}^{\perp\prime})] + T_4^q(y)[M_1 \mathcal{C}(\bar{w}_1 H z_2 \bar{H}_1^\perp) + M_2 \mathcal{C}(w_1 z_1 H_1^\perp \bar{H}^{\perp\prime})]\}$$

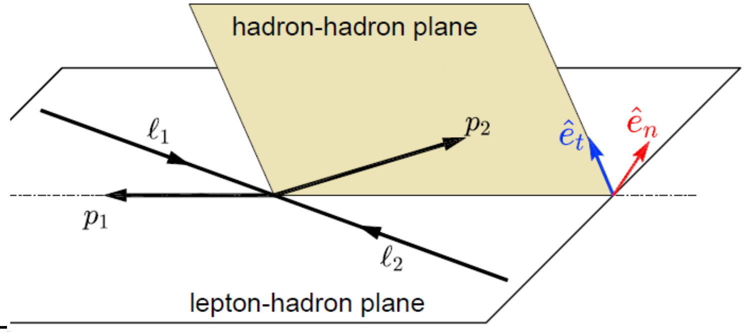
Cahn effect (DIS)

$$\langle \sin \varphi \rangle_U^{(1)} = \frac{2D(y)}{z_1 z_2 Q \sum_q T_0^q(y)\mathcal{C}[D_1 \bar{D}_1]} \times \sum_q \{T_3^q(y)[M_1 \mathcal{C}(w_1 G^\perp z_2 \bar{D}_1) - M_2 \mathcal{C}(\bar{w}_1 z_1 D_1 \bar{G}^\perp)] + 2c_3^e c_2^q [M_1 \mathcal{C}(\bar{w}_1 E z_2 \bar{H}_1^\perp) - M_2 \mathcal{C}(w_1 z_1 H_1^\perp \bar{E})]\}$$

P-odd

Accessing tensor polarized 3-D FFs in $e^+e^- \to V\pi X$

■ Hadron polarizations



At twist-2

Longitudinal polarization	$\langle\lambda\rangle^{(0)} = \frac{2}{3}\frac{\sum_q P_q(y) T_0^q(y)\mathcal{C}[G_{1L}\bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}$	$\langle S_{LL}\rangle^{(0)} = \frac{1}{2}\frac{\sum_q T_0^q(y)\mathcal{C}[D_{1LL}\bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}$
Transverse polarization w.r.t. hadron-hadron plane	$\langle S_T^t\rangle^{(0)} = -\frac{2}{3}\frac{\sum_q P_q(y) T_0^q(y)\mathcal{C}[w_1 G_{1T}^\perp\bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}$ $\langle S_{LT}^n\rangle^{(0)} = \frac{2}{3}\frac{\sum_q P_q(y) T_0^q(y)\mathcal{C}[w_1 G_{1LT}^\perp\bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}$ $\langle S_{TT}^{nt}\rangle^{(0)} = -\frac{2}{3}\frac{\sum_q P_q(y) T_0^q(y)\mathcal{C}[w_{dd}^{tt} G_{1TT}^\perp\bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}$	$\langle S_T^n\rangle^{(0)} = \frac{2}{3}\frac{\sum_q T_0^q(y)\mathcal{C}[w_1 D_{1T}^\perp\bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}$ $\langle S_{LT}^t\rangle^{(0)} = -\frac{2}{3}\frac{\sum_q T_0^q(y)\mathcal{C}[w_1 D_{1LT}^\perp\bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}$ $\langle S_{TT}^{nn}\rangle^{(0)} = -\frac{2}{3}\frac{\sum_q T_0^q(y)\mathcal{C}[w_{dd}^{tt} D_{1TT}^\perp\bar{D}_1]}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}$
Characteristic	Quark polarization dependent, P-odd	Quark polarization independent, P-even

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- **Energy dependence of hadron polarizations**
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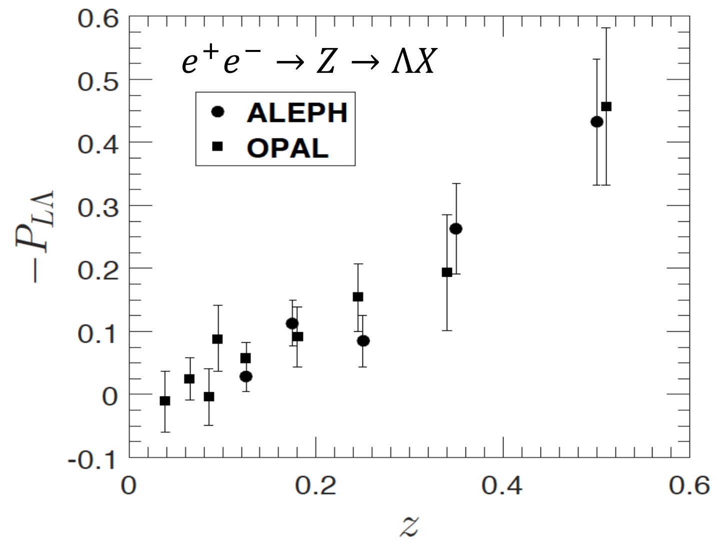
Energy dependence of hadron polarizations

■ Energy dependence of hadron polarizations in $e^+e^- \to \gamma^*/Z \to hX$

Longitudinal polarization of Λ hyperon

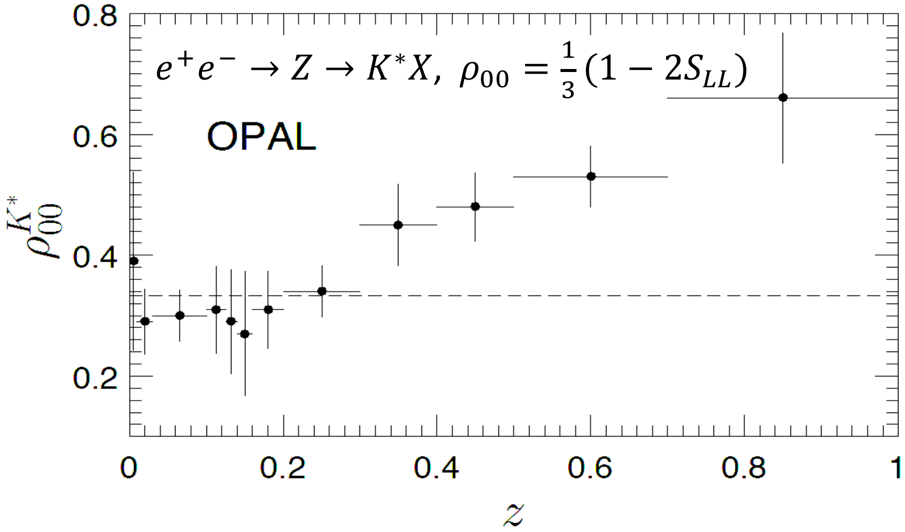


$$P_{L\Lambda} = \frac{\sum_q \bar{P}_q W_q G_{1L}^{q\to\Lambda}(z,Q^2)}{\sum_q W_q D_1^{q\to\Lambda}(z,Q^2)}$$

Quark polarization dependent

Strong energy dependence expected

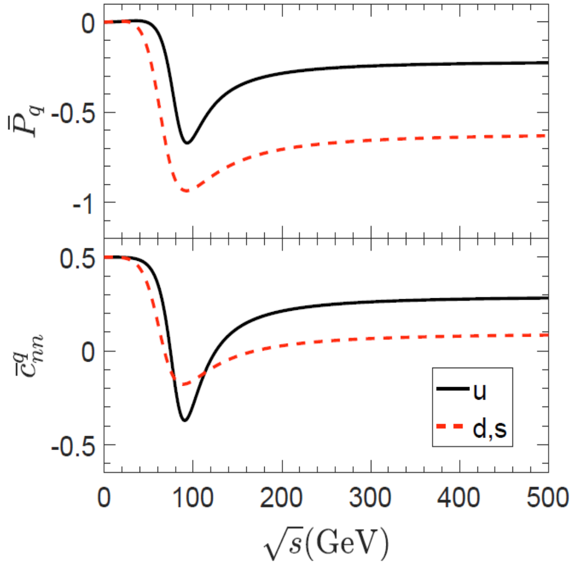
Spin alignment of K^*



$$\bar{\rho}_{00}^{K^*} = \frac{1}{3} - \frac{\sum_q W_q D_{1LL}^{q\to K^*}(z,Q^2)}{3\sum_q W_q D_1^{q\to K^*}(z,Q^2)}$$

Quark polarization independent

Weak energy dependence expected



- Leading twist
- Leading order evolution

Energy dependence of hadron polarizations

Scale evolution of polarized FFs

DGLAP

$$\begin{cases} \frac{\partial G_{1L}^{q_i\to h}(z,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_z^1 \frac{dy}{y}\left[G_{1L}^{q_i\to h}(z/y,Q^2)\Delta P_{qq}(y) + G_{1L}^{g\to h}(z/y,Q^2)\Delta P_{gq}(y)\right] \\ \frac{\partial G_{1L}^{g\to h}(z,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_z^1 \frac{dy}{y}\left[G_{1L}^{g\to h}(z/y,Q^2)\Delta P_{gg}(y) + \sum_{j=1}^{2N_f} G_{1L}^{g\to h}(z/y,Q^2)\Delta P_{qg}(y)\right] \end{cases}$$

$G_{1L}(z,Q^2)$

$$\Delta P_{qq}(y) = C_F\left[\frac{1+y^2}{(1-y)_+}+\frac{3}{2}\delta(1-y)\right]$$

$$\Delta P_{gq}(y) = C_F\left[1-(1-y)^2\right]/y$$

$$\Delta P_{qg}(y) = \left[y^2-(1-y)^2\right]/2$$

$$\Delta P_{gg}(y) = N_C\left[(1+y^4)\left(\frac{1}{y}+\frac{1}{(1-y)_+}-\frac{(1-y)^3}{y}\right)\right] + \frac{11N_C-2N_f}{6}\delta(1-y)$$

$D_{1LL}(z,Q^2)$

$$P_{qq}(y) = \Delta P_{qq}(y) = C_F\left[\frac{1+y^2}{(1-y)_+}+\frac{3}{2}\delta(1-y)\right]$$

$$P_{gq}(y) = C_F\left[1+(1-y)^2\right]/y$$

$$P_{qg}(y) = \left[y^2+(1-y)^2\right]/2$$

$$P_{gg}(y) = N_C\left[\frac{2y}{(1-y)_+}-2(y^2-y-\frac{1}{y}+1)\right] + \frac{11N_C-2N_f}{6}\delta(1-y)$$

Energy dependence of hadron polarizations

Parametrization of polarized FFs

$$\begin{cases} G_{1L}^{s\to\Lambda}(z) = z^a D_1^{s\to\Lambda}(z) \\ G_{1L}^{u/d\to\Lambda}(z) = N z^a D_1^{u/d\to\Lambda}(z) \\ a > 0, |N| \leq 1 \end{cases}$$

D. de Florian, M. Stratmann, and W. Vogelsang, PRD (1998)

$$\begin{cases} D_{1LL}^{u\to K^*}(z) = c_1 D_1^{u\to K^*}(z) \\ D_{1LL}^{d/s\to K^*}(z) = (c_1 + c_2 z) D_1^{d/s\to K^*}(z) \\ -3/2 \leq c_1,\ c_1 + c_2 z \leq 3 \end{cases}$$

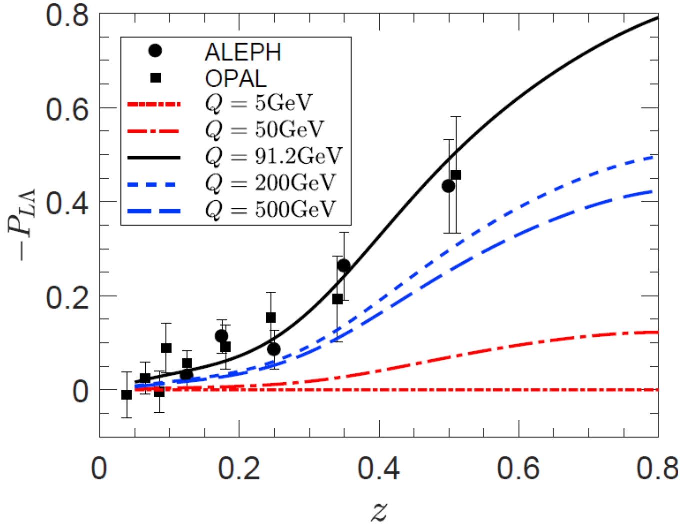
s quark contribution larger than u or d

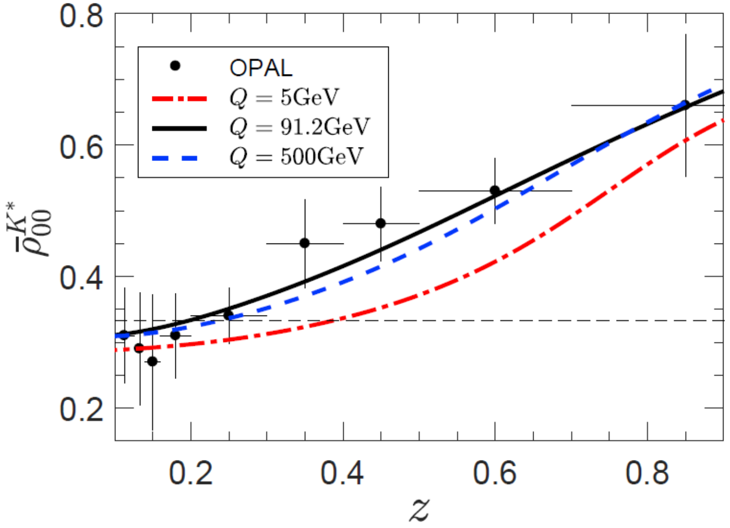
Valance quark dominant at large-z

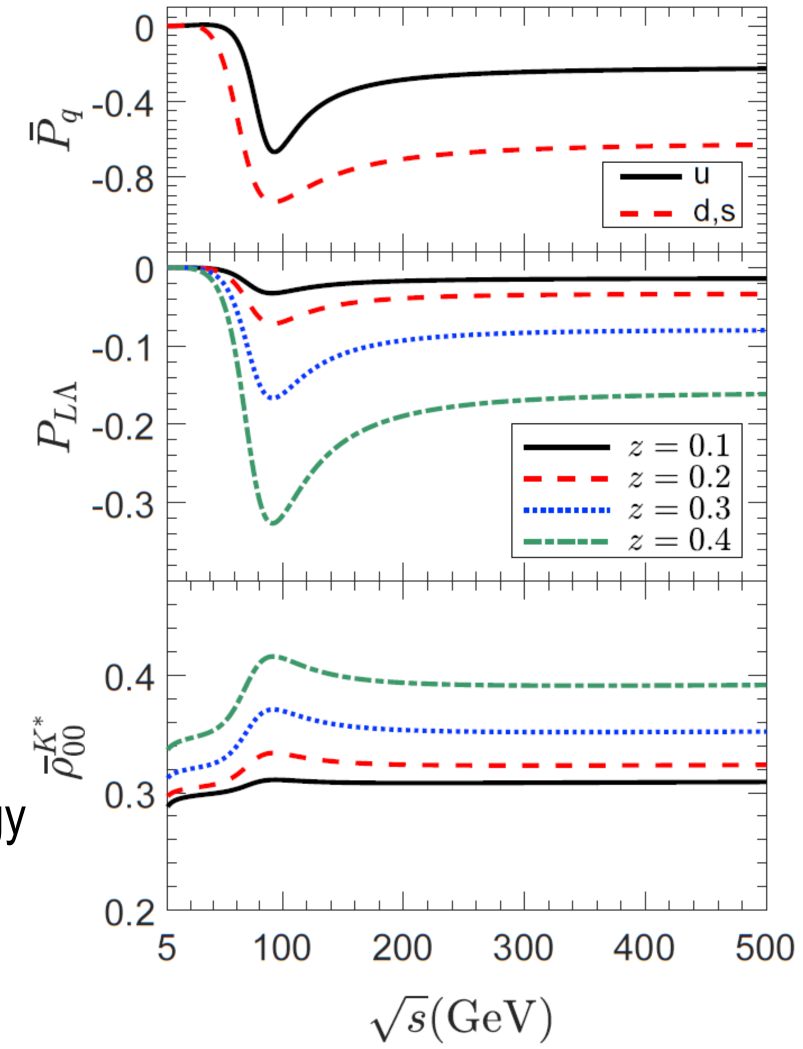
Convergence of different flavors at small-z

Energy dependence of hadron polarizations

■ Numerical results







✓ Strong energy dependence

✓ Polarization vanishes at low energy

✓ Weak energy dependence

✓ Sizable spin alignment at low energy

Possibility to be checked at BES or BELLE.

K.B. Chen, W.H. Yang, Y.J. Zhou and Z.T. Liang, PRD 95, 034009 (2017)

Energy dependence of hadron polarizations

■ Spin alignment in SIDIS

$$\langle \rho_{00}^{V} \rangle = \frac{1}{3} - \frac{\left(c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)\right)\mathcal{C}[f_1(x,k_{iT}) D_{1LL}(z_h,k_{fT})]}{3\left(c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)\right)\mathcal{C}[f_1(x,k_{iT}) D_1(z_h,k_{fT})]}$$

Take Gaussian form for TMDs:

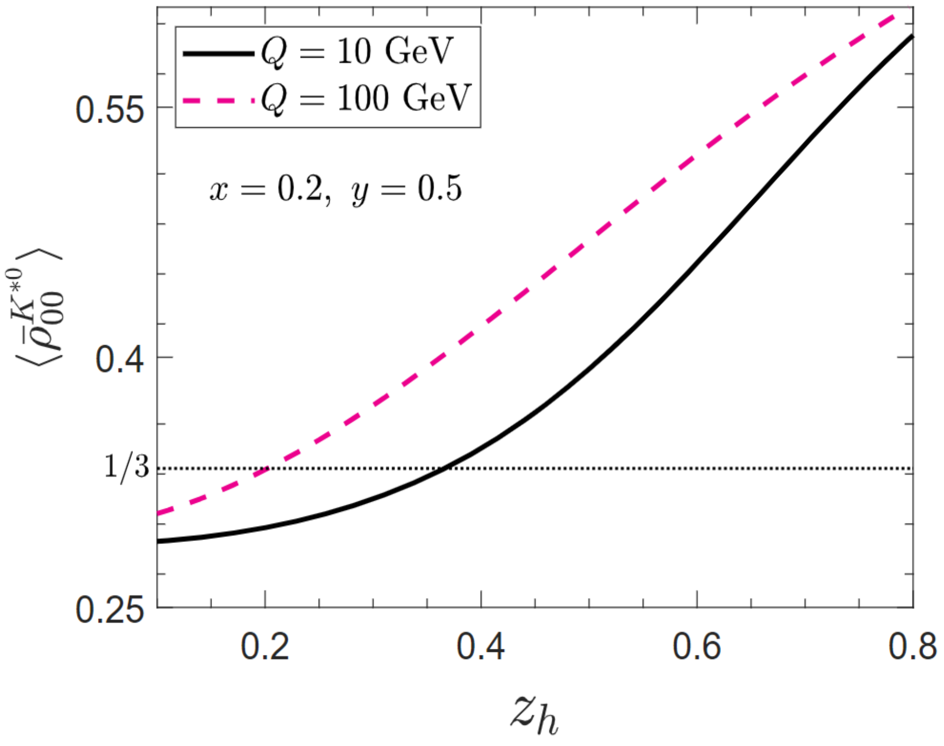
$$f_1(x,k_{iT}) = f_{1q}(x)\frac{1}{\pi\Delta_f^2} e^{-\vec{k}_{iT}^2/\Delta_f^2},$$

$$D_1(z_h,k_{fT}) = D_{1q}^{K^{*0}}(z_h)\frac{1}{\pi\Delta_D^2} e^{-\vec{k}_{fT}^2/\Delta_D^2},$$

$$D_{1LL}(z_h,k_{fT}) = D_{1LLq}^{K^{*0}}(z_h)\frac{1}{\pi\Delta_{LL}^2} e^{-\vec{k}_{fT}^2/\Delta_{LL}^2}.$$

$p_{h\perp}$ integrated

$$\langle \bar{\rho}_{00}^{K^{*0}} \rangle = \frac{1}{3} - \frac{\left[c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)\right] f_{1q}(x) D_{1LLq}^{K^{*0}}(z_h)}{3\left[c_{11}^{\text{ew}} A(y) + c_{33}^{\text{ew}} C(y)\right] f_{1q}(x) D_{1q}^{K^{*0}}(z_h)}$$



A rough estimate of the spin alignment for K^{*0} production in SIDIS

X.S. Jiao and K.B. Chen, PRD 105, 054010 (2022)

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Summary

- We give a complete decomposition of 3-D FFs from quark-quark correlator for spin-1 hadron.

- General kinematic analysis for $e^+e^- \to V\pi X$ leads to 81 structure functions.

- Parton model calculation for $e^+e^- \to V\pi X$ is carried out up to twist-3 level. Structure functions, azimuthal asymmetries and hadron polarizations are expressed in terms of the convolution of the 3-D FFs.

- Energy dependences of Hyperon longitudinal polarization and vector meson spin alignment are very much different.

Thank you!

Twist-3 横向极化

$$\langle S_T^x \rangle^{(1)} = -\frac{8M_1}{3z_1Q}\frac{\sum_q \tilde{T}_3^q(y)\mathcal{C}[\mathcal{G}_T^\perp \bar{D}_1] + \cdots}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]} \qquad \langle S_{LT}^x \rangle^{(1)} = -\frac{8M_1}{3z_1Q}\frac{\sum_q \tilde{T}_2^q(y)\mathcal{C}[\mathcal{D}_{LT}^\perp \bar{D}_1] + \cdots}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}$$

$$\langle S_T^y \rangle^{(1)} = \frac{8M_1}{3z_1Q}\frac{\sum_q \tilde{T}_2^q(y)\mathcal{C}[\mathcal{D}_T^\perp \bar{D}_1] + \cdots}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]} \qquad \langle S_{LT}^y \rangle^{(1)} = \frac{8M_1}{3z_1Q}\frac{\sum_q \tilde{T}_3^q(y)\mathcal{C}[\mathcal{G}_{LT}^\perp \bar{D}_1] + \cdots}{\sum_q T_0^q(y)\mathcal{C}[D_1\bar{D}_1]}$$

单举反应

$$\langle S_T^x \rangle_{in}^{(1)} = -\frac{8M_1}{3z_1Q}\frac{\sum_q \tilde{T}_3^q(y)G_T}{\sum_q T_0^q(y)D_1}$$ *P*-odd, *T*-even

$$\langle S_{LT}^x \rangle_{in}^{(1)} = -\frac{8M_1}{3z_1Q}\frac{\sum_q \tilde{T}_2^q(y)D_{LT}}{\sum_q T_0^q(y)D_1}$$ *P*-even, *T*-even

$$\langle S_T^y \rangle_{in}^{(1)} = \frac{8M_1}{3z_1Q}\frac{\sum_q \tilde{T}_2^q(y)D_T}{\sum_q T_0^q(y)D_1}$$ *P*-even, *T*-odd

$$\langle S_{LT}^y \rangle_{in}^{(1)} = \frac{8M_1}{3z_1Q}\frac{\sum_q \tilde{T}_3^q(y)G_{LT}}{\sum_q T_0^q(y)D_1}$$ *P*-odd, *T*-odd